1-13-2011

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Episodic tremors and slip in Cascadia in the framework of the Frenkel-Kontorova model

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Received 27 August 2010; revised 20 October 2010; accepted 28 October 2010; published 13 January 2011.

[1] The seismic moment for regular earthquakes is proportional to the cube of rupture time. A second class of phenomena, collectively called slow earthquakes, has very different scaling. We propose a model, inspired from the phenomenology of dislocation dynamics in crystals, that is consistent with the scaling relations observed in the Cascadia episodic tremor and slip (ETS) events. Two fundamental features of ETS are periodicity and migration. In the northern Cascadia subduction zone, ETS events appear every 14.5 months or so. During these events, tremors migrate along-strike with a velocity of 10 km/day and simultaneously zip back and forth in the relative plate-motion direction with a typical velocity of 50 km/h. Our model predicts the formation of a sequence of slip pulses on the boundary of the plates, which describes the major features of fault dynamics, including periodicity and the migration pattern of tremors. Citation: Gershenzon, N. I., G. Bambakidis, E. Hauser, A. Ghosh, and K. C. Creager (2011), Episodic tremors and slip in Cascadia in the framework of the Frenkel-Kontorova model, Geophys. Res. Lett., 38, L01309, doi:10.1029/2010GL045225.

1. Introduction

[2] The average slip rate of tectonic plates is a few cm per year. The temporal and spatial distribution of the actual slip rate depends on local frictional processes and recent dynamical history. As a result, some parts of a fault are locked for many years after large crustal earthquakes; other parts slide more or less freely. Tremors occur over a limited depth range within the transition zone. Hence this zone may hold key clues to better understand the fault dynamics. How are the stress and strain redistributed between zones? The answer could lie in the inelastic wave (succession of slip pulses) generated at the boundary of the plates [Savage, 1971; Ida, 1974; Bykov, 2001; Gershenzon et al., 2009].

[3] Tremor and slow slip are coupled seismic and geodetic phenomena that are broadly correlated in space and time, and strikingly periodic in the northern Cascadia subduction zone. In this zone, tremor is located between the surface projection of 30 and 45 km depth contours of the plate interface. Each episodic tremor and slip (ETS) event releases a moment that is equivalent to a magnitude 6–7 earthquake, and accompanied by ~30 mm of slip at the plate interface.

[4] Ghosh et al. [2009] studied tremor with the data from a dense seismic array installed directly above the ETS zone. They showed that slip–parallel tremor bands sweep Cascadia along the strike direction with a velocity of 10 km/day, which is associated with long-term tremor migration [Kao et al., 2006; Ghosh et al., 2010a]. Tremor also propagates rapidly and continuously in the slip–parallel direction, both up- and down-dip at a velocity of ~50 km/hr, which is associated with short-term tremor migration [Ghosh et al., 2010b]. Such long- and short-term tremor migrations are also observed in the subduction zone of southwest Japan [e.g., Obara, 2009; Shelly et al., 2007].

[5] The rate-and-state approach of modeling nucleation of regular earthquakes has been recently developed for ETS events as well [Ampuero and Rubin, 2008; Liu and Rice, 2005, 2007; Rubin, 2008; Rubin and Ampuero, 2009; Shibazaki and Shimamoto, 2007]. Here we introduce a complementary approach. To describe the fault dynamics we apply the Frenkel-Kontorova (FK) model [Frenkel and Kontorov, 1938], a phenomenological model that describes the dynamics of dislocations in crystals. The motivation for using this model is as follows. The spasmodic local motion along a fault, occurring due to earthquakes and slow slip events (SSE), requires substantially less external stress than spatially and temporally uniform motion. The process is analogous to plastic deformation in crystals, which is realized by the movement of edge dislocations. Such movement requires only a small fraction of the stress necessary for uniform relative displacement of planes of crystal atoms. A difference between a fault and a crystal plane is the size of the substrate, which is a typical distance between neighboring microasperities in the former and the distance between neighboring atoms in the latter. The dynamics of edge dislocations is effectively described by the FK model [Hirth and Lothe, 1982; Gershenzon, 1994]. In the continuum limit the FK model is described by the sine–Gordon (SG) equation [Lamb, 1980], which is a widely used and well-developed nonlinear equation in mathematical physics.

[6] We will show that an inelastic nonlinear wave, in terms of the FK model, can intrinsically describe the scaling law of SSEs as well as the periodicity and migration pattern of tremors in ETS events.

2. Model

[7] In order to apply the FK model to plate dynamics, we suppose that the plate surfaces are covered quasi-periodically by microasperities with a typical spacing b between them. The value of b should be comparable to the typical slip in a
slow event, roughly a few cm. Among many different typical asperity sizes of fault materials (from 100 μm to tens of meters) b may represent the typical size of granules in the “cohesive granular layer” of Sagi and Brodsky [2009]. Note that the microasperities need not be perfectly periodic, since the main features of the FK model are preserved in the case of a randomized distribution of microasperities [Braun and Kivsky, 2004]. The derivation of SG equation from the FK model can be found in the book by Lamb [1980] and specifically relating to plate dynamics in the article by Gershenzon et al. [2009]. The dimensionless SG equation can be expressed in the form:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial u}{\partial t} = A^2 \sin(u),$$

where x and t are the spatial and temporal coordinates along the fault in units of b and b/c respectively, u is the relative displacement of the plate surfaces in the x direction in units of b/(2π), c is P-wave velocity, and A is a dimensionless empirical scaling factor which incorporates the roughness between the plates. The sinusoidal term at the right side of the equation reflects the presence of bumps on plate surfaces of effective height A. Essentially A is a complicated (basically unknown) function of geometry and the adhesive properties of asperities. If A is negligibly small the plates move almost freely relative to each other. For values of A closer to 1 (like in crystals [Hirth and Lothe, 1982; Gershenzon, 1994]), the distinction between the mechanical properties of the material between the plates and inside the plates disappears, and the equation describes the relative motion of two parts of a homogeneous medium. Gershenzon et al. [2009] showed that the value of A could be found based on two measurable parameters such as rupture velocity and displacement rate (the average slip divided by the rupture time) during a regular earthquake. Since the velocity of rupture for regular shallow earthquakes is about 0.7–0.8 of the S-wave velocity [Kanamori et al., 1998] and the displacement rate is about 0.1 m/s (the latter can be estimated from the scaling law for regular crustal earthquakes), the value of A is equal to ~10^{-3} and is almost universal for well-developed faults worldwide. Note that A << 1, which reflects the well-known fact that plate boundaries have mechanical properties distinct from those of the material inside the plate [e.g., Ben-Zion, 2008].

$$A = 0$$ the wave equation is recovered from SG equation. The FK model has some common features with the Burridge-Knopoff model. In the latter the complex spasmodic movement of the blocks is obtained by selecting a specific nonlinear relationship between the frictional force and the velocity. In contrast, in the FK model the nonlinear behavior is implicit.

The basic solutions of the SG equation are phonons and solitons [Lamb, 1980]. In the context of plate dynamics, phonons describe seismic radiation and solitons describe slip pulses (dislocations). Solitons have the remarkable feature that they can move with any velocity from zero up to the P-wave velocity, in contrast to solutions of the standard wave equation (i.e., A = 0), which describes the propagation of disturbances with seismic velocities only. The propagation of solitons may generate phonons. Another remarkable feature of the SG equation is that a constant or slowly time-varying boundary condition (i.e., rate of relative plate motion) may generate a sequence of pulses. Thus the continuum FK model predicts the existence of inelastic waves describing a succession of slip pulses generated at the boundary between two plates.

According to our model, slip occurs due to the movement of dislocations, which appear as a response to shear stress. A dislocation is a two-dimensional structure with length (along the front) about the size of the characteristic extent of a SSE and a width (perpendicular to the front) spanning thousands of intervals between successive microasperities (see below). The passage of one dislocation through a particular point on the plate surface corresponds to a relative plate shift for a typical substrate size (exactly the same way that an edge dislocation shifts atomic planes in a crystal relative to each other). Typically one dislocation “produces” one ETS event, shifting plates by a characteristic amount b, relieving a certain amount of stress and generating a massive sequence of tremors.

The parameters of a dislocation (amplitude of stress σ₀ and strain ε₀ associated with the presence of dislocation) are entirely defined by the crust/fault parameters and do not depend on the process parameters: σ₀ = μA(2π) and ε₀ = A(2π) [Gershenzon et al., 2009], where μ is the rigidity. If μ = 30 GPa than σ₀ ≈ 5 MPa and ε₀ ≈ 1.6 · 10^{-3}. Another useful parameter is the characteristic width d of a dislocation, the distance where the anomaly stress due to a dislocation is e⁻¹ times the maximum stress σ₀. To estimate the value of d we will use the periodic solution of the SG equation expressed through the Jacobian elliptic function [Gershenzon et al., 2009]. It can be shown that d is practically independent of the process parameters and can be estimated by the simple relation: d ≈ 2πb/A. If b = 3 cm and A = 10⁻³ then d ≈ 190 m. It appears to be a very small pulse width compared with the width supported by geodetic data and tremor locations in Cascadia. However it is obvious that any value much smaller than the pulse depth cannot be resolved by existing geodetic data. Moreover, the mechanism of tremor-triggering by a slip pulse isn’t known. Does tremor occur on top of the pulse front, or on the tail of the pulse after some delay time? Furthermore, a sizeable amount of tremor occurs outside of the sliding plane [Kao et al., 2006]. Thus there is no contradiction between the predicted pulse width and the observed data.

The periodicity of tremors and ETS events has been observed in subduction zones [Brudzinski and Allen, 2007;...
Obara, 2009; Rogers and Dragert, 2003] as well as at transform faults [Nadeau and Guilhem, 2009]. If we suppose that an ETS is triggered in a certain area by a dislocation, we have to assume that the interval between two such events is the same as the interval between two successive dislocations arriving at this area. So the periodicity interval is just $b$ divided by the averaged slip rate. Since recurrence period and slip rate are measurable parameters we can estimate $b$ (about 30 mm in Cascadia). The periodicity interval varies for different zones and even for different areas in the same zone, ranging from 50 days to 22 months. Since the average slip rate is approximately the same in different subduction zones, how does our model explain the wide range of periodicity intervals? Although the slip rate averaged over a large time interval would not be expected to vary from place to place, the short-term rate could vary considerably due to the spasmodic nature of plate sliding. So the current periodicity interval in any particular area depends not only on averaged slip rate and value of $b$, but on the local dynamical history of the fault, namely, on how large and how far removed in time and space was the most recent closest earthquake. In such a model, the periodicity interval should increase with time after the earthquake. This effect indeed was observed after the 2004 Parkfield earthquake [Nadeau and Guilhem, 2009]; the periodicity interval of tremors increased from 50 days shortly after the earthquake to 100 days a few years after the earthquake.

3.2. Tremor Migration

Tremor migration may simply reflect dislocation movement, and the tremor migration speed is the speed of the dislocation. Figure 2a depicts the time evolution of tremors during ETS in January 2007 in Cascadia. It also shows slow along-strike tremor migration, which has been observed both in Cascadia [e.g., Ghosh et al., 2010a] and the Nankai subduction zone [e.g., Obara, 2009]. As already mentioned, we suppose that a dislocation is generated on the plate boundaries between free-slipping and transition zones and propagates up-dip. The dislocation serves to cover the entire transition zone and thus release stress by local slip. Even so, the dislocation should (hypothetically) line-up in the strike direction and propagate in the up-dip direction; the actual shape of the dislocation, hence the (local) direction of movement, could be more complicated for various reasons. We discuss one possible reason here. As typical for processes of plastic deformation in crystals, the dislocation could be pinned in some places forcing the dislocation to

![Figure 1. Schematic representation of the (a) subduction zone, (b) spatial distribution of slip velocity and (c) shear stress in Cascadia. Note the difference between actual (brown) and spatially averaged slip velocity and shear stress (blue broken lines). The distance between the left edge of the ETS zone and the locked zone is exaggerated.](image1)

![Figure 2. (a) Location of a migrating tremor during an ETS episode in January 2007 in the Cascadia subduction zone. The successive positions of the dislocation (black curves) from before 14th January until after 30th January are shown (based on a figure by A. Wech). (b) The solid black curve depicts a dislocation front with kinks moving parallel (red arrows) to the dislocation front. The expanded region to the right shows a slip-parallel tremor streak in the Cascadia subduction zone with rapid down-dip short-term migration of the tremor with a velocity of 65 km/hr. Colored circles on the maps represent tremor locations. Time is color-coded to show tremor migration. The black solid square marks the seismic array used to observe the tremor streak. The arrow indicates the overall slip direction of the Cascadia subduction zone. Dashed contour lines show plate interface depth in km.](image2)
change shape and direction of movement up to the direction opposite to the shear stress direction \[ \text{[Hirth and Lothe, 1982].} \]

The idea of dislocation pinning is supported by segmentation of tremor in strike direction observed in Cascadia and Nankai active tremor belts \[ \text{[Brudzinski and Allen, 2007; Obara, 2009; Ghosh et al., 2010a].} \] As a result of pinning, some parts of a dislocation would propagate in the strike direction both to the north and south (see dislocation positions at January 20th and 24th in Figure 2a), explaining the tremor migration in both directions at the same time. Since the characteristic length of the active tremor belt is usually much larger in the strike direction than in the dip direction, it is easier to resolve tremor migration in the former direction although the actual migration pattern may be more complicated. There is a simple relation between slip velocity \( W \), pulse velocity \( U \), and the accumulated shear stress \( \Sigma \): \( W = U\Sigma / (c^2) \) \[ \text{[Gershenzon et al., 2009].} \] From this relation we can estimate \( \Sigma \), which is of order 10–20 kPa if \( W = 30 \text{ mm}/(2–3 \text{ weeks}) \) and \( U = 10 \text{ km/day} \).

[14] Rapid, slip-parallel tremor propagation has been observed in Cascadia \[ \text{[Ghosh et al., 2010b] (see an example in Figure 2b). An explanation may again be found in the analogy between processes of plate sliding and plastic deformation in crystals. Indeed the actual movement of a dislocation occurs by local jogs (kinks) \text{[Hirth and Lothe, 1982].} \] Due to local inhomogeneities, a pair of kinks may appear at a dislocation line (see Figure 2b). Both kinks move along the dislocation line but in opposite directions, providing a slow average movement of the dislocation as a whole in the direction perpendicular to the dislocation line. As a result, the front of a moving dislocation is not a straight line, but rather a line with multiple kinks (in much the same way as edge dislocations move in crystals). The velocity of kink propagation \( U_k \) along a dislocation is larger than the average velocity of a dislocation itself. These velocities are connected by the relation, \( U_k = U / (2 n d) \), where \( n \) is the average number of kink pairs on a piece of dislocation of width \( d \) and length \( l \). Supposing \( U = 10 \text{ km/day}, l = 50 \text{ km}, d = 190 \text{ m} \) and \( n = 1 \) we find that \( U_k = 55 \text{ km/hour} \), which is in good agreement with the observed value 50 km/hour.

### 3.3. Scaling Law for Slow Earthquakes

[15] It has been found that there is a different scaling law for SSE than for regular earthquakes \[ \text{[Ide et al., 2007],} \] namely the seismic moment \( M_0 \) is proportional to the rupture time \( T \) for the slow earthquakes and to the cube of rupture time for regular earthquakes. The seismic moment can be expressed in terms of the characteristic rupture area \( S \) and the average slip \( D \),

\[
M_0 = \mu \cdot D \cdot S.
\]

For regular earthquakes, \( D \) and \( S \) are proportional to \( T \) and \( T^2 \), respectively, which explains the \( M_0 \sim T^3 \) scaling law. For slow events, however, \( D \) according to our model is proportional to the characteristic size between microasperities and does not depend on \( T \). Since a dislocation is a linear object and moves with more or less constant velocity \( V \), the time of dislocation propagation through the whole slipping area is proportional to the characteristic dimension in the direction of dislocation propagation and does not depend on the dimension in the transverse direction. Assuming \( D = b \)

\[
S = b \cdot L \cdot W
\]

and \( S = b \cdot T \cdot V \) we can find from the above equation \( M_0 = \mu b l V T \). If \( l = 50 \text{ km} \) and \( V = 10 \text{ km/day} \) then \( M_0 \approx 6 \cdot 10^{11} \text{T Nm} \), a value consistent with the scaling law for SSEs \[ \text{[Ide et al., 2007].} \] Note that this scaling law holds for the fast tremor swarms along-dip as well. In this case \( l = d = 190 \text{ m} \) and \( V = V_{\text{kin}} = 50 \text{ km/h} \) then \( M_0 \approx 2 \cdot 10^{12} \text{T Nm}. \)

### 4. Conclusions

[16] It has been shown that some features of plate dynamics, such as the scaling law for SSE, periodicity of ETS events and migration pattern of tremors, can be described by the FK model. In the framework considered, the model predicts the appearance of inelastic wave generated on the boundary of plates. Besides the standard elastic parameters, our model requires only two adjustable parameters: the characteristic distance between microasperities and the scaling factor \( A \). Furthermore, we have shown that the value of \( A \) is essentially universal for the upper crust, so this parameter need not be readjusted for every type of seismic event.

[17] The governing dynamical equation is the sine-Gordon equation. The analytical solutions obtained from this equation are appropriate for describing one or a few interacting pulses \[ \text{[Lamb, 1980]} \] as well as a sequence of many pulses \[ \text{[Gershenzon, 1994; Gershenzon et al., 2009], allowing a unified analytical treatment of various seismic events, such as regular earthquakes, ETS, SSEs and creep. It is interesting that self-similar slip pulses have been recently derived from the rate-and-state model as well \text{[Rubin and Ampuero, 2009].} \]

[18] The dislocation parameters of stress amplitudes, and strain amplitude and dislocation width depend only on the intrinsic parameters of the medium and do not depend on the process parameters. Thus the values of the dislocation parameters are effectively universal, at least for the upper crust, regardless of the actual physical mechanism causing a particular type of seismic event.

[19] In this model, an ETS is equivalent to the passage of a dislocation moving up-dip through an entire ETS zone, which results in a few cm of slip. The phenomenon of ETS is known to be periodic. The periodicity interval should depend on the average slip rate in the area considered and its seismic history.

[20] The model describes a dislocation as a two-dimensional object located on the boundary between two plates. The width of the fault is not included in this model. However, it is obvious that the stress disturbance related to the dislocation presence extends in the direction perpendicular to the plate boundary by at least the width of the dislocation (~200 m). It is interesting that faults such as San-Andreas have typical widths of this order \[ \text{[Korneev et al., 2003].} \]

[21] The presence of a dislocation is accompanied by a strong shear stress anomaly (up to 5 MPa). That is why it is not surprising that the movement of a dislocation may generate tremor. It is surprising, however, that tremor may be triggered by the much smaller stresses produced by seismic waves from distant large earthquakes \[ \text{[Rubinstein et al., 2009]} \] or even by tilt variations \[ \text{[Rubinstein et al., 2008].} \] This phenomenon could be explained if we assume that small transient stresses can affect the production of tremor by changing the motion of dislocation. This suppo-
sion is based on the well known phenomenon of mechanical impulse transfer from phonons to dislocation in crystals [Hirth and Lothe, 1982].

[22] Acknowledgments. We thank the referees J.-P. Ampuero, A.M. Rubin, and J. Savage for their insightful comments and suggestions.

References


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