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Multi-User Signal Classification via Cyclic Spectral Analysis

Brent Edward Guenther

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Multi-User Signal Classification Via Cyclic Spectral Analysis

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

by

Brent E. Guenther
Department of Electrical Engineering
B.S.E.E., Wright State University, 2009

2010
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Brent E. Guenther ENTITLED Multi-User Signal Classification Via Cyclic Spectral Analysis BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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This research develops and evaluates several different methods for multi-user signal classification via cyclic spectral analysis. The first method uses the spectral correlation function (SCF) through temporal smoothing with the use of higher order cyclic statistics (HOCS) to allow for modulation classification. The second method uses the cyclic temporal cumulant function (CTCF) and the spectral coherence function (SOF) through frequency smoothing. Using a feature-based pattern recognition technique with the SOF can not only determine the number of signals present in the received signal, but can also give signal parameter estimation and group classification performance. The last method conducts further modulation classification by using second and fourth order Cyclic Cumulants. Cyclostationary processing is the foundation of this research and requires no \textit{a priori} information about the incoming signal parameters; including but not limited to symbol rate, carrier frequency and phase offset. Monte Carlo simulations completed in MATLAB and performance results are given for all aforementioned methods.
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<td>$x(t)$</td>
<td>Complex-Valued Received Signal</td>
</tr>
<tr>
<td>$R_x(t, \tau)$</td>
<td>Autocorrelation Function (ACF)</td>
</tr>
<tr>
<td>$R_x^\alpha(\tau)$</td>
<td>Cyclic Autocorrelation Function (CAF)</td>
</tr>
<tr>
<td>$S_x^\alpha(f)$</td>
<td>Spectral Correlation Function (SCF)</td>
</tr>
<tr>
<td>$C_x^\alpha(f)$</td>
<td>Spectral Coherence Function (SOF)</td>
</tr>
<tr>
<td>$R_x(t, \tau)_{n,q}$</td>
<td>Temporal Moment Function (TMF)</td>
</tr>
<tr>
<td>$R_x^\alpha(\tau)_{n,q}$</td>
<td>Cyclic Temporal Moment Function (CTMF)</td>
</tr>
<tr>
<td>$C_x^\alpha(\tau)_{n,q}$</td>
<td>Cyclic Temporal Cumulant Function (CTCF)</td>
</tr>
<tr>
<td>$C_x(\tau)_{n,q}$</td>
<td>$n$th-order/q-conjugate Cyclic Cumulant</td>
</tr>
<tr>
<td>$\Gamma_{\gamma}(\gamma, \tau)_{n,q}$</td>
<td>Modulus of $n$th-order/q-conjugate Cyclic Cumulant</td>
</tr>
<tr>
<td>$S_{x_T}^\alpha(t, f)$</td>
<td>Cyclic Periodogram Estimate</td>
</tr>
<tr>
<td>$S_{x_T}(n, f_0)_{\Delta t}$</td>
<td>Time-Smoothed Cyclic Periodogram</td>
</tr>
<tr>
<td>$S_{x_T}(n, f_0)_{\Delta f}$</td>
<td>Frequency-Smoothed Cyclic Periodogram</td>
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I would also like to thank my friends and family for their support and encouragement over the past years to help me become the person I am today.
Dedicated to

My wife-to-be Mandy
Introduction

1.1 Research Motivation

As the available frequency spectrum diminishes the need to communicate in allocated spectrum will become a necessity. The biggest problem when communicating in allocated spectrum is interfering with the primary user(s). There are two different ways one can avoid interfering with the primary user. The first being to simply transmit where the primary user is not located. However this will still limit the number of users being able to co-locate a frequency band. Therefore, the second method is to extract information about the primary users signal to allow the secondary user to tailor their signal (reduce power or change modulation type).

One example of a technology which allows us to avoid interference with primary users is cognitive radio (CR). Since first introduced in 1999 [2] CR has been touted as the future of wireless communication systems by having the ability to sense the environment and autonomously adapt its transmission so not to interfere with primary users. One advantage to CR is its ability to autonomously change it’s transmission parameters fast enough so that even if a primary user is using a frequency hopping modulation technique the CR will still not interfere with the primary user. To give the CR the best opportunity to avoid interference with the main user it must know as much information contained within the frequency band as possible.

Several different spectrum sensing techniques have been employed, in the past, to ex-
tract information. Energy detectors scan the frequency spectrum and compare the energy received to a threshold to determine whether a signal is present. The downside is that energy detection lacks the ability to determine how many signals are present and therefore is susceptible to uncertainties in background noise [3]. If more information is known about the signal, such as carrier frequency and bandwidth, a matched filter followed by thresholding can be used. However, knowledge about the primary signal is not always available. Therefore we turn to cyclostationary processing, which does not require knowledge of the number of signals nor their parameters (carrier frequency, symbol rate, etc), to extract information. Research has shown [?] [?] cyclostationary processing to be the most robust and best performing technique for spectrum sensing.

There has been much research dealing with single user signal classification using cyclostationary processing [4] [5] [6] but little in multi-user classification [7]. The research presented in [7] only covers the case with two users. The research in this thesis will cover the case where there are more than two users with little requirement of a priori information to be known.

1.2 Problem Statement

The goal of this research is to develop a classification process able to classify the modulation technique of multiple overlapping signals with little to no a priori information known about the carrier frequency, symbol rate, or phase offset.

1.3 Research Approach

The research throughout falls under the category of cyclostationary processing and its ability to extract the cyclic features of man made cyclostationary signals.
1.4 Thesis Outline

Chapter 2 gives background information about cyclostationary processing, which includes information about the spectral correlation function (SCF) and the spectral coherence function (SOF) and the necessary information required to implement the SCF and the SOF. This chapter also covers the theory behind higher order cyclic statistics (HOCS) with its previous uses. There are many different pattern recognition processes which will also be discussed and a new combination pattern recognition method will be explained.

Chapter 3 describes three separate methods for multi-user signal classification. The first method uses a two step process consisting of using the temporal-smoothing FFT accumulation method (FAM) for producing the SCF and the use of HOCS to further classify the signals. The second method uses the cyclic temporal cumulant function to calculate the important cycle frequencies to produce the SOF where a feature-based pattern recognition technique classifies the signals. The last method is an extension of the second process where HOCS are used to further classify the groups of signals into individual modulation techniques.

Chapter 4 gives numerical results from all three methods of multi-user classification and compares these results to other multi-user and single user signal classification techniques.

Finally, Chapter 5 gives a summary of the research as well as recommendations for further research.
Background

2.1 Introduction

This chapter covers the fundamental principles used in this research for multi-user signal classification. Section 2.2 provides information regarding cyclostationary processing, including the theory behind the SCF and SOF. It also provides the theory behind HOCS and the cyclic temporal cumulant function (CTCF) and how their fundamental principles allow for the classification of multiple signals. Section 2.3 details the process of estimating the SCF and SOF using temporal smoothing and frequency smoothing processes. In section 2.4 the framework of pattern recognition used for previous signal classification is overviewed and a new pattern recognition technique is introduced.

2.2 Cyclostationary Processing

When modulating a signal there are three parameters which can be changed: amplitude, phase, and frequency or any combination thereof. A technique must be used which can distinguish between different variations of these parameters. This technique is called cyclostationary processing. Researchers have stated that cyclostationary processing is advantageous because of its insensitivity to low signal to noise ratios (SNRs) and little requirement of a priori knowledge [8].
Random signals received have periodic keying of amplitude, phase, or frequency, or some combination thereof. These statistical parameters of the signal vary in time with one or more periodicities. This is due to the fact that most man-made signals are not stationary. A time-series function is said to exhibit second-order cyclostationarity (in the wide sense), if finite-strength additive sinewave components can be created through a stable quadratic time-invariant transformation of the function [9] [10]. The ability to produce spectral lines from the received signal comes from using some Nth-order homogeneous polynomial transformation [1]. While stationary signals can only produce spectral lines at cyclic (cycle) frequency equal to zero ($\alpha = 0$) [1], cyclostationary signals have the ability to produce spectral lines at a cyclic frequency other than zero. This is why cyclostationary analysis is useful when noise (which can be considered stationary) is present. By applying the cyclic autocorrelation function (CAF) to a cyclostationary signal and then taking its Fourier Transform we are able to create a three-dimensional plot of the magnitude of that Fourier Transform as a function of cycle frequency ($\alpha$) and spectral frequency ($f$) [10] (termed the SCF).

Time, frequency, and phase offsets created by the channel can be eliminated by taking the modulus and normalization of the SCF to produce the SOF. The SOF, proven later, is insensitive to channel effects thereby providing a strong foundation for multi-user signal classification.

We adopted the Cyclic Fraction of Time (CFOT) Probabilistic Framework view of random signals because the theory behind second-order cyclostationarity has a more natural development for the time-series framework [9]. This allows us to accommodate signals whose nth-order products exhibit periodic components. This section will first talk about 2nd order cyclostationary processing and the theory behind the SCF and SOF. It will then talk about higher order cyclostationary processing and its ability to extract the timing parameters of multiple received signals.
2.2.1 Second Order Cyclostationary Processing

The information given in this subsection provides a foundation for 2nd order cyclostationary processing and describes information regarding the estimation of the SCF. We first begin by describing our digitally modulated received signal as

\[
x(t) = e^{j2\pi f_c t} e^{j\phi} \sum_{k=-\infty}^{\infty} S_k p(t - kT_s - t_0) + n(t)
\]  

(2.1)

Where

- \( x(t) \): Complex-Valued Received Signal
- \( f_c \): Carrier Frequency
- \( \phi \): Carrier Phase
- \( t_0 \): Signal Time Offset
- \( n(t) \): Additive White Gaussian Noise
- \( p(t) \): Pulse Shape
- \( T_s \): Symbol Period
- \( S_k \): Digital Symbol Transmitted at time \( t \in (kT - T/2, kT + T/2) \)

We assume that the symbols, \( S_k \), are zero mean, independent and identically-distributed random variables. We begin by describing the autocorrelation function (ACF) of a cyclostationary signal in terms of its Fourier series components [9]

\[
R_x(t, \tau) = E\{x(t + \tau/2)x^*(t - \tau/2)\} = \sum_{\alpha} R_x^\alpha(\tau)e^{j2\pi\alpha t},
\]  

(2.2)

where \( \{\alpha\} \) is the set of Fourier Components and \( R_x^\alpha(\tau) \) is denoted the cyclic autocorrelation function (CAF). The CAF completely characterizes the measure of the spectral correlation between time shifted versions of a cyclostationary process and is defined as:
\[ R^\alpha_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t, \tau) e^{j2\pi\alpha t} dt \] (2.3)

As previously stated, the Fourier Transform of the CAF produces the SCF, which represents the correlation between the spectral components of a received signal. The SCF is defined as.

\[ S^\alpha_x(f) = \int_{-\infty}^{\infty} R^\alpha_x(\tau) e^{-j2\pi fr} d\tau \] (2.4)

It can be seen in Equation (2.4) that the SCF is a true measure of the correlation between spectral components of the received signal.

Because the SCF is dependent on signal strength, the ability for one signal to suppress the other is a possibility. Therefore a normalized version of the SCF is needed which will have a proper coherence value with a magnitude of [0,1]. This normalized version is known as the SOF and takes the form of, [11]:

\[ C^\alpha_x(f) = \frac{S^\alpha_x(f)}{[S^0_x(f + \alpha/2)^*S^0_x(f - \alpha/2)]^{1/2}} \] (2.5)

The SOF of different signals produces a unique image and can be used as a spectral fingerprint for signal classification. The SOF is said to be insensitive to channel effects [9]. Taking this into consideration we can see that the SCF of a received signal with channel response \( h(t) \) is given by:

\[ S^\alpha_Y(f) = H(f + \frac{\alpha}{2})H^*(f - \frac{\alpha}{2})S^\alpha_x(f), \] (2.6)

\[ y(t) = x(t) \otimes h(t) \]

By plugging Equation (2.6) into Equation (2.5) is is obvious that the channel effects...
are removed and the SOF with channel effects is identical to the SOF without channel effects. This allows for reliable feature generation used for signal classification.

When calculating the SCF of the received signal, if multiple independent signals are present, the resultant SCF is simply the addition of the individual SCF’s [9]. To prove this empiracally we will combine a BPSK signal with \( Fc/Fs = 0.25 \) and a QPSK signal with \( Fc/Fs = 0.13 \) with a sampling length of 4096 and \( 1/Ts = Fs/10 \). Figure 2.1 shows the SCF of a BPSK signal and Figure 2.2 shows the SOF of a BPSK signal. Figures 2.3 and 2.4 represent the SCF and SOF of the QPSK signal respectively. The combined SCF and SOF of a BPSK signal and QPSK signal are represented by Figures 2.5 and 2.6 respectively.

Figure 2.1: SCF of a BPSK Signal in AWGN Channel at 5 dB SNR with \( 1/Ts = Fs/10 \) \( Fc = 0.25Fs \) no samples=4096
Figure 2.2: SOF of a BPSK Signal in AWGN Channel at 5 dB SNR with $1/T_s=10$,
$F_c=0.25 F_s$, no samples=4096

Figure 2.3: SCF of a QPSK Signal in AWGN Channel at 5 dB SNR with $1/T_s=10$
$F_c=0.25 F_s$, no samples=4096
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Figure 2.5: SCF of a BPSK Signal in AWGN Channel at 5 dB SNR with $1/T_s=Fs/10$ $Fc=.25Fs$, no samples=4096
Figure 2.6: SOF of a BPSK Signal in AWGN Channel at 5 dB SNR with $1/T_s = F_s/10$
$F_c = 0.25F_s$, no samples=4096
2.2.2 **Higher Order Cyclic Statistics**

Higher Order Cyclic Statistics (HOCS) have been used for differentiating received signals before the turn of the century. HOCS allow for the ability to determine specific modulation types based off the raw received data’s cyclic cumulants and moments. This is proven by the authors in [12], [13], and [14] by using 2nd and 4th order cyclic cumulants to distinguish signals. Other authors have used even higher order cumulants such as in [15], [16], [17], and [18] using 8th order cyclic cumulants. These papers all look at the case where only one signal is present whereas this thesis introduces the idea behind using higher order, (4th), cyclic cumulants to classify multiple received signals. The remainder of this section will detail the theory behind HOCS and how it can be applied to multiple signal classification.

We begin by describing the nth-order/q-conjugate temporal moment function (TMF):

\[
R_x(t, \tau)_{n,q} = E\{\prod_{i=1}^{n} x^{(*)}_i(t + \tau_i)\}
\]  

(2.7)

Where \((*)_i\) denotes the one of q total conjugations. For the case of n=2, q=1 we can see that Equation (2.7) turns into Equation (2.2). We can describe a signal \(x(t)\) as being nth-order wide sense cyclostationary if Equation (2.7) is periodic with some period \(T_0\) such that

\[
R_X(t + T_0, \tau)_{n,q} = R_X(t, \tau)_{n,q}
\]

\(\forall t \in \mathbb{R}\) \(\forall (\tau_1, ..., \tau_{n-1}) \in \mathbb{R}^{n-1}\)

\(\forall n = 1, ..., N \quad q = 0, ..., n\)

(2.8)

and the output of a quadratic transformation produces at least one finite-strength additive sine-wave component at a nonzero frequency. The frequencies found are also known as cycle/cyclic frequencies. To transform Equation (2.7) for specific cyclic frequencies we can express the equation in terms of it’s Fourier coefficients:
\[ R_x(t, \tau)_{n,q} = \sum_{\alpha} R_x^\alpha(\tau)_{n,q} e^{j2\pi \alpha t} \]

\[ R_x^\alpha(\tau)_{n,q} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t, \tau)_{n,q} e^{-j2\pi \alpha t} dt, \quad (2.9) \]

Where \( R_x^\alpha(\tau)_{n,q} \) is termed the cyclic temporal moment function (CTMF). We can isolate the nth order cyclic features by using the moment-to-cumulant formula to find the nth-order/q-conjugate temporal cumulant:

\[ C_x(t, \tau)_{n,q} = \text{Cum}\{x^{(s)}_1(t + \tau_1)\ldots x^{(s)}_n(t + \tau_n)\} \]

\[ = \sum_{P_n} (-1)^{z-1}(z-1)! \prod_{z=1}^{Z} m_x(t, \tau_z)_{n,z,q_z} \quad (2.10) \]

where \( P_n \) represents a set of distinct partitions, \((1, 2, 3, \ldots, n)\), and \((n_z,q_z)\) represents the number of elements and the number of conjugated terms in the subset \( P_z \), respectively. By using the nth-order/q-conjugate TCF we are able to isolate the cyclic features present in the signal to use for classification. The CTCF of a cyclostationary signal is a periodic function with its Fourier components described by [9]:

\[ C_x^\gamma(\tau)_{n,q} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} C_x(t, \tau)_{n,q} e^{-j2\pi \gamma t} dt, \quad (2.11) \]

where \( C_x(\tau) \) is the nth-order/q-conjugate cyclic cumulant of \( x(t) \). Because of the computational complexity of performing a multi-dimensional Fourier Transform of Equation (2.11) we instead manipulate equations to produce the cyclic cumulant of a received signal. This is done by plugging Equation (2.1) into Equations (2.10) and (2.11) to obtain the nth-order/q-conjugate cyclic cumulant of a cyclostationary data sequence defined as:

\[ C_x^\gamma(\tau)_{n,q} = C_s,n,q T_s^{-1} e^{j2\pi \beta_0} e^{j(n-2q)\phi} e^{j2\pi f_0} \sum_{u=1}^{n-1} (-)_{u\tau_u} \times \]

14
\[
\int_{-\infty}^{\infty} p^{(s)n}(t) \prod_{u=1}^{u=n-1} p^{(s)n}(t + \tau_u) e^{-j2\pi\beta} dt,
\]
\[(2.12)\]

\[
\gamma = \beta + (n - 2q)f_c,
\]
\[
\beta = \frac{k}{T_s}
\]

where \(C_{s,n,q}\) is the nth-order/q-conjugate cumulant value of the discrete data sequence. We then take the modulus of (2.12) to remove the time, carrier frequency, and phase offset created by the channel. By taking the modulus the resultant nth-order/q-conjugate cyclic cumulant is defined as:

\[
\Gamma_{\gamma}(\gamma, \tau)_{n,q} = C_{s,n,q} T_s^{-1} \times \int_{-\infty}^{\infty} p^{(s)n}(t) \prod_{u=1}^{u=n-1} p^{(s)n}(t + \tau_u) e^{-j2\pi\beta} dt
\]
\[(2.13)\]

where,

\[
\gamma = \beta + (n - 2q)f_c,
\]
\[
\beta = \frac{k}{T_s}
\]

Where \(p^{(s)n}\) represents the pulse shape of the received signal. Because the pulse shape will alter the overall value of the cyclic cumulant we will assume a rectangular pulse shape for the temporal smoothing section and a raised cosine pulse shape with 50\% excess bandwidth for the frequency smoothing section. It has been shown in [19] that by assuming a raised cosine pulse shape the maximum of Equation (2.13) occurs at \(\tau = \vec{0}_n\), where \(\vec{0}_n\) is an n-dimensional zero vector. The authors of [19] also show that at \(\tau = \vec{0}_n\) the function decreases with increasing \(k\), therefore we chose \(k = 1\) to maximize the values of (2.13).

2.2.3 Cyclic Temporal Cumulant Function

By using the side knowledge of (2.13) and calculating the CTCF using Equation (2.11) the parameters regarding the received raw signal can be calculated. By using various or-
der/conjugate pairs this low complexity fast algorithm allows for parameter estimation of
the received signal, such as carrier frequency and symbol rate. The algorithm is as follows:

1. Calculate Cyclic Cumulants

   Use Equation (2.10) for given n,q pair

2. Calculate Cyclic Temporal Cumulant Function

   Calculate CTCF for various cyclic frequencies $\gamma$ using Equation (2.10)

The first step of the algorithm requires the calculation of the cyclic cumulant of the
received signal. They are calculated by using Equation (2.10). Table 2.1 gives a breakdown
of how to calculate the nth-order/q-conjugate cyclic cumulant values.

<table>
<thead>
<tr>
<th>$C_{n,q}$</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{2,0}$</td>
<td>$m_{2,0}$</td>
</tr>
<tr>
<td>$C_{2,1}$</td>
<td>$m_{2,1}$</td>
</tr>
<tr>
<td>$C_{4,0}$</td>
<td>$m_{4,0} - 3C_{2,0}^2$</td>
</tr>
<tr>
<td>$C_{4,1}$</td>
<td>$m_{4,1} - 3C_{2,0}C_{2,1}$</td>
</tr>
<tr>
<td>$C_{4,2}$</td>
<td>$m_{4,2} - C_{2,0}C_{2,0}^* - 2C_{2,1}^2$</td>
</tr>
<tr>
<td>$C_{6,0}$</td>
<td>$m_{6,0} - 15C_{2,0}C_{4,0} - 15C_{2,0}^3$</td>
</tr>
<tr>
<td>$C_{6,1}$</td>
<td>$m_{6,1} - 10C_{2,0}C_{4,1} - 5C_{2,1}C_{4,0} - 15C_{2,1}C_{2,0}^2$</td>
</tr>
<tr>
<td>$C_{6,2}$</td>
<td>$m_{6,2} - C_{2,0}^2C_{4,0} - 8C_{2,1}C_{4,1} - 6C_{2,0}C_{4,2} - 3C_{2,0}^2C_{2,1}^2 - 12C_{2,0}^2C_{2,1}$</td>
</tr>
<tr>
<td>$C_{6,3}$</td>
<td>$m_{6,3} - 3C_{2,0}C_{4,1} - 9C_{2,1}C_{4,2} - 3C_{2,0}C_{4,1}^* - 9C_{2,0}C_{2,1}C_{2,0} - 6C_{2,1}^3$</td>
</tr>
<tr>
<td>$C_{8,0}$</td>
<td>$m_{8,0} - 28C_{2,0}C_{6,0} - 35C_{4,0} - 210C_{2,0}C_{4,0} - 105C_{4,0}^2$</td>
</tr>
</tbody>
</table>

Table 2.1: Cyclic Cumulant Formula

where $m_{n,q}$ represents the moment of the received signal and is defined as [17]:

$$m_{n,q} = E[x(k)^{n-q}(x^*(k))^q]$$

(2.14)

Higher order cyclic cumulants were left out for simplicity. The second step uses the
calculated cyclic cumulant value in Equation (2.11) at different values of $\gamma$. For each value
of $\gamma$ the numerical integration is carried out where the output of this algorithm is a complex
valued vector. The modulus of this complex vector at different values of $\gamma$ can be used to
determine carrier frequency and symbol rate. The magnitude of the periodic components of the CTCF output represent the periodicities present in a received signal. Therefore the CTCF algorithm will produce a large magnitude at the periodicities present in the received signal. If multiple signals are present then there will be multiple periodic large magnitudes.

As previously stated the CTCF will show periodic components of carrier frequency and symbol rate depending on the order(n)/conjugate(q) pair chosen. An example of this can be seen by choosing \( n = 4, q = 2 \) which makes the side knowledge of equation (2.13),

\[
\gamma = \frac{k}{T_s} + (4 - 2 \ast 2)f_c = \frac{k}{T_s}.
\]

This means that the CTCF of the received signal will have peak values at \( \gamma = \frac{k}{T_s} \). This is proven by Figure 2.7 where a combined BPSK and QPSK signal with \( T_{s1} = 10, T_{s2} = 5 \) and \( f_{c1} = 10, f_{c2} = 14 \) respectively with \( F_s = 100 \) will have a peak located at \( \frac{1}{T_s \ast F_s} = .1 \) and .05

![CTCF Magnitude for Calculating Ts](image)

Figure 2.7: CTCF of Combined BPSK and QPSK Signal with n=4, q=2

The carrier frequency can also be determined similarly by choosing \( n = 4, q = 0 \) resulting in CTCF peak magnitudes developing at \( 4f_c \). From Figure 2.8 we can see that the
top two CTCF magnitudes are $4F_s$ and $.56F_s$. By equating the carrier frequency knowledge with the top two magnitudes we can see that $f_{c1} = \frac{4F_s}{4} = 10$ and $f_{c2} = \frac{.56F_s}{4} = 14$ just as expected.

If two signals are combined the resultant cyclic cumulant will be the addition of each individual cumulant [9]. Because of this, the resolution of the carrier frequency and the symbol rate is limited to the number of cyclic frequencies chosen to represent the CTCF. The resolution for the parameters for test purposes was $\frac{F_s}{100}$.

While the CTCF is a very powerful tool, it lacks the ability to determine the number of signals received and the modulation type. Therefore we need to use the SCF/SOF as a means of determining how many signals and what modulation types are received.
2.3 Producing SCF/SOF

In reality, the cyclic-spectral density cannot be measured exactly [1]. An estimation of the cyclic-spectral density is created through one of two ways: temporal or frequency smoothing. Both methods are based upon the cyclic periodogram estimate of the SCF:

\[
S_{XT}^\alpha(t, f) = X_T(t, f + \frac{\alpha}{2})X_T^*(t, f - \frac{\alpha}{2})
\]  

(2.15)

where \(X_T(t, f)\) is defined as

\[
X_T(t, f) = \int_{-\infty}^{\infty} \alpha_T(t - u)x(u)e^{-j2\pi fu}du
\]

(2.16)

and \(\alpha_T(t)\) is a data tapering window of width \(T\).

2.3.1 Temporal Smoothing Methods

Smoothing the cyclic periodogram estimate of the SCF, (2.15), through temporal methods is based upon the time smoothed periodogram, expressed digitally as:

\[
S_{XT}^\alpha(n, f_0)_\Delta t = \sum_{r=-N/2}^{N/2-1} X_T(r, f_1)X_T^*(r, f_2)g_{\Delta t}(n-r)
\]

(2.17)

where the sliding FFT window is defined as

\[
X_T(n, f_k) = \sum_{r=-N'/2}^{N'/2-1} a_T(r)x(n-r)e^{-j2\pi f_k(n-r)T_s}
\]

(2.18)

where \(a_T(r)\) is a data tapering window of width \(\Delta t \gg 1/(\Delta f) \approx T\) and is the time resolution of Equation (2.18). \(T_s\) is the sampling rate, \(f_k \in 1/T \cdot \{-N'/2, ..., N'/2 - 1\}\), \(\alpha_0 \equiv f_1 - f_2, f_0 = (f_1 + f_2)/2\). It is assumed that \(N > N'\) and are both even. We can define the frequency and time resolutions of the estimate by \(\Delta t = NT_s\) and \(\Delta f = 1/T = \ldots\)
The above temporal smoothing algorithm creates an estimate of the bi-frequency plane. The bi-frequency plane is a representation of the cyclic spectral density \((f, \alpha)\). To get a better understanding of the equations and what the bi-frequency plane is we will take a look at a graphical representation of the temporal smoothing algorithm.

A signal’s \(x(t)\) frequency components are determined over a time window \(T_W\) with a sliding Fast Fourier Transform (FFT) over the entire observation time interval \(\Delta t\). Figure 2.9 shows how a sliding FFT is used over the entire time window of our received signal. Notice that there is overlap, \(L\), between each FFT. This is done to avoid cyclic leakage and aliasing so we define \(L \leq T_W/4\) [1].

![Figure 2.9: Pictorial Illustration of the Time-Variant Spectral Periodogram [1]](image)

The resultant spectral components of each short-time FFT produces a sequence of products which get summed and divided by the total time defined by Equation (2.17). The bifrequency plane is finally calculated and shown in Figure 2.10 where the zoomed in
sections are the spectral frequency, $\Delta f$, and cycle frequency resolutions, $\Delta \alpha$.

The FFT accumulation method (FAM) is an example of the temporal smoothing method. This method divides Figure 2.10 into smaller areas called channel-pair regions for which we can control the accuracy of the time-smoothing periodogram. These regions are then passed through a FFT to estimate the cyclic spectral density. In other words to make the process computationally and time efficient we are estimating discrete regions instead of the entire bi-frequency plane. This is shown by taking the estimates of Equations (2.17) and (2.18) respectively:

\[
S_{X_N}^\gamma(n, k)_N = \frac{1}{N} \sum_{n=0}^{N-1} \left[ \frac{1}{N'} X_{N'}(n, k + \frac{\gamma}{2}) X_{N'}^*(n, k - \frac{\gamma}{2}) \right] (2.19)
\]
\[ X_{N'}(n, k + \gamma/2) = \sum_{n=0}^{N'-1} w[n]x[n]e^{-j2\pi kn N'} \]  

(2.20)

where,

\( w[n] \): Data Taper (Hamming) Window

\( N' \): Short Time FFT Size

\( N \): Observation Time

Grendander’s Uncertainty Condition (GUC) gives requirements for statistical reliability for the estimation of the periodogram [20] for both continuous and discrete cases and can be seen in Table 2.2.

<table>
<thead>
<tr>
<th>Name</th>
<th>Continuous</th>
<th>Discrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCF</td>
<td>( S_{xTW}^\alpha (t, f)\Delta t )</td>
<td>( S_{XN'}^\gamma (n, k)_N )</td>
</tr>
<tr>
<td>Short FFT Size</td>
<td>( T_W )</td>
<td>( N' )</td>
</tr>
<tr>
<td>Observation Time</td>
<td>( \Delta t )</td>
<td>( N )</td>
</tr>
<tr>
<td>Time</td>
<td>( t )</td>
<td>( n )</td>
</tr>
<tr>
<td>Frequency</td>
<td>( f )</td>
<td>( k )</td>
</tr>
<tr>
<td>Cycle Frequency</td>
<td>( \alpha )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>GUC</td>
<td>( M = \frac{\Delta f}{\Delta \alpha} \gg 1 )</td>
<td>( M = \frac{N}{N'} \gg 1 )</td>
</tr>
</tbody>
</table>

Table 2.2: Comparison of the Estimated Time-Smoothed Periodogram for Continuous and Discrete time

As explained before the goal of this research is to be able to extract information about received signals which may have overlapping power spectral densities. In order to obtain the best results the spectral frequency resolution must be small. The frequency resolution can be calculated by \( k_{res} = \frac{f_s}{N'} [1] \). A graphical representation of the bi-frequency plane when there are \( N^2 \) channel pair regions is found in Figure 2.11.

As the graph and equation show we want to reduce \( k_{res} \) so that we can have a fine resolution. However, in order to accomplish this we must have either a low sampling fre-
Figure 2.11: Division of the bi-frequency plane in channel pair regions [1]

quency or a very large FFT size, which both conditions will cause undesirable results. We are forced to keep the sampling rate at the Nyquist frequency and therefore cannot reduce that number. We are also limited by the FFT size because to obtain statistical reliability (according to GUC) the observation time must be much greater than the FFT size. A smaller resolution will require a larger FFT size, resulting in an increased observation time.

### 2.3.2 Frequency Smoothing Methods

Just as the temporal smoothing method was based on time smoothed cyclic periodogram the frequency smoothing method is based on the frequency smoothed cyclic periodogram. We can digitally express the frequency smoothed cyclic periodogram as:

$$S_{X_T}^{00}(n, f_0)_{\Delta f} = \sum_{r=\pm N'/2}^{N'/2-1} X_T(n, f_1 + r/T)X_T^*(n, f_2 + r/T)\alpha_{\Delta f}(r)$$  \hspace{1cm} (2.21)
\[ X_T(n, f_k) = \sum_{r=-N/2}^{N/2-1} g_T(r) x(n-r)e^{-j2\pi f_k(n-r)T_s} \] (2.22)

is the Discrete Fourier Transform of \( x(n) \) and \( w(n) \) is a rectangular window of \( N \) points of the FFT related to the total observation time \( \Delta t \), and \( \alpha_0 = f_1 - f_2, f_0 = (f_2 + f_2)/2 \). It is assumed that \( N' \) and \( N \) are even with \( N > N' \). The time resolution of the equation can be defined as \( \Delta t = T = NT_s \) and the frequency resolution can be defined as \( \Delta f = N'/T = N'/(NT_s) \). The time frequency resolution product is \( \Delta t \Delta f = N' \) which implies, under GUC, that \( N' >> 1 \).

The equations can be carried out by finding the cross spectral correlation of the FFT frequency components. The ability to spectrally smooth the periodogram comes at a cost to either the statistical reliability or the spectral resolution [21]. A large amount of spectral smoothing is desired for a statistically reliable estimate of the SCF. However, a large amount of smoothing will not allow for the fine spectral resolution of the SCF. This ultimately is a design parameter so that we can use the largest amount of smoothing while still obtaining a fine spectral resolution.

Because the pattern recognition process, described later, is based upon the locations and shapes of the magnitudes of the SCF the ability to provide a statistically reliable estimation of the SCF while still having a fine spectral resolution is very important. By estimating the SCF through the frequency smoothing method we have the ability to choose specific cycle (\( \alpha \)) frequencies. This allows us to choose a smaller set of cycle frequencies to estimate the SCF, in turn becoming a more efficient algorithm, than temporal smoothing, for creating high-accuracy estimates of the SCF [22] [23] [24].
2.4 Pattern Recognition Techniques

2.4.1 Introduction

A pattern recognition process can be broken down into three steps: data acquisition and preprocessing, feature generation and extraction, and decision making. For the classification of multiple signals presented in this thesis, data acquisition and preprocessing is a sampled representation of the received signal. The feature generation is completed by calculating the SCF and SOF, where only the frequency band of interest is known. The decision process determines how many signals are present from each modulation group. The signal classification process can be completed by several different methods. Papers utilizing pattern recognition for signal classification have used Supported Vector Machines (SVM) [6] [25] [26], Hidden Markov Models (HMM) [4] [27] [28], and Artificial Neural Networks (ANN) [5] [29] [30].

Methods for carrying out pattern recognition can be categorized into decision (statistical) and syntactic (linguistic/structural). These two methods are not independent and can sometimes be interpreted as the same pattern recognition method. Each method’s advantages and disadvantages are analyzed below.

Statistical pattern recognition transforms the information received into an $n$-dimensional feature vector [31] where various techniques in discriminate analysis and statistical decision theory are applied for classification. This form of pattern recognition uses the numerical valued feature vectors as a means of pattern recognition. For our case this would be specific locations in the SCF/SOF. A problem arises because the feature vector produced from different classes of signals can have very similar feature vector values. This would result in a large number of misclassifications.

A syntactic pattern recognition approach is based on the explicit or implicit representation of a classes structure [32]. A structure is defined, conceptually, by the way its subpatterns (elements) are related and configured to create a more complex pattern. Be-
cause the equations for the locations of the peaks for each signal are known, we will use the explicit representation of its classes structure. There are several different modeling techniques which can model a received signal. However, this requires statistical knowledge of the time-series function \textit{a priori}. Interferences such as noise and multi-path fading introduced to the received signal will alter the time-series function and will diminish the syntactic pattern recognition’s ability to correctly model the signal.

By combining the advantages of both approaches we can better optimize the pattern recognition process. The statistical approach gives the ability to effectively model the received signal using the SCF/SOF but lacks the ability to classify. The syntactic approach lacks the ability to model the received signal but has the ability to classify. Therefore, we combined the statistical approach’s ability to create a model of the received signal and the syntactic approach’s ability to classify. The procedure is as follows:

1. Produce Feature Vector (SCF/SOF)
   Uses Temporal/Frequency Smoothed Periodogram

2. Extract Features
   Lowering Threshold (Temporal Smoothing)
   OR: Adaptive Threshold (Frequency Smoothing)

3. Classify Signal Based on Peak Interrelationships

   The two groups of signals the temporal smoothing technique was attempting to classify between were real constellation valued digital signals and complex constellation valued digital signals. The real valued signals were Binary Phase Shift Keying (BPSK) and 4 Amplitude Shift Keying (4-ASK). The complex valued signals were Quadrature Phase Shift Keying (QPSK), 8 Phase Shift Keying (8-PSK), and 16 Quadrature Amplitude Modulation (16-QAM).
The frequency smoothing method attempted to distinguish between real constellation valued digital signals and complex constellation valued digital signals. The signals with real values were BPSK, 4-ASK, and 8-ASK. The complex valued signals were QPSK, 8-PSK, 16-PSK, 16-QAM, and 64-QAM.

While not an exhaustive representation of the combinations of received signals, both multi-user signal classification techniques provide a foundation for further research.

2.4.2 Previous Methods

A Hidden Markov Model (HMM) is a stochastic process which models a system assumed to be a Markov process with an unobserved state. Each model is created from a Markov process using the Baum-Welch algorithm. This theory was applied to the SOF in [?] for signal detection and classification. A theoretical model was created using the feature vector of the SOF for each signal. After training, the likelihood of an unknown incoming signal’s feature vector is calculated for each model. The model which created the Maximum Log-likelihood was selected as the correct signal type. Expanding this theory to multi-signal classification, a model would need to be created for each combination of signal types. This would increase the number of computations because the received model would need to be compared to a larger number of theoretical models. Also, a HMM is dependent on the statistical knowledge of the pre-experimentation and therefore has a poor ability to classify the data [4].

An Artificial Neural Network (ANN) uses artificial neurons to find patterns in data. An ANN has the ability to generalize patterns so that even as the figure (SOF) changes the network still has the ability to classify the signal. The authors in [5] and [29] both utilize an ANN and the SOF for signal classification. Where just the cyclic profile of the was used in [5], both the cyclic and spectral profiles were used by [29]. However, just as HMM, an ANN is needed for each desired signal combination; resulting in more computations.

A Support Vector Machine (SVM) is a process for which a hyperplane is created to
separate classes in feature space. The hyperplane is calculated so that the margin between one class and another is identical. Once this optimum hyperplane has been calculated the decision theoretic is defined as a SVM. A SVM’s information processing mechanism has the inability to classify time-varying process signals directly. Therefore the authors of [6] expand the classification mechanism and information processing of a SVM to the time-domain to create a process SVM (PSVM).

Similar to the research of this thesis, the authors in [26] use cyclostationary processing as the foundation for signal classification. They were able to prove their two step hybrid HMM/SVM classifier outperforms an ANN and a HMM. However, this method also requires multiple steps for all signal combinations, resulting in longer computation time.

In conclusion, all aforementioned previous methods require an increase in computations when introduced to multiple received signals. The feature-based pattern recognition technique introduced in this thesis requires no knowledge of the number of received signals. Also, the number of computations does not increase as the number of signals increases.

### 2.4.3 Pattern Recognition Process

Two different pattern recognition techniques, using the same basic feature generation properties, were implemented in this research. Both pattern recognition techniques employ a combination statistical/syntactical approach for signal classification.

The process begins by taking the received sampled signal and transforming it into feature space by finding the signals SCF/SOF. From the introduction of this chapter we know that a statistical approach requires the transformation of the received signal into feature space to create a feature vector. Therefore we can classify the transformation via temporal/frequency smoothed periodogram of the received signal as a statistical process.

A threshold based on the numerical value of the spectral coherence is then set to extract the locations of the peaks. The process used to define the location of this threshold is different for each pattern recognition technique and will be defined later.
A syntactic approach requires the ability to break down a complex pattern into simpler patterns used for classification [33]. A received signals SOF can be perceived as a complex pattern and the peak locations can be perceived as the simpler patterns (primitives). Classification based on the interrelationship of the subpatterns is what defines this process as a syntactic (structural) process. A pre-defined structure is created for each signal type through the interrelationships of the primitives. A training set defines the collection of all the pre-defined structures.

Because of real world nuances, such as additive white gaussian noise and multi-path fading, the primitives of the structure will not always carry the same interrelationships causing the structure to not be exact. This leads to the development of a spectral frequency tolerance for each peak location. We defined our tolerances for temporal smoothing and frequency smoothing as \( \varepsilon_1 = 5 \text{Hz} \) and \( \varepsilon_2 = 0.03 f / F_s \) respectively. These tolerances were determined from numerous simulations of the estimation of the SCF/SOF. This approach is similar to an ANN’s ability to generalize a pattern for classification. As previously defined, the training set is a compilation of the the pre-defined structures and can seen below:

<table>
<thead>
<tr>
<th>Group</th>
<th>Cycle Frequency</th>
<th>Spectral Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>0</td>
<td>( \pm f_c \pm \varepsilon_1 )</td>
</tr>
<tr>
<td></td>
<td>2( f_c )</td>
<td>( 0 \pm \varepsilon_1 )</td>
</tr>
<tr>
<td>Complex</td>
<td>0</td>
<td>( \pm f_c \pm \varepsilon_1 )</td>
</tr>
</tbody>
</table>

Table 2.3: Training Set Including Tolerance for Time Smoothing

<table>
<thead>
<tr>
<th>Group</th>
<th>Cycle Frequency</th>
<th>Spectral Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>( 1/(T_s F_s) )</td>
<td>( \pm f_c / F_s \pm \varepsilon )</td>
</tr>
<tr>
<td></td>
<td>( 2 f_c / F_s )</td>
<td>( 0 \pm \varepsilon_2 )</td>
</tr>
<tr>
<td>Complex</td>
<td>( 1/(T_s F_s) )</td>
<td>( \pm f_c / F_s \pm \varepsilon_2 )</td>
</tr>
</tbody>
</table>

Table 2.4: Training Set Including Tolerance for Frequency Smoothing

Using a combined statistical/syntactic approach enables us to classify multiple received signals. The statistical pattern recognition estimation of the the SCF/SOF allows for
the independent construction of every individual signal. Because the SCF/SOF structure changes as the signal parameters change a syntactic approach based off the interrelationships among the cyclic/spectral features is necessary.

2.4.4 Temporal Smoothing Pattern Recognition

The temporal smoothing pattern recognition method was based on the SCF where we assumed we knew the symbol rate of the transmitted signals. The real signals exhibit large peaks at \((\alpha = 0, f = \pm f_c)\) and \((\alpha = \pm 2f_c, f = 0)\) and small peaks at \((\alpha = \pm \frac{1}{T_s}, f = \pm f_c)\) and \((\alpha = 2f_c \pm \frac{1}{T_s}, f = 0)\) where \(T_s\) is the symbol duration, as seen in Figure 2.12. The complex signals exhibited large peaks at \((\alpha = 0, f = \pm f_c)\) and small peaks at \((\alpha = \pm \frac{1}{T_s}, f = \pm f_c)\) [11], as seen in Figure 2.13. These peak location equations define the primitive interrelationships. Using the method described in section 2.3.1 the SCF is used to generate the features.

Figure 2.12: SCF of BPSK
Once the features are generated, a lowering threshold algorithm is used to extract the
features. However, before defining a threshold we need to understand the shape of the peaks. By taking a look at the spectral profile of a combined BPSK and QPSK signal at $\alpha = 0$ (better known as the power spectral density (PSD)) we can see that the values are monotonically increasing/decreasing. Figure 2.15 is simply the values of Figure 2.14 at a cycle frequency equal to zero.

We begin by setting a spectral coherence threshold and determining what values of the SCF lie above this threshold (the lowering threshold will be described later). By setting this threshold we break the spectral profile of the SCF into different sections. From each section we extract the spectral frequency associated with the maximum spectral correlation value. A graphical representation of the spectral profile being broken down into sections can be seen in Fig 2.16 where the threshold was set at 1.5. This resulted in the creation of 4 sections which lie above the threshold. For each section the spectral frequency at the largest magnitude value was chosen as the peak location. This method of setting a threshold and determining the peak location is done for every cycle frequency. This creates a 3 dimensional data set which contains the cycle frequency, spectral frequency, and spectral coherence (peak value) from which we can determine how many signals from each group are present.

The location of the threshold is paramount when determining how many signals are present. When the PSD of the received signals are overlapping we can very easily mistake two signals as one signal. We note here that the cyclic spectrum is an even function about the cyclic axis for all signals. Figure 2.17 is broken down to show how the correct location of the threshold is necessary. On the right side of the figure we can see that by setting a threshold at 1.5 (just as on Figure 2.16) the program only sees two locations where the PSD crosses the threshold, making the program believe that there is only 1 signal present when there is actually two signals present. The left side of the figure shows how by the proper placement of the threshold we are able to obtain two separate sections and thereby
Figure 2.15: Spectral Profile of Combined BPSK and QPSK at $\alpha = 0$

Figure 2.16: Peak Calculating
determining that there are two signals present instead of one. To effectively choose the location of the threshold a lowering threshold algorithm was developed.

Figure 2.17: Threshold Testing
The process begins by setting the threshold at a value which is just above the maximum spectral coherence value of the SCF (=maximum($S_x^{\alpha}(f)$)). For every cycle frequency ($\alpha$), between -300 to 300 Hz, we find any spectral frequencies which lie above that threshold and determine the peak locations and magnitudes. The interrelationships of all combinations of these values are then compared to the theoretical interrelationships defined by the training set of structures. If a signal is deemed present then that signal is erased from the SCF. Once this process is complete we lower the threshold by .1 and repeat the process until the threshold limit, 1.0, is reached. A better approach may have been to make the threshold limit a percentage of the maximum value of the SCF. The output of the feature-based pattern recognition step will be the number of signals present from each group along with the estimated carrier frequencies. Broken down into steps, the above algorithm is:

Temporal Pattern Recognition with Lowering Threshold Algorithm

1. Generate SCF

2. Find maximum value of SCF

   =maximum($S_x^{\alpha}(f)$). Set as threshold

3. Find all values above threshold and section off

   $S_x^{\alpha}(f) >$ threshold $\forall \alpha$

   $\Delta f =$ separation between sections

   If $\Delta f \geq 5$ Hz, then different sections

   If $\Delta f < 5$ Hz, then same section

4. Find max value for each section and store:

   Cycle Frequency, Spectral Frequency, Spectral Coherence

5. Determine Signals Present
Compares interrelationships of found peaks with training set

if signal=found, then erase from SOF

6. Final Step

if threshold > threshold limit

threshold=threshold-.1

Return to Step Three

if threshold < threshold limit

Exit

2.4.5 Frequency Smoothing Pattern Recognition

The first step of the process is the feature generation. Using the side knowledge of (2.13) and calculating the CTCF using (2.11) we are able to extract the carrier frequency and symbol rate of the received signal. Based upon these values important cycle frequencies are calculated, defined later. The SCF is generated from the cross spectral correlation of the FFT frequency components described by (2.21). The SOF is then calculated from the SCF defined by (2.5). When the SOF is calculated, the normalization makes the PSD, $\alpha = 0$, of the received signal equal to one. This can be seen in Figure 2.4.

The next step in the process is feature extraction. Because features are no longer overlapping, from the aforementioned generation of the SOF, the algorithm for a lowering threshold is no longer necessary. The threshold is found by implementing an adaptive algorithm based on the estimation error of the SCF.

Because the SCF and SOF are estimated, as section 2.3 described, there will be noise-like spectral coherence values located in the SCF/SOF where the signal does not contain spectral correlation. We can consider this estimation error and we define our adaptive threshold relative to this error value. We begin the threshold determination by calculating
the SOF at 4 cycle frequencies deemed not important. At each cycle frequency the max-
imum spectral coherence value is stored into a vector. The second maximum value was
used as a base value to define our threshold. By simply using the base value as the thresh-
old a high possibility of the system thinking a signal is present although it isn’t, causing
a false alarm. If we add a scalar value onto the base value the percentage of a false alarm
decreases. However, this will cause a small decrease in performance, evaluated later.

Once a threshold has been defined, the peak locations need to be extracted. One
difference between the temporal smoothing and the frequency smoothing methods is the
shape of the peaks. While the maximum value in each section can be considered the peak
location for temporal smoothing it is not true for frequency smoothing. An example of
this is the peak located at \((\alpha = 2f_c, f = 0)\) for a real signal, see Figure 2.18. As the
figure shows there exists multiple points above the threshold which are the maximum value.
Therefore, to calculate the peak location we take the locations of where the SOF crosses
the threshold and find its midpoint. This is done at each important cycle frequency with the
same threshold and the peak locations are stored in a matrix.

The final step is to determine the number of signals from each group. This is done by
comparing the interrelationships of the found peaks with a training set of structures. The
output of this process will give the number of signals present from each group along with
carrier frequency and symbol rate estimations.

There are several advantages to using the frequency approach for classification. The
first deals with the ability of the frequency approach to produce a more accurate repre-
sentation of the SCF. This finer resolution allows for the carrier frequencies of multiple
signals to be closer to each other and still have the ability to separate the signals. Another
advantage is the speed of classification. With the same number of time-series samples,
the temporal method described above will take approx 30 seconds for each test while the
frequency approach above takes less than .2 seconds. This is due to the fact that there are
less cycle frequencies for which the SCF is calculated. The final advantage of using the
Figure 2.18: Spectral Profile of BPSK Signal at $\alpha = 2f_c$

The frequency smoothing approach is its insensitivity to channel effects, proved in subsection 2.2.1.
Simulation Process

As described earlier there are two methods of producing the SCF/SOF. For each method there are multiple steps of obtaining classification results and different assumptions were made for each technique.

3.1 Classification via Temporal Smoothing and HOCS

3.1.1 Introduction

A two step process was used to determine the carrier frequencies and the modulation types of the received signal(s). The first step involves a feature-based pattern recognition technique with an adaptive threshold along with the SCF to determine how many signals from each group are present in the received signal. HOCS are then computed using the received signal, using (2.13), and are used to further classify the received signal into a specific signal modulation type.

The CAF demonstrated that there are unique SCF’s for each modulation scheme. Two factors contribute to the SCF’s uniqueness: peak location, and spectral coherence. A two-dimensional pattern matching technique based on cyclic and spectral frequency is insufficient; because peaks will move as carrier frequency and symbol rate vary. However, by knowing the interrelationship of the peaks we are able to generalize the groups.
### 3.1.2 Two-Step Temporal Signal Classification

For test purposes, the simplest case was taken where a real (BPSK or 4-ASK) and a complex (QPSK) signal were mixed together. Their power spectral densities (PSD’s) had a zero percent frequency overlap. With a sampling frequency of 500 Hz the real signal had a carrier frequency of 160 Hz and the QPSK signal had a carrier frequency of 70 Hz. Even though the carrier frequencies were kept constant the modulated data changed every test iteration and the pattern recognition signal classifier did not know value of the carrier frequency. The only information known was that the symbol rate of all transmitted signals was 50 Hz. The PSD of the combined signals is given by Figure 2.15.

The two signals were mixed together and the SCF was produced using the temporal smoothing method. The temporal smoothing method produces complex values for the estimation of the cyclic spectral density. Therefore, the magnitude of the resultant estimation was used and plotted vs cyclic frequency and spectral frequency. Figure 2.14 shows the SCF of the combined BPSK and QPSK signals using the temporal smoothing method.

Once the SCF is produced we need to extract the features of the received signal using the method described in subsection 2.4.4. The features extracted from the SCF are then compared to a training set to determine how many signals from each group (real or complex) are present in the received signal. A training set contains a set of patterns from which information can be compared. Figure 3.1 shows the pattern recognition process of how the SCF is produced and we determine that \( N \) real and \( M \) complex signals are present in the signal. By knowing this we can then proceed to step two, which implements HOCS.

![First Step Flow Chart](Image)

**Figure 3.1: First Step Flow Chart**
The output of the pattern recognition process determines whether there is a single signal \((N + M = 1)\) or are multiple signals present \((N + M > 1)\). It also determines the number of real and complex signals. Figure 3.2 shows the flow chart for a single user case. It shows how the pattern recognition technique determines whether there is a real or complex signal and then HOCS are used to determine the specific modulation technique.

Figure 3.3 shows the multiple user case where the pattern recognition process determines the number of real and complex signals present in the received signal. An example of the process once the number of real/complex signals has been identified is shown by Figure 3.4.

The second-order/zero-conjugate cyclic cumulant values were used for classification. Simulations were completed to estimate the average HOCS for all signal combination pairs and the PDF of those values where plotted. Figure 3.5 is the PDF for the second order/zero conjugate cyclic cumulant values. The location between the individual PDF’s where the possibility of a false positive and a false negative are equal was chosen as the decision boundary. With this set of signals, we can see that there is a possibility of separation. When more signals get introduced the ability to separate these PDF’s will become harder and harder, creating a near impossible classification probability.

![Flowchart of Second Step Single User Case](Image)

**Figure 3.2: Second Step Single User Case**

The problem with using the HOCS is that as the SNR decreases the values of the
HOCS increases, which leads to a misclassification. Figure 3.6 shows how for a 16-QAM and 4-ASK combined signal the values for the second-order/zero-conjugate at a high SNR is close to the theoretical average but at a low SNR is not close. The figure was normalized so that the y-axis is a ratio of the actual cyclic cumulant average compared to the theoreti-
Figure 3.5: 2\textsuperscript{nd} Order Zero Conjugate Cyclic Cumulant

cumulant value $NormalizedValue = (EmpiricalValue)/(TheoreticalValue)$. The theoretical value was obtained by finding the cyclic cumulant value without being passed through any channels.

Figure 3.6: HOCS Values of Combined 16-QAM and 4-ASK With Varying SNR


## 3.2 Frequency Smoothing

The frequency smoothing method requires that a smaller set of cycle frequencies be used when calculating the SOF. By knowing the carrier frequency and symbol rate of the received signals we can calculate the required cycle frequencies. Whether there is one or multiple signals, the carrier frequency and symbol rate estimates are acquired by employing the CTCF.

As subsection 2.2.3 described we are restricted in resolution to determine the differences in symbol rate. We assume that the symbol rate of the two signals, while not known, is the same value for all signals being transmitted. Because we do not know the number of signals being received the top four carrier frequencies, \( F_{c1}, F_{c2}, F_{c3}, F_{c4} \), and the top two symbol rates, \( T_{s1} \) and \( T_{s2} \), were calculated using the CTCF algorithm described by subsection 2.2.3. These values were then used to calculate the important cycle frequencies:

\[
\begin{align*}
\alpha_1 &= \frac{1}{T_{s1}F_s} \\
\alpha_2 &= \frac{1}{T_{s2}F_s} \\
\alpha_3 &= \frac{2 \times F_{c1}}{F_s} \\
\alpha_4 &= \frac{2 \times F_{c2}}{F_s} \\
\alpha_5 &= \frac{2 \times F_{c3}}{F_s} \\
\alpha_6 &= \frac{2 \times F_{c4}}{F_s}
\end{align*}
\]

Using the cycle frequency values calculated by the CTCF the SOF is produced at those discrete cycle frequencies. The SOF is produced using the frequency smoothing algorithm from a pre-determined number of samples of our received signal. By increasing the sample length, the more accurate the frequency smoothing method is at estimating the cyclic spectral density (SCF). After the production of the SOF the pattern recognition process described in subsection 2.4.5 allows for the determination of the number of signals and to which group the received signal(s) belong.
The next step is determining the specific modulation type; this is done by using HOCS. By knowing the number of real/complex signals present we are able to use different order/conjugate pairs to create a hierarchical method of determining the signals present. If the pattern recognition process determines that there are two signals present (using the SOF) then the process follows that of Figure 3.7 to determine the real/complex pair.

If it is determined that there are two real signals present the second-order/first-conjugate cumulant, C(2,1), is used to determine which two combined real signals are present, shown by Figure 3.8. If it is determined that two complex signals are present then the process follows that of Figure 3.9. This process uses both C(2,1) and C(4,2), whereas if it were determined that a real and complex signal were present an additional C(2,0) was used. Figure 3.10 shows a three step process to determine which real/complex configuration is present in the received signal.

**Figure 3.7: Process of Determining Number of Real & Complex Signals**

**Figure 3.8: Process of Determining Combined Real & Real Signal**
Figure 3.9: Process of Determining Combined Complex & Complex Signal
Figure 3.10: Process of Determining Combined Real & Complex Signal
Results

4.1 Introduction

This chapter will describe the simulated results for the three aforementioned methods of multi-user signal classification. It will also compare multi-user signal classification results from [7] and will show how this method allows for multi-user signal classification with no change to the classifier.

4.2 Temporal Smoothing Results

The results for the temporal smoothing will first be broken down into individual steps and then the system as a whole will be analyzed. Testing was completed using Monte Carlo simulations with one real signal (BPSK or 4-ASK) and one complex signal (QPSK) with 1001 samples at a sampling rate of 500 Hz. By increasing the number of samples we would get a better approximation of the SCF. One hundred simulations were ran for each SNR value. As stated before this technique was being implemented to determine the feasibility of using a temporal smoothing algorithm for multi-user signal classification, therefore the simulation curves will not be as smooth as if we were running more simulations.

A correct classification from the first step means that the groups from which the signals came were identified correctly. It also means the carrier frequencies were correctly
identified. The real signal was variably chosen but the complex signal remained a QPSK signal. The classification performance of the first step is given by Figure 4.1. As that figure depicts even at low SNR values the ability to extract the number of signals and the signal parameters is still feasible. The limiting factor for this approach was the ability of the pattern recognition scheme to determine real signals. This is proven by Figures 4.2 and 4.3 which show first step classification performance for a complex and a real signal respectively.

As described in subsection 3.1.2 as the SNR decreases the ability of the system to correctly identify the specific modulation type using HOCS becomes more difficult. This is proven by simulations in Figure 4.4 which shows that at a low SNR the ability to correctly classify the signal is near impossible. This graph assumes it knows the correct real/complex signal combination and was not based upon the pattern recognition technique.

However, as the SNR increases, the second step using HOCS outperforms the feature-based pattern recognition technique. It begins to outperform around 11 dB in Figure 4.5. Below 11 dB the limiting factor is the HOCS but equal to or above 11 dB the pattern recognition scheme is the limiting factor. The final two step process can be summarized by Figure 4.6.

Considering the results found in Figure 4.6 it was deemed that the ability of the temporal smoothing method along with HOCS was not a reliable technique for multi-user signal classification. One downfall being the inability of the temporal smoothing method to produce an accurate estimate of the SCF for a real signal. The other downfall is the HOCS being effected by the noise and ultimately not being able to further classify the signals. Therefore a secondary method to signal classification was researched using the CTCF with the frequency smoothing algorithm for multi-user signal classification.
Figure 4.1: First Step Temporal Smoothing Classification Results

Figure 4.2: First Step Complex Classification Results
Figure 4.3: First Step Real Signal Classification Results

Figure 4.4: Second Step Classification Performance
Figure 4.5: HOCS vs Pattern Recognition

Figure 4.6: Temporal Smoothing Two Step Classification Results
4.3 Frequency Smoothing Results

This chapter describes the results of the two remaining methods for multi-user signal classification. The first being the determination of the number of signals and group classification. The second being further classification through the use of HOCS.

4.3.1 Pattern Recognition Using CTCF Without HOCS

The first testing completed was to determine whether the CTCF was able to correctly identify the carrier frequency and the symbol rate of the received signal(s). Figure 4.7 shows the symbol rate detection performance of the CTCF using the second-order/zero-conjugate cumulant for multiple signal lengths and Figure 4.8 shows the carrier frequency detection performance. As the figures show, the ability of the CTCF program to correctly identify the carrier frequency and the symbol rate of the received signal combination is very successful even at low SNR values. The testing was completed using two random real/complex signals which were spectrally overlapped by 35-75 percent using 1024, 2048, 4096, and 8192 samples at a sampling rate of $1/T_s = F_s/10$. Being able to successfully determine the carrier frequency and symbol rate of the received signal(s) is a requirement for the pattern recognition process to correctly identify how many and what type of signals are present.

Before classification results are given for signals which will have spectral overlap a baseline performance where signals are not spectrally overlapping needs to be determined. Results are given in Table 4.1 which show the classification performance of one, two, and three signals without spectral overlap. Signals were deemed correctly classified if the pattern recognition technique correctly identified the carrier frequency and symbol rate of the signal being sent. Testing was completed using 4096 samples of the received combined real/complex signals which had variable carrier frequencies. Simulations were completed assuming a 10% false alarm rate. More will be discussed later on the false alarm rate and
how it can be adjusted.

<table>
<thead>
<tr>
<th># Signal</th>
<th>-10 dB</th>
<th>-7 dB</th>
<th>-4 dB</th>
<th>-1 dB</th>
<th>2 dB</th>
<th>5 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>69.4%</td>
<td>85.6%</td>
<td>97.6%</td>
<td>99.6%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Two</td>
<td>47.6%</td>
<td>80.4%</td>
<td>95.4%</td>
<td>99.8%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Three</td>
<td>48.5%</td>
<td>79.8%</td>
<td>92.5%</td>
<td>99.2%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 4.1: Classification Performance Without Overlap

We can further break the table down into the classification performance for real and complex signals for each SNR value. Table 4.2 shows the breakdown of the classification performance for the individual real/complex signals. As the table shows the probability of a real signal being classified is slightly less than that of a complex signal for nearly all SNR values.

A good understanding of spectral overlap can be seen by Figures 4.9, 4.10 and 4.11 which show the PSD of 4096 samples of two complex signals with a 30, 50 and 70 percent spectral overlap respectively. As the latter figures show it is hard even with the naked eye
to determine not only how many signals are present but also their carrier frequencies. However, by implementing the SOF it can be seen in Fig 4.12 that there are still distinguishable features even with a 70 percent overlap. These graphs, along with the remainder of the graphs, assume equal power from both signals.

As described in subsection 2.4.5 when determining the adaptive threshold value a scalar value is added to a base value. The base value is calculated from the second maxi-
Figure 4.9: PSD With 30 Percent Spectral Overlap

Figure 4.10: PSD With 50 Percent Spectral Overlap
Figure 4.11: PSD With 70 Percent Spectral Overlap

Figure 4.12: SOF With 70 Percent Spectral Overlap
mum spectral coherence value of random cycle frequencies. To determine the scalar value which was to be added, a test was performed to determine the classification performance and false alarm rate for several various scalar values. Various real/complex signals were sent over an AWGN channel with a 35 to 75 percent overlap where 4096 samples were taken at the receiver. Figure 4.13 shows the classification performance when the scalar value (defined in subsection 2.4.5) is .015, .025, .035, and .05. Figure 4.14 shows the false alarm rate Vs the scalar value.

![Threshold Testing](image)

**Figure 4.13: Classification Performance Vs. Threshold**

As Figures 4.13 and 4.14 show when the scalar value is increased the classification performance and the false alarm rate decreases. By choosing a large scalar value we decrease the false alarm rate but lose classification performance by almost 3 dB. By choosing a small scalar value the false alarm rate becomes unacceptable. Therefore by choosing a scalar value in the middle we will be better able to get acceptable classification performance with a low false alarm rate. For all simulations a scalar value of .025 was added to
the chosen maximum peak for a ten percent false alarm rate.

The next testing was performed to determine the percentage of overlap in the frequency domain (PSD) compared to the performance of the pattern recognition process. Two random real/complex signals were combined with a sample length of 4096 and the percentage of overlap was controlled to determine the results. The carrier frequency was varied while still keeping the correct percentage of overlap and the symbol rate was, while not known, fixed. The results can be seen by Figure 4.15. As expected, the figure shows that as the amount of spectral overlap increases, the classification performance decreases.

The benchmark comparisons made for the next several graphs come from [7] where the author utilizes the SCF along with a multi-pole classifier to classify the time-frequency overlapped signals. As previously proven by increasing the sample length we are able to obtain a better estimate of the SCF/SOF, resulting in better classification performance. Figure 4.16 compares benchmark classification performance from [7], where two signals are overlapped in a single channel, to the simulated results found by using the pattern
recognition technique developed in subsection 2.4.5. There were one hundred simulations ran for each SNR at %20 and %50 overlap with 8192 samples.

Figure 4.15: Group Classification Performance Vs. Percentage Overlap

Figures 4.17 and 4.18 show the classification performance of two random mixed signals with spectral overlap with different sampling lengths. The graph compares the results from [7] with the simulated results using the aforementioned pattern recognition technique. There were 250 Monte Carlo simulations with simulated lengths of 2048, 4096, 8192, and 16384 for each SNR value with varying carrier frequency. As expected, the figures show that as the length of the sampled received signal increases the better the classification performance will be.

Group classification performance analysis shows the ability to classify a given signal with very little a priori information using cyclic spectral analysis. Table 4.3 shows the performance when: only real signals are present, only complex signals are present, both
Figure 4.16: Signal Classification Comparison for Percentage Overlap

Figure 4.17: Group Classification Performance Vs Length of Signal
real and complex signals are present. Simulations were completed using 4096 samples of the received signal with 35 to 75 percent spectral overlap.

<table>
<thead>
<tr>
<th>Group</th>
<th>-10 dB</th>
<th>-7 dB</th>
<th>-4 dB</th>
<th>-1 dB</th>
<th>2 dB</th>
<th>5 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>19.8%</td>
<td>51.2%</td>
<td>72.4%</td>
<td>81.6%</td>
<td>89.8%</td>
<td>94.4%</td>
</tr>
<tr>
<td>Complex</td>
<td>41.4%</td>
<td>66.4%</td>
<td>83.2%</td>
<td>89.8%</td>
<td>93.0%</td>
<td>96.2%</td>
</tr>
<tr>
<td>Real &amp; Complex</td>
<td>70.2%</td>
<td>79.0%</td>
<td>85.6%</td>
<td>89.6%</td>
<td>93.6%</td>
<td>96.0%</td>
</tr>
</tbody>
</table>

Table 4.3: Group Classification Performance with 35-75 Percent Spectral Overlap

### 4.3.2 Pattern Recognition Using CTCF With HOCS

To determine whether the usage of HOCS was going to be feasible we began by assuming we know the correct real/complex signal combination and knowing that only two signals are present. We then made the situation more realistic by not knowing the number of signals or the correct real/complex signal combination. The scalar value, when determining the fixed threshold, was increased so that there was only a 5% false alarm rate. The scalar value is
increased because a false alarm will automatically make the HOCS step incorrect. Figure 4.19 shows the performance when we know the real/complex signal combination vs not knowing the combination. Five Hundred simulations were completed for each SNR using random real/complex signals with various carrier frequencies, 4096 samples, and a 30% spectral overlap.

As Figure 4.19 shows the results are not promising and the requirement of a high SNR is needed for any sort of reliable classification performance. There are two factors which greatly decrease the performance of the HOCS classification process. The first is the inability of HOCS to distinguish 16-PSK from 16-QAM. This is proven by Figure 4.20 which shows the probability density function (PDF) for an 8-ASK signal combined with a 16-PSK or 16-QAM signal for fourth-order/second-conjugate cumulant values. Where the two PDF’s overlap will be the region where the HOCS has the inability to distinguish between the two groups of signals. The second factor which limits HOCS ability to determine
the modulation type is the false alarm rate. Because the HOCS requires the exact knowledge of the number of real/complex signals present in the received signal, a false alarm signal will completely eliminate the chance of a correct classification through HOCS.

Figure 4.20: PDF of 8-ASK and 16-QAM/16-PSK Cumulant Values
Conclusion

5.1 Restatement of Research Goal

The goal of this research was to develop a classification process able to classify the modulation technique of multiple overlapping signals with little to no \textit{a priori} information known about the carrier frequency, symbol rate, or phase offset.

5.2 Conclusions

Three different methods of modulation classification of multiple overlapping signals based on the spectral correlation function and spectral coherence function were tested. Cyclostationary processing was used because of its insensitivity to noise and its ability to extract the inherent cyclic features in a communication signal to allow for classification.

The method using the temporal smoothed periodogram to estimate the SCF was used because of its ability to create the cyclic spectrum estimate faster than the frequency smoothed periodogram. The problem lies in the temporal smoothing method’s inability to obtain an accurate representation of the cyclic spectrum. This results in classification performances less than ideal and takes a step backward from previous works. The second step for this classification process was based off the SCF using Higher Order Cyclic Statistics and was tested and deemed not reliable at low SNR values.
The process utilizing the SOF along with a pattern recognition technique with an adaptive threshold was proven to work on multiple overlapping signals with no \textit{a priori} information known. Some of the advantages to using a feature based pattern recognition technique are: less complex than HMM, SVM, or ANN, does not require to be trained by sample test data, and requires less computation time.

The final method utilizing HOCS with the frequency smoothed SOF was proven to require high SNR to work and does not appear to be a promising technology.

5.3 Recommendations for Further Research

There are several main topics of focus for further research.

The first topic being the increased spectral resolution of the SCF/SOF. The current frequency smoothing method provides the most accurate results, but once the percentage of overlap exceeds a certain value the method is unable to distinguish the cyclic features from two signals.

The second topic for further research would be to expand the number of signals and channel types. The addition of wideband signals, such as OFDM or CDMA, and analog signals would allow for a more encompassing set of signal types. Receiving a signal in an AWGN channel is an ideal case in wireless communications. Passing the signals through different channels to determine the effects on the SCF/SOF should also be included in this area.

The last topic for further research is to further classify the received signals into specific modulation techniques. The research presented in this paper provided a solid foundation for determining not only the number of signals present but also some signal parameters.
Bibliography


