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The Wright Stuff: The Mathematics of the Wright Brothers

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Introduction

On a cold and blustery day in December of 1903 near the town of Kitty Hawk, North Carolina, two brothers from Dayton, Ohio, Wilbur and Orville Wright, haul an ungainly looking craft from a shed and get ready to attempt a feat that has never been accomplished before - to fly a heavier-than-air powered airplane with a person on board.

The weather conditions that day were terrible. The temperature was 34° F and the winds were steadily blowing between 20 and 30 miles an hour [McFarland, 395].[1] The wind chill (according to modern tables) was around 8° F [McCullough, 113]. [2] The Wrights had originally come to Kitty Hawk because of the strong and steady winds, which were generally between 10 and 20 miles an hour [Wescott and Degen, 24], [3] but the winds on this day were stronger than usual. One of their friends, William Tate, didn't even come to watch their attempted flight saying "… no one but a crazy man would try to fly in such a wind" [Kelly, 61]. [4]

And yet they tried. Why? It was late in the season and the weather was predicted to get worse. They had also planned to be home for Christmas and time was running out. However, the main reason they tried was the confidence the brothers had that flight was possible under these conditions. I believe that the main reason for their confidence was mathematics.

I would like to discuss some of the mathematical experiments and calculations performed by the Wright brothers before this famous date.

The Forces of Flight

There are 4 basic forces of flight that act on an airfoil in flight:

- **weight:** a force pulling the aircraft toward the ground
- **lift:** a force acting perpendicular to the flight path, counterbalancing weight
- **drag:** a force acting opposite to the direction of the flight path, caused by air friction and pressure distribution
- **thrust:** a force acting in the general direction of the flight path

In simple terms, flight will be possible whenever there is enough lift to overcome the weight and enough thrust to overcome the drag. When the Wright brothers first became interested in the possibility of flight, they studied the previous experiments and experimenters. In particular, Otto Lilienthal of Germany had experimented with gliders and had developed tables for the lift achieved for many different airfoils. He used the well-established formulas for lift and drag [McFarland, 575-576] [5]

$\mathbf{L = kSV^2c_L}$ **and** $\mathbf{D = kSV^2c_D}$

where the symbols represent:

- **L**= the lift generated (in pounds)
- **D**= the drag generated (in pounds)
- **k**= the Smeaton coefficient of air pressure
- **S**= the surface area of the airfoil (in square feet)
- **V**= the velocity relative to the wind (in miles per hour)
- $\mathbf{c_L}$= the coefficient of lift
- $\mathbf{c_D}$= the coefficient of drag

These are the two equations that the Wrights used, and they are the formulas that caused them the most trouble. While surface area and velocity were easy to calculate, the other components of these formulas were the major stumbling blocks to flight. Let us follow Wilbur and Orville's attempts to make sense of these formulas.

The Coefficient of Lift

The coefficient of lift, c_L, gave a ratio of the pressure on an airfoil to the pressure on a flat plane perpendicular to the wind [McFarland, 553]. [6] Its value was different for different airfoil shapes and different angles of attack. The angle of attack is the angle between the relative wind and the chord of the wing [Welch, 545]. [7] Lilienthal's tables calculated c_L for many different angles of attack.

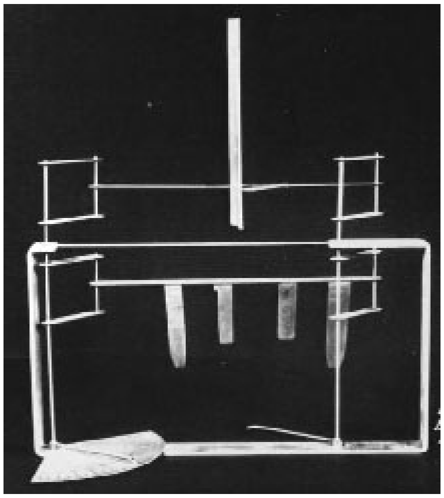
The early Wright gliders of 1900 and 1901 used wings similar to Lilienthal's and used his calculations for c_L. The results were very disappointing. The gliders did not produce nearly the lift or drag predicted by the equations [McFarland, 130-133]. [8] The Wrights returned to Dayton totally discouraged. Wilbur offered the opinion that if flight were possible, it wouldn't be in a thousand years, certainly "… not within my lifetime" [Crouch, 213]. [9]

Perhaps the most historic "site" in the story of the Wright brothers is not to be found in the sands of Kitty Hawk or on the fields of Huffman Prairie, but, rather, in the front parlor of their home on 7 Hawthorn Street in Dayton, Ohio, where they thought about the problems of flight and came up with many ideas to test their conjectures. The house is now preserved in Greenfield Village in Michigan.

The Wright House in Greenfield Village

In the winter of 1901-02, the Wrights began to rethink all that they had been through.

Could Lilienthal have been wrong? Using a horizontal wheel mounted on a bicycle, they rode around the hills of Dayton taking primitive pressure measurements. On this horizontal wheel they attached two airfoils at right angles to each other: one curved wing with a surface area of one square foot set at an angle of attack of 5°, and the other a flat plate with a surface area of two-thirds of a square foot. If Lilienthal was correct, the pressure of the two airfoils would balance each other and the wheel wouldn't move. These crude experiments indicated that there were serious errors in Lilienthal's tables. The pressure on the flat plate was considerably greater than the lift produced by the curved wing. However, the Wrights needed a more precise method for measuring c_L. So they designed a wind tunnel [McFarland, 552-553]. [10]



The lift balance.
(Franklin Institute)

In their first experiment, they tested more than 50 different airfoils at 14 different angles of attack. A "lift balance" was created to measure the lift of their airfoil in terms of the pressure on a square plate of equivalent area normal (perpendicular) to a wind of equal velocity. After an adjustment to eliminate the effects of drag, the wind was turned on. In the Wright's own words: "The lift is thus balanced against the normal pressure on the resistance surfaces … Therefore, the lift … at the given angle of incidence (angle of attack) is to the pressure on a square plane of equal area at 90° as the sine of the angle indicated by the pointer (on the lift balance) is to one". The sine of the angle in question is c_L [McFarland, 207]. [11]

For airfoil #12 (similar to the final design of the Wright Flyer), at an angle of attack of 5° (similar to the angle of attack attempted for the first flight), the value of c_L was 0.515.

The Coefficient of Drag

The Wrights then conducted a second experiment to determine c_D. They used the word drift to refer to what we would now call drag. The value of c_D was not measured directly. Using the wind tunnel and a "drift balance", the Wrights measured an angle they called the tangential, which (in their words) "gives the inclination of the chord [of the wing] above or below the horizon" [McFarland, 197]. [12] In modern terms, this tangential would be equivalent to the pitch angle [Welch, 545]. [13] This tangential angle (which is often negative) is then added to the angle of attack to find what the Wrights called the gliding angle, which is the angle between the horizontal and the relative wind [McFarland, 576]. [14] In modern terms, this gliding angle would be equivalent to the flight path angle [Welch, 545]. [15] For example, using airfoil #12 and a 5° angle of attack, the tangential was measured to be 1°, so the gliding angle was 6°.

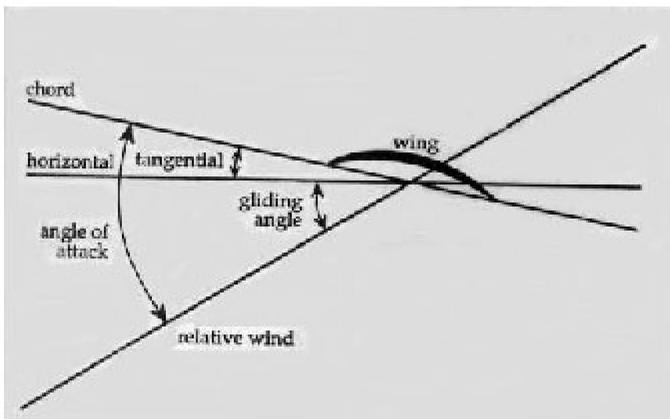
Since the forces of lift and drag are perpendicular to each other, the tangent of the gliding angle represents the ratio of drag to lift. Using this information, c_D can be calculated.

$$\tan G = \frac{D}{L} = \frac{c_d}{c_l}, \text{ so } c_d = c_l \tan G$$

,

For airfoil #12 at an angle of attack of 5°, c_D is computed as follows:

$$c_d = c_l \tan G = 0.515) \tan 6° = (0.515)\ (0.1051) = 0.0541$$



The Wright angles.

Although these three angles (angle of attack, tangential and gliding angle) all have modern-day equivalents, the signs of these angles as used by the Wright brothers may differ from the signs of the angles used today. To the Wrights, the gliding angle was an angle of descent and was always positive. The angle of attack was also always positive, but the tangential was considered negative if it were above the horizon. The relationship between these angles is shown in the figure, which is based on an original sketch from the Wright brothers' notebook [McFarland, 197, 199]. [16]

The original wind tunnel, lift and drift balances and many other artifacts from the Wright's experiments are currently displayed at the Franklin Institute in Philadelphia, the first scientific organization to recognize the Wright's accomplishments.

When the Wright brothers compared their results with those of Lilienthal, they found some disagreement, but not as much as they expected. As Wilbur states in his diary for October 16, 1901: "It would appear that Lilienthal is very much nearer the truth then we have heretofore been disposed to think." [Wolko, 21]. [17] The formulas were still not producing the lift and drag that were actually being produced. The only other possible source of error in these equations was the Smeaton coefficient of air pressure.

The Smeaton Coefficient

John Smeaton was an English civil engineer, sometimes called the father of engineering, whose list of accomplishments is quite impressive. These achievements include navigational improvements to rivers, strengthening piers, designing a pumping engine and experimental work on the design of steam engines [Crouch, 175-176]. [18]

He became interested in improving the efficiency of windmill blades and published a paper in 1759 that won him the prestigious Royal Society's Gold Medal. In an appendix to this paper he included a value, later called the Smeaton coefficient, which when multiplied by the square of the velocity (in miles per hour), gives the pressure in pounds per square foot on any flat surface presented at right angles to the wind. Its value was determined by Smeaton to be a constant with a value of approximately 0.00492, rounded off to 0.005 for ease of calculation [Combs, 374]. [19]

There is an inconsistency in units used in this coefficient. Velocity is expressed in miles per hour, while pressure is given in pounds per square foot. These were the units used by experimenters in Smeaton's time and at the time of the Wright brothers. The Smeaton coefficient takes these incompatible units into account [McFarland, 612]. [20]

Perhaps it was because Smeaton had such a great reputation that this coefficient became accepted as fact and used in many textbooks for over 100 years, even though it was incorrect.

The difficulty in this calculation is mentioned by the Wright brothers in an article that appeared in Century Magazine in September, 1908: "The practical difficulties of obtaining an exact measurement of this force have been great … When this simplest of measurements presents so great difficulties, what shall be said of the troubles encountered by those who attempt to find the pressure at each angle as the plane is inclined more and more edgewise to the wind?" [McFarland, 572] [21]

Otto Lilienthal had actually used an approximate metric equivalent of Smeaton's coefficient of 0.13 (although 0.12 would have been closer to Smeaton's value). Lilienthal referred to the Smeaton coefficient as being "generally known" [Lilienthal, 12]. [22] On my trip to Wright State University, I studied Orville Wright's copy of Lilienthal's book and discovered that Orville had underlined the words "generally known" and put a question mark in the column next to them.

On September 18, 1901, after numerous tests of lift using kites, Wilbur writes "that the well-known Smeaton coefficient of $0.005V^2$ for the wind pressure at 90 degrees is probably too high by at least 20%" [McFarland, 112]. [23]

They became convinced that the inaccuracy of the Smeaton coefficient was the main cause for the discrepancies between theory and practice that they were getting. If they had known at the beginning of their experiments that this coefficient was so much in error, Tom Crouch writes, "they would never have begun" [Crouch, 214]. [24]

Octave Chanute became a close friend and correspondent with the Wrights. In 1894 he wrote a book, Progress in Flying Machines, in which he describes Samuel Langley's experiments in flight. Chanute states that Langley's experiments for determining lift and drag yield values somewhat lower than would be calculated "… when taken in connection with Smeaton's table of wind pressures." [Chanute, 126]. [25] Perhaps it was these discrepancies that led Chanute to perform his own calculations for the Smeaton coefficient. He came up with nearly fifty different values ranging from 0.0027 to 0.0054 [McFarland, 612]. [26] He suggested to Wilbur that the value of Smeaton's coefficient might be different in natural wind (0.005) than in still air (0.003). Wilbur quickly replied that "It would seem that still air is really as effective as natural wind in actual practice, and I can see no theoretical reason why it should not be." [McFarland, 132]. [27]

The Wright's value for the Smeaton coefficient was 0.0033. The value currently accepted for this coefficient is 0.00327. This is a "truly astonishing agreement" according to Marvin McFarland [McFarland, 575]. [28] How did Wilbur and Orville achieve such an accurate value?

Although no record of the exact calculations exists, the Wrights averaged a series of results from a glider in 1902. Combined with their wind tunnel tests to determine c_L and c_D, by directly measuring the drag on the glider using a spring scale and restraining ropes, they were then able to directly calculate the Smeaton coefficient k from the equation for drag [McFarland, 574]. [29]

$$D = kSV^2c_d, \text{ so solving for k gives}$$

$$k = \frac{D}{SV<^2c_d}$$

According to Harry Combs, this calculation was "the key to the whole show" [Combs, 374]. [30]

The Wright's Propellers and Engine

Now that lift and drag could be measured with confidence, the Wrights turned their attention to an engine to provide the thrust needed for flight and to the propellers that would be needed to transform the power of the engine into forward thrust. Their engine produced 12 horsepower and weighed 180 pounds. Their chief competitor, Samuel Langley of the Smithsonian Institution, had an engine weighing only 125 pounds and producing 52 horsepower. If the Wrights had possessed such an engine, they wouldn't have had to make all of these precise calculations. Perhaps it was better for science that they didn't [McFarland, 380-381]. [31]

Finding no reliable theory on propellers for air travel, the Wright's devised their own formulas and, using their wind tunnel results, were able to calculate the thrust of their propellers to within 1% of the actual thrust obtained [Combs, 182]. [32]

The Wrights used numerous formulas in their attempt to calculate the thrust of their engines. To get an idea of the complexity of the problem, we will consider one of their formulas [McFarland, 612]: [33]

$$T = S^2 \cos(B + t)kc_r,$$

where

- **T** = thrust (in pounds)
- **B** = blade angle (the angle between the chord and the plane of rotation of the propellers)
- **t** = the tangential
- **k** = the Smeaton coefficient using velocity in ft/sec (0.001534)
- c_r = the resultant pressure (c_l / cos(gliding angle))
- **S** = the speed at any point of blade (gross speed x csc (B + t)

- **gross speed** = the forward velocity (v) + throwdown (x)
- **throwdown** = the loss of efficiency of the engine, given by

$$\frac{-v + (v^2 + 1704P)^{1/2}}{2}$$

- **P** = the pressure per square foot of propeller disc area.

In any event, the Wrights directly measured the thrust of their engine in November 1903, using a spring balance, and recorded a total thrust of 132 pounds [McFarland, 389]. [34]

Katharine Wright's Mathematical Role

After my presentation on the Wright brothers at the ICIAM conference in 1991, I received a phone call and subsequently a letter from Dr. Rolfe A. Leary of St. Paul, Minnesota. Dr. Leary had been a good friend of Edward Haskell, a nephew of Katharine Wright and Harry Haskell, editor of the *Kansas City Star*. According to Dr. Leary, Edward Haskell claimed that it was "Aunt Katharine" who did all of the mathematics for her brothers. Unfortunately, Mr. Haskell had recently passed away, so I was unable to contact him. However, Mr. Leary wrote to Henry Haskell, another nephew of Katharine, and received a response indicating that he agreed that Aunt Katharine had probably done the mathematics for her brothers but was too modest to talk about it.

I have read extensively on the Wright brothers, including all of the references at the end of this paper, and none of these documents mention Katharine Wright as the mathematician.

At Wright State University, I studied the high school report cards of Wilbur, Orville and Katharine, being particularly interested in the mathematics courses. Wilbur, the oldest of the three, took algebra, geometry and trigonometry and received high scores in all three. Orville also took algebra, geometry and trigonometry, receiving somewhat lower scores, but nonetheless receiving proficiency in all three. Katharine apparently took algebra and geometry in high school, and she scored higher in algebra than her brothers.

Katharine was the only one of the three to go to college. She went to the prestigious Oberlin College in Ohio and received a teaching degree in 1898. I found her college trigonometry text, indicating that she took this course and was, therefore, as well educated in mathematics as were her brothers.

Many of the letters of correspondence between members of the Wright family have been preserved, and they offer an insight into this discussion. There seems to be no mention of Katharine taking a scientific interest in the experiments of her brothers. She certainly was a devoted sister who encouraged and supported her brothers, but I find no evidence in the letters that she was involved in their calculations. In a letter to her father on September 3, 1901, she writes:

"We don't hear anything but flying machine and engine from morning till night. I'll be glad when school begins so I can escape" [McFarland, 92]. [35] At this time Katharine was a teacher at Steele High School in Dayton, where she taught classical subjects [McFarland, 92]. [36] Her main interest in teaching seemed to be Latin and other languages, not mathematics.

In the book *Three Together* by Lois Mills, there is an account of Wilbur and Orville trying to explain the principles of flight to Katharine with little success [Mills, 126-127]. [37]

When the Wright brothers and their sister visited Europe in 1909, author Tom Crouch in *The Bishop's Boys* states that "Stories about Katharine abounded. She was said to have financed the work on the airplane, solved abstruse mathematical problems for her brothers, and to be familiar with every inch of the machine. It mattered little that none of it was true. They admired her for her wit and honesty, not for her supposed contributions to the invention of the airplane" [Crouch, 387]. [38]

In a letter to Alexander Klemin on April 11, 1924, Orville responds to this "persistent legend", which he said made a "pretty story" [McFarland, 27]: [39] "… Katharine was always a loyal sister who had great confidence in her brothers … but she never contributed anything either in money or mathematics" [McFarland, 27]. [40]

According to Wilkinson Wright, a grandson of Loren Wright, an older brother of Wilbur and Orville, and an expert on the Wright brothers, "Katharine's role was that of a very devoted younger sister who took great interest in their work and encouraged them in any way she could" [McCullough 1994, appendix]. [41] He also stated that the Wrights inherited their mathematical ability from their mother, Susan Katherine Koerner. She attended college at Huntington, Indiana, which was "a most unusual attainment for a lady in the mid 19th century".

I must conclude that although Katharine Wright was an intelligent and interesting person, she probably did not substantially help her brothers mathematically, although she probably could have contributed to their discussions had she wanted to do so.

December 17, 1903

Let us return to the sands of Kitty Hawk and see if the Wright Flyer can get off the ground. The surface area of the Wright Flyer is 512 ft^2. If we assume a wind velocity of 25 miles per hour and a ground speed of 7 miles per hour (the ground speed actually achieved on December 17), the velocity relative to the wind becomes 32 miles per hour. The calculations follow:

lift: $(0.0033)\ (512)\ (32)^2\ (0.515) = 891$ pounds
weight: Kitty Hawk Flyer: 605 pounds Orville (in suit and tie): 140 pounds
Total: 745 pounds

drag: $(0.0033)\ (512)\ (32)^2\ (0.515) = 94$ pounds
thrust: 132 pounds

Since lift is greater than weight and thrust is greater than drag, flight is possible! Armed with information similar to this, Orville and Wilbur attempted flight. Their success was no great surprise to themselves. As Wilbur later wrote in a letter to Chanute: "One of the most gratifying features of the trials was the fact that all of our calculations were shown to have worked out with absolute exactness …" [Wescott and Degen, 119]. [42]

Both lift and drag increase with velocity. If we assume a wind velocity of 22 mph for the flight, the formula gives a lift of only 732 pounds, not enough to overcome weight. Similarly, if we assume a wind velocity of 32 mph for the flight, the drag generated is 139 pounds, which is more than the thrust.

The Wright Flyer was only marginally flyable, and the fact that the Wright brothers figured out these margins is a tribute to their genius. Perhaps all they proved in 1903 was that flight was possible on a cold and windy day in North Carolina, but that was enough! Harry Combs writes: "We rob ourselves of a part of our heritage when we fail to elevate Wilbur and Orville Wright to the high reaches of scientific genius" [Combs, 185]. [43] I agree with these sentiments.

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