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Functional Principal Components Analysis and the Capacity Coefficient

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Functional Principal Component Analysis and the Capacity Coefficient

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Purpose

- The capacity coefficient[3, 4] is a well established measure of the efficiency of processing combined sources of information.
- The capacity coefficient is a *function*.
- Typical clinical analyses, such as structural equation modeling, use scalar values or vectors with limited length as input.
- We were interested in finding an efficient way of reducing the information contained in the capacity coefficient to a scalar.

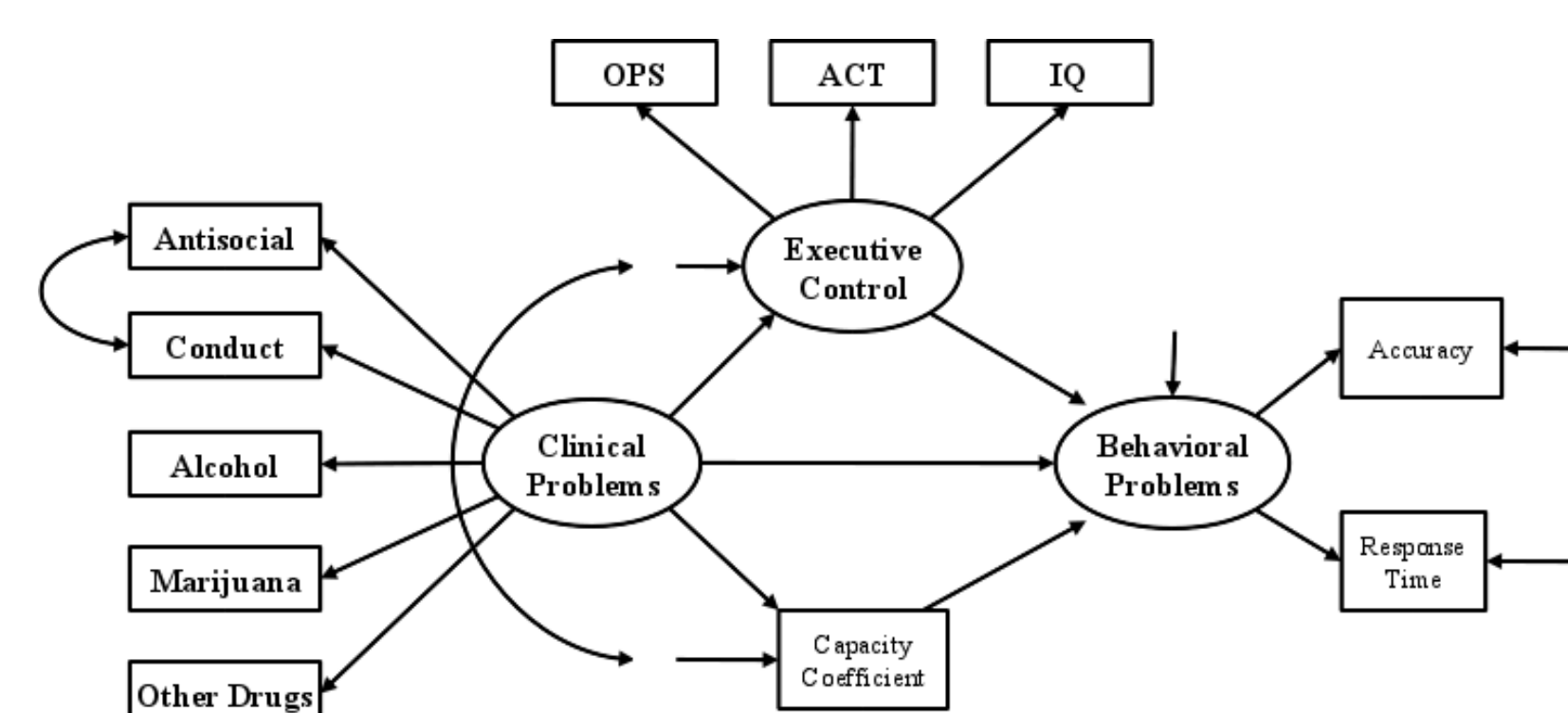
The Capacity Coefficient

The capacity coefficient ($C(t)$) is a measure of processing efficiency that looks at how reaction times change under a workload manipulation. The measure compares processing speed for the "whole" compared to the sum of its parts. In this case the parts are memory search for color (VS) and for letter (PL).

$$H(t) = \ln S(t) = \ln(1 - F(t))$$

$$C(t) = \frac{H_{VS+PL}(t)}{H_{VS}(t) + H_{PL}(t)}$$

The Data

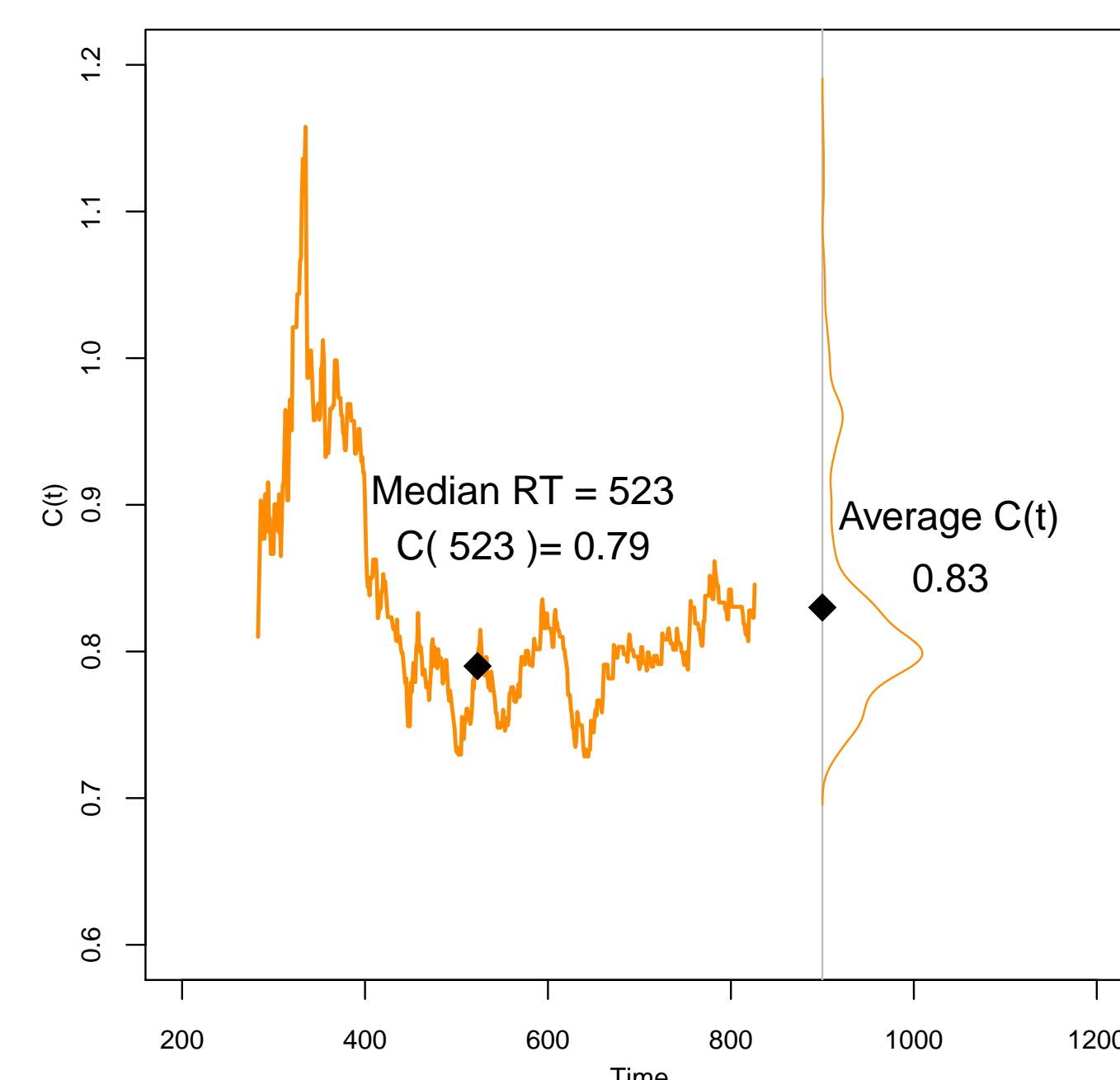


- Executive processes mediate the association between lifetime clinical problems and day to day behavioral problems.
- We are interested in the capacity coefficient as an alternative to standard working memory capacity measures (ACT; OPS)

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Alternative Approaches



fPCA

fPCA is straight forward extension of PCA that operates on functional data (e.g. $C(t)$) and returns component functions, $\xi(t)$, along with scalar score values, f .

Thus, if N is the number of participants, the model can be written as,

$$f_i = \int \xi(t)C_i(t) dt, i = 1, \dots, N.$$

Finding the principal components and associated scores proceeds in the same manner as PCA.

For each $\xi_i(t)$,

$$\text{maximize } N^{-1} \sum_i \left(\int \xi_j(t)C_i(t) dt \right)^2$$

such that

$$\int \xi_j^2(t) dt = 1 \text{ and } \int \xi_k(t)\xi_j(t) dt = 0, k < j.$$

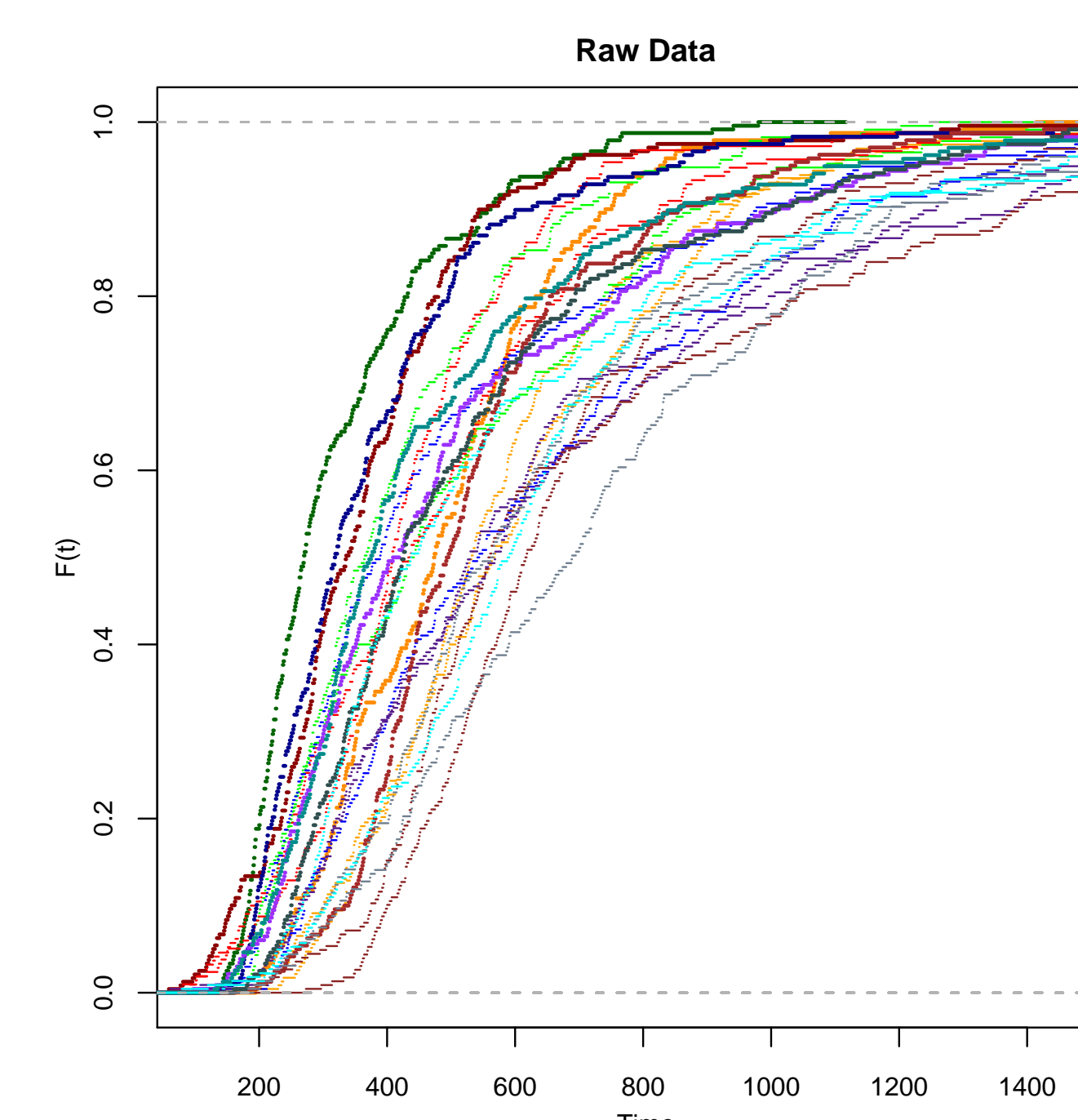
Varimax

- The components form a basis of the space.
- This basis can be rotated to concentrate variance and improve interpretability.

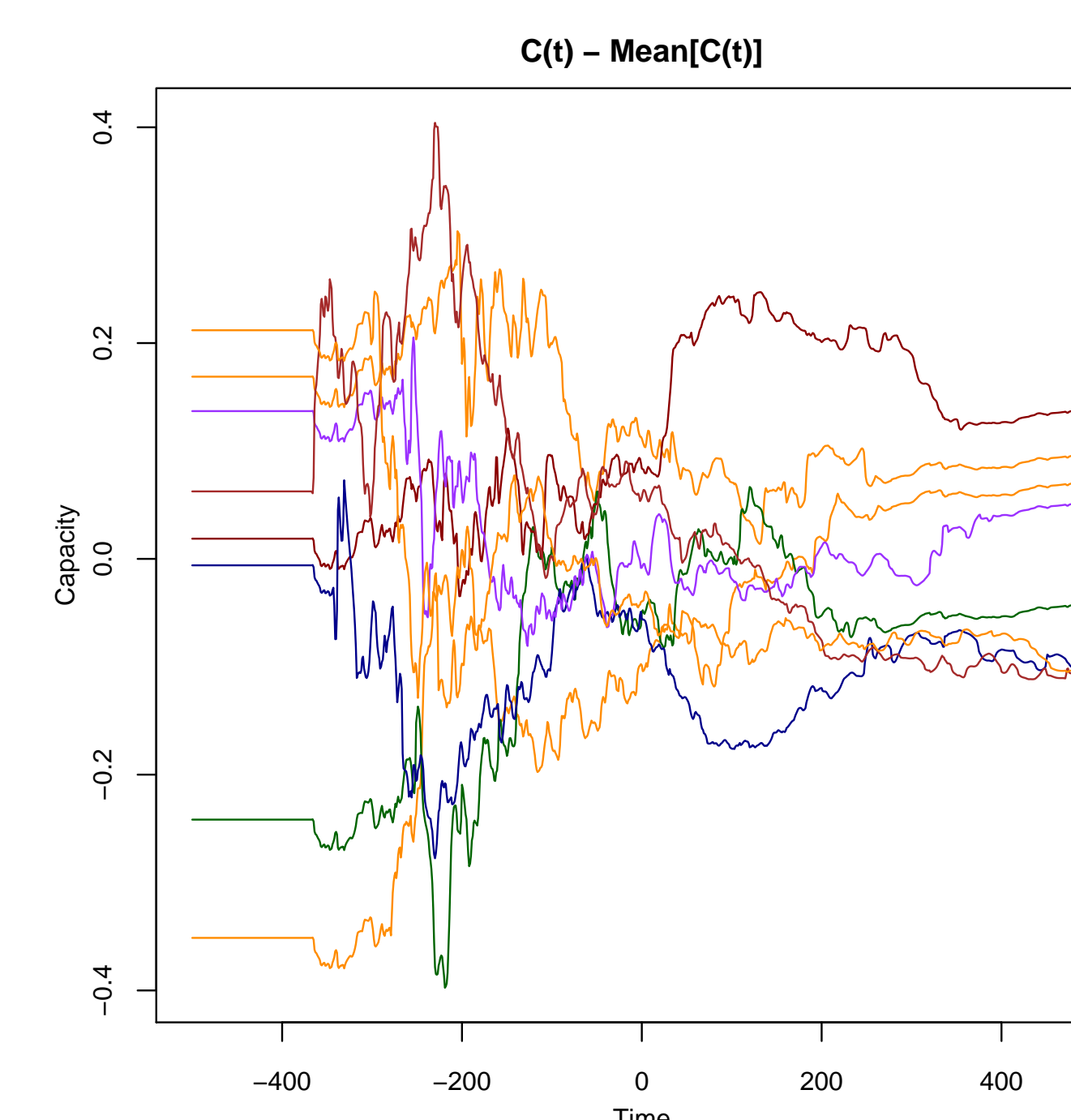
Let B be a $K \times N$ matrix representing the first K principal component functions ξ_1, \dots, ξ_k and T any orthogonal matrix of order K . Find $A = TB$ in order to maximize the variation in the values $a_{m,j}^2$.

Pre-processing

1. Estimate empirical CDF's.

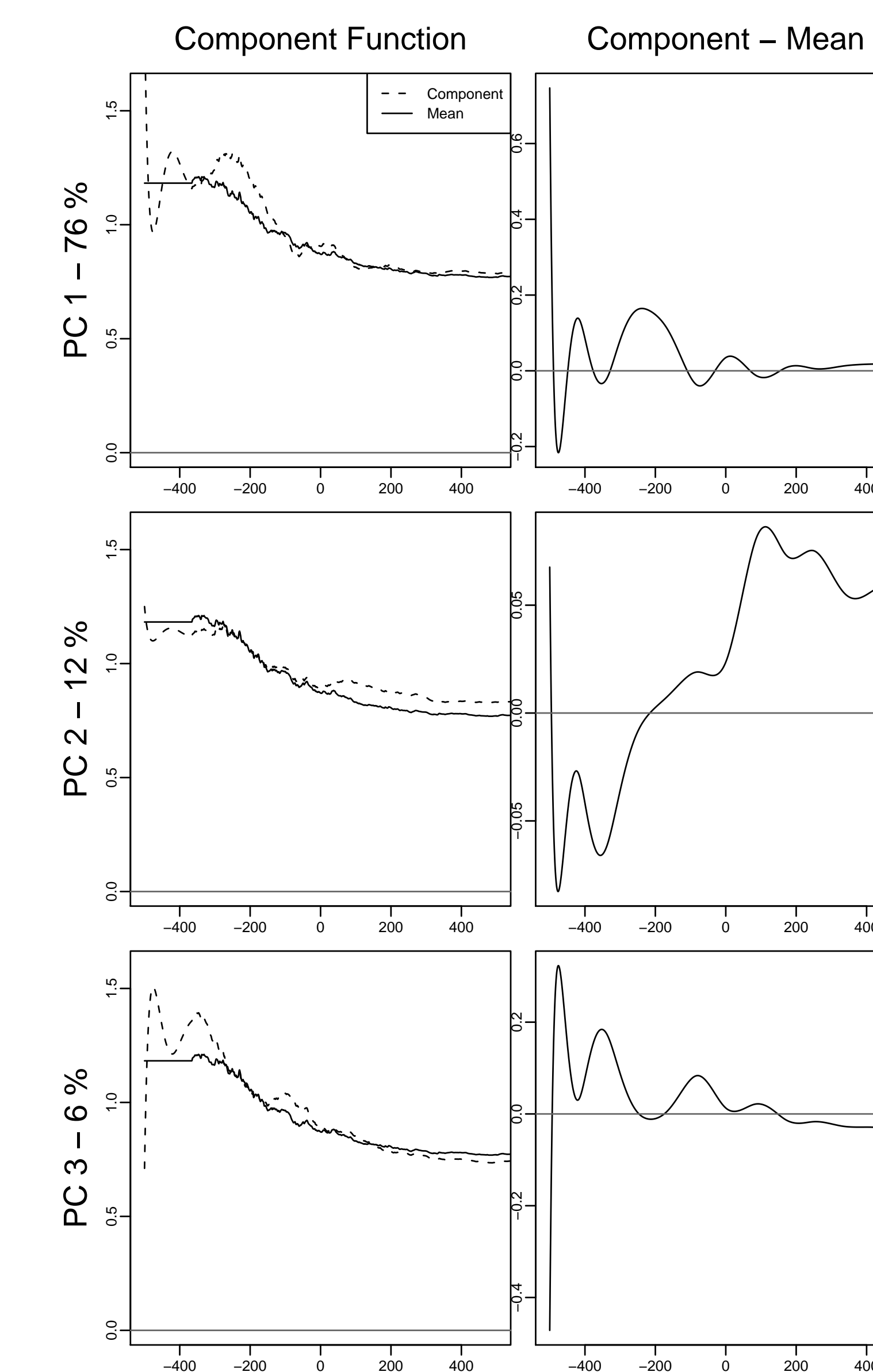


2. Register data. Subtract each participant's median RT across all single and double target trials to align CDF's.
3. Calculate smoothed $H(t)$. Transform the CDF's into integrated hazard functions and fit with a monotone Hermite spline (or similar) to smooth.
4. Compute $C(t)$. Take the ratio of the smooth $H(t)$ to obtain smoothed $C(t)$ estimates.
5. Subtract means. Calculate the mean capacity function across participants and then subtract to better see individual variance.



6. Determine weighting function. To place more emphasis on regions with the most data, compute a weighting function across all participants based on registered RT density.

Principal Components



Principal component functions, $\xi_j(t)$, $j = 1, 2, 3$, after varimax rotation.

Results

	PC 1	PC2	$C(t_{\text{median}})$	Mean $C(t)$
ACT	0.85	0.22	0.65	0.55
OPS	0.53	-0.09	0.45	0.19

Correlations between the standard working memory tasks and different reductions of the capacity coefficient.

- The first PC has higher correlation with the ACT score and the OPS than both $C(t)$ at the median RT and $C(t)$ averaged across time.

References

- [1] Ramsay, J. O., Hooker, G. & Graves, S. (2009). *Functional Data Analysis with R and MATLAB*. New York: Springer.
- [2] Ramsay, J. O. & Silverman, B. W. (2005). *Functional data analysis* (2nd ed.). New York: Springer.
- [3] Townsend, J. T. & Nozawa, G. (1995). "Spatio-temporal properties of elementary perception: An investigation of parallel, serial and coactive theories." *Journal of Mathematical Psychology*, 39, 321-360.
- [4] Townsend, J. T. & Wenger, M. (2004). "A theory of interactive parallel processing: New capacity measures and predictions for a response time inequality series." *Psychological Review*, 111, 1003-1035.