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Bayesian Analyses of the Survivor Interaction Contrast

Joseph W. Houpt, Andrew Heathcote, Ami Eidels and James T. Townsend







Psychological and Brain Sciences
Cognitive Science Program

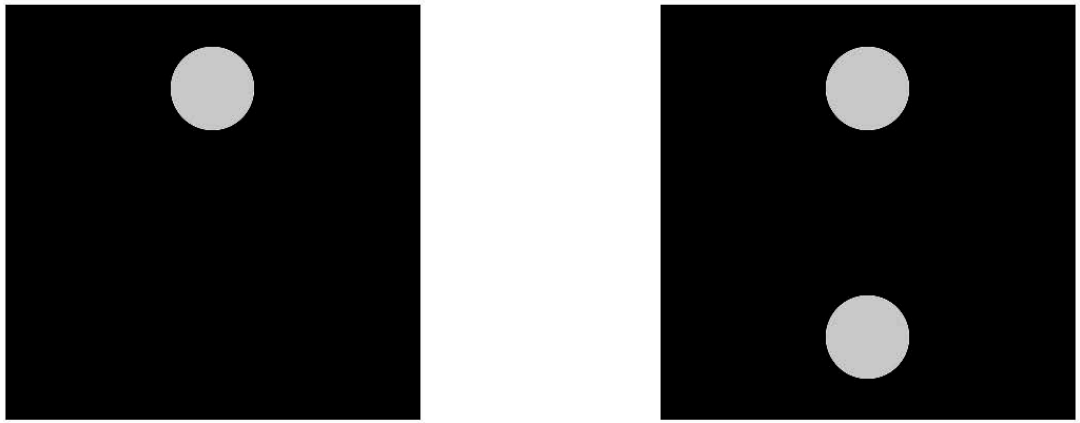
Society for Mathematical Psychology Annual Meeting
Columbus, Ohio
July 22, 2012

Outline

1. Introduction
2. Parametric Test
   - Model
   - Simulation
3. Nonparametric Test
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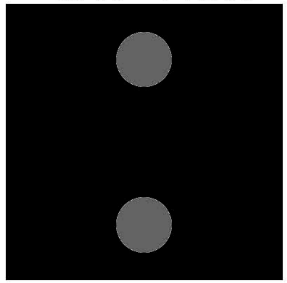


- How do different sources of information combine in mental processing?
  - Are both sources used concurrently, or do we use one at a time?
  - How many sources are enough to respond?

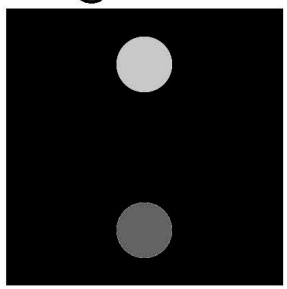
Salience

- To test architecture and stopping rule, without conflating them with workload capacity, factorially speed up and slow down the processing of each source of information.

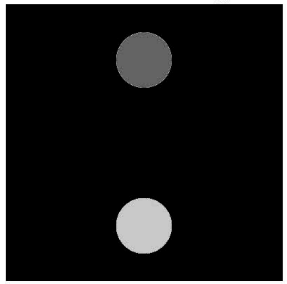
Low–Low



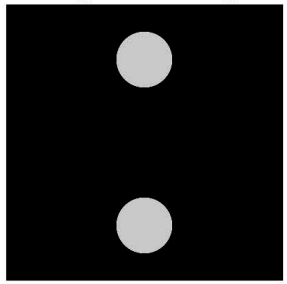
High–Low



Low–High



High–High



Survivor Interaction Contrast

- Indicates architecture and stopping rule.

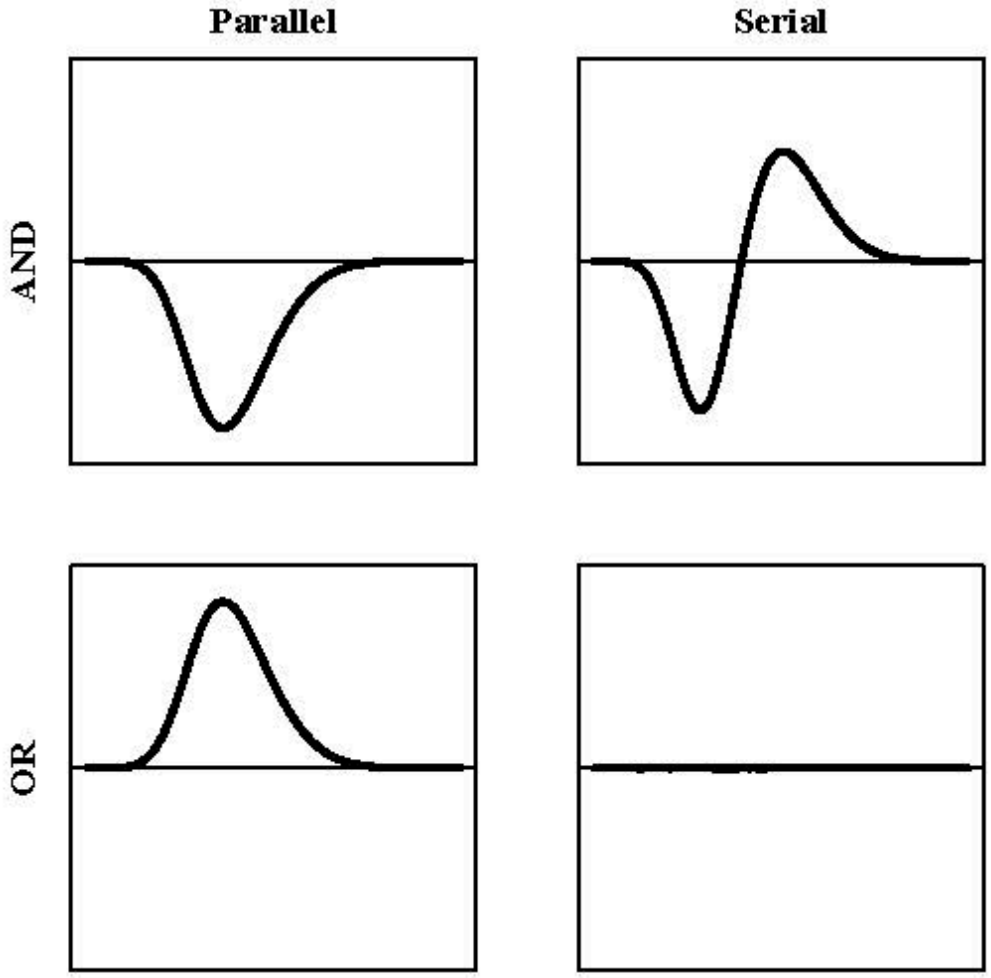
Survivor Interaction Contrast

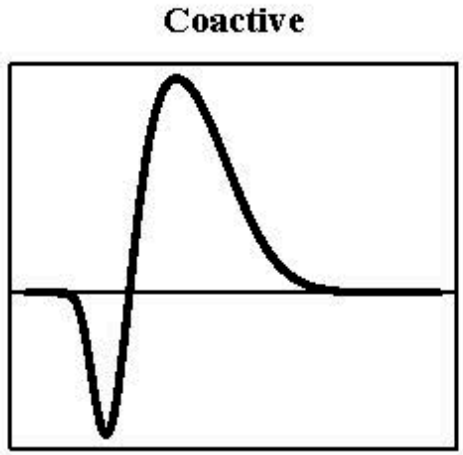
- Indicates architecture and stopping rule.
- The SIC is interaction between the salience manipulations.
  - Instead of just using the mean time, we use the survivor function: $S(t) = \Pr\{T > t\} = 1 - F(t)$.

$$\mathrm{SIC}(t) = [\mathrm{S_{LL}}(t) - \mathrm{S_{LH}}(t)] - [\mathrm{S_{HL}}(t) - \mathrm{S_{HH}}(t)]$$

Here, the subscripts indicate the salience of each source of information.

Survivor Interaction Contrast

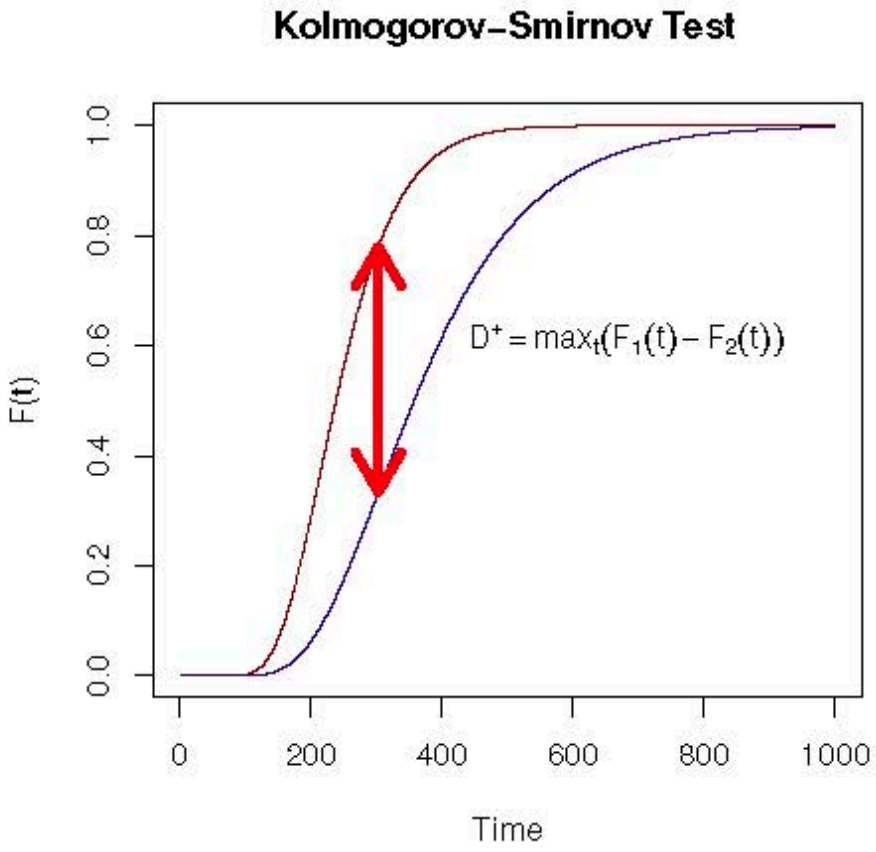


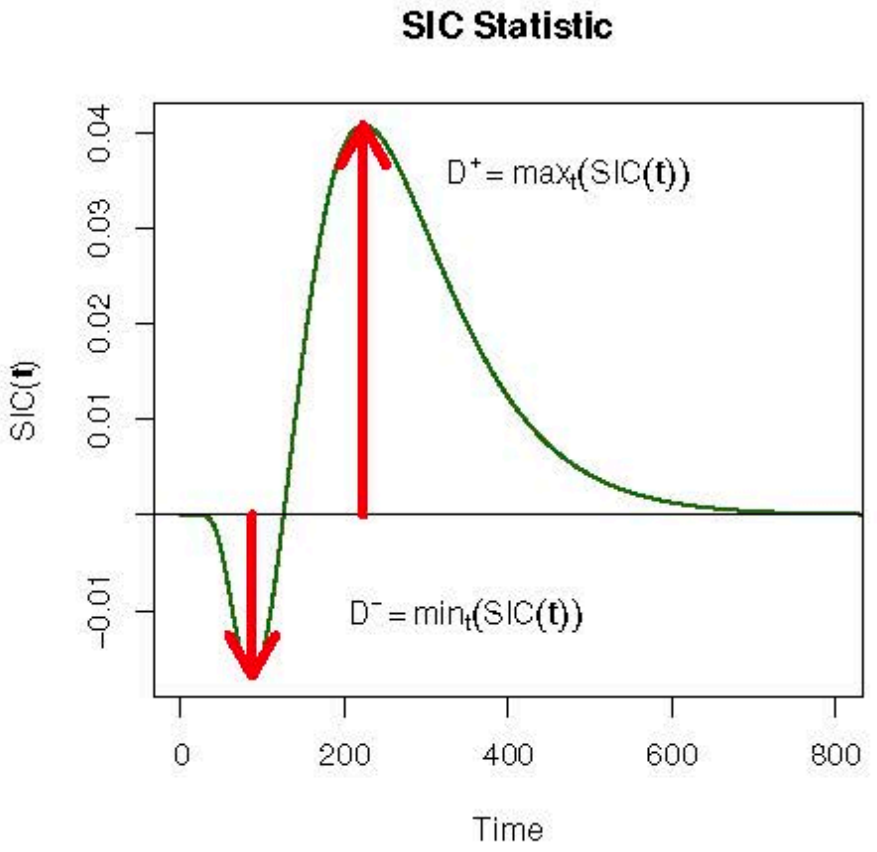


Townsend & Nozawa (1995)
Schweickert, Giorgini & Dzhafarov (2000)

Dzhafarov, Schweickert & Sung (2004)
Houpt & Townsend (2011)

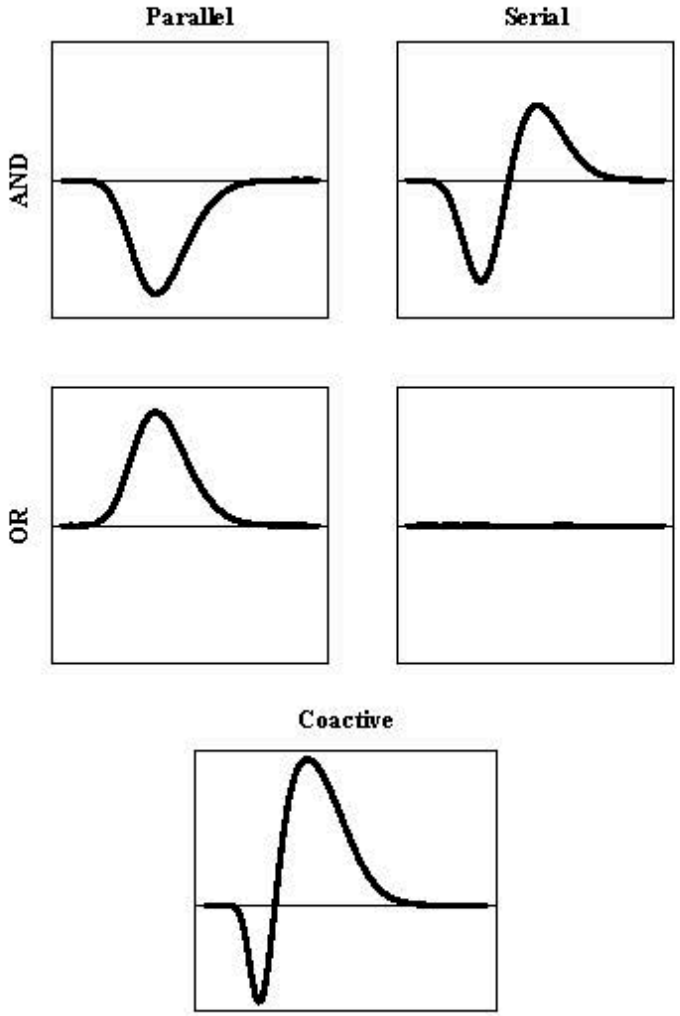
Null Hypothesis Test





$$\lim_{N\to\infty} \Pr\{\sqrt{N}D^+ \geq x\} = \Pr\{\sqrt{N}D^- \geq x\} = e^{-2x^2}$$

$$N_{KS} = \frac{1}{1/m + 1/n} \qquad N_{SIC} = \frac{1}{1/k + 1/l + 1/m + 1/n}$$



Model	$\hat{D}^+$	$\hat{D}^-$	Mean Interaction
Serial-OR	$\emptyset$	$\emptyset$	$\emptyset$
Serial-AND	✓	✓	$\emptyset$
Parallel-OR	✓	$\emptyset$	✓
Parallel-AND	$\emptyset$	✓	✓
Coactive	✓	✓	✓

✓: Reject null hypothesis
$\emptyset$: Fail to reject null hypothesis

Shortcomings

- Tests positive and negative deflections *not* SIC form.
  - Requires two separate tests.
- Only can gain evidence against a lack of positive or negative deflection.
- Only get a yes/no answer, not relative evidence.

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$f(t)$: Density (PDF) $F(t)$: Cumulative Distribution (CDF)

Parallel-OR $f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$

$f(t)$: Density (PDF) $F(t)$: Cumulative Distribution (CDF)

Parallel-OR $f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$

Parallel-AND $f_{12}(t) = f_1(t)F_2(t) + f_2(t)F_1(t)$

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Parallel-OR $f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$

Parallel-AND $f_{12}(t) = f_1(t)F_2(t) + f_2(t)F_1(t)$

Serial-OR $f_{12}(t) = pf_1(t) + (1 - p)f_2(t)$

$f(t)$: Density (PDF) $F(t)$: Cumulative Distribution (CDF)

Parallel-OR $f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$

Parallel-AND $f_{12}(t) = f_1(t)F_2(t) + f_2(t)F_1(t)$

Serial-OR $f_{12}(t) = pf_1(t) + (1 - p)f_2(t)$

Serial-AND $f_{12}(t) = f_1(t) * f_2(t)$

$$T_{i;H} \sim \mathcal{IG}\left(\frac{\alpha}{\nu_H}, \alpha^2\right) \qquad \eta \sim \text{Exponential}(100)$$

$$T_{i;L} \sim \mathcal{IG}\left(\frac{\alpha}{\nu_L}, \alpha^2\right) \qquad \nu_L \sim \Gamma(4, 0.1)$$

$$\alpha \sim \Gamma(4, 0.1) \qquad \nu_H = \nu_l + \eta$$

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$$f_i(t; \nu_i, \alpha) = \sqrt{\frac{\alpha^2}{2\pi t^3}} \exp\left[\frac{-(t\nu_i - \alpha)^2}{2t}\right]$$

$$F_i(t; \nu_i, \alpha) = \Phi\left[\sqrt{\frac{\alpha^2}{t}}\left(\frac{t\nu_i}{\alpha} - 1\right)\right] + \exp\left[2\alpha\nu_i\right]\Phi\left[-\sqrt{\frac{\alpha^2}{t}}\left(\frac{t\nu_i}{\alpha} + 1\right)\right]$$

Simulation Parameters

$$T_i = \inf\{t : X_i(t) \geq \alpha\}$$

$$T_i \sim \mathcal{IG}\left(\frac{\alpha}{\nu_i}, \frac{\alpha}{\sigma^2}\right)$$

$$\alpha = 30 \qquad \nu_H = 0.3$$

$$\sigma^2 = 1 \qquad \nu_L = 0.1$$

$$p = 0.5$$

Simulation Results

	Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial-OR	1.00	0	0	0	0
Serial-AND	0	0.99	0	0.01	0
Parallel-OR	0	0	0.98	0	0.02
Parallel-AND	0	0	0	1.00	0
Coactive	0	0	0	0	1.00

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- Approach: Model the response time *distributions*
  - (as opposed to the RT generating process).
- Assume each RT distribution is an independent sample from a Dirichlet process prior.
- Compare the Bayes factor of each SIC form in the posterior relative to encompassing prior.

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  - (as opposed to the RT generating process).
- Assume each RT distribution is an independent sample from a Dirichlet process prior.
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$$\alpha_l \sim \mathcal{DP}(\beta)$$
$$\mathrm{RT}_{l(i)} \sim \alpha_l.$$

Simulation

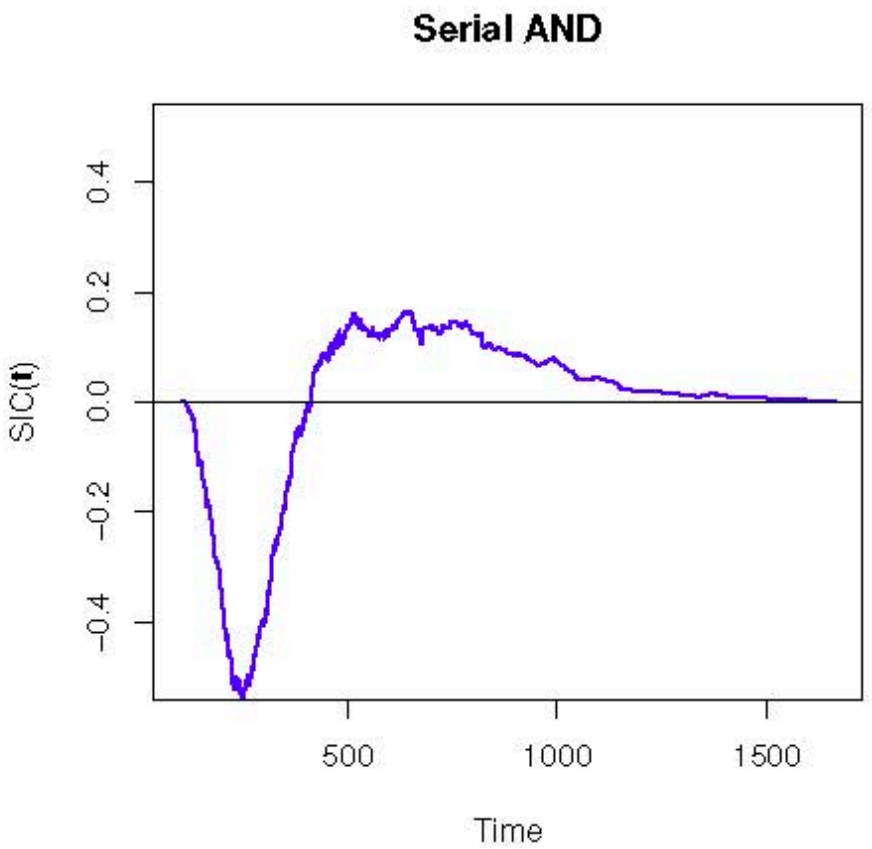
- Tested on same models as parametric-Bayesian test (but with 1000 rounds rather than 100).
  - Used region of probabilistic equivalence $\pm.1$ for SIC and $\pm.3$ for MIC.

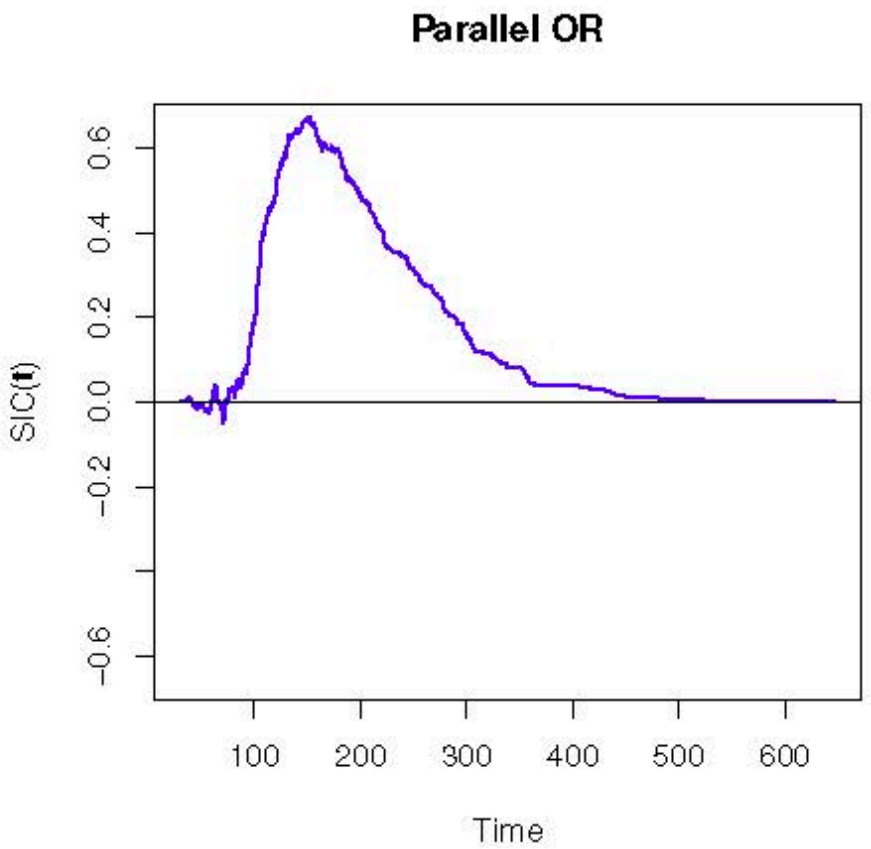
Simulation

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	Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial OR	1.00	0	0	0	0
Serial AND	0	0.79	0	0.21	0
Parallel OR	0	0	0.93	0	0.07
Parallel AND	0	0	0	1.00	0
Coactive	0	0	0	0	1.00

Example SICs





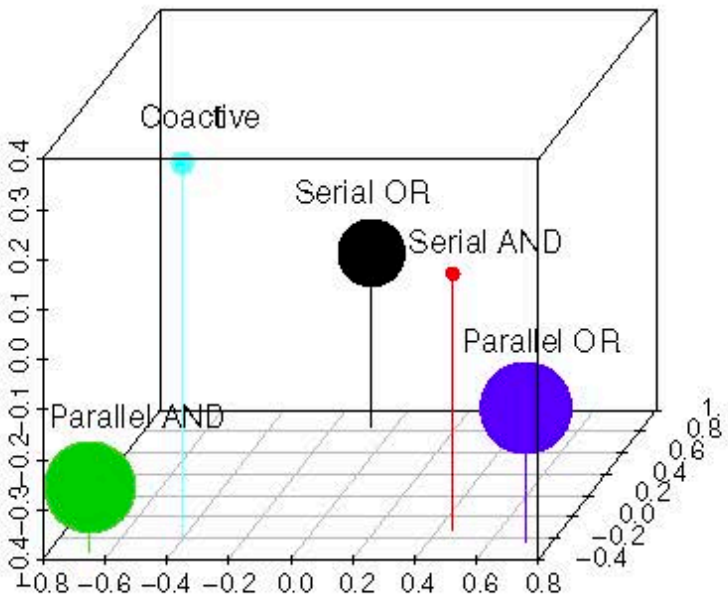
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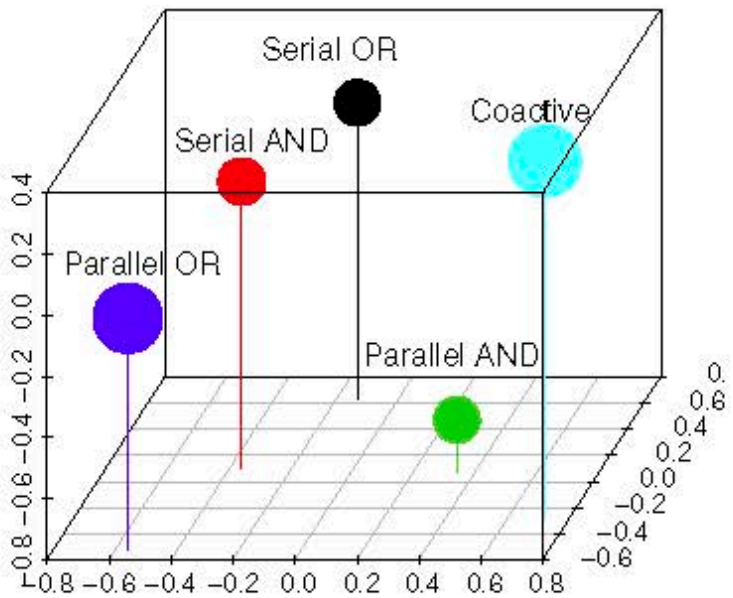
		Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial OR	KS	0.96	0	0	0.04	0
	DP	1.00	0	0	0	0
	BUGS	1.00	0	0	0	0
Serial AND	KS	0	0.80	0	0.15	0.05
	DP	0	0.79	0	0.21	0
	BUGS	0	0.99	0	0.01	0
Parallel OR	KS	0	0	1.00	0	0
	DP	0	0	0.93	0	0.07
	BUGS	0	0	0.98	0	0.02
Parallel AND	KS	0	0	0	1.00	0
	DP	0	0	0	1.00	0
	BUGS	0	0	0	1.00	0
Coactive	KS	0	0	0.02	0	0.98
	DP	0	0	0	0	1.00
	BUGS	0	0	0	0	1.00

		Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial OR	KS	0.93	0	0.05	0.02	0
	DP	0.79	0.18	0.02	0.01	0
Serial AND	KS	0	0.41	0	0.56	0.03
	DP	0	0.77	0	0.23	0
Parallel OR	KS	0	0	1.00	0	0
	DP	0	0	0.79	0	0.21
Parallel AND	KS	0	0	0	1.00	0
	DP	0	0.04	0	0.96	0
Coactive	KS	0	0	0.50	0	0.50
	DP	0	0	0	0	1.00

KS Test



DP Test



KS Test

Participant	OR Task		AND Task	
	$\sqrt{N}\hat{D}^{+}$	$\sqrt{N}\hat{D}^{-}$	$\sqrt{N}\hat{D}^{+}$	$\sqrt{N}\hat{D}^{-}$
1	4.86***	0.11	0	4.65***
2	1.11	0.04	0.04	2.73***
3	4.87***	0.14	0	3.61***
4	2.12***	0.77	0.07	3.30***
5	2.59***	0.22	0.21	4.24***
6	3.52***	0.04	0.16	2.79***
7	1.44*	0.11	0.04	2.04***
8	3.64***	0.24	0.11	2.10***
9	3.86***	0.07	0.07	4.98***

Parametric Bayes

	OR Task				
	Serial		Parallel		
	OR	AND	OR	AND	Coactive
1	7991	7985	**7869**	8012	7964
2	8489	8489	**8394**	8486	8488
3	7831	7792	**7623**	7920	7746
4	9480	9504	9530	**9464**	9505
5	9347	9351	**9274**	9352	9335
6	8870	8875	8885	**8830**	8867
7	9210	9216	**9192**	9201	9214
8	8624	8636	**8531**	8638	8620
9	8830	8850	**8828**	8837	8837

Parametric Bayes

	AND Task				
	Serial		Parallel		
	OR	AND	OR	AND	Coactive
1	7861	7863	7872	**7817**	7890
2	7832	7833	**7791**	7871	7836
3	7246	7249	**7242**	7297	7265
4	8883	8880	8922	**8789**	8890
5	9390	9370	**9350**	9360	9380
6	7434	7426	7441	**7374**	7426
7	7853	7857	**7815**	7858	7861
8	8272	8269	**8229**	8250	8273
9	8011	7998	**7968**	8009	8010

Nonparametric Bayes

	OR Task					
	Serial		Parallel			
	OR	AND	OR	AND	Coactive	Np
1	1	0.17	**7.26**	0	0.05	0
2	**160**	2.57	7.24	0.03	0.15	0.02
3	1	0.20	**6.98**	0	0.31	0
4	1	0.12	**3.19**	0	0	0
5	1	0.25	**7.02**	0	0.70	0
6	1	0.25	**7.45**	0	0	0
7	**72**	0.29	7.25	0	0.01	0
8	1	0.25	**7.19**	0	0.13	0
9	1	0.25	**7.22**	0	0.01	0

Nonparametric Bayes

	AND Task					
	Serial		Parallel			
	OR	AND	OR	AND	Coactive	Np
1	1	0.50	0	**7.41**	0	0
2	1	0.25	0	**7.51**	0	0
3	1	0.17	0	**7.69**	0	0
4	1	0.50	0	**7.26**	0	0
5	1	1.00	0	**7.36**	0	0
6	1	0.17	0	**7.37**	0	0.24
7	1	0.50	0	**7.22**	0	0.04
8	1	0.25	0	**7.37**	0	0.48
9	1	0.50	0	**7.31**	0	0

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  - Parametric conclusions diverged from NHST and nonparametric tests on human data.

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  - Hierarchical models.

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Thank you.