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# A philological examination of Eratosthenes' calculation of Earth's circumference

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## Introduction

- A Greek mathematician named Eratosthenes calculated an accurate measurement of the Earth's circumference, that being 250,000 stades or close to Earth's actual circumference of 40,120 km,<sup>1</sup> in Hellenistic Alexandria, Egypt circa 240 B.C.E.<sup>2</sup>
- To do this, Eratosthenes' utilized the distance between Alexandria and Syene and gnomon measurements taken in Syene and Alexandria at noon on the summer solstice.<sup>3</sup>

## Cleomedes Account of Eratosthenes Proof

Eratosthenes began with five assumptions:

- Syene and Alexandria lie on the same meridian geographically.
- The distance between Syene and Alexandria is 5,000 stades apart.
- The Sun's rays hit the Earth in a parallel fashion.
- Due to the Sun's rays being parallel, alternate angles outside and inside these lines are equal.
- Arcs subtended by equal angles are similar.<sup>4</sup>

With this in mind, Eratosthenes deduced the following:

Statements	Reasons
1. Alexandria and Syene are 5,000 stades apart.	1. Given
2. The angle of the shadow at Alexandria is 7.2°, whereas the angle of the shadow at Syene is 0°.	2. Given
3. $\theta' = 7.2^\circ$	3. Euclid's Proposition 29 <sup>5</sup>
4. $\frac{7.2}{360} = \frac{5,000}{x}$	4. Rule of Proportion
5. $7.2x = 1,800,000$	5. Cross-multiplication
6. $\therefore x = 250,000$	6. Division

## Errors in Eratosthenes Assumptions

Two of Eratosthenes' five assumptions are either mistaken or questionable at best:

- Assumption (1) is incorrect. The longitudinal difference between Syene and Alexandria is about 3 degrees.<sup>6</sup>
- Assumption (2) is questionable since we do not know how long a stadion is.
  - The distance between Alexandria and Syene (Aswan, Egypt) is about 844 km apart,<sup>7</sup> meaning a stadion would have to measure around 169 m.

## Estimates on the Length of a Stadion

- According to Pliny the Elder, 1 schoenus = 40 stades.<sup>8</sup>
  - Contemporary philologists estimate that 1 schoenus = 20,000 royal cubits of 0.525 or 0.5237 m each.<sup>9</sup>
    - Therefore, 1 stade = 262.5 m
- Earlier philologists estimate that 1 schoenus = 12,000 royal cubits.<sup>10</sup> Therefore, 1 stade = 157.5 m under the same cubit-meter conversion.
- Philologist Gyula Priskin claims that the schoenus-stade conversion should be 1:50.<sup>11</sup>
  - If 1 schoenus = 20,000 royal cubits, then 1 stade  $\approx$  210 m
- According to Strabo, 1 Roman mile = 8 stades.<sup>12</sup>
  - 1 Roman mile = 1,478 m<sup>13</sup>
  - Therefore, 1 stade = 184.75 m

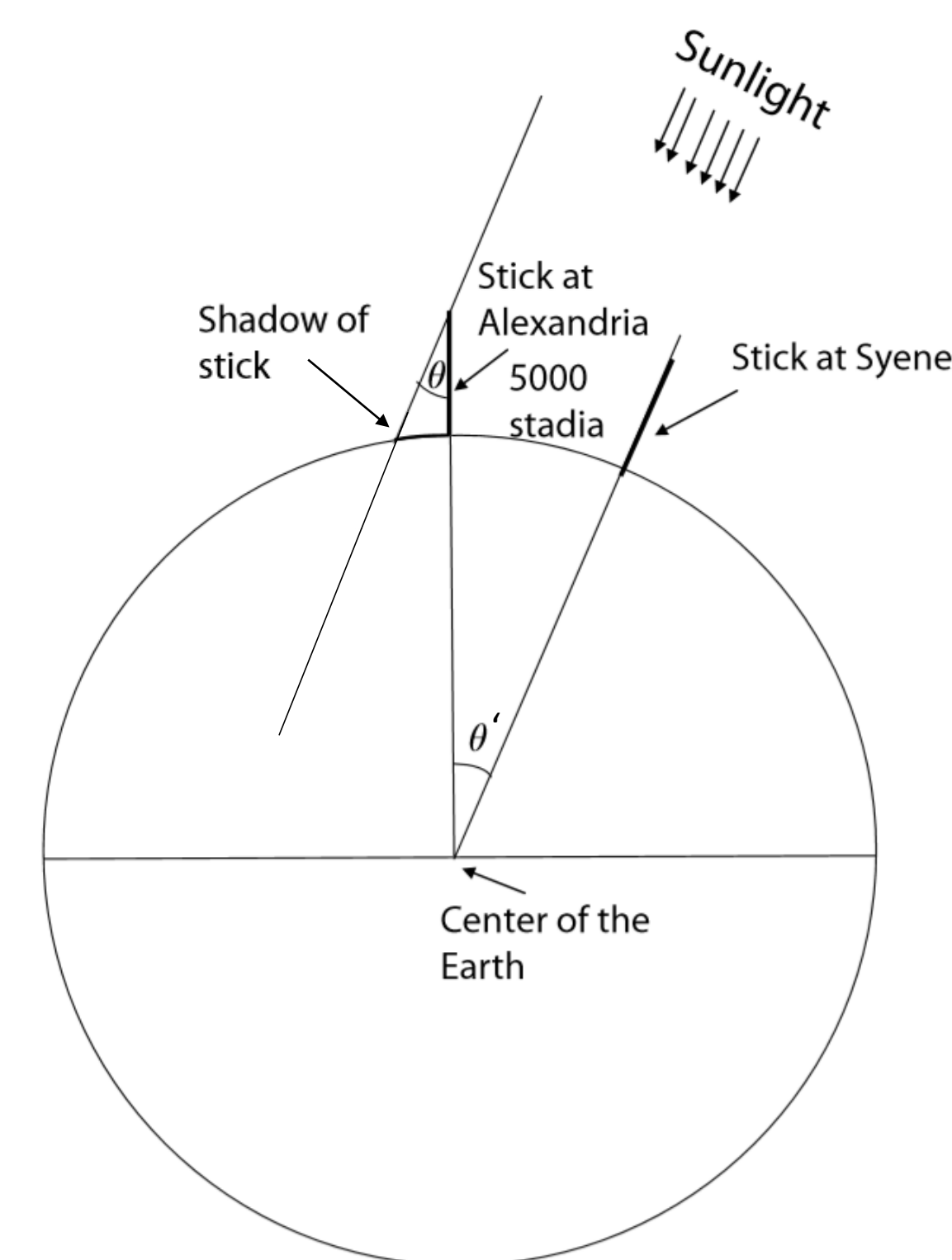


Diagram of proof including only Alexandria and Syene; Adapted from Brown and Kumar.<sup>25</sup>

## Historical Inconsistencies of Eratosthenes Final Result

- Cleomedes and John Philoporus state Eratosthenes' result was 250,000, whereas many others such as Vitruvius, Martianus Capella, Strabo, and many others state it was 252,000 stades.<sup>14</sup>
  - Benefits of 252,000 stades:
    - Yields a clean 700 stades per circular degree.<sup>15</sup>
    - 252,000 was already valued because of Pythagoras.<sup>16</sup>
    - Esoteric—Plato recognized 5,040 (distance between Alexandria and Syene) as an ideal number because of its divisibility by all digits from 1 to 10.<sup>17</sup>

## Other Inconsistencies in Historical Accounts

- Accounts like Martianus Capella<sup>18</sup> and Strabo<sup>19</sup> state that a third city was taken into consideration, Meroë. Syene formed the midpoint between Alexandria and Meroë.
- Gnomon measurements may have been observed at the equinoxes, according to Martianus Capella<sup>20</sup> and Vitruvius.<sup>21</sup>
- Recorded gnomon measurements were likely cited from other sources such as Vitruvius rather than being Eratosthenes' own measurement.<sup>22</sup>
- Eratosthenes' method was not original to him—Posidonius<sup>23</sup> and an unauthored proof given by Cleomedes demonstrate the same method.<sup>24</sup>

## Conclusion

- Despite the popularity of Cleomedes' account, the original measurement was likely rounded to 252,000 stades according to a majority of sources from antiquity.
- Although the measurement of a stadion is still undetermined, we can conclude a lower and upper limit of 157.5 m and 262.5 m, respectively.
  - Measurement of stadion is likely between 170 m and 210 m provided by Priskin.

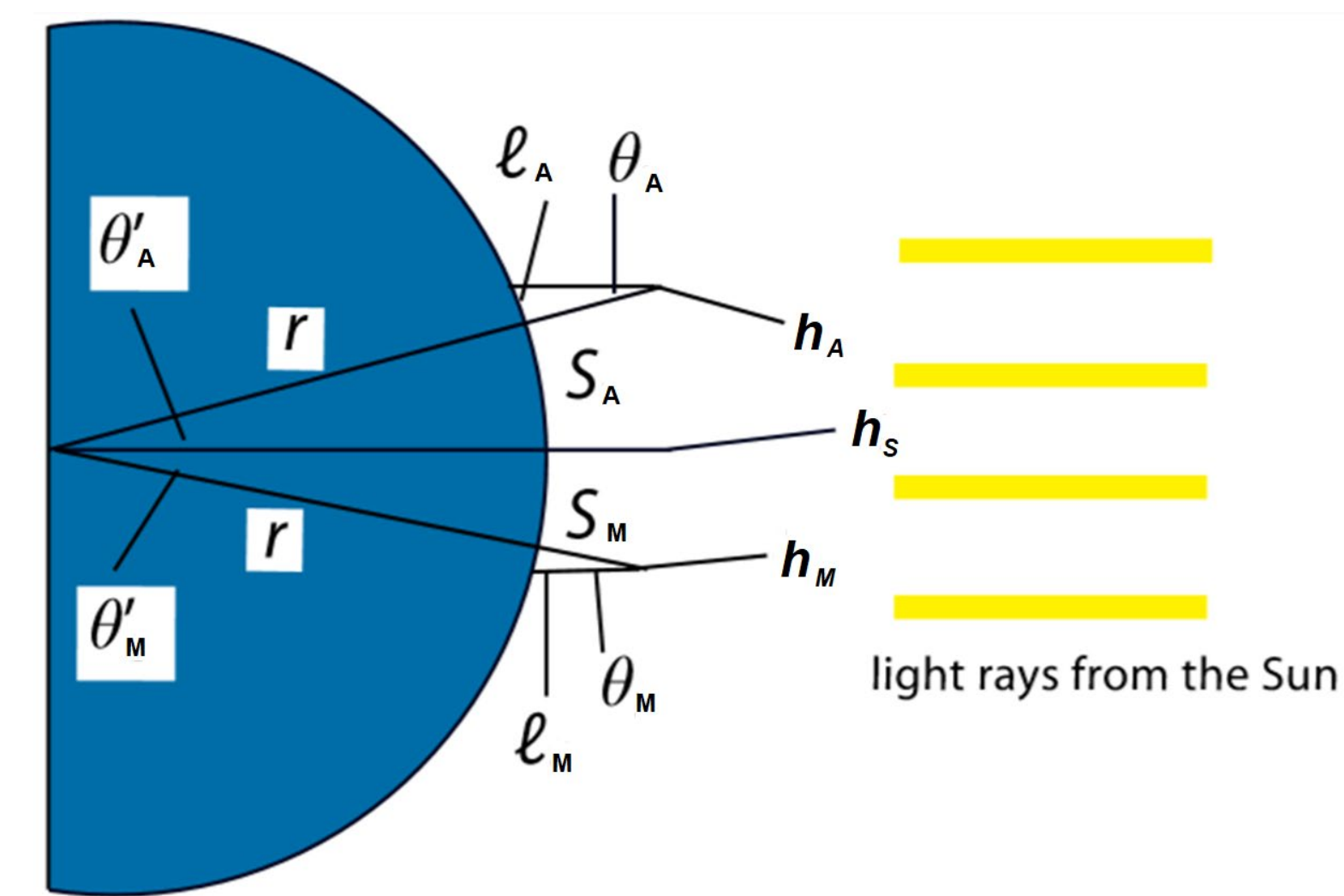


Diagram of proof including Alexandria, Syene, and Meroë; Adapted from Takemae, Kirwin, and McIntosh.<sup>26</sup>

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