2008

Design and Analysis of a Mach 3 Dual Mode Scramjet Combustor

Christopher Ryan Corbin
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DESIGN AND ANALYSIS OF
A MACH 3 DUAL MODE
SCRAMJET COMBUSTOR

A thesis submitted in partial fulfillment of the requirements for the degree of
Master of Science in Engineering

By

CHRISTOPHER RYAN CORBIN
B.S., Syracuse University, 2003

2008
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Christopher Ryan Corbin ENTITLED Design and Analysis of a Mach 3 Dual Mode Scramjet Combustor BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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ABSTRACT

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Design and Analysis of a Mach 3 Dual Mode Scramjet Combustor

Low speed operation of a dual mode scramjet engine is important to the development of a two stage to orbit reusable launch vehicle. This study investigates the Mach 3 operation of a dual mode scramjet engine. SRGULL, a one-dimensional cycle code for scramjet engines, and VULCAN, a computational fluid dynamics code capable of solving reacting flows, are used in this study. Staged injection is investigated to allow more heat release at a low flight Mach number condition so that more thrust can be achieved and inlet unstart is avoided. The nominal case has one injector located 1.067 meters downstream of the inlet with a fuel equivalence ration of 0.488. An increase in thrust of 11.6% is shown in this study by injecting a fuel equivalence ratio of 0.437 upstream and a fuel equivalence ratio of 0.369 at a location 0.8 meters downstream of the first injector.
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CHAPTER 1
INTRODUCTION

Motivation

There has been a change in how outer space is being viewed over the past couple of decades. As space technologies increase, so does the interest in inexpensive, responsive space access. The current systems available for access to space are very costly and are unable to provide quick response. Space access is of great importance to the United States military, and although space may never be a true battleground in the same way as land, sea, and air; it is not a far stretch to say that space will play a key role to the war fighter. The infrastructure located in space performs very important surveillance, reconnaissance, and communications missions and therefore it is easy to see the importance of protecting these assets.1 Responsive access to space is critical to the protection of these assets, and is necessary because of the extent to which the United States depends on them.2 The Commission to Assess United States National Security Space Management and Organization, commonly referred to as the “Space Commission,” has issued some findings as to the importance of space to national security. Some of these findings are as follows.

- The present extent of U.S. dependence on space, the rapid pace at which this dependence is increasing and the vulnerabilities it creates, all demand that U.S. national security space interests be recognized as a top national security priority.
- The U.S. government—particularly, the Department of Defense and the Intelligence Community—is not yet arranged or focused to meet the national security space requirement of the 21st century.
• We know from history that every medium–air, land and sea–has seen conflict. Reality indicates that space will be no different. Given this virtual certainty, the U.S. must develop means to both deter and to defend against hostile acts in and from space.2

In the realm of space applications, the military of the United States has been very involved in the creation of space vehicles and space assets. The most intriguing advancement for space flight is the reusable launch vehicle (RLV) and more recently the space operations vehicle (SOV).4

The defense challenges posed in the 21st century demands a responsive space capability that provides near-real-time global force application based on critical intelligence, surveillance, and reconnaissance (ISR). This entails “launch on warning” non-nuclear weapons and space based sensors that are available for and responsive to the National Command Authority (NCA) and the war fighter. The NCA requires timely, accurate, and responsive intelligence information for informed decision-making in crises and wartime. For the war fighter, timely execution of NCA direction could be accomplished through the use of space-delivered weapons on alert and ready to strike targets in less than 100 minutes from launch and rapidly deploys space-based sensors that can become available for use by the war fighter within three hours of launch.4

For military applications the desire is to develop a Military Space Plane (MSP) to be an asset delivery vehicle. In the near future an MSP will probably be a Two Stage to Orbit (TSTO) vehicle, with the first stage being an SOV. The SOV will be a versatile vehicle capable of performing various missions with a fast turnaround time, on the order of hours rather than the months necessary for the U.S. Space Shuttle.3

**Quicksat**

The *Quicksat* SOV vehicle has been designed as the first element in a TSTO RLV. Figure 1 shows an illustration of the *Quicksat* vehicle.
**Quicksat** uses air breathing propulsion which enables the following capabilities which cannot be accomplished with current chemical rocket engines: fly out, abort, and loiter. *Quicksat* also uses non cryogenic fuels. This helps reduce ground time and increase number of sorties, leading to the ability for on demand and responsive launches.

*Quicksat* is also a horizontal take-off and landing vehicle. The propulsion system of *Quicksat* is a Turbine Based Combined Cycle (TBCC) engine. The first component is a combination of 6 turbine engines, which accelerate the vehicle to Mach 3.75 with the assistance of tail rockets during the transonic region. The second component consists of 4 dual-mode scramjet (DMSJ) engines which accelerate the vehicle from Mach 3.75 to Mach 8 where staging occurs.³

It can be seen from the results of the initial design that the DMSJ engines accelerate more quickly than the turbine engines. 2/3’s of the total time to orbit comes from the turbofan acceleration up to Mach 3.75. The time to accelerate up to Mach 3.75 is 1145 s. It takes only 230 more seconds to accelerate from this point up to Mach 8.⁵ If the transition from turbine engine power to ramjet/scramjet power can occur at Mach 3.0 as opposed to Mach 3.75 the time to staging could be decreased by up to 175 seconds if the acceleration from Mach 3.0 to Mach 3.75 under ramjet power provides the same
acceleration as the current acceleration from Mach 3.75 to Mach 8. This can be seen in figure 2.

The result of this is an up to 13% decrease in time to orbit. From the turbine engine standpoint, there are also advantages to moving the transition Mach number up to Mach 3.0. First of all less fuel will be needed, leading to a lower vehicle weight or a higher payload. Secondly, there will be less turbine engine complexity needed, leading to potentially lighter and less expensive turbine engines. In addition, a smaller number of turbine engines may be required which will increase the mission and payload options.
CHAPTER 2
LITERATURE REVIEW

Introduction

Hypersonic air breathing propulsion is a fairly recent field in engine technology, and offers great increases in specific impulse, $I_{sp}$, compared to Rockets and flight Mach number range compared to turbojets. This is illustrated in figure 3.

Figure 3. Specific Impulse versus Mach Number for different engine types (Ref. 6)

In a hypersonic air breathing propulsion system, the shocks formed by the compressible high speed flow act as the compressor component of the engine. They attain the
deceleration of and the pressure rise in the flow required by the combustor. There are
two basic types of hypersonic air breathing propulsion systems, the ramjet and the
scramjet. In a ramjet the flow is subsonic by the time it gets to the combustor. In a
scramjet the flow remains supersonic through the combustor. The dual mode scramjet
bridges the gap between the ramjet and scramjet. It uses the same combustor geometry
for both the ramjet and scramjet modes, but operates with a thermal throat in ramjet
mode. Figure 4 shows schematics of these three engines.

Figure 4. Schematics of a) scramjet, b) ramjet, and c) dual mode scramjet operating in
ramjet mode engines and the Mach number profile for each (Ref. 7)
The goal of this work will be to examine and expand the low speed operation range of a dual mode scramjet engine.

**Scramjet and Dual Mode Scramjet History and Operation**

The idea for Scramjet engine first came to be realized in the early 1960’s. Though operating at lower Isp’s than the conventional ramjet engines, the Scramjet could theoretically attain much higher Mach numbers up to Mach 25. When originally researched, Mach 8 was considered to be the beginning operation point of a pure Scramjet engine.6 It was latter shown that scramjet operation can begin in the Mach 5 to 7 range for a pure scramjet. At speeds lower than this a ramjet is more effective. Figure 5 shows the specific impulse versus Mach number for subsonic and supersonic combustion ramjets and it can easily been seen where the range of operation for both lay.

![Figure 5. Performance of Subsonic and Supersonic Modes (Ref. 8)](image-url)
The value \( V_0 - V_2 \) is a measure of the compression obtained by the inlet, \( V_0 \) is the freestream velocity and \( V_2 \) is the velocity at the exit of the inlet.

In order to increase the low speed operating range of the scramjet engine, the dual mode scramjet was engineered and patented by Curran and Stull in the early 1960’s.\(^\text{9} \) As the flight Mach number decreases, the flow in the combustor becomes subsonic. When heat is added at low supersonic speeds in the combustor the flow becomes choked and a shock is formed upstream. In a normal scramjet engine this shock would travel out the inlet and unstart the engine, to counter this issue the Dual Mode Scramjet concept has an isolator placed between the combustor and the inlet. This isolator holds the shock avoiding interaction with the inlet and combustor.

Another difficulty that arises, when the flight Mach number is low enough that the flow becomes choked, is a limitation on heat addition. Because of the shock that is formed upstream the flow in the combustor is subsonic and the flow upstream of combustion is affected by the heat addition. In a constant area combustor, the heat addition to the flow forms a shock train upstream of the combustor. If too much heat is added then the shock train moves out of the inlet and the engine is unstarted. One way to alleviate this restriction on heat addition is to have a diverging duct combustor. In the low speed operating range it is necessary to add enough heat such that a sonic condition exists before the exit of the combustor. The relationship between area change and heat release can be understood by looking at this one-dimensionally through following equation\(^\text{7},\)

\[
\frac{dM}{dx} = M \left(1 + \frac{\gamma - 1}{2} M^2 \right) \left(- \frac{1}{A} \frac{dA}{dx} + \frac{1 - \gamma M^2}{2} \frac{dT}{T} \frac{dT}{dx} \right),
\]  

(1)
for the case where $M=1$, the quantity

$$
\left( -\frac{1}{A} \frac{dA}{dx} + \frac{1 - \gamma M^2}{2} \frac{1}{T_i} \frac{dT_i}{dx} \right),
$$

must equal zero, therefore there must be a balance between area change and heat release, this condition is referred to as a thermal throat. At the low end of this range the diverging duct provides too much area change for the heat release to provide a thermal choke. For low subsonic operation in the combustor a contraction in the combustor exit is needed to accelerate the flow to sonic when a thermal throat cannot be established.\(^8\)

**Previous Research**

A study performed by Bauer et. al. in 1998 showed a comparison of a ramjet and a dual mode scramjet operating in the Mach 3 to 6 range. This study used a one-dimensional analysis that assumed a real gas relationship with a fuel equivalence ratio of 1. For the ramjet a combustion efficiency of 95% was used and the combustion efficiency in the dual mode ramjet case was 90%. No reaction was assumed in the nozzle. Thrust was look at as the defining parameter of operability in the study. At Mach 3 conditions the ramjet had much higher thrust than the dual mode scramjet. Conversely at Mach 6 the dual mode scramjet had greater thrust. A result of the study can be seen in figure 6. It is obvious that in order to operate past Mach 6, a dual mode scramjet must be used. It is clear the Mach 3 performance must be improved in order to efficiently use a dual mode scramjet at such speeds.\(^10\)
The low Mach number operating range of the Dual Mode Scramjet, which to date has been the range between Mach 4 and Mach 8, is the key technological area of this research. There have been many research programs worldwide that have been investigating this low speed operation range. The majority of these programs start operation at Mach 4, assuming some other means of propulsion to get the vehicle to that point. Around Mach 6.5 the engine is in full Scramjet mode, with no subsonic flow in the combustor.9

Direct connect testing is a common technique for experimental scramjet combustor studies. In direct connect setups a facility nozzle provides flow to the
combustor at conditions simulating the inlet exit conditions for a specific flight Mach number. Research Cell 22 (RC22) at the Air Force Research Laboratory Aerospace Propulsion Division (AFRL/RZA) is a direct connect facility with facility nozzles available to supply Mach 1.8, 2.0, and 2.2. The incoming flow can be heated so that a specific flight Mach number can be studied by prescribing the combustor inlet Mach number and the total temperature of the flow.

One of the biggest difficulties in a scramjet engine is to get effective fuel-air mixing from injection. Two intrusive techniques that are used are strut injectors and ramp injectors. Both cause good mixing to occur because of the vorticity they induce in the flow by their obstruction. But this obstruction also causes drag and exposes the injector to the hot flow field requiring it to be cooled. Another non obtrusive technique is wall injectors. In this technique the fuel is injected, usually at an angle, through the wall into the flow. This alleviates the requirement for cooling of the injector and doesn’t cause increased drag. This is the injection scheme that this study will use.

A 1990 study by Vinogradov et. al. in Russia at the Central Institute of Aviation Motors showed some results of direct connect testing of a rectangular dual mode scramjet combustor using hydrogen fuel. This study used both strut and wall injectors to study flight Mach numbers from 4 to 6. For the Mach 4 case only wall injectors were used. For the Mach 4 case the study shows maximum combustion efficiency around 0.75 for a fuel-air equivalence ratio of about 0.2-0.6. Later studies in 2001 by Ogorodnikov et. al. were performed on a round combustor, again using hydrogen fuel, for a flight Mach number of 3.5. The combustion efficiency was slightly higher for the round Mach 3.5 case than for the rectangular Mach 4 case. This study also showed a maximum fuel-air
equivalence ratio of 0.75 for the Mach 3.5 case before the combustor inlet conditions became affected.\textsuperscript{12}

Liquid hydrocarbon fuels are primarily used in the low Mach range DMSJ studies. They offer a higher fuel density than Hydrogen fuel allowing storage at room temperature, removing the need for cryogenic fuel storage. This allows for easier handling for military applications. The fuel tanks will also be able to be smaller as well.\textsuperscript{13,14}

In a 1999 study by Mathur et. al. a direct connect study was performed on a rectangular combustor with wall injectors. The study used condition equivalent to a flight Mach number between 4 and 5. A cavity was used behind the injection location for flame holding. An almost linear relationship between the thrust and the fuel equivalence ratio was shown between fuel equivalence ratios of 0.25 and 0.75. At a fuel equivalence ratio of 0.75 the combustion efficiency is shown to be approximately 0.8.\textsuperscript{15}

The initial results of an initiative for numerical and experimental studies at the Air Force Research Laboratory were presented in 1998 by Baurle et. al. The study showed some good and promising comparison between experimental and numerical work. This study looked at a rectangular combustor with wall injectors and a flame holding cavity using ethylene fuel.\textsuperscript{14} In a 2001 study by Eklund et. al. the computational results of a study on an aerodynamic ramp injector yielded an important conclusion for scramjet computational work. It showed a large dependence on Turbulent Schmidt and Prandtl number. Good matching was shown with experimental work using the proper Turbulent Schmidt and Prandtl numbers. It also showed a combustion efficiency between 0.8 and 0.6 for a flight Mach number of 4.\textsuperscript{16} In a later study in 2001 Baurle and Eklund
performed a computational study on an ethylene fueled dual mode scramjet at Mach 4 and 6.5 flight conditions. This study showed the same mixing efficiency for both cases but the Mach 6.5 case had better combustion efficiency. This study also showed the dependence on Turbulent Schmidt and Prandtl numbers.17

Staged injection is a technique for achieving more thrust and allowing more fuel injection. In the past decade testing has been done to show improvements due to staging fuel injection. Staged fuel injection has been shown to significantly increase the thrust while keeping the shock in the isolator and preventing engine unstart.11,18-21 Kobayashi et al.18 showed experimentally for Mach 4 flight conditions that the total injected equivalence ratio of fuel could be doubled and the Thrust could be increased to 2230 N from 1380N due to the addition of a second stage fuel injector. Fan et al.19 showed that at Mach 2.5 flow into the combustor, for the same total equivalence ratio, the thrust increment could be increased by staging a portion of the fuel downstream.

To this point the majority of research has been in the area of rectangular 2D geometry combustors. A main reason for this is the ability to inject fuel to the center of the flow field of a high aspect ratio rectangular geometries. As of late studies have began at the Air Force research Labs to investigate scramjet combustors with round geometries. There are a few advantages that round geometries offer over rectangular geometries the first is a decrease in structural weight for the same aspect ratio. The heat load on the combustor is also lower with an axisymmetric design. A final advantage is the ability to have the cavity for the wall injector setup to stretch around the entire outer wall of the combustor; in the rectangular setups there was a cavity on the top wall and sometimes one on the bottom wall.
A computational study performed by Tam et. al.\textsuperscript{22} at AFRL/RZA using a 3D sector analysis in VULCAN investigated round combustor geometries for use in RC22. The sector was 1/16\textsuperscript{th} of the entire round geometry with one symmetry plane halfway between injectors and the other through the center of an injector. The round flow paths were designed and analyzed for Mach 5 flight conditions. This study will employ the baseline design from the 3D VULCAN study as the baseline flow path. RC22 is a modular facility and will offer the ability to create a module with the modifications of this study to compare the computational results to actual test data.

The baseline combustor is 2.438 meters long with 8 equally spaced fuel injectors located 1.067 meters downstream of the inlet throat. The combustor has a flame holding cavity located behind the injector in the flow path. The flow path can be seen in figure 7.

![Figure 7. RC22 round geometry combustor flow path](image)

The combustor operated with ethylene fuel at a fuel-air equivalence ratio of 0.5 in the results presented here.

The goal of this study will be to investigate ways to operate the geometry from Tam’s study at Mach 3 flight conditions. The study will look at the use of staged
injection. The major performance characteristic that will be considered is the uninstalled thrust.
CHAPTER 3

METHODOLOGY

Introduction

The study will use two computational tools to study the low speed operation of a
dual mode scramjet. The first is a one-dimensional cycle code developed at NASA
Langley called SRGULL and the second is a Computational Fluid Dynamics program
also developed at NASA Langley called VULCAN. Sample input files for these
programs can be seen in Appendices A and B.

SRGULL

SRGULL\textsuperscript{23} is a cycle analysis code for ramjets and scramjets, created at NASA Langley.
SRGULL uses a two-dimensional solver for the inlet and nozzle and a one-dimensional
solver for the combustor. Since this study will just be looking at the combustor, the
nozzle will be neglected. The inlet will still be needed in order to supply the conditions
to the throat.

Inlet

The inlet is modeled by solving the two-dimensional Euler equations following the
method of Salas.\textsuperscript{24} The two-dimensional Euler Equations are,

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \quad (3)
\]
\[
\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x} \left( \rho u^2 + p \right) + \frac{\partial}{\partial y} \left( \rho uv \right) = 0, \quad (4)
\]

\[
\frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial x} \left( \rho uv \right) + \frac{\partial}{\partial y} \left( \rho v^2 + p \right) = 0, \quad (5)
\]

and,

\[
\frac{\partial E}{\partial t} + \frac{\partial}{\partial x} \left[ (E + p)u \right] + \frac{\partial}{\partial y} \left[ (E + p)v \right] = 0. \quad (6)
\]

The equations are solved in the radial direction at each x-location from values at the previous x-location. Before moving to the next location a check is made to see if there is a shock at that x-location. If so, the equations are solved on the low pressure side of the shock, then the high pressure side of the shock is computed. The Euler equations are then re-solved based on the new conditions behind the shock. The solution is stepped to the next x-location. In order to account for viscous effects an integral boundary layer code is used. The momentum and energy equations are integrated across the boundary layer in order to determine the coefficient of friction and heat transfer at the wall. The Spalding and Chi\textsuperscript{,21,25,26} method is used to determine the coefficient of friction, \(C_f\), and is of the form,

\[
C_f = \frac{0.242C_F}{0.242 + 0.8686\sqrt{C_F}F_c}, \quad (7)
\]

and \(C_F\) is the average friction coefficient defined by,

\[
C_F = \frac{1}{F_c} \left[ \frac{0.242}{\log_{10}(2F_cR_\theta)} \right]^2. \quad (8)
\]
The Values $F_c$ and $F_r$ are compressibility corrections defined by,

$$F_r = t_w^{-1.474} t_{aw}^{0.772}$$

and,

$$F_c = \frac{t_{aw} - 1}{\left( \sin^{-1} \left( \frac{2 \sqrt{t_{aw} - 1} (t_{aw} - t_w) - \sqrt{t_w (2 - t_{aw} - t_w)})}{(t_{aw} + t_w)^2 - 4t_w} \right) \right)^2}$$

where $t_{aw}$ is the ratio of adiabatic wall enthalpy to boundary layer edge enthalpy and $t_w$ is the ratio of wall enthalpy to boundary layer edge enthalpy. The Reynolds number based on momentum thickness, $R_\theta$, is defined as,

$$R_\theta = \frac{\rho u_\infty \theta}{\mu}$$

with the momentum thickness, $\theta$, given by,

$$\theta = \int_0^\infty \frac{\rho}{\rho_x} \frac{u}{u_x} \left( \frac{1 - u}{u_x} \right) d \frac{y}{\delta}$$

using the free stream density, $\rho$, and velocity, $u_\infty$, and the boundary layer thickness $\delta$.

The velocity profile used for turbulent flows in SRGULL is,

$$\frac{u}{u_x} = \left( \frac{y}{\delta} \right)^{\frac{1}{2}}$$

The surface heat transfer, $q_w$, is determined from,
where $h_{aw}$ is the adiabatic wall enthalpy, $h_w$ is the wall enthalpy, and $Z$ is the Karman factor which is a function of the ratio of specific heats and coefficient of friction.

**Combustor**

The combustor is modeled using a control volume method. The combustor is broken down into 220 steps. The first 140 steps are used to model the isolator. There are two options available to model the isolator and in this study the shock train model is used. This model is the most representative of what really occurs in the engine. A separation bubble is modeled in the region behind the shock that occurs in the isolator. The flow reattaches in the region behind the fuel injection due to the heat release from combustion. The flow reattachment region is modeled using 57 steps with reattachment at step 197. The remaining steps model the region where the flow is reattached and there is no distortion. The flow through the combustor is solved using the one-dimensional continuity, momentum, and energy equations which are defined in SRGULL\textsuperscript{23} as,

\begin{equation}
\dot{m} = \rho_2 A_2 V_2, \tag{15}
\end{equation}

\begin{equation}
V_2 = \frac{\left(\dot{m}V_1 + P_1 A_1\right) - \int_{1}^{2} q_1 C_{f_{1,2}} ds - P_2 A_2 + \int_{1}^{2} P dA}{\dot{m}}, \tag{16}
\end{equation}

and,

\begin{equation}
h_2 = h_{1i} - \frac{V_2^2}{2} = \frac{\int Q ds}{\dot{m}}, \tag{17}
\end{equation}

19
where \( \dot{m} \) is the mass flow rate, \( \rho \) is the density, \( A \) is the area, \( V \) is the velocity, \( q \) is the dynamic pressure, \( C_f \) is the coefficient of friction, \( h \) is the enthalpy, \( h_t \) is the total enthalpy, and \( Q \) is the heat load in the combustor. The integrals are evaluated by taking the sum up to the computational step being computed.

In the flow separation region a fourth equation is added, a momentum balance on the separation bubble. This equation is needed because of the introduction of an additional variable \( \eta_A \), which is the ratio of the flow area to the total area. The location of the shock is originally guessed as an input in the input file. The non-separated equations are solved up to that point by guessing a value for \( P_2 \). Equations 16 and 17 are then solved to determine \( V_2 \) and \( h_2 \). The values of \( h_2 \) and \( P_2 \) are used with the input combustion efficiency, \( \eta_c \), and fuel equivalence ratio, \( \phi \), in the equilibrium solver to determine \( \rho_2 \). Now the mass flow rate, \( \dot{m}_2 \), can be calculated from equation 15. If \( \dot{m}_2 \) is not equal to \( \dot{m}_1 \) then the value for \( P_2 \) is changed and the procedure is repeated. This iteration on pressure is performed for each step until the shock location is reached, and then the momentum balance equation is added. Again \( P_2 \) is guessed, but it is also used to determine \( \eta_A \) which then goes into the continuity equation to determine convergence. This procedure is continued through the end of the end of the combustor. If a solution cannot be determined the shock is moved one step closer to the inlet and the process is repeated.

**Initial SRGULL Studies**

In order to gain familiarity with SRGULL some studies were performed using the *Quicksat* vehicle geometry. Since the original study on Quicksat was performed using an earlier version of SRGULL, it was studied using the current version of the program.
Additional studies were performed by changing some parameters to see the effect on the solution. Results of these studies can be seen in Appendix C.

**VULCAN**

Optimal designs will be analyzed using the Viscous Upwind Algorithm for Complex Flow Analysis (VULCAN)\textsuperscript{28,29} computational fluid dynamics (CFD) program. It “is a cell-centered finite-volume, structured grid, multi-block code which solves the equations governing inviscid and viscous flow of a calorically perfect gas or of an arbitrary mixture of thermally perfect gasses undergoing non-equilibrium chemical reactions.”\textsuperscript{29} The VULCAN code solves the Favre Averaged Navier-Stokes (FANS). A Favre averaged quantity is defined by,

\[ \tilde{f} = \overline{\frac{\rho f}{\rho}}, \]  \hspace{1cm} (18)

where \( \rho \) is the density, \( f \) is any conserved variable, and the over bar represents a time averaged quantity defined by,

\[ \tilde{f} = \frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t} f dt, \]  \hspace{1cm} (19)

where, \( t \) is time, \( \Delta t \) is a time increment, and \( t_0 \) is some initial time. All properties are then defined as an average quantity plus a fluctuation, either,

\[ f = \tilde{f} + f', \]  \hspace{1cm} (20)

for time averaging, or,
\[ f = \tilde{f} + f^*, \quad (21) \]

for Favre averaging, where \( f' \) and \( f'' \) are the respective fluctuations.

**Governing Equations**

When the average values and fluctuations are inserted into the Navier-Stokes equations, the following equations result after Favre averaging the equations, the double over bar represents a Favre averaged product:

1. Continuity

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho \tilde{u}_j) = 0, \quad (22)
\]

2. Conservation of momentum

\[
\frac{\partial}{\partial t}(\rho \tilde{u}_i) + \frac{\partial}{\partial x_j}(\rho \tilde{u}_i \tilde{u}_j) = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \tilde{\tau}_{ij} - \rho u'_i u'_j \right), \quad (23)
\]

3. Conservation of energy

\[
\frac{\partial}{\partial t}(\rho \tilde{E}) + \frac{\partial}{\partial x_j}(\rho \tilde{u}_j \tilde{H}) = \frac{\partial}{\partial x_j} \left( \tilde{u}_i \tilde{\tau}_{ij} + u'_i \tilde{\tau}_{ij} - \rho u'_i u'^*_j \right) - \frac{\partial q_j}{\partial x_j}, \quad (24)
\]

where, \( \tilde{u}_j \) is the average velocity vector, \( x_j \) is the position vector, \( \tilde{p} \) is the average pressure, \( \tilde{\tau}_{ij} \) is the molecular stress tensor, \( -\rho u'_i u'_j \) is the turbulent Reynolds stress tensor, \( \tilde{E} \) is the average total energy, \( \tilde{H} \) is the average total enthalpy, and \( q_j \) is the heat release from conduction and energy release from diffusion. The average total pressure is
defined by the equation of state for a thermally perfect gas neglecting fluctuations in the composition as,

\[ \bar{p} \approx \bar{\rho} R_u \bar{T} \sum_{i=1}^{N_{CS}} \frac{\bar{Y}_i}{MW_i}, \]  

(25)

where \( R_u \) is the universal gas constant, \( N_{CS} \) is the number of conserved species, \( MW_i \) is the molecular weight of species \( i \), and \( \bar{Y}_i \) is the average mass fraction of species \( i \) defined by,

\[ \bar{Y}_i = \frac{m_i}{m}, \]  

(26)

with, \( m_i \) as the mass of species \( i \) and \( m \) as the total mass of the mixture. The molecular stress tensor is defined by,

\[ \bar{\tau}_{ij} \approx \mu \left[ \left( \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \tilde{u}_k}{\partial x_k} \right], \]  

(27)

assuming,

\[ \tilde{u} = \bar{u}, \]  

(28)

where, \( \mu \) is the laminar viscosity neglecting any fluctuations and defined by Wilke’s\textsuperscript{29} formula as,

\[ \mu = \sum_{i=1}^{N_{CS}} \frac{\chi_i \mu_i}{\sum_{j=1}^{N_{CS}} \chi_j \phi_j}, \]  

(29)
and, \( \chi_i \) is the mole fraction of species \( i \) defined by,

\[
\chi_i = \frac{n_i}{n},
\]

having \( n_i \) be the number of moles of the \( i \)-th species and \( n \) be the total number of moles.

The species laminar viscosity, \( \mu_i \), is defined by Sutherland's formula as,

\[
\mu_i = \mu_{0,i} \left( \frac{T}{T_{0,i}} \right)^{\frac{3}{2}} \left( \frac{T_{0,i}}{T} \right)^{\frac{i}{2}} + S_i,
\]

where \( \mu_{0,i} \), \( T_{0,i} \), and \( S_i \) are constants for each species \( i \), and

\[
\phi_{ij} = \left[ \frac{1 + \left( \frac{\mu_i}{\mu_j} \right)^{\frac{1}{2}} \left( \frac{MW_j}{MW_i} \right)^{\frac{1}{4}}}{\sqrt{8} \left( 1 + \frac{MW_i}{MW_j} \right)^{\frac{1}{2}}} \right]^{2}.
\]

The Kronecker delta function, \( \delta_{ij} \), is defined as,

\[
\delta_{ij} = \begin{cases} 
1, i = j \\
0, i \neq j
\end{cases}.
\]

The average total energy is given by,

\[
\bar{E} = \bar{H} - \frac{\bar{p}}{\bar{\rho}},
\]

and the average total enthalpy is defined by,
\[ \bar{H} = \bar{h} + \frac{1}{2}\left(\overline{u_ju_j} + D\right), \]  
(35)

where \( \bar{h} \) is the static enthalpy defined by,

\[ \bar{h} = \sum_{i=1}^{N_{cs}} \bar{h}_i \bar{Y}_i, \]  
(36)

with the species static enthalpy, \( \bar{h}_i \), defined by,

\[ \bar{h}_i = \bar{h}_i^0 + \int_{T_0}^{\bar{\bar{T}}} c_{p,i} dT, \]  
(37)

where \( \bar{h}_i^0 \) and \( T_0 \) are values at a reference temperature and the constant pressure specific heat for each species \( i \), \( c_{p,i} \), is described by a polynomial curve fits of McBride et al\(^{29}\) defined by,

\[ c_{p,i} = \frac{R_u}{MW_i} \left( A_i + B_i \bar{T} + C_i \bar{T}^2 + D_i \bar{T}^3 + E_i \bar{T}^4 \right), \]  
(38)

or,

\[ c_{p,i} = \frac{R_u}{MW_i} \left( A_i \left( \frac{1}{\bar{T}} \right)^2 + B_i + C_i + D_i \bar{T} + E_i \bar{T}^2 + F_i \bar{T}^3 + G_i \bar{T}^4 \right). \]  
(39)

Where \( A_i, B_i, C_i, D_i, E_i, F_i, \) and \( G_i \) are coefficients defined for each species, and the mixture specific heat is

\[ c_p = \sum_{i=1}^{N_{cs}} c_{p,i} \bar{Y}_i. \]  
(40)
\( \tilde{k} \) is the turbulent kinetic energy defined by,

\[
\tilde{k} = \frac{1}{2} u_i' u_i' .
\] (41)

Two more terms from the energy equation that must be modeled are,

\[
\frac{\partial}{\partial x_j} (u_i' \tau_{ij}) \approx \frac{\partial}{\partial x_j} \left( \mu \frac{\partial \tilde{k}}{\partial x_j} \right),
\] (42)

and,

\[
\frac{\partial}{\partial x_j} \left( \frac{\rho u_i' u_i'}{H} \right) = \frac{\partial}{\partial x_j} \left( \frac{\mu_T}{\Pr_T} \frac{\partial h}{\partial x_j} - u_i \left( - \frac{\rho u_i' u_i'}{Pr} \right) \frac{\mu_T}{\Pr_T} \frac{\partial \tilde{k}}{x_j} \right),
\] (43)

with,

\[
\frac{\mu_T}{Pr_T} \frac{\partial h}{\partial x_j} = \frac{c_p \mu_T}{Pr_T} \frac{\partial \tilde{T}}{\partial x_j} + \frac{\mu_T}{Sc_T} \sum_{i=1}^{N_c} h_i \frac{\partial \tilde{Y}_i}{\partial x_j} .
\] (44)

The heat release from conduction and energy release from diffusion is defined as,

\[
\bar{q}_j = -\lambda \frac{\partial \tilde{T}}{\partial x_j} - \frac{\mu}{Sc} \sum_{i=1}^{N_c} h_i \frac{\partial \tilde{Y}_i}{\partial x_j} ,
\] (45)

where the mixture conductivity, \( \lambda \), is defined using Wassiljewa’s formula as,

\[
\lambda = \frac{\sum_{i=1}^{N_c} \chi_i \tilde{\lambda}_i}{\sum_{j=1}^{N_y} \Phi_{ij}} ,
\] (46)
where, $\lambda_i$ is the species conductivity defined by Sutherland’s formula as,

$$
\lambda_i = \lambda_{0,i} \left( \frac{T}{T_0} \right)^{\frac{3}{2}} \left( \frac{\mathcal{F}_{0,i}}{T} \right) + S_i,
$$

with the species specific constants $\lambda_{0,i}$, $\mathcal{F}_{0,i}$, and $S_i$, and

$$
\Phi_y = \left[ 1 + \left( \frac{\lambda_i}{\lambda_j} \right) \left( \frac{MW_j}{MW_i} \right) \right]^{\frac{1}{2}} \left( \frac{MW_i}{MW_j} \right)^{\frac{1}{3}}\sqrt{\frac{8}{1 + \left( \frac{MW_i}{MW_j} \right)^{\frac{1}{2}}}}.
$$

The Prandtl number, $Pr$, is used to relate conductivity to viscosity by,

$$
Pr = \frac{c_p \mu}{\lambda i}.
$$

In order to relate molecular diffusion, $\mathcal{D}$, to known quantities the Schmidt number, $Sc$, is used which is defined by,

$$
Sc = \frac{\mu}{\rho \mathcal{D}}.
$$

Values for laminar and turbulent Prandtl and Schmidt numbers that are used in VULCAN for this study can be seen in table 1.
Since the laminar conductivity is determined from Wassiljewa’s formula, the value for $Pr_L$ is not used. Values for turbulent Prandtl and Schmidt number are in line with values from Baurle and Eklund.\textsuperscript{17}

**Chemistry Modeling**

In addition to the RANS equations there is a conservation of species equation for each of the chemical species included.

\[
\frac{\partial}{\partial t} \left( \overline{\rho Y_i} \right) + \frac{\partial}{\partial x_j} \left( \overline{\rho Y_i u_j} + \overline{\rho Y_i \dot{u}_j} + \overline{\rho Y_i v_j} \right) = \dot{m}_i^*,
\]

where $Y_i$ is the mass fraction of each species, $v_j$ is the diffusion velocity defined by Fick’s law as,

\[
v_j = \frac{D}{Y_i} \frac{\partial Y_i}{\partial x_j},
\]

and,

\[
\overline{\rho Y_i v_j} \approx -\frac{\mu}{Sc} \frac{\partial \overline{Y_i}}{\partial x_j}.
\]
The term $\overline{\rho \nu_j}$ is the Reynolds mass flux and is defined as,

$$\overline{\rho \nu_j} = -\frac{\mu_T}{Sc_T} \frac{\partial Y_i}{\partial x_j},$$  \hspace{1cm} (54)

$m_i^\prime$ is the production of species $i$ on a mass basis defined by,

$$m_i^\prime = \dot{\omega}_i MW_i,$$ \hspace{1cm} (55)

where $\dot{\omega}_i$ is the species production term defined by,

$$\dot{\omega}_i = \sum_{j=1}^{N_r} (v_{ji}^\prime - v_{ji}^\prime') \left( \prod_{k=1}^{N_s} [X_k]^j - k_{ri} \prod_{i=1}^{N_s} [X_i]^{j\prime} \right),$$ \hspace{1cm} (56)

where $v_{ji}^\prime$ is the stoichiometric coefficient of the $i$-th species in the $j$-th chemical reaction on the RHS, $v_{ji}^\prime'$ the stoichiometric coefficient of the $i$-th species in the $j$-th chemical reaction on the LHS, $k_{fi}$ is the forward reaction rate of the $i$-th species, $k_{ri}$ is the reverse reaction rate of the $i$-th species, $N_r$ is the number of reactions, $N_s$ is the number of species, and $[X_i]$ is the species concentration defined by,

$$[X_i] = \chi_i \frac{P}{R_u T}.$$ \hspace{1cm} (57)

This study uses a 19 species, 267 reaction reduced model developed by Law at Princeton.\textsuperscript{30}
Turbulence Modeling

The turbulence model adds two more conservation equations and a set of closure coefficients in order to determine the Reynolds stresses. This study will use Menter’s BSL turbulence model \(^3\) which uses a k-\(\omega\) model near the wall and a k-\(\varepsilon\) model away from the walls. The two models are made into one set of equations using a blending function, \(F_1\). The two equations are,

\[
\frac{\partial \rho k}{\partial t} + \mathbf{u}_i \frac{\partial \rho k}{\partial x_i} = -\rho u'_i u'_j \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_k \mu_T \right) \frac{\partial k}{\partial x_j} \right], \tag{58}
\]

and,

\[
\frac{\partial \rho \omega}{\partial t} + \mathbf{u}_i \frac{\partial \rho \omega}{\partial x_i} = \frac{\gamma}{v_t} \left( -\rho u'_i u'_j \frac{\partial u_i}{\partial x_j} \right) - \beta \rho \omega^2 \\
+ \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_\omega \mu_T \right) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \rho \sigma_{\omega^2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, \tag{59}
\]

where \(\omega\) is the turbulence frequency and the following constants and equations are used,

\[
\beta^* = 0.09, \tag{60}
\]

\[
\sigma_k = 1.0 - 0.5F_1, \tag{61}
\]

\[
\beta = 0.0828 - 0.0078F_1, \tag{62}
\]

\[
\sigma_\omega = 0.856 - 0.356F_1, \tag{63}
\]

\[
\sigma_{\omega^2} = 0.856, \tag{64}
\]
\[
\gamma = \left( \frac{0.0828}{0.09} - \frac{0.856 \times 0.41^2}{\sqrt{0.09}} \right) - \left( \frac{0.0078}{0.09} - \frac{0.356 \times 0.41^2}{\sqrt{0.09}} \right) F_1, \tag{65}
\]

\[
F_1 = \tanh(\arg_i^4), \tag{66}
\]

\[
\arg_i = \min \left[ \max \left( \frac{\sqrt{k}}{0.09 \omega y}, \frac{500 \nu}{\omega y^2}, \frac{4 \times 0.856 \rho k}{CD_{k\omega} y^2} \right) \right], \tag{67}
\]

\[
CD_{k\omega} = \max \left( 2 \times 0.856 \rho \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_j} ; 10^{-20} \right), \tag{68}
\]

\[
\mu_T = \rho \frac{k}{\omega}, \tag{69}
\]

and

\[
- \rho \mu' \overline{u_j} = \mu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i} + \frac{2}{3} \frac{\partial \bar{u}_k}{\partial x_k} \delta_{ij} \right). \tag{70}
\]

In order to alleviate grid density requirements at the walls, wall matching is used. This methodology used was developed by Wilcox.\textsuperscript{32} Wall matching is an extension of the traditional wall functions but includes considerations for the pressure gradient. Wall matching is based on the fact that flows behave in a certain way within the inner 10% of the boundary layer.

**Computational Methods**

The VULCAN code solves the RANS equations using a cell-centered finite volume method. This particular study uses a MUSCL scheme with a \( \kappa \) value of 1/3. This defines
the stencil and points that are used in the finite volume approximation. The van Leer Total Variation Diminishing (TVD) flux limiting scheme is used in the stencil on higher order terms in order to limit the effect of discontinuities in the flow on the conserved variables. Because of the large storage requirements, the diagonalized approximate factorization (DAF) scheme is used to march the solution in time. This scheme offers the least storage requirements and requires fewer operations than the other methods available in VULCAN. The Low-Diffusion Flux-Splitting Scheme (LDFSS) developed by Edwards\textsuperscript{33} is used for flux splitting. Flux splitting is necessary in order to be able to accurately predict strong shocks. The LDFSS splits the fluxes into convection and pressure components. The CFL number is defined as a relationship between time step size and grid size. In this study when the solution is run at a constant CFL, the CFL number is used to determine the time step size for each grid cell, the time step size is defined by,

\[
\frac{1}{\Delta t} = \frac{1}{\Delta t_x} + \frac{1}{\Delta t_y} + \frac{1}{\Delta t_z} + \frac{1}{\Delta t_v},
\]  

(71)

where,

\[
\Delta t_x = \frac{CFL \cdot \Delta x}{\max(u, u + a, u - a)},
\]  

(72)

where \(a\) is the speed of sound,

\[
\Delta t_y = \frac{CFL \cdot \Delta y}{\max(v, v + a, v - a)},
\]  

(73)
\[ \Delta t_z = \frac{CFL \cdot \Delta z}{\max(w, w + a, w - a)}, \]  

and,

\[ \Delta t_v = \frac{1}{2} \frac{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}{\max\left(\mu, \frac{\mu}{Pr}, \frac{\mu}{Sc}\right) + \max\left(\frac{\mu_T}{Pr_T}, \frac{\mu_T}{Sc_T}\right)}. \]

One-Dimensionalization

In order to make comparisons between SRGULL, the axisymmetric VULCAN, and 3-D VULCAN the axisymmetric and 3-D case are one-dimensionalized using a utility provided by VULCAN. The results are one-dimensionalized using mass-averaging. This is done using the mass flux through a cell and is defined by,

\[ \dot{f}_{iD} = \frac{\int \rho uy dy}{\int \rho uy dy}. \]

The one-dimensional utility also calculates the chemical efficiencies. This study will use 2 of the 5 methods for determining combustion efficiency. The first is

\[ \eta^1_c = 1 - \frac{\dot{m}_F}{\dot{m}_{F,inj}}, \]

where \( \dot{m}_F \) is the mass flow rate of the fuel at a specific x-location and \( \dot{m}_{F,inj} \) is the injected mass flow rate of the fuel. This is a measure of efficiency based on the amount of fuel that is depleted. The second method used is,
\[ \eta_C = \frac{h(T_{REF}, Y_i) - h(T_{REF}, Y_{REF})}{h(T_{REF}, Y_{i,\text{IDEAL}}) - h(T_{REF}, Y_{REF})}, \quad (78) \]

which is a measure of efficiency based on how much heat is released versus an ideal heat release. \( Y_{i,\text{IDEAL}} \) is determined from the local static pressure and enthalpy. \( T_{REF} \) and \( Y_{REF} \) are reference values from the inlet throat. Another important parameter that the one-dimensionalization utility calculates is the stream thrust, \( F_{ST} \), which is defined by,

\[ F_{ST} = \int (\rho u^2 + P) dA. \quad (79) \]

The total uninstalled thrust can then be determined by subtracting the stream thrust at the exit from the stream thrust at the inlet.
CHAPTER 4

RESULTS

Introduction

The geometry used by Tam et. al.\textsuperscript{22} in their study will first be analyzed using SRGULL and then by VULCAN using the axisymmetric option. These results will be compared to the 3-D sector results of Tam et. al. in order to see the compatibility of the lower order methods for design purposes. The computational grid used by Tam will be utilized for this portion of the study. The next step will be to use SRGULL to determine potential fuel staging locations for study in axisymmetric and 3-D VULCAN at Mach 3 flight conditions. The conditions used in this study are given in table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>(M_\infty)</td>
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<tr>
<td>(Q) (psf)</td>
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<td>(T_T) (K)</td>
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<td>(P_T) (kPa)</td>
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<td>(P_{throat}) (kPa)</td>
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For Mach 3 flight conditions the throat Mach number would be 1.536 with a total pressure of 337 kPa and a total temperature of 670 K. The static pressure and temperature at the throat are 59.05 kPa and 414.7 K. Because the lowest Mach number facility nozzle in RC22 is 1.8, the total conditions were changed such that the same static conditions would exist at the throat.

In order to perform the VULCAN CFD study the computational grid supplied by Tam will be modified with the coordinates for the Mach 1.8 facility nozzle. A second injector and cavity will then be added at the new downstream location determined by SRGULL. First an axisymmetric study in VULCAN will be performed without the second injector in order to get a baseline to compare too. This will be done at a fuel equivalence ratio of approximately 0.5. This will also be performed as a sector analysis to get more comparative data between axisymmetric and 3-D VULCAN results. Then the combustor will be studied using the staged injection, injecting a fuel equivalence ratio of approximately 0.4 in the first injector and fuel equivalence ratios in the second injector to give total fuel equivalence ratios of approximately 0.5, 0.6, and 0.8. The $\phi=0.8$ case will then be studied without the cavity to see if similar performance can be achieved. This case will also be studied with a 3-D sector analysis. The $\phi=0.8$ case will also be used to study grid convergence and time-step convergence. A study using a coarsening factor of 2 as well as a refining factor of 2 for the grid will be done. The convergence study will also be run at a time step of $1/4^{th}$ and a time step 4 times the baseline 3e-7 second time step.
Grid and Boundary Conditions

The 2-D axisymmetric one injector simulations were performed using 15,800 grid cells and the 3-D sector simulation had 269,875 grid cells. For the 2 injector cases the 2-D grid had 21,835 grid cells and the 3-D grid had 372,350 grid cells. A subsonic inflow boundary condition is used at the facility nozzle inlet. The total density, 1.9134 kg/m3, and total pressure, 378445 Pa, are defined to give the proper conditions at the throat. Turbulence intensity is 0.01 and there is a turbulent viscosity ratio of 0.1. The walls of the combustor are set as adiabatic no slip walls. The exit of the combustor is a supersonic outflow. The injector inflow is defined using a subsonic inflow boundary condition. For 2-D flows static density and total temperature are define to give the desired fuel mass flow rate. For 3-D flows the subsonic inflow boundary condition defining total density and total pressure is used. The injector walls are set up as slip walls.

Solution Methodology

First a “cold flow” solution is obtained without reactions. Once a converged solution is determined the injection block is turned on. This is accomplished by applying a temperature of 2500 K to the cavity, the first cavity only in staged configurations. After approximately 2000 iterations the injection block is turned off. The solution is then run at a CFL of 4.0 to convergence. Convergence is determined mainly by observing the mass conservation. Mass is considered to be conserved when the 1-D mass flow rate remains constant except for a rise when fuel is injected. The residual, $L_2$, is defined by,

$$L_2 = \sqrt{\frac{\sum (RHS)^2}{N_{\text{cells}}}},$$

(80)
where $N_{\text{cells}}$ is the number of cells. RHS represents the right hand side of the continuity, momentum, and energy equations where the left hand side is the unsteady term. So ideally when steady state is achieved, the RHS would be zero. This residual is also observed for convergence. A steadily declining residual is an indication of a solution that is converging. In some cases the residual will get to a point where it oscillates. At this point the solution is set to a constant time step of $3\times 10^{-7}$ seconds, which is between the minimum and maximum time step sizes from the constant CFL solution. If the nature of the residual remains oscillatory a time averaged solution is taken over one period if the oscillations are periodic. If the oscillations are not periodic, then the solution is monitored every 20,000 iteration for 100,000 iterations if the shock location remains constant then a time averaged solution is taken over 50,000 and 100,000 iterations if these two results are the same a pseudo steady state solution is assumed. A representative case can be seen in figures 8 through 10.
Figure 8. Mass Flow Rate at different numbers of iterations

Figure 9. Pressure at different numbers of iterations
Convergence Studies

In order to assess the accuracy of the results time and grid convergence studies were performed on the total fuel equivalence ratio of 0.8 case. Time convergence studies were performed with a four times larger time step and a one fourth smaller time step. The larger time step was time averaged over 25,000 iterations and the smaller time step was time averaged over 400,000 iterations such that all three were averaged over the same total time. The one-dimensional properties seen in Figures 11 through 13 show that the time step used was not too large, and in fact in future studies it may be possible to use a larger time step to decrease computational time.
Figure 11. One-dimensional Mach number for time convergence study

Figure 12. One-dimensional Pressure for time convergence study
The grid convergence study was performed by refining the grid by a factor of two and by coarsening it by a factor of 2. A comparison on the one-dimensional properties can be seen in Figures 14 through 16. The coarse grid showed a great disparity with the nominal grid in terms of the shock location from the first injection whereas the fine grid matches well with the nominal grid almost everywhere. The only place where there is a big difference between the fine grid and the nominal grid is in the location of the shock from the second injection. Even with this disparity there was only a 1.1% difference in the combustion efficiency and a 1.6% difference in the net stream thrust. The difference between coarse and fine was 4.1% for efficiency and 5.0% for stream thrust. Plots of this can be seen in figures 17 and 18.
Figure 14. One-dimensional Mach number for grid convergence study

Figure 15. One-dimensional Pressure for grid convergence study
Figure 16. One-dimensional Temperature for grid convergence study

Figure 17. $\eta_c^3$ for different grid sizes
The times required for attaining the solution for the three grids can be seen in table 3.

TABLE 3

COMPUTATION TIMES FOR VARIOUS GRID DENSITIES

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<th>Grid</th>
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<tr>
<td>Coarse</td>
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<tr>
<td>Nominal</td>
<td>150</td>
</tr>
<tr>
<td>Fine</td>
<td>630</td>
</tr>
</tbody>
</table>
The nominal grid takes only about 2.5 times as long to run and still takes less than a week, but yields results that are much closer to the fine grid which takes about 3 and a half weeks to run.

**Mach 5 Validation Study**

The first thing accomplished was to compare results from Tam et al.\textsuperscript{22} with SRGULL and axisymmetric VULCAN results. This case was at a flight Mach number of 5 and a fuel equivalence ration of 0.5. The SRGULL study was performed by changing the combustion efficiency distribution and separation reattachment location until results matched well with the 3D sector analysis. Figures 19 through 21 show plots of the one-dimensional pressure, temperature, and Mach number versus distance for all three calculations.

![Figure 19. One-dimensional Mach Number for Mach 5 case](image-url)
Figure 20. One-dimensional Pressure for Mach 5 case

Figure 21. One-dimensional Temperature for Mach 5 case
The shock location and peak pressure rise match well for SRGULL and the sector analysis after proper selection of the combustion efficiency and reattachment location in SRGULL. Since the proper combustion efficiency and reattachment location for matching are known, the SRGULL code will be used with these values to study various injector and cavity locations and mechanical throat designs to increase the operability down to Mach 3. Downstream of the cavity flame holder (x=1.21m) there is better agreement between the axisymmetric and sector analyses. This is because the SRGULL case had less heat release from combustion than the VULCAN cases. One way to look at heat release is to look at how much the total temperature changes. This can be seen in figure 22.

Figure 22. One-dimensional Total Temperature for Mach 5 case
There is a significant difference between the shock locations for the sector and axisymmetric VULCAN cases. This difference can be attributed to the decrease in mixing efficiency in the axisymmetric case because the fuel is being modeled as a slot all the way around the combustor instead of 8 separate injectors. In order to make use of the three-dimensional grid to create the two-dimensional grid, the diameter of the circular injector from the sector case became the width of an injector slot that is rotated all the way around the combustor. Since the mass flow rate of fuel is kept constant the fuel in the axisymmetric case is moving much slower since the area is much larger. This leads to less penetration in to the flow by the fuel and also less disturbance of the flowfield by the fuel making the mixing less efficient. Also in the 3D model there is swirl that aids in mixing. Agreement could be improved by using a smaller turbulent Schmidt number to increase turbulent mixing or by adding more fuel to increase the amount of heat release in the axisymmetric simulation.

There is also a large difference in the location of the flame for the axisymmetric and sector cases. This can be seen in the static temperature contours line and the fuel equivalence ratio contours from the axisymmetric and sector models are shown in figures 23 through 26. The temperature contours also show the line where the fuel equivalence ratio is equal to 1; this is where the fuel is most likely to ignite if hot enough.
Figure 23. Temperature contours with $\phi=1$ line for axisymmetric Mach 5 case

Figure 24. Temperature contours with $\phi=1$ line for sector Mach 5 case (Tam 2007)
Figure 25. $\phi$ contours for axisymmetric Mach 5 case

Figure 26. $\phi$ contours for sector Mach 5 case (Tam 2007)
In the axisymmetric case, combustion is initiated upstream of the cavity flame holder, whereas in the sector case ignition occurs in the cavity. This stabilization of the flame at the injector is due to a coincidence of perfect conditions. The way the fuel is ignited using the cavity as an ignition block along with the flow disturbance cause by the cavity creates a high temperature reverse flow region. The reversed flow convects the high temperature products upstream. In the axisymmetric case the fuel equivalence ratio equal to 1 line is located closer to the top boundary so when the hot products move upstream they ignite the fuel. In the sector case the fuel penetrates farther into the flow field so when the hot products convect upstream they are in a fuel rich region and the flow cannot be ignited. Contours of temperature and heat release for the ignition sequence case can be seen in figures 27 and 28.

Figure 27. Mach 5 axisymmetric ignition sequence Temperature contours
Figure 28. Mach 5 axisymmetric ignition sequence Heat Release contours

**Mach 3 Baseline Study**

The Mach 3 flight conditions have been run with the original geometry in both SRGULL and in a VULCAN axisymmetric case. In the SRGULL study the injection location was moved to different locations downstream. It was determined that moving the injection downstream allowed for a higher efficiency distribution, i.e. greater overall heat release. Results of this can be seen in figure 29,
where, $\eta_{\text{max}}$ refers to the efficiency distribution used in the Mach 5 case. From this study it was determined that a second injector and cavity should be added 0.8 meters downstream of the initial injector and cavity for study in VULCAN. Before doing this the original geometry was studied axisymmetrically in VULCAN and will be studied as a 3D sector in order to have a baseline for comparison.

In the axisymmetric case the shock occurred approximately 0.1 meters downstream of the inlet throat avoiding a case of inlet unstart. The flame was also stabilized in the cavity and there was no blowout. These results were surprising because it was thought that a thermal throat would not be achieved at this condition, but it is also known from the Mach 5 case that the sector geometry yields increased mixing. Hence, it
is expected that the results of the Mach 3 sector case will have the precombustion shock train extend beyond the inlet throat. As can be seen in the comparisons of the 1-D properties, the shock is right at the throat for the sector case. With the exception of the shock location though there is very good agreement between the axisymmetric and sector results. This can be seen in figures 30 through 32.

Figure 30. One-dimensional Mach number for baseline Mach 3 case
Figure 31. One-dimensional Pressure for baseline Mach 3 case

Figure 32. One-dimensional Temperature for baseline Mach 3 case
The first rise in temperature can be attributed to the shock formed by the combustion in the subsonic flow. The next large rise in temperature is from the heat release of the combustion. The decrease in temperature at the end is from the expansion of the flow out the end of the combustor. Contours of the temperature with the fuel equivalence ratio of 1 line can be seen in figures 33 and 34.

Figure 33. Static Temperature contours with $\phi=1$ line for baseline axisymmetric case
A three-dimensional view of the temperature contours of the sector in Fig. 35 shows the shape of the flame. The temperature rise in the flow downstream eventually spreads to the halfway point between injectors. It can also be seen how the high temperature core of the flow spreads out as the flame moves downstream. The contours of the fuel-air equivalence ratio can be seen in Figure 36. There is a core of fuel-rich flow along the path of fuel injection. This core is surrounded by the region where the local fuel-air equivalence ratio equals one. This is where the fuel will burn if there is enough heat. The streamlines in the injector and cavity region shown in Fig’s. 37 through 40 reveal how the injector affects the free stream flow and how the flow recirculates in the cavity for flameholding. At a distance one quarter the way from the injector midplane to the plane halfway between injectors there is no longer any turning of
the flow from injection. In Fig. 40, it can be seen that at an x location of 1.22 m, there remains hardly any velocity in the z-direction.

Figure 35. Three-dimensional Temperature contours

Figure 36. Three-dimensional $\phi$ contours
Figure 37. Three-dimensional Temperature contours with streamlines

Figure 38. U-velocity contours and streamlines at midplane of injector
Figure 39. U-velocity contours and streamlines at one-quarter distance between midplane of injector and plane halfway between injectors
Mach 3 Staged Injection Study

The next series of studies will include Mach 3 axisymmetric case with the new injector and cavity. In this study a fuel equivalence ratio of approximately 0.4 will be injected from the first injector because the fuel equivalence ratio of 0.5 produced a shock.
right at the throat. Fuel will then be injected from the second injector to yield a desired total fuel equivalence ratio. The three total fuel equivalence ratios studied are 0.5, 0.6, and 0.8. Plots of pressure, temperature, and Mach number versus combustor length for the baseline case and the three staged injection cases can be seen in figures 41 through 43.

Figure 41. One-dimensional Mach number for axisymmetric Mach 3 cases
Figure 42  One-dimensional Pressure for axisymmetric Mach 3 cases

Figure 43. One-dimensional Temperature for axisymmetric Mach 3 cases
It can be seen from the one-dimensional properties that with staged injection if the fuel equivalence ratio in the first injector remains the same the shock location is constant, and in this case is approximately 0.2 meters downstream of the inlet throat. A reason for this can be seen in figure 41; the shock formed by the second injection does not move past the thermal throat of the first injection. Therefore in these cases there are actually two thermal throats in the combustor.

Another means of comparison of the different cases is to look at the combustion efficiency. The combustion efficiencies can be seen in figures 44 and 45.

![Figure 44. $\eta_c^1$ for axisymmetric Mach 3 cases](image)

$\eta_c^1$ for axisymmetric Mach 3 cases
The $\eta_c^3$ values are pretty much the same for each case, but for $\eta_c^3$ as the value of $\phi$ increases the combustion efficiency decreases. To get a better understanding of which cases are burning the most fuel the reacted $f$ is calculated by,

$$\phi_{\text{reacted}}^3 = \eta_c^3 \phi,$$

(81)

the results of this can be seen in figure 46.
The values of Stream Thrust of the different cases can be seen in table 4.

<table>
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<th>Case</th>
<th>$F_a$ (N)</th>
<th>% increase</th>
</tr>
</thead>
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</table>

It can be seen that as long as combustion can be maintained, better performance is achieved by injecting all of the fuel upstream. The advantage is the ability to inject more
fuel by using staged injection. In a system level study it would be important to determine if the added weight from more fuel and the extra injector and cavity would be worth the increase in thrust that is gained. The temperature contours with fuel equivalence ratio equal to 1 line for the three staged injection cases can be seen in figures 47 through 50.

Figure 47. Static Temperature contours with $\phi=1$ line for first injector for all staged injection cases
Figure 48. Static Temperature contours with $\phi=1$ line for second injector for $\phi_2=0.051$ case

Figure 49. Static Temperature contours with $\phi=1$ line for second injector for $\phi_2=0.184$ case
When looking at the temperature contours it can be seen that the fuel from the second injector ignites right along the fuel equivalence ratio of 1 line due to the amount of heat in the flow from the fuel combustion of the first injector. Because of this it may be unnecessary to have the cavity for flame holding. The total fuel equivalence ratio of approximately 0.8 was run without the second cavity, but a converged solution was unable to be attained using the nominal grid from this study.
CHAPTER 5

CONCLUSION

A study was performed to increase the range of a dual mode scramjet combustor to Mach 3. A combustor geometry that has been studied at Mach 5 flight conditions was run at Mach 3 flight conditions with a fuel equivalence ratio of 0.488 and showed sustained combustion. In order to increase the amount of thrust, staged injection was used with a second injector and cavity located 0.8 meters downstream of the first injector. A fuel equivalence ratio of 0.437 was injected at the first location and fuel equivalence ratios of 0.051, 0.184, and 0.369 were injected at the second location. For the same total fuel equivalence ratio more thrust was produced when all of the fuel is injected upstream, but when more total fuel is injected more thrust can be achieved. For the fuel equivalence ratio of 0.369 injected from the second injector, an 11.6% increase in thrust was achieved. It was seen that downstream combustion was stabilized at the injector not in the cavity, so it may be possible to operate without the second cavity. Studies of time step size showed no change in the one-dimensional properties for the different time step sizes. The grid size study showed a much greater difference in shock location between the coarse and nominal case than between the fine and nominal case. Downstream of the shock there was little variation between the three grids with the exception of the second shock location. This has little effect on the performance characteristics between the nominal and fine case, but in future studies more refinement is needed in the second
injector region. A comparison of two-dimensional and three-dimensional results for the nominal one injector case showed very little variation in the one-dimensional properties other than the shock location.

**Future Study**

After this study, there is still much more work in the realm of low speed dual mode scramjet engines. The first thing to be done is to get a converged solution of the combustor for the case without the second cavity. If the removal of the cavity can be shown to not adversely affect performance than it would be able to be left out of any other future studies. Also related to this study, follow on work is needed to determine if more fuel can be added at the second injector, and the effect that has on the thermal throat. More study should also be done in VULCAN to look at the second injector location to determine if more thrust can be produced moving the location upstream without affecting the shock location. A fuel less reactive than ethylene, which is the most reactive of the hydrocarbons, such as JP-7 should be studied since sustaining combustion would be more difficult at the low supersonic speeds. It is also necessary to look at extending the operability of the dual mode scramjet engine down to Mach 2.5. This may require additional complexity to the flow path geometry such as a physical throat at the combustor exit in order to help accelerate the flow to sonic.
### APPENDIX A

#### SRGULL INPUT FILE

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3000.0 394.00
5.0 0.1524 0.0 0.1524 400.0 015.0 100.0
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0.0254 3.078 1.0 0.1524 0.1524 0.1524
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122.18425 0.00
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APPENDIX B

VULCAN INPUT FILE

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'**
'** Input control data  **
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'm18nozz_2inj_axi_split.grd'
'GRID FORMAT'  3.0  (1=sb form, 2=sb bin, 3=mb form, 4=mb bin)
'RESTART IN'  0.0  (input restart file name to follow)
'restart.out'
'RESTART OUT'  0.0  (input restart file name to follow)
'restart.out'
'RESTART OUT INTERVAL'  200.0  200.0
'WARNING MESSAGES'  3.0  (0=off, 1=wf warnings, 2=temp warnings, 3=all)
'PLOT ON'  4.0  (1=sb form, 2=sb bin, 3=mb form, 4=mb bin)
'32 BIT BINARY'  0.0  (write plot files as 32 or 64 bit binary)
'PLOT NODES'  0.0  (nodes or cells; multi-block needs nodes)
'PLOT FUNCTION'  11.0  (no greater than no. of conserved variables)
'DENSITY'
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'HEAT RELEASE'
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'C2H6' 'CH2CO' 'C3H6' 'N2'

$------------------- Chemical species information --------------------$

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$---------------------- Reference condition data ---------------------$

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'TOTAL TEMP.'  3.00000e+02
'TOTAL RHO'  2.00000e+01
'REFERENCE LENGTH'  1.00000e+00
'LAM. PRANDTL NO.'  7.20000e-01
'LAM. SCHMIDT NO.'  1.00000e+00
'TURB. PRANDTL NO.'  8.90000e-01
'TURB. SCHMIDT NO.'  5.00000e-01
'TURB. MODEL'  0.0
'MENTER'  0.0
'TURB. INTENSITY'  1.0e-02
'TURB. VISC. RATIO'  1.0e-01
'NO 2/3 RHOK IN REY. STRESS'  0.0
'INIT. MIN. DIST.'  0.0
$----------------------- Turbulence model data -----------------------$

'PROCESSORS'  08.0

75
| ONE | 26 | J | MAX | I | MIN | 22 | K | MIN | MAX | 0 | SYMM |
| ONE | 22 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 | SYMM |
| ONE | 26 | J | MAX | I | MIN | 22 | K | MIN | MAX | 0 | SYMM |
| ONE | 21 | J | MIN | I | 11 | MAX | K | MIN | MAX | 0 | SYMM |
| ONE | 25 | J | MAX | I | 69 | MAX | K | MIN | MAX | 0 | SYMM |
| ONE | 21 | J | MIN | I | MIN | 11 | K | MIN | MAX | 0 | SYMM |
| ONE | 25 | J | MAX | I | 63 | 69 | K | MIN | MAX | 0 | SYMM |
| ONE | 20 | J | MIN | I | MIN | 11 | K | MIN | MAX | 0 | SYMM |
| ONE | 25 | J | MAX | I | 32 | 63 | K | MIN | MAX | 0 | SYMM |
| ONE | 19 | J | MIN | I | MIN | 11 | K | MIN | MAX | 0 | SYMM |
| ONE | 25 | J | MAX | I | 17 | 48 | K | MIN | MAX | 0 | SYMM |
| ONE | 17 | J | MIN | I | MIN | 17 | K | MIN | MAX | 0 | SYMM |
| ONE | 25 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 | SYMM |
| ONE | 18 | J | MIN | I | MIN | 32 | K | MIN | MAX | 0 | SYMM |
| ONE | 17 | J | MIN | I | MIN | 17 | K | MIN | MAX | 0 | SYMM |
| ONE | 25 | J | MAX | I | MIN | 21 | K | MIN | MAX | 0 | SYMM |
| ONE | 11 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 | SYMM |
| ONE | 26 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 | SYMM |
| ONE | 22 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 | SYMM |
| ONE | 26 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 | SYMM |

| OUTFLO | 27 | I | MAX | J | MIN | MAX | K | MIN | MAX | 0 | EXTRAP |
| OUTFLO | 24 | I | MAX | J | MIN | MAX | K | MIN | MAX | 0 | EXTRAP |
| CUT NAME | BLK | FACE | PLACE | IND1 | BEG | END | IND2 | BEG | END | IN-ORD |
| ONE | 1 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 4 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 2 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 4 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 3 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 4 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 5 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 14 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 6 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 14 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 7 | J | MIN | I | MIN | 27 | K | MIN | MAX | 0 |
| ONE | 14 | J | MAX | I | MIN | 53 | K | MIN | MAX | 0 |
| ONE | 7 | J | MIN | I | MIN | 27 | K | MIN | MAX | 0 |
| ONE | 15 | J | MAX | I | MIN | 6 | K | MIN | MAX | 0 |
| ONE | 8 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 15 | J | MAX | I | MIN | 6 | K | MIN | MAX | 0 |
| ONE | 9 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 15 | J | MAX | I | MIN | 6 | K | MIN | MAX | 0 |
| ONE | 10 | J | MIN | I | MIN | 12 | K | MIN | MAX | 0 |
| ONE | 15 | J | MAX | I | MIN | 6 | K | MIN | MAX | 0 |
| ONE | 10 | J | MIN | I | MIN | 12 | K | MIN | MAX | 0 |
| ONE | 16 | J | MAX | I | MIN | 21 | K | MIN | MAX | 0 |
| ONE | 11 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 |
| ONE | 16 | J | MAX | I | MIN | 21 | K | MIN | MAX | 0 |
| ONE | 11 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 |
| ONE | 17 | J | MIN | I | MIN | 17 | K | MIN | MAX | 0 |
| ONE | 17 | J | MIN | I | MIN | 17 | K | MIN | MAX | 0 |
| ONE | 17 | J | MIN | I | MIN | 17 | K | MIN | MAX | 0 |
| ONE | 25 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 19 | J | MIN | I | MIN | MAX | K | MIN | MAX | 0 |
| ONE | 25 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 25 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 25 | J | MAX | I | MIN | 32 | K | MIN | MAX | 0 |
| ONE | 21 | J | MIN | I | MIN | 11 | K | MIN | MAX | 0 |
| ONE | 25 | J | MAX | I | MIN | 69 | K | MIN | MAX | 0 |
| ONE | 21 | J | MIN | I | MIN | 11 | K | MIN | MAX | 0 |
| ONE | 26 | J | MAX | I | MIN | 22 | K | MIN | MAX | 0 |
| ONE | 22 | J | MIN | I | MIN | 16 | K | MIN | MAX | 0 |
| ONE | 26 | J | MAX | I | MIN | 22 | K | MIN | MAX | 0 |
```
| ACUT59 | 43 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT58 | 42 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT57 | 41 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT56 | 40 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT55 | 39 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT54 | 38 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT53 | 37 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT52 | 36 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT51 | 35 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT50 | 34 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT49 | 33 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT48 | 32 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT47 | 31 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT46 | 30 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT45 | 29 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT44 | 28 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT43 | 27 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT42 | 26 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT41 | 25 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT40 | 24 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT39 | 23 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT38 | 22 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT37 | 21 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT36 | 20 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT35 | 19 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT34 | 18 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT33 | 17 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT32 | 16 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT31 | 15 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT30 | 14 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT29 | 13 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT28 | 12 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT27 | 11 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT26 | 10 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT25 | 9  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT24 | 8  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT23 | 7  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT22 | 6  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT21 | 5  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT20 | 4  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT19 | 3  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT18 | 2  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT17 | 1  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT16 | 0  | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT15 | 53 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT14 | 52 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
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| ACUT12 | 50 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT11 | 49 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT10 | 48 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT9  | 47 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT8  | 46 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT7  | 45 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT6  | 44 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT5  | 43 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT4  | 42 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT3  | 41 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT2  | 40 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ACUT1  | 39 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| ONE    | 38 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| SEVEN  | 37 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| EIGHT  | 36 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| SEVEN  | 35 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| EIGHT  | 34 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| SEVEN  | 33 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
| EIGHT  | 32 | I | MIN | J | MIN | MAX | K | MIN | MAX | 0  
```

APPENDIX C
SRGULL INITIAL STUDIES

Initial studies were performed with SRGULL in order to gain a familiarity with the program. The studies used the Quicksat geometry. The first study was a comparison between some results that were obtained from a previous version of SRGULL and results obtained from the current version. Table C1 shows good agreement of some of the flow variables at different locations in the engine.

<table>
<thead>
<tr>
<th>Location</th>
<th>Mach</th>
<th>Dynamic Pressure (Pa)</th>
<th>Static Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ver. 1</td>
<td>Ver. 2</td>
<td>%</td>
</tr>
<tr>
<td>Free Stream</td>
<td>4.2500</td>
<td>4.2500</td>
<td>0.00</td>
</tr>
<tr>
<td>Cowl Leading Edge</td>
<td>3.1099</td>
<td>3.1097</td>
<td>0.01</td>
</tr>
<tr>
<td>Inlet Throat</td>
<td>2.3593</td>
<td>2.3276</td>
<td>1.34</td>
</tr>
<tr>
<td>Combustor Exit</td>
<td>1.8179</td>
<td>1.7736</td>
<td>2.44</td>
</tr>
</tbody>
</table>

There was however very poor agreement in the combustor for friction and heat loss. That comparison can be seen in table C2. There must have been a change in the way these two parameters were calculated from version 1 to version 2 to account for such
a big difference. Another useful comparison is that of the thrust and specific impulse. SRGULL calculates these parameters two ways one is using additive drag and the other uses the pressure integral. The values determined from the pressure integral in

<table>
<thead>
<tr>
<th></th>
<th>Version 1</th>
<th>Version 2</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustor Friction Loss (N)</td>
<td>32,153</td>
<td>17,526</td>
<td>45.49187</td>
</tr>
<tr>
<td>Combustor Heat Loss (J/s)</td>
<td>101,770,000</td>
<td>56,395,000</td>
<td>44.58583</td>
</tr>
</tbody>
</table>

both versions match well, but the values determined by the additive drag method show a big discrepancy between versions 1 and 2. In version 2 the thrust and specific impulse calculated by both methods match well. These comparisons can be seen in table C3.

<table>
<thead>
<tr>
<th>Version</th>
<th>Thrust (N)</th>
<th>Specific Impulse (s)</th>
<th>% Difference</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Additive Drag</td>
<td>Pressure Integral</td>
<td>Additive Drag</td>
<td>Pressure Integral</td>
</tr>
<tr>
<td>1</td>
<td>1661100</td>
<td>1464400</td>
<td>13.43</td>
<td>1583.3</td>
</tr>
<tr>
<td>2</td>
<td>1444100</td>
<td>1472600</td>
<td>1.94</td>
<td>1376.2</td>
</tr>
<tr>
<td>% Change</td>
<td>13.06</td>
<td>0.56</td>
<td></td>
<td>13.08</td>
</tr>
</tbody>
</table>

The second study performed looked at the differences between the normal shock isolator model and the shock train isolator model. This study used a flight Mach number of 4.25, a free stream dynamic pressure of 2000 psf, and a fuel equivalence ratio of 0.82. The Mach number and pressure through the combustor can be seen in figures C1 and C2.
The normal shock model produces a much higher pressure rise that the shock train model and also decelerates the flow significantly more. In fact in the shock train model the flow does not go subsonic anywhere in the combustor.

Figure C1. Mach number for isolator comparison
The next study performed looked at the effect of reattachment location on the solution. This study also used a flight Mach number of 4.25, a free stream dynamic pressure of 2000 psf, and a fuel equivalence ratio of 0.82. This study also used the shock train model for the isolator. The Mach number and pressure comparisons can be seen in figures C3 and C4. It can be seen that as the reattachment location moves farther downstream the shock moves farther upstream.
Figure C3. Mach number for reattachment location comparison

Figure C4. Pressure for reattachment location comparison
The final study looked at the effect of the fuel equivalence ratio on the solution. This study also used a flight Mach number of 4.25 and a free stream dynamic pressure of 2000 psf. The shock train model was used with a reattachment location of 3.9423 m. Mach number and pressure comparisons can be seen in figures C5 and C6. It can be seen that for a fuel equivalence ratio of 0.5 not enough heat is released to produce the shock train upstream.

Figure C5. Mach number for fuel equivalence ratio study
Figure C6. Pressure for fuel equivalence ratio study
REFERENCES


