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Effect of Material Anomalies on Fatigue Life of Turbine Disks

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EFFECT OF MATERIAL ANOMALIES ON FATIGUE LIFE OF TURBINE DISKS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

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ABSTRACT

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There is an economic need to extend the fatigue life of turbine engine rotor disks. The probability of failure during the operation life must be quantified as components remain in service beyond their traditional safe-life limit. Fracture mechanics based probabilistic methods are utilized to predict the probability of failure of components containing manufacturing and fatigue anomalies. Total fatigue life is defined in terms of crack initiation and propagation phases. A micromechanical initiation model uses material properties at the micro-scale to characterize the initiation phase, while short and long fatigue-crack growth models predict the crack propagation phase. A Monte Carlo simulation determines the fatigue-life variability by modeling random material properties in the fatigue models. This methodology is applied to a representative α+β alloy (Ti-6Al-4V) fan disk to quantify the probability of failure due to manufacturing and fatigue induced anomalies. It is concluded that fatigue damage increases the risk beyond the safe-life limit, but proper inspection planning can maintain the risk and enhance the life of components.
LIST OF SYMBOLS

\( a \)  Crack depth
\( a_c \)  Critical crack size
\( a_i \)  Initial crack size
\( a_0 \)  Short crack parameter
\( \sigma_{ij} \)  Stress tensor
\( \Delta K_I \)  Mode I stress intensity factor range
\( \Delta K_{th} \)  Threshold stress intensity factor range
\( K_{lc} \)  Fracture toughness
\( \sigma_{ys} \)  Yield stress
\( \sigma_e \)  Endurance limit
\( F(a) \)  Boundary-correction factor
\( R \)  Stress ratio
\( \mu \)  Shear modulus
\( \lambda \)  Universal constant of 0.005 for the initiation model
\( h \)  Slipband width
\( D \)  Grain size
\( \alpha \)  Initiation model exponent
\( r_p^* \)  Monotonic or cyclic plastic zone size
\( N_r \)  Total fatigue life
\( N_i \)  Fatigue initiation life
\( N_p \)  Fatigue propagation life
\( p_c^f \)  Conditional probability of failure
\( P_f \)  Unconditional probability of failure
\( a_i \)  Anomaly occurrence rate
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1. Introduction

Turbine disks are regarded as one of the most critical flight safety components in aero-engines. Traditionally, turbine disks are designed for a finite life on the basis of a low-cycle fatigue (LCF) criterion under the safe-life approach. The safe-life approach considers components to be automatically exhausted after 0.1% of the components have initiated a crack of 0.79mm (1/32 in) in length [1]. In some turbine disk materials, premature fatigue crack initiation occurs due to handling, machining damage, fretting, or inherent defects causing cracks to propagate and become critical during the predicted "safe-life" [2]. Turbine disk failures resulting from anomalies, such as the catastrophic crash in Sioux City in 1989, have led to the introduction of an additional damage tolerance approach in the life management process. The damage tolerance approach employs fracture mechanics to assure adequate fatigue crack propagation lives between inspection intervals.

Due to the conservative nature of these methods, there is a considerable economic incentive to extend the service life of these components. The United State Air Force (USAF) has developed a Retirement for Cause (RFC) method to allow components to be extended beyond their traditional LCF life [6]. The RFC method utilizes the concepts of damage tolerance to establish inspection schedules and retires components when a defect is found during an inspection. Although RFC reduces component life-cycle cost, the potential risk of failure increases as components remain in service longer. Thus, the risk of failure must be accurately quantified before components are successfully extended.
DARWIN is a Federal Aviation Administration (FAA) approved design certification tool created to quantify the risk of components subject to inherent anomalies. In this study, DARWIN’s limitations are addressed by including fatigue damage in the risk analysis to improve the risk assessment for component life extension. Fatigue damage is assumed to consist of the entire range of damage accumulation from crack initiation to final fast fracture. Chapter 2 discusses fatigue damage concepts that are used later in the study to develop fatigue damage models. Fatigue models are developed in Chapter 3 to model crack initiation and propagation stages. A micromechanical initiation model relates crack initiation life to crack depth, while the fatigue propagation life is modeled using both long and short crack growth models. Finally, the fatigue damage methodology is applied to DARWIN to determine the probability of failure of a fan disk subject to fatigue induced damage.
2. Literature Review

The purpose of this chapter is to provide the reader with an overview in the concepts of design and analysis of turbine disks. The first section discusses the three major design philosophies addressing each of their advantages and limitations. Next, fatigue damage mechanisms in turbine disk material are overviewed. The following sections review the analytical modeling of fatigue damage using linear elastic fracture mechanics (LEFM) and its application to fatigue crack growth models. The role of short crack growth in life management of turbine engines and the limitations of LEFM are addressed. The fatigue damage concepts established in this chapter are utilized in the following chapter to predict the total fatigue life of turbine disks.

2.1 Life Management Methods

Since the late 1960’s, the safe-life design approach has been utilized to determine the LCF life of rotating turbine components [1]. Under this method, component lives are estimated from statistical data obtained from limited material and component fatigue testing. These tests determine the number of cycles needed to initiate a crack, typically a 0.8 mm surface crack, using a load spectrum that best represents the service flight profile. The estimated life has a built-in safety factor to account for unknowns such as loading conditions, scatter in test results, material property variations, and existence of initial defects [2]. Components are retired from service when one out of 1000 components are assumed to have developed a detectable crack (Figure 1).
Figure 1: Safe-life design approach [3]

One of the disadvantages of the safe-life approach is the over conservative estimation of fatigue life. By definition, 999 out of 1000 components are retired with remaining residual fatigue life. Kappas [4] estimated that over 80% of engine rotor disks have ten or more LCF lives remaining when retired under safe-life. Replacing expired component populations is costly and is a significant contribution factor to life cycle costs. However, the main shortcoming of safe-life, in the life management of rotor components, is the failure to explicitly address inherent defects. Service or manufacturing rare defects can grow to a critical size faster than the cycles to initiation predicted under safe-life. Rare defects have led to several incidents including the loss of the DC-10 at Sioux City in 1989 due to a hard alpha defect [5]. These incidents have led to the application of a damage tolerance approach in the life management process.

The damage tolerance design method applies fracture mechanics principles to predict component fatigue life and quantify the inspection intervals. Unlike crack initiation in safe-life, damage tolerance assumes components contain inherent material or service-induced defects in fracture critical locations. Fracture mechanics is used to determine the safety limit (SL), or the number of cycles for an initial crack size, $a_i$, to
grow to a critical crack size, \( a_c \). The initial crack size is typically taken to be just below the detection limit of non-destructive inspection (NDI) techniques. Inspection schedules are established based on the safe inspection interval (SII) by applying a safety factor to the SL, as demonstrated in Figure 2. After each SII, the disks are removed from service for inspection. If indications of cracks are found during these inspections, the component will be removed from service and replaced with a new component.

**Figure 2:** Damage tolerance design method [3]

Damage tolerance accounts for inherent defects by ensuring that no cracks will reach a critical size between the inspection intervals. However, damage tolerance can be costly since an elaborate NDI infrastructure is required to support component inspections [3]. In addition, damage tolerance is not seen as an alternative to the safe-life approach, but an additional procedure for an enhanced life management process. Although the combination of these approaches has been successful in preventing failures, both use conservative methods to determine component life. The high cost of component
replacement has led to the economic need for component life extension. As discussed next, the retirement for cause method was developed for cost savings.

Retirement for cause (RFC) allows the service life of components to be extended beyond their LCF based safe-life. The RFC method utilizes the damage tolerance concepts to set the safe inspection intervals (SII) and to retire components from service only when there is a specific reason, or cause, for removal. Typically, retirement occurs at an inspection when a flaw of a certain allowable size is detected using NDI techniques, as shown in Figure 3.

![Figure 3: Retirement for Cause][3]

In 1985, the United States Air Force (USAF) implemented a RFC program to manage its F100 aircraft engines. This system replaced classical low-cycle fatigue approaches where entire populations of components were retired, regardless of condition. The economic implications of RFC were substantial. Harris [6] estimated that a cost savings of $1 billion over a nineteen-year period would result from the RFC program, and an additional $655 million savings due to labor and maintenance fuel savings would
result indirectly from the program. Although cost savings are high, the RFC concept is highly dependent on the quality of the NDI techniques. The risk of missing a critical crack during inspection increases as the life of the component is extended [7]. Therefore, a balance must be established between cost savings and the increasing risk of failure. The risk of failure must be accurately quantified before components can be extended beyond their traditional LCF life. The component failure is a result of fatigue damage as discussed in the following sections.

2.2 Fatigue Damage Overview

The main differences among the life-management methods often rest on how fatigue damage is quantitatively treated. There are different stages of fatigue damage where defects may nucleate in an initially undamaged section and propagate to failure. In general, the total process of fatigue failure can be divided into the following five stages: (1) early cyclic formation and damage (2) micro-crack nucleation (3) short crack propagation (4) macro-crack propagation and (5) final critical failure. Typically, the first three stages are referred to as the 'initiation period' while the macro-crack propagation stage is referred as the 'crack growth period' [13]. Initiation and propagation of cracks depend on material, geometry, and stress levels.

Crack initiation typically occurs at a free surface and results from the formation of intrusions, extrusions, and persistent slip bands. At low stress values, the fatigue life is mainly contributed by crack nucleation called high cycle fatigue (HCF). The durability of components subject to HCF can be characterized by S-N curves giving the number of cycles to failure at a specific magnitude of stress. The fatigue limit on the S-N curve defines a loading criterion under which no macroscopic crack will form or the initiated
crack will be arrested [15]. Surface conditions have a large effect on the crack initiation since imperfections such as inclusions, small scratches, and dents can generate stress concentrations that produce crack initiation sites [16]. The initiation period is followed by a transition of crack growth perpendicular to the maximum applied stress [14].

At high stress values, cyclic plastic deformation takes place, and the total fatigue life is dominated by strain accumulation, known as low cycle fatigue (LCF). Fatigue crack propagation behavior is generally characterized by a log-log plot of the crack growth rate (da/dN) versus stress-intensity factor range, ΔK, as shown in Figure 4. The crack propagation curve contains three distinct regions: Region I is mainly short crack, Region II is steady propagation, and Region III is rapid crack propagation leading to failure. Paris et al. [17] was the first to describe the linear region of crack growth using the Irwin stress intensity factor range ΔK.

\[
\frac{da}{dN} = C(\Delta K)^n, \quad \Delta K_{th} < \Delta K < \Delta K_{ic}
\]

(1)

where ΔK_{th} is the fatigue threshold below which the crack will not grow, ΔK_{ic} is the fracture toughness of the material, and C and n are experimentally determined material properties. In the next section, the stress-intensity factor concept is discussed in detail.

**Figure 4:** Three stages of fatigue growth behavior [18]
2.3 Linear Elastic Fracture Mechanics

Linear Elastic Fracture Mechanics (LEFM) is based on a mathematical description of the stress field near the crack tip developed by Irwin [8]. Using isotropic linear elastic assumptions, Irwin expressed the crack tip stress field as a series solution. Equation 2 gives the stress field using polar coordinates with the origin at the crack tip, as shown in Figure 5.

$$\sigma_{ij}(r, \theta) = \left(\frac{k}{\sqrt{r}}\right)f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{m} g_{ij}^{(m)}(\theta)$$  \hspace{1cm} (2)

where $\sigma_{ij}$ is the stress tensor, $r$ and $\theta$ are polar coordinates, $k$ is a constant, and $f_{ij}$ is a dimensionless function of $\theta$ that depends on load and geometry. For distances close to the crack tip, the higher order terms may be neglected and the first term is proportional to $1/\sqrt{r}$. As $r \to 0$ the first term approaches infinity, and stress near the crack tip varies with the $1/\sqrt{r}$ singularity. At this point, it is convenient to replace $k$ by the stress intensity factor $K$, where $K = k\sqrt{2\pi}$.

![Figure 5: Stress field at of the crack tip](image)

A crack can experience three types of loading termed mode I, II and III as illustrated in Figure 6. Mode I is the tensile opening mode, Mode II is the in-plane sliding
mode, and Mode III is the tearing or anti-plane shear mode. Mode I is the most frequent, but a cracked body can experience a combination of two or three modes, referred to as mixed mode loading. The stress intensity factor is given a subscript to indicate the mode of loading. In design, it is assumed that the material can withstand crack tip stresses up to a critical stress intensity factor, $K_{IC}$, before rapid propagation occurs.

![Mode I (Opening), Mode II (In-Plane Shear), Mode III (Out-of-Plane Shear)](image)

**Figure 6:** The three basic modes of loading [9]

Since the stress field approaches infinity at the crack tip ($r \to 0$), it is clear that equation 2 is valid only for a limited region around the crack tip. Rather than bearing an infinite stress, the crack tip material will yield and form a plastic zone surrounding the crack tip. The plastic zone size under monotonic tensile stress can be approximated by

$$r_{pm}^* = \frac{1}{2\pi} \left( \frac{K_I}{\sigma_{ys}} \right)^2$$

(3)

where $\sigma_{ys}$ is the yield stress and $K_I$ is the mode I stress intensity factor [10]. In reality, yield stress will be redistributed across the crack tip, and the plastic zone will be larger than predicted by equation 3 [11-12]. When considering cyclic crack growth, the extent of the cyclic plastic zone size, $r_{pc}$, is approximately a quarter of the size of the monotonic zone as illustrated in Figure 7.
An important result of cyclic crack tip plasticity is known as the crack closure phenomenon. Elber [20] was the first to recognize the crack closure effect by observing the compliance (dΔ/dP) of the fatigue specimens at various loads. At high loads, the compliance agreed with standard formulas for fracture mechanics. While at low loads, the compliance was close to that of a specimen without a crack. According to Elber, the change in compliance was due to contact between the crack surfaces at low loads, referred to as crack closure. He proposed that the crack closure effect decreased the fatigue crack growth by reducing the effective stress-intensity range.

\[
\frac{da}{dN} = C(\Delta K_{eff})^n; \quad \Delta K_{eff} \equiv K_{max} - K_{op}
\]  

(4)

where \(K_{op}\) is the stress intensity factor when the crack opens.

Since Elber's study, there have been five identified mechanisms for fatigue crack closure: plasticity-induced closure, transformation-induced closure, roughness-induced closure, oxide-induced closure, and closure induced by viscous fluid. Plasticity-induced closure and transformation-induced closure mechanisms are considered to be true crack closure mechanisms since they produce residual stresses that force crack faces to close.
Other closure mechanisms involve crack wedging and are better described as residual crack openings rather than crack closures [9]. Crack closure has been used to explain short crack behavior, load interactions under spectrum loading, the influence of residual stresses, microstructure, and environment on fatigue crack growth rate [21]. The next section discusses short crack behavior and the limitations of linear elastic fracture mechanics when predicting crack growth.

2.4 Short Crack Growth

Short fatigue cracks have been known to display anomalous behavior in the short crack region, Region I, of the fatigue crack growth curve, as seen in Figure 4. The growth rates of short cracks can exceed those of large cracks at the same applied stress intensity factor range, and short cracks can propagate at applied stress intensities less than the fatigue threshold for large cracks. Due to the inconsistent behavior, short cracks produce a large amount of scatter in the total fatigue life.

Although there is no precise definition of what crack size is considered "short", the short crack regime corresponds to the transition from the crack nucleation regime to the long crack regime where fracture mechanics concepts apply. Ritchie and Lankford [23] have classified small cracks according to the factors responsible for the deviation from long crack behavior.

1. Microstructurally short cracks ($a<20\ \mu m$) are on the order of grain size and have a continuum mechanics limitation.

2. Mechanically short cracks ($20\ \mu m < a < 1\ mm$) are comparable to the plastic zone size and have a linear-elastic fracture mechanics limitation.
Short cracks play a critical role in the life management of engine components and must be considered to accurately predict component life. A damage tolerant analysis of short cracks using LEFM could over-predict the fatigue life of components. Leis [22] has shown that higher growth rates of short cracks could lead to a magnitude lower lifetime than predicted by LEFM. In addition, the improvement of non-destructive inspection (NDI) techniques may reduce the detectable crack size below short crack behavior. These improvements may lead to important effects in damage tolerance calculations. The requirements of LEFM to accurately predict short crack growth is discussed next.

Fracture mechanics concepts and ΔK threshold start to break down near the short crack region. The fracture mechanics parameter describing the mechanical driving force for fatigue crack growth is based on the ability of that parameter to characterize the actual crack tip conditions. The predicted ΔK value should correspond to the true stress-strain conditions at the crack tip, producing identical fatigue crack growth rates. If this is the case, similitude is said to exist. Similitude implies equivalent crack tip plastic zones and equivalent elastic stress fields. LEFM is based on the formulation of parameters which express crack tip similitude. Similitude must be considered when applying fracture mechanics to fatigue analysis.

Leis et al. [24] have recently reviewed similitude requirements for the application of LEFM to fatigue crack growth. Similitude requires: (1) crack tip plastic zone size is small compared to other dimensions, (2) the plastic zone size is small with respect to the distance over which the first term of the stress field solution is dominate, (3) equivalent $K_{\text{max}}$ and ΔK, (4) equivalent constraint, i.e. plane stress or plane strain, and (5) equivalent crack closure fields.
3. Methods

The fatigue and design of turbine disks are subject to numerous uncertainties. The fatigue damage and crack growth of turbine disk materials show a large amount of scatter due to service conditions (speed, temperature, etc.) and structural properties (material properties, geometry, etc.) [29]. Traditionally, deterministic design methods have been used by assuming worst case scenarios and applying a safety factor to account for these uncertainties [30]. Deterministic methods often produce inconclusive or unrealistic results; therefore, probabilistic methods are employed to quantify random variables and enhance component life reliability.

The first section of this chapter gives an overview of the probabilistic design tool, DARWIN. This design tool is validated using a calibration test provided by the FAA. Next, fatigue damage models are established to describe the entire range of fatigue life from crack initiation to final failure. A Monte Carlo simulation is performed on these fatigue models to determine the fatigue-life variability of a Ti-6Al-4V alloy. Finally, a fatigue damage program is developed to predict the probability of failure due to fatigue induced cracks. This program uses the Monte Carlo simulation, fatigue models, and DARWIN. The program is applied to a representative fan disk geometry to determine the probability of failure due to inherent and fatigue defects. DARWIN is described first since it serves as a basis of the probability of failure calculations used in the fatigue damage program.
3.1 DARWIN

Southwest Research Institute (SwRI) developed probabilistic lifing methods for materials used in gas turbine engines. The introduction of the probabilistic lifing methods was originally motivated by uncontained engine failures at Sioux City, Iowa in 1989 and Pensacola, Florida, in 1996. As a response to the incidents, the Federal aviation Administration (FAA) requested that the Aerospace Industries Association (AIA) Rotor Integrity Sub-Committee (RISC) review available techniques to determine whether a damage tolerance approach could be introduced to reduce the rate of uncontained rotor events. Under the guidance of RISC, SwRI collaborated with four major gas turbine manufactures to address emerging technologies and developed a damage tolerance probabilistic design code, called DARWIN (Design Assessment of Reliability With INspection). This section will discuss the implementation of the probabilistic damage tolerance methodology utilized by DARWIN and the validation process utilized to verify compliance of the assessment tools with respect to the FAA requirements.

3.1.1 Overview

DARWIN is a probabilistic design code that integrates the anomaly distribution data, the probability of detection for inspection techniques, material properties, and finite element analysis to predict the probability of fracture of components containing inherent defects. One of two probabilistic methods, Monte Carlo Simulation or Importance Sampling method, is used to randomly sample the variables from the statistical input distributions to determine if the failure condition is satisfied. DARWIN determines the probability of failure with or without in-service inspection. The various inputs and results are shown on the flowchart in Figure 8.
**Figure 8**: DARWIN random input variables and results [32]

DARWIN's initial capabilities were focused on the hard alpha titanium problem, which led to the Sioux City incident. Schafrik et al. [31] have shown that rare hard alpha defects can be introduced in titanium alloys during the manufacturing processes and serve as crack initiation sites. An anomaly distribution is used to characterize the number of anomalies that exceed a particular area for a given amount of material. The anomaly distribution defines the occurrence rate per disk, which is assumed to correspond to the exceedance value at the minimal anomaly size, $a_{\text{min}}$, as illustrated in Figure 9 [33]. The crack size cumulative distribution function (CDF) is calculated from the anomaly distribution using equation 5 [32].

$$CDF = 1 - \frac{N_d(a) - N_d(a_{\text{max}})}{N_d(a_{\text{min}}) - N_d(a_{\text{max}})}$$  \( (5) \)
where $a_{\min}$ and $a_{\max}$ are the minimum and maximum crack sizes of the anomaly distribution, respectively; and $N_{d}(a_{\min})$ and $N_{d}(a_{\max})$ are the exceedance of the minimum and maximum crack sizes, respectively.

![Anomaly distribution graph](image)

**Figure 9:** Anomaly distribution describes the frequency and size of hard alpha defects

DARWIN utilizes probabilistic sampling methods to generate random defect areas from the anomaly size CDF. The defect area is converted to crack dimensions assuming that the initial crack is circular for embedded cracks, semi-circular for surface cracks, and quarter-circular for corner cracks. Once a crack is defined, a fatigue module determines the fatigue crack growth life using linear elastic fracture mechanics. DARWIN contains libraries of stress intensity factor solutions of various crack configurations and multiple models for FCG rate calculations. The component fatigue life is defined as the number of cycles needed to grow a given crack size until the stress intensity factor reaches the fracture toughness of the material. Failure is assumed to occur when the limit state $g(x)$ reaches zero.

$$g(x) = K_c - K(x)$$ (6)
Inherent anomalies can be located anywhere within the component, so an approximate solution is used to address the uncertainty associated with the anomaly location. The component is subdivided into regions of approximately equal risk, called zones. Each zone is assigned a crack type and is placed in the location that minimizes the crack fatigue life, also called the life limiting location, to maintain a conservative risk estimate. DARWIN makes the assumption that there is only one anomaly per zone. Therefore, the unconditional probability of failure of the disk, \( p_f \), is computed by summing the risk of all the zones.

\[
p_f(\text{disk}) = \sum_{i=1}^{n} p_f^c[\text{zone } i \mid \text{given a defect in zone } i] \cdot a_i \tag{7}
\]

where \( a_i = N_{dl}[a_{min}] \frac{W_i}{W} \)

where \( p_f^c \) is the conditional probability of failure or the condition having a defect; \( a_i \) is occurrence rate in each zone scaled by the zone’s weight, \( W_i \) with the weight of the disk, \( W \).

DARWIN determines the reduction in the risk of components which are subject to in-service inspections using non-destructive inspection (NDI) techniques. Components are retired from service once a crack like defect is detected. The probability of a NDI method detecting a crack of a given size is defined by the probability of detection (POD) curve, as illustrated in Figure 10. The lower and upper limits of the POD correspond to the smallest detectable defect and minimal defect size for a 100% detection rate, respectively. The probability that at least one inspection will detect an anomaly with a size greater than \( a \) subject to \( k \) inspections is,

\[
POD(a_1 \ldots a_k) = 1 - \prod_{i=1}^{k} PND_i(a_i) \tag{8}
\]
where $PND$ (probability of non-detection) is the complementary function of the POD where $a$ can be either area or length.

![Ultrasonic POD curve for a #3 FBH with a 1:1 reject calibration](image)

**Figure 10:** Ultrasonic POD curve for a #3 FBH with a 1:1 reject calibration [33]

### 3.1.2 Validation

DARWIN has been created to address specific advisory circulars (AC) issued by the FAA, including the AC 33.14 for titanium hard alpha and AC 33.70-2 for circular holes. Both advisories include a generic calibration test which allows the manufacturer to assess their analytical tools for the risk calculations. The calibration tests provide an acceptable means for showing compliance with the FAA requirements.

The FAA AC 33.14 calibration test was conducted using DARWIN to validate the level of acceptability of the risk calculations and to gain insights on the intermediate results. The test case includes all the necessary information to determine the probability of fracture of a rotating Ti-6Al-4V disk (Table 1). The disk geometry is subject to a zero to maximum rotational speed of 6,800 RPM and an external pressure load of 7.25 ksi to
simulate blade loading, as shown in Figure 11. The design life of the disk was given to be 20,000 flight cycles. A single ultrasonic inspection is to be performed at 10,000 cycles, where all detected defects are removed.

**Table 1: AC 33.14 Calibration Test Input Values [33]**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Input Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture Mechanics Method</td>
<td>Flight Life – Paris Crack Growth – No Mean Stress</td>
</tr>
<tr>
<td>Fatigue Crack Growth Constant (C)</td>
<td>5.248E-11</td>
</tr>
<tr>
<td>Fatigue Crack Growth Exponent (α)</td>
<td>3.87</td>
</tr>
<tr>
<td>Risk Computation Method</td>
<td>Monte Carlo Simulation @10,000 samples</td>
</tr>
<tr>
<td>Failure Toughness Factor</td>
<td>1.0</td>
</tr>
<tr>
<td>Stress Multiplying Factor Median</td>
<td>1.0</td>
</tr>
<tr>
<td>Stress Multiplying Factor COV</td>
<td>0.0</td>
</tr>
<tr>
<td>Defect Distribution</td>
<td>Post 1995 #3/#3 FBH Defect Dist</td>
</tr>
<tr>
<td>PoD Curve</td>
<td>1-1 #3 FBH PoD Curve</td>
</tr>
<tr>
<td>Life Scatter Factor Median</td>
<td>1.0</td>
</tr>
<tr>
<td>Life Scatter Factor COV</td>
<td>0.0</td>
</tr>
<tr>
<td>Inspection Time Mean</td>
<td>10,000 cycles</td>
</tr>
<tr>
<td>Inspection Time COV</td>
<td>0.0</td>
</tr>
<tr>
<td>Percentage of Samples Not Inspected</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Figure 11: AC 33.14 Test Case Geometry [33]**

The finite element analysis of the titanium disk was performed using ABAQUS 6.9-1. The FEA stress results were input into DARWIN and zones were defined. The FAA AC 33.14 recommends refining zones based on 5 ksi stress intervals as a starting point, so regions with high stresses may require further zone refinement. Risk
calculations converge when the initial crack has the same fatigue lifetime anywhere within the zone. The fatigue lifetime is mainly influenced by the stress history, crack type, and proximity to the surface. Near-surface embedded cracks can grow and transition to the surface boundary, growing more rapidly as surface or corner cracks. Therefore, zones must be refined near the surface. The FAA AC 33.14 recommends creating 20 mil "onion skin", surface zones, around the component to simulate the approximate depth limit of surface defects. Three zone refinements were investigated and are provided in Figure 12.

![Figure 12](image)

**Figure 12**: Calibration test stress results with three zone refinement levels

A number of manufacturers have performed the FAA AC 33.14 calibration test to define an acceptable statistical range of results. Test case results are considered acceptable if they fall within the ranges from 1.27E-9 to 1.93E-9 without inspection and from 8.36E-10 to 1.53E-9 with inspection. A Monte Carlo simulation was run for each of the three refinement cases to determine the POF with and without the inspection. The contribution factors for each refinement case show how the risk converges as zones have approximately equal risk, as demonstrated by Figure 13. The POF results for with and
without inspection are given in Figure 14. The converged results obtained are considered to be acceptable since they fall within the specified range.

![Figure 13: Zone contribution factors show the level of risk convergence](image)

**Figure 13:** Zone contribution factors show the level of risk convergence

![Figure 14: AC 33.14 converged probability of failure with and without inspection](image)

**Figure 14:** AC 33.14 converged probability of failure with and without inspection

### 3.2 Fatigue Damage

The risk of failure increases as components are extended beyond their traditional "safe-life". Under the safe-life method, components are retired when an accepted probability of initiating a crack is reached, see Figure 1. As component service life reaches the safe-life limit, fatigue damage will result in crack initiation which will lead to
crack propagation and eventual component failure. DARWIN was created to address specific advisory circulars, whose purpose was to develop a generic damage tolerance design approach to compliment the safe-life method for an enhanced life management process [33]. The probability of failure analysis in DARWIN only accounts for defects prior to service and does not address component damage induced during service. Therefore, fatigue damage must be included in the overall risk assessment if DARWIN is to accurately manage component life past traditional limits. In addition, the associated risk of component life extension can be quantified when fatigue damage is included in the risk assessment.

Damage mechanisms such as creep, corrosion, erosion, fatigue, fretting, and oxidation lead to crack initiation and growth [29]. However, low-cycle fatigue (LCF) is the primary damage mechanism in turbine disks [34] and will be the focus of this study. Fatigue damage is assumed to be the entire range of damage accumulation from crack initiation to final fast fracture. Fatigue models are developed to model crack initiation and propagation stages. A Monte Carlo simulation is performed utilizing the fatigue models to determine the fatigue life scatter resulting from material property variability. These methods are applied to a Ti-6Al-4V alloy, in order to compare the predicted results against the experimental data. Once the methods are established, fatigue damage is implemented into DARWIN to determine the probability of failure of a Ti-6Al-4V fan disk.
3.2.1 Fatigue Models

The total fatigue life \( N_f \) is assumed to be comprised of two parts: (1) number of cycles to initiate a crack of a specific characteristic length and (2) number of cycles to propagate a crack to a critical length. The total fatigue life is determined by,

\[
N_f = N_i + N_p
\]

where \( N_i \) initiation life and \( N_p \) is propagation life. The two parts of the fatigue life will be discussed in detail below. First, the crack initiation model is established.

The fatigue-life variability in turbine engine materials have been investigated in several studies [27,35,36]. These studies indicate the importance of microstructure properties in the fatigue life. Tanaka and Mura [37] developed a crack initiation model that explicitly addresses microstructure properties. The model was developed by considering the dislocation-dipole mechanism along the slip band operating in a surface or subsurface grain, which ultimately leads to crack nucleation. Tanaka and Mura's model was recently extended by Chan [38] to include crack initiation depth (\( \alpha \)) and other relevant microstructure parameters.

\[
(\Delta \sigma - 2\Delta \sigma_e)N_i^{\alpha} = \left[ \frac{8M^2 \mu^2}{\lambda \pi(1-\nu)} \right]^{1/2} \left( \frac{h}{D} \right) \left( \frac{\alpha}{D} \right)^{1/2}
\]

where \( \Delta \sigma \) is the stress range, \( \Delta \sigma_e \) is the endurance limit, \( \mu \) is the shear modulus, \( \lambda \) is a universal constant (0.005), h is the slipband width, D is the grain size, and \( \alpha \) is not a constant but depends on the degree of slip irreversibility and the stacking-fault energy, where \( 0 < \alpha \leq 1 \). These inputs will be further examined in the following sections. Now that the crack initiation model has been established, the crack propagation model will be discussed.
An accurate calculation of fatigue crack propagation life is a critical step in predicting risk of fracture of engine components. As mentioned earlier, short crack growth exhibits increased scatter, accelerated growth rates relative to large crack growth, and growth at nominal $\Delta K$ values below the large-crack threshold [28]. For these reasons, the growth of both short and large cracks is included in the analysis of the crack propagation phase. A wide variety of different analysis methods have been proposed to calculate an enhanced crack tip driving force for short cracks to predict their accelerated growth rates. These methods apply fracture mechanics to short cracks by either adjusting LEFM to account for plasticity or by explicitly considering residual crack tip plasticity and crack closure [24]. In this study, three short crack growth models are investigated: (1) microstructure-based model, (2) El Haddad, and (3) bilinear Paris law. The short crack growth models are utilized in conjunction with the long crack Paris equation (Eq 1) to determine the propagate life of an initiated crack.

The microstructure-based short crack model is derived from equation 11 by differentiating crack length with respect to fatigue cycles, which gives the growth rate of a newly initiated short crack.

$$\frac{da}{dN} = 2\alpha \left( \left[ \frac{1}{8M} \left( \frac{\Delta \sigma - 2\Delta \sigma_x}{\mu} \right) \left( \frac{D^3}{h^2} \right) \right]^{a^{2a-1}} \right)^{1/2a}$$

Equation 11 indicates that the crack growth rate of small cracks is independent of crack length when $\alpha = 1/2$; increases when $\alpha > 1/2$; and decreases when $\alpha < 1/2$. At $\Delta K$ ranges below the large crack threshold, the growth of small cracks is described by equation 11. Upon reaching the large crack threshold, the crack is assumed to be large enough to grow according to the Paris law.
El Haddad et al. [26] proposed a simple short crack model based on the Kitagawa diagram, shown in Figure 15. Kitagawa and Takahashi [25] related the large crack fracture mechanics threshold, $\Delta K_{ch}^{lc}$ to the smooth specimen endurance limit, $\Delta \sigma_e$. The intersection of the two parameters on the diagram defines the small crack parameter.

$$a_0 = \frac{1}{\pi} \left( \frac{\Delta K_{ch}^{lc}}{\Delta \sigma_e F(a)} \right)^2$$

where $F(a)$ is the crack boundary correction factor. The KT diagram shows that short cracks must be able to grow at nominal stress intensity factor ranges less than the large crack threshold, since smooth specimens fail by the initiation and growth of micro cracks. El Haddad postulated that short cracks grow below the large crack threshold because they have a larger effective driving force than predicted by LEFM. He proposed an effective driving force by replacing the actual crack size, $a$ with the sum $(a + a_0)$ in the stress intensity calculation.

$$\Delta K_{eff} = \Delta \sigma F(a) \sqrt{\pi(a + a_0)}$$

The small crack parameter $a_0$ has a large contribution in the stress intensity solution when the actual crack size is small. As the actual crack size increases, the parameter has a negligible effect. The effective stress intensity factor, $\Delta K_{eff}$, is used in Paris equation (Eq 1) to determine crack growth from the initiated crack size until it reaches the fracture toughness.
Figure 15: Kitagawa diagram relating fatigue endurance limit and crack threshold [28]

The third propagation model used a bilinear Paris equation to characterize short and long crack growth rates, as illustrated in Figure 16. The bilinear relationship is written as

\[
\frac{da}{dN} = C_1 (\Delta K)^{n_1} \quad \Delta K \leq \Delta K^* \\
\frac{da}{dN} = C_2 (\Delta K)^{n_2} \quad \Delta K \geq \Delta K^*
\]

(14)  
(15)

where $\Delta K^*$ is the "knee" in the bilinear curve where the two equations intersect, and it is assumed to be equal to the large stress intensity factor, $\Delta K_{th}^{lc}$. Equation 14 and 15 describes the short and long crack growth rate, respectively. The short crack growth constants, $C_1$ and $n_1$ are derived from short crack data. The short crack stress-intensity factor threshold, $\Delta K_{th}^{sc}$ is assumed to be zero for a "worse case" scenario since all cracks are assumed to grow. It should also be noted that the bilinear relationship of short and long crack growth rates may not exist for certain materials. However, Goswami [58] has
shown a bilinear relationship for Regime I to Regime II in Ti-6Al-4V alloy forging, which will be used in this study.

![Bilinear Paris FCG equation](image)

**Figure 16:** Representative of the Bilinear Paris FCG equation [39]

### 3.2.2 Monte Carlo Simulation

A probabilistic framework has been developed in Matlab to determine the fatigue life variability due to the variations in material properties. The Matlab code is given in Appendix A. The Monte Carlo simulation was performed by sampling random variables and deterministically calculating the fatigue life using the fatigue models described above. The random variables are assumed to be statistically uncorrelated. The probabilistic method can be broken into five steps:

**Probabilistic Fatigue Life:**

1. Determine initiated crack size $a_i$ which minimizes the total fatigue life
2. Sample random variables from material property distributions
3. Compute the deterministic fatigue life and store answer
4. Repeat 2 and 3 for a sufficient amount of times
5. Compute the mean and standard deviation of the fatigue life
The first step of the process is to determine the crack size at which a crack is considered to have been "initiated" and treated as a long crack in the FCG models. Determining the crack length at which the fatigue crack length is considered to transition from the initiation phase to growth phase is not a trivial task. The initiated crack size has a significant impact on the total computed life. If the estimated crack size is not reasonable, it may lead to a large error in the fatigue life calculation. As an example, Figure 17 shows the effect of two different initial crack sizes on the fatigue life required to reach a critical size, $a_c$. The figure shows that an initial crack size of 100μm has a smaller predicted fatigue life than an initial crack size of 15 μm. The initiation model (Eq 10) predicts that it will take more time to initiate a longer crack, but the longer crack will give a much shorter fatigue life predicted by the FCG models.

**Figure 17:** Fatigue life of two different crack initiation sizes
Since the fatigue life is sensitive to the initiated crack size, a worst case scenario is assumed in the analysis by minimizing the total fatigue life. Either the crack initiation or propagation phase will be dominant at some time during the life of the component. A variable initiation length model provides a method of determining which mechanism is dominating the life and gives the corresponding initial crack size. This approach is based on the summation of an initial life curve and propagation life curve as a function of initial crack size.

\[
N_f(a_i) = N_i(a_i) + N_p(a_i)
\]

where \(a_i\) is the initial crack size. The initiation life curve is created by calculating the initiation life required to initiate a crack of size \(a_i\). The propagation life curve is created by calculating the cycles to grow the initial crack size, \(a_i\) to a critical length, \(a_c\). The worst case scenario is the minimum life on the total fatigue curve as illustrated in Figure 18. A polynomial interpolation optimization method was utilized to find the minimum life and the corresponding initial crack size [41]. The optimization scheme is given in Appendix A.2.

![Figure 18: Variable initiation length method to determine the initial crack size [42]](image)
Once the initial crack size is determined, the Monte Carlo simulation determines the fatigue life variability by sampling random variables from material property distributions and calculating the fatigue initiation life and propagation life deterministically. The initiation life is still a function of material properties even though the initial crack size has been set. In this study, lognormal distributions are used for unknown variables since Annis [43] has shown that fatigue lives are lognormal. The mean and standard deviation of the random material properties can be obtained from experimental data. Since the variables are lognormal random variables, the logarithmic of the variables are Gaussian random variables. The mean and standard deviation of the logarithmic can be determined [44] as seen in equations 17 and 18, respectively.

\[
\mu_{il} = 2 \log(\mu_i) - \frac{1}{2} \log(\mu_i^2 + \sigma_i^2) \tag{17}
\]

\[
\sigma_{il} = \left[ -2 \log(\mu_i) + \log(\mu_i^2 + \sigma_i^2) \right]^{\frac{1}{2}} \tag{18}
\]

where \( \mu_i \) and \( \sigma_i \) are the mean and standard deviation of the random variable, respectively.

### 3.2.3 Application to a \( \alpha+\beta \) Alloy

Titanium superalloys have been widely used in the aircraft industry due to their favorable strength-to-weight ratio [31]. The developed probabilistic fatigue damage method has been applied to a \( \alpha+\beta \) forged Ti-6Al-4V alloy to predict the fatigue life scatter due to material variability. Together, the initiation and propagation fatigue models determine the number of cycles required to initiate and grow a surface crack to a critical depth in a round fatigue bar specimen. Loading is assumed to be a single fatigue cycle with a stress ratio \( R=0.1 \). The predicted fatigue life scatter is compared against experimental S-N data, and the three fatigue propagation models are contrasted.
Oberwinkler et al. [45] has shown that the grain size is the dominate microstructure parameter in the crack initiation life, thus the initiation life model uses grain size as a random variable, whereas other microstructure parameters are assumed to be constant. Figure 19 shows the typical microstructure of Ti-6Al-4V, which is comprised of 60% primary α grains and 40% Widmänstatten α+β colonies. Both α grain size and α+β colony size were treated as equivalent sizes and fitted onto a single probability density function. The mean and standard deviation of grain size were determined to be 13.7 μm and 4.4 μm, respectively, using the experimental data sources [19,46-48]. The impact of grain size on the predicted initiation life is illustrated in Figure 20 showing the number of cycles needed to initiate a crack depth of two times the average grain diameter at various grain sizes. The figure shows that a larger grain size will reduce the number of fatigue cycles for initiating a crack of a specific depth. This phenomenon is consistent with literature since larger grain sizes favor crack initiation [14]. Fine-grained microstructure tends to slow the propagation rates of small cracks due to the high density of grain boundaries or α plates acting as obstacles [56].

**Figure 19:** Microstructure of Ti-6Al-4V is comprised of 60% primary α grain (light phase) and 40% of α+β Widmanstatten colonies (dark phase) [49]
The fatigue crack propagation life is determined as the number of cycles needed for the initiated crack size \((a_i)\) to grow to a critical crack size \((a_f)\). The Paris equation is used to determine the fatigue life.

\[
N_f = \int_0^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^n}
\]  \hspace{1cm} (19)

The stress intensity factor solution for a semielliptical surface crack was used to determine the stress intensity factor range in the Paris equation according to Murakami [51].

\[
\Delta K = F_l \left(\frac{c}{r}, \frac{c}{a}\right) \Delta \sigma \sqrt{\pi a}
\]  \hspace{1cm} (20)

where \(\Delta \sigma\) is the stress range. \(F_l\) is the boundary correction factor which is a polynomial function of crack depth \((c)\), radius of specimen \((r)\), and crack surface length \((2a)\). Numerical integration of the Paris equation is required since the boundary correction factor depends on crack size. A crack aspect ratio of \(c/a\) is assumed to be 1. The critical crack size, \(a_f\) is defined by [52].
where $K_{Ic}$ is the fracture toughness, $\sigma_{max}$ is the maximum stress. Table 2 shows the material properties used in the initiation and propagation models. Five of the material properties are modeled as lognormal random variables, while the rest are treated as deterministic values. The mean and standard deviation of the normal distribution for the random variables are given in Table 2. The Paris equation constants are derived from Ti-6Al-4V small [40,53,54] and large [27,52] crack growth rate data at room temperature with $R=0.1$ as shown in Figure 21. Golden et al. [40] have shown that small crack data regression line deviates from the long crack data below $\Delta K$ values of 10MPa$\sqrt{m}$, thus a bilinear Paris relationship with $\Delta K^* = 10$MPa$\sqrt{m}$ is used in this study. In addition, the short crack threshold, $\Delta K^{sc}_{th}$, is assumed to equal zero for a "worse case" scenario since all the initiated cracks are assumed to grow until failure.

Table 2: Ti-6Al-4V material Properties for fatigue models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initiation Model [27]</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Grain Size</td>
<td>μm</td>
<td>13.7</td>
<td>4.4</td>
</tr>
<tr>
<td>M</td>
<td>Taylor Factor</td>
<td>-</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>λ</td>
<td>Universal Constant</td>
<td>-</td>
<td>0.005</td>
<td>-</td>
</tr>
<tr>
<td>μ</td>
<td>Shear Modulus</td>
<td>MPa</td>
<td>4.40E+04</td>
<td>-</td>
</tr>
<tr>
<td>ν</td>
<td>Poisson Ratio</td>
<td>-</td>
<td>0.333</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \sigma_e$</td>
<td>Endurance Limit</td>
<td>MPa</td>
<td>490</td>
<td>10</td>
</tr>
<tr>
<td>α</td>
<td>Life Exponent</td>
<td>-</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>h</td>
<td>Slipband Width</td>
<td>μm</td>
<td>5.00E-02</td>
<td>-</td>
</tr>
<tr>
<td><strong>Propagation Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>Bilinear Constant (Short)</td>
<td>SI</td>
<td>1.72E-09</td>
<td>7.00E-10</td>
</tr>
<tr>
<td>$n_1$</td>
<td>Bilinear Exponent (Short)</td>
<td>-</td>
<td>1.85</td>
<td>-</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Bilinear Constant (Long)</td>
<td>SI</td>
<td>1.02E-11</td>
<td>4.50E-12</td>
</tr>
<tr>
<td>$n_2$</td>
<td>Bilinear Exponent (Long)</td>
<td>-</td>
<td>3.84</td>
<td>-</td>
</tr>
<tr>
<td>$K_{TH,LC}$</td>
<td>Long Crack Threshold</td>
<td>MPa$\cdot$m$^{1/2}$</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Fracture Toughness</td>
<td>MPa$\cdot$m$^{1/2}$</td>
<td>67</td>
<td>-</td>
</tr>
<tr>
<td>R</td>
<td>Stress Ratio</td>
<td>-</td>
<td>0.1</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 21: Ti-6Al-4V bilinear Paris equation for small and large crack growth rates

The initial crack size was determined by finding the crack size that corresponds to the minimum on the total fatigue life curve. An initial crack size of approximately 30 μm was determined using the Bilinear Paris FCG model at a stress range of 800 MPa as seen in Figure 22. Studies have shown that naturally initiated crack sizes for titanium alloys are on the order of one to two times the grain diameter [27,45]. The fact that the initiated crack sizes are so small reinforces the need for short crack growth models in the fatigue life assessment.
Figure 22: The computed total fatigue life shows a minimum at a crack depth of 2.97e-005m, at a stress range of 800 MPa.

The Monte Carlo method formulated in the previous sections was run using an initial crack size of 30μm with 10,000 samples. Each of the three FCG models contains two random variables resulting in the fatigue life scatter calculations. The El Haddad uses the endurance limit and long fatigue threshold; the bilinear Paris equation uses C1 and C2; and the microstructure-based model uses grain size and endurance limit as random variables. Since the probability distributions are assumed to be uncorrelated, only the C values in the Bilinear Paris equation are modeled as random variables. In reality, the parameter estimates for n and C are jointly distributed. If they are modeled as separate distributions (as assumed in this study) an unrealistic error would result in the fatigue life [43].
The Paris law was utilized to determine the number of cycles needed to grow each initial crack size to its critical size using the three FCG models. Figure 23 compares the predicted mean life of the three different FCG models with experimental S-N data [19,46-48] at two stress range values. As seen from the figure, the three FCG models give a similar mean fatigue life. The Monte Carlo method along with the three FCG models is given in Appendix A.

![Figure 23](image.png)

**Figure 23:** The mean fatigue life of the three FCG models (ai = 30μm).

The fatigue life scatter was determined at a stress range of 750 and 800 Mpa to compare the three FCG models. Figure 24 shows the predicted total fatigue life scatter compared to the experimental data. As seen from Figures 23 and 24, the predicted fatigue life of the FCG models start to deviate from the experimental data as the stress range decreases. This deviation may be due to the assumption that fatigue lives are lognormal, instead they often appear to be bimodal distributions. Golden et al. [40] have shown a bimodal failure occurs in Ti-6Al-4V since either initiation or propagation starts to
dominate the fracture. For the purpose of this study, components are assumed to fail according to the low life mode (high stress) of the bimodal distribution. The low life mode is effectively modeled using the proposed FCG models.

Figure 24: Probability density of the total life at 750 MPa (top) and 800 MPa (bottom)
3.3 Fatigue Damage Program

As mentioned earlier, DARWIN is a probabilistic design tool that uses a generic damage tolerant method to account for inherent defects and does not address induced fatigue cracks. Therefore, fatigue damage must be included in the overall risk assessment if DARWIN is to accurately manage component life. In addition, the associated risk of component life extension can be quantified when fatigue damage is included in the risk assessment.

Now that the fatigue models have been established, fatigue damage can be introduced into DARWIN. A probabilistic fatigue damage program has been developed in Matlab to determine the probability of failure (POF) of components subject to fatigue induced cracks. The program is automated to write and read files through the DARWIN framework. It assumes that fatigue damage is defined as the entire range of damage accumulation from crack initiation to final failure. The initiated fatigue cracks are accounted for in DARWIN through the use of the anomaly distribution. Once the fatigue anomaly distribution is created, DARWIN determines the probability of failure as a function of flight cycles using its integrated fatigue propagation solutions. Component failure is assumed to be the result of a single dominating fatigue crack located at its life limiting location. Furthermore, each component in the population is assumed to have initiated a fatigue crack. The conditional POF is used since each component contains a defect (i.e. \( \alpha_i = 1 \) in equation 7). Thus, the POF of components containing fatigue induced defects is assumed to be

\[
p_f(disk) = \frac{N_f}{N_{samp}}
\] (22)
where $N_f$ is the number of failed samples and $N_{\text{samp}}$ is the total number of samples. The fatigue damage program is provided in Appendix B.

3.3.1 Fatigue Anomaly Distribution

The induced fatigue cracks are accounted for in DARWIN through the use of a conditioned anomaly distribution. The anomaly distribution defines the probability of exceeding a given crack size per component area or volume by accounting for the crack size distribution. DARWIN uses the anomaly distribution to determine the crack size CDF, which is used to randomly sample initial crack sizes during the probabilistic analysis. Therefore, utilizing the anomaly distribution allows fatigue initiated cracks to be represented as a distribution of crack sizes, which explicitly accounts for fatigue-life variability in the initiation phase.

The fatigue anomaly distribution is created by using the probabilistic fatigue initiation model to determine an initial crack size distribution. In the previous sections, the initiation model used the initial crack size as a deterministic value. DARWIN uses the anomaly distribution to represent the initial crack size as a random variable. Therefore, the initiation life is treated as a constant while the initial crack size distribution is determined. The crack size can be found by rearranging equation 1

$$a = \frac{D}{\sqrt{8M^2\mu^2}} \left(\frac{h}{D}\right) \frac{(\Delta \sigma - \Delta \sigma_e)^2 N_f^2}{\Delta \sigma \geq \sigma_e}$$

(23)

The Monte Carlo simulation developed in Section 3.2.2 is performed to determine the initial crack size distribution by treating one or more of the material parameters as random. Once the initial crack area is known, the crack size CDF is calculated and used to determine the exceedance values of the anomaly distribution. Equation 5 can be rearranged to solve for the exceedance values.
\[ \frac{N_d(a)}{a} = \frac{N_d(a_{min})}{a} - \text{CDF}(a)[N_d(a_{min}) - N_d(a_{max})] \] (24)

where \( N_d(a_{min}) \) and \( N_d(a_{max}) \) are the exceedance of the minimum and maximum crack sizes, respectively; and \( \text{CDF}(a) \) is CDF of the crack size distribution. The exceedance values are only used to create the anomaly distribution, they and are not utilized in the actual fatigue POF calculations since the conditional POF is used. Therefore, exceedance values are normalized in the created fatigue anomaly distribution.

The probability of failure resulting from the created fatigue anomaly distribution is shifted by the initiation life. DARWIN is used to determine the propagation life of the sampled crack population from the anomaly distribution starting at zero cycles. This starting point in the POF analysis can be considered a relative position since the total fatigue life is comprised of the initiation and propagation life \((N_f = N_i + N_p)\). Therefore, the determined POF from the fatigue anomaly distribution is shifted by the initiation life, as demonstrated by Figure 25.

\[ \text{Shifted} \]

**Figure 25:** The POF is shifted by the crack initiation life
3.3.2 Short Crack

The anomaly distributions contain very short cracks (a~30 μm) and fall into the category of mechanically short fatigue cracks. As mentioned earlier, short crack growth data has been shown to exhibit increased scatter, accelerated growth rates, and growth at nominal ΔK values below the large-crack threshold. Accurate POF calculations depend on the ability to predict short crack growth rates. DARWIN currently has no explicit way to address short crack behavior. However, the created fatigue damage program accounts for short crack behavior to enhance the POF analysis. The increase in short crack growth rate and scatter is predicted using both the El Haddad method and bilinear Paris equation. DARWIN already includes the bilinear Paris equation in its library of FCG methods, so short crack growth rates are included in this method by applying experimental data to the lower bilinear curve (Figure 16). Furthermore, the constants C1, and C2 are modeled as lognormal random variables to determine the effects of short and long crack growth scatter, respectively.

The fatigue damage POF analysis includes the El Haddad method to account for the accelerated short crack behavior. The stress-intensity factors solutions in DARWIN are integrated and cannot be manipulated. However, the El Haddad short crack parameter, \( a_0 \) can be added to the initiated crack sizes to bring the short crack data in line with the corresponding long crack results. Adding the short crack parameter to the initiated cracks sizes produces an effective stress intensity factor, \( \Delta K_{eff} \).

\[
a_0 = \frac{1}{\pi} \left( \frac{\Delta K_{IC}}{\Delta \sigma_{eff} (a+a_0)} \right)^2
\]

(25)

\[
\Delta K_{eff} = \Delta \sigma F(a) \sqrt{\pi (a + a_0)}
\]

(26)
where the geometry correction factor, \( F(a+a_0) = 0.67 \) is assumed for consistency with the smooth specimen endurance limit data in section 3.2.3. Note that \( F(a) \) is the boundary correction factor for the geometry and does not include \( a_0 \). In addition, the scatter in short crack growth for this method is determined using the large stress-intensity and endurance limit as lognormal variables.

### 3.4 Fan Disk Analysis

The methodologies developed in this study are used to determine the POF of a generic Ti-6Al-4V fan disk geometry subject to both inherent defects and induced fatigue damage. The analysis was conducted for the United States Air Force (USAF). The purpose of this analysis was to determine the increase in risk when the disk remains in service beyond the safe-life limit. The component life is assumed to be extended to a "double life" of 16,000 cycles. The "double" life represents the extension of the LCF life from 8,000 total accumulative cycles (TACS) to 16,000 TACS. The increased risk of the component failure during service life is maintained and delayed through the use of scheduled component inspection. Inspections are performed at 4,000, 8,000, and 12,000 cycles using both eddy current and ultrasonic non-destructive inspection techniques. The first section discusses the finite element stress analysis of the representative fan disk geometry using ABAQUS.

#### 3.4.1 Finite Element Stress Analysis

Finite element analysis (FEA) code ABAQUS 6.9-1 was utilized to determine the stress contour of the representative fan disk geometry. A three-dimensional FE model was created to determine the stress concentration factors around the bolt holes. The stress
concentration factors were applied to a two-dimensional model that was input into DARWIN. Table 3 gives the Ti-6Al-4V material properties used in the FEA.

Table 3: Ti-6Al-4V material properties [49]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>0.16 lb/in³</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.31</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>16900 Ksi</td>
</tr>
</tbody>
</table>

The three-dimensional model consists of a 1/14th section of the disk and was meshed using 4-node linear tetrahedral elements. Since the bolt hole location had the highest stress, the mesh was refined around the bolt hole to ensure accurate results. Symmetry boundary conditions were applied to the cut surfaces of the disk. Another boundary condition was applied at the aft side of the disk to restrain the displacement in the direction perpendicular to the loading. Boundary conditions on the three-dimensional are illustrated in Figure 26.

Figure 26: Three-dimensional disk model with boundary conditions
The disk was run using a maximum speed of 7,110 RPM, and a blade pressure load was simulated by applying a body force on pins located in the bolt holes as shown in Figure 26. A blade load of 83,000 lbs/in$^3$ was applied in the radial direction on the pins, and a normal contact interaction was used with a 0.05 frictional factor for FE convergence. No thermal stresses were determined since the disk analysis was assumed to be at isothermal temperature conditions.

The maximum hoop stress converged at approximately 140 Ksi (965 MPa) located at the bolt hole shown in Figure 27. A two-dimensional model was analyzed using the same rotational speed and boundary conditions; however, the bolt hole and stress concentration factors were defined in DARWIN to match the three-dimensional stress results. The two-dimensional model was utilized in DARWIN for the inherent risk analysis, as discussed next.

![Figure 27: Bolt hole stress concentration in the hoop direction](image)
3.4.2 Inherent Probability of Fracture

The probability of failure (POF) results in this section serve as the baseline to compare to the POF increase past the safe-life limit of 8,000 TACS when fatigue damage is included. The risk of failure due to inherent hard alpha defects was determined using the procedure outlined in the FAA 33.14. As mentioned earlier, DARWIN was originally created to address the hard alpha problem. The hard alpha anomaly data is represented by the FAA 33.14 titanium anomaly distribution (Figure 9), which describes the frequency and size of the defects. The Ti-6Al-4V crack growth relationship was input in the form of the Paris equation [50].

\[
\frac{da}{dN} = 9.68E - 10(\Delta K)^{3.11} \quad (27)
\]

Once the FE stress results were input into DARWIN, the fan disk geometry was discretized into zones of approximately equal risk. Hard alpha defects can be located anywhere within the disk, so both surface and subsurface zones were created. Surface zones were created using a 20 mil depth to simulate defects just below the component surface. All zones were initially partitioned based on 5 Ksi intervals, but were further refined until the POF converged. In addition, the cracks were placed in the life limiting location in each zone to ensure a conservative risk prediction. The final disk zone refinement consists of 133 surface and embedded zones (Figure 28).
The fan disk being considered is subject to eddy current and ultrasonic inspections of surface and subsurface regions, respectively. Actual POD information was not available for the use of this analysis, so the POD curves from the FAA 33.14 AC calibration tests were used. The eddy current POD curve (Figure 29) was utilized to indicate surface cracks, while embedded cracks were indicated by the ultrasonic POD curve (Figure 10). Three in-service inspections were performed at 4,000, 8,000, and 12,000.

\[ \sigma_{\text{max}} = 139.5 \text{ Ksi} \]

**Figure 28:** The fan disk volume is comprised of 133 surface and embedded zones

**Figure 29:** Eddy current POD curve for machined surfaces with a 2:1 reject calibration
A Monte Carlo simulation using 10,000 samples was performed to determine the POF with and without inspections. Figure 30 shows the risk contribution factors of each zone to the total disk risk. The risk contribution factors are helpful to identify the zones that contribute the most to the total disk POF. To avoid over conservative risk results, the zones with the highest contribution factor are further refinement until they have approximately equal contributions.

**Figure 30:** Contribution factors for the inherent fan disk POF

The converged disk POF results with and without inspection are shown in Figure 31. The POF results are significantly higher than the acceptable range defined by the AC 33.14 calibration test, which are the ranges from 1.27E-9 to 1.93E-9 without inspection and from 8.36E-10 to 1.53E-9 with inspection. The high POF is a result of the fast crack growth rates of Ti-6Al-4V material. Also, the stress concentration around the bolt hole limits the component life.
Figure 31: Fan disk inherent POF with and without inspections

The POF results divided by flight cycles are shown in Figure 32 compared to the estimated titanium design target risk (DTR). The Aerospace Industries Association (AIA) Rotor Integrity Sub-Committee has established a DTR of $1 \times 10^{-9}$ events per cycle for the design of new titanium rotating components. As seen from the figure, the POF due to inherent defects is much higher than the DTR. However, the established DTR was based off the titanium hard alpha distribution which is roughly correlated to industry experience, so these DTR values are not "absolute" [57]. In the next section, the fatigue damage program is used to determine the POF due to induced fatigue damage.
Figure 32: Inherent POF/Flights with and without inspections showing the DTR

3.4.3 Fatigue Probability of Fracture

The fatigue damage program has been run to determine the POF due to fatigue induced cracks. It is a standard assumption that a single crack located in the highest stress region will dominate component failure. Multiple micro cracks will either coalesce into a single crack or one crack will dominate over the others. The fatigue analysis was conducted on a corner crack (DARWIN crack type CC08) located at the bolt hole, which corresponds to the location of the highest stress of 140 Ksi (965 MPa). Figure 33 shows the corner crack on the FEA model and fracture mechanics plate used by DARWIN to determine the stress intensity factor solutions.
The fatigue damage program created and analyzed the fatigue anomaly distribution for three initiation lives: 250, 500, and 1,000 cycles. Microstructure material properties for the Ti-6Al-4V fan disk were kept consistent with section 3.2.3, as shown in Table 2. A Monte Carlo simulation of 10,000 samples was used to determine the initiated crack size distributions modeling grain size and endurance limit as lognormal random variables. The initiated fatigue crack distribution and corresponding anomaly distributions for the three initiation lives are shown in Figures 34 and 35, respectively. The fatigue induced anomaly distributions shows that the initiated crack sizes increase exponentially as the initiation life is increased, which is consistent with initiation model (Eq. 23) showing that the mean initial crack size is proportional to $N_i^{2\alpha}$. 

Figure 33: Fatigue damage performed on a corner crack located at the bolt hole.
Figure 34: Initial crack size CDF for three initiation lives ($N_i$)

Figure 35: Created anomaly distributions for the three initiation lives.
The POF analysis was performed using the three created anomaly distributions with mean FCG values in bilinear Paris equation. Figure 36 shows the three anomaly distributions have a large impact on POF calculations. The figure also shows the total life ($N_f = N_i + N_p$) of the maximum crack sizes in the created anomaly distributions. As seen from the figure, the POF starts increasing rapidly at the point where the maximum defect reaches its total life. This trend enforces the importance of the crack size assumed to be "initiated", which has a major impact on the total life affecting the POF of components. Naturally initiated cracks are shown to be very small [45]; therefore, the anomaly distribution with the initiation life of 250 cycles with a mean of 50µm will be used in the remainder of the POF analysis. Next, the short crack growth rates were investigated in the POF calculations.

![Figure 36: Probability of failure increases as the initiation life increases](image)

<table>
<thead>
<tr>
<th>$N_f (a_{max})$</th>
<th>3,572</th>
<th>6,835</th>
<th>11,575</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$</td>
<td>250</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>
The fatigue damage program was run using the bilinear Paris equation and El Haddad method to account for accelerated short crack growth rates in the POF calculations. Figure 37 compares the results of two short crack growth models and the long crack growth model (Eq. 1). The figure also shows the total life of the maximum crack size for the three FCG models. The Bilinear Paris and El Haddad short FCG models have a 20% and 30% shorter fatigue life compared to the long crack growth model. The El Haddad results are more conservative since the anomaly distribution has been shifted by the mean short crack parameter, $a_0 = 74\mu m$, which further demonstrates that the POF is sensitive to crack size. The scatter of the two short crack growth equations is discussed next.

**Figure 37:** Two short FCG models have a large impact on the POF calculation
The fatigue damage program has been run using random variables for the two short crack growth models to show the impact of the POF. The 3-sigma confidence bounds for the El Haddad and bilinear Paris equation are shown in Figure 38. Although the mean POF is lower for the EH method, the range in POF using the bilinear Paris equation is much greater. This shows that the fatigue scatter is more sensitive when random variables are directly applied to the FCG rates, compared to the El Haddad method which uses the endurance and large stress-intensity factor threshold as random variables. Next, the increase in risk from fatigue damage is compared to the inherent risk.

Figure 38: POF with FCG scatter using El Haddad (top) and bilinear Paris (bottom)
The inherent POF results determined in the previous section are compared to the increase in POF due to fatigue induced damage, as seen in Figure 39. The inherent POF analysis serves as the base POF for comparison, whereas the fatigue damage program has been run using the mean bilinear Paris equation to determine the POF increase. Whitney-Rawls [59] has shown that the total POF can be comprised of the linear superposition of inherent POF plus fatigue POF.

The total POF results at the end of life (16,000 TACs) are 0.0654 without inspection and 8.8e-4 with inspection. As seen from the figure, there is a rapid increase in total POF past 11,000 TACs compared to the inherent POF. In order to explain this phenomenon, the process of risk analysis must be examined in more detail. The fatigue damage program determined the POF by dividing the number of failed samples by the total number of samples, whereas the inherent POF is determined by using the defect occurrence rate (Eq. 7). Therefore, the large increase in POF is due to the "scaling" differences in the POF results by the occurrence rate. In addition, the initial crack sizes in the fatigue anomaly distribution have similar sizes, as seen from the CDF in Figure 34. Since the majority of the initiated cracks are comparable in size, they will have very similar fatigue lifetimes. Thus, the POF due to fatigue damage will increase rapidly at a certain point in time.

The increase of component risk during service life can be maintained and delayed through the use of scheduled component inspection. For demonstration purposes, the POF results are compared to the safe-life risk level (1/1000). As seen from the figure, the total POF reaches the safe-life limit at 11,100 cycles, but when inspections are applied the components can be left in service until 16,000 cycles to maintain the same risk level.
Therefore, if the acceptable risk level is established to be 1/1000, component service life can be increased by 44% through proper inspection planning.

**Figure 39:** Increase in manufacturing POF due to fatigue damage
4. Conclusion

Turbine disk rotors have been traditionally managed by the combination of safe-life and damage tolerance methods. Although these methods provide a safe way of determining the operation life of components, they are inherently conservative. There is a substantial economic need for component life extension as components reach their LCF limit. RFC allows component life extension through the use of damage tolerance methods and NDI methods to ensure safety through scheduled component inspection. However, the risk of failure increases as components are continued in service beyond their safe-life. Therefore, risk must be quantified before component life extension concepts are implemented.

The FAA approved tool, DARWIN, uses probabilistic concepts to quantify the risk of components in service subject to inherent defects. However, induced defects must be accounted for if DARWIN is to quantify the risk for components reaching their LCF life. The inclusion of fatigue damage in the analytical risk assessment will improve accuracy of the current risk analysis procedure and allow for successful adoption of component life extension.

Fatigue damage is assumed to be the entire range of damage accumulation from crack initiation to final fast fracture of the component. Fatigue models were developed to model crack initiation and propagation stages. A micromechanical initiation model relates crack initiation life to crack depth by using grain-size material parameters. The fatigue propagation life is modeled using both long and short crack growth models. Short
fatigue crack growth models: El Haddad, microstructure-based, and bilinear Paris equation were utilized to account for the anomalous behavior of small initiated cracks, while the Paris equation calculates the long crack growth behavior.

A Monte Carlo simulation was developed using the fatigue models to determine the fatigue-life variability of a Ti-6Al-4V alloy. Grain size was established as the dominant microstructure parameter in the crack initiation phase and was modeled as a lognormal random variable. Together, the initiation and propagation models were used to predict the total fatigue life confidence bounds due to material variability or uncertainties in material properties. The proposed FCG equations effectively compared against experimental data at high stress, which corresponds to the LCF life of turbine components.

The developed fatigue damage code predicts the probability of failure of components subject to fatigue induced damage. The initiation equation determines the naturally initiated crack distribution by modeling grain size and endurance limit as random variables. The distribution of initiated cracks is inserted into DARWIN through the use of a created anomaly distribution. The fatigue damage program accounts for short crack growth rates and scatter using the El Haddad correction method and the bilinear Paris FCG equation. The El Haddad equation uses long crack growth threshold and endurance limit as random variables, whereas the Paris equation uses the constants to determine the scatter in FCG.

The methodology was applied to a Ti-6Al-4V representative fan disk geometry to determine the probability of failure due to inherent and fatigue induced anomalies. The POF results show that the initial crack size was a major factor in the risk assessment. In
addition, the POF is sensitive to fatigue crack growth scatter, material properties, and analytical modeling of short crack growth. The advantage of the fatigue damage program is its usefulness in estimating the expected initial crack size for the POF analysis, which can further be used to establish an enhanced inspection planning. Inspections can reduce and maintain risk of failure as component life is extended beyond their low-cycle fatigue limit established under the safe-life method.
References:


Appendix A: Monte Carlo Simulation

% JACE CARTER: MONTE CARLO SIMULATION: PREDICTION OF FATIGUE VARIABILITY
clc; clear all; global matprop stress

% Random variables are modeled with lognormal or normal probability
densities. Initiation, short, and long crack growth phases are
predicted for round bar fatigue specimen geometry (FI(a)). Three short
crack growth models are implemented: El Haddad, Bilinear Paris
equation, and microstructure-based model(Chan).

% Choose crack propagation model (ElHaddad, BilinearParis, or Chan)
Smallcrack = 'BilinearParis';

%%%%%%%%%%%%%%%%%  DETERMINISTIC VARIABLES %%%%%%%%%%%%%%%%%%%%
M = 2; % Taylor factor
lamda = 0.005; % universal constant
u = 4.4e4; % shear modulus (MPa)
v = 0.333; % poissons ratio
alpha = 0.6; % life exponent
h = 5e-8; % slipband width (m)
Kc = 67; % Fracture toughness (Mpa-sqrt(m))
Kb = 10; % Bilinear "Knee" (Mpa-sqrt(m))
n2 = 3.845; % Long Crack Paris exponent
R = 0.1; % Stress ratio

%%%%%%%%%%%%%%%%%%%  PROBABILISTIC RANDOM VARIABLES %%%%%%%%%%%%%%
type = 'lognormal'; % lognormal or normal distribution types

% Mean (mu) and standard deviation (sigma)
muD = 13.7e-6; sigmaD = 4.4e-6; % Grain size, D (M)
muKthl = 5.0; sigmaKthl = 0.7; % Large crack threshold(Mpa-sqrt(m))
muKe = 490; sigmaKe = 10; % Fatigue limit (MPa)
muC1 = 1.72e-9; sigmaC1 = 7e-10; % Paris constants (Short Crack)
muC2 = 1.024e-11; sigmaC2 = 4.5e-12; % Paris constants (Long Crack)

samples = 1; % number of samples in simulation
str = 800; % Stress Range (MPa)

for j=1:length(str)

stress = str(j);
iter = 2;
Dprop = [M lamda u alpha h Kc n1 n2 muD muKthl muKe muC1 muC2];
ai = Optimization(Dprop,iter,Smallcrack);

for i=1:samples % Monte Carlo simulation
sigma = [sigmaD sigmaKthl sigmaKe sigmaC1 sigmaC2];
mu = [muD muKthl muKe muC1 muC2];
if strcmp(type,'lognormal')

    sigmalog = sqrt(log(1+(sigma./mu).^2));
    mulog = log(mu) - 1/2.*log(1+(sigma./mu).^2);

    D(i) = lognrnd(mulog(1),sigmalog(1));
    Kthl(i) = lognrnd(mulog(2),sigmalog(2));
    Ke(i) = lognrnd(mulog(3),sigmalog(3));
    C1(i) = lognrnd(mulog(4),sigmalog(4));
    C2(i) = lognrnd(mulog(5),sigmalog(5));

elseif strcmp(type,'normal') % (make sure all are positive)

    D(i) = normrnd(mu(1),sigma(1));
    Kthl(i) = normrnd(mu(2),sigma(2));
    Ke(i) = normrnd(mu(3),sigma(3));
    C1(i) = normrnd(mu(4),sigma(4));
    C2(i) = normrnd(mu(5),sigma(5));
end

% El Haddad crack size (ao)
f1 = @(x)1/pi()*(Kthl(i)/Ke(i)/CorFactor(x))^2 - x;
ao(i) = fzero(f1,30e-6);

% Critical crack size (af)
f2 = @(x)CorFactor(x)*stress/(1-R)*sqrt(pi()*x) - Kc;
af(j) = fzero(f2,0.02);

% Crack size corresponding to Kthl (ath)
f3 = @(x)CorFactor(x)*stress*sqrt(pi()*x) - Kthl(i);
ath(i) = fzero(f3,0.0001);

% Crack size corresponding to K* (ab)
f4 = @(x)CorFactor(x)*stress*sqrt(pi()*x) - 10;
ab(j) = fzero(f4,0.0005);

% Short crack growth in Bilinear Paris Equation (n1)
n1(i) = (log10(C2(i)) - log10(C1(i)) + n2*log10(Kb))/log10(Kb);

% Global material properties for crack growth models
matprop = [M lamda u v alpha h Kc n1(i) n2 D(i) Kthl(i)
Ke(i) C1(i) C2(i) ao(i)];

% Crack Initiation Life
Ni(i) = Initiation(ai);

% Crack Propagation Life
if strcmp(Smallcrack,'ElHaddad')
    Npsc(i) = quad(@(ElHaddad,ai,af(j));
    Nplc(i) = 0;
elseif strcmp(Smallcrack, 'BilinearParis')
    Npsc(i) = quad(@BilinearParis,ai,ab(j));
    Nplc(i) = quad(@Growth,ab(j),af(j));

elseif strcmp(Smallcrack, 'Chan')
    Npsc(i) = quad(@Chan,ai,ath(i));
    Nplc(i) = quad(@Growth,ath(i),af(j));
end

% Total Life
Nf(i) = Ni(i) + Npsc(i) + Nplc(i);

% 3-Sigma confidence bounds in life models
confNi(j,1:2) = prctile(Ni,[0.3 99.7]); confNi(j,3) = mean(Ni);
confNpsc(j,1:2) = prctile(Npsc,[0.3 99.7]); confNpsc(j,3) = mean(Npsc);
confNplc(j,1:2) = prctile(Nplc,[0.3 99.7]); confNplc(j,3) = mean(Nplc);
confNf(j,1:2) = prctile(Nf,[0.3 99.7]); confNf(j,3) = mean(Nf);

y1 = logspace(log10(confNf(j,1)),log10(confNf(j,2)),50);
f5 = ksdensity(Nf,y1,'function','pdf');
f6 = ksdensity(Nf,y1,'function','cdf');
f5 = f5/norm(f5);
figure (1); % Plot Nf PDF on S-N curve
semilogx(cycles,stressrange,'o',y1,f5*100+stress,'r-'); hold on;

end

semilogx(confNf(:,3),str,'r+',confNi(:,3),str,'b+'); hold off

Appendix A.1: Fatigue Models

function y = BilinearParis(a)
global matprop stress
n1 = matprop(8); C1 = matprop(13);
y = 1./(C1.*(CorFactor(a).*sqrt(pi().*a)).^n1);
end

function Nscp = Chan(a)
global matprop stress
M = matprop(1); lamda = matprop(2); u = matprop(3); v = matprop(4);
alpha = matprop(5); h = matprop(6); D = matprop(10); Ke = matprop(12);
x = (lamda*pi()*(1-v)/8*M);
y = ((stress-Ke)/u)^2;
z = (D^3/h^2)*a.^(2*alpha-1);
Nscp = 1./(2*alpha.*(x.*y.*z).^(1/2/alpha));
end

function y = ElHaddad(a)
global matprop stress
C2 = matprop(14); n2 = matprop(9); ao = matprop(15);
\[ y = \frac{1}{(C2 \cdot (\text{CorFactor}(a) \cdot \text{stress} \cdot \sqrt{\pi(a + a_0)})^n)^2}; \]

function \( Ni = \text{Initiation}(ai) \)

global matprop stress

M = matprop(1); lamda = matprop(2); u = matprop(3); v = matprop(4);
alpha = matprop(5); h = matprop(6); D = matprop(10); Ke = matprop(12);
A = (8 \cdot M^2 \cdot u^2 \cdot \pi / (1 - v))^0.5;
B = (h / D) \cdot (ai / D)^0.5;
C = stress - Ke;
Ni = (A \cdot B / C)^{1 / alpha};
end

Appendix A.2: Optimization Scheme

function \( ai = \text{Optimization}(Dprop, iter, Smallcrack) \)

% This function determines the crack size that corresponds to the minimum on the total life curve. Total life is found by summing the initiation and propagation life. Three propagation models can be used via Monte Carlo simulation (El Haddad, Bilinear Paris, and chan (micro-based model).

format long

global matprop stress

M = Dprop(1); lamda = Dprop(2); u = Dprop(3); v = Dprop(4); alpha = Dprop(5);
h = Dprop(6); Kc = Dprop(7); n1 = Dprop(8); n2 = Dprop(9); D = Dprop(10);
Kthl = Dprop(11); Ke = Dprop(12); C1 = Dprop(13); C2 = Dprop(14);

% El Haddad crack size (ao)
f1 = @(x)1/pi()*(Kthl/Ke/CorFactor(x))^2 - x; % No cor-factor
ao = fzero(f1, 30e-6);
% El Haddad crack size (ao)
f1 = @(x)1/pi()*(Kthl/Ke/CorFactor(x))^2 - x;
ao = fzero(f1, 30e-6);

% Critical crack size (af)
f2 = @(x)CorFactor(x)\cdot\text{stress}/(1-0.1)\cdot\sqrt{\pi(x)} - \text{Kc};
af = fzero(f2, 0.02);

% Crack size corresponding to Kthl (ath)
f3 = @(x)CorFactor(x)\cdot\text{stress}\cdot\sqrt{\pi(x)} - \text{Kthl};
ath = fzero(f3, 0.0001);

% Crack size corresponding to K* (ab)
f4 = @(x)CorFactor(x)\cdot\text{stress}\cdot\sqrt{\pi(x)} - 10;
ab = fzero(f4, 0.0001);

matprop = [M lamda u v alpha h Kc n1 n2 D Kthl Ke C1 C2 ao];

data = 15; % number of data points for poly interpolation
for k=1:data
    crack = logspace(log10(1e-6), log10(af*0.95), data);
a = crack(k);

    % Crack Initiation Life
    Ni(k) = Initiation(a);
    % Crack Propagation Life
if strcmp(Smallcrack,'ElHaddad')
    Npsc(k) = quad(@(ElHaddad,a,af);
    Nplc(k) = 0;

elseif strcmp(Smallcrack, 'BilinearParis')
    if a<=ab
        Npsc(k) = quad(@(BilinearParis,a,ab);
        Nplc(k) = quad(@(Growth,ab,af);
    else
        Npsc(k) = 0;
        Nplc(k) = quad(@(Growth,a,af);
    end

elseif strcmp(Smallcrack, 'Chan')
    Npsc(k) = quad(@(Chan,a,ath);
    Nplc(k) = quad(@(Growth,ath,af);
end
% Propagation life
Np(k) = Npsc(k) + Nplc(k);

end

% Minimum of the polynomial interpolation in log10-space (Matlab cannot
% perform exponential regression fit)
X = log10(crack);
Yin = log10(Ni);
Yprop = log10(Np);
Ytot = log10(Ni + Np);
for n=1:iter
    fit = polyfit(X,Ytot,2);
    minL(n) = -1/(2*fit(1))*fit(2); % minimum guess in log10-space
    YminL(n) = (fit(3)+fit(2)*minL(n)+fit(1)*minL(n)^2);
    ac = 10^minL(n);
    % Actual Initiation life at min life guess (MinL)
    newin(n) = Initiation(ac);
    % Crack Propagation Life
    if strcmp(Smallcrack,'ElHaddad')
        NpscA = quad(@(ElHaddad,ac,af);
        NplcA = 0;
    elseif strcmp(Smallcrack, 'BilinearParis')
        if ac<=ab
            NpscA = quad(@(BilinearParis,ac,ab);
            NplcA = quad(@(Growth,ab,af);
        else
            NpscA = 0;
            NplcA = quad(@(Growth,ac,af);
        end
    elseif strcmp(Smallcrack, 'Chan')
        NpscA = quad(@(ElHaddad,ac,ath);
        NplcA = quad(@(Growth,ath,af);
% Actual Initiation life at min life guess (MinL)
newprop(n) = NpscA + NplcA;
newtot(n) = log10(newprop(n) + newin(n));

if X(2) <= minL(n)
    if Ytot(2) <= newtot(n)
        X(1) = X(1);
        X(3) = minL(n);
        Ytot(3) = newtot(n);
        X(2) = X(2);
    else
        X(1) = minL(n);
        Ytot(1) = newtot(n);
        X(3) = X(3);
        X(2) = X(2);
    end
else
    if Ytot(2) <= newtot(n)
        X(1) = minL(n);
        Ytot(1) = newtot(n);
        X(3) = X(3);
        X(2) = X(2);
    else
        X(1) = X(1);
        X(3) = X(2);
        Ytot(3) = Ytot(2);
        X(2) = minL(n);
        Ytot(2) = newtot(n);
    end
end

end

% Convert back from log
x = [crack 10.^minL];
yin = [10.^Yin newin];
yprop = [10.^Yprop newprop];

% Sort data for plotting
[yprop p] = sort(yprop);
x = x(p);
yin = yin(p);
ytot = yin + yprop;
figure (1)
loglog(x,yin,'b-o',x,yprop,'r-o',x,ytot,'k-o'); hold on
loglog(10^minL(n),10^newtot(n),'r+','MarkerSize',11,'LineWidth',2);
grid on;
xlabel('Crack depth (m)','FontSize',18)
ylabel('Cycles','FontSize',18)
title(['Minimum fatigue life occurs at a crack depth of ','num2str(10^minL(n),'%10.2e'),'m'],'FontSize',18)
h=legend('Initiation','Propagation','Total',['min life= ','num2str(10^newtot(n),'%10.2e')]); hold off;
set(h,'FontSize',14)
clear matprop
[m l] = min(ytot);
ai = x(l)
end
Appendix B: Fatigue Damage Code

% JACE CARTER: FATIGUE DAMAGE PROGRAM: INDUCED CRACKS IN DARWIN
clc; clear all; global matprop stress

% This code (1) creates fatigue anomaly distribution from initiated cracks using the initiation model, random variables, and Monte Carlo simulation; (2) writes input file for DARWIN analysis; (3) performs DARWIN analysis; (4) reads POF results from DARWIN results file

% The El Haddad and Bilinear Paris short crack growth models are included. El Haddad short growth model uses a conditioned anomaly distribution by adding the short crack parameter, ao, to the initiated crack sizes. The bilinear equation is built and written to the input file for DARWIN analysis.

% Running: (1) define inputs: all material property units must be in SI units; however, DARWIN base file can be in either SI or US units. (2) specify Monte Carlo sample size, initiation life, stress at crack location (3) Run the program, select the input (DAT) DARWIN file being analyzed.

Smallcrack = 'Bilinear' ; % FCG models: ElHaddad or Bilinear
type = 3 ; % Confident bounds for FCG models: Lower(1), Upper(2), Mean(3)
units = 'US' ; % DARWIN Input File Units: US (Ksi-in) or SI (MPa - m)

% DETERMINISTIC VARIABLES
M = 2; % Taylor factor
lamda = 0.005; % universal constant
u = 4.4e4; % shear modulus (MPa)
v = 0.333; % poisson's ratio
alpha = 0.6; % life exponent
h = 5e-8; % slipband width (m)
Kc = 67; % Fracture toughness (Mpa-sqrt(m))
n2 = 3.845; % Long crack Paris exponent
Kb = 10; % Bilinear Elbow (Mpa-sqrt(m))
Kths = 0.01; % Small crack threshold
Temp = 75; % FCG temperature
Kc = 67; % Fracture Toughness (Mpa-sqrt(m))

% PROBABILISTIC RANDOM VARIABLES
muD = 13.7e-6; sigmaD = 4.4e-6; % Grain size, D (M)
muKthl = 10.0; sigmaKthl = 0.7; % Large crack threshold (Mpa-sqrt(m))
muKe = 490; sigmaKe = 10; % Fatigue limit (MPa)
muC1 = 1.72e-9; sigmaC1 = 7e-10; % Paris constants (Short Crack)
muC2 = 1.024e-11; sigmaC2 = 4.5e-12; % Paris constants (Long Crack)
samples = 10000; % number of samples (must be the same used in DARWIN)
stress = 958; % Stress at life-limiting location (MPa)
Ni = 250; % Initiation(a); Initiation Life

% Select the input (DAT) file
fprintf('\nSelect the DARWIN input file');
[filename,filepath]=uigetfile('*.dat','Select the DARWIN input file.\n');
oldpath = pwd;
% DARWIN path
darwinpath="C:\Program Files (x86)\DARWIN-7-0\bin\w32\RAC\darwin.exe";

% MONTE CARLO SIMULATION
for i=1:samples
    sigma = [sigmaD sigmaKthl sigmaKe sigmaC1 sigmaC2];
    mu = [muD muKthl muKe muC1 muC2];
    sigmalog = sqrt(log(1+(sigma./mu).^2));
    mulog = log(mu) - 1/2.*log(1+(sigma./mu).^2);
    D(i) = lognrnd(mulog(1),sigmalog(1));
    Kthl(i) = lognrnd(mulog(2),sigmalog(2));
    Ke(i) = lognrnd(mulog(3),sigmalog(3));
    C1(i) = lognrnd(mulog(4),sigmalog(4));
    C2(i) = lognrnd(mulog(5),sigmalog(5));
    
    % El Haddad crack size (ao)
    f1 = @(x)1/pi()*(Kthl(i)/Ke(i)/0.67).^2 - x;
    ao(i) = fzero(f1,30e-6); % Crack size distribution
    ai(i) = (D(i).*(stress/Ke(i)).^2.*Ni^(2*alpha))./((8*M^2*u^2)./(lamda*pi().*(1-v)).*(h./D(i)).^2);
end
confD(1:2) = prctile(D,[1 99]); confD(3) = mean(D);
confKthl(1:2) = prctile(Kthl,[1 99]); confKthl(3) = mean(Kthl);
confKe(1:2) = prctile(Ke,[1 99]); confKe(3) = mean(Ke);
confC1(1:2) = prctile(C1,[1 99]); confC1(3) = mean(C1);
confC2(1:2) = prctile(C2,[1 99]); confC2(3) = mean(C2);
confn1(1:2) = prctile(n1,[1 99]); confn1(3) = mean(n1);
confao(1:2) = prctile(ao,[1 99]); confao(3) = mean(ao);

if strcmp(Smallcrack,'ElHaddad')
    a = ai + confao(type); % El Haddad correction
elseif strcmp(Smallcrack,'Bilinear')
    a = ai;
end
confa(1:2) = prctile(a,[1 99]); confa(3) = mean(a);

% CREATE FATIGUE ANOMALY DISTRIBUTION:
x1 = logspace(log10(confa(1)),log10(confa(2)),50);
CDF = ksdensity(a,x1,'function','cdf');
PDF = ksdensity(a,x1,'function','pdf');
Nd_amin = 1; Nd_amax = 0; Nd = Nd_amin - CDF.*(Nd_amin - Nd_amax);
if strcmp(units,'US')
    area = (x1.^2.*pi()*39.3700787^2)./4*1e6; % Area in in^2*1e6
end

if strcmp(units,'SI')
    area = (x1.^2.*pi()*1000^2)./4; % Area in mm^2
end

CracksizeCDF = [x1' CDF'];
anomDIS = [area' Nd'];

figure (1)
figure(1) % Initial Crack CDF and Anomaly Distirbution
subplot(1,2,1);
plot(x1,CDF,'o'); grid on;
xlabel('Crack depth'); ylabel('CDF');

subplot(1,2,2);
loglog(area,Nd,'o'); grid on;
xlabel('Anomaly area'); ylabel('Exceedence');

% Define Crack growth data
if strcmp(units,'US')
    % Paris equation for DARWIN: [n1,n2,C1,C2,Kths,Kc,temp]
    ac = confa(2)*39.3700787; % maximum crack in anomaly Distirbution
    if strcmp(Smallcrack,'ElHaddad')  % Long FCG only
        Paris = [n2 n2 39.3700787*confC2(type) 39.3700787*confC2(type)
                0.01 60 Temp];
    elseif strcmp(Smallcrack,'Bilinear')
        n1 = (log10(39.3700787*confC2(type)) -
             log10(39.3700787*confC1(type))) +
             n2*log10(Kb))/log10(Kb);
        Paris = [n1 n2 39.3700787*confC1(type) 39.3700787*confC2(type)
                0.01 60 Temp]; % mean
    end
elseif strcmp(units,'SI')
    ac = confa(2);
    if strcmp(Smallcrack,'ElHaddad')  % Long FCG only
        Paris = [n2 n2 confC2(type) confC2(type) 0.01 60 Temp];
    elseif strcmp(Smallcrack,'Bilinear')
        n1 = (log10(confC2(type)) -
             log10(confC1(type))) +
             n2*log10(Kb))/log10(Kb);
        Paris = [n1 n2 confC1(type) confC2(type) 0.01 60 Temp]; % mean
    end
end

inspections = [4000-Ni 0; 8000-Ni 0; 12000-Ni 0]; % Shift Inspections

% WRITE RUN READ DARWIN
[newinput] = 
    writePROBinput(filename,filepath,oldpath,anomDIS,Paris,inspections,ac);
[resultsfile] = RunDARWIN(newinput,filepath,oldpath,darwinpath);
[dim Inspl SL UncondPOF CondPOF prop]=readPROBresults(resultsfile,filepath,oldpath);
% ConPOF: 1) Cycles; 2) Number fractured with Insp; 3) Number fractured % w/o Inspection; 4) Number removed; 5) Number escaped; 6) POF w/o % Inspection; 7) POF w/ Inspection

cycles = [0 SL(1):SL(3):SL(2)];
conpofwo = CondPOF([length(cycles)+1:2*(length(cycles))],3);
conpofw = CondPOF([length(cycles)+1:2*(length(cycles))],2);
POFw = conpofw./(samples);
POFwo = conpofwo./(samples);
TotalPOF = [Ni+cycles' POFwo POFw];

figure (2)
plot(Ni+cycles',POFwo,'b--','LineWidth',2); hold on
plot(Ni+cycles',POFw,'r--','LineWidth',2); grid on;
axis([0 SL(2)+Ni 0 .6]);

Appendix B.1: Write DARWIN Input File

function [newinput] = writePROBinput(filename,filepath,oldpath,anomDIS,
Paris,inspections,ac)
% New input file name
newinput = 'Newinput.dat';
% Change to input file directory
cd(filepath);
% Open old input file for read only
fidold=fopen(filename,'r');
% Open/Create new input file for write/read only
fidnew=fopen(newinput,'w+');
% End of file status (end=1)
eofstat=0;
% Material Properties
n1=Paris(1); n2=Paris(2); C1=Paris(3); C2=Paris(4);
Kth=Paris(5); Kc=Paris(6); Temp=Paris(7);
while eofstat == 0
    pass=0;
    % Get next line
tline = fgetl(fidold);
    [a,b] = size(tline);
    % Condition line for fprintf (Doubling '\' & '%' Characters)
i=1;
    while i <= b
        if tline(1,i)=='%';
            clear newline
            newline(1,1:i)=tline(1,1:i);
            newline(1,i+1)='%';
            newline(1,i+2:length(tline)+1)=tline(1,i+1:length(tline));
            clear tline;
            tline=newline;
            i=i+1;
        end
        if tline(1,i)=='\';
            clear newline;
            newline(1,1:i)=tline(1,1:i);
            newline(1,i+1)='\';
            newline(1,i+2:length(tline)+1)=tline(1,i+1:length(tline));
            clear tline;
            tline=newline;
            i=i+1;
        end
        % Process line
    end
    % Print line
    fprintf(fidnew,tline);
end
% End of file
fclose(fidold);
close(fidnew);
end
clear tline;
tline=newtline;
i=i+1;
end
[a,b]=size(tline);
i=i+1;
end

% Change initial crack size

% Change anomaly distribution number of points

% Change anomaly data

% Change initial crack size
fprintf(fidnew, '    %6.5e  %.5f  %6.5e  %.5f  %6.5f  %.5f  %6.5f
', C1, n1, C2, n2, Kth, Kc, Temp);
tline = fgetl(fidold);
tline = fgetl(fidold);

% Change inspection information
insp= 'MEAN_CYCLES   STANDARD_DEVIATION';
if b >= length(insp) & tline(1:length(insp)) == insp
    pass = 1;
    inspfound = 1;
    fprintf(fidnew, tline);
    fprintf(fidnew, '
');
    % Print first inspection
    fprintf(fidnew, '1.0f   %lf
', inspections(:,1), inspections(:,2));
    % Print the remaining inspections
dots = '!..........................

    % Print unchanged text
    if pass == 0
        fprintf(fidnew, tline);
        fprintf(fidnew, '
');
    end
end

eofstat = feof(fidold);
fclose(fidold);
fclose(fidnew);
cd(oldpath);
Appendix B.2: Run DARWIN Input File

```matlab
function [resultsfile] = RunDARWIN(newinput, filepath, oldpath, darwinpath)
    cd(filepath);
    lic='XXXXXXXXXXXXXXXX';
    fprintf('\n------------------------\n
    Running Input File: %s', newinput);
    fprintf('\n------------------------\n
    d=length(darwinpath);
    n=length(newinput);
    D=length(lic);
    clear K
    K(1,1:d)=darwinpath;
    K(1,d+1)=
    K(1,d+2:1+d+n)=newinput;
    K(1,d+n+2)=
    K(1,d+n+3:d+n+2+D)=lic;
    system(K);
    cd(oldpath);
    resultsfile='Newinput.ddb';
end
```

Appendix B.3: Read DARWIN Results File

```matlab
function [dim Insp SL UncondPOF CondPOF prop]=readPROBresults(resultsfile, filepath, oldpath)
    cd(filepath);
    fidresults=fopen(resultsfile, 'r');
    i = 0;
    j = 0;
    k = 0;
    l = 0;
    m = 0;
    eofstat = 0;
    while eofstat == 0
        tline = fgetl(fidresults);
        [a,b] = size(tline);
        if b>=length(dimensionsinfor) &
            tline(1:length(dimensionsinfor)) == dimensionsinfor
            dim = sscanf(tline, '<dimensions zones="%f" defects="%f"
            materials="%f" pods="%f" inspections="%f" missions="%f"/>');
            if dim(5)==0
                Insp = 'None';
            end
            end
        % Read Inspection information (mean, stdev)
        Inspinfor = '<inspectionCurve mean';
        if b>=length(Inspinfor) & tline(1:length(Inspinfor)) == Inspinfor
            Inspec=sscanf(tline, '<inspectionCurve mean="%f" stdev="%f"/>');
            % Store Inspection information
            i = i+1;
```

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Insp(i,1) = Inspec(1); % Mean
Insp(i,2) = Inspec(2); % Stdev

% Read service life information (beginning, ending, increment)
SLcond = '        <serviceLife beginning';
if b>=length(SLcond) & tline(1:length(SLcond))==SLcond
    SL=sscanf(tline,'        <serviceLife beginning="%f"
ending="%f" increment="%f"'/);
end

% Read conditional information for each zone
ConPOF= '                    <conditionalRiskState cycles=';
if b>=length(ConPOF) & tline(1:length(ConPOF))==ConPOF
    POF = sscanf(tline,'                    <conditionalRiskState
cycles="%f" numFracturedWithInspection="%f"
numFracturedWithOutInspection="%f" numRemoved="%f"
numEscaped="%f" probReduced="%f"
probFailWithOutInspection="%f" probFailWithInspection="%f"'/>);
end

% Read disk unconditional POF (sum of zone's uncond POF)
UnconPOF='                <unconditionalRiskOfDiskState cycles=';
if b>=length(UnconPOF) & tline(1:length(UnconPOF))==UnconPOF
    POF = sscanf(tline,'                <unconditionalRiskOfDiskState
cycles="%f" probFractureWithInspection="%f"
probFractureWithInspectionLower="%f" probFractureWithInspectionUpper="%f"
probFractureWithOutInspection="%f"
probFractureWithOutInspectionLower="%f" probFractureWithOutInspectionUpper="%f"
probFractureReducedByInspection="%f"'/>);
end

% Read propagation life
propagation = '            <avsn>';
if b>=length(propagation) &
tline(1:length(propagation))==propagation
tline = fgetl(fidresults);
j = j+1;
    prop(:,j)=sscanf(tline,'            <avsnFractureResult
formationLife="0" propagationLife="%f"'/>);