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The Effects of Multicollinearity in Multilevel Models

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THE EFFECTS OF MULTICOLLINEARITY IN MULTILEVEL MODELS

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ABSTRACT

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This study examined a method for calculating the impact of multicollinearity on multilevel modeling. The major research questions concerned a) how the simulation design factors affect (multilevel variance inflation factor) MVIF, b) how MVIF affects standard errors of regression coefficients, and c) how MVIF affects significance of regression coefficients. Monte Carlo simulations were conducted to address these questions. Predictor relationships were manipulated in order to simulate multicollinearity. Findings indicate that a) increases in relationships among Level 1 predictors and also relationships among Level 2 predictors led to increased MVIF for those specific variables, b) as MVIF increases for a predictor, the standard errors for the regression coefficients also increase., and c) when MVIF values for the regression coefficients were 5 or higher, margins of error were around .20, and therefore any coefficients around .20 or lower will become non-significant.

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The Effects of Multicollinearity in Multilevel Models

Multicollinearity occurs when one or more of the predictor variables highly correlates with the other predictor variables in a regression equation (Cohen, Cohen, West, & Aiken, 2003). Multicollinearity is an issue that has been widely discussed in the context of OLS regression. For example, regression textbooks discuss issues of multicollinearity (e.g., Cohen et al., 2003), and numerous studies using regression detail how researchers deal with multicollinearity. Multicollinearity within the multilevel modeling (MLM) framework has not received the same attention, however. There has been little discussion of the effect of multicollinearity on issues related to MLM. This seems to be a large gap in the literature as MLM is used in many domains, including but not limited to psychology, education, biology, and medicine. Multicollinearity in MLM has been mentioned in textbooks introducing MLM (Kreft & De Leeuw, 1998), but the issue of how parameter estimates and standard errors are impacted is virtually undocumented (see Shieh & Fouladi, 2003 for an exception). One goal of this paper is to examine how issues of multicollinearity impact the parameter estimates and standard errors of multilevel models.

A second goal of this paper is to develop a measure of the magnitude of multicollinearity that is present at all levels in the model called a multilevel variance inflation factor (MVIF) that is similar to the variance inflation factor (VIF) used in ordinary least squares (OLS) regression. In the following sections, I provide a simulated example and explain how these MVIF values will be calculated and how they should be interpreted for multilevel models.

Multilevel Modeling

Multilevel models are used to analyze data that has a hierarchical structure, meaning that these models require that data be measured on at least two different levels. When the data is on two different levels, it is typical to refer to the lower level data as being nested within the higher level. Some examples of multilevel data include students nested within classrooms, classrooms nested within schools, and schools nested within districts. The discussion that follows focuses on a model with data at two levels, but can be extended to more than two levels.

Multilevel models can be thought of as an extension of OLS regression models. When data have a hierarchical or multilevel structure, using OLS regression will lead to negatively biased standard errors and alpha inflation. Random coefficient regression is the alternative to OLS regression and should be used to analyze data with a multilevel structure. MLM estimates the parameters for all levels simultaneously, but it is useful to present the linear model for each level separately. A general form of a MLM with two levels is:

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + r_{ij}$$

(1)

In this equation, β_{0j} is the group intercept and β_{1j} is the group regression slope for predicting *Y* using *X*. The Level 1 error is labeled r_{ij} and its variance is labeled σ^2, which represents within group variance not explained by the model. The subscripts *i* and *j* refer to the Level 1 and 2 units, respectively. The Level 2 equations model variance in β_{0j} and β_{1j}:

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_j + u_{0j}$$

(2)

$$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_j + u_{1j}$$

(3)

where Z_j is a Level 2 predictor, the *γ*'s are Level 2 regression coefficients, and the u's represents Level 2 error terms. The regression coefficients are referred to as fixed effects; the Level 2 error terms are random effects. The variance of u_{0j} is labeled τ_{00} and represents intercept variance not explained by Z_j. Similarly, the variance of u_{1j} is labeled τ_{11} and represents the unexplained variance in the Level 1 slope.

An example of data with a multilevel structure is salespeople nested within managers. The example used here is simplified and borrowed from Mathieu, Ahearne, and Taylor (2007). In this example, there are two Level 1 predictors and one Level 2 predictor. The two Level 1 predictors are technology self-efficacy (TechSE) and use of technology (TechUse) and the Level 2 predictor is manager commitment (LeadComm). The outcome variable is performance. Substituting these variables into the previous equations produces the Level 1 equation

$$\text{Performanceij} = \beta_{0j} + \beta_{1j}\text{TechSE} + \beta_{2j}\text{TechUse} + r_{ij}$$

(4)

At Level 2, the equations are

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\text{LeadComm} + u_{0j}$$

(5)

$$\beta_{1j} = \gamma_{10} + \gamma_{11}\text{LeadComm} + u_{1j}$$

(6)

$$\beta_{2j} = \gamma_{20} + \gamma_{21}\text{LeadComm} + u_{2j}$$

(7)

Based on these equations, each employee's performance is a function of (a) the average performance of the group or class intercept (β_{0j}), (b) the regression slope for TechSE (β_{1j}) multiplied by the employees' TechSE, (c) the regression slope for TechUse (β_{2j}) multiplied by the employees' TechUse, and (d) the residual error term for the individual (see Equation 4). From this, the average performance of the group is a function of the overall grand mean (γ_{00}), a fixed coefficient (γ_{01}) multiplied by the manager's commitment, and a group level error term (see Equation 5). Equation 6 states that the relationship between the outcome variable, performance, and TechSE depends on the amount of commitment of the manager. Similarly, Equation 7 states that the relationship between performance and TechUse is dependent on manager commitment. More specifically, if γ_{11} or γ_{21} are positive, the effect of the predictor is larger with more committed managers. Conversely, if either γ_{11} or γ_{21} are negative then the effect of the predictor is smaller with more committed managers. In these equations, γ_{10} and γ_{20} are the group level average of TechSE and TechUse, respectively. Also, u_{1j} and u_{2j} are the slope variances.

Writing out the full equation we get:

$$\text{Performance}_{ij} = \gamma_{00} + \gamma_{10}\text{TechSE}_{ij} + \gamma_{20}\text{TechUse}_{ij} + \gamma_{01}\text{LeadComm}_{j} + \gamma_{11}\text{TechSE}_{ij} * \text{LeadComm}_{j} + \gamma_{21}\text{TechUse}_{ij} * \text{LeadComm}_{j} + u_{1j}\text{TechSE}_{ij} + u_{2j}\text{TechUse}_{ij} + u_{0j} + r_{ij}$$

In this equation, the terms '$\gamma_{11}\text{TechSE}_{ij} * \text{LeadComm}_{j}$' and '$\gamma_{21}\text{TechUse}_{ij} * \text{LeadComm}_{j}$' are cross-level interaction terms that appear as a result of modeling the varying regression slopes of TechSE and TechUse with manager commitment. The interpretation of the cross-level interaction terms is much simpler when the predictors are centered. Interested readers should refer to Hox (2010) for an in-depth discussion of methods for better interpreting cross-level interactions.

Multicollinearity in OLS Regression

The effects of multicollinearity in OLS regression are well known: High standard errors, overly sensitive or nonsensical regression coefficients, and low t-statistics are likely to occur. These effects make interpretation of the coefficients under study nearly impossible. In most real world cases, there is some relationship between all variables in the study. Exact multicollinearity occurs when one predictor variable can be perfectly predicted from the remaining predictors. This has the effect of making the model empirically under identified, which means that not all of the parameters can be estimated.

In practice, exact multicollinearity is rare. However, it is not necessary for multicollinearity to be exact in order for problems to exist.

A more common scenario is when two or more of the predictor variables are moderately or strongly related to one another. There is no agreed upon definition of too high a correlation between predictors. By Cohen's definition, a correlation of greater than 0.37 would be considered large. This amount of correlation between predictors is fairly common in social science research. For example, using meta-analysis techniques, Schmidt and Hunter (1998) found that the corrected correlations between general mental ability (GMA) and job knowledge is .48, GMA and bio data is .50, GMA and assessment center ratings is .50, and GMA and education is .55. It would not be surprising to see these predictors paired together when trying to predict an outcome variable such as performance, and based on the findings of Schmidt and Hunter, issues of multicollinearity may cause some problems.

Measures of Multicollinearity in OLS Regression

The variance inflation factor (VIF) provides a measure of how the variance of the parameter estimate changes relative to a model in which all predictor variables are uncorrelated. The equation for the VIF is

$$\text{VIF} = \frac{1}{1 - R^2_{i..k}} \quad (8)$$

Where i is a predictor variable, k is 1 minus the number of predictor variables in the model because I am assessing the amount of variance accounted for in the predictor of interest by all of the other predictors, and R^2 is the squared multiple correlation between X_i and the other predictor variables in the regression equation. This formula is drawn from the standard error formula and is the square of the third term

$$SE_{B_i} = \frac{sd_y}{sd_{xi}} \sqrt{\frac{1-R^2_{Y.12...k}}{n-k-1}} \sqrt{\frac{1}{1-R^2_{i.12...(i)...k}}}$$

(9)

where $1-R^2_{i.12...(i)...k}$ is the squared multiple correlation between X_i and the other predictor variables in the regression equation. Based on this formula we can see that as the relationship among predictors increases ($R^2{}_i$), the standard errors will also increase.

A VIF is calculated for each predictor in the regression equation. A common rule of thumb is that a VIF of more than 10 provides evidence of severe multicollinearity. However, Cohen et al. (2003) stated that they “believe that this common rule of thumb guideline is too high (lenient) for most behavioral science applications” (p. 423). Cohen et al. also noted that “there is no good statistical rationale for the choice of any of the traditional rule of thumb threshold values for separating acceptable from unacceptable levels of multicollinearity. For example, some authors have proposed values of 6 or 7 as a threshold value for the VIF” (p. 424). Cohen et al. also provided a table that demonstrates how parameter estimates and standard errors may noticeably change at VIF values around 5. These issues are discussed in much more detail in the coming sections.

Tolerance is another measure of multicollinearity and is simply the reciprocal of the VIF. Tolerance describes how much of the variance in a particular predictor variable is independent of the other IVs. The general rule of thumb is that tolerance values less than .10 (equivalent to a VIF of 10) indicate a serious multicollinearity problem in the equation (Cohen et al., 2003). Belsley (1984, 1991) discussed a measure of multicollinearity called condition number. Essentially the condition number is derived from a set of orthogonal dimensions composed of a correlation matrix of the predictors. These orthogonal dimensions share no variance in common and are completely nonoverlapping. Statistical programs will perform the decomposition of the correlation matrix into orthogonal dimensions, called a principal components analysis. The result of this analysis is a set of eigenvalues for each predictor that indicates the amount of shared variance between the predictors. The condition number, often (κ) kappa, is defined as the square root of the ratio of the largest eigenvalue divided by the smallest eigenvalue. A traditional rule of thumb is that κ values 30 or larger indicate severe multicollinearity; however, some researchers suggest values as low as 15 or 20. The condition number is primarily used in econometrics.

For illustration purposes, I have included some examples of how multicollinearity among predictors affects parameter estimates and standard errors. For these examples, I have generated the data using techniques similar to the ones described in the Method section of this paper. The primary difference is that for these examples I generated the data using an OLS model instead of a MLM. The first example includes two predictors (X_1 and X_2) and the population correlation between X_1 and X_2 is .30. The population parameter estimates for X_1 and X_2 were 0.5 and 0.3, respectively. The parameter estimates and standard errors for one sample using this data for X_1 and X_2 are 0.46 and 0.29, respectively. The standard errors for both estimates are 0.03.

For the next example, I calculated the parameter estimates and standard errors for one sample from a population with three predictor variables using matrix inversion. Cohen et al. (2003) describe a method for calculating the VIF using something they call the Doolittle solution. This method involves computing the inverse of the correlation matrix among the predictors. The method for calculating the inverse of a matrix is straightforward and able to be accomplished by hand or with a computer program. The population parameter estimates for all of the predictors are 0.5. The mean and standard deviation of the predictors are 1 and 0, respectively. The only other specification for the population in this example is that X_1 and X_2 are correlated .30 and X_2 and X_3 are correlated .90, with X_1 and X_3 assumed to be correlated 0. Based on the data I just described, a correlation matrix for one sample is in Table 1. Using the methods described by Cohen et al., the inverse of this matrix is found in Table 2.

The parameter estimates are β_{X1}=0.43, β_{X2}=0.58, and β_{X3}=0.40. The standard errors of the parameter estimates are 0.03 for β_{X1}, and .08 for β_{X2} and β_{X3}. As mentioned by Cohen et al., the VIFs are on the diagonals of the inverse matrix. As shown Table 2, the VIF for β_{X1} is slightly elevated from 1 due to the correlation of .30 between X_1 and X_2. For the other two predictors, the VIFs are around 9 and the standard errors are also elevated to nearly three times as large as the standard error for β_{X1}. Generally, severe multicollinearity problems arise when there are significant correlations between three or more predictors for the same reason that R^2 values increase in OLS regression with the addition of predictors more highly correlated with the outcome variable. There would be little doubt that if a correlation between X_1 and X_2 was higher and if a correlation was added between X_1 and X_3 that the VIFs would exceed 10. One can also determine the proportion of variance in each predictor that is shared with the other predictors by

$$R_i^2 = 1 - \frac{1}{VIF_i} \quad (10)$$

Where i is the predictor variable of interest and VIF is calculated in the same manner as in the previous equation.

Multicollinearity in OLS regression does not reduce the power or reliability of the model; however, it affects the standard errors and parameter estimates of the individual predictors. As seen in the example, the standard errors for the variables highly correlated with other variables in the model are elevated. This has the effect of reducing the likelihood that those parameter estimates will be significant. The main concern of high multicollinearity in practice is that the standard errors of the parameter estimates may be high, meaning that they show a lot of sample-to-sample variation and are therefore unreliable. Large standard errors typically result in a wide confidence interval around the parameter estimate and difficulty achieving significance.

The goal of the present study is to model how multicollinearity impacts multilevel models. Within the multiple regression framework, only the correlations between the variables and the sample size can be manipulated. Within the multilevel framework, correlations across levels (cross-level interactions), as well as the number of groups, the size of the groups, the variance at the group level, as well as slope means and variances can all be manipulated.

Multicollinearity in Multilevel Models

The previously mentioned issue with multicollinearity in OLS regression is that many textbooks describe the problem, but very few describe ways to diagnose and handle the issue. This issue is even more pronounced in the context of multilevel regression because not only do researchers not describe the problem, there is currently no measure of multicollinearity so there is no way to even begin diagnosing and handling issue. This problem becomes more pronounced when you consider that many researchers in recent years have framed their research questions around a multilevel framework. Most of these studies overlook the topic of multicollinearity altogether or mention that it is an issue and fail to discuss it further.

The only relevant articles in the literature that thoroughly examined the issue of multicollinearity in MLMs are by Kubitschek and Hallinan (1999) and Shieh and Fouladi (2003). Kubitschek and Hallinan (1999) examined three different statistical models with different degrees of correlations among predictors. Their results indicated that the standard errors of the parameter estimates increased with multicollinearity. They also found that the effect of certain predictors varied greatly from sample to sample as other predictors were added or removed from the model. Shieh and Fouladi (2003) conducted a Monte Carlo simulation examining the effects of varying degrees of correlations between predictors, number of groups, group size, and intraclass correlation on parameter estimates and standard errors. They detailed a number of relevant findings. They found that convergence of the model improved as the number of groups, group size, and sample size increased, and as the intraclass correlation and correlation between the Level 1 predictors decreased. They found that the Level 2 parameter estimates were not biased under various levels of multicollinearity in Level 1 predictor variables. The variance-covariance components at Level 2, however, do show bias under conditions of multicollinearity among Level 1 predictor variables. They also found that multicollinearity introduced bias into the standard errors of the parameter estimates. These findings illustrate that multicollinearity in the predictor variables at Level 1 do have an impact on the model. Kubitschek and Hallinan (1999) used real world data to examine the results from multiple models and including Level 1 predictor variables with varying levels of multicollinearity. Shieh and Fouladi (2003), while employing a simulation design, also only manipulated Level 1 predictor correlations.

The issue of cross-level interactions has not received substantial attention in terms of multicollinearity in MLMs. The above studies examined the impact of relationships between Level 1 predictors, but the only known researchers to discuss cross-level interactions and multicollinearity are Kreft and De Leeuw (1998). They used an extended example to illustrate the effects of multicollinearity on parameter estimates and standard errors. They showed that multicollinearity makes the interpretation of model coefficients difficult, especially when dealing with cross-level interactions. They found that small changes in the model led to large changes in the coefficients and standard errors for correlated variables. The primary conclusions they drew were first, group mean centering seems to improve the multicollinearity situation because correlations between Level 2 predictors and both Level 1 predictors and cross-level interactions are zero. Therefore, the only correlations to think about are between the cross-level interactions and the corresponding Level 1 predictors. Second, even in fixed coefficient models, the use of cross-level interactions is problematic.

Based on the above discussion, there still seems to be a gap in the literature because each of these studies ignored the issue of what happens when there is a relationship between the Level 2 predictors. My plan is to thoroughly examine the impact of multicollinearity when there are relationships between the 1) Level 1 predictors, 2) the cross-level interaction(s) and the Level 1 predictors, and 3) finally when there are relationships between the Level 2 predictors. I am unaware of any work in the literature even mentioning this topic.

In summary, there is a lack of discussion and research on the effects of multicollinearity in MLMs. Among researchers, there is a solid understanding of when multicollinearity is an issue in OLS regression (Cohen et al., 2003), but there is a need to have that level of understanding for MLMs. My goal is to provide researchers with a measure of multicollinearity for multilevel models called MVIF. This measure allows researchers to determine whether or not their models suffer from the problems associated with multicollinearity.

Multilevel Variance Inflation Factor

I propose using a multilevel version of the VIF that can be calculated in a manner similar to OLS models. As with the VIF for OLS, the MVIFs are the diagonals of the inverse of the predictor correlation matrix. For example, with two Level 1, (X_1 and X_2) and two Level 2 predictors (Z_1 and Z_2), the resulting correlation matrix will be four by four (X_1, X_2, Z_1, Z_2) and the four numbers on the diagonals of the inverse are the MVIFs. When there are cross-level interactions, the product term is created and included in the correlation matrix. For example, adding a cross-level interaction between X_1 and Z_1 would produce a 5 x 5 correlation matrix.

As an illustrative example, I calculated the parameter estimates and standard errors for one sample from a population with a multilevel data structure. I simulated these data in a similar manner to the previous OLS regression example. I included three Level 1 variables (X_1, X_2, and X_3) and one Level 2 variable (Z). In the population, X_1 and X_2 were correlated .30, and X_2 and X_3 were correlated .90 with no correlation between X_1 and X_3 and no correlation between the X's and Z. The within group variances of X_1, X_2, and X_3 were set to 1, as was the between group variance of Z. The population parameter estimates were .50 for γ_{10}, γ_{20}, γ_{30}, and γ_{01}, and the cross-level interaction estimates were 0. In this example, there are 30 groups of 30 people. Based on the data I just described, a correlation matrix in this situation would be seven by seven (γ_{10}, γ_{20}, γ_{30}, γ_{01}, γ_{11}, γ_{22}, γ_{33}). The correlation matrix for this data is presented in Table 3. The inverse of this matrix is presented in Table 4.

Once again, the VIFs for γ_{20} and γ_{30} were elevated, 10.42 and 9.37 respectively. This was expected because those variables correlated at .90. The fact that X_2 also has a moderate correlation with X_1 is reflected in the higher VIF for γ_{20} than γ_{30}, a value that is actually over the widely accepted significant value. It is interesting to note that the VIFs for the cross-level interactions are also highly elevated. For instance, the cross-level interaction for γ_{22} and γ_{33} are 10.43 and 9.58, respectively. This is an issue that has been largely unexplored in the literature. The effect of multicollinearity on cross-level interactions is a topic that researchers generally avoid. This avoidance appears to be as a result of the difficulty in interpreting cross-level interactions, especially when there are issues of multicollinearity that can muddle the interpretation of seemingly straightforward coefficients. Based on this example, I can say that multicollinearity does have an effect on the VIFs in a MLM context. Now the question becomes whether elevated VIFs or more specifically, multicollinearity, impacts the standard errors and parameter estimates in this sample. The parameter estimates and standard errors are in Table 5. The standard errors for the estimates involving one or both of the highly correlated Level 1 predictor variables (X_2 and X_3) are elevated compared to the standard errors involving X_1. This includes the standard errors for the cross-level interactions. This example seems to illustrate that when there is severe multicollinearity between variables used in a MLM that the cross-level interactions will become even more difficult than usual to interpret. I will discuss this topic in greater detail in the last section of this paper. Table 5 also reveals that some of the parameter estimates are nonsensical and not able to be reliably interpreted (4.87 for γ_{20}).

The primary goal of this paper is to expand on the MVIF example discussed earlier. A more comprehensive simulation is needed to fully examine the performance of MVIF's. Based on these preliminary analyses I expect that as the proportion of variance in each predictor that is shared with the other predictors (R^2) increases, MVIF and the standard error will also increase. Specifically, I am interested primarily in a) how the simulation design factors affect MVIF, b) how MVIF affects standard errors of regression coefficients, and c) how MVIF affects significance of regression coefficients.

Method

I conducted a series of Monte Carlo simulations to study the impact of multicollinearity on the regression coefficients and standard errors in a MLM under various conditions: including varying sample sizes, mean slopes, predictor relationships (Level 1 and Level 2), and slope variances.

Multilevel Model Structure

The MLMs consisted of Level 1 (X_1, X_2, and X_3) and Level 2 (Z_1, Z_2, and Z_3) predictors of the Level 1 outcome (*Y*). The simulated level 1 model was

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + r_{ij}$$

(11)

The simulated level 2 model was

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \gamma_{02} Z_{2j} + \gamma_{03} Z_{3j} + u_{0j}$$

(12)

$$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_{1j} + \gamma_{12} Z_{2j} + \gamma_{13} Z_{3j} + u_{1j}$$

(13)

$$\beta_{2j} = \gamma_{20} + \gamma_{21} Z_{1j} + \gamma_{22} Z_{2j} + \gamma_{23} Z_{3j} + u_{2j}$$

(14)

$\beta_{3j} = \gamma_{30} + \gamma_{31} Z_{1j} + \gamma_{32} Z_{2j} + \gamma_{33} Z_{3j} + u_{3j}$

(15)

The estimated coefficients were all of the fixed effect coefficients (γ's) and the Level 1 error variance component, σ^2_r. For each model, all of the predictors were specified to have a mean of 0 and a variance of 1. Y had a mean of 0. In addition, the intraclass correlation coefficient (ICC) for *Y* was fixed at .20 across all conditions and slope variances (τ_{11}, τ_{22}, τ_{33}) were fixed at .05.

Simulation Design Factors

I generated and analyzed the data using the free software environment R. I loosely based the study design on Shieh and Fouladi (2003). The study was a fully crossed 3 x 3 x 4 x 5 x 5 x 2 design with 1800 conditions. For each condition, I conducted 100 replications.

Sample size. In order to assess the effect of sample size in the study, I varied group size and the number of groups. The three sampled group sizes were 5, 10, or 20 and the three number of group manipulations were 50, 100, or 200. This resulted in total sample sizes from 250 to 4000. These values were largely based on Shieh and Fouladi (2003), and are consistent with values found in organizational literature.

Mean slope. I varied values of the mean slope for X_1, γ_{10}; possible values were either 0 or .30 creating two mean slope conditions. The mean slope of .30 is based on LaHuis and Ferguson (2009). The other mean slopes (γ_{20} and γ_{30}) were fixed to .30.

Level 2 regression coefficient. I varied values of the regression coefficient for Z_1, γ_{01}; possible values were either 0 or .30 creating two Level 2 regression coefficient conditions (LaHuis and Ferguson, 2009). The other Level 2 regression coefficients (γ_{02} and γ_{03}) were fixed to .30.

Predictor relationship. I also varied the magnitude of the relationship between the predictors. As previously mentioned an R^2 value can be calculated for each predictor that indicates the proportion of variance in the predictor of interest (X_1 and Z_1) that is shared with the other predictors. These values were 0, .25, .49, .81, and .90. In other words, 0%, 25%, 49%, 81%, or 90% of the variance in X_1 or Z_1 was accounted for by the other predictors in the model, X_2 and X_3 or Z_2 and Z_3, respectively. More specifically, in the .25 condition for X_1, 12.5% of the variance in X_1 is explained by X_2 and 12.5% of the variance is explained by X_3. This was the primary multicollinearity manipulation. The 0% condition was viewed as a Type I error condition, because there should be no multicollinearity in this condition and VIFs should be around 1.00. Five different magnitudes of predictor relationship at Level 1 and also Level 2 creates ten predictor relationship conditions. These values were chosen because they represent a sampling of the full range of possibilities. There were no cross-level relationships among the predictors.

Cross level interaction. Values of the cross-level interaction coefficient, γ_{11}, were varied to equal either 0 or .20 creating two different cross-level interaction conditions. The value .20 is a moderate effect size when Level 1 variance is standardized (LaHuis & Ferguson, 2009; Raudenbush & Liu, 2000). The other cross-level interaction coefficients (γ_{12}, γ_{13}, γ_{21}, γ_{22}, γ_{23}, γ_{31}, γ_{32}, γ_{33}) were fixed at .20 across all conditions.

Analyses

I split the analyses into essentially three parts. I conducted analyses focusing on the Level 1 predictors, Level 2 predictors, and cross-level interactions. See Appendix 1 for a breakdown of the analysis plan. For the Level 1 analyses, there were no Level 2 predictors in the model and the outcome variable, Y, had between group variance. Within the Level 1 analyses, I modeled: A) with all slopes and variances free to vary. For the Level 2 analyses, B) there were no Level 1 predictors in the model. For the cross-level interaction analyses, I ran three different models: C) where Z_1 is predicting all three Level 1 variables, D) where all three Level 2 variables are predicting X_1, and E) where all three Level 2 predictors were predicting all three Level 1 variables.

Results

In the following, I address the research questions at this point in order. First, I examine whether any of the simulation design factors have an impact on MVIF. In order to test this I calculated the correlation between each of the design factors and MVIF. Please see Figures 1a-8b as well as Appendices 2-9 for a more detailed explanation of these findings.

Effect of Design Factors on MVIF

Effect of Design Factors on Level 1 predictor MVIF. I found no relationship between Level 1 predictor (X_1) MVIF and the design factors for number of groups (r = -.01), number of participants in each group (r = -.01), mean slope (r = .01), and cross level interaction (r = .00). However, as expected, I did, find a relationship between the X_1 MVIF and the amount of variance accounted for in X_1 by X_2 and X_3 (r = .87). This finding suggests that increasing the relationships among Level 1 predictors increases MVIF.

Effect of Design Factors on Level 2 MVIF. I found no relationship between Level 2 predictor (Z_1) MVIF and the design factors for number of participants in each group (r = -.00), level 2 regression coefficient (r = .01), and cross level interaction (r = .00). I did however find a relationship between Z_1 MVIF and the number of groups (r = -.03, p < .05) and the amount of variance accounted for in Z_1 by Z_2 and Z_3 (r = .87, p < .05). This finding suggests that increasing the relationships among Level 2 predictors increases MVIF.

Effect of Design Factors on Cross-Level Interaction MVIF. I found a relationship between the cross-level interaction predictor MVIF and the design factors for number of groups (r = -.02) and number of participants in each group (r = -.01). I also found a relationship between MVIF and the amount of variance accounted for in X_1 by X_2 and X_3 (r = .34) as well as the relationship between MVIF and the amount of variance accounted for in Z_1 by Z_2 and Z_3 (r = .34). These lower correlations are expected because the relationship between X_2, X_3, and X_1Z_1 includes Z_1 and naturally drives down the amount of variance accounted for in by the other Level 1 predictors and vice versa.

These findings were expected and provided evidence that the relationship between multicollinearity and variance inflation in OLS regression holds in MLM. Overall, about 76% of the variance in MVIF for the Level 1 (r = .87) and Level 2 (r = .87) predictors was accounted for by the relationship among the predictors. Essentially, there was a relationship between collinearity among the predictors and MVIF such that increasing collinearity lead to higher MVIF values.

MVIF and Predictor Standard Errors

Effect of Level 1 Predictor MVIF on Standard Errors. Next, I examined whether or not MVIF leads to increases in standard errors of the parameter estimates for the regression coefficients. To test this question I calculated the relationship between MVIF and standard error and then I split MVIF into ranges and calculated the standard errors at each range to see how they change. It should be kept in mind that the general formula for the effect of VIF on standard errors in the OLS regression case is that the square root of the VIF is equal to the increase in standard errors. For example, if a coefficient has a VIF of 9, then the standard error for that coefficient will be 3 times as high than if that coefficient had a VIF of 1.

The overall correlation between the Level 1 predictor (X_1) MVIF and the standard error for γ_{10} is .66. This is a strong correlation and indicates that around 44% of the variance in standard errors of the parameter estimate for the Level 1 regression coefficient is due to MVIF. To further explore this relationship I examined how the standard errors would change across different ranges of MVIF. These changes can be found in Figures 3a-5b. As expected, the standard errors increase as MVIF increases. In Figures 1a-1b, which includes analyses done using the Level 1 model (A), standard errors range from .06 for the lowest amount of MVIF (between 1 and 2) to .13 for the highest amount of MVIF (between 10 and 11). Figures 3a-5b, which include analyses done using Level 1 and Level 2 variables show that standard errors range from .05 to .13 across all three models (C, D, and E). As expected, these findings demonstrate that there is a relationship between increasing amounts of MVIF for the Level 1 predictor variable and increases in standard errors for the regression coefficient. In other words, if the MVIF for X_1 is elevated, the standard error of γ_{10} will also be elevated. Examining Figures 1a-8b reveals that the standard errors increase by the expected amount. This means that if the MVIF for the predictor is 9.00, then the standard error for that predictor will be roughly three times higher than if the MVIF is 1.00.

Effect of Level 2 Predictor MVIF on Standard Errors. The overall correlation between the Level 2 predictor (Z_1) MVIF and the standard error for γ_{01} is .76. Similar to the results for the Level 1 variable, this is a strong correlation and indicates that around 58% of the variance in standard errors of the parameter estimate for the Level 2 regression coefficient is due to MVIF. Interestingly though this relationship is higher than the relationship found in Level 1. The results of the effects of Z_1 MVIF on standard errors can be found in Figures 2a-5b. In Figures 2a-2b, which includes analyses done using the Level 2 model (B), standard errors range from .06 for the lowest amount of MVIF (between 1 and 2) to .27 for the highest amount of MVIF (between 12 and 13). Figures 3a-4b show that standard errors range from .05 to .19 across models C and D, and Figures 5a-5b show that standard errors range from .05 to .20 for model E with all three Level 2 variables predicting all three Level 1 variables. These findings suggest that, like Level 1, there is a relationship between increasing amounts of MVIF for the Level 2 predictor variable and increases in standard errors for the regression coefficient. As expected, these findings demonstrate that there is a relationship between increasing amounts of MVIF for the Level 2 predictor variable and increases in standard errors for the regression coefficient. In other words, if the MVIF for Z_1 is elevated, the standard error γ_{01} will also be elevated.

Effect of Cross-Level Interaction MVIF on Standard Errors. The overall correlation between the cross-level interaction predictor MVIF and the standard error for γ_{11} is .76. This is a strong correlation and indicates around 58% of the variance in standard errors of the cross-level interaction term is due to MVIF. This is slightly larger than the relationships found at Level 1 and Level 2. The results of the effects of cross-level MVIF on standard errors can be found in Figures 6a-8b. For Models C and D, the MVIFs for X_2Z_1, X_3Z_1, X_1Z_2, and X_1Z_3 never increases above 6.99, and the standard errors increase as expected. The MVIFs for γ_{11} increase to greater than 10, and again the standard errors increase along with the MVIF. The standard errors for the Level 1 and 2 predictors when the MVIFs are less than 3.00 never increase above 0.13. Similarly, the standard errors for the cross-level interactions never increase about 0.13 when the MVIFs are less than 3.00. Also, when MVIFs are at least 10, standard errors never increase about 0.30 for any sample. For model E, the standard errors for the cross-level interaction coefficient, $\gamma_{11,}$ are substantially inflated (Figures 8a-8b) and range as high as 0.80. This finding demonstrates that as the number of cross-level interaction effects increases from three (Models C and D) to nine (Model E), the standard errors increase and therefore the cross-level interaction coefficients become less stable. This simulation demonstrates the perils of interpreting cross-level interaction coefficients in models with multiple cross-level interactions specified.

MVIF and Significance of Regression Coefficients

Effect of Level 1 Predictor MVIF on Significance of Regression Coefficients. Finally, I tested whether the standard errors increased to a level that would change the resulting significance of the Level 1 regression coefficient. In order to test this I assessed the significance of the mean slope by dividing the estimate for the mean slope, γ_{10} by its standard error. Note that 720 of the 900 conditions in which the mean slope was simulated to equal 0.30 were also simulated to have some level of multicollinearity among Level 1 predictors. In the Level 1 model (A), 78 of the 720 (10.8%) averaged samples where the mean slope is simulated to equal 0.30 are nonsignificant due to large standard errors. Compare this to the 180 averaged samples in which γ_{10} was simulated to equal 0.30 and the predictor relationship was simulated to equal 0 (no multicollinearity) where I found none of the mean slopes to be nonsignificant (0.0%). In the three models including both Level 1 and Level 2 variables (C, D, and E), I found similar results. Specifically, for model C, 74 of the 720 (10.3%) averaged samples were nonsignificant. For model D, 79 of the 720 (11.0%) averaged samples were nonsignificant. And for model E, 76 of the 720 (10.6%) averaged samples were nonsignificant. Compare these results to the no multicollinearity samples where none of the mean slopes were found to be nonsignificant (0.0%). This implies that when multicollinearity among Level 1 predictors is present, relatively large (0.30) regression coefficients can become nonsignificant due to the increase in the standard errors.

To further illustrate the effect of MVIF, I calculated 95% margins of error (standard error * 1.96) for the Level 1 coefficients (Figures 1a-8b). This may provide a guideline for researchers as to when MVIF becomes a problem causing their estimated coefficients to be nonsignificant. In Figures 1a-1b for the model with only Level 1 predictors, when there is very little MVIF (< 2.00) then the margins of error are relatively narrow. This follows directly from the standard errors discussion earlier. For example in Figure 1a, a margin of error of 0.12 indicates that any estimated regression coefficient less than 0.12 will be nonsignificant, and vice versa. The margins of error then increase to 0.16 with MVIFs between 2.00 and 3.00, and then level off at 0.20 between MVIFs of 5.00 and 9.00. The margins of error then increase to 0.25 when the MVIFs reach 10.00. This finding indicates that when MVIFs are in the 5.00 to 9.00 range, that regression coefficients must be approximately 67% larger ([.20 - .12] / .12) to be found significant. Also, when MVIFs increase to 10.00, regression coefficients must be approximately 108% larger ([.25 - .12] / .12), or slightly over double, to be found significant. This same general pattern is repeated for models C, D, and E involving the Level 2 predictors (Figures 3a-5b).

Effect of Level 2 Predictor MVIF on Significance of Regression Coefficients. I was also interested in the effect of the standard errors on the significance of the Level 2 regression coefficient. I tested this by assessing the significance of the Level 2 regression coefficient by dividing the estimate for the coefficient, γ_{01} by its standard error. In the Level 2 model (B), 175 of the 720 (24.3%) averaged samples where the Level 2 regression coefficient is simulated to equal 0.30 are nonsignificant due to large standard errors. Compare this to the 180 averaged samples in which γ_{01} was simulated to equal 0.30 and the predictor relationship was simulated to equal 0 where I found none of the regression coefficients to be nonsignificant (0.0%). In the three models including both Level 1 and Level 2 variables (C, D, and E), I found somewhat different results.

Specifically, for model C, 86 of the 720 (11.9%) averaged samples were nonsignificant. For model D, 88 of the 720 (12.2%) averaged samples were nonsignificant. And for model E, 89 of the 720 (12.4%) averaged samples were nonsignificant. This implies that when multicollinearity among Level 2 predictors is present, relatively large (0.30) regression coefficients can become nonsignificant due to the increase in the standard errors.

I also calculated margins of error for the Level 2 coefficients (Figures 1a-8b). In Figures 2a-2b for the model with only Level 2 predictors, when there is very little MVIF (< 2.00) then the margins of error are relatively narrow. This follows directly from the standard errors discussion earlier. For example in Figures 2a-2b, a margin of error of 0.12 indicates that any estimated regression coefficient less than 0.12 will be nonsignificant, and vice versa. The margins of error then increase to 0.16 with MVIFs between 2.00 and 3.00, and then increase to 0.25 when MVIFs reach 5.00, and 0.41 when MVIFs reach 6.00. This is possibly an anomalous finding due to the low number of samples with MVIFs in this range because the margins of error are found to be 0.25 again when the MVIFs reach 9.00. Similar to the Level 1 predictor findings, the margins of error increase again when MVIFs reach 10.00. This illustrates the compounding effect of MVIF at high levels. This same general pattern is repeated for models C, D, and E involving the Level 1 predictors (Figures 3a-5b).

In general, under the assumption that a regression coefficient of .10 reflects a small effect size and .20 to .30 medium effect sizes, it appears that it will be difficult even in conditions of little to no MVIF to find a regression coefficient of .10 to be significant. In addition, when MVIFs are around 5.00, then even medium sized regression coefficients will begin to be classified as nonsignificant. This finding would appear to be a problem for most researchers using MLM who would hope to detect medium effect size regression coefficients.

Effect of Cross-Level Interaction MVIF on Significance of Regression Coefficients. Finally, I was also interested in the effect of the standard errors on the significance of the cross-level interaction coefficient. In order to test this I assessed the significance of the cross-level interaction coefficient by dividing the estimate for the coefficient, γ_{11} by its standard error. Note that 864 of the 900 conditions in which the cross-level interaction coefficient was simulated to equal 0.20 were also simulated to have some level of multicollinearity among Level 1 or Level 2 predictors. In model C, 66 of the 864 (7.6%) averaged samples where the cross-level interaction coefficient is simulated to equal 0.20 are nonsignificant due to large standard errors. In model D, 91 of the 864 (10.5%) averaged samples were found to be nonsignificant. In model E, 455 of the 864 (52.7%) averaged samples were found to be nonsignificant. The results for models C and D are approximately equal to the findings for the Level 1 and Level 2 results, but the results for model E show that for a full model with three Level 1 and three

Level 2 predictors over half of the significant cross-level interaction coefficients become nonsignificant due to large standard errors.

I also calculated margins of error for the cross-level interaction coefficients (Figures 6a-8b). The results for the cross-level coefficients are very similar to the results for γ_{01} and γ_{10}. For those coefficients we found that margins of error increased to around .20 at MVIF values of 5.00, and we see that for γ_{11} the margins of error for the three Models (C, D, and E) are .18, .19, and .17 respectively. This indicates that finding cross-level interaction coefficients when MVIF values are 5.00 or higher becomes difficult because the standard errors increase to a point where they are no longer significant.

Discussion

In this Monte Carlo simulation study, I was interested in expanding the literature on the impact of multicollinearity among a set of predictors in a multilevel modeling (MLM) context. As previously stated, there is a lack of focus and research on the effects of multicollinearity in MLMs. Among researchers, there is an understanding of when multicollinearity is an issue in OLS regression (Cohen et al., 2003), but there is a need to have that level of understanding for MLMs. In order to address this gap in the literature, I described a method of calculating MVIF, similar to the method of calculating VIF in an OLS regression framework. I then varied aspects of the data (number of groups, number of people per group, relationships among Level 1 and Level 2 predictors, values of cross-level effects, mean slopes, and cross-level interaction coefficients) to assess the impact on MVIF. I focused primarily on three research questions.

The first research question that I was interested in was whether any of the manipulations impacted MVIF for Level 1 variables. I primarily wanted to assess whether increasing the amount of variance in a predictor accounted for by the other predictors would increase MVIF. I did find that increases in relationships among Level 1 predictors led to increased MVIF for those specific variables. This is exactly what I expected because the very definition of multicollinearity is the presence of relationships among predictor variables. This finding provides evidence that the MVIF statistic is a valid measure of the relationships among predictor variables. Due to the fact that no other study has discussed the topic of MVIF, there was no evidence to suggest that any of the other manipulations would have an effect on MVIF and I found that no other relationships were significant. I was also interested in whether any of the manipulations impacted MVIF for Level 2 variables. Again, I found that increases in relationships among Level 2 predictors led to increased MVIF values. No other study had assessed the impact of multicollinearity among Level 2 variables before, and it appears based on the above findings that the impact on MVIF is similar in magnitude to the findings at Level 1.

The second research question involved the relationship between MVIF and standard errors of the regression coefficients. Shieh and Fouladi (2003) demonstrated that multicollinearity among Level 1 predictors introduced bias into the standard errors of the regression coefficients. I wanted to expand on this finding by demonstrating the relationship between the MVIF statistic and increases in standard errors. I found a strong, positive relationship between MVIF and standard errors for the regression coefficients at both Level 1 and Level 2. Therefore, as MVIF increases for a predictor, the standard errors for the regression coefficients also increase. In other words, as the amount of variance accounted for in a predictor variable by the other predictors in the model increases, the parameter estimates for that predictor become less reliable. Essentially, when an inflated MVIF value is calculated, the standard error for that particular predictor should be expected to be larger than normal. Normal in this context refers to a case where multicollinearity is within an expected range. This will be discussed in further depth in the next paragraph.

The third research question dealt with the significance of the regression coefficients. I found that multicollinearity did impact regression coefficients to the point where they were no longer significant. In general for models only including Level 1 variables (Appendix 1 Model A, Figures 1a-1b), when MVIF values are around 5 or higher, regression coefficients around .20 or lower will become non-significant. Also when MVIF values are around 10 or higher, regression coefficients around .25 or lower will become non-significant. For models only including Level 2 variables (Appendix 1 Model B, Figures 2a-2b), when MVIF values are around 5 or higher, regression coefficients around .25 or lower will become non-significant. Also when MVIF values are around 10 or higher, regression coefficients around .30 or lower will become non-significant. The findings for the models including both Level 1 and Level 2 variables (Appendix 1 Models C-E, Figures 3a-5b) were consistent with the above findings. This provides researchers with a guideline for interpreting and using the MVIF statistic. The findings were very similar for the cross-level interaction coefficient (γ_{11}). Specifically, when MVIF values for the cross-level coefficients were 5 or higher, margins of error were around .20, and therefore any coefficients around .20 or lower will become non-significant. When MVIF values for cross-level coefficients reach values of 6 or higher, then the margins of error were even larger (.30) and therefore significance was even more difficult to achieve. This implies that caution should be used when interpreting the cross-level interaction coefficients involving predictors with high (>5) MVIF values.

Considering that the only design factor in this study that had a significant relationship with MVIF was the amount of variance accounted for, I can provide no simple guidelines in terms of sample size for example that will reduce the impact of multicollinearity. However, I can provide some guidance in terms of how this study can help combat issues related to multicollinearity. When creating a multilevel model, researchers can now calculate an MVIF value for each parameter included in the model. It may be possible for some models to be revised so that the degree of multicollinearity is reduced. Perhaps the remedy would be as simple as combining a few of the highly related predictors. Another possible solution would be to drop a predictor from the model that has a large MVIF value (>5). This decision should make sense in terms of the theory being tested however, and is not always straightforward because removal of one of the predictors has implications for the rest of the model, especially if it is included in an interaction. Removing a Level 1 variable and retaining the cross-level interaction coefficient has the effect of confounding the interaction with the effect of the Level 1 variable. Essentially, MVIF will not tell the researcher indisputably which predictors should be removed from the model, but it will provide an idea about the sources of multicollinearity and the power required to find significant effects. For example, if a regression coefficient has an MVIF value of 10, then even a practically significant effect can be found to be not statistically significant due to the impact of multicollinearity. The decision to collect additional data can also have the effect of reducing multicollinearity for one or more of the predictors in the model. In making the researcher aware that multicollinearity issues exist, MVIF becomes an invaluable tool to MLM practitioners.

Implications

Based on the results of the simulation studies and the following discussion, researchers can now begin to understand how relationships among the predictors in a MLM context impact their results. Previously, the problem tended to be ignored or assessed in inappropriate ways. With this demonstration of MVIF, researchers now have a tool to help improve their models and understand the impact of including certain variables. There is a premium placed on the parsimony in statistical modeling and decisions about which effects to model and variables to include can sometimes be difficult. This statistic can be used as an aid in making these decisions in terms of determining which predictors and/or cross-level effects could potentially be removed from the model. As previously mentioned these decisions should always make sense and agree with the theory being tested. Also, the decision to remove variables from a model becomes more difficult the more complex the model becomes.

In order to be an effective aid for researchers to rely on, a general cutoff for understanding what different values of MVIF indicate was necessary. Based on the results from the simulation studies, I can suggest that researchers should use caution when including parameters with MVIF values greater than 5. In addition, the further the MVIF value gets from 5, the more unreliable that estimate becomes. Using this information, a researcher can then either exclude it from the model or re-evaluate the methodology behind collecting the data for that particular predictor. Knowing the scale and having an idea of expected values greatly increases the utility of MVIF because researchers can calculate the value, know what sort of results to expect, and make changes to the model accordingly.

As previously mentioned, the issue of how parameter estimates and standard errors are impacted was virtually undocumented in the literature. Therefore a major goal of this paper was to examine how issues of multicollinearity impact the parameter estimates and standard errors of multilevel models. The findings indicate that the impact of multicollinearity in a MLM context is similar to a linear regression context. This means high standard errors, overly sensitive or nonsensical regression coefficients, and low t-statistics are likely to occur using multilevel models as well. The take home message is that multicollinearity within MLM is not an issue to be ignored because it can have a rather large impact on the overall model.

Limitations and Future Research

Future simulation research on this topic should examine the results of using different sample sizes. For example we only had group sizes ranging from 5 to 20. There are certainly examples of larger groups in the literature and researchers should test if the results are consistent across larger sample sizes. In this study, the ICC and slope variances were kept constant. Perhaps future research can include additional conditions by varying these aspects of the model. Also, researchers should examine the issue of cross-level interactions further in future studies by including more manipulations of the cross-level interaction coefficient and including cross-level relationships among predictors.

Researchers should test methods to overcome the problem of multicollinearity in MLM. For example, there are OLS regression techniques that can provide different, and preferably better, estimates of the regression coefficients. These techniques (ridge regression and principal components regression; Cohen et al., 2003) allow researchers to obtain better models without removing predictors or altering the plan for testing the theory in any way and can be the superior option when it comes to correcting for multicollinearity. Now that I have demonstrated that multicollinearity has an impact in the MLM context, the field can begin testing ways to reduce and overcome that impact.

More research is necessary with MVIF in order to fully understand the impact of multicollinearity in multilevel models. Specifically, researchers should begin to calculate and report MVIF when using multilevel models to further refine the guidelines and expected values of the statistic. This will give the field a more thorough understanding of what values are truly expected based on actual data. Researchers should implement changes in their models based on the calculated values of MVIF and determine if the changes make sense. This will add to the reliability evidence for MVIF to demonstrate findings in applied settings outside of this simulation study. Then exploration can begin in terms of how the resulting model has changed from the original model and the actual impact of multicollinearity. While the results of this one simulation study lay the foundation for using the MVIF statistic, it will take numerous published articles reporting the statistic to get a more solid understanding of the cutoff values. Ideally then a meta-analytic report can be published examining the findings.

Conclusion

The primary goal of this paper was to introduce the idea of the multilevel variance inflation factor (MVIF) into the literature and determine its utility. A comprehensive simulation was necessary to fully examine the performance of the MVIF statistic. Based on preliminary and follow-up analyses, I found that as the proportion of variance in each predictor that is shared with the other predictors (R^2) increases, MVIF and the standard error also increased. Specifically, I found that a) increases in relationships among Level 1 predictors and also relationships among Level 2 predictors led to increased MVIF for those specific variables, b) as MVIF increases for a predictor, the standard errors for the regression coefficients also increase, and c) when MVIF values for the regression coefficients were 5 or higher, margins of error were around .20, and therefore any coefficients around .20 or lower will become non-significant.

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Table 1

Simulated correlation matrix of three Level 1 variables.

	β_1	β_2	β_3
β_1	1.00	0.24	-0.05
β_2	0.24	1.00	0.89
β_3	-0.05	0.90	1.00

Table 2

Simulated inverse correlation matrix of three Level 1 variables. VIFs are on the diagonals and highlighted.

	β_1	β_2	β_3
β_1	1.84	-2.85	2.66
β_2	-2.85	9.67	-8.84
β_3	2.66	-8.84	9.09

Table 3

Simulated correlation matrix of three Level 1 variables, 1 Level 2 variable, and three cross-level interactions.

	γ_{10}	γ_{20}	γ_{30}	γ_{01}	γ_{11}	γ_{22}	γ_{33}
γ_{10}	1.00	0.31	0.02	0.02	0.16	0.06	0.02
γ_{20}	0.31	1.00	0.90	-0.03	0.07	0.13	0.10
γ_{30}	0.02	0.90	1.00	-0.03	0.02	0.10	0.10
γ_{01}	0.02	-0.02	-0.03	1.00	0.02	-0.05	-0.04
γ_{11}	0.15	0.07	0.02	0.02	1.00	0.27	0.02
γ_{22}	0.06	0.13	0.10	-0.05	0.27	1.00	0.91
γ_{33}	0.02	0.10	0.10	-0.04	0.02	0.91	1.00

Table 4

Simulated inverse correlation matrix of three Level 1 variables, one Level 2 variable, and three cross-level interactions. MVIFs are on the diagonals and highlighted.

	γ_{10}	γ_{20}	γ_{30}	γ_{01}	γ_{11}	γ_{22}	γ_{33}
γ_{10}	1.90	-3.02	2.69	-0.04	-0.29	0.58	-0.54
γ_{20}	-3.02	10.42	-9.33	0.07	0.52	-2.22	2.00
γ_{30}	2.69	-9.33	9.37	-0.04	-0.48	1.98	-1.88
γ_{01}	-0.04	0.07	-0.04	1.01	-0.08	0.22	-0.16
γ_{11}	-0.29	0.52	-0.48	-0.08	1.71	-2.67	2.40
γ_{22}	0.58	-2.22	1.98	0.22	-2.67	10.43	-9.45
γ_{33}	-0.54	2.00	-1.88	-0.16	2.40	-9.45	9.58

Table 5

MVIFs, parameter estimates, standard errors, and t-values for the simulated multilevel model.

Variable	MVIF	Estimate	Standard Error	T-value
γ_{10}	1.90	0.50	0.04	11.87
γ_{20}	10.42	4.87	0.10	50.38
γ_{30}	9.37	0.60	0.09	6.37
γ_{01}	1.01	0.40	0.18	2.30
γ_{11}	1.71	-0.01	0.04	-0.33
γ_{22}	10.43	-0.02	0.09	-0.20
γ_{33}	9.58	0.02	0.09	0.19

Figure 1a. Margins of error in each MVIF range based on a multilevel model with only Level 1 variables.

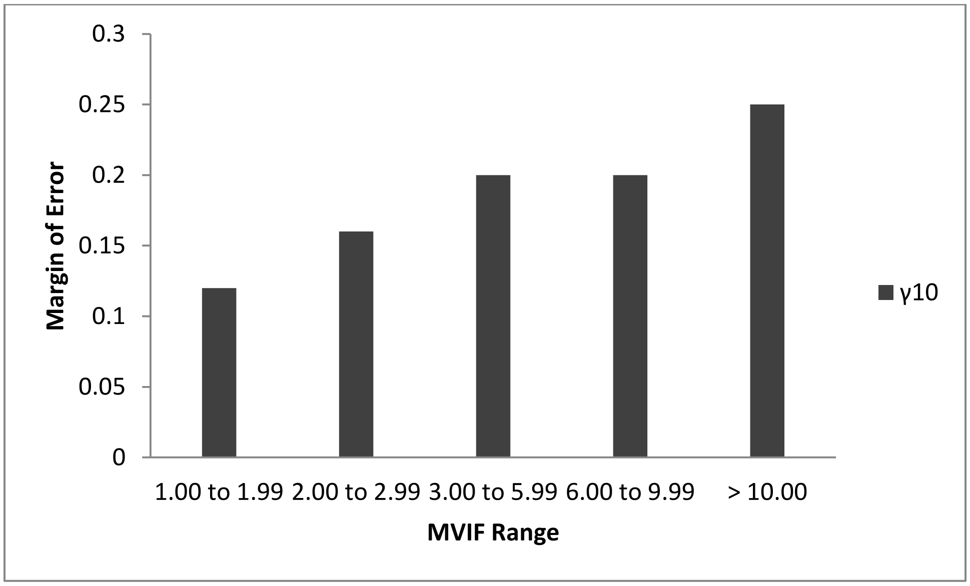


Figure 1b. Standard error means in each MVIF range based on a multilevel model with only Level 1 variables.

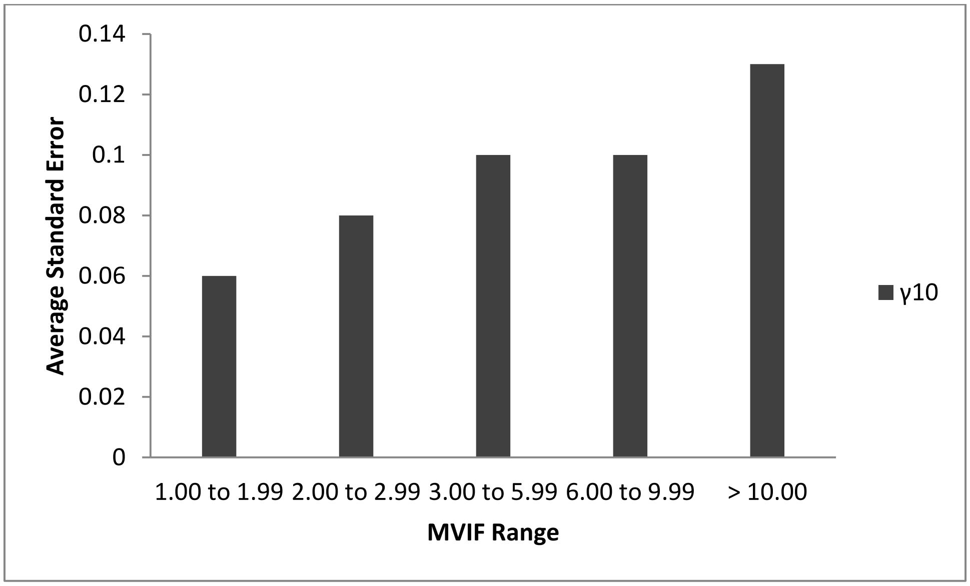


Figure 2a. Margins of error in each MVIF range based on a multilevel model with only Level 2 variables.

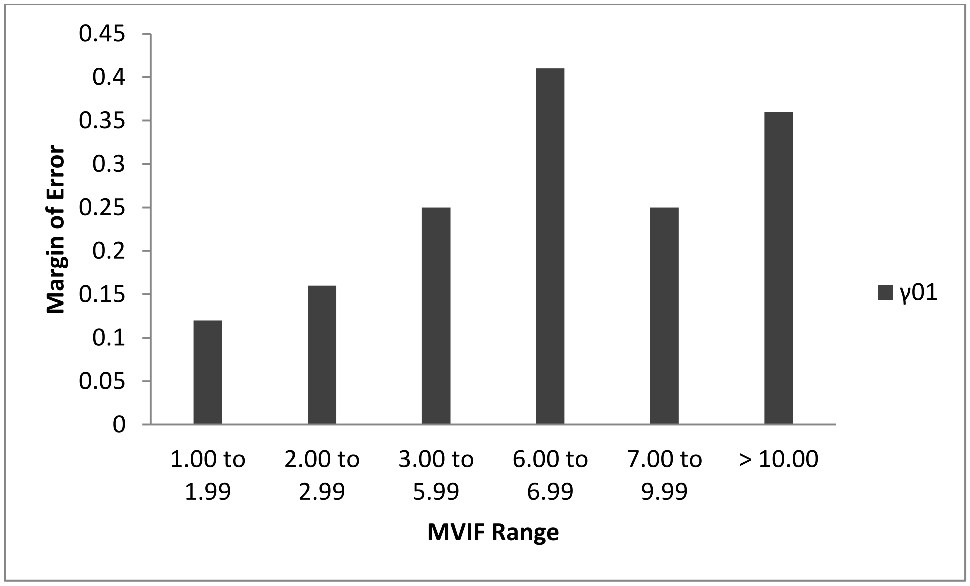


Figure 2b. Standard error means in each MVIF range based on a multilevel model with only Level 2 variables.

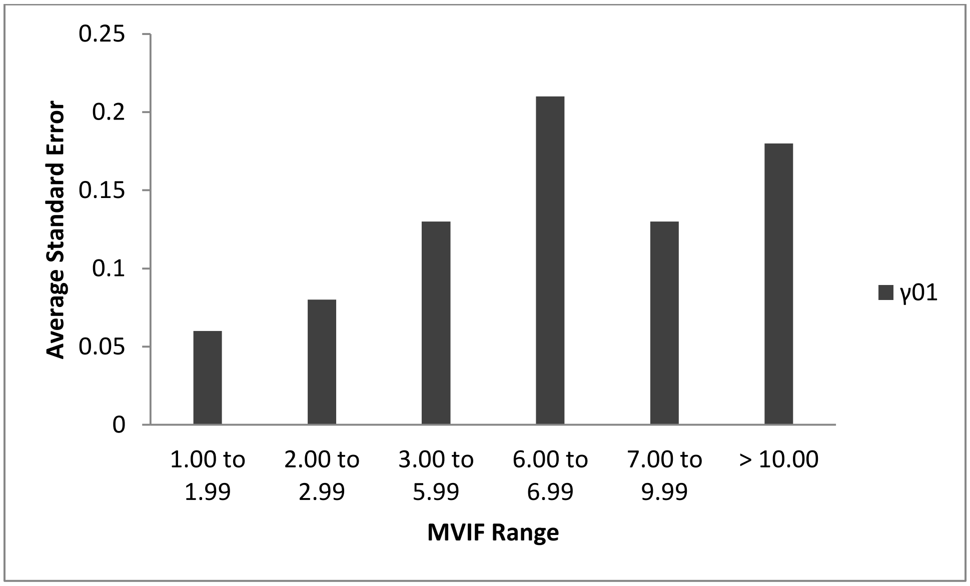


Figure 3a. Margins of error in each MVIF range based on a multilevel model with a Level 2 variable predicting the slopes of all three Level 1 variables.

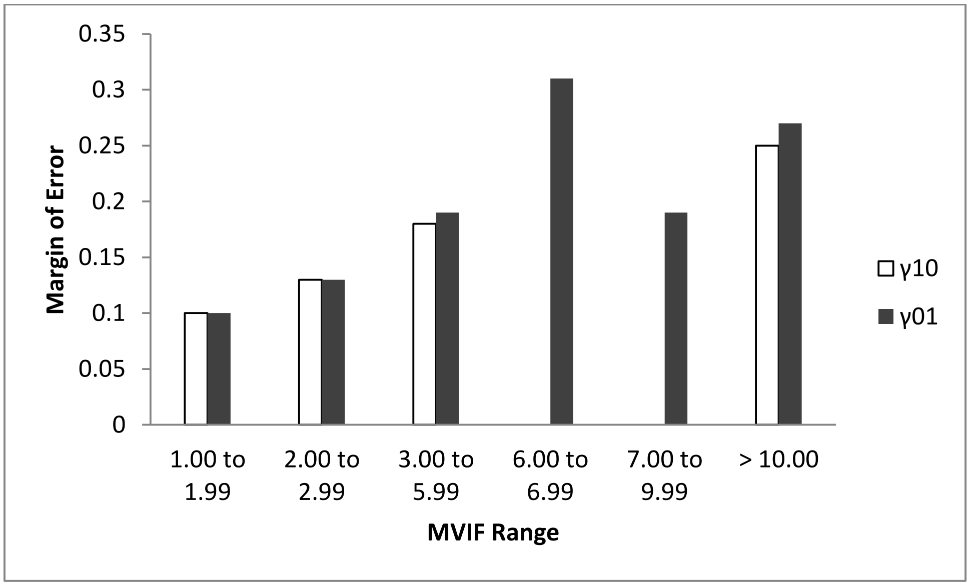


Figure 3b. Standard error means in each MVIF range based on a multilevel model with a Level 2 variable predicting the slopes of all three Level 1 variables.

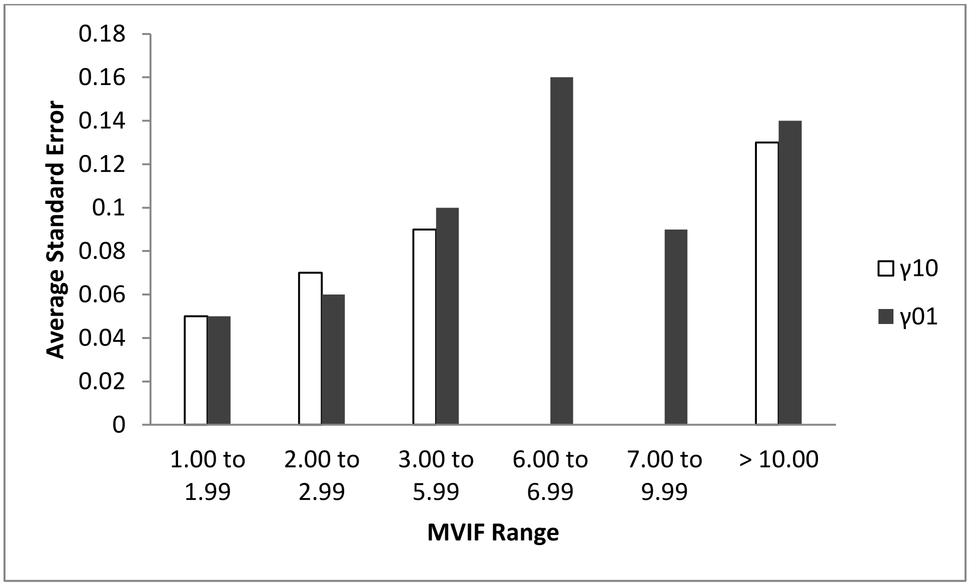


Figure 4a. Margins of error in each MVIF range based on a multilevel model with all three Level 2 variables predicting the slopes of one Level 1 variable.

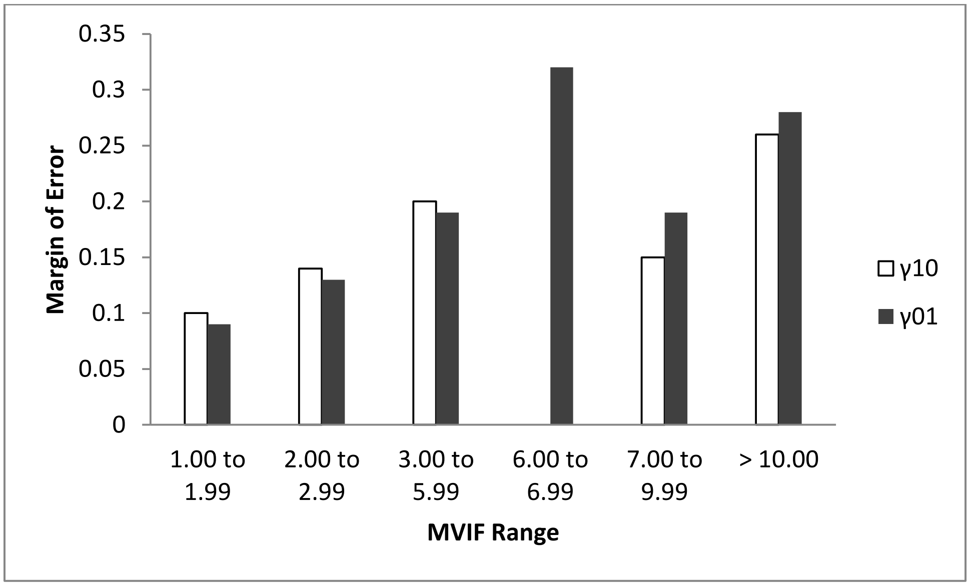


Figure 4b. Standard error means in each MVIF range based on a multilevel model with all three Level 2 variable predicting the slopes of one Level 1 variable.

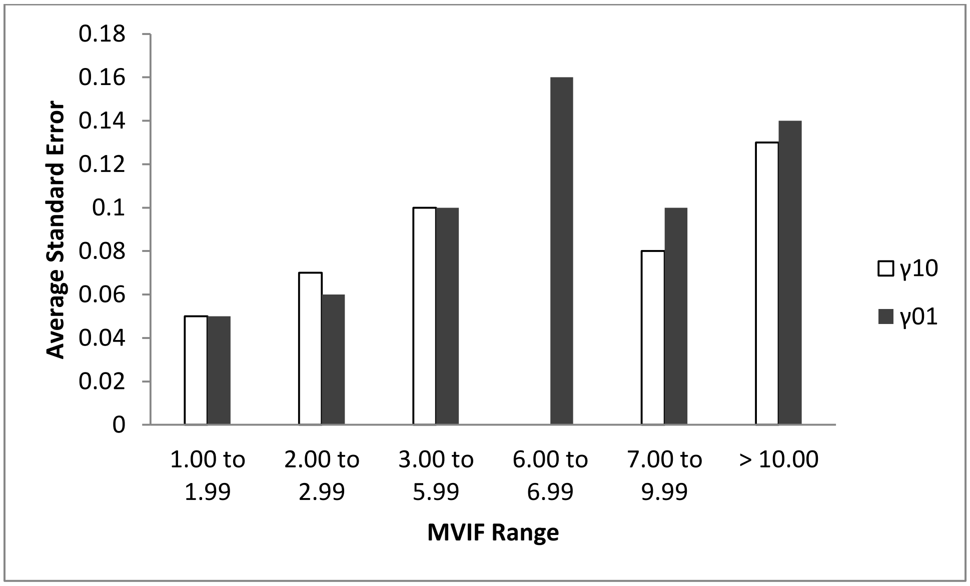


Figure 5a. Margins of error in each MVIF range based on a multilevel model with all three Level 2 variables predicting the slopes of all three Level 1 variables.

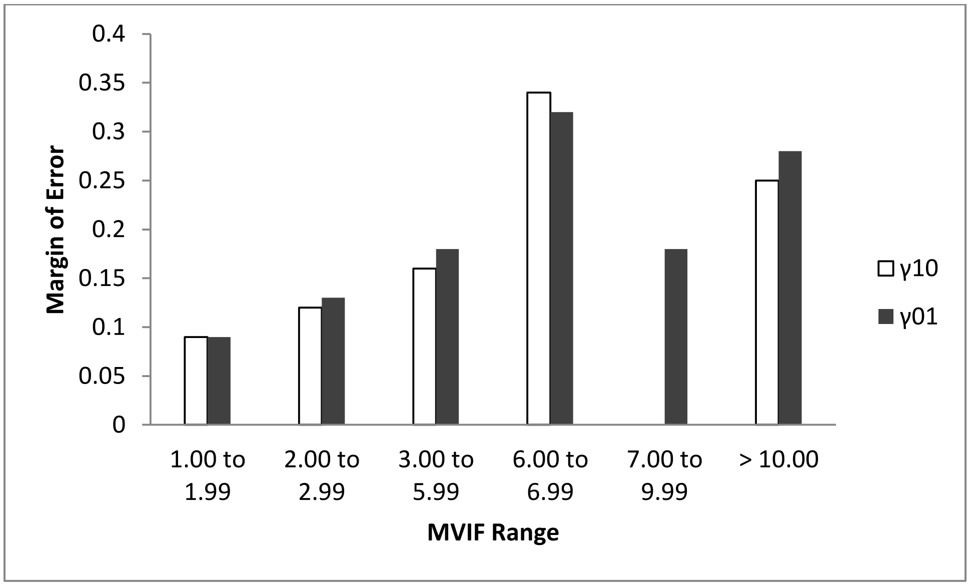


Figure 5b. Standard error means in each MVIF range based on a multilevel model with all three Level 2 variable predicting the slopes of all three Level 1 variables.

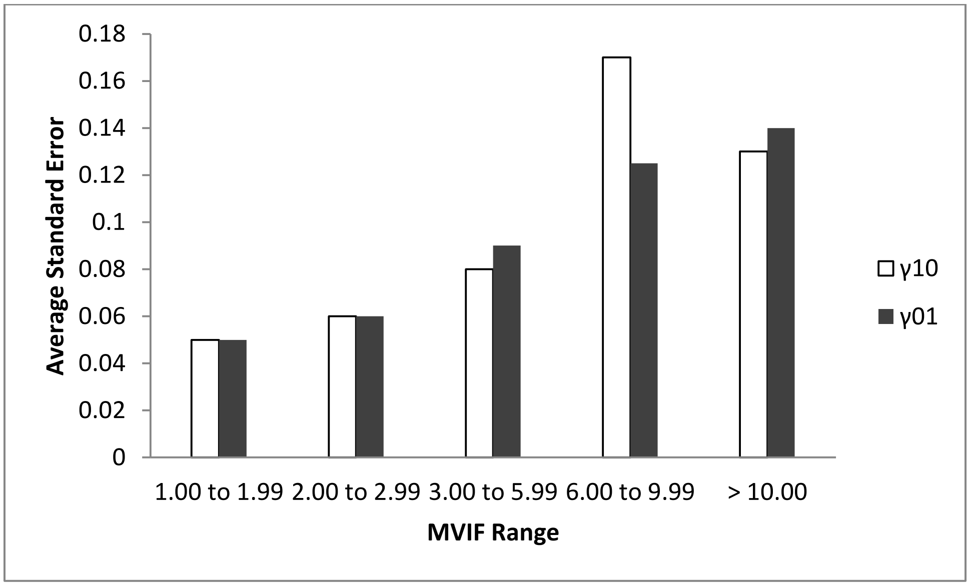


Figure 6a. Margins of error of cross-level interaction coefficients in each MVIF range based on a multilevel model with a Level 2 variable predicting the slopes of all three Level 1 variables.

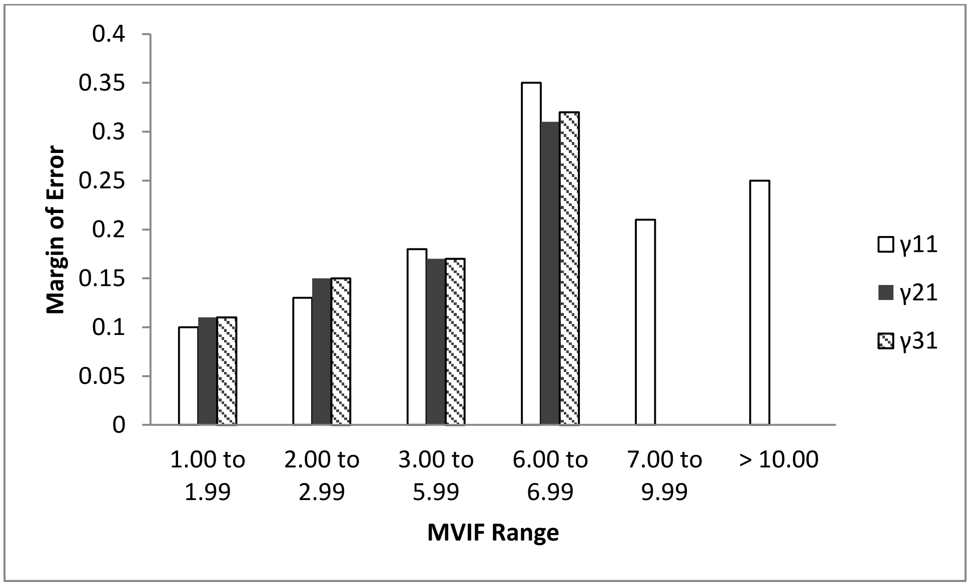


Figure 6b. Standard error means of cross-level interaction coefficients in each MVIF range based on a multilevel model with a Level 2 variable predicting the slopes of all three Level 1 variables.

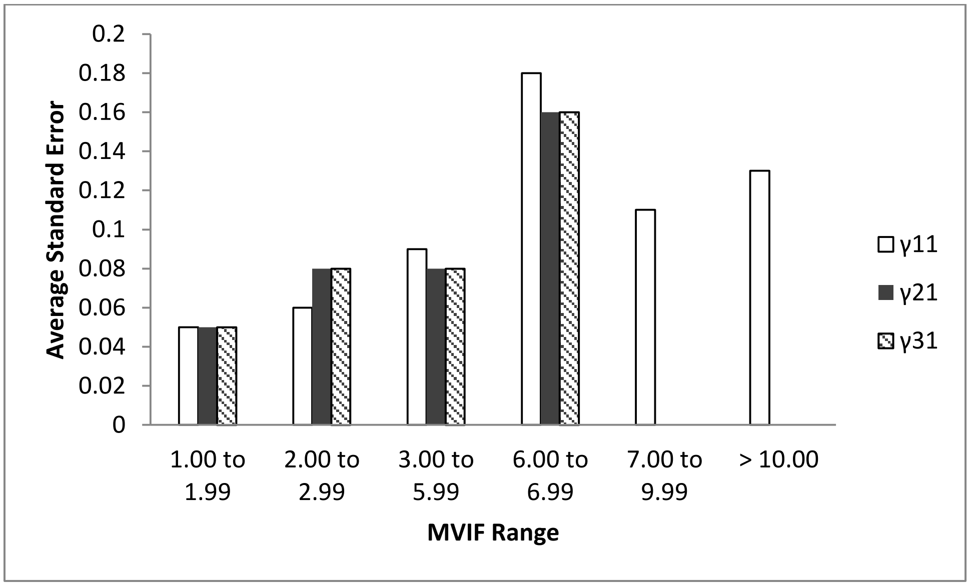


Figure 7a. Margins of error of cross-level interaction coefficients in each MVIF range based on a multilevel model with all three Level 2 variables predicting the slopes of one Level 1 variable.

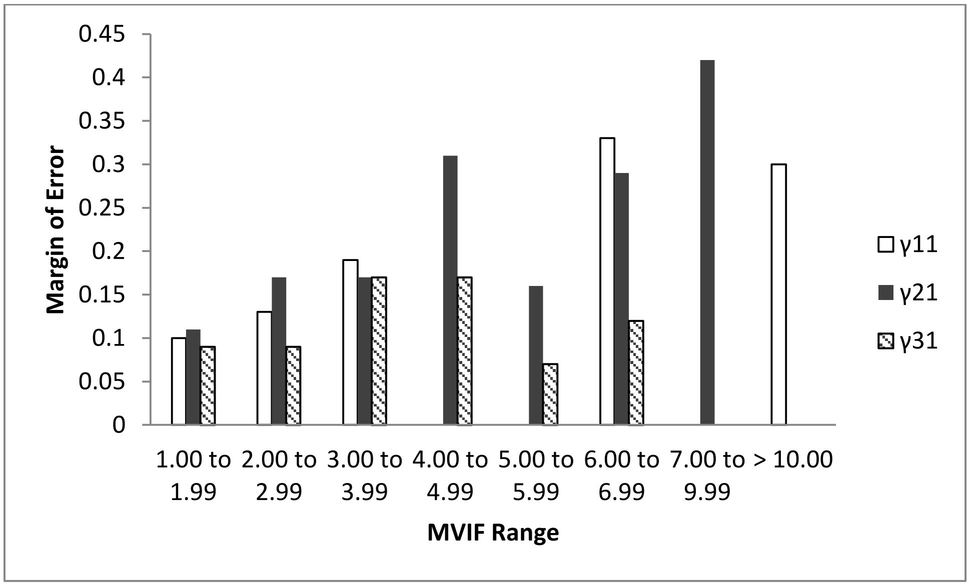


Figure 7b. Standard error means of cross-level interaction coefficients in each MVIF range based on a multilevel model with all three Level 2 variable predicting the slopes of one Level 1 variable.

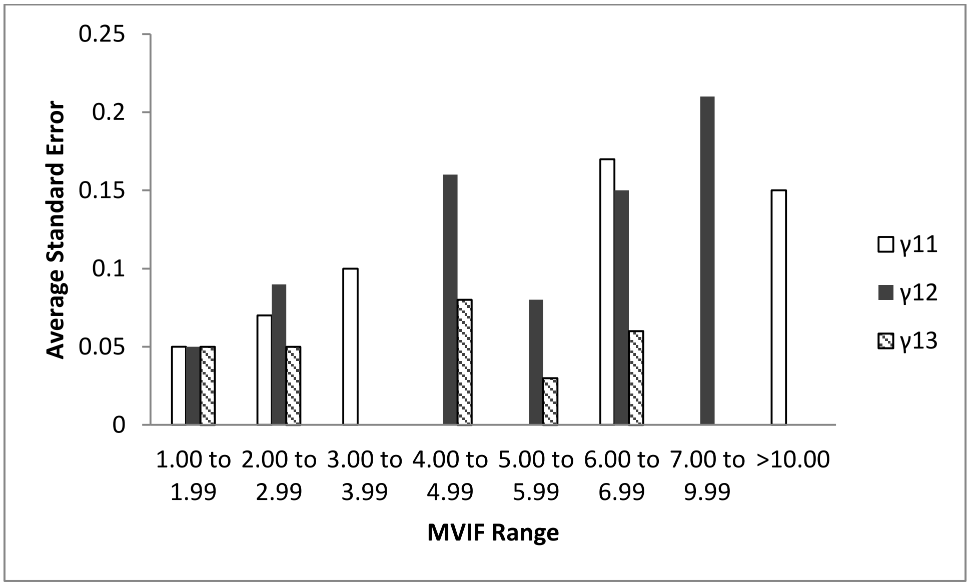


Figure 8a. Margins of error of cross-level interaction coefficients in each MVIF range based on a multilevel model with all three Level 2 variable predicting the slopes of all three Level 1 variables.

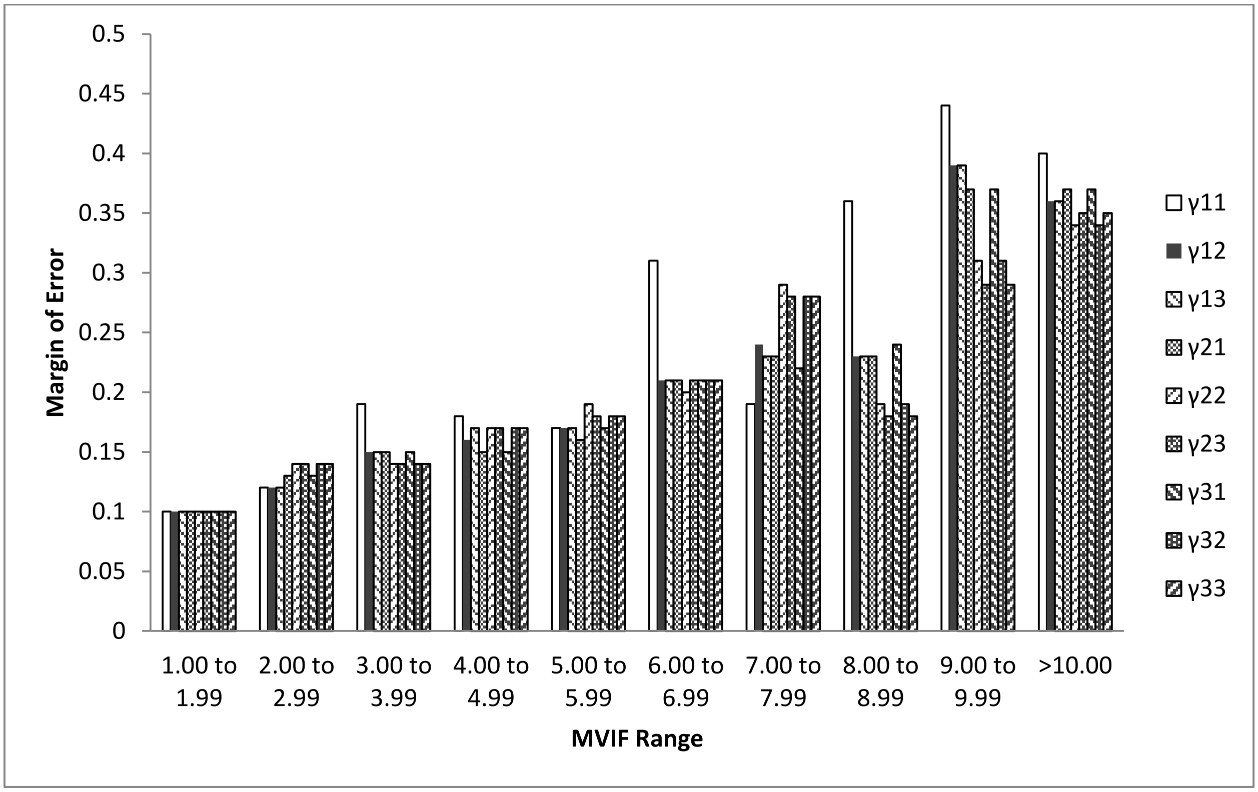
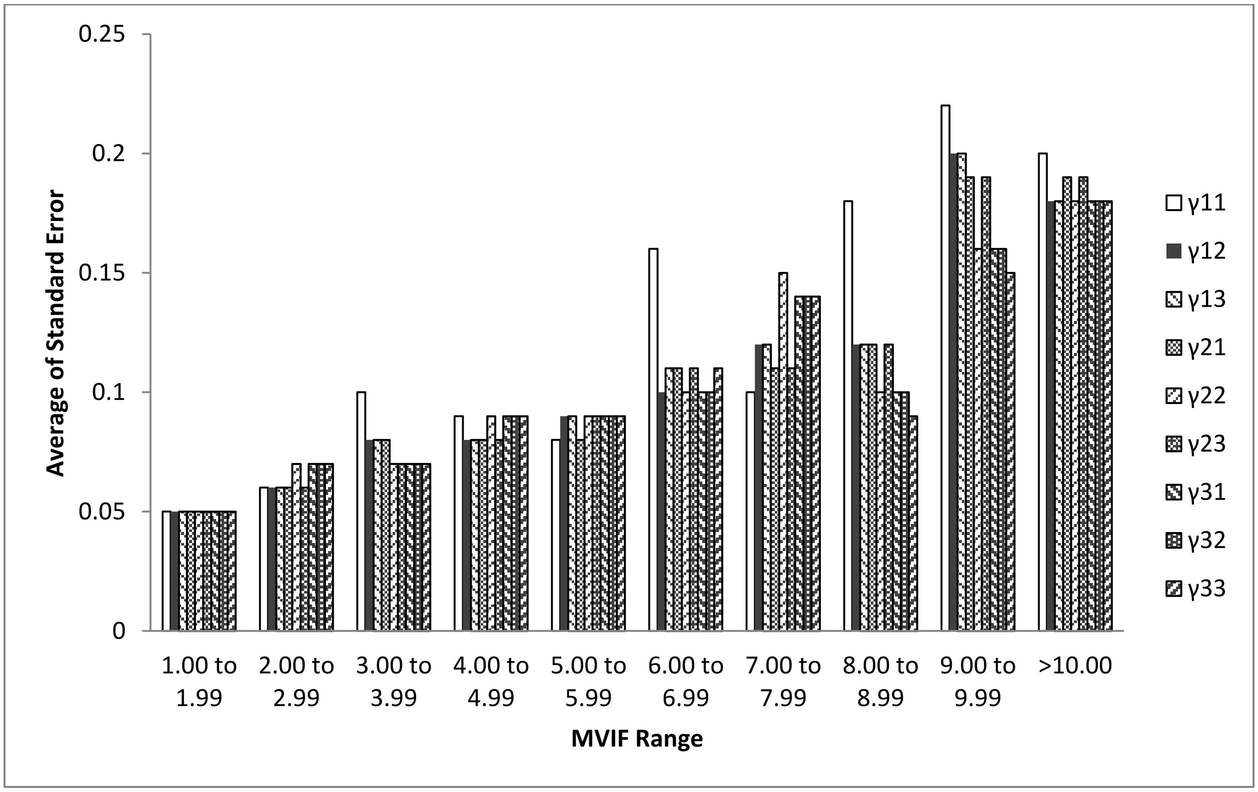


Figure 8b. Standard error means of cross-level interaction coefficients in each MVIF range based on a multilevel model with all three Level 2 variable predicting the slopes of all three Level 1 variables.



Appendix 1

Plan for analyses

Level 1 model analyses

A. $Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + r_{ij}$

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10} + u_{1j}$

$\beta_{2j} = \gamma_{20} + u_{2j}$

$\beta_{3j} = \gamma_{30} + u_{3j}$

Level 2 model analyses

B. $Y_{ij} = \beta_{0j} + r_{ij}$

$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \gamma_{02} Z_{2j} + \gamma_{03} Z_{3j} + u_{0j}$

Cross-level analyses

C. $Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + r_{ij}$

$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \gamma_{02} Z_{2j} + \gamma_{03} Z_{3j} + u_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_{1j} + u_{1j}$

$\beta_{2j} = \gamma_{20} + \gamma_{21} Z_{1j} + u_{2j}$

$\beta_{3j} = \gamma_{30} + \gamma_{31} Z_{1j} + u_{3j}$

D. $Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + r_{ij}$

$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \gamma_{02} Z_{2j} + \gamma_{03} Z_{3j} + u_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_{1j} + \gamma_{12} Z_{2j} + \gamma_{13} Z_{3j} + u_{1j}$

$\beta_{2j} = \gamma_{20} + u_{2j}$

$\beta_{3j} = \gamma_{30} + u_{3j}$

E. $Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + \beta_{3j} X_{3ij} + r_{ij}$

$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_{1j} + \gamma_{02} Z_{2j} + \gamma_{03} Z_{3j} + u_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_{1j} + \gamma_{12} Z_{2j} + \gamma_{13} Z_{3j} + u_{1j}$

$\beta_{2j} = \gamma_{20} + \gamma_{21} Z_{1j} + \gamma_{22} Z_{2j} + \gamma_{23} Z_{3j} + u_{2j}$

$\beta_{3j} = \gamma_{30} + \gamma_{31} Z_{1j} + \gamma_{32} Z_{2j} + \gamma_{33} Z_{3j} + u_{3j}$

Appendix 2

Standard error ranges, means, margins of error, and number of conditions falling in each MVIF range based on a model with Level 1 variables only.

γ_{10} MVIF Range	γ_{10} Std. Err. Range	γ_{10} Avg. Std. Err.	Avg. Margin of Error	Number of Conditions
1.00 to 1.99	.03 to .12	.06	± .12	831
2.00 to 2.99	.03 to .13	.08	± .16	249
3.00 to 5.99	.05 to .19	.10	± .20	360
6.00 to 9.99	.06 to .17	.10	± .20	4
>10.00	.06 to .25	.13	± .25	356

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges as there were no MVIFs between 3 and 5, and also between 6 and 9.

Appendix 3

Standard error ranges, means, margins of error, and number of conditions falling in each MVIF range based on a model with Level 2 variables only.

γ_{01} MVIF Range	γ_{01} Std. Err. Range	γ_{01} Avg. Std Err	Avg. Margin of Error	Number of Conditions
1.00 to 1.99	.03 to .13	.06	± .12	725
2.00 to 2.99	.04 to .16	.08	± .16	325
3.00 to 5.99	.06 to .27	.13	± .25	348
6.00 to 6.99	.13 to .27	.21	± .41	12
7.00 to 9.99	.09 to .16	.13	± .25	5
>10.00	.09 to .37	.18	± .36	355

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges as there were no MVIFs between 3 and 5, and also between 7 and 9.

Appendix 4

Standard error ranges, means, margins of error, and number of conditions falling in each MVIF range based on a model with a Level 2 variable predicting the slopes of all three Level 1 variables.

MVIF Range	γ_{10} Std. Err. / γ_{01} Std. Err. Range	γ_{10} Avg. Std. Err. / γ_{01} Avg. Std. Err.	Avg. Margin of Error	Number of Conditions γ_{10} / γ_{01}
1.00 to 1.99	.02 to .10 / .02 to .10	.05 / .05	± .10 / .10	799 / 751
2.00 to 2.99	.03 to .12 / .03 to .11	.07 / .06	± .13 / .13	281 / 329
3.00 to 5.99	.04 to .18 / .05 to .18	.09 / .10	± .18 / .19	360 / 337
6.00 to 6.99	NA / .10 to .19	NA / .16	NA / ± .31	0 / 23
7.00 to 9.99	NA / .07 to .13	NA / .09	NA / .19	0 / 5
>10.00	.06 to .25 / .07 to .25	.13 / .14	± .25 / .27	360 / 355

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges as there were no MVIFs between 3 and 5, and also between 7 and 9. Also see Figures 3a and 3b.

Appendix 5

Standard error ranges, means, margins of error, and number of conditions falling in each MVIF range based on a model with all three Level 2 variables predicting the slopes of one Level 1 variable.

MVIF Range	γ_{10} Std. Err. / γ_{01} Std. Err. Range	γ_{10} Avg. Std. Err. / γ_{01} Avg. Std. Err.	Avg. Margin of Error	Number of Conditions γ_{10} / γ_{01}
1.00 to 1.99	.02 to .11 / .02 to .10	.05 / .05	± .10 / .09	772 / 752
2.00 to 2.99	.03 to .13 / .03 to .12	.07 / .06	± .14 / .13	308 / 328
3.00 to 5.99	.05 to .19 / .05 to .19	.10 / .10	± .20 / .19	360 / 335
6.00 to 6.99	NA / .10 to .19	NA / .16	NA / ± .32	0 / 25
7.00 to 9.99	.07 to .09 / .07 to .13	.08 / .10	± .15 / .19	2 / 5
>10.00	.06 to .25 / .07 to .27	.13 / .14	± .26 / .28	358 / 355

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges as there were no MVIFs between 3 and 5, and also between 7 and 9. Also see Figures 4a and 4b.

Appendix 6

Standard error ranges, means, margins of error, and number of conditions falling in each MVIF range based on a model with all three Level 2 variables predicting the slopes of all three Level 1 variables.

MVIF Range	γ_{10} Std. Err. / γ_{01} Std. Err. Range	γ_{10} Avg. Std. Err. / γ_{01} Avg. Std. Err.	Avg. Margin of Error	Number of Conditions γ_{10} / γ_{01}
1.00 to 1.99	.02 to .09 / .02 to .10	.05 / .05	± .09 / .09	732 / 740
2.00 to 2.99	.03 to .11 / .03 to .12	.06 / .06	± .12 / .13	348 / 340
3.00 to 5.99	.04 to .18 / .05 to .18	.08 / .09	± .16 / .18	322 / 301
6.00 to 6.99	.12 to .18 / .11 to .19	.17 / .16	± .34 / ± .32	38 / 59
7.00 to 9.99	NA / .09	NA / .09	NA / .18	0 / 1
10.00 to 10.99	.05 to .25 / .07 to .26	.13 / .14	± .25 / .28	360 / 359

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges as there were no MVIFs between 3 and 5, and also between 7 and 9. Also see Figures 5a and 5b.

Appendix 7

Standard error ranges and means, and number of conditions for cross-level interactions falling in each MVIF range based on a model with a Level 2 variable predicting the slopes of all three Level 1 variables.

Cross-Level Coefficient	MVIF Range	Std. Err. Range	Avg. Std. Err.	Avg. Margin of Error	Number of Conditions
γ_{11}	1.00 to 1.99	.02 to .10	.05	± .10	737
	2.00 to 2.99	.03 to .12	.06	± .13	343
	3.00 to 5.99	.04 to .19	.09	± .18	353
	6.00 to 6.99	.17 to .19	.18	± .35	7
	7.00 to 9.99	.06 to .16	.11	± .21	2
	> 10.00	.06 to .26	.13	± .25	358
γ_{21}	1.00 to 1.99	.02 to .11	.05	± .11	1080
	2.00 to 3.99	.03 to .15	.08	± .15	360
	4.00 to 5.99	.04 to .19	.08	± .17	296
	6.00 to 6.99	.09 to .20	.16	± .31	64
γ_{31}	1.00 to 1.99	.02 to .11	.05	± .11	1080
	2.00 to 3.99	.03 to .15	.08	± .15	360
	4.00 to 5.99	.04 to .18	.08	± .17	300
	6.00 to 6.99	.09 to .20	.16	± .32	60

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges.

Appendix 8

Standard error ranges and means, and number of conditions for cross-level interactions falling in each MVIF range based on a model with all three Level 2 variables predicting the slopes of one Level 1 variable.

Cross-Level Coefficient	MVIF Range	Std. Err. Range	Avg. Std. Err.	Avg. Margin of Error	Number of Conditions
γ_{11}	1.00 to 1.99	.02 to .10	.05	± .10	723
	2.00 to 2.99	.03 to .13	.07	± .13	357
	3.00 to 5.99	.05 to .20	.10	± .19	281
	6.00 to 6.99	.11 to .21	.17	± .33	79
	7.00 to 9.99	NA	NA	NA	0
	> 10.00	.07 to .29	.15	± .30	360
γ_{12}	1.00 to 1.99	.02 to .11	.05	± .11	1080
	2.00 to 3.99	.04 to .16	.09	± .17	356
	4.00 to 4.99	.16 to .16	.16	± .31	4
	5.00 to 5.99	.05 to .14	.08	± .16	181
	6.00 to 6.99	.08 to .21	.15	± .29	178
	7.00 to 7.99	.21	.21	± .42	1
γ_{13}	1.00 to 1.99	.02 to .09	.05	± .09	1080
	2.00 to 3.99	.02 to .09	.05	± .09	358
	4.00 to 4.99	.08 to .08	.08	± .17	2
	5.00 to 5.99	.02 to .06	.03	± .07	170

6.00 to 6.99	.03 to .09	.06	± .12	190

Note: Margins of error calculated as 1.96 multiplied by the standard error. Collapsed across some ranges.

Appendix 9

Standard error ranges and means, and number of conditions for cross-level interactions falling in each MVIF range based on a model with all three Level 2 variables predicting the slopes of all three Level 1 variables.

Cross-Level Coefficient	MVIF Range	Std. Err. Range	Avg. Std. Err.	Avg. Margin of Error	Number of Conditions
γ_{11}	1.00 to 1.99	.02 to .10	.05	± .10	273
	2.00 to 2.99	.03 to .12	.06	± .12	279
	3.00 to 3.99	.04 to .14	.10	± .19	36
	4.00 to 4.99	.04 to .16	.09	± .18	60
	5.00 to 5.99	.04 to .14	.08	± .17	115
	6.00 to 6.99	.11 to .20	.16	± .31	28
	7.00 to 7.99	.05 to .20	.10	± .19	118
	8.00 to 8.99	.12 to .22	.18	± .36	26
	9.00 to 9.99	.22	.22	± .44	1
	>10.00	.06 to .80	.20	± .40	864
γ_{12}	1.00 to 1.99	.02 to .11	.05	± .10	360
	2.00 to 2.99	.03 to .13	.06	± .12	220
	3.00 to 3.99	.03 to .15	.08	± .15	139
	4.00 to 4.99	.04 to .17	.08	± .16	65
	5.00 to 5.99	.04 to .17	.09	± .17	105
	6.00 to 6.99	.04 to .20	.10	± .21	160
	7.00 to 7.99	.05 to .21	.12	± .24	73

	8.00 to 8.99	.05 to .22	.12	± .23	85
	9.00 to 9.99	.11 to .23	.20	± .39	17
	>10.00	.06 to .59	.18	± .36	576
γ_{13}	1.00 to 1.99	.02 to .11	.05	± .10	360
	2.00 to 2.99	.03 to .13	.06	± .12	219
	3.00 to 3.99	.03 to .15	.08	± .15	138
	4.00 to 4.99	.04 to .17	.08	± .17	66
	5.00 to 5.99	.04 to .17	.09	± .17	109
	6.00 to 6.99	.04 to .20	.11	± .21	160
	7.00 to 7.99	.05 to .21	.12	± .23	69
	8.00 to 8.99	.05 to .22	.12	± .23	87
	9.00 to 9.99	.15 to .23	.20	± .39	16
	>10.00	.06 to .59	.18	± .36	576
γ_{21}	1.00 to 1.99	.02 to .10	.05	± .10	360
	2.00 to 2.99	.03 to .13	.06	± .13	220
	3.00 to 3.99	.03 to .14	.08	± .15	140
	4.00 to 4.99	.04 to .16	.08	± .15	66
	5.00 to 5.99	.04 to .16	.08	± .16	102
	6.00 to 6.99	.05 to .19	.11	± .21	156
	7.00 to 7.99	.05 to .21	.11	± .23	84
	8.00 to 8.99	.06 to .21	.12	± .23	74
	9.00 to 9.99	.12 to .23	.19	± .37	19
	>10.00	.06 to .59	.19	± .37	579

γ_{22}	1.00 to 1.99	.02 to .10	.05	± .10	555
	2.00 to 2.99	.03 to .12	.07	± .14	93
	3.00 to 3.99	.03 to .15	.07	± .14	224
	4.00 to 4.99	.04 to .16	.09	± .17	131
	5.00 to 5.99	.04 to .18	.09	± .19	157
	6.00 to 6.99	.05 to .20	.10	± .20	144
	7.00 to 7.99	.08 to .21	.15	± .29	60
	8.00 to 8.99	.05 to .21	.10	± .19	93
	9.00 to 9.99	.08 to .23	.16	± .31	49
	>10.00	.06 to .44	.18	± .34	294
γ_{23}	1.00 to 1.99	.02 to .10	.05	± .10	556
	2.00 to 2.99	.03 to .12	.07	± .14	92
	3.00 to 3.99	.03 to .15	.07	± .14	224
	4.00 to 4.99	.04 to .16	.09	± .17	133
	5.00 to 5.99	.04 to .18	.09	± .18	155
	6.00 to 6.99	.05 to .20	.11	± .21	144
	7.00 to 7.99	.08 to .21	.14	± .28	64
	8.00 to 8.99	.05 to .16	.09	± .18	85
	9.00 to 9.99	.06 to .23	.15	± .29	53
	>10.00	.06 to .44	.18	± .35	294
γ_{31}	1.00 to 1.99	.02 to .11	.05	± .10	360
	2.00 to 2.99	.03 to .13	.06	± .13	220
	3.00 to 3.99	.03 to .14	.07	± .15	140

	4.00 to 4.99	.04 to .16	.08	± .15	65
	5.00 to 5.99	.04 to .16	.09	± .17	105
	6.00 to 6.99	.05 to .20	.11	± .21	155
	7.00 to 7.99	.05 to .21	.11	± .22	83
	8.00 to 8.99	.06 to .22	.12	± .24	76
	9.00 to 9.99	.12 to .23	.19	± .37	18
	>10.00	.06 to .60	.19	± .37	578
γ_{32}	1.00 to 1.99	.02 to .10	.05	± .10	555
	2.00 to 2.99	.03 to .12	.07	± .14	93
	3.00 to 3.99	.03 to .15	.07	± .14	224
	4.00 to 4.99	.04 to .16	.09	± .17	133
	5.00 to 5.99	.04 to .18	.09	± .18	153
	6.00 to 6.99	.05 to .20	.10	± .21	149
	7.00 to 7.99	.08 to .21	.14	± .28	56
	8.00 to 8.99	.05 to .21	.10	± .19	95
	9.00 to 9.99	.08 to .23	.16	± .31	50
	>10.00	.06 to .44	.18	± .34	292
γ_{33}	1.00 to 1.99	.02 to .10	.05	± .10	556
	2.00 to 2.99	.03 to .12	.07	± .14	92
	3.00 to 3.99	.03 to .15	.07	± .14	228
	4.00 to 4.99	.04 to .16	.09	± .17	124
	5.00 to 5.99	.04 to .18	.09	± .18	162
	6.00 to 6.99	.05 to .20	.11	± .21	144

7.00 to 7.99	.08 to .21	.14	± .28	61
8.00 to 8.99	.05 to .21	.09	± .18	83
9.00 to 9.99	.06 to .23	.15	± .29	56
>10.00	.06 to .44	.18	± .35	294

Note: Margins of error calculated as 1.96 multiplied by the standard error.