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A Stochastic Production Planning Model Under Uncertain Demand

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A Stochastic Production Planning Model Under Uncertain Demand

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY MEENAKSHI PRAJAPATI ENTITLED A Stochastic Production Planning Model Under Uncertain Demand BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering

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Abstract

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Production planning plays a vital role in the management of manufacturing facilities. The problem is to determine the production loading plan-consisting of the quantity of production and the workforce level - to fulfill a future demand. Although the deterministic version of the problem has been widely studied in the literature, the stochastic production planning problem has not. The application of production planning models could be limited if the stochastic nature of the problem, for example, uncertainty in future demand, is not addressed. This study addresses such a stochastic production planning problem under uncertain demand and its application in an enclosure manufacturing facility.

The thesis first addresses the forecast of the demand where seasonal fluctuation is present. A decomposition model is utilized in the forecast and compared with other forecasting methods. Although forecast models could be used to improve the accuracy of forecast, error and uncertainty still exists. To deal with this uncertainty, a two stage stochastic scenario based production planning model is developed to minimize the total cost consisting of production cost, labor cost, inventory cost and overtime cost under uncertain demand.

The model is solved with data from a local manufacturing facility and the results are compared with various deterministic production models to show the effectiveness of the developed stochastic model. Parametric analyses are performed to derive managerial insights related to issues such as overtime usage and inventory holding cost and the proper selection of scenarios under pessimist, neutral and optimist forecasts. An extension of the stochastic model, i.e., a robust model is also solved in an effort to minimize changes in the solutions under various scenarios.

The stochastic production planning model has been implemented in the manufacturing facility, provided guidance for material acquisition and production plans and has dramatically increased the company's bottom line. As a result, it's estimated an approximately annual savings of $340,000 in inventory cost can be achieved for the company in the next few years.

Contents

List of Figures

List of Tables

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I am immensely indebted to my parents for their nurture, upbringing and unconditional love. I am also thankful to my sisters Sarla and Renu and my parents-in-law for their moral support and encouragement.

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Dedicated to:

My Husband and My Parents

Chapter 1

Introduction

1.1 Production Planning in Manufacturing Environment

Manufacturing planning helps direct the acquisition and allocation of available resources to production activities so as to satisfy customer demand over a specific time horizon. The production planning problem aims to match production and sourcing decisions to meet future customer demand subject to production capacity, workforce availability and overtime restrictions and is inherently an optimization problem. The objective of the problem is to minimize the total cost or to maximize profit.

Mathematical models for production planning problems can be broadly classified into two categories: deterministic models and stochastic models. Deterministic models assumes that the data are known and typically model the uncertainty using ”best guesses” of uncertain values. Although various human judgment based and quantitative models have been developed to forecast these variables with uncertainty such as demand, these deterministic models typically end up solving ”mean-value” or ”worst-case” problems. The solution to such ”worst-cast” or ”mean-value” problems are often inadequate – large error bounds arise when one solves ”mean value” problems and ”worst-case” formulations that can produce very conservative and expensive solutions (Birge (1982)). Without considering uncertainty, the deterministic production planning models, though widely studied in the literature, are less acceptable and deployed in practice. For some examples, please see Qiu and Burch (1997) and Tabucanon (1988).

Several uncertainties in manufacturing exist and can be categorized into two categories: environmental uncertainty and system uncertainty. The former refers to uncertainties that are beyond the scope of control of the production process, such as supply and demand uncertainty, while the latter refers to uncertainties that relate to the production process, such as operation yield uncertainty, production lead time uncertainty, quality uncertainty and production failures. Though various analytic and simulation models have been proposed, see Mula *et al.* (2006) for a survey, performing an all-inclusive study of production planning with all the uncertainties is almost impossible – simulation models are descriptive in nature and generally do not provide optimal solutions. In the case of production planning, a more viable approach is to identify and address one or a few uncertainties within a model which can be used to derive optimal solutions. This is the focus of this study.

1.2 Motivation and Contribution

This thesis focuses on the stochastic production planning with uncertain demand. The problem arises in a local enclosure manufacturing facility producing thermal cooling units where seasonal fluctuation in demand and uncertainty in company growth are present. Seasonal demand creates more complex production problems than stationary demand because firms that face seasonal demands typically do not have sufficient capacity to meet demand in the high demand season and must build inventory in the low demand season in anticipation of high demand later (Krane and Braun (1991) and Fair (1989)). The uncertainty of company and market growth raises even more uncertainty in the demand of products.

To make things more complicated, the company purchase raw material and semi-finished goods from overseas and a lead time of 40-45 working days is the norm. With more manufacturing are being outsourced overseas, the long lead time in purchasing materials has a dramatic impact on production planning – purchasing decisions and production plans for future (3$\sim$4) months has to be made before demand is realized. The uncertainty in demand and company growth calls for the solution of a stochastic production planning problem. The goal is to develop a production plan for the next year to minimize the expected total inventory, overtime and workforce costs by considering the manufacturing capacity and workforce level under different scenarios.

This thesis develops a two stage stochastic production planning model that yields solutions which serve as the baseline for different economic growth scenarios. The model is solved with real data from a local company and compared with deterministic production models on the basis of robustness and effectiveness. A parametric analysis is used to derive managerial insights related to issues such as overtime usage, inventory holding cost and the proper selection of scenarios under pessimistic, neutral and optimistic forecasts. An extension of the stochastic model i.e., a robust model is also formulated and solved in an effort to minimize changes in the solutions under various scenarios.

The stochastic production planning model is currently implemented in the manufacturing facility, provided guidance for material acquisition and production plans and has significantly increase the company's bottom line. It's estimated an approximately annual savings of $340,000 in inventory can be achieved for the company in the next few years.

1.3 Thesis Organization

This thesis is organized as follows. Chapter 2 reviews the relevant literature. Chapter 3 presents the decomposition forecast model and comparison with the company's current human judgmental forecast. Chapter 4 describes the two stage stochastic production planning model, followed by a robust production planning model. Chapter 5 presents the computational results using data from the company. The results show the effectiveness and efficiency of the stochastic model as compared with the deterministic model. The various parametric analysis have been preformed. Finally, conclusions and implementation are outlined in Chapter 6.

It is noteworthy that although the stochastic programming has been successfully applied on a wide variety of problems ranging from financial planning (Carino *et al.* (1998)), airline crew scheduling (Schaefer *et al.* (2005)), the solution to production planning using stochastic programming has only recently received some attention (Karabuk and Wu (2008)). While most of these studies focus on the strategic issues such as capacity planning, this thesis focuses more on an operational production planning model with long purchasing lead time that has not been addressed in the literature.

Chapter 2

Literature Survey

This chapter presents an overview of the literature on deterministic production planning, stochastic programming and stochastic production planning problems.

2.1 Deterministic Demand Production Planning

The classical deterministic production planning problem has been the focus of and received a significant attention in the optimization literature. For a recent survey, please see Fogarty *et al.* (1984), for some of the well-known models, their computational characteristics and solutions approaches. The basic mathematical model for deterministic production planning, also known as the lot sizing problem, can be formulated as follows.

Parameters

$f(t)$	fixed cost of producing in period t
$C_p(t)$	unit production cost in period t
$C_h(t)$	storage cost in period t
C	manufacturing capacity
$D(t)$	demand in period t

Decision variable

$P(t)$	amount produced in period t
$I(t)$	inventory level at the end of period t
$y(t)$	1 if production occurs in t; 0 otherwise

Deterministic Production Planning or Lot Sizing Model

Minimize

$$\sum_{t=1}^{n} C_p P(t) + \sum_{t=1}^{n} f(t) y(t) + \sum_{t=1}^{n} C_h I(t) \tag{2.1}$$

Subject to

- Inventory balance constraints

$$I(t-1) + P(t) = D(t) + I(t) \qquad \forall t = 1..n \tag{2.2}$$

- Capacity constraints

$$P(t) <= C y(t) \qquad \forall t = 1..n \tag{2.3}$$

- Non-negativity and integer constraints

$$P(t) \geq 0, \quad y(t) \in \{0,1\} \qquad \forall t = 1..n \tag{2.4}$$

The objective (2.1) here is to minimize the total cost including the labor cost per year (term 1), production cost per year (term 2) and inventory cost (term 3). Constraint (2.2) ensures that the available inventory in any period plus the demand in that period equals to the summation of inventory from the previous period and production during the current period. Constraint (2.3) states the capacity limitation while constraint (2.4) ensures that the decision variables $P(t)$ is non-negative and $y(t)$ is binary. The deterministic production planning problem under capacity limitation is proven to be a NP-hard problem (Garey and Johnson (1979)). Deterministic production planning (Swoveland (1975)) has received much attention from the literature, some applications are listed below.

Homburg (1996) modeled a production planning model for decentralized organization. The model was formulated as a linear program with multiple objectives. Lee *et al.* (2005) used production planning model to develop an optimal production schedule for a manufacturer of hard-disk drives. Liu and Tu (2008) solve a production planning problem where the production quantity is limited by inventory storage capacity. The problem further assumes that stock outs are allowed and that inventory storage capacity is constant. Huang and Xu (1998) transformed the multi-stage, multi-item aggregate scheduling problem into a static job-assignment problem. They used an adapted Frank Wolfe algorithm to solve their modified problem. Qiu and Burch (1997) modeled a hierarchical production planning problem for a yarn-fiber facility. Kanyalkar and Adil (2005) formulated a linear programming model for an integrated multi-item, parallel multi-plant production and dynamic distribution problem. It also considered safety stocks and capacity aggregation to minimize the effect of uncertainties in demand and supply.

The deterministic production planning model and its solution quality depends heavily on the accuracy of forecasted demand requirements – a difficult goal in a noisy and uncertain information environment (see Tabucanon (1988)). As mentioned, ”average demand” or ”worse case” demand estimates often leads to inferior solutions. To model uncertainty, stochastic production planning have to be developed.

2.2 Stochastic Programming

Stochastic programs, originated in the late fifties with the pioneering works of Dantzig (1955), Beale (1955), and Birge (1959), are mathematical programs where some of the data in the objective or constraints are uncertain. Uncertainty is usually characterized by a probability distribution on the parameters. Although the uncertainty is rigorously defined, in practice it can range in detail from a few scenarios to specific probability distributions.

2.2.1 Scenario Analysis

Rockafellar and Wets (1991) introduced a method for optimizing against an uncertain future. This technique, called scenario analysis, requires the user to specify a finite number of scenarios, each representing a different environment for the time period considered. Rockafellar and Wets (1991) then posed the problem of optimizing the expected objective value of this collection of problems: that is, the sum of the objective values of the individual scenario problems, each multiplied by the probability of that scenario's occurrence. To analyze how the optimal solution would change with respect to different scenarios, scenario analysis can be carried out by solving a set of optimization models with different sets of data as well as the blending of all of the scenario-dependent solutions into one solution.

2.2.2 Two-stage Stochastic Production Planning Model

Each decision and its execution can be subdivided into several stages, so that the stochastic problems will represent a multi-stage optimization problem. The two-stage stochastic model makes decisions using a two stage framework. The first stage decision variables are optimized before a realization of the uncertain and random variables. After realization of the random variables, the second stage variables are optimized.

The first stage decision variables are called the *structural* component that is fixed in the second stage and free of any uncertainty in its input data. The second stage decision variables are called the *control* component that is subjected to uncertain input data.

In this study, two sets of variables are introduced to define the two-stage stochastic model:

x, denotes the vector of decision variables whose optimal value is not conditioned on realization of the uncertain parameters. These are *design* variables. Variables in this set cannot be adjusted once a specific realization of the data is observed.

y, denotes the vector of *control* decision variables that are subjected to adjustment once the uncertain parameters are observed. Their optimal value depends both on the realization of uncertain parameters, and on the optimal value of the design variables.

The definition of *design* and *control* variables is borrowed from the flexibility analysis of production and distribution processes Seider *et al.* (1991). In a manufacturing environment, a facility location, facility capacity, supplier selection, product purchasing decisions and demand realization comes in phases. Depending on the problem and context, a two-stage models can be defined in several ways. For example, the first stage could incorporate variables related to the location and capacity of the facility such as workforce size, production capacity and the second stage could represent how, given these decisions, the detailed production plans should be carried out or inventory levels managed under the realized demand. This results in a *strategic production capacity planing model* or the stochastic capacity planning model as seen in Eppen *et al.* (1989) and Karabuk and Wu (2008).

The general form of the two-stage stochastic programming model is expressed as follows:

$$Min_x \qquad c^T x + E[Q(x,\xi)] \tag{2.5}$$

$$s.t \qquad Ax = b \tag{2.6}$$

$$x \geq 0 \tag{2.7}$$

Where $Q(x,\xi)$ is the optimal value of the second stage problem

$$Min_y \qquad q(\omega)^T y \tag{2.8}$$

$$s.t \qquad T(\omega)x + Wy = h(\omega) \tag{2.9}$$

$$y \geq 0 \tag{2.10}$$

The second stage problem depends on the data $\xi_\omega \equiv (q(\omega), h(\omega), T(\omega))$, elements of which can be random, while the matrix W is assumed to be known beforehand. The matrices $T(\omega)$ and W are called the technology and recourse matrices, respectively. The expectation $E[Q(x,\xi)]$ is taken with respect to the random vector $\xi = \xi(\omega)$ whose probability distribution is assumed to be known. The above formulation originated in the works of Dantzig (1955) and Beale (1955). Equation (2.6) denotes the structural constraints whose coefficients are fixed and free of uncertainty, so they are processed in the first stage. Equation (2.9) denotes the control constraints and they are processed in the second stage. Therefore, Equations (2.5)-(2.7) represent the first-stage model and Equations (2.8)-(2.10) represent the second-stage model. c is the vector of cost coefficient at the first stage. A is the first-stage coefficient matrix and b is the corresponding right-hand side vectors. q is the vector of cost (recourse) coefficient vectors at the second stage. W is the second-stage (recourse) coefficient matrix and $h(\omega)$ is the corresponding right-hand side vector. $T(\omega)$ is the matrix that ties the two stages together. In the second-stage model, the random constraint defined in (2.9), $h(\omega) - T(\omega)x$, is the goal constraint: violations of this constraint are allowed, but the associated penalty cost, $q^T y$, will influence the choice of x.

2.2.3 Multi-Scenario Stochastic Model

Consider the special case when random data have a discrete distribution with a finite number K of possible realizations $\xi_k = (q_k, h_k, T_k), k = 1, ..., K$, (called scenarios), with the corresponding probabilities p_k. In that case $E[Q(x,\xi)] = \sum_{k=1}^{K} p_k Q(x,\xi_k)$, where

$$Q(x,\xi_k) = Min \;\; q_k^T y_k : T_k x + W y_k, y_k \geq 0 \tag{2.11}$$

The above two-stage problem can be formulated as:

$$Min \qquad c^T x + \sum_{k=1}^{K} p_k (q_k^T y_k) \tag{2.12}$$

$$s.t \qquad Ax = b \tag{2.13}$$

$$T_k x + W y_k = h_k \qquad s = 1, ..., S \tag{2.14}$$

$$x, y_k \geq 0 \qquad k = 1, ..., K \tag{2.15}$$

The vector of first-stage decision variables, x, is scenario independent. The vectors of second-stage decision variables, y_k are introduced to control the random constraints with minimal recourse penalty cost.

Stochastic programming and scenario analysis has been applied in a number of different areas, such as financing (Carino *et al.* (1994), Carino *et al.* (1998), Carino and Ziemba (1998) and Consigli and Dempster (1998)), agriculture planning (Maatman *et al.* (2002)), logistics planning (Dempster *et al.* (2000)) and airline crew scheduling (Schaefer *et al.* (2005) and Yen and Birge (2001)). In the manufacturing domain, several stochastic programming models have been proposed on capacity planning and production planning.

Eppen *et al.* (1989) studied the capacity planning issues for a General Motors plants. They formulated a model to aid in making decisions about capacity for four of GM's auto lines. The model incorporates elements of scenario planning, integer programming, and risk analysis.

Escudero and Kamesam (1955) presented two approaches for modeling the aggregate production planning problem under stochastic non-stationary demand, capacity constraints and dual sourcing.

Kira *et al.* (1997) proposed a stochastic linear programming approach for hierarchical production planning under uncertain demand. They demonstrated that the shape of the distribution that describes demand is an important component in production decision-making.

Karabuk and Wu (2003) develop a strategic capacity planning model for a major semi-conductor manufacturer. They formulate a multi-stage stochastic program with recourse where demand and capacity uncertainties are incorporated via a scenario structure.

Karabuk and Wu (2008) develop a stochastic programming model that explicitly includes uncertainty in the form of discrete demand scenarios for a production planning problem in textile manufacturing. The paper describe the development of the model and illustrate its application with numerical examples.

Alonso-Ayuso *et al.* (2003) presented a modeling framework for two-stage production planning under uncertainty in the main parameters.

Clay and Grossmann (1997) solves a stochastic linear programming models for production planning where cost coefficient and RHS term uncertainties are considered. The decisions are taken in two-stages for deciding production levels.

Escuderoa *et al.* (1999) proposes a two-stage model for a manufacturing, assembly and distribution supply chain planning problem under uncertainty in product demand, component supplying cost and delivery time.

Hill and Sawaya (2004) presents a stochastic model for finding the optimal dates to stop production of an existing product and to start production of a new product in the presence of an uncertain approval date of medical devices.

Fleten and Kristoffersen (2008) develop a short-term multi-stage stochastic production plan for a price-taking hydro-power plant operating under uncertainty.
Feiring and Sastri (1990) includes an aggregate production planning model of manufacturing resources in order to satisfy stochastic demand for a family of products to minimize total costs that include production and inventory holding costs over a rolling horizon. The demand in this paper is assumed to be normally distributed.

Leung *et al.* (2006) addresses the production planning problem with additional constraints, such as production plant preference selection. To deal with the uncertain demand data, a stochastic programming approach is proposed to determine optimal medium-term production loading plans under an uncertain environment. A set of data from a multinational lingerie company in Hong Kong is used to demonstrate the robustness and effectiveness of the proposed model.

Chen *et al.* (2002) addressed the solution of linear stochastic planning problems. In this paper, they consider the role of product mix flexibility, defined as the ability to produce a variety of products, in an environment characterized by multiple products, uncertainty in product life cycles and dynamic demands. Using a scenario based approach for capturing the evolution of demand, they develop a stochastic programming model for determining technology choices and capacity plans.

Reaychen and Fang (2001) solve the production planning problem where unit cost to subcontract, work force level, production capacity and market demands are uncertain and random.

Chapter 3

Demand Forecast

An accurate forecast of demand is critical to an effective operation of a manufacturing facility. It affects the decisions regarding the capacity of the facility, the workforce size and composition, the production plans and the setting of safety stock. For production planning, it serves as an input to an optimization model and significantly affects the final result – an inaccurate forecast results in unreliable production plans with over- or under-stock problems. The set of data provided to us by the manufacturing facility reveals a strong seasonality in the market. When a demand is highly seasonal, it is unlikely an accurate forecast can be obtained without the use of an appropriate model.

There are generally two families of forecasting methods: quantitative and qualitative. Quantitative methods are those which rely on quantitative data such as previous sales figures to make predictions about the future. The forecast methods in this group include Time-Series and Causal models and are typically used for middle term forecast, from a few months to a year.

Qualitative methods mainly rely on subjective data gathered from executives, sales team and consumers that predict future economic conditions for a business.

In this study, a quantitative decomposition model is used to predict the seasonal demand for the manufacturing company to find the trend and seasonality factors. The forecast model was implemented in Microsoft Excel and was able to obtain reasonable results with a Mean Absolute Percentage Error (MAPE) of 7%. These results are comparable to those provided by the Statistics Analysis System (SAS) software package. This forecast was combined with subjective forecasts from the sales team to derive pessimistic, neutral and optimistic scenarios used in the stochastic production planning model.

3.1 Seasonal Decomposition/Time-Series Model

In time series analysis, demand data are considered as the combination of a systematic pattern and some random noise superimposed to that pattern. A forecasting approach can be improved if the underlying factors of a data pattern can be identified and forecasted separately. Breaking down the data into its component parts is called decomposition. A decomposition approach to a forecast model typically assumes that sales are affected by four factors: the general trend, general economic cycles, seasonality and irregular or random occurrences. In most analyses, the random noise is considered zero-mean, white and Gaussian. A forecast model need to find these separate components - trend, seasonal, cyclical and random - and combine then to provide the systematic pattern for a time series (Box *et al.* (1976) and Yaffee and McGee (2000)).

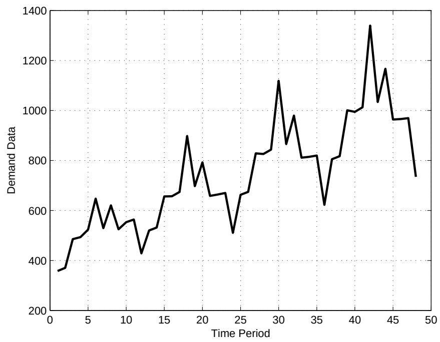

Figure 3.1: Observed Value of Series

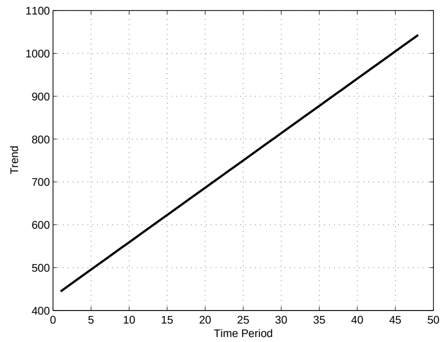

Figure 3.2: Trend of Series

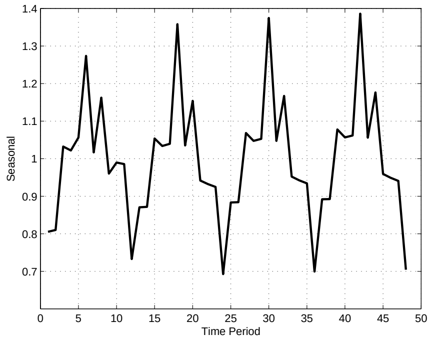

Figure 3.3: Seasonal Component of Series

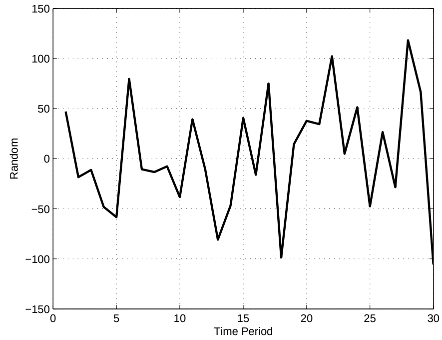

Figure 3.4: Random Component of Series

Specifically the components can be represented by:

- Trend-Cycle(TC_t): A trend is a general long-term pattern observed over the entire data set. The cycle is a cyclical variations around the trend line.

- Seasonal(S_t): A seasonal component is a cyclical variation with a fixed frequency, such as yearly cycles in the sales of lawnmowers. Seasonal effects repeat on a regular and predictable basis.

- Random(R_t): A random component is a residual component left behind after the other three components have been accounted for.

The time series Y_t can be a function of these components:

$$Y_t = f(TC_t, S_t, R_t) \tag{3.1}$$

where function f may take several forms:

$$\textit{Additive} \qquad Y_t = TC_t + S_t + R_t \tag{3.2}$$

$$\textit{Multiplicative} \qquad Y_t = TC_t \times S_t \times R_t \tag{3.3}$$

$$\textit{Pseudo-Additive} \qquad Y_t = TC_t(S_t + R_t - 1) \tag{3.4}$$

Figures (3.1) to (3.4) illustrate how a series may be decomposed into three components. In this study, a multiplicative model is used for the forecast of the company's data.

3.2 Forecast Demand and Results

The company manufactures a variety of products in the thermal cooling unit group, however, individual product in the group shares sub-assemblies or components with other products and follows a similar seasonal demand pattern. As a result, it is more effective to forecast the aggregate demand of the thermal cooling products as a group. The forecasting of an aggregate demand is not only more accurate than that of the individual product (Nahmias (1993)), but also more suited for production planning purpose, since these products share components and subassemblies.

Demand Data and Forecast Calculation: The aggregate demand data for the past two and a half years of the manufacturing company from January 2005 to June 2007 are presented in Table (3.1). Here the first column represents the time period and the second column the demand in that period.

The decomposition forecast is developed and implemented in Microsoft Excel to find the trend and seasonality associated with data.

- Based on the data from January, 2005 to December, 2006, the values of intercept and slope are calculated as 417.5 and 13.58 respectively by using the INTERCEPT and SLOPE function embedded in Microsoft Excel. The trend of the series is then calculated as:

$$Trend = Intercept + Slope \times TimePeriod$$

- The seasonal factors are calculated by dividing the consumption (actual demand) value by trend value. The average seasonal factors are calculated as the average of seasonal factors in the respective months.

- Finally, the forecast is calculated by multiplying the trend with the average seasonal factors.

The trend, seasonal factors, the average factor and the forecast of 2007 are listed in Table (3.1) under the corresponding columns.

Decomposition Forecast versus Company Forecast: To evaluate the accuracy of the forecast model, we adopt the absolute percentage error (APE) as the measure of the accuracy. The absolute percentage error (APE) is defined as:

$$APE = \frac{\|Actual\ Demand - Forecast\ Data\|}{Actual\ Demand}$$

Table (3.2) gives the forecast results and the absolute percentage errors of the decomposition forecast as well as the company's judgmental forecast for the first six month of 2007. The decomposition forecast model was providing reasonable good results.

As we can see from Table (3.2), the maximum APE of the decomposition forecast model was 17%, with an average of 7%. The maximum APE of the company's judgement forecast was 38% with an average APE of 23%. As a side story, at the end of June 2007, we decided to re-run the model using data from January, 2005 to June, 2007 and updated the forecast for July and August of 2007. The updated forecast for July 2007 was 878 units and the real demand was 881 units – a 3 unit difference. The updated forecast for August 2007 was 994 units and the real demand was 991 units – a 13 unit difference. The forecast has gained the trust of the company.

Figure (3.5) shows the results of the various forecast models from January 2005 to June 2008. Here, the line with plus sign represents the actual demand, the dotted line the company's forecast, and the line with circle the decomposition forecast. As we can see, the decomposition model forecast, the line with circle, matches closely with actual demand.

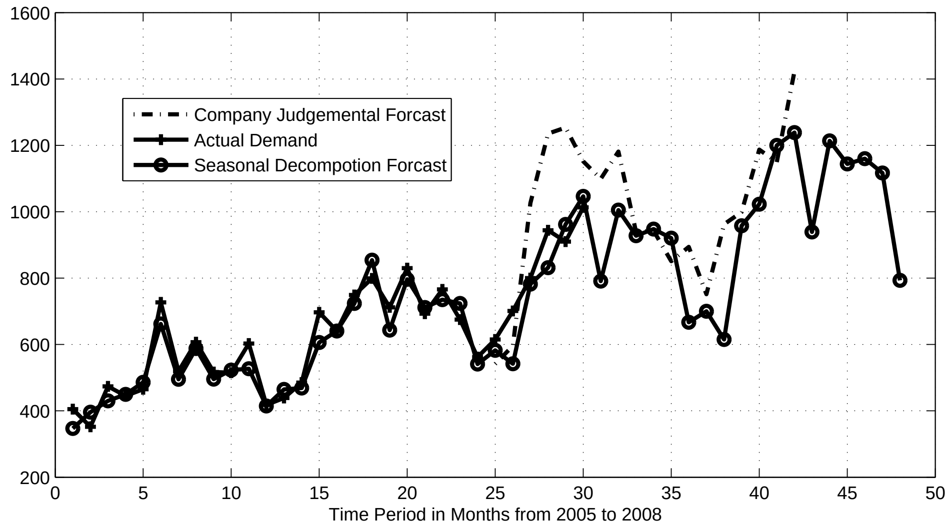

Figure 3.5: Forecast Comparison of Company Data

The Decomposition Forecast versus SAS Forecast: To see how the decomposition forecast model compares with other forecast models, we input the same demand data from January 2005 to December 2006 into Statistics Analysis System (SAS) software package. SAS uses different types of forecast models and the various forecast results and their mean square errors provided are listed in (3.3). For details of these modeles, please refer to (Brocklebank and Dickey (2003)). Based on the mean square error (MSE), it can be seen that the seasonal decomposition model has the lowest forecast error, provides the best result and is used in the comparison.

Table (3.4) shows the results of the the decomposition forecast results and that of the decomposition model from SAS. As we can see, the errors from both the forecast model are rather close although SAS provided slightly better results, the reason seems to be the additional feature embedded in the SAS system such as smoothing of the demand to derive the trend etc.

Final Forecast of Demand from September 2007 to August 2008: Based on the data from January, 2005 to June, 2007, we updated the forecast of the demand from September 2007 to August 2008. Here, the values of intercept and slope are calculated as 409.71 and 14.22 respectively. The trend, seasonal factor, average factor and the forecast of the year 2007 and 2008 are listed in Table (3.5).

Conclusion: Although more complicated forecast model can be developed to discover the root of the fluctuation, there is no guarantee that they will provide more accurate forecasts than simple models. Beside, market changes could significantly change these forecast values – the key here is to develop stochastic models to be used under various scenarios. The above model provides insight in the business trend and is used together with company's forecast to form pessimistic, neutral and optimistic scenarios as inputs to the stochastic production planning model to be discussed in the next chapter.

Table 3.1: Demand Data & Decomposition Forecast Results for 2007

Year	Consumption	Trend	Seasonal Factors	Average Factors	Forecast
Jan-05	405	431	0.94	0.84	362
Feb-05	352	445	0.79	0.79	353
Mar-05	474	458	1.03	1.08	494
Apr-05	445	472	0.94	0.98	461
May-05	465	485	0.96	1.06	513
Jun-05	727	499	1.46	1.33	665
Jul-05	519	513	1.01	1.03	530
Aug-05	607	526	1.15	1.18	620
Sept-05	517	540	0.96	0.97	525
Oct-05	515	553	0.93	1.00	553
Nov-05	603	567	1.06	0.99	564
Dec-05	418	581	0.72	0.74	428
Jan-06	439	594	0.74	0.84	499
Feb-06	485	608	0.80	0.79	483
Mar-06	697	621	1.12	1.08	670
Apr-06	641	635	1.01	0.98	620
May-06	749	648	1.16	1.06	685
Jun-06	799	662	1.21	1.33	882
Jul-06	712	676	1.05	1.03	698
Aug-06	830	689	1.20	1.18	813
Sept-06	693	703	0.99	0.97	683
Oct-06	766	716	1.07	1.00	716
Nov-06	675	730	0.92	0.99	726
Dec-06	562	743	0.76	0.74	549
Jan-07		757		0.84	635
Feb-07		771		0.79	613
Mar-07		784		1.08	846
Apr-07		798		0.98	779
May-07		811		1.06	857
Jun-07		825		1.33	1099
Jul-07		839		1.03	866
Aug-07		852		1.18	1005
Sept-07		866		0.97	841
Oct-07		879		1.00	879
Nov-07		893		0.99	888
Dec-07		906		0.74	669

Table 3.2: Forecast Result Comparison with Company's Judgmental Model

Year 2007	Consumption	Our Forecast	APE (Forecast Model)	Company Forecast	APE (Company's Model)
Jan-07	615	635	0.03	542	0.12
Feb-07	701	613	0.13	596	0.15
Mar-07	800	846	0.06	1026	0.28
Apr-07	944	779	0.17	1236	0.31
May-07	910	857	0.06	1253	0.38
Jun-07	1013	1099	0.08	1152	0.14
Jul-07	881	878	0.00	1099	0.25
Aug-07	981	994	0.01	1181	0.20
Sum (Error)			**0.54**		**1.83**
Average (Error)			**0.07**		**0.23**

Table 3.3: Forecast Errors of the Models

Forecast Model	MSE
Linear Trend with Autoregressive Errors	77.58
Seasonal Decomposition	41.79
Seasonal exponential Smoothing	59.40
Winters Method-Additive	43.76
Winters Method-Multiplicative	61.83
ARIMA(0,1,1)(1,0,0)s NO INT	92.63
ARIMA(2,1,0)(0,1,1)s NO INT	85.74
Log Linear Trend with Autoregressive Errors	77.29
Log Linear Trend with Seasonal Terms	44.19
Log Seasonal Exponential Smoothing	64.51
Log Winters Method-Additive	46.27
Log Winters Method-Multiplicative	65.80
Log ARIMA(0,1,1)(1,0,0)s NO INT	92.86
Log ARIMA(2,1,0)(0,1,1)s NO INT	108.65

Table 3.4: Forecast Data Comparison with SAS Results

Year 2007	Consumption	Decomposition Forecast	APE (Forecast Model)	SAS Forecast	APE (SAS's Model)
Jan-07	615	635	0.03	663	0.07
Feb-07	701	613	0.13	689	0.02
Mar-07	800	846	0.06	833	0.04
Apr-07	944	779	0.17	853	0.11
May-07	910	857	0.06	884	0.03
Jun-07	1013	1099	0.08	1023	0.01
Jul-07	881	878	0.00	880	0.00
Aug-07	981	994	0.01	983	0.00
Sum (Error)			**0.54**		**0.28**
Average (Error)			**0.07**		**0.03**

Table 3.5: Forecast Data & Results of the Decomposition Model for September 2007 to August 2008

Year	Consumption	Trend	Seasonal Factors	Average Factors	Forecast
Jan-05	405	424	0.96	0.83	353
Feb-05	352	438	0.80	0.83	365
Mar-05	474	452	1.05	1.06	479
Apr-05	445	467	0.95	1.04	487
May-05	465	481	0.97	1.07	517
Jun-05	727	495	1.47	1.29	640
Jul-05	519	509	1.02	1.03	526
Aug-05	607	523	1.16	1.18	616
Sep-05	517	538	0.96	0.97	522
Oct-05	515	552	0.93	1.00	550
Nov-05	603	566	1.07	0.99	561
Dec-05	418	580	0.72	0.73	426
Jan-06	439	595	0.74	0.88	520
Feb-06	485	609	0.80	0.88	533
Mar-06	697	623	1.12	1.06	658
Apr-06	641	637	1.01	1.03	659
May-06	749	651	1.15	1.04	677
Jun-06	799	666	1.20	1.36	903
Jul-06	712	680	1.05	1.03	702
Aug-06	830	694	1.20	1.15	798
Sept-06	693	708	0.98	0.94	664
Oct-06	766	722	1.06	0.93	669
Nov-06	675	737	0.92	0.92	676
Dec-06	562	751	0.75	0.69	516
Jan-07	615	765	0.80	0.88	670
Feb-07	701	779	0.90	0.88	682
Mar-07	800	794	1.01	1.06	838
Apr-07	944	808	1.17	1.03	836
May-07	910	822	1.11	1.04	854
Jun-07	1013	836	1.21	1.36	1134
Jul-07		850		1.03	878
Aug-07		865		1.15	994
Sept-07		879		0.94	823
Oct-07		893		0.93	827
Nov-07		907		0.92	833
Dec-07		921		0.69	633
Jan-08		936		0.88	819
Feb-08		950		0.88	831
Mar-08		964		1.06	1018
Apr-08		978		1.03	1013
May-08		993		1.04	1032
Jun-08		1007		1.36	1365
Jul-08		1021		1.03	1054
Aug-08		1035		1.15	1190

Chapter 4

Stochastic Production Planning Models and Its Formulation

Product demand is one of the key inputs and a major source of uncertainty to a production planning problem. As we have seen, forecast will never be 100% accurate and market changes could significantly change the demand in the future. As such, developing stochastic production planning that accounts for demand uncertainity is necessary for effectively running a production facility and to deal with any specific realization of the demand uncertainty.

In this chapter, a two-stage stochastic production planning model is presented. The model is then extended to a robust production planning model. While stochastic programming model is designed to minimize only the expected costs, the robust programming model is designed to provide solutions that are consistent across all scenarios to reduce solution variability.

4.1 Problem Statement

The Definition of The First and Second Stage Decisions: In a two-stage stochastic model, the definition of the first and the second stage variable are problem and context dependent and accordingly a two-stage stochastic models can be defined in several ways. For example, the first stage could incorporate variables related to the location and capacity of the facility and the second stage could represent how, given these decisions, the detailed production plans should be carried out or inventory levels managed under the realized demand. This results in a *strategic capacity planing model* as seen in Eppen *et al.* (1989) and Karabuk and Wu (2008).

In an operation environment, when the capacity is fixed, purchasing and production decisions also come in phases. For our client, a US subsidiary of a global manufacture, most of the materials and sub-assemblies are either purchased from its parent company in Europe or from east Asia due to global outsourcing activities to utilize low labor cost. As a result, a long lead time of more than 40∼45 working days is the norm, which means purchasing decision and production decisions has to be made (3∼4) months in advance before the demand is realized. The first stage decision in this operational environment is therefore related to the purchasing decisions and production decision for the next few (3∼4) months and the second stage decision is to decide how, given these decisions, the detailed inventory, overtime and production schedules under realized demand. This results in an *stochastic operational production planning model*.

To be specific, it is assumed that the maximum production capacity, defined as maximal number of units per time period, remain unchanged. The first stage decision is how much material to purchase for the production of the next 4 months, with an eye of the future seasonal demand. When a possible realization of demand is revealed, the second stage decisions are made to find, given these first stage purchasing and production decisions, the detailed production schedules, overtime usage and inventory level managed for these months.

In practice, of course, only the purchasing to support the next 4 months are taken and the model needs to be solve every few months. The problem is therefore a rolling plan capacity allocation model under uncertain demand.

The Generation of Pessimistic, Neutral and Optimistic Scenarios: The decomposition forecast, though accurate, seems to gives the lowest values and is used as the pessimistic forecast of future. The company's judgmental forecast, although always seems to be an overestimate, does represent an optimistic view of the future and therefore as the optimistic forecast. A third forecast was obtained by randomly choosing a value between pessimistic and optimistic forecast and serves as a neutral forecast. An equal probability is given to all these three forecasts or scenarios.

Other Operations: In developing the production planning model, the follow options are used:

1. Overtime. Overtime can be used to create a temporary increase in capacity without the added expense of hiring additional workers. Due to the labor union regulation, overtime is limited to 20% of the maximum regular production capacity.

2. Inventory. Finished-goods inventory is used to fill demand during periods of high demand. A minimum safety stock of 200 units has been decided to hedge against spikes in a period.

3. Hire/lay off. Laying off and rehiring are strategic decisions and is thus not considered in this study. Workers cross trained and in a low demand seasons, can be transferred in other production groups.

4. Back-Log. Back-log orders and subcontracting are not considered in this study.

Notice again that the production plan for first four months are kept the same, this represents the first stage decisions. When the purchasing decision are known and demand are realized, the second-stage decisions are the detailed overtime decisions and inventory level decisions that changes under each scenario.

4.2 A Two-stage Stochastic Production Planning Model

4.2.1 Mathematical Model

The above production planning model belongs to the class of multi-scenario, two-stage stochastic production planning model. The goal is to find a optimal production plan that minimizes the overall production, inventory and overtime cost to satisfy the demand.

The following notation are defined in the development of the model.

Parameters

Indices:

s	Index of scenario
t	Index of time period

Sets:

S	Set of scenario
T	Set of time period (12 months)

Deterministic Parameters:

$d(s,t)$ Demand forecast of thermal cooling unit product in period t under scenario s

C_r Regular production cost of worker per year ($36000/year)

C_o Overtime production cost of worker ($/man-hour)

C_h Annual holding cost per unit of product

N Average number of days in the period

I_0 Initial inventory

τ Daily production of units per person

α Overtime ratio

β Average Inventory cost

δ Minimum Production quantity in time period t

w Number of workers

ρ Efficiency of workforce (ρ = 0.9)

Recourse Parameters:

$p(s)$ Probability of occurrence of scenario

$SS(s,t)$ Safety stock of products in period t under scenario s

Decision variables

$P(s,t)$	Number of units produced in the regular hours in period t under scenario s (Units/hours)
$O(s,t)$	Number of units produced in the overtime hours in period t under scenario s (Units/hours)
$I(s,t)$	Inventory of product in end of period t under scenario s (Units)
$y(s,t)$	1 if overtime is used, otherwise 0 in period t under scenario s

A Two Stage Stochastic Production Planning Model

Minimize

$$\sum_{s=1}^{S} p(s)\left[C_r.w + \sum_{t=1}^{T} O(s,t).C_o + \sum_{t=1}^{T}(I(s,t)/12).C_h\right] \tag{4.1}$$

Subject to

- Inventory balance constraints

$$I(s,0) + P(s,t) + O(s,t) = d(s,t) + I(s,t) \quad \forall s \in S, t = 1 \tag{4.2}$$

$$I(s,t-1) + P(s,t) + O(s,t) = d(s,t) + I(s,t) \quad \forall s \in S, t = 2..T \tag{4.3}$$

- Capacity constraints

$$P(s,t) \leq N.\tau.\rho.w \qquad \forall s \in S, t \in T \tag{4.4}$$

- Overtime and production occurrence constraints

$$P(s,t) \geq N.\tau.\rho.w.y(s,t) \qquad \forall s \in S, t \in T \tag{4.5}$$

$$O(s,t) \leq \alpha.N.\tau.\rho.w.y(s,t) \qquad \forall s \in S, t \in T \tag{4.6}$$

- Minimum production constraints

$$P(s,t) \geq \delta.N.\tau.\rho.w \qquad \forall s \in S, t \in T \tag{4.7}$$

- Safety stock of finished product constraints

$$I(s,t) \geq SS(s,t) \qquad \forall s \in S, t \in T \tag{4.8}$$

- Same production constraints

$$P(s,t) = P(s+1,t) \qquad \forall s = 1..S-1, t = 1..4 \tag{4.9}$$

- Non-negativity conditions

$$P(s,t),\ O(s,t),\ I(s,t) \geq 0 \qquad \forall s \in S, t \in T \tag{4.10}$$

$$y(s,t)\ is\, binary \qquad \forall s \in S, t \in T \tag{4.11}$$

The objective (4.1) is to minimize the total cost, including the fixed annual labor cost (term 1), overtime cost (term 2), and inventory cost (term 3). Constraints (4.2) and (4.3) ensure that the beginning inventory plus production and overtime during the current period equals the demand plus ending inventory. Constraint (4.4) ensures that the total production by regular-time workers during period t and scenarios s is limited by the production capacity. Constraints (4.5) and (4.6) ensure that the overtime occurs only when the regular production is at its maximum level and overtime cannot not exceed the maximum limit, which is set at α% of maximum regular production. Constraint (4.7) ensures that the production under all scenarios remains greater than the minimum production. Constraint (4.8) ensures that the ending inventory at each period is not less than the minimum safety stock. Constraint (4.9) ensures that the productions under all scenarios remains the same for first four months–the first stage decisions. Constraint (4.10) ensures that all decision variables are non-negative and constraint (4.11) ensures that the decision variables $y(s,t)$ is binary. This problem is an integer linear programming (INLP) problem and is NP-hard (Garey and Johnson (1979)).

4.3 Robust Production Planning Formulation

The above stochastic program aims to minimize the expected value, the first moment, of inventory and production cost and does not include any high order moments or risk attribute of the decision maker, or the distribution of the objective values. In an effort to minimize the second or higher moments of the inventory and production cost and to reduce the variability of solutions under different scenarios, a robust model is used. The robust model aims to obtain a solution that will not differ substantially among different scenarios and to achieve this, minimizes fist order as well as higher order moments.

Two popular approaches for handling risk are: mean/variance models (Markowitz (1959)), and von Neumann-Morgenstern expected utility models (Keeney and Raiffa (1976) and Bai *et al.* (1997)). The former includes minimizing variance as part of the objective whereas the latter presents a more general approach for handling risk aversion. In this study, a mean/variance model by (Mulvey and Vladimirou (1991)) with a additional variance constraint is used. This approach requires that the distribution of the random variable be symmetric around its mean. Third and higher moments are ignored in the model.

To do so, a variance of cost is added to the model for robustness and the following additional notations and variable are used in the development of the model.

Parameters

λ	Weighting scale
ε_1	Lower bound
ε_2	Upper bound

Variables

ξ_s	Variable with probability p_s under scenario s

Mathematical Model

The robust production planning model can be formulated as follows.

$$Min \qquad \sum_{s=1}^{S} p_s \xi_s + \lambda \sum_{s=1}^{S} p_s \left(\xi_s - \sum_{s'=1}^{S} p_s^{'} \xi_s^{'} \right)^2 \tag{4.12}$$

$$Where \qquad \xi_s = c^T x + \sum_{k=1}^{K} p_k (q_k^T y_k) \tag{4.13}$$

$$Subject \ \ to \ \ \varepsilon_1 \leq \frac{(\sum_{s=1}^{S} p_s \xi_s - \sum_{s'=1}^{S} p_s^{'} \xi_s^{'})}{(\sum_{s=1}^{S} p_s \xi_s)} \leq \varepsilon_2 \tag{4.14}$$

$$Equation (2.13) - (2.15) \tag{4.15}$$

The first term of the objective function is the same as the stochastic model, the expected total cost.

The second term in the objective function, $\lambda \sum_{s=1}^{S} p_s \left(\xi_s - \sum_{s'=1}^{S} p_s^{'} \xi_s^{'} \right)^2$, is used to penalize variations of production plan cost under various scenarios. Constraint (4.14) is used to keep the difference between scenarios cost within ε_1 and ε_2. The objective function here is to minimize the deviation in an effort to get solutions that are less sensitive to change in the demand data under all scenarios.

Objective function

- Regular production cost

$$PC = C_r.w \tag{4.16}$$

- Overtime production cost

$$OC(s) = \sum_{t=1}^{T} O(s,t).C_o \tag{4.17}$$

- Inventory cost

$$IC(s) = \sum_{t=1}^{T} (I(s,t)/12).C_h.\beta \tag{4.18}$$

Minimize

$$\sum_{s=1}^{S} p(s)\left[PC + OC(s) + IC(s)\right] \tag{4.19}$$
$$+\lambda \sum_{s=1}^{S} p(s)\left[(PC + OC(s) + IC(s)) - \sum_{s'=1}^{S} p(s')\left(PC + OC(s') + IC(s')\right)\right]^2$$

Subject to

Equation (4.2)-(4.11)

- Scenario difference constraints

$$\varepsilon_1 \leq \frac{(\sum_{s=1}^{S} p(s)\left[PC + OC(s) + IC(s)\right] - \sum_{s'=1}^{S}\left[PC + OC(s') + IC(s')\right])}{(\sum_{s=1}^{S} p(s)\left[PC + OC(s) + IC(s)\right])} \leq \varepsilon_2$$
$$\forall s \in S \tag{4.20}$$

The first part of objective function in equation (4.19) is the total cost associated with the labor cost per year, overtime workers cost and the inventory cost. The second part in objective function is the square of the difference between first part of the cost and each individual scenario cost. Constraint (4.20) ensures that the difference between each individual scenario cost should remain between ratio ε_1 and ε_2 and average cost.

This problem is a integer non-linear integer programming (NILP) problem, where all unknown variables are all required to be integers and some of the constraints or objective is non-linear.

Chapter 5

Computational Results

All the stochastic and robust production planning models are implemented in Xpress and solved using its integer linear and nonlinear solvers on a Pentium IV 1.8MHZ computer with 512 MB of RAM. The computation time for most of the models are within 4 to 5 seconds and has not posed a great challenge. The results from various models are presented derive managerial insights.

5.1 Input Parameters

The Generation of Pessimistic, Neutral and Optimistic Scenarios:

As mentioned, the decomposition forecast, although accurate, seems to provide the smaller forecast values and is used as the pessimistic forecast (S(L)) of the future. The company's judgmental forecast, although always seems to be an overestimate, does represent an optimistic view of the future and therefore as the optimistic forecast (S(H)). A third forecast was obtained by randomly choosing a value between pessimistic and optimistic forecast and serves as a neutral forecast(S(M)). An equal probability is given to each of these three forecasts or scenarios. The product quantities required under different scenarios in each month are shown in Table (5.1).

Table 5.1: Company's Demand Data of the Scenarios

Scenario (s)	1	2	3	4	5	6	7	8	9	10	11	12
S1	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
S2	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
S3	869	885	824	633	836	950	1012	795	1036	1375	1385	1248

Initial Inventory and Inventory Cost: The initial inventory, I_{s0}, is the finished goods inventory at the end of August 2007. This value is currently set at 1500 units. Notice that this is a rather large value due to the lack of proper forecast and stochastic production planning models in the past. The annual inventory holding cost is provided by company and is set at 10% per year and the average cost per unit is $1000. The minimum inventory is set at 200 units as per company requirement.

Production Capacity: The number of workers for the production group (w) is 18 and is fixed. The average daily production rate, τ, is 2.89 unit and with 90% efficiency, it becomes 2.60 unit. The company operates 8 hours a day, 5 days a week, thus the regular production capacity is calculated as 983 units per month with an average of 21 working days.

Production and Overtime Costs: The regular production cost (C_r) and overtime production cost (C_o) are constant throughout the planning horizon and are estimated to be $18 per hour and $27 per hour respectively. The overtime ratio is set at 20%.

5.2 Optimal Production Plan and Cost Analysis

Given these parameters, the optimal level of production, inventory and overtime for different models are discussed presently.

5.2.1 Solution Under Perfect Information

Under perfect information, it is assumed that each of the scenario can happen with the given probability, but the manager knows beforehand exactly what scenario will happen. If this is the case, the manager would then take optimal solution for each scenario as given in Table (5.2). This would leave him with a cost of $731,475 for the pessimistic scenario, $750,608 for the optimistic scenario, and $745,267 for the neutral scenario. The average cost in the long run would be the average of three costs, namely $742,450 per year. This is the cost that incurs under perfect information when the future demand is perfectly known.

Table 5.2: Deterministic Production Planning Model

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Produce	650	650	650	650	657	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	967	984	822	974	939	909	860	478	407	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Produce	650	650	891	983	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1168	924	994	1133	1279	1273	1254	1447	1300	874	467	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Produce	650	650	650	734	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1281	1046	872	973	1120	1153	1124	1312	1259	867	465	200

5.2.2 Two-Stage Stochastic Solution with the Same Four Month Production

Of course, to know future demand is impossible, in that case, the best a manager can do is to follow the production plan as given by the stochastic production planning model shown in Table (5.3). This gives an optimal cost of $747,145 per year, which includes production cost, inventory cost and overtime cost. From the Table (5.3), we can see that most of the products are produced from regular-time. The production for first four month is the same in all scenarios.

Table 5.3: Two-Stage Stochastic Model Production Plan (4 Month)

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Produce	650	650	650	983	650	731	983	909	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	967	1317	1148	1048	1013	909	860	478	407	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Produce	650	650	650	983	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	44	197
Inventory	1168	924	753	892	1038	1032	1013	1206	1059	633	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Produce	650	650	650	983	650	956	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	111
Inventory	1281	1046	872	1222	1036	1042	1013	1201	1148	756	354	200

The *expected value of perfect information* (EVIP) is defined the value of knowing the future with certainty. The value can be calculated as the cost of stochastic model ($747,145) minus the cost of solutions under perfect information ($742,450), which equals $4,695. This is the cost we have to pay for not knowing the future or the maximum money we would like to pay to know the future.

5.2.3 Two-Stage Stochastic Model with the Same 12 Month Production

A two stage stochastic model can also be derive to find solutions where all the 12 month productions remain the same. Such a model would be useful to provide purchase forecast for supplier or for supply contract negotiation. In such a model, we again set the first stage decision as the regular production, but set all the production to be the same across the planning horizon. When demand are realized, overtime and inventory can be adjusted to meet the requirements. The cost of such a two stage stochastic model, as shown in Table (5.4), is $754,169. The value is higher than two-stage stochastic model with same 4 moth production as it has more constraints. The expected value of perfect information and value of stochastic solution for the two two-stage models are given in Table (5.5)

Table 5.4: Two-Stage Stochastic Model Production Plan with Same 12 Month Production

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	650	650	966	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	967	984	1131	1283	1248	1218	1169	787	716	509
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	650	650	966	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	197	197	197
Inventory	1168	924	753	559	688	682	663	856	709	480	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	650	650	966	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	101
Inventory	1281	1046	872	889	1019	1052	1023	1211	1158	766	364	200

Table 5.5: EVIP and VSS Value for the Two Two-Stage Stochastic Models

Model	Total Cost	EVIP	VSS (Expected Value)	VSS (Expected Demand)
Two-Stage (Same 12 Month)	754169	11719	-3305	-1730
Two-stage (Same 4 Month)	747145	4695	-10329	-8754

5.2.4 Expected Value Solution

Yet another approach to derive production plans is to choose the production plan as the average of the production plan in all three scenarios, when they are solved individually. This results in what is called *the expected value solution*. This approach is common in optimization field but can have unfavorable results. Table (5.6) shows the inventory level and overtime usage for such a solution under the three scenarios, \$763,033 in the first scenario, \$760,598 in the second scenario and \$748,792 in the third scenario. The average total cost in long run is \$757,474 per year.

The *value of the stochastic solution*(VSS) is the possible gain from solving the stochastic model and is calculated as the expected value solution cost (\$757,474) minus stochastic model cost (\$747,145) and equals \$10,329. This is the benefit of solving the stochastic model over the average model, in this case, the expected value solution.

5.2.5 Solution Under Expected Demand

Lastly, the most common approach, the deterministic model, is to solve the problem with the average of the demand of the three scenarios. This approach is also called *the solution under expected demand*. Table (5.7)

Table 5.6: Inventory and Overtime Based on Expected Value

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	730	789	874	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	1047	1203	1258	1410	1375	1345	1296	914	843	636
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	730	789	874	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	70	197	197
Inventory	1168	924	833	778	815	809	790	983	836	480	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	730	789	874	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1281	1046	952	1108	1146	1179	1150	1338	1285	893	491	226

shows the production plan under such a model. The cost of this expected demand solution for the three scenarios are shown in Table (5.8).

The value of the stochastic solution (VSS) is calculated as the expected demand solution cost ($755,899) minus stochastic model cost ($747,145) and equals $8,754. This is the benefit of solving the stochastic model over the average model, in this case, the expected demand solution.

It has to mention that, the *expected value solution* was to combine a solution from the each individual solution, where as the *solution under expected demand* is to solve the problem with the expected demand.

Table 5.7: Production Plan Based on Expected Demand

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Demand	891	869	826	703	831	923	1011	866	1066	1383	1276	1229
Production	650	650	650	760	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1259	1040	864	921	1073	1133	1105	1222	1139	739	446	200

Table 5.8: Inventory and Overtime for All Scenarios Based on Expected Demand Production

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	650	760	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	967	1094	1258	1410	1375	1345	1296	914	843	636
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	650	760	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	70	197	197
Inventory	1168	924	753	669	815	809	790	983	836	480	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	650	760	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1281	1046	872	999	1146	1179	1150	1338	1285	893	491	226

As a conclusion, Table (5.9) shows the inventory cost, overtime cost, production cost, total cost and expected cost for different models discussed in this section.

5.3 Sensitivity Analysis

5.3.1 Change of Probability Associated with the Scenarios

In this section, we considered the changes in the total costs by changing the probability associated with the scenarios. Three cases have been considered in this study. In case I, each scenario has equal probability. In case II, the probability distribution is skewed towards the pessimistic forecast (S(L)). In case III, the probability distribution is skewed towards the optimistic forecast (S(H)). Table (5.10) shows the probability of each scenarios and Table (5.11) shows the optimal costs in these cases.

As can be seen, the total cost is reduced in Case II since a large prob-

Table 5.9: Cost Comparison of Various Models

Model	Scenario	Production Cost	Overtime Cost	Inventory Holding Cost	Total cost	Expected cost
Perfect Information Solution						
	S(L)	648000	0	83475	731475	
	S(H)	648000	0	102608	750608	742450
	S(M)	648000	0	97267	745267	
Two-Stage Stochastic Model (4 Month)						
	S(L)	648000	0	90200	738200	
	S(H)	648000	20022	84900	752922	747145
	S(M)	648000	9221	93092	750313	
Two-Stage Stochastic Model (12 Month)						
	S(L)	648000	0	104075	752075	
	S(H)	648000	49098	66267	763365	754169
	S(M)	648000	8391	90675	747066	
Expected Value Solution						
	S(L)	648000	0	115033	763033	
	S(H)	648000	38548	74050	760598	757474
	S(M)	648000	0	100792	748792	
Expected Demand Solution						
	S(L)	648000	0	113458	761458	
	S(H)	648000	38548	72475	759023	755899
	S(M)	648000	0	99217	747217	

Table 5.10: Probability Associated with Each Scenario

Case	Scenario S(L)	Scenario S(H)	Scenario S(M)
Case I	0.33	0.33	0.33
Case II	0.5	0.2	0.3
Case III	0.2	0.5	0.3

ability of a low demand is present. In contrast, the total cost is increased in case III since a large probability of high demand is present. Not surprisingly, changes of scenario probability changes the mean of future demand, high (low) demand will result in high (low) costs because more units are produced, more (less) overtime used, and more (fewer) inventory required to avoid shortages. The detailed production plans associated with case II and case III are shown in Table (5.12) and (5.13) respectively.

Table 5.11: Sensitivity Analysis- Change of Probability Distribution

Case	Production Cost	Overtime Cost	Inventory Holding Cost	Total cost
Case I	648000	9748	89397	747145
Case II	648000	10293	85936	744229
Case III	648000	9853	91241	749094

Table 5.12: Inventory, Overtime and Production Under Case II

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	650	882	650	758	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	967	1216	1047	974	939	909	860	478	407	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	650	882	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	27	0	0	0	118	197
Inventory	1168	924	753	791	937	931	939	1132	985	559	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	650	882	650	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	185
Inventory	1281	1046	872	1121	935	968	939	1127	1074	682	280	200

5.3.2 Change from Scenario Generation Method

In all previous experiments, scenarios S(L) and S(H) represents lower and upper bound and scenario S(M) has random values between S(L) and S(H). In this experiment, rather than choose lower and higher demand S(L) and S(H) as the scenarios, three random scenarios are generated with demand uniformly distributed between S(L) and S(H). The detailed production plans are shown in Table (5.14). The cost associated with the three scenarios are \$688,883, \$689,542, \$688,292 and the average is \$688,906.

It is interesting to notice that a significant lower cost of \$688,906 is obtained. The reason we believe is as follows. Supper a low demand of 0 and

Table 5.13: Inventory, Overtime and Production Under Case III

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	694	983	650	731	983	865	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1327	1150	1011	1361	1192	1092	1057	909	860	478	407	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	694	983	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	197
Inventory	1168	924	797	936	1082	1076	1057	1250	1103	677	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	694	983	650	956	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	67
Inventory	1281	1046	916	1266	1080	1086	1057	1245	1192	800	398	200

a high demand of 1, if the three scenarios (S(L),S(H),S(M)) are used, it represents the lower, upper and medium scenario, shown in Figure (5.1) with a discrete distribution. The mean is of the demand 0.5 and the variance is 0.408. The random sample of scenarios from low and high, however, represents the medium demand, (shown in Figure (5.1) by uniform distribution). The mean of the demand is 0.5 and variance is 0.288. Though the mean demand of the two experiments are the same, the latter has a much smaller variance, thus a much smaller cost.

As we can see, the magnitude of variance in the uncertainty have a dramatic impact on the solution of a stochastic program. While in practice, it is common to specify optimistic, neutral, and pessimistic scenarios, whether to use optimistic and pessimistic or to choose random samples between the optimistic and pessimistic requires detailed investigation.

Table 5.14: Inventory, Overtime and Production Under 3 Random Scenarios

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (M1)												
Demand	970	854	892	968	905	916	866	878	943	969	825	915
Production	650	650	650	650	689	916	866	878	943	969	825	915
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1180	976	734	416	200	200	200	200	200	200	200	200
Scenario (M2)												
Demand	940	855	972	846	965	974	937	899	942	971	853	941
Production	650	650	650	650	678	974	937	899	942	971	853	941
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1210	1005	683	487	200	200	200	200	200	200	200	200
Scenario (M3)												
Demand	904	978	929	857	947	825	923	901	941	872	850	894
Production	650	650	650	650	715	825	923	901	941	872	850	894
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1246	918	639	432	200	200	200	200	200	200	200	200

5.3.3 Change of Overtime Ratio

In this experiment we study the change of costs with respect to different overtime ratios. These overtime ratios are set 0%, 5%, 10%, 15%, 20%, 25% and 30% respectively. Table (5.15) and Figure (5.2) shows the changes the costs associated with overtime ratio.

Table 5.15: Sensitivity Analysis-Overtime Ratio Varies

Case	Production Cost	Overtime Cost	Inventory Holding Cost	Total cost
0%	648000	0	103228	751228
5%	648000	7062	93161	748223
10%	648000	9748	89833	747581
15%	648000	9748	89536	747284
20%	648000	9748	89397	747145
25%	648000	9748	89397	747145
30%	648000	9748	89397	747145

As we can see, an overtime ration of 50%, representing no overtime allowed, gives the highest cost of $751,228. When overtime ratio increases, the total cost drops from $751,228 to $ 747,145. However, when overtime

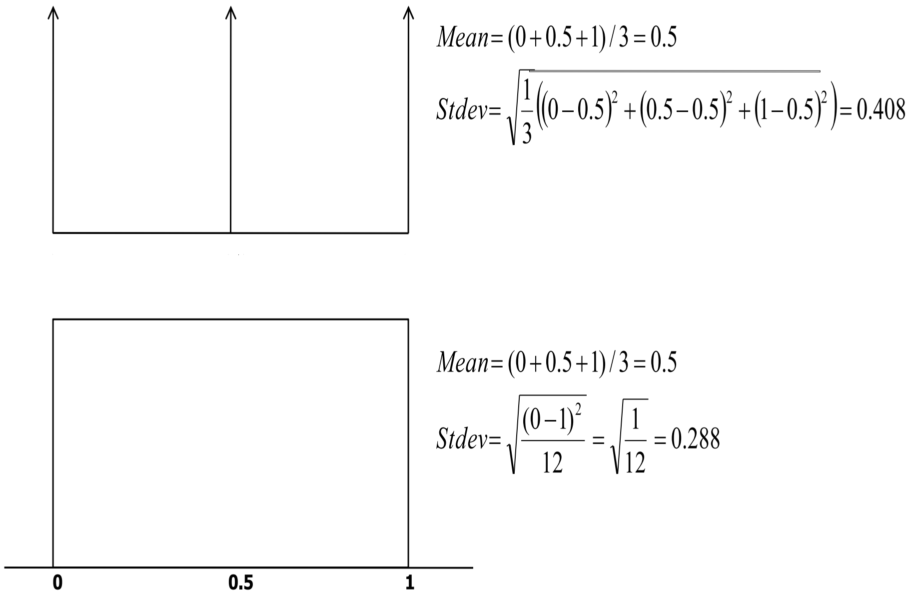

Figure 5.1: Mean and Variance of two random samples

ratio exceeds 20%, the total cost remains the same.

This experiment shows that certain use of overtime is beneficial. However, as excessive use of overtime may not be optimal – as overtime ratio raise up to 20%, an 100% preminum overtime pay is incurred, raising the overtime cost to $36 per hour. These overtime is costly and will not appear in the optimal solution.

5.3.4 Change of Inventory Holding Cost

In this experiment, we studied the changes of total cost with respect to different inventory holding costs. The inventory holding cost are set at 0%, 5%, 10%, 15%, 20%, 25%, and 30% respectively. Table (5.16) shows the total costs associated with the inventory holding values and the detailed production plans for inventory holding cost of 20% and 30% are shown in Table

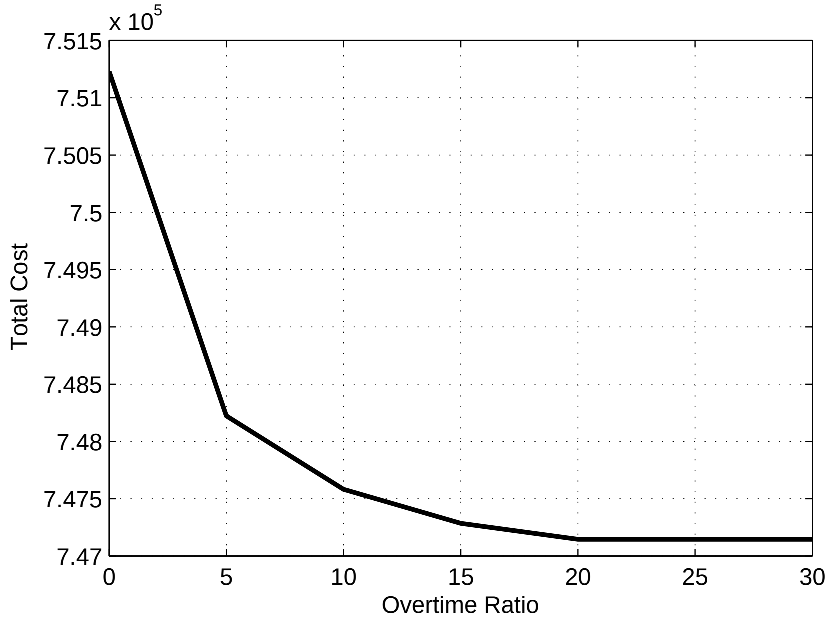

Figure 5.2: Sensitivity Analysis-Overtime Ratio Varies

(5.17) and Table (5.18) respectively.

Table 5.16: Sensitivity Analysis-Inventory Holding Cost Varies

Case	Production Cost	Overtime Cost (Units)	Inventory Holding Cost (Units)	Total cost
5%	648000 (10575)	0 (0)	51614 (13887)	699614
10%	648000 (10457)	9748 (117)	89397 (12228)	747145
15%	648000 (10192)	31957 (383)	108833 (10222)	788790
20%	648000 (10149)	35363 (426)	140883 (9955)	824246
25%	648000 (10016)	46578 (559)	164174 (9392)	858752
30%	648000 (9939)	53141 (636)	191292 (9131)	892433

As we can see, when inventory holding cost increases, the total cost increases, more units are produced in overtime and the average units stored for future demand is decreased. It is interesting to notice that as inventory holding cost increases, even regular production for first few months is kept at the minimum and overtime is used to fulfill the demand. This seems to correspond to a lean manufacturing notion that ”extra production, if not needed, is considered as a waste”.

Table 5.17: Inventory, Overtime and Production with Inventory Holding Cost 20%

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	650	650	650	731	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	62	197
Inventory	1327	1150	967	984	815	715	680	650	601	219	210	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	650	650	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	180	197	197
Inventory	1168	924	753	559	705	699	680	873	726	480	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	650	650	650	956	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	50	197	197
Inventory	1281	1046	872	889	703	709	680	868	815	473	268	200

5.4 Robust model

In the computation of the robust model, parameters λ is set at 0.01, ε_1 and ε_2 at 0.05. The optimal level of production, inventory and overtime is shown in Table (5.19) and the optimal total cost is $750,714.

Though the robust solution has a slightly high total cost, it however, has dramatically less changes in the cost for each scenario. The cost for S(L) is $750,674, for S(M), $750,667, for S(H), $750,801. The small cost difference among each scenarios may be preferable if a manager does not want dramatic changes in its cost over years. The cost difference for all models are listed in Table (5.20)

Table 5.18: Inventory, Overtime and Production with Inventory Holding Cost 30%

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	650	650	650	650	850	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	195	81	197
Inventory	1327	1150	967	984	815	634	466	436	387	200	210	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	650	650	769	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	197	197	197	197
Inventory	1168	924	753	559	491	485	466	659	709	480	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	650	650	650	742	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	67	197	197	197
Inventory	1281	1046	872	889	703	495	466	654	668	473	268	200

Table 5.19: Robust Model Production Plan

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	650	650	791	983	707	753	904	983	934	983	828	964
Overtime	0	0	0	0	0	0	0	4	0	4	0	0
Inventory	1327	1150	1108	1458	1346	1268	1154	1128	1030	652	426	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	650	650	791	983	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	100
Inventory	1168	924	894	1033	1179	1173	1154	1347	1200	774	367	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	650	650	791	983	650	956	983	953	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	1281	1046	1013	1363	1177	1183	1154	1312	1259	867	465	200

Table 5.20: Cost Difference of Various Models

Model	Total Cost	cost under S(L)	cost under S(H)	cost under S(M)
Perfect Information Solution	742450	731475	750608	745267
Two-Stage Stochastic Model (4 Month)	747145	738200	752922	750313
Two-Stage Stochastic Model (12 Month)	754169	752075	763365	747066
Robust Stochastic Model	750674	750801	750667	750714
Expected Value Solution	757474	763033	760598	748792
Expected Demand Solution	755899	761458	759023	747217

Chapter 6

Conclusions and Implementation

6.1 Conclusions

Production planning plays a vital role in manufacturing companies, but uncertainty and randomness in demand pattern makes it difficult for the managers to take full advantage of production planning model. In this study, a two-stage stochastic production planning model is proposed to deal with uncertain demand.

The model is solved with real data from a local company and compared with deterministic production models on the basis of robustness and effectiveness. A parametric analysis is used to derive managerial insights related to issues such as overtime usage, inventory holding cost and the proper selection of scenarios under pessimistic, neutral and optimistic forecasts. An extension of the stochastic model i.e., a robust model is also formulated and solved in an effort to minimize changes in the solutions under various scenarios. The stochastic model provide the best overall results and help making sound decision under uncertainty.

6.2 Implementation

The initial inventory of thermal cooling unit , as collected from real data of work-in-progress or finished good inventories at the beginning of the planning horizon (September 2007), is 1500 units. This makes the average inventory from September 2007 to August 2008 to 894 units.

Table 6.1: Production Plan From September 2008 to August 2009

Time Period	1	2	3	4	5	6	7	8	9	10	11	12
Scenario (L)												
Demand	823	827	833	633	819	831	1018	1013	1032	1365	1054	1190
Production	982	894	983	983	715	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	0
Inventory	359	426	576	926	822	974	939	909	860	478	407	200
Scenario (H)												
Demand	982	894	821	844	837	989	1002	790	1130	1409	1390	1250
Production	982	894	983	983	983	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	120	197	0	0	0	118	197
Inventory	200	200	362	501	647	761	939	1132	985	559	270	200
Scenario (M)												
Demand	869	885	824	633	836	950	1012	795	1036	1375	1385	1248
Production	982	894	983	983	940	983	983	983	983	983	983	983
Overtime	0	0	0	0	0	0	0	0	0	0	0	185
Inventory	313	322	481	831	935	968	939	1127	1074	682	280	200

To find the optimal level of initial inventory, we resolve the model and leave I_0 as a variable and kept it equal to final inventory. The optimal level of initial inventory is 200 units. As such, the average inventory from September 2008 to August 2009, as shown in Table (6.1) will be 551 units. This translates into a reduction of inventory by 38% and more than $350,000 of inventory cost savings for the company. This is achieved through the use of better forecast and stochastic production planning model with uncertain demand. In addition, due to the purchasing of materials at the right time, air frieght expition cost has been reduced and service level has slightly increased.

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