High-Power Energy Scavenging for Portable Devices

Simon J. Tritschler
Wright State University

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HIGH-POWER ENERGY SCAVENGING FOR PORTABLE DEVICES

A dissertation submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

By

SIMON JOSEF TRITSCHLER

B.S.E.E., Wright State University, 2001
M.S.Egr., Wright State University, 2003

2010
Wright State University

Marian K. Kazimierczuk, Ph.D.  
Dissertation Director

Ramana V. Grandhi, Ph.D.  
Director, Ph.D. in Engineering Program

Committee on Final Examination

Marian K. Kazimierczuk, Ph.D.

Fred D. Garber, Ph.D.

Raymond E. Siferd, Ph.D.

Ronald A. Coutu, Ph.D.

Brad S. Bryant, Ph.D.

Jack A. Bantle, Ph.D.  
Vice President for Research and Graduate Studies  
and Interim Dean of Graduate Studies
ABSTRACT


Portable electronic devices and crafts such as unmanned aerial vehicles (UAV’s) may benefit greatly from the ability to extract power from overhead distribution power lines on a temporary basis to power electronics or charge on-board batteries. However, most of the current literature on the subject of energy scavenging is focused on micropower and other small-scale applications. Several high-power energy scavenging methods are investigated here with an emphasis on relating physical sensor dimensions with output power. A novel power scavenging mechanism is introduced that shows excellent correlation between theoretical and experimental performance. In addition, a universal power supply is proposed which may be interfaced with an overhead distribution line of 4.16 – 34.5 kVAC to create a temporary source of high-quality regulated power for portable device electronics and battery charging.
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ACKNOWLEDGEMENT

The author would like to give acknowledgement and thanks to the following people:

To my dissertation advisor, Dr. Marian K. Kazimierczuk, with whom my ten-year association has borne much academic fruit and who made me want to teach electrical engineering as a profession;

To my dissertation committee chief, Dr. Fred Garber, my original freshman-year inspiration in the academic field of electrical engineering and a very important continuing source of technical and moral support;

To my committee members and friends, Drs. Ron Coutu and Ray Siferd, for invaluable insight and constructive input; and especially to Dr. Brad Bryant, esteemed colleague and guitar picker extraordinaire;

To Mr. Barry Woods, my academic advisor and kindred spirit in all things nuclear and audio, who stuck with me from the first day I set foot on campus in 1997 and kept me going through the toughest times;

To Ms. Vickie Slone, without whom the EE department wouldn’t be the slightest bit operational, who always makes sure I get paid, helped me get a house, and keeps the show running;

To Ms. Marie Donohue, who manages to patiently deal with every kind of student and faculty member and gets the job done perfectly, no matter how utterly ridiculous (how many do I owe you?)
To Roger Hughes at Midwest Surplus Electronics in Fairborn, Ohio, who will NEVER address me as Doctor; and to Dave Horner, who might think about it;

To Dr. Leo Finkelstein, Jr., for seeing to it that I didn’t blow myself up with 12,000 volts and after whose class I could write a sentence or two;

To all my friends, relatives, and colleagues, especially the ones who haven’t seen me much the past year;

To my father and EE lab manager Mr. Tony Tritschler, who didn’t complain too much about the burning plywood and ozone; and

To my mother, Ms. Elizabeth Tritschler, who can help wind a 300-turn choke and then make cookies for the defense.
1. Introduction

1.1 Overview of High-Power Energy Scavenging

*Energy scavenging* is an important and fast-growing field. While the more-familiar and currently-fashionable term *energy harvesting* is often used to describe the harnessing and storing of energy from sun, wind, and water as an alternative to fossil fuels for the generation of electricity to power homes and businesses, energy scavenging (and the interchangeable term *power scavenging*) is used here to describe the extraction of power on a more localized basis, typically to be used with a single portable device rather than to distribute power.

The subject of energy scavenging for portable electronic devices is of considerable interest in the field of electrical engineering and has attracted significant attention over the last five years [1-5]. Numerous methods have been proposed: one paper alone discusses five, including photonic energy scavenging using photovoltaic cells, kinetic-flow energy from wind and water currents, thermal energy from various natural and industrial sources of thermal differentials, electromagnetic energy scavenging from power lines, and autophagous structure-power [6]. Of the many sources of energy available, the focus here will be upon extracting power from an existing man-made infrastructure; specifically, the use of energy from power lines.

Often synonymous with the subject of portable-device energy scavenging is its application to battery-charging [7-8]. Of particular interest
is the ability to charge batteries in small crafts such as unmanned aerial vehicles (UAV’s). The ability to charge batteries remotely in these devices could offer a tremendous advantage in many applications; for example, a power utility company could use UAV’s equipped with measurement hardware for monitoring power line currents with virtually unrestricted flight range. Outside the scope of aircraft technology and battery charging, applications which may benefit from the conversion of a power line into a convenient power supply for electronic devices could be manifold: one might envision portable cameras for border surveillance and patrol over long periods of time with minimal maintenance, for example. In either case, a reliable, repeatable source of power, most likely in the form of regulated low-voltage DC, is the desired output.

At the time of this writing, most research into energy scavenging (including that which relates to battery charging) is focused on micropower and very small-scale applications such as wireless phones. Comparatively few publications exist on the subject of high-power energy scavenging. The term “high-power” is defined here as being around the order of magnitude of single watts; i.e. from tenths of a watt to tens of watts. By comparison, most energy-scavenging systems, such as those used in wireless phones, are in the lower milliwatt-range at best. Larger portable devices, such as cameras and small unmanned aircraft, require commensurately more operating and charging power. For this reason, most of the current literature is of limited
relevance. With regards to battery-charging, power is defined as the exchange of energy per unit time; therefore, higher charging power means less charging time. One may intuitively understand why it would be desirable to charge batteries as quickly as possible within the limits of current battery technology in many applications. Thus the desire for maximum power from a given technology and form factor is acknowledged.

A dissertation on the topic of high-power energy scavenging is presented here.

1.2 Dissertation Objectives

The primary objective of this dissertation is to investigate the subject of high-power energy scavenging as a whole and offer meaningful technical insight into a range of power-scavenging technologies. This objective may be further broken down into two primary components: to research and analyze the current state of high-power energy scavenging and its applications, with an emphasis on rigorously examining technical merits and shortcomings of candidate methods and their proposed solutions; and to advance the state of the art by proposing alternative technologies in the context of a rigorous evaluation and experimental verification.
1.3 Scope

The intended scope of this dissertation includes an investigation into the theory and implementation of various technologies on the general topic of high-power energy scavenging, including an expansion of the quantitative understanding of this subject in the electrical engineering discipline at the time of writing; and the design and analysis of a power-scavenging system that offers an alternative to those currently in use. The exploration of possibilities for practically implementing an energy-scavenging power supply in battery-charging applications, including the design of any conditioning or processing circuitry required to interface the components of the overall system, is included.

1.4 Areas Intentionally Excluded

Areas of research specifically not within the scope of this dissertation include but are not limited to the following: 1) batteries and associated power management technology, specifically limiting understanding, where applicable, to proper use and application, not reinvention; 2) mechanical issues, bearing in mind the general application is that of portable devices in considerations such as mass and physical size but intentionally not considering, for example, problems such as guidance systems and ballistic issues with power line attachment; 3) power scavenging from high-voltage and extra-high-voltage transmission and sub-transmission lines and the
problems associated with these voltages, particularly in deference to experimental issues, except as briefly noted: and 4) resonant, pulsed, or other novel power loading methods, focusing on resistive loading for all work presented here. With regard to the first of these areas, there is indeed much research being done in the field of battery design to allow increasingly fast charging without damage to the battery; lithium polymer designs seem to be leading the way [9], but this is outside the scope of this dissertation. Limitations of scope have been placed on more specific areas within the dissertation and these are detailed in the respective chapters. All excluded areas may certainly form the basis of meaningful future research.
2. Power-Line Energy Scavenging Mechanisms

2.1 Power Distribution Systems in the United States and Abroad

In the United States and in many foreign countries, power is transmitted and distributed to industry and homes via conductive lines as sinusoidal 60-Hz alternating current. The 50-Hz line frequency is also used in many parts of the world. These lines are typically of the “overhead” type on above-ground poles, as opposed to underground distribution. Domestic transmission line voltages fall into three main categories: extra-high-voltage (generally from 500 – 765 kV for long-range propagation from power plants), high-voltage (230 – 345 kV), and sub-transmission (69 – 169 kV), whereas distribution lines are typically less than 110 kV. Common U.S. distribution line voltages include 4.16 kV, 7.2 kV, 12.47 kV, 13.2 kV, 14.4 kV, 23.9 kV, and 34.5 kV [10-12]. Line currents can range from several hundred amperes for densely-populated urban distribution systems to a few amperes for high-voltage transmission or end-user lines, varying according to power consumption requirements at any given time for a given location [13]. This dynamic range, both in voltage and in current, could pose a considerable challenge if a universal system of scavenging energy from overhead lines is the goal. As stated in Sect. 1.4, the scavenging of power from high-voltage and extra-high-voltage transmission and sub-transmission lines is specifically excluded from this dissertation except where briefly noted; thus, the focus is upon power distribution lines.
2.2 Power Line Energy-Scavenging Methods and Scope

To extract power efficiently from overhead distribution lines implies four possible principal mechanisms which constitute basic physics: direct current sampling, direct voltage sampling, magnetic field (transformer) coupling, and electric field (capacitor) coupling. Secondary mechanisms created by the practical consequences of electrical transmission and distribution, such as line vibration due to magnetostriction and the piezoelectric effect [14·15], have not been considered but may be of future interest. Also not considered is power extraction via electromagnetic radiation as the size of a suitable antenna at 60 Hz is not likely to be practical, although, interestingly, solar energy in the THz range has been scavenged in this manner [16].

At the time of writing, the vast majority of research into power line energy scavenging uses the electromagnetic principle, including at least one commercial device [17]; this is discussed further in Section 2.5 and in Chapter 3.

2.3 Current Sampling

To sample current directly from a power line implies cutting the line and inserting a load resistance in series with it, through which power is extracted directly as $P = I^2R$. This is not practical in a portable or mobile
system as it requires a complex and time-consuming installation. For this reason, it is not given further consideration in this dissertation.

2.4 Voltage Sampling

To sample voltage directly, the difference in potential between distribution lines of differing phase or from a line to ground or neutral is used to develop power into a load resistance by \( P = \frac{V^2}{R} \). Power line cables are generally un-insulated, encouraging the feasibility of a mechanical coupling and thus electrical connection [18]. The recent U.S. Patent #7563124 also describes a method of connecting to live, insulated overhead lines [19]. Most power distribution lines are referred to a common neutral line that is connected to Earth ground, potentially simplifying connection [20]. The direct voltage sampling method forms the basis of Chapter 7.

2.5 Magnetic Field Coupling

To harness a magnetic field implies mechanically coupling an inductance to a power line such that the pair forms a current transformer whose primary is the line itself and whose secondary is the coupled inductance, from which the field-generated AC secondary current is applied to a load resistance. Ampère’s Law states that a time-varying current in a conductor, such as that passing through an overhead power line, will induce a time-varying magnetic field; Faraday’s Law in turn states that a time-
varying magnetic field may be used to produce a time-varying current. It is this mechanism by which power may be transferred. This coupling of line current may be considered a reactive transfer rather than direct. This method is the basis of Chapters 3 and 4.

2.6 Electric Field (Capacitive) Coupling

To harness an electric field from an overhead power line implies mechanically coupling an insulated conductor to the line such that the pair forms a capacitor whose AC current may be developed into a load resistance. The electric field is created by the voltage difference between conductors and the area over which the potential exists. As such, it may be considered reactive voltage coupling rather than direct voltage sampling.

The smallest voltage likely to be found overhead in rural and residential areas is 4.16 kV which, even at 60 Hz, is significant. One theoretical advantage to electric field coupling over magnetic field coupling is that a given line voltage is designed to be regulated and therefore may yield more predictable and repeatable results than any method relying upon power line current. The author speculates that line voltage information may even be incorporated into GPS devices used by energy-seeking crafts in order to calibrate the device for an expected source voltage.

Capacitive power line coupling has not been found in the literature at time of writing and is the basis of Chapters 5 and 6.
3. Current Transformer Power Scavenging Theory

3.1 Current Transformer Overview

The current-transformer method of power line energy scavenging is by far the most published and widely used. Paradoxically, little meaningful information has been published to date relating physical parameters of a power-scavenging current transformer to its expected performance in an actual application. At least one commercial device has been advertised using this technology, as mentioned in Sect. 2.5: it is assumed that proprietary information in this area exists but has not yet reached the literature.

Consider a wound magnetic core coupled to a power distribution line such that it acts as a current transformer, with the line itself serving as a single-turn primary and the core’s winding as the secondary. The analysis and design of a current transformer is significantly different from that of a conventional transformer. Current transformers are often used as sense or measurement devices, for which they are called current probes. The application described here is also different than the usual manner in which current transformers are used in that current probes are not designed to transfer appreciable power: on the contrary, it is usually desirable to draw as little power as possible when performing measurements. Here, our objective is to use a current transformer in a power supply and this requires a significant re-thinking of current transformer implementation.
3.2 Current Transformer Model

The basic model for a current transformer, as described by Kondrath [21], is shown in Figure 3.2.1, including parasitic components.

![Current transformer model, including parasitic components.](image)

Figure 3.2.1: Current transformer model, including parasitic components.

$I_P$ is the steady-state AC RMS primary current, $N$ is the number of secondary turns, $I_S$ is the secondary AC current, $L_M$ is the secondary magnetizing inductance, $r_c$ is the core parallel equivalent resistance, $L_l$ is the total leakage inductance reflected to the secondary winding, $r_s$ is the resistance of the secondary winding, $C$ is the total stray capacitance reflected to the secondary circuit, $R_L$ is the load resistance (often referred to as a burden resistor or sense resistance in measurement and instrumentation applications), and $L_L$ is any series inductance in the load resistance. All AC voltage and current signal amplitudes in this dissertation will be expressed as RMS values; thus, all AC circuit models will resemble their DC counterparts for clarity.
3.3 Mechanical Interface of Current Transformer to Line

Ahola et al. describe a power-scavenging current transformer attached to one phase conductor of a low-voltage induction motor [22]. The transformer, driving a switch-mode power supply, is used as a maintenance-free source of power for online motor condition-monitoring equipment. The phase conductor carrying 100 A of 50-Hz alternating current was shown to produce approximately 12 W of usable output power from a core of modest size. As 100 amperes may easily be of the order of magnitude found in overhead power-line currents under some conditions, the application of this basic concept to overhead lines is immediately apparent. The form factor of the current transformer described in the Ahola paper is that of a wound, ungapped toroidal core through which the conductor is presumably inserted and then connected to the motor. This specific mechanical interface does not lend itself to direct adaptation to portable devices for the same reason that direct current sampling was deemed impractical in Sect. 2.3; it requires installation and does not directly facilitate mobility nor portability.

The form factor of a toroidal core that may be instantaneously interfaced with an overhead power line is that of a core which is cut into two mating halves, allowed to be hinged or otherwise maneuvered around the line, and subsequently re-joined, completing the magnetic circuit. The actual prototype demonstrated in the Ahola paper is described as being constructed
of two U-cores, indicating that a two-piece core is a viable form factor. The cut-core approach is considered in the following section.

Consider the initial form factor of a power-scavenging current transformer as a high-permeability magnetic core which is cut and then placed around a current-carrying overhead power line in such a way that mutual coupling takes place between it and a secondary winding wound around the core. The degree of coupling is initially assumed to be perfect for simplicity of analysis. The cut core results in an unavoidable air gap which will be shown to affect power transfer directly. Although the Ahola paper does make reference to a two-piece core for its prototype, no mention is made of the air gap and its effect on power output. Closer analysis has shown the gap to have a quite significant effect, as predicted in the theoretical expression for output power in Section 3.5 and confirmed experimentally in Section 4.6. Refer to figure 3.3.1.

![Diagram of cut-core current transformer](image)

**Fig. 3.3.1: Interface of cut-core current transformer to line.**
3.4 1st-order LF Model of Current Transformer Circuit

At low frequencies, the dominant component of the current transformer model is its magnetizing inductance. As the operating frequency is fixed at 50 or 60 Hz we will consider this model in detail. Refer to the first-order low-frequency circuit shown in Fig. 3.4.1, which neglects such parasitic components as leakage inductance, load inductance, winding resistance, core resistance, and inter-winding and stray capacitances.

![Diagram of first-order low-frequency current transformer model](image)

**Figure 3.4.1: First-order low-frequency current transformer model.**

Primary current is defined as $I_P$, secondary current is $I_S$, magnetizing inductance is defined as $L_M$ through which the current $I_L$ flows, and load resistance is defined as $R_L$ through which flows the load current $I_R$.

If the number of primary turns is taken to be unity, then the number of secondary turns becomes simply $N$ and the secondary current is
\[ I_S = \frac{I_p}{N}. \] (3.4.1)

This secondary current is divided between the magnetizing inductance \( L_M \) and load resistance \( R_L \) by Kirchoff’s Current Law. The effect of the magnetizing inductance is to cause an increasing proportion of secondary current to flow through it with decreasing frequency. From the perspective of the load resistance, this forms a high-pass filter whose corner frequency \( \omega_L \) is defined as

\[ \omega_L = \frac{R_L}{L_M}. \] (3.4.2)

The operating frequency of the system may be defined as \( \omega_0 \) and thus the impedance of the magnetizing inductance at a given operating frequency is \( j \omega_0 L_M \). The voltage across the parallel circuit consisting of \( L_M \) and \( R_L \) may be determined by multiplying secondary current \( I_S \) by the effective parallel circuit impedance \( Z_{eff} \). As this voltage may be taken as that across the load resistance, we define it as \( V_R \) (rather than \( V_L \) to avoid confusion with inductor voltage).

\[ V_R = I_S Z_{eff.} = I_S \left( \frac{j \omega_0 L_M R_L}{j \omega_0 L_M + R_L} \right) = I_S R_L \left( \frac{j \omega_0 L_M}{j \omega_0 L_M + R_L} \right) \] (3.4.3)

By rearranging and substituting eqn. (3.4.2) into (3.4.3), we can define the load voltage in terms of the corner frequency, \( \omega_L \).

\[ V_R = I_S R_L \left( \frac{j \omega_0 L_M}{j \omega_0 L_M + R_L} \right) = I_S R_L \left( \frac{1}{1 + \frac{R_L}{j \omega_0 L_M}} \right) = I_S R_L \left( \frac{1}{1 - j \frac{R_L}{\omega_L}} \right) \] (3.4.4)
From (3.4.4),

\[ V_R = I_S R_L \left( \frac{1}{1-j\frac{\omega_L}{\omega_0}} \right) \left( 1+j\frac{\omega_L}{\omega_0} \right) = I_S R_L \left[ \frac{1+j\frac{\omega_L}{\omega_0}}{1+\left(\frac{\omega_L}{\omega_0}\right)^2} \right]. \]  

(3.4.5)

### 3.5 Output Power of Current Transformer Circuit

Because power is being delivered into a resistive load (from Sect. 1.4) from a purely inductive source (assuming we consider the first-order model), we may intuitively expect maximum power to occur when the load resistance equals the magnetizing inductive reactance at the frequency of operation. Output power is derived in this section.

Complex power \( S \) may be defined as \( S = I^*V \), where \( V \) is a complex voltage vector and \( I^* \) is the complex conjugate current vector. Alternatively, if power is to be determined into a purely resistive load, we may consider the equivalent forms \( S = II^*R \) or \( S = \frac{VV^*}{R} \).

From (3.4.5),

\[ V_R V_R^* = I_S^2 R_L^2 \left[ \frac{1+j\frac{\omega_L}{\omega_0}}{1+\left(\frac{\omega_L}{\omega_0}\right)^2} \right] \left[ \frac{1-j\frac{\omega_L}{\omega_0}}{1+\left(\frac{\omega_L}{\omega_0}\right)^2} \right] = I_S^2 R_L^2 \left[ \frac{1+\left(\frac{\omega_L}{\omega_0}\right)^2}{1+\left(\frac{\omega_L}{\omega_0}\right)^2} \right] = I_S^2 R_L^2 \left[ \frac{1}{1+\left(\frac{\omega_L}{\omega_0}\right)^2} \right]. \]  

(3.5.1)

Output power may now be defined.

\[ P_O = S = \frac{V_R V_R^*}{R_L} = I_S^2 R_L \left[ \frac{1}{1+\left(\frac{\omega_L}{\omega_0}\right)^2} \right] \]  

(3.5.2)
It will be noted that this is the classical expression for a first-order high-pass filter with respect to power transfer into a resistive load in terms of $\omega_0$ and $\omega_L$. The resemblance of the low-frequency first-order current transformer model (Fig. 3.4.1) to that of an audio-frequency filter is affirmed.

If we use eqn. (3.4.1) to find the output power terms of primary current $I_P$, the expression becomes

$$P_O = \left(\frac{I_P}{N}\right)^2 R_L \left[\frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2}\right] = \frac{I_P^2 R_L}{N^2} \left[\frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2}\right]. \quad (3.5.3)$$

If the permeability of the core is sufficiently high to neglect any magnetomotive force dropped across it compared to that dropped across its air gap, then the inductance is that of an air-cored inductor. An equation describing the magnetizing inductance $L_M$ in terms of core geometry is as follows [23], where $\mu_0$ is the permeability of free space ($\mu_R$ for air $\approx 1$), $A_c$ is the area of the core, $l_g$ is the length of the air gap, and $N^2$ is the square of the turns ratio.

$$L_M = \frac{\mu_0 A_c N^2}{l_g} \quad (3.5.4)$$

Rearranging eqn. (3.5.4) for $N^2$,

$$N^2 = \frac{L_M l_g}{\mu_0 A_c}. \quad (3.5.5)$$

It should be noted that if we consider the total magnetic path through which magnetic flux flows, the cut-core transformer effectively has two gaps in series due to the two core-face pairs. As magnetic permeance may be modeled as an electrical resistance (detailed further in Sect. 4.5), the two gap
lengths may be summed to form an effective core air gap of twice the core-
half spacing. It will be important to keep this mind when computing
theoretical power with an actual air gap dimension.

Substituting eqn. (3.5.5) into (3.5.3),

\[
P_O = \frac{i_{\Phi R_L}}{N^2} \left[ \frac{1}{1 + (\frac{\omega_L}{\omega_0})^2} \right] = \frac{i_{\Phi R_L} \mu_0 A_c}{L_M i_g} \left[ \frac{1}{1 + (\frac{\omega_L}{\omega_0})^2} \right].
\]  

(3.5.6)

Noting that \(\omega_L = \frac{R_L}{L_M}\) from eqn. (3.4.1), the final equation describing the
output power of a resistively-loaded air-gapped current transformer is

\[
P_O = \frac{i_{\Phi} \mu_0 A_c \omega_L}{i_g} \left[ \frac{1}{1 + (\frac{\omega_L}{\omega_0})^2} \right].
\]  

(3.5.7)

Because magnetizing inductance is directly proportional to the square
of the turns ratio, it effectively becomes inversely proportional to output
power. In conjunction with the load resistance, output power becomes a
function of the system corner frequency \(\omega_L\), as shown. This is a very
important component of the final power equation because it allows for
optimization with regards to maximum power transfer.

If we group frequency terms together, the output power becomes

\[
P_O = \frac{i_{\Phi} \mu_0 A_c}{i_g} \left[ \frac{\omega_L}{1 + (\frac{\omega_L}{\omega_0})^2} \right].
\]  

(3.5.7a)

The placement of the corner frequency term \(\omega_L\) into the numerator of the
power equation changes the characteristic shape of the power frequency
response such that it reaches maximum value when \(\omega_L = \omega_0\). Refer to figure

Tritschler
3.5.1, which shows output power with respect to $\omega_L$, normalized by $\omega_0$ and $K = \frac{i_0^2 \mu_0 A_c}{i_g}$, respectively.

As shown, maximum power is transferred when the corner frequency of the circuit is placed at the operating frequency. Note that as $\omega_0 = \omega_L = \frac{R_L}{L_M}$ from Eqn. 3.4.1, maximum power is transferred when $R_L = \omega_0 L_M$, the inductive reactance of the winding at the operating frequency: this is analogous to Thévenin’s Theorem of Maximum Power Transfer but involving a reactive source and resistive load, as intuitively expected at the beginning of this section.

![Figure 3.5.1: Normalized output power with corner frequency.](image)

Tritschler

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Substituting $\omega_L = \omega_0$ into eqn. (3.5.7a) and solving for $\omega_0$ (which is fixed in a given geographical area; for example, 120π rad./s in the United States),

$$P_{o(\text{MAX})} = \frac{i_0^2 \mu_0 A \omega_0}{2l_g}.$$  \hfill (3.5.8)

Thus, an expression for maximum output power from a resistively-loaded current transformer of given dimensions coupled to a power line of a given current amplitude has been derived.

Notice that maximum power is in linear proportion to operating frequency for a given core area and air gap length. Intuitively, this stands to reason because lower frequencies (and thus longer wavelengths) would require a commensurately larger core area with all other factors being equal. For applications requiring small size and low mass (such as aerial applications), the direct power dependence on core area is of concern. Finally, it is apparent that output power is in inverse proportion to the air gap length and this could pose a considerable mechanical challenge as well: this is given further consideration in Chapter 4.

### 3.6 Output Power Examples of Current Transformer Circuit

As an example of how much theoretical power in watts may be expected from a gapped toroidal current transformer of reasonable size and weight, assuming its inductance has been “matched” to the ideal load resistance required for maximum power at a given operating frequency, refer to the following conditions, extrapolated directly for the sake of comparison.
from the Ahola paper: \( A_c = 512 \times 10^6 \text{ m}^2 \), \( I_p = 100 \text{ A} \), and \( \omega_0 = 100\pi \text{ rad/s} \) plus an arbitrary air gap length of 0.05 mm for a total effective gap of 0.1 mm.

From eqn. 3.5.8,
\[
P_{o(\text{MAX})} = \frac{l_0^2 \mu_0 A_c \omega_0}{2 l_g} = \frac{100^2 \times 4\pi \times 10^{-7} \times 512 \times 10^{-6} \times 100\pi}{2 \times 0.1 \times 10^{-3}} \equiv 10 \text{ W}.
\]

As the output power claimed by Ahola et al. was 12 W, it is inferred that their effective air gap may have been comparable to that arbitrarily assumed.

For maximum power transfer, \( \omega_L = \omega_0 = 100\pi \text{ rad/s} \) in this example. For an arbitrary load resistance \( R_L = 50 \Omega \), the required inductance is (from eqn. 3.4.2)
\[
L_M = \frac{R_L}{\omega_L} = \frac{50}{100\pi} \equiv 160 \text{ mH}.
\]

This results in a required number of turns (from eqn. 3.5.5)
\[
N = \sqrt{\frac{L_M l_g}{\mu_0 A_c}} = \sqrt{\frac{160 \times 10^{-3} \times 0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 512 \times 10^{-6}}} \equiv 160.
\]

This, coincidentally, is the number used by Ahola et al.

Load voltage, \( V_R \), is
\[
V_R = \sqrt{P_o R_L} = \sqrt{10 \times 50} = 22.36 \text{ V}.
\]

The load current, \( I_L \), is
\[
I_L = \frac{P_o}{V_R} = \frac{10}{22.36} = 0.4472 \text{ A}.
\]
It will be noted that this current is –3 dB from the expected secondary current (from Eqn. 3.4.1) of \( I_S = \frac{I_p}{N} = \frac{100}{160} = 0.6250 \) A, indicating that the system corner frequency has been correctly placed at the operating frequency.

The line current figure of 100 A might be of roughly the order of magnitude encountered with a 7.2·kV power line feeding a neighborhood or business district during moderate-use hours, for example [24]. The total effective gap length of 0.1 mm is arbitrary and implies fairly precise machining of the two core halves with minimal surface imperfections or debris on the core faces. If we consider that the smallest line current likely encountered may be on the order of five amperes (final distribution to a few homes or businesses under low-current conditions such as the middle of the night), the output power becomes

\[
P_o = \frac{5^2 \times 4\pi \times 10^{-7} \times 512 \times 10^{-6} \times 100\pi}{2 \times 0.1 \times 10^{-3}} \approx 25 \text{ mW}.
\]

It is worth mentioning that this figure implies a perfect core with infinite relative permeability, no core losses such as those due to Eddy currents, no copper losses, perfect line-to-secondary coupling and no higher-order effects. Of particular concern is again the dependence of output power upon air gap length, which could be considerably variable due to machining tolerances, contamination or debris, or environmental factors such as corrosion and other sources of surface degradation.
3.7 Implications of Theoretical Results

Based on the results of the derivation of output power from a cut-core current transformer, two limiting factors are clear. First, core surface area, which implies proportionality of size and mass, causes power output to be in direct conflict with the desire for low mass in a portable device such as a small aircraft. Second, air gap length causes output power to be in direct conflict with the practical extent to which the core may be precisely manufactured and maintained, implying proportionality of cost. These considerations are given further mention in Sect. 7.2.
4. Experimental Power-Scavenging Current Transformer

4.1 Introduction

Experimental work regarding the current-transformer method of power scavenging has been conducted with the objective of validating theoretical models and computations of output power with real-world devices placed in real-world service, at least to the extent that power lines may be practically simulated in a laboratory situation. For these experiments, it has not been attempted to simulate the very wide dynamic range of power distribution line currents that may be encountered in practical use. As a consequence, the effects of very high current on output power with a practical device with regard to core saturation, for example, may not be determined experimentally at this time; however this may form the basis of future research.

4.2 Test Apparatus for Measuring Transformer Output Power

In order to test the output power of a cut-core current transformer of the type described in Chapter 3, the author constructed a test bed consisting of a source of AC current and a length of overhead power line of the type normally used for distributing power at 12.47 kV. A small power transformer with a 10-V$_{RMS}$ secondary rated at 8 A (Stancor P-6139) was connected in series with a composite 1-Ω resistor consisting of six 6-Ω, 50-W resistors wired in parallel. The 1-Ω resistor helps to regulate current; additionally, it
serves as a convenient means of sensing current by measuring its voltage drop under test. 5%-tolerance resistors were used; the composite resistance measured 1.0 Ω using a Wavetek 320B Digital Multimeter. This source was connected to a 24” length of 12.47-kV line via. 12-gauge solid-core copper wire of the type used for domestic household wiring. The power cable is composed of an aluminum alloy, thus precluding the use of regular soldered electrical connections; therefore, a mechanical interference joint was employed and the copper ends of the connecting wires tinned with solder to reduce the likelihood of galvanic corrosion. A variable autotransformer was connected to the filament transformer’s primary to facilitate adjustment of line current to the desired magnitude under load, up to the maximum eight amperes. It is assumed that sufficient experimental utility will be gained by conducting tests at the lower end of the current scale. It is further assumed that sufficient power transfer may cause a voltage drop across the section of interfaced line such that it will necessitate adjustment.

4.3 Prototype Current Transformer

A practical current transformer was wound by the author in order to compare theoretical analyses with those obtained with a real-world device. Of particular interest is to qualify the use of a first-order model in simple computations of output power.
A strip-wound double-C core of 79%-nickel “Supermalloy” composition was procured as a surplus item and used for the experiment [25]. This material has very high relative permeability ($\mu_r \approx 40,000 – 100,000$) and was chosen for this property in the hopes of qualifying the assumption stated in Sect. 3.5 that any magnetomotive force dropped across the length of the core is negligible compared to that dropped across the air gap. Strip winding of the core material produces a laminated core which greatly reduces the magnitude of eddy currents over that of a solid core. Core cross-section dimensions are $1.28 \text{ cm} \times 0.98 \text{ cm}$ for a core area of $1.225 \times 10^{-4} \text{ m}^2$. The process by which the core was cut and lapped was not specified and therefore neither the condition of the lamination stack nor the flatness of the core faces after cutting may be determined with precision; however, visual inspection suggests that the core faces are ground with sufficient precision that any practical magnetic gap inserted will not likely be affected by core face irregularities. 33 turns of 18-awg copper magnet wire were wound around each core half using PVC electrical tape to protectively insulate the winding from the core for a total of 66 turns when the two coils are connected in series. A single layer of 0.15-mm PVC electrical tape was placed on each face of one core half to impose an initial gap whose relative permeability is very close to that of air; the two gaps result in an effective series gap length of 0.3 mm. Refer to Figure 4.3.1. The theoretical inductance is (from 3.4.9)

$$L_M = \frac{\mu_0 A_C N^2}{l_g} = \frac{4\pi \times 10^{-7} \times 1.225 \times 10^{-4} \times 66^2}{0.3 \times 10^{-3}} = 2.24 \text{ mH}.$$
The measured inductance, using a Tenma 72-1025 LCR Meter at 100 Hz (the closest available frequency to the desired 60 Hz), was 2.15 mH for a negligible deviation from the theoretical value. The measured DC winding resistance, using a Wavetek 320B Digital Multimeter, was 70 mΩ.

![Prototype current transformer](image)

**Figure 4.3.1: Prototype current transformer.**

### 4.4 Inductance of Prototype Current Transformer

In order to test the inductance of the prototype current transformer with respect to different gap lengths up to 4.0 mm, non-magnetic spacers were incrementally inserted between the transformer core halves and the inductance measured at 100 Hz for each gap. Spacer material was 0.10-mm
plastic transparency sheet, verified in thickness by measurement. It is assumed that the plastic sheet material has relative permeability comparable to that of air; this property is typical of plastics. As a spacer spanning both core faces of the current transformer constitutes two effective gaps in the magnetic core path length, incremental gap length is effectively 0.2 mm per spacer (from Section 3.5). Refer to Fig. 4.4.1, which also includes theoretical inductance from Eqn. 3.4.9.

At an air gap length of 0.2 mm, measured inductance is less than expected, possibly due to irregularities in the geometry of the core faces resulting in a larger-than-expected effective gap. As gap length increases, the inductance is greater than expected and the deviation between expected and measured inductance also increases. The source of this increase is fringing magnetic flux, a phenomenon by which magnetic lines of force repel each other when traversing a non-magnetic medium. Note that the initial chosen gap of 0.3 mm lies between negative and positive deviations from theory; the very close agreement between measured and theoretical values is affirmed. One experimental observation of note is that the sensitivity of the inductance measurement became very significant at small air gaps, say less than 1.0 mm. Very small variations in alignment of the two core halves, both in azimuth and zenith, produced quite large changes in the value of inductance; even squeezing the core halves together produced noticeable changes. It is speculated that this is partly due to irregularities in the
lapping and precision of the core faces. If maintaining a small gap is necessary for transferring maximum power consistently in the power-scavenging current transformer, this sensitivity at small gaps is an undesirable feature of the arrangement.

<table>
<thead>
<tr>
<th>$I_g$</th>
<th>$L_M$</th>
<th>$L_M$ (measured)</th>
<th>% Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 mm</td>
<td>3.433 mH</td>
<td>3.13 mH</td>
<td>-8.8%</td>
</tr>
<tr>
<td>0.4 mm</td>
<td>1.717 mH</td>
<td>1.85 mH</td>
<td>+7.8%</td>
</tr>
<tr>
<td>0.6 mm</td>
<td>1.144 mH</td>
<td>1.36 mH</td>
<td>+19%</td>
</tr>
<tr>
<td>0.8 mm</td>
<td>0.858 mH</td>
<td>1.12 mH</td>
<td>+31%</td>
</tr>
<tr>
<td>1.0 mm</td>
<td>0.687 mH</td>
<td>0.95 mH</td>
<td>+38%</td>
</tr>
<tr>
<td>1.2 mm</td>
<td>0.572 mH</td>
<td>0.85 mH</td>
<td>+49%</td>
</tr>
<tr>
<td>1.4 mm</td>
<td>0.490 mH</td>
<td>0.77 mH</td>
<td>+57%</td>
</tr>
<tr>
<td>1.6 mm</td>
<td>0.429 mH</td>
<td>0.71 mH</td>
<td>+65%</td>
</tr>
<tr>
<td>1.8 mm</td>
<td>0.381 mH</td>
<td>0.66 mH</td>
<td>+73%</td>
</tr>
<tr>
<td>2.0 mm</td>
<td>0.343 mH</td>
<td>0.62 mH</td>
<td>+81%</td>
</tr>
<tr>
<td>2.2 mm</td>
<td>0.312 mH</td>
<td>0.60 mH</td>
<td>+92%</td>
</tr>
<tr>
<td>2.4 mm</td>
<td>0.286 mH</td>
<td>0.57 mH</td>
<td>+99%</td>
</tr>
<tr>
<td>2.6 mm</td>
<td>0.264 mH</td>
<td>0.55 mH</td>
<td>+108%</td>
</tr>
<tr>
<td>2.8 mm</td>
<td>0.245 mH</td>
<td>0.53 mH</td>
<td>+116%</td>
</tr>
<tr>
<td>3.0 mm</td>
<td>0.229 mH</td>
<td>0.51 mH</td>
<td>+123%</td>
</tr>
<tr>
<td>3.2 mm</td>
<td>0.215 mH</td>
<td>0.50 mH</td>
<td>+133%</td>
</tr>
<tr>
<td>3.4 mm</td>
<td>0.202 mH</td>
<td>0.48 mH</td>
<td>+138%</td>
</tr>
<tr>
<td>3.6 mm</td>
<td>0.191 mH</td>
<td>0.47 mH</td>
<td>+146%</td>
</tr>
<tr>
<td>3.8 mm</td>
<td>0.181 mH</td>
<td>0.46 mH</td>
<td>+155%</td>
</tr>
<tr>
<td>4.0 mm</td>
<td>0.172 mH</td>
<td>0.45 mH</td>
<td>+162%</td>
</tr>
</tbody>
</table>

Fig. 4.4.1: Inductance ($L_M$) vs. air gap ($I_g$).
The positive implication of this increase in inductance over that expected for a given air gap length is that the reduction in power with increasing air gap length examined in Section 3.7 may be tempered by fringing flux.
4.5 Fringing Flux in Prototype Current Transformer

As mentioned in Section 4.4, fringing flux is a mechanism by which magnetic lines of force repel each other and bulge outward when traversing a non-magnetic medium, increasing inductance. With respect to a gapped-core inductor, the non-magnetic medium is the air gap. Consider the magnetic circuit of a gapped-core inductor, in which the reluctance of the fringing flux (analogous to electrical resistance) is effectively in parallel with the reluctance of the gap, reducing the equivalent reluctance which will be shown to increase inductance; refer to Fig. 4.5.1.

![Diagram of Magnetic Circuit](image)

**Fig. 4.5.1:** Magnetic circuit of gapped-core inductor.
As shown, $\phi$ is the magnetic flux flowing through the core, $R_c$ is the reluctance of the core, $\phi_g$ is the flux in the gap, $R_g$ is the reluctance of the gap, $\phi_f$ is the fringing flux, and $R_f$ is the fringing reluctance.

The permeance of the gap, $P_g$, is the reciprocal of the gap reluctance and is given by

$$P_g = \frac{1}{R_g} = \frac{\mu_0 A_c}{l_g} \quad (4.5.1)$$

where $l_g$ is the gap length, $A_c$ is the core (and thus gap) area, and $\mu_0$ is the permeability of free space ($= 4\pi \times 10^{-7}$ H/m), sufficiently close to the permeability of air to neglect its relative permeability $\mu_R$ in the expression. Note that magnetic permeance is further analogous to electrical conductance in that it is proportional to area and inversely proportional to length.

The outward bulge of fringing flux results in an increase in the length and cross-sectional area of the magnetic flux in the gap. As the geometry of this increase is complex and may be visualized as a gradually-decaying field with increasing distance from the core, consider the integration of this field into a mean fringing flux width and length. Define $w_f$ as the mean width of the fringing flux and $l_f$ as the mean length of the fringing flux. Refer to Fig. 4.5.2.

The permeance of the fringing flux region, $P_f$, is the reciprocal of the fringing reluctance and is given by

$$P_f = \frac{1}{R_f} = \frac{\mu_0 A_f}{l_f} \quad (4.5.2)$$

where $A_f$ is the total area of the fringing flux.
Figure 4.5.2: Effect of fringing flux on magnetic geometry

If the permeability of the core is sufficiently high that any magnetomotive force dropped across it is negligible compared to that dropped across its air gap, as assumed in Sect. 3.5, then the ideal inductance $L$, not including the fringing flux (and therefore involving only the gap permeance), is that of an air-cored inductor and is given by

$$L = P_g N^2$$

(4.5.3)

where $N$ is the number of turns of the winding.
The total permeance $P$ of the air gap and fringing region is, in the manner of conductors in parallel,

$$P = P_g + P_f. \quad (4.5.4)$$

The inductance including fringing flux, $L_f$, is

$$L_f = PN^2. \quad (4.5.5)$$

Define the fringing flux factor, $F_f$, such that

$$L_f = F_f L \quad (F_f > 1). \quad (4.5.6)$$

$$F_f = \frac{L_f}{L} \quad (4.5.6a)$$

Plugging (4.5.5) and (4.5.3) into (4.5.6a),

$$F_f = \frac{PN^2}{P_g N^2} = \frac{p_g + p_f}{p_g} = 1 + \frac{p_f}{p_g} = 1 + \frac{\mu_0 A_f}{\mu_0 A_c} \frac{l_f}{l_g}.$$ \hspace{1cm} (4.5.7)

Consider a magnetic core with a rectangular cross section of dimensions $a$ and $b$. The core area, $A_c$, is

$$A_c = ab. \quad (4.5.8)$$

The core perimeter is

$$P_c = 2a + 2b. \quad (4.5.9)$$

The cross-sectional area of the fringing flux, referring to Fig. 4.5.2, is

$$A_f = (a + 2w_f)(b + 2w_f) - ab.$$  

$$A_f = ab + 2aw_f + 2bw_f + 4w_f^2 - ab$$

$$A_f = (2a + 2b)w_f + 4w_f^2$$

$$A_f = P_c w_f + 4w_f^2 \quad (4.5.10).$$
Plugging (4.5.11) into (4.5.8),

\[ F_f = 1 + \frac{A_f l_g}{A_c l_f} = 1 + \left(\frac{P_c w_f + 4w_f^2}{A_c}ight) \left(\frac{l_g}{l_f}\right) . \]

\[ F_f = 1 + \left(\frac{P_c w_f + 4w_f^2}{A_c l_f}\right) l_g \]

\[ F_f = 1 + \left[\left(\frac{P_c}{A_c}\right) \left(\frac{w_f}{l_f}\right) + \left(\frac{4}{A_c}\right) \left(\frac{w_f^2}{l_f}\right)\right] l_g \]  \hspace{1cm} (4.5.11)

From this expression, we have two known terms, \(\frac{P_c}{A_c}\) and \(\frac{4}{A_c}\), and two unknown terms, \(\frac{w_f}{l_f}\) and \(\frac{w_f^2}{l_f}\).

Plugging (4.5.1) into (4.5.3), we get an expression for \(L\), the initial inductance not considering fringing flux (in fact, the standard textbook expression for an air-cored inductor first introduced in Eqn. 3.5.4).

\[ L = P_g N^2 = \frac{\mu_0 A_c N^2}{l_g} \]  \hspace{1cm} (4.5.12)

Plugging (4.5.11) and (4.5.12) into (4.5.6), we get an expression for inductance including fringing flux, \(F_f\):

\[ L_f = LF_f \]

\[ L_f = L \left(1 + \left[\left(\frac{P_c}{A_c}\right) \left(\frac{w_f}{l_f}\right) + \left(\frac{4}{A_c}\right) \left(\frac{w_f^2}{l_f}\right)\right] l_g \right) \]

\[ L_f = L + \frac{\mu_0 A_c N^2}{l_g} \left[\left(\frac{P_c}{A_c}\right) \left(\frac{w_f}{l_f}\right) + \left(\frac{4}{A_c}\right) \left(\frac{w_f^2}{l_f}\right)\right] l_g \]

\[ L_f = L + \mu_0 N^2 \left[ P_c \left(\frac{w_f}{l_f}\right) + 4 \left(\frac{w_f^2}{l_f}\right)\right] \]

\[ L_f = L + \mu_0 N^2 \left[ P_c \left(\frac{w_f}{l_f}\right) + 4 \left(\frac{w_f^2}{l_f}\right)\right] \]  \hspace{1cm} (4.5.13)
Thus, total inductance for a given gap length is shown to consist of the original inductance not including fringing flux $L$ plus an additional inductance that is no longer in terms of $L_0$; a very surprising result.

A regression of experimental inductance measurement data from Sect. 4.4 was performed [26] and it was determined that the increase in inductance with increasing gap length is essentially linear, as predicted; in fact, a t-statistic hypothesis test shows that the quadratic term of the expression is insignificant within a 99% confidence interval. Refer to Fig. 4.5.3.

![Graph](image)

**Fig. 4.5.3:** Regression of experimental inductance measurement data.
Therefore, we may discard the quadratic term and the total inductance is

\[ L_f = L + \mu_0 N^2 P_c . \quad (4.5.14) \]

With only one set of experimental data, it is impossible to determine what correlation, if any, the theoretical parameters \( w_i \) and \( l_f \) may have to air gap length independently. As a combined quantity, however, \( w_i/l_f \) is shown to have a constant value of 1.13 across the range of measurement with respect to the linearized model.

Define a new term \( L_\delta \), which is called the \textit{constant excess inductance}. Likewise, define a new term \( \delta \) (lower-case delta), which is called the \textit{fringing coefficient}. Thus, the final inductance, including fringing flux, is

\[ F_f = L + L_\delta \quad (4.5.15) \]

where

\[ L_\delta = \mu_0 N^2 P_c \delta . \quad (4.5.16) \]

For the experimental inductor, where \( \delta = w_i/l_f = 1.13 \),

\[ L_\delta = \mu_0 N^2 P_c \delta = 4\pi \times 10^{-7} \times 66^2 \times 4.36 \times 10^{-2} \times 1.13 = 0.270 \text{ mH}. \]

Plotting the difference between measured and theoretical ideal inductance, it appears as a constant value of 0.270 mH: refer to figure 4.5.4.

The final expression for fringing flux factor, as applied to eqn. (4.5.6), is

\[ F_f = 1 + \frac{P_c \delta \delta_0}{\Lambda_c} . \quad (4.5.17) \]
Fig. 4.5.4 Difference between theoretical and measured inductance.

Thus, an expression for fringing flux factor has been derived that shows excellent correlation between experimental and theoretical inductance over the range of air gaps used for measurement. Traditional expressions for fringing flux show a logarithmic relationship between air gap length and inductance [27]; however, the author’s inductance measurement data does not support traditional models. It is hypothesized that the fringing coefficient, $\delta$, is a property of a given core material which may vary with composition, possibly in relation to other parameters such as relative permeability. More experiments with different materials will be necessary to support or reject this hypothesis, but from a physical standpoint it seems intuitively reasonable that a material’s crystalline and metallurgical
properties would influence the geometric behavior of the fringing flux. If different core materials can be statistically shown to have significant non-linearity in inductance with increasing gap length, then it is suggested that \( w_r \) and \( I_r \) may be used as separate fringing coefficients to achieve the desired system order. With regards to the core perimeter component \( P_c \) in the constant excess inductance term (Eqn. 4.5.16), it is intuitively reasonable that a core face with high aspect ratio will exhibit more fringing flux as the flux is positioned closer to the outside of the core; conversely, a core with the same cross-sectional area but square in proportion would have more flux located towards the interior, reducing the likelihood of fringing.

![Graph](image)

**Fig. 4.5.5: Measured inductance vs. new fringing inductance model.**
Measured inductance is shown in Fig. 5.4.5 to be low at very small gap lengths in comparison to the final theoretical fringing inductance. It is speculated that this is due to imperfections in the cutting and lapping of the core faces, as mentioned in Sect. 4.3, resulting in uneven spacing which would become significant at small gap lengths. Sensitivity to core alignment was shown to be significant in Sect. 4.4; therefore, it is possible that measurement error may largely account for the variation. Replication of these measurements for this and other core materials, and with these and other core dimensions, may form the basis of very significant future research.

4.6 Output Power of Prototype Current Transformer

The required load resistance for maximum power at 60 Hz using the initial measured inductance of 2.15 mH with 0.3-mm gap is (from 3.4.2)

$$ R_L = L_M \times \omega_0 = 2.15 \times 10^{-3} \times 120\pi = 811 \text{ mΩ}. $$

Compared to the parasitic winding resistance of the coil of 70 mΩ, copper loss will be considerable.

The theoretical maximum power with an arbitrary 8-A input is (from 3.4.12)

$$ P_{O(MAX)} = \frac{I_0^2 \mu_0 A_C \omega_0}{2l_g} = \frac{8^2 \times 4\pi \times 10^{-7} \times 1.225 \times 10^{-4} \times 120\pi}{2 \times 0.3 \times 10^{-3}} = 6.190 \text{ mW}. $$

The corresponding total voltage at this output power, not considering the voltage drop across the winding resistance, is

$$ V_O = \sqrt{P_O R_L} = \sqrt{6.190 \times 10^{-3} \times 811 \times 10^{-3}} = 70.85 \text{ mV}_{RMS}. $$
This voltage will be reduced by the winding resistance to

\[ V_{L(\text{WITH COPPER LOSS})} = V_o \frac{R_L}{R_L + r_s} = 70.85 \frac{811}{811 + 70} = 65.22 \text{ mV}_{\text{RMS}}. \]

This results in a power output, including copper loss, of

\[ P_o = \frac{V_L^2}{R_L} = \frac{(65.22 \times 10^{-3})^2}{811 \times 10^{-3}} = 5.245 \text{ mW}. \]

Note that there is an error introduced into this computation by the change in effective load resistance seen by the inductive circuit, which will shift the corner frequency and change theoretical maximum output power. This effect will be shown to be minimal later in this section.

The core was interfaced with the test apparatus detailed in section 4.2 carrying 8 A at 60 Hz and an accurate variable load resistance (General Radio 1432-N) connected in parallel. Voltage was monitored with an Agilent Technologies DSO5012A oscilloscope and measured with a Wavetek 320B digital voltmeter. Voltage into various loads spaced approximately ½-octave apart is given in Fig. 4.6.1.

<table>
<thead>
<tr>
<th>( R_L )</th>
<th>( V_{L(\text{RMS})} )</th>
<th>( P_L = \frac{V_L^2}{R_L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30 Ω</td>
<td>30 mV</td>
<td>3.0 mW</td>
</tr>
<tr>
<td>0.40 Ω</td>
<td>37 mV</td>
<td>3.4 mW</td>
</tr>
<tr>
<td>0.60 Ω</td>
<td>48 mV</td>
<td>3.8 mW</td>
</tr>
<tr>
<td><strong>0.80 Ω</strong></td>
<td><strong>56 mV</strong></td>
<td><strong>3.9 mW</strong></td>
</tr>
<tr>
<td>1.1 Ω</td>
<td>65 mV</td>
<td>3.8 mW</td>
</tr>
<tr>
<td>1.6 Ω</td>
<td>72 mV</td>
<td>3.2 mW</td>
</tr>
<tr>
<td>2.4 Ω</td>
<td>78 mV</td>
<td>2.6 mW</td>
</tr>
</tbody>
</table>

*Fig. 4.6.1: Output power vs. load resistance.*
Maximum power was reached when the load resistance was 0.80 Ω, close to the theoretical value of 811 mΩ. Of note is that sensitivity to load resistance is not particularly great; a mismatch of a factor of 2 in either direction reduced power by less than 1 dB. Actual maximum power output was 3.9 mW, some 1.3 dB less than that calculated, even with copper losses taken into consideration. Possible reasons for this additional loss are very likely the following two sources: eddy current losses due to imperfect core laminations and the finite number thereof, quantified by the shunt core resistance parameter $r_{cl}$ and imperfect coupling between core and line and thus effectively between primary and secondary, resulting in leakage inductance $L_\ell$, both parameters used in the full model (refer to figure 3.3.1). Inter-winding shunt capacitance is not a likely source of loss at 60 Hz.

4.7 Comparison of Theoretical and Experimental Performance

As shown in Section 4.4, measured inductance was shown to be in close agreement with theoretical inductance for a nominal gap of 0.3 mm. Output power and inductance over a range of air gaps show significant deviation from theory. Fringing flux was shown in Section 4.5 to account very largely for variation in inductance. Excess deviation in power was suggested in Section 4.6 to be caused by higher-order components of the current transformer model. For this reason, it is suggested by the author that higher-order current transformer model components, particularly leakage...
inductance $L_i$ and shunt core resistance $r_C$, should be investigated and this may form the basis of future research. Additional comments pertaining to the practical implementation of a power-scavenging current transformer are given in Sect. 7.2.
5. Coupling-Capacitor Power Scavenging Theory

5.1 Overview of Capacitive Coupling

Contactless capacitive coupling to overhead power distribution lines has not been found in any literature.

Consider an overhead power line cable of the type used to distribute power. A typical cable of this type consists of several aluminum conductors in a twist (stranded) pattern. As a first-order approximation of the capacitance that may be expected from a very basic coupling of a secondary conductor to the line through a dielectric, consider the placement of a semi-cylindrical conductor adjacent to the power line in partial concentricity. Assume that the primary conductor may be considered essentially cylindrical. Therefore, the two conductors have constant spacing and may be taken to be parallel plates separated by air, the inner “plate” being the section of power line and the outer being the secondary conductor. Refer to Fig. 5.1.1.

Fig. 5.1.1 Basic arrangement of capacitive coupling.
5.2 Determination of Capacitance

The effective area of the inner conductor of a semi-concentric pair of
cylindroids is that of a half-cylinder of radius $r$ and length $l$, where $l$ is the
length of the coupled plates. Therefore the area is

$$A = \frac{1}{2}(2r\pi l) = \pi rl .$$

Assume that the outer plate is sliced circumferentially such that its effective
plate area is the same as that of the inner plate. The spacing between conductors is the thickness of the dielectric material, given as $d$. The
capacitance of a parallel-plate capacitor used for line-coupling is denoted as
$C_C$ and is

$$C_C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

where $\varepsilon_0$ is the permittivity of free space and $\varepsilon_r$ is the relative permittivity of
the dielectric. Substituting 5.2.1 into 5.2.2,

$$C_C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon_0 \varepsilon_r \pi rl}{d} .$$

5.3 Mechanical Interface of Secondary Conductor to Line

As capacitive power scavenging has not been found in any literature by
the author at the time of this writing, the form factor of a high-power
capacitive coupling device is strictly theoretical in utility. Consider a semi-
concentric outer conductor placed adjacent to a power line such that
capacitive coupling takes place between the conductors. In order for this
coupling to take place, it is necessary that a voltage difference exist between conductors. Therefore, the secondary conductor must be at a different potential than the power line. In the case of a balanced three-phase floating system in which the power line in question is one of the phases and there is no explicitly defined neutral line, the issue becomes one of interfacing between two of the three lines with an effective potential difference of $\sqrt{3} \times V_{LINE}$, the mechanical issues of which are beyond the scope of this dissertation.

Conversely, the neutral line of a conventional single- or polyphase distribution system is referenced to Earth ground. Therefore, some means of achieving electrical conductivity to Earth is required with an effective potential difference of simply $V_{LINE}$. The required impedance of this connection will be discussed in Section 5.6.

5.4 First-Order LF Model of Coupling Capacitor Circuit

If a power-coupling capacitor is placed in series with a load resistance such that AC current flows through the circuit, then the circuit may be modeled as an RC voltage divider whose upper impedance is the line-to-secondary-conductor coupling capacitance and the lower impedance is the load resistance, with the line voltage connected across the series circuit. Such capacitor parasitic components as ESR (equivalent series resistance), ESL (equivalent series inductance), leakage resistance, and frequency-
dependent non-linearities such as skin effect are initially disregarded for simplicity of analysis. Refer to Figure 5.4.1. The RMS line voltage $V_{LINE}$ is divided between the coupling capacitance $C_C$ and the load resistance $R_L$ by Kirchoff’s Voltage Law. The effect of the coupling capacitance is to cause an increasing proportion of line voltage to be dropped across it with decreasing frequency. From the perspective of the load resistance, this forms a high-pass filter (as in the first-order current-transformer model) whose corner frequency $\omega_L$ is defined as

$$\omega_L = \frac{1}{R_L C_C},$$

(5.4.2)

![Figure 5.4.1: First-order low-frequency model of coupling capacitor circuit.](image)
The operating frequency may be defined as $\omega_L$ and thus the impedance of the coupling capacitor at a given operating frequency is $\frac{1}{j\omega_L C_C}$. The current through the series circuit consisting of $C_C$ and $R_L$ may be determined by dividing the line voltage $V_{\text{LINE}}$ by the total series impedance $Z_{\text{series}}$. As this current may be taken to be the load current, we define it as $I_L$.

$$I_L = \frac{V_{\text{LINE}}}{Z_{\text{series}}} = \frac{V_{\text{LINE}}}{R_L + \frac{1}{j\omega_L C_C}} = \frac{V_{\text{LINE}}}{R_L} \left( \frac{1}{1 + \frac{1}{j\omega_L R_L C_C}} \right) \quad (5.4.3)$$

By rearranging and substituting eqn. (5.4.2) into (5.4.3), we may define the load voltage in terms of the corner frequency $\omega_L$.

$$I_L = \frac{V_{\text{LINE}}}{R_L} \left( \frac{1}{1 + \frac{1}{j\omega_L R_L C_C}} \right) = \frac{V_{\text{LINE}}}{R_L} \left( \frac{1}{1 + \frac{\omega_L}{\omega_0}} \right) = \frac{V_{\text{LINE}}}{R_L} \left( \frac{1}{1 + \frac{\omega_L}{\omega_0}} \right) \left( 1 + \frac{j\omega_L}{\omega_0} \right) \quad (5.4.4)$$

From (5.4.4),

$$I_L = \frac{V_{\text{LINE}}}{R_L} \left( \frac{1}{1 - j\frac{\omega_L}{\omega_0}} \right) \left( 1 + j\frac{\omega_L}{\omega_0} \right) = \frac{V_{\text{LINE}}}{R_L} \left[ 1 + j\frac{\omega_L}{\omega_0} \right] \quad (5.4.5)$$

### 5.5 Output power of Coupling Capacitor Circuit

Complex power $S$ may be defined as $S = I\bar{I}^* R$. Thus,

$$I_L I_L^* = \frac{V_{\text{LINE}}^2}{R_L^2} \left[ \frac{1 + \frac{\omega_L}{\omega_0}}{1 + \left( \frac{\omega_L}{\omega_0} \right)^2} \right] = \frac{V_{\text{LINE}}^2}{R_L} \left[ \frac{1 + \left( \frac{\omega_L}{\omega_0} \right)^2}{1 + \left( \frac{\omega_L}{\omega_0} \right)^2} \right] = \frac{V_{\text{LINE}}^2}{R_L} \left[ 1 + \left( \frac{\omega_L}{\omega_0} \right)^2 \right] \quad (5.5.1)$$

Output power may now be defined as

$$P_O = I_L I_L^* R_L = \frac{V_{\text{LINE}}^2 R_L}{R_L^2} \left[ \frac{1}{1 + \left( \frac{\omega_L}{\omega_0} \right)^2} \right] = \frac{V_{\text{LINE}}^2}{R_L} \left[ \frac{1}{1 + \left( \frac{\omega_L}{\omega_0} \right)^2} \right] \quad (5.5.2)$$
Note that this is the expression for a first-order high-pass filter with respect to power transfer into a resistive load in terms of $\omega_L$ and $\omega_0$, just as in the current transformer example.

The load resistance in terms of $\omega_L$ is (from 5.4.2)

$$\omega_L = \frac{1}{R_L C_C} \rightarrow R_L = \frac{1}{\omega_L C_C}. \quad (5.4.2a)$$

By substituting eqn. (5.4.2a) into eqn. (5.5.2), we may find the output power in terms of coupling capacitance $C_C$.

$$P_O = \frac{V_{\text{LINE}}^2}{R_L} \left( \frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2} \right) = \frac{V_{\text{LINE}}^2}{\omega_L C_C} \left[ \frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2} \right] = V_{\text{LINE}}^2 \omega_L C_C \left[ \frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2} \right]. \quad (5.5.3)$$

Note that this places the corner frequency $\omega_L$ into the power equation, as with the current transformer in section 3.5. Substituting the expression for parallel-plate capacitance (5.2.3) into (5.5.3), we get

$$P_O = V_{\text{LINE}}^2 \omega_L C_C \left[ \frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2} \right] = V_{\text{LINE}}^2 \omega_L \frac{\varepsilon_0 \varepsilon_r \pi r l}{d} \left[ \frac{1}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2} \right]. \quad (5.5.4)$$

If we group frequency terms together, the output power becomes

$$P_O = V_{\text{LINE}}^2 \frac{\varepsilon_0 \varepsilon_r \pi r l}{d} \left[ \frac{\omega_L}{1 + \left(\frac{\omega_L}{\omega_0}\right)^2} \right]. \quad (5.5.5)$$

The placement of the corner frequency term $\omega_L$ into the numerator has the effect of causing maximum power to be achieved at $\omega_L = \omega_0$. Refer to Fig. 5.5.1, which shows output power with respect to $\omega_L$, normalized by $\omega_0$ and $K = V_{\text{LINE}}^2 \frac{\varepsilon_0 \varepsilon_r \pi r l}{d}$. Thus, maximum power is transferred when the corner frequency of the circuit is placed at the operating frequency, exactly as the

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current transformer case in section 3.5. Substituting $\omega_L = \omega_0$ into eqn. (5.5.5) and solving for $\omega_0$, which is fixed in a given geographical area,

$$
P_{O(MAX.)} = V_{LINE}^2 \frac{\varepsilon_0 \varepsilon_r \pi r l}{d} \left[\frac{\omega_0}{1 + \left(\frac{\omega_0}{\omega_0}\right)^2}\right] = V_{LINE}^2 \frac{\varepsilon_0 \varepsilon_r \pi r l \omega_0}{2d}. \quad (5.5.6)
$$

![Graph showing normalized output power with corner frequency.](image)

**Fig. 5.5.1**: Normalized output power with corner frequency.

From 5.5.6, maximum power is shown to be in linear proportion to operating frequency for a given capacitive contact length, permittivity, and dielectric thickness. Intuitively, this makes sense because, much like the current transformer proposed for magnetic field coupling, lower frequencies
exhibit longer wavelengths and therefore require commensurately larger sensing devices. The dependence of power upon power line radius is of concern. Also shown is that, among parameters that are within our design control, it is desired to have high relative permittivity, small gap size, and long contact length. The former two may involve the science of dielectric materials, while the latter is most certainly mechanical in nature.

It is important to note that this derivation holds only for configurations in which the electric field geometry is that of a concentric parallel-plate capacitor: for different secondary conductor shapes and/or and increasing distances from the cylindrical line, the field geometry may become much more complex and difficult to model. This is given further consideration in Section 5.6.

### 5.6 Output Power Examples of Capacitive Coupling Circuit

For the first example detailed in this section, consider the construction of a secondary conductor semi-cylindrical in shape coupled semi-concentrically to a power distribution line 1 cm. in diameter at a potential of 4.16 kV. Assume its capacitance has been “matched” to the ideal load resistance for maximum power at the line frequency of 60 Hz (120π rad./s). Also assume that the cylinder is 20 cm in length, separated from the line by a PVC insulator ($\varepsilon_R \approx 4.5$) with a thickness of 0.15 mm; this value was chosen with respect to the dielectric strength of PVC ($\approx 40$ MV/m) at the maximum-
possible 4.16-kV line voltage \((\approx 5.89 \text{ kV}_{\text{PK}})\), recognizing that this strength may not be practically achievable but would only be necessary in the event of a fault such as a shorted load. Finally, assume the lower leg of the voltage divider has been connected to system neutral such that the full 4.16 kV is placed across the series circuit consisting of coupling capacitance and load resistance. The power is (from 5.5.6)

\[
P_{O(\text{MAX})} = V_{\text{LINE}}^2 \frac{\varepsilon_0 \varepsilon_r \pi r l \omega_0}{2d} \\
= (4.16 \times 10^3)^2 \frac{8.854 \times 10^{-12} \times 4.5 \times \pi \times 0.5 \times 10^{-2} \times 20 \times 10^{-2} \times 120\pi}{2 \times 0.15 \times 10^{-3}} = 2.722 \text{ W.}
\]

The capacitance of the coupling is (from 5.2.3)

\[
C_C = \frac{\varepsilon_0 \varepsilon_r \pi r l}{d} = \frac{8.854 \times 10^{-12} \times 4.5 \times \pi \times 0.5 \times 10^{-2} \times 20 \times 10^{-2}}{0.15 \times 10^{-3}} = 834.5 \text{ pF.}
\]

Maximum power occurs when \(\omega_0 = \omega_L = \frac{1}{R_L C_C}\). Therefore

\[
R_L(\text{MAX POWER}) = \frac{1}{\omega_0 C_C} = \frac{1}{\frac{1}{120\pi \times 834.5 \times 10^{-12}}} = 3.179 \text{ M\Omega.}
\]

The load voltage, \(V_L\), is

\[
V_L = \sqrt{P_O R_L} = \sqrt{2.722 \times 3.179 \times 10^6} = 2.942 \text{ kV.}
\]

Note that this voltage is \(-3 \text{ dB}\) with respect to the original line voltage of 4.16 kV, indicating that the operating frequency is at the corner frequency of the system.

The load current, \(I_L\), is

\[
I_L = \frac{P_O}{V_L} = \frac{2.722}{2.942 \times 10^3} = 925.2 \mu\text{A.}
\]
In the second example, consider a semi-concentric conductor placed in proximity to a 34.5-kV distribution power line with a SiO₂ glass insulator \((\varepsilon_r \approx 7)\) having a thickness of 4 mm; this value was chosen with respect to the dielectric strength of glass \((\approx 13 \text{ MV/m})\) and recognizing that the peak line voltage is nearly 50 kV. It is recognized that as the dielectric strength of air is only \(\approx 3 \text{ MV/m}\), this will need to be considered, especially if a craft is involved; but please bear with the author for the sake of comparison. It is also recognized that at this voltage, distribution line radius is likely to be larger than the 0.5 cm found with the lower-voltage line.

\[
P_{O(MAX.)} = V_{LINE}^2 \frac{\varepsilon_r \varepsilon_0 \pi r l \omega_0}{2d} = (34.5 \times 10^3)^2 \frac{8.854 \times 10^{-12} \times 7 \times \pi \times 0.5 \times 10^{-2} \times 20 \times 10^{-2} \times 120 \pi}{2 \times 4 \times 10^{-3}}
\]

\[
= 10.92 \text{ W}.
\]

The capacitance of the coupling is (from 5.2.3)

\[
C_C = \frac{\varepsilon_0 \varepsilon_r \pi r l}{d} = \frac{8.854 \times 10^{-12} \times 7 \times \pi \times 0.5 \times 10^{-2} \times 20 \times 10^{-2}}{4 \times 10^{-3}} = 48.68 \text{ pF}.
\]

Maximum power occurs when \(\omega_0 = \omega_L = \frac{1}{R_L C_C}\). Therefore,

\[
R_L(MAX.POWER) = \frac{1}{\omega_0 C_C} = \frac{1}{120 \pi \times 48.68 \times 10^{-12}} = 54.49 \text{ M}\Omega.
\]

The load voltage, \(V_L\), is

\[
V_L = \sqrt{P_O R_L} = \sqrt{10.92 \times 54.49 \times 10^6} = 24.39 \text{ kV}.
\]

Note that this voltage is \(-3 \text{ dB}\) with respect to the original line voltage of 34.5 kV.

The load current, \(I_L\), is

\[
I_L = \frac{V_L}{R_L} = \frac{24.39 \times 10^3}{54.49 \times 10^6} = 447.6 \mu A.
\]
5.7 Implications of Theoretical Results

Equation 5.5.6 shows that output power from an overhead line coupling capacitor of a semi-concentric, semi-cylindrical configuration is dependent on two geometric parameters within limited control for a given line radius: the length of the coupling and the spacing between conductors. The former will, obviously, be dictated by the limits of size and weight imposed on the design for a given application, while the latter will likely be constrained by the precision of the geometry of the conductors as well as the line voltage itself, as the possibility of dielectric breakdown and arcing must certainly be considered when working with power lines. One parameter that will be given consideration in Chapter 6 is the effect of alternative dielectric materials interposed between conductors, the increased permittivity of which will be shown to be beneficial as permittivity directly influences output power, as shown in equation 5.5.6.

Mentioned in Section 5.3 is the necessity of referencing the load to system neutral if a single contactless line coupling is the desired objective. As many overhead distribution line systems are referenced to earth ground, this is a viable option for achieving this connectivity. In both examples of capacitive coupling mechanisms, the required load resistance was on the order of megohms; therefore, one may expect that the requirements for connectivity are not particularly stringent and may be accomplished by, for example, dropping a weighted spike onto the ground from a tether which
could then be retracted after use [28]. It is speculated by the author that atmospheric and ground conditions may only marginally affect the integrity of the load connection should a capacitive power scavenging device be implemented in this manner.

The very useful output power of 10.92 W theoretically achievable from the 34.5 kV line voltage with a cheap glass insulator is of particular interest. The difference in line voltage between 4.16 kV and 34.5 kV constitutes a dynamic range of some 18 dB. At these voltages, this range is of particular concern and will be dealt with in Chapter 7.
6. Experimental Power-Sca|veng|ing Coupl|ing Capac|tor

6.1 Introduction

Experiments have been conducted in the interest of qualifying theoretical derivations regarding capacitive power scavenging, at least to the extent that high-voltage overhead power lines may be simulated in a university laboratory setting. The final line voltage of 12.47 kV found in many commercial and residential areas was chosen as the highest practical voltage that may be tolerated in the lab; and even this voltage rather frightened the author. Although distribution line voltages up to 34.5 kV may be encountered in service, it was deemed impractical to explore the high end of this range in a standard university laboratory not equipped for this kind of work.

6.2 Test Apparatus for Measuring Coupling Capacitor Output Power

A 12-kV, 30-mA power transformer of the type used to power neon advertisement signs was procured by the author in surplus and used for all capacitive power-scavenging experiments. The same overhead distribution line cable used in the current-transformer experiments was used here, composed of stranded aluminum alloy with an approximate overall diameter of 1.0 cm, verified with a dial caliper. A 38-cm section of power line was connected to one terminal of the 12-kV power transformer secondary winding by directly bolting one strand of the cable to the ¼”-20 mounting stud. The
free end of the cable was lashed to the power transformer’s chassis with PVC electrical tape to reduce the strain on the connected terminal.

### 6.3 Capacitance of Prototype Coupling Capacitor

A prototype coupling capacitor was constructed by placing a 23-cm cylindrical PVC tube with an inside diameter of 1.2 cm around the line and a semi-cylindrical 15-cm section of galvanized steel tubing in close-fitting concentricity. The reason for a cylindrical insulator rather than semi-cylindrical was mainly convenience; it also served as a means of ensuring against the possibility of arcing through the air gap created at the edge of the semi-concentric pair; this issue is further explored in Chapter 7. The same rationale applies to the extension of the dielectric tube beyond the length of the secondary conductor, similar in concept to the construction of tubular plastic film capacitors. The wall thickness of the PVC tube is 1.8 mm. The theoretical capacitance of this configuration, temporarily ignoring the distributed air gap caused by the difference in diameter between the line and tube as well as the twist pattern of the stranded construction, is (from 5.2.3)

\[
C_C = \frac{\varepsilon_0 \varepsilon_r \pi r l}{d} = \frac{8.854 \times 10^{-12} \times 4.5 \times \pi \times 0.5 \times 10^{-2} \times 15 \times 10^{-2}}{1.8 \times 10^{-3}} = 52.15 \text{ pF}.
\]

The measured capacitance, using a Tenma 72-8155, was 22 pF, compensating for the residual component of the meter and checking accuracy with a precision capacitor of similar value. This indicates that the air gap, which
effectively places a small capacitor in series with the theoretical one, is significant. Refer to Fig. 6.3.1.

![Diagram](image)

**Fig. 6.3.1: Geometry of air gap interposed between line and secondary.**

The capacitance of an equivalent air gap of 1 mm, representing the difference in radius between the cable and PVC tube and remembering that the dielectric is cylindrical in this case (and not semi-cylindrical), is

\[
C_{\text{AIR}} = \frac{\varepsilon_0 \varepsilon_r 2\pi rl}{d} = \frac{8.854 \times 10^{-12} \times 1 \times 2\pi \times 0.5 \times 10^{-2} \times 15 \times 10^{-2}}{1 \times 10^{-3}} = 41.72 \ \text{pF}.
\]

The two capacitors in series have a theoretical equivalent capacitance of

\[
C_{\text{EQ.}} = \frac{C_C C_{\text{AIR}}}{C_C + C_{\text{AIR}}} = \frac{41.72 \times 52.15}{41.72 + 52.15} = 23.18 \ \text{pF}.
\]

This is sufficiently close to the measured value of 22 pF.
6.4 Output Power of Prototype Coupling Capacitor

Maximum output power at 60 Hz from the measured coupling capacitance of 22 pF is theoretically attained with a load resistance of (from 5.4.2a)

\[ R_{L(MAX\_POWER)} = \frac{1}{\omega_0 C_c} = \frac{1}{120\pi \times 22 \times 10^{-12}} = 120.6 \text{ M}\Omega. \]

A composite 120-MΩ load resistor consisting of twelve 10-megohm 1-W carbon composition resistors was connected between the secondary conductor described in Section 6.3 and the second terminal of the 12-kV power transformer secondary winding. The effective measured composite resistance, determined by measuring each resistor and then summing the resistances, was 122.1 MΩ. As no available soldering iron had sufficient thermal mass to heat the galvanized steel secondary conductor directly, a tinned solder lug was physically screwed into the pipe to form a mechanical joint with excellent electrical conductivity. A 1-kΩ precision resistor was inserted in series with the load resistance to serve as a current-sensing resistor, across which a millivoltmeter was connected via a pair of cables constructed from 40-kV test-probe wire. The center tap of the power transformer is internally connected to its chassis, which is Earth-grounded at the 120-VAC input in order to conform to IEC Class-I wiring specifications for safety; thus each terminal of the transformer secondary is \( \approx 6 \text{ kV}_{\text{RMS}} \) with respect to ground, necessitating non-conductive work surfaces and careful
attention paid to insulating test equipment from all impedances to ground (such as tile/concrete flooring). Refer to Figure 6.4.1.

![Prototype coupling capacitor with load under test.](image)

**Figure 6.4.1: Prototype coupling capacitor with load under test.**

The power transformer used in the experiment was rated for 12 kV at 30 mA nominal load current. Because the expected load current with the prototype coupling capacitor is only on the order of a hundred microamperes or so, it can be expected that the transformer secondary voltage will be somewhat higher than anticipated, necessitating calibration; this is easily accomplished using the variable autotransformer connected to the transformer's primary. To avoid the potential problems (pun intended) of a direct voltage measurement of this magnitude, the author temporarily placed the 122.1-MΩ load resistor directly across the secondary winding and
adjusted the autotransformer for a voltage reading across the current-sense resistor of 102.1 mV; this sets the calibration load current to 102.1 μA and thus the secondary voltage to 12.47 kV. The load was then reconnected to its normal position. As the actual load current will be somewhat less than 102.1 μA, the secondary voltage will still rise very slightly above 12.47 kV; but the effect is likely to be insignificant compared to the potentially large error of the virtually unloaded secondary with full line voltage applied. DC resistance of the power transformer’s secondary winding was measured to be on the order of 10 kΩ, validating this assumption as a 30-mA load would normally drop some 300 V.

The measured load current with the application of full line voltage was measured to be 79.3 μA (79.3 mV across the precision 1-kΩ current-sensing resistor) for a load voltage of

\[ V_L = I_L R_L = 79.3 \times 10^{-6} \times 122.1 \times 10^6 = 9.681 \text{ kV}. \]

This is some 0.8 dB higher than the 8.818 kV we would expect at the corner frequency with a 12.47-kV source, possibly due to excess leakage currents caused by electron tunneling through an impure dielectric; empirical rationale for this speculation is audible high-frequency “hissing” emanating from the energized device and the smell of ozone.
Theoretical power is

\[ P_{L\text{(THEORETICAL)}} = \frac{V_L^2}{R_L} = \frac{8818^2}{122.1 \times 10^6} = 0.6368 \text{ W}. \]

Experimental power is

\[ P_{L\text{(EXPERIMENTAL)}} = \frac{V_L^2}{R_L} = \frac{9681^2}{122.1 \times 10^6} = 0.7676 \text{ W} \]

which is 0.8 dB in excess, as noted. For the sake of comparison, theoretical maximum power of the coupling device with no air gap between the line and dielectric is (from 5.5.6)

\[ P_{O\text{(MAX)}} = \frac{V_{LINE}^2 \varepsilon_0 \varepsilon_r \pi rl \omega_0}{2d} \]

\[ = (12.47 \times 10^3)^2 \frac{8.854 \times 10^{-12} \times 4.5 \times 10^{-2} \times 0.5 \times 15 \times 10^{-2} \times 120 \pi}{2 \times 1.8 \times 10^{-3}} = 1.528 \text{ W}. \]

We may conclude that the air gap has effectively reduced maximum output power by some 3-4 dB.

6.5 Comparison of Theoretical and Experimental Performance

As shown in Section 6.4, there is considerable agreement between theoretical and measured capacitance of the prototype coupling capacitor once the air gap between the line and dielectric is taken into consideration. Furthermore, there is considerable agreement between theoretical power and actual power when the air gap is considered; discrepancy between theoretical and measured results was shown to be less than 1 dB. In order to maximize the amount of available power output, it will be necessary to accomplish any or as many of the following objectives as possible, within the limits of
practicality: a) increase the permittivity of the dielectric; b) decrease the
spacing between the conductors; c) increase the length of the secondary
conductor; and d) eliminate potential air gaps by fitting the dielectric to the
line as closely as possible. The latter of these objectives may be extended to
the interface between the secondary conductor and the dielectric: as the fit
was quite close in the prototype coupling capacitor, it is assumed that it was
justifiably taken for granted.
7. Design of a Complete Energy-Scavenging System

7.1 Introduction

This chapter involves the design of a complete energy-scavenging system to be used as a power supply whose main intended purpose is to power electronics or charge batteries in small portable devices such as unmanned aerial vehicles (UAV’s). A justification of the power-scavenging method employed, total system architecture overview, design of the raw power supply and required interface circuitry, and connection to the load and its associated power management circuitry are explained in detail. Particular emphasis is given to reasonable maximization of performance of the interface circuit with recommendations for further areas of research.

7.2 Justification of Power-Scavenging Method

The current-transformer method of power scavenging was theoretically investigated in Chapter 3, with an experimental unit built and tested in Chapter 4. Mechanical issues were shown to be of significant importance; of these, the dependence of output power on the length of the air gap of the two-piece transformer core was of considerable concern. Fringing flux was shown in Section 4.4 to somewhat temper the reduced inductance of increasing air gap widths; however, at very small gaps, mechanical alignment of the two core halves with respect to azimuth and zenith proved very sensitive in practice and made achieving a consistently high inductance problematic.
Furthermore, as noted by Dr. Fred Garber rhetorically [29], the cost and weight of iron-cored transformers and their associated windings in general make them particularly undesirable for aircraft use; and as noted in Chapter 3, increased power may only be attained with increased core size. Finally, the variability in line current of practical overhead power lines causes extreme uncertainty of attainable output power; at very low practical line currents, power was shown in Section 3.6 to be on the order of tens of milliwatts, hardly sufficient for battery charging of the desired order of magnitude.

The coupling-capacitor method of power scavenging was investigated in Chapter 5 and a practical unit constructed and tested in Chapter 6. Coupling-capacitor power scavenging does not particularly suffer from any of the key disadvantages associated with current-transformer power scavenging, but there are still inherent problems with this technology. Primary advantages over current-transformer power scavenging are threefold: electrostatic materials are typically inexpensive and lightweight, in contrast to ferromagnetic materials, especially those of high permeability and laminated construction; the gap between the power line and dielectric, while to be minimized, is manageable and does not require precise (read: expensive) manufacture; and as power output is dependent upon line voltage, which is carefully regulated by the power utility company, quite repeatable results may be attained once the line voltage is known, which may then be
documented for a given geographical location. Thus, between the two reactive power sensing mechanisms, capacitive coupling certainly shows much more promise. Its inherent disadvantages include power dependence on three parameters: line radius, permittivity of the material and its associated dielectric strength, and contact length, which still relates to size and therefore mass, despite the advantageous material properties. A crippling disadvantage is the necessity of a ground connection for the required potential difference.

For both electromagnetic and electrostatic power scavenging, there is uncertainty in the magnitude of the signal available for converting into useful power. As shown in Chapters 3 and 5, even if performance is fully optimized in terms of loading and mechanical coupling, the maximum amplitude of attainable current and voltage, respectively, is always at best 3 dB less than the original amplitude. Much more importantly, practical considerations result in a serious limitation of available power in either case; a transducer of given size operating at a given frequency is only capable of finite power transfer. Conversely, direct sampling of voltage or current may yield virtually unlimited power. Although capacitive power scavenging is decidedly superior to current transformer power scavenging in many respects, the necessity of a direct connection to ground deems it no more desirable than a system requiring direct connection to the line. For these reasons, research into current-transformer and capacitive power scavenging
systems for overhead distribution power lines has been discontinued for the remainder of the dissertation. Direct voltage sampling is the method employed in this chapter for the proposed system.

With regard to future research into capacitive power scavenging, particularly the scavenging of power from high-voltage (HV) and extra-high-voltage (EHV) transmission and sub-transmission lines, the author envisions a capacitive system wherein the craft itself is made conductive and forms the secondary conductor for electrostatic coupling, the field intensity being controlled by the distance the craft maintains from the line. It is speculated by the author that the increasingly accurate and sophisticated navigational systems available to the field of small-craft aviation may be of considerable utility in controlling the degree of capacitive coupling and, thus, battery charging in flight.

7.3 Power-Scavenging System Architecture Overview

As explained in Chapter 2, overhead power distribution line voltages vary domestically in magnitude, typically from 4.16 kV to 34.5 kV. This represents some 18 dB of dynamic range. More importantly, these voltages must be converted into a signal appropriate for charging a battery or powering a portable electronic device; therefore the overall system must incorporate a raw power supply and interface circuit that converts high voltage/low current 60-Hz AC into low voltage/high current DC. In deference
to both of the intended applications, the author has chosen to design a regulated 12-VDC power supply which may power many mobile electronic devices directly, or may be interfaced with a battery-charging power-management integrated circuit of the type used in conjunction with one- or two-cell Lithium-Ion (Li-ion) or Lithium-Polymer (Li-poly) batteries, for which this voltage is nearly ideal [30]. For other output voltages, such as the ever-decreasing power supply rail voltages found in CMOS RF and computing devices (possibly lower than 3.3 V), it is desired that the system be easily modified without changing quality of performance. The fundamental block diagram of the energy scavenging system as applied to a universal power supply is shown in Fig. 7.3.1.

![Block diagram of practical energy-scavenging system.](image)

The fundamental blocks of the power supply therefore consist of the following: the overhead distribution power line itself, whose voltage varies between 4.16 and 34.5 kV; the means of power scavenging to be employed, including necessary line or ground connectivity as required; a raw power supply which scales the range of voltages into one appropriate for further processing, as well as converting them into DC; an interface circuit which
performs the task of processing the raw DC into a stable, repeatable, regulated 12 V; and finally the load, a portable electronic device or battery management integrated circuit.

### 7.4 Power Line Interface

As discussed in Chapter 2 (and again in Chapter 5 as the subject relates to capacitive power scavenging), a voltage difference is required in order to scavenge power. This may be accomplished one of three ways in direct voltage sampling: line-to-ground, in which an electronic device or craft is physically placed near and connected to the line and the required voltage difference created by a flying tether to earth ground; ground-to-line, in which a device is placed on and connected to ground and the voltage difference created by a flying tether to an overhead line; and line-to-line, in which a device is connected to a line and a flying tether connects to an adjacent line, which may be neutral or another phase, the latter of which creates a voltage difference of \( \sqrt{3} \ V_{\text{LINE}} \) in the case of three-phase systems, as discussed in Section 5.3. The three attachment methods are illustrated in Figs. 7.4.1 – 7.4.3.
Fig. 7.4.1 – Line-to-ground direct voltage sampling from overhead line.

Fig. 7.4.2 – Ground-to-line direct voltage sampling from overhead line.
As stated in Section 1.4, the ballistic issues associated with line attachment are outside the scope of this dissertation but should be the basis of future research.

7.5 Raw Power Supply Considerations

The task of converting a 60-Hz overhead distribution power line operating at tens of thousands of volts into regulated 12 V is not a trivial one. Following the physical achievement of electrical connectivity to the AC line voltage signal, a raw power supply is necessary to step down the range of voltages and convert them into DC for further processing.

One method of accomplishing this conversion, brute-force in philosophy, would be to use a linear power supply with a tapped step-down power transformer, rectifier, and smoothing capacitor to perform the required
conversion. Taps on the power transformer primary may be used to select an expected line voltage, thus maintaining a fairly constant input to the rectifier circuit. The advantage to this type of system is that its design is time-tested and very well understood; furthermore, power transformers capable of handling tens of kilovolts on their primaries may be readily designed with the rest of the components off-the-shelf. Refer to Fig. 7.5.1.

![Diagram of power supply system]

**Fig. 7.5.1: Option number one: brute-force power supply.**

The use of an autotransformer to accomplish voltage step-down would have a number of advantages over the use of a traditional transformer with separate primary and secondary windings. We may use an autotransformer in this application because there is likely no need for DC isolation. First, an
autotransformer would be cheaper and easier to design than a standard transformer; a final tap near the end of the winding provides low-voltage AC output. Second, coupling between primary and secondary windings, both desirable mutual coupling and undesirable capacitive coupling, would be virtually a non-issue: minimization of leakage flux is now solely dependent on core material and design. Finally, an autotransformer is likely to be smaller and lighter for a given power level than a standard transformer due to a more compact core design and no separate secondary winding.

A more elegant solution in general to the problem of power conversion is the pulse-width-modulated switch-mode power supply, which, inherent in its use of a high switching frequency, may use much smaller (and lighter) reactive components than a power supply operating at 60 Hz [31]. Particularly advantageous to many switch-mode DC-DC converters is the elimination of a bulky and massive 60-Hz power transformer. In fact, with adequate control circuitry, it is often possible to integrate the raw power supply and regulator into one transformerless circuit. Switch-mode regulation is more efficient than linear regulation, largely because linear regulation is inherently wasteful in its operation: load current is drawn directly from the source and any voltage difference across the regulator causes $VI$ heating. Unfortunately, it is not practical to expect a transformerless switch-mode power supply to perform a DC-DC conversion directly with a transfer function on the order of 4,000 or more, as would be
the case with the application of the full rectified 34.5 kV line voltage to the input of such a converter attempting to deliver 12 V on the output. Even a transformer-coupled converter design, such as a flyback converter, would be inherently impractical in this application because the voltage stresses on the semiconductor devices are largely commensurate with the extremely high input voltage. Furthermore, even the raw power supply would be challenging to build: the need to rectify and smooth up to 34.5 kVAC directly would require very expensive components as the peak voltage is nearly 50 kV.

A compromise between the brute-force and purely switch-mode power supply solutions is the use of an autotransformer to convert the range of AC line voltages into a range of AC voltages with may be rectified, smoothed, and applied to the input of a transformerless switch-mode DC-DC converter. The switch-mode power supply converts the range of DC inputs into 12 VDC directly, completely circumventing the need to switch transformer taps for different line voltages, and with adequate control circuitry, may be designed to be efficiently self-regulated. The raw power supply may use conventional, inexpensive components. The 18-dB dynamic range, if presented in a reasonable order of magnitude, is easily handled by a correctly-designed switch-mode power supply. Refer to Fig. 7.5.2.
Fig. 7.5.2 Option number two: hybrid power supply.

Taking this concept a step further for the sole purpose of battery charging, it would be possible with a little ingenuity to integrate the DC-DC converter into the power management I.C. itself, as the latter usually employs an internal switch-mode power supply operating at several hundred kilohertz. In fact, all that is needed is a power management I.C. design with sufficient input dynamic range to accept the 18-dB line variation directly. No such devices yet exist; the maximum rated input voltage found so far for a lead-acid battery management I.C. is 50 V, with most Li-ion and Li-poly designs limited to 28 V for 0.5 to 2 amperes charging current. This concept should certainly be the basis for future research.
7.6 Design of Buck PWM DC-DC Converter

A variety of pulse-width-modulated DC-DC power conversion schemes is available to convert a range of incoming DC voltages into a regulated DC output voltage. As the task of voltage stepdown is to be accomplished externally by an autotransformer, it is desired to use a transformerless converter design to avoid increasing the already considerable bulk. Intuitively, it seems more logical to split the duty of voltage stepdown between the transformer and the converter, rather than transforming the voltage in excess and boosting it back up; therefore, a buck converter will be employed to further reduce an 18-dB range of input voltages to the desired 12-V output.

A buck PWM converter of the type described by Kazimierczuk is shown in Fig. 7.6.1.

![PWM buck DC-DC converter diagram](image)

**Fig. 7.6.1:** PWM buck DC-DC converter.
$V_I$ is the input DC voltage, $S$ is the switching MOSFET across which is dropped $v_S$ and through which flows current $i_S$. $D_I$ is the switching diode with corresponding $v_D$ and $i_D$. $L$ is an inductor with $v_L$ and $i_L$. $C$ is a smoothing capacitor, and $R_L$ is the DC load.

For simplicity of design and analysis, assume that the converter will be held in continuous conduction mode (CCM). Rationale for this design decision is especially encouraged by the wide input voltage dynamic range and range of possible output currents. Initially assume an efficiency of 90%. Because of the 18-dB input dynamic range, the range of duty cycle $D$ applied to the gate of the switching MOSFET will also be on the order of 18 dB. As the duty cycle of a practical buck PWM converter cannot reach 0\% nor 100\%, set $D_{min} = 0.10$. Therefore,

$$M_{V_{DC_{min}}} = \eta \times D_{min} = 0.9 \times 0.10 = 0.090 \cdot$$ (7.6.1)

$$V_{I_{max}} = \frac{V_o}{M_{V_{DC_{min}}}} = \frac{12}{0.090} = 133.3 \text{ V} \quad (7.6.2)$$

Assuming this voltage will be supplied from a bridge rectifier and capacitor-input filter connected to the output of the initial stepdown autotransformer, the approximate required RMS AC voltage, neglecting the regulation of the raw filter but including the 1.4-V drop of the full-wave bridge rectifier utilizing silicon diodes, is

$$V_{I_{AC_{max}}} = \frac{V_{I_{max}} + 1.4}{\sqrt{2}} = \frac{133.3 + 1.4}{1.414} = 95.27 \text{ VAC} \cdot$$ (7.6.3)

The required stepdown ratio of the autotransformer is

$$N = \frac{V_{LINE_{max}}}{V_{I_{AC_{max}}}} = \frac{34.5 \times 10^{3}}{95.27} \approx 360 \cdot$$ (7.6.4)
At minimum line voltage, the input to the rectifier and filter is

\[ V_{IAC_{\text{min}}} = \frac{V_{\text{LINE}_{\text{min}}}}{N} = \frac{4.16 \times 10^3}{360} = 11.56 \text{ VAC}. \]  \hspace{1cm} (7.6.5)

The resulting minimum DC input to the converter is

\[ V_{I_{\text{min}}} = V_{IAC_{\text{min}}} \times \sqrt{2} + 1.4 = 11.56 \times 1.414 - 1.4 = 14.94 \text{ V}. \]  \hspace{1cm} (7.6.6)

The corresponding DC transfer function is

\[ M_{V_{\text{DC}_{\text{max}}}} = \frac{V_{O}}{V_{I_{\text{min}}}} = \frac{12}{14.94} = 0.803. \]  \hspace{1cm} (7.6.7)

Finally, the maximum duty cycle is

\[ D_{\text{max}} = \frac{M_{V_{\text{DC}_{\text{max}}}}}{\eta} = \frac{0.803}{0.9} = 0.892. \]  \hspace{1cm} (7.6.8)

Choose a switching frequency of 600 kHz. This frequency is arbitrarily chosen as that which may be found in many battery-management integrated circuits. Assume that we would like a maximum of 12 W available output power with a minimum of 1 W at idle. The range of load current is

\[ I_{L_{\text{min}}} = \frac{P_{0_{\text{min}}}}{V_{O}} = \frac{1}{12} = 83.3 \text{ mA}. \]  \hspace{1cm} (7.6.9)

\[ I_{L_{\text{max}}} = \frac{P_{0_{\text{max}}}}{V_{O}} = \frac{12}{12} = 1 \text{ A}. \]  \hspace{1cm} (7.6.10)

The corresponding effective load resistances are

\[ R_{L_{\text{min}}} = \frac{V_{O}}{I_{L_{\text{max}}}} = \frac{12}{1} = 12 \Omega \]  \hspace{1cm} (7.6.11)

and

\[ R_{L_{\text{max}}} = \frac{V_{O}}{I_{L_{\text{min}}}} = \frac{12}{0.833} = 144 \Omega. \]  \hspace{1cm} (7.6.12)

The minimum inductance required to keep the converter in CCM is

\[ L_{\text{min}} = \frac{R_{L_{\text{max}}}(1-D_{\text{min}})}{2f_{S}} = \frac{144(1-0.10)}{2 \times 600 \times 10^3} = 108 \mu\text{H}. \]  \hspace{1cm} (7.6.13)

Choose \( L = 120 \mu\text{H}. \) Specify a winding resistance no more than 70 mΩ.
The maximum inductor ripple current is

$$\Delta i_L \max = \frac{V_o(1-D_{\min})}{f_s L} = \frac{12(1-0.10)}{600 \times 10^3 \times 120 \times 10^{-6}} = 150 \text{ mA} \quad (7.6.14)$$

Suppose that we would like less than 0.5\% ripple in the output waveform.

The ripple voltage is

$$V_r = V_o \times 0.005 = 60 \text{ mV} \quad (7.6.15)$$

Assume that at this frequency, the output smoothing capacitor’s ESR will be the predominant impedance across which the ripple voltage will be developed. The maximum permissible ESR is

$$r_C \max = \frac{V_r}{\Delta i_L \max} = \frac{60}{150} = 400 \text{ m\Omega} \quad (7.6.16)$$

Assume $r_C = 300 \text{ m\Omega}$. The minimum value of filter capacitance that will ensure that most of the ripple is across its ESR is

$$C_{\min} = \max \left\{ \frac{D_{\max}}{2f_s r_C}, \frac{1-D_{\min}}{2f_s r_C} \right\} = \frac{1-D_{\min}}{2f_s r_C} = \frac{1-0.1}{2 \times 600 \times 10^3 \times 300 \times 10^{-3}} = 2.5 \mu\text{F} \quad (7.6.17)$$

Choose $C = 2.7 \mu\text{F} / 25 \text{ V}$ with an ESR of 300 m\text{\Omega} or better.

The corner frequency of the output low-pass filter is

$$f_0 = \frac{1}{2 \pi \sqrt{L C}} = \frac{1}{2 \pi \sqrt{120 \times 10^{-6} \times 2.7 \times 10^{-6}}} = 8.84 \text{ kHz} \quad (7.6.18)$$

The power MOSFET and diode voltage and current stresses are

$$V_{SM \max} = V_{DM \max} = V_{I \max} = 133.3 \text{ V} \quad (7.6.19)$$

$$I_{SM \max} = I_{DM \max} = I_{O \max} + \frac{\Delta i_L \max}{2} = 1 + \frac{150 \times 10^{-3}}{2} = 1.075 \text{ A} \quad (7.6.20)$$

As this is also the maximum current flowing through the inductor, it is necessary to design the inductor to avoid core saturation at this current level.
Choose IRF7465, a 150-V, 1.9-A n-channel SMD HEXFET. From the manufacturer’s data sheet [32], \( r_{DS} = 280 \, \text{m}\Omega \) and \( C_o = 76 \, \text{pF} \). The low output capacitance (and thus minimal switching loss at the high 600 kHz frequency) makes this component very attractive for this specific application. Also choose MBRS3201, a 200-V, 3-A fast, soft-recovery Schottky barrier diode with \( V_F = 0.4 \, \text{V} \) and \( R_F = 140 \, \text{m}\Omega \) [33].

Efficiency is calculated at \( D_{\min} \) and \( I_{L\,\max} \). Conduction and switching losses in the MOSFET are

\[
P_{r_{DS}} = D_{\min} r_{DS} I_{O\,\max}^2 = 0.10 \times 280 \times 10^{-3} \times 1^2 = 28 \, \text{mW} \quad (7.6.21)
\]

\[
P_{SW} = f_s C_o V_{i\,\max}^2 = 600 \times 10^3 \times 76 \times 10^{-12} \times 133.3^2 = 810 \, \text{mW}. \quad (7.6.22)
\]

Forward drop and dynamic resistance losses in the diode are

\[
P_{VF} = (1 - D_{\min}) V_F I_{O\,\max} = (1 - 0.10) \times 0.5 \times 1 = 450 \, \text{mW} \quad (7.6.23)
\]

\[
P_{RF} = (1 - D_{\min}) R_F I_{O\,\max}^2 = (1 - 0.10) \times 140 \times 10^{-3} \times 1^2 = 126 \, \text{mW}. \quad (7.6.24)
\]

Power loss in the inductor with a DC winding resistance of 70 m\( \Omega \) is

\[
P_{r_L} = r_L I_{O\,\max}^2 = 70 \times 10^{-3} \times 1^2 = 70 \, \text{mW}. \quad (7.6.25)
\]

Power loss in the capacitor with ESR of 300 m\( \Omega \) is

\[
P_{r_C} = \frac{r_C \Delta L_{\,\max}^2}{12} = \frac{300 \times 10^{-3} \times (150 \times 10^{-3})^2}{12} = 563 \, \mu\text{W}. \quad (7.6.26)
\]

Total power loss in the converter is

\[
P_{\text{loss}} = P_{r_{DS}} + P_{SW} + P_{VF} + P_{RF} + P_{r_L} + P_{r_C}.
\]

\[
P_{\text{loss}} = (28 + 810 + 450 + 126 + 70 + 0.563) \times 10^{-3} = 1.485 \, \text{W} \quad (7.6.27)
\]

Efficiency of the converter at full load is therefore

\[
\eta = \frac{P_o}{P_o + P_{\text{loss}}} = \frac{12}{12 + 1.485} = 89\%. \quad (7.6.28)
\]
This efficiency is sufficiently close to the original approximation of $\eta = 90\%$.

For the initial raw power supply design, assume that the degree of filtering is not critical and ripple may be made as high as 10% as the control circuitry for the buck PWM DC-DC converter will reject this to an acceptable level. As the PWM buck converter uses control circuitry to vary the duty cycle of the signal applied to the MOSFET switch - the very mechanism by which it may tolerate a wide dynamic range of input signals - correct design will enable it to track a 120-Hz ripple waveform of reasonable magnitude. This will keep the value of primary filter capacitance as small as possible, a desirable goal at 60 Hz.

As maximum current will be drawn from the raw power supply by the PWM buck converter at $V_{\text{min}} = 14.94$ V, maximum permissible ripple is calculated at this condition. With an efficiency of 89%, actual input power at full load is

$$P_{l\max} = P_O + P_{\text{loss}} = 12 + 1.485 = 13.49 \text{ W.} \quad (7.6.29)$$

Maximum input current is therefore

$$I_{l\max} = \frac{P_{l\max}}{V_{l\min}} = \frac{13.49}{14.94} = 0.90 \text{ A.} \quad (7.6.30)$$

For 10% ripple, the peak-to-peak voltage is

$$v_{r\,\text{RAW}} = 14.94 \times 0.10 = 1.494 \text{ V}_{\text{P-P}}. \quad (7.6.31)$$

The required value of filter capacitance is therefore [34]

$$C_{\text{RAW}} = \frac{I_{l\max}}{2f v_{r\,\text{RAW}}} = \frac{0.90}{2 \times 60 \times 1.494} = 5037 \mu\text{F}. \quad (7.6.32)$$
In deference to the $V_{\text{max}}$ of 133.3 V, choose a 5,600 µF, 160-V capacitor. This value is available as a relatively small snap-in aluminum electrolytic. For the four-diode bridge rectifier operating at 60 Hz, it is adequate to use garden-variety 1N4004 diodes with a PIV rating of 400 V.

For other output voltages besides 12 V, the range of input voltages will likely need to be re-scaled, possibly with a different output tap on the autotransformer. The duty cycle variation of the existing circuit, on the order of 10-90%, is close to the practical limit of a buck PWM DC-DC converter. If 3.3-V output is desired, for example, attempting to accomplish this output with the existing circuit simply with feedback control of the gate pulse width will require another 11 dB of dynamic range which is not likely to be achievable, particularly at high input voltages as the duty cycle would be very close to cutting off.
7.7 Simulation of Proposed Power Supply

The final design of a 12-V regulated power supply which may be interfaced with a power distribution line of 4.16 – 34.5 kV, excluding the necessary control signal for the MOSFET switch, is shown in Fig. 7.7.1.

![Diagram of 12-V regulated power supply driven from overhead line.](image)

**Fig. 7.7.1:** 12-V regulated power supply driven from overhead line.

The output transient response of the PWM buck converter was SPICE-simulated using DesignSoft TINA and associated models [35] at the maximum output current of one ampere and maximum input voltage of 133.3 V. Fig. 7.7.2 shows the output voltage waveform at turn-on. The duty cycle $D_{min}$ applied to the gate of the MOSFET was adjusted slightly from 0.10 to 0.083 for an output voltage of 12 V. At no time does the voltage exceed the 25-V rating of the output capacitor.
Similarly, the PWM buck converter was SPICE simulated at the minimum output current of 83.3 mA and minimum input voltage of 14.94 V. Fig. 7.7.3 shows the output voltage waveform at turn-on. $D_{max}$ was adjusted slightly from 0.892 to 0.78 for an output of 12 V. The waveform shows less damping than the one-ampere load current waveform; this is to be expected as the LC low-pass filter is less-damped by the higher effective load resistance. At no time does the output exceed the capacitor voltage rating, nor does it exceed the 28-V input limit of many power-management integrated circuits.
In a practical PWM buck DC-DC converter, the duty cycle is regulated by closed-loop control circuitry to ensure constant output voltage. Such control circuitry is outside the scope of this dissertation. However, the buck PWM DC-DC converter has been shown in simulation to have sufficient design capability to handle the expected input voltage and output current ranges with a duty cycle between 0.083 and 0.78, well within the practical limits of easily-designed control circuitry. The raw power supply is of such straightforward design that it has not been simulated due to the ease by which it may be modified in the event of unexpected operation in practice.
8. Conclusion

8.1 Summary of Dissertation

The subject of high-power energy-scavenging for portable devices has been explored in this dissertation in several key areas, particularly power scavenging mechanism technology, output power derivations, and power electronics.

The objective, scope, and limitations of this dissertation were stated in Chapter 1, with an emphasis on the current state of power-scavenging technology and proposed areas of study. Power-scavenging mechanisms were defined in Chapter 2, with a brief technical description of each of the four principal methods by which power may be sourced from overhead distribution lines, and an investigation into the voltages and currents likely to be encountered. The current-transformer method of power scavenging was theoretically discussed in Chapter 3 with an emphasis on correlation of physical dimensions and power transfer, and an experimental power-scavenging current-transformer was built and analyzed in Chapter 4.

Capacitive power scavenging was explored in theory in Chapter 5, again with an emphasis on correlation of physical dimensions and power transfer, with an experimental coupling capacitor constructed and analyzed in Chapter 6. Chapter 7 offered an analysis of the research into the two reactive power scavenging methods and justified the use of neither for the specific application described. A power supply based on direct-connection to a power
distribution line offered one possible solution to the powering of electronics and batteries in portable devices with suggestions for improvement.

8.2 Contribution to the Field of Electrical Engineering

The specific areas in which this dissertation may contribute to the field of electrical engineering are manifold. Rather than addressing one area of research or solving one particular problem, the intention has been to investigate the field of high-power energy scavenging as a discipline. As this area of research is relatively new (few publications exist on the topic prior to 2005), there is much unchartered territory and a comparatively small amount of useful information in current literature. The novel ways in which this dissertation may contribute to the field are as follows:

- The definition of principal power scavenging methods in Chapter 2, including brief mention of secondary mechanisms such as magnetostriction which have not yet been investigated but should form the basis of future research;
- The derivation of current-transformer output power for a given core size and gap length and the required loading for maximum power in Chapter 3, the results of which were later confirmed experimentally in Chapter 4 and could be of key interest as this technology is further commercialized;
• A new derivation for fringing flux factor in Chapter 4 which shows superior correlation between experimental and theoretical results than any expression for fringing flux known to date and is detailed in a paper entitled “Fringing Flux Factor in Rectangular High-Permeability Gapped-Core Inductors” (S. J. Tritschler, primary author, W. R. Earick and M. K. Kazimierczuk, secondary and tertiary authors, respectively), to be submitted June 2010;

• The introduction of the concept of capacitive power scavenging in Chapter 5, a completely new method of transferring power from overhead lines, including a derivation of output power for a given capacitor size and line voltage and its required loading, the experimental confirmation of such in Chapter 6, and the proposal of an extension of capacitive power scavenging to HV and EVH transmission lines in Chapter 7, all for which patents and/or other intellectual property rights will be investigated and a paper is proposed entitled “Capacitive Power Scavenging,” S. J. Tritschler, primary author; and

• The design of a power supply for application to portable devices in Chapter 7 that may handle the full 18-dB dynamic range of power distribution lines in the United States, for which a paper is proposed entitled “Universal 12-V Power Supply from Overhead Distribution Lines,” S. J. Tritschler, primary author.
8.3 Suggestions for Further Research

That this dissertation has covered so many topics and investigated so many possibilities for further research, refinement, and experimentation has been an exhilarating experience for the author. He would dearly like to be able to spend several years on each subject; but until the invention of the forty-hour workday, he must be content with proposing further areas of study. These include:

- Secondary power scavenging mechanisms;
- Theoretical work with current-transformer power scavenging, including closer examination of higher-order model components and their effects on output power in an effort to achieve better correlation between theoretical and experimental output;
- Much more research into capacitive power scavenging, particularly with HV and EHV transmission lines, including the dream of completely contactless, ground-less power transfer;
- Many more experiments into inductor core materials and gap lengths in the interest of better quantifying fringing flux factor, with the objective of being able to predict final inductance for a variety of core shapes, sizes, and materials; and
- The physical construction, testing, and refinement of an actual power supply prototype, particularly the critical range-scaling
autotransformer, and the elegant implementation of an integrated circuit for direct battery charging.
References


[29] Garber, Fred D. Comment made at research proposal defense. Wright State University, Dayton, Ohio, 25 September 2009.


