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Characterization and Improvement of a Cone-Beam CT Scanner for Quantitative Imaging

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Characterization and Improvement of a Cone-Beam CT Scanner for Quantitative Imaging

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

by

JIMISH JOSHI
B.E., University of Mumbai, 2007

2010
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Jimish Joshi ENTITLED Characterization and Improvement of a Cone-Beam CT Scanner for Quantitative Imaging BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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ABSTRACT

Joshi Jimish. M.S.Egr., Department of Biomedical Industrial and Human Factors Engineering, Wright State University, 2010. Characterization and Improvement of a Cone-Beam CT Scanner for Quantitative Imaging.

Computed tomography (CT) has various applications in different fields. In our case, the cone-beam CT scanner is used in the industrial field for qualitative and quantitative assessment of Procter & Gamble products. The Flash CT scanner, on which we do our research, has a microfocus x-ray tube, a detector size of 30 cm x 40 cm and a rotating stage for the object to be imaged. The x-ray tube can be operated up to 225 kV. Non-linearities in the response of the detector, scatter and beam hardening may cause false interpretations of the image values. To allow appropriate assessment of the images, the scanner needs to be characterized. We measured the linearity of the detector and the geometric resolution of the scanner. To reduce the effect of scatter, we designed a collimator. For the beam-hardening effect we propose a software solution to improve the accuracy of the CT values in the reconstructed images.

The linearity of the detector elements was tested by acquiring images with no objects in the beam at voltages between 50-150 kVp in intervals of 25 kVp and at a number of anode current settings. An R-squared test of detector reading versus anode current allowed the identification of bad pixels, and we found 5,660 pixels that were below the decided threshold (R-squared value of 0.98).
No correction was done for this, as the manufacturer’s software already provides a correction, and this was confirmed as non-linear pixels did not appear in the calibrated file format.

A forearm phantom made from Plexiglas and aluminum was reconstructed to perform modulation transfer function (MTF) measurements. Because a step phantom creates streaks in the reconstructed images, a cylindrical phantom (forearm phantom) is preferable for this purpose. The images contained five circular profiles, and the MTFs of these five profiles were measured by using an error spread function (ESF)-based fitting procedure, in which parameters are optimized using a non-linear least-squares method. The 10% MTF value was higher at a stage distance of 300 mm from the source (7.2 cycles/mm to 8.5 cycles/mm) than at a stage distance of 550 mm from the source (2.9 cycles/mm to 3.5 cycles/mm) and at a stage distance of 700 mm from the source (2.2 cycles/mm to 2.76 cycles/mm). To obtain a higher cut-off value for the 10% MTF, the stage position should be as close as possible to the source.

A step phantom made from Delrin and the forearm phantom made from Plexiglas and aluminum were used for scatter measurements in the projections and reconstruction images, respectively. Three stage positions (300 mm, 550 mm and 700 mm from the source) and three regions of the detector (region 1, region 2 and region 3) were used to conduct experiments for this part of our research. A precise collimator was designed to limit the cone beam to the detector size by calculating the source position and determining the necessary size of the cone beam. Data for both the step phantom in projection images and the forearm phantom in reconstructed images were compared with and without collimator. Scatter causes an increase in the photon counts at the detector, resulting in decreased projection values and decreased reconstructed image values. The collimator prevents scatter from structures outside the x-ray cone resulting in increased projection and image values. At a distance of 300 mm from the source, the rotating stage was completely outside of the collimated beam; therefore, scatter emanating from the rotating stage was not...
present for data collected at this stage position. Our results indicate that the effect of scatter is more prominent for stage positions closer to the detector.

The software beam-hardening correction was based on a fourth-order polynomial, representing measured projection value versus step-wedge thickness. As the beam-hardening correction needs to only correct for beam-hardening and not scatter, the scatter collimator was in place for the measurement of the step-phantom projection values. The correction applied to the forearm phantom image removed the cupping artifact (decreased reconstructed values near the center of the object) associated with the beam-hardening effect. As the beam-hardening correction is based on the step-phantom material Delrin, it will work only for materials close to the composition and density of Delrin and will over correct for objects with densities lower than that of Delrin and under correct for objects with densities higher than that of Delrin.

Characterizing and improving the CT scanner gives us a better understanding and knowledge of the scanner. It not only corrects for certain non-linearities but also helps us to design better experiments.
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Dedicated to

my parents and my sister
1. INTRODUCTION

1.1 Computed Tomography

Computed tomography (CT) is an imaging tool that reveals information about objects based on their density and atomic number. Unlike plain radiography, CT can provide images in all the planes and is also used for 3D rendering. X-rays penetrate the object and are then detected by single or two-dimensional detector arrays. Projections are formed based on the detector readouts. These projections contain information based on the thickness and density of the material, and a CT image is reconstructed from these projections.

1.1.1 Generations of CT Scanners

There are five generations of CT scanners (M.Paslawski 2007), (G.Michael 2001). The first-generation scanner uses a collimated pencil beam of x-rays, which is detected by a single detector on the other side of the object. The source and detector are translated in a straight line, after which they are rotated, and the process is repeated. Measurements are taken over 180° to generate a sufficient number of projections for reconstruction. The measurement time with such a system is about 5 minutes. The second-generation scanners use multiple detectors, measuring multiple projections in one translation. Measurement times with such systems are in the order of one minute. The third-generation scanners use only a rotating motion of the source and detectors around the object. The x-ray tube is collimated to a fan-shaped x-ray beam, which is large enough to cover the whole object cross-section and is detected by an arc-shaped row of detector elements. Scan times of this generation are in the order of seconds. The fourth-generation scanners use a
rotating source with a stationary detector of around a thousand or more detector elements forming a ring around the object. The fifth generation scanners contain a semi-circular arc of x-ray sources and stationary detectors; therefore, no mechanical motion is required, and very fast data acquisition is possible. The cost of the fourth and fifth generation CT scanner is very high due to the large number of detector elements and possibly multiple x-ray sources associated with them.

1.1.2 Applications of Computed Tomography

CT is widely used in medicine for such diverse diagnostic applications as finding blood clots in the brain, tumors, respiratory diseases, abdominal diseases, and bone fractures.

Industrial CT is used to test for cracks and defects. In the field of hydrology, CT is used to map the fluid distribution or to test fluid retention capability of a sample (e.g. coarse sand) (D. Wildenschilda 2002). In manufacturing, CT is used to study deformations of parts or to learn more about mechanical properties, structure and composition of various materials (E. Bayraktara 2008).

1.1.3 Differences between medical and industrial CT

In the medical field, the photon energies commonly used are between 30 and 150 keV, whereas in the industrial field they range from a few keV to several MeV. Radiation dose is an important issue, which is addressed by minimizing the intensity or number of photons generated in the medical field; there are no such concerns in the industrial field. The size of the object in medical CT is limited to that of human body parts; in industrial CT, the object size might vary from a few millimeters to meters. Industrial scanners often have a rotating stage with the x-ray source and detector in a fixed position, whereas in the medical field the gantry moves around the patient. The reconstructed image values are expressed in Hounsfield units in the medical field, but there is no
common scale in the industrial field. Image matrix sizes in the medical field are typically 512 x 512 pixels (rows x columns), whereas in the industrial field they range from 512 x 512 to 2048 x 2048 pixels or higher.

1.2 Industrial Computed Tomography

Industrial CT is used for non-destructive testing in various industries. The object size, shape and density may vary depending on the application. Therefore, scanner designs satisfy the special needs of individual applications.

Robot CT is a mobile system that is used in x-ray laminographic inspection of bulky objects like aircraft fins. It contains two arms, one with an x-ray source and another with a flat panel detector, and the two are not connected to each other. A large number of detector elements with high sensitivity is very important for quantitative assessment of carbon fibers present in the fins.

Nano CT systems contain x-ray sources with 100 nanometer or smaller focal-spot sizes and detector-element size below 10 micrometers. Inline CT uses high-speed area detectors, which provide images with high signal-to-noise ratio and enable readout frequencies of 20 frames per second or higher. Inline CT is used for fast inspection of manufacturing parts during production. (R. Hanke 2008)

Synchrotron radiation-based microCT scanners are useful in applications like geosciences and hydrology, where samples are required to be assessed at the highest resolution (D. Wildenschilda 2002) (F. Mees 2003). These scanners use synchrotron radiation instead of x-ray tube generated photons because they require a narrow energy selective bandwidth of x-rays. X-ray energies of 20-40 keV are typically used in these applications (A. A. Ternov 1967) (U. Bonse 1996). CT with x-ray energies between 0 and 400 keV are most commonly used in industries like manufacturing, food processing etc., where the size and density of the object are not high in
density or size. For large objects and high-density materials like automobile engines, solid fuel rocket motors, etc: x-rays at high energies are used. The high photon energies are produced by linear electron accelerators, which yield x-ray energies of 4-12 MeV (S.Izumi 1993). For our research, we use a FlashCT® scanner with an x-ray source, rotating stage and a flat panel detector. (Edge cabinet, Hytec, Los Alamos, New Mexico). Detailed information on the scanner is provided in the following chapter.
2. BACKGROUND

2.1 Flash-CT Scanner

FlashCT® technology started as a research and development project between the Los Alamos National Laboratory and Imtec® Corporation. FlashCT® scanners are based on area-detector and cone beam technology and have widespread applications in various industries like aerospace, bomb disposal squad etc. The FlashCT® scanner, on which we are conducting our research, is the ILUMA® Edge cabinet system. This system comes with a microfocus x-ray tube with focal spot size of five micrometers, voltage ranging from 0 to 225 keV, anode current from 0 to 3000 µA and a flat panel amorphous silicon detector with a pixel spacing of 194 µm (assuming square pixels) and a size of 30 x 40 cm. It contains a rotating stage with a diameter of 15 cm, which also translates both horizontally and vertically with the help of stepper motors. The source-to-detector distance is 88 cm. The detector response is stored in three types of file formats: .raw, cal and .att. The .raw file format is the raw response of the detector containing raw count data without any calibration. The .cal is the version that stores the files after calibrating the detector at the same settings. The calibration is done by measuring the dark current when the tube is off, and these values are subtracted on a pixel-by-pixel basis in the calibrated format. An open-field file is also stored at the desired voltage and current setting as part of the calibration process. The .att file format calculates the attenuation values based on log (I₀/I). I₀ is calculated from the .cal file and is the mean of columns 2013 to 2048 in the detector, I is the value given to each pixel after accounting for dark current in the .cal file, and, therefore, the .att file is also calculated on a pixel-
by-pixel basis. FFT-based rapid algorithms are used for reconstruction of the projection files (Imtec 2009)

2.2 X-Ray Physics

2.2.1 X-Ray Generation

X-ray generation in x-ray tubes takes place at the target (anode), which is positioned at an angle to the incoming electrons, and it is most often made up of tungsten. In most x-ray tubes, the x-rays are emitted on the same side of the target as the electrons enter the target. In special applications, like the CT scanner used for our project, the useful x-rays are emitted on the opposite side of the target, which needs to be rather thin for this purpose. There are two types of radiation generated by bombarding the target. When an electron interacts with the target material, there is sudden deceleration of the electron, which produces radiation known as bremsstrahlung. Characteristic x-rays are produced when bombarding electrons dislodge electrons from a shell of the target and electrons from outer shells fill the vacancy, which produces x-rays of energy equal to the difference in the binding energies between the two electron shells.

2.2.2 X-Ray Attenuation

In an imaging application, x-rays traverse through the medium, where they are attenuated. The number of attenuated photons is dependent on the number traversing through the medium as well as the density and thickness of the object. Under the conditions that the beam is monoenergetic and narrow and the transmitted beam contains no scattered photons, the number of photons $I$ penetrating a slab of thickness $x$ is given by the Beer-Lambert’s equation,

$$ I = I_0 e^{-\mu x}, \quad (2.1) $$
where

\( \mu \): linear attenuation coefficient of the medium [cm\(^{-1}\)],

\( I_0 \): number of photons entering the medium

The probability \( e^{-\mu x} \) that a photon traversing through a slab of thickness \( x \) does not interact with the material is given by the product of the probabilities from coherent scattering (\( \omega \)), photoelectric absorption (\( \tau \)), Compton scattering (\( \sigma \)) and pair production (\( \kappa \)):

\[
\mu = \mu_0 \cdot e^{-\mu x} \cdot e^{-\tau x} \cdot e^{-\alpha x} \cdot e^{-\kappa x},
\]

resulting in \( \mu = \omega + \tau + \sigma + \kappa \). (2.3)

The linear attenuation coefficient \( \mu \) is dependent on the density \( \rho \) of the medium and the mass attenuation coefficient \( \mu_m \):

\[
\mu = \frac{\mu_m}{\rho}
\]

### 2.2.2.1 Photoelectric Absorption

During photoelectric absorption, incoming photons of energy \( E \) transfer their energy to an inner electron of the atom. As a result of this, the electron is ejected from the inner shell with energy \( E_k \),

\[
E_k = E - E_b
\]

where

\( E_b \): binding energy of the electron

### 2.2.2.2 Coherent Scattering

In coherent scattering, also known as Rayleigh scattering, the photons are deflected or scattered with minimum loss of energy. The incident photon deposits minimum energy in the medium and is scattered in approximately the same direction as the incident photon. The probability of
coherent scattering is higher at low energies (a few hundred keV) and is more prominent with higher atomic numbers (Z).

### 2.2.2.3 Incoherent or Compton Scattering

During this type of interaction, part of the incident photon energy is transferred to the loosely bound electron within the medium. The energy of the electron released is equal to the energy lost by the incident photon, assuming the binding energy of the electron is negligible. This electron is known as the recoil electron. The incident photon is scattered at angle $\theta$, and the electron is scattered at angle $\Phi$.

The expression that relates the energy of the incoming photon $h\nu$ and the photon scatter angle $\theta$ to the scattered photon energy $h\nu'$ is,

$$ h\nu' = \frac{h\nu}{1 + \alpha(1 - \cos \theta)} $$

(2.6)

where

$\alpha = \frac{h\nu}{m_0c^2}$: ratio of the energy of the photon to the rest mass energy of the electron

$\nu'$: Energy of the scattered photon

The probability of Compton scattering per atom is directly proportional to the number of electrons available for scattering and is therefore a function of $Z$ (atomic number). (G.F.Knoll 1999)

### 2.2.2.4 Pair Production

Photons may interact by pair production while near a nucleus in an attenuating medium. A pair of electrons, one positive and one negative, is produced, equivalent to 0.51 MeV each. Therefore, to
produce one pair of electrons, the minimum energy required is 1.02 MeV. Consequently, photons below this energy do not interact by pair production.

\[ E = 1.02 + E_{ke_-} + E_{ke_+} \]  

(2.7)

where

\[ E: \text{energy of the incoming photon} \]

\[ E_{ke_-} = E_{ke_+} : \text{kinetic energies of the positron and electron} \]

2.3 Detector

There are various ways to detect incoming photons. Since for our research we use a flat-panel amorphous silicon detector, we will describe in some detail how the flat-panel detector works. The flat-panel detector consists of amorphous silicon thin film transistor/diodes at the core. These are coupled to an x-ray scintillator. Cesium iodide (CsI) is often used as the scintillator material because of its efficiency to convert x-ray photons to visible-light photons. Images are collected using a two-dimensional matrix of amorphous silicon thin film transistors/diodes. Images may be collected at 30 frames per second or more, making rapid acquisition possible. When a beam of x-ray photons strikes the CsI layer, the x-ray photons get converted to visible-light photons. These photons are converted to electrons in the transistors/diodes, which represent the pixels in the amorphous silicon layer. This generates electronic data, which can be read out. Each pixel in the layer can accumulate a certain amount of charge before it gets saturated. Saturation causes a lack in pixel response for increasing charge, and the response of the detector becomes non-linear. Some pixels may have a higher sensitivity, which causes them to saturate earlier than other pixels, or they may have a lower sensitivity, causing a reduced response to radiation, or there may be dead pixels that give no response at all. (P. R.Aufrichtig 2000) (J. H.Siewerdsen 1999).
2.4 Effect of Scatter

There are two types of scatter, coherent and incoherent, which occur depending on the interaction of the incoming photons with the material they are passing through. The amount of scatter detected in an image depends on (T.N.Hangartner 1978):

1) Energy of the incident beam
2) Type of the material in the beam path
3) Detector’s energy-selection capability
4) Collimation geometry

X-ray attenuation is described by the exponential law shown in equation (1); this reflects an ideal case, in which scatter is not considered. Due to coherent and incoherent scattering, the path of the x-rays changes, causing them to be detected by neighboring detector elements or the original detector element in the case of multiple scatter. Scatter causes non-linearities in the CT projection and reconstructed data that need to be corrected. To minimize the necessary corrections, a collimator should be used in front of the detector, if possible, to reduce the number of scattered photons reaching the detector. (T.N.Hangartner 1978) (H.Kanamori 1985) (J.H.Siewerdsen 2001).

The following calculations give us an idea as to how scatter causes non-linearities in the projection and reconstructed data. The projection data for a CT scanner are given by,

Case 1: No Scatter

\[ P_1 = \ln \frac{I_o}{T} = \mu_1 d \]  

\[ (2.8) \]

where

- \( P_1 \): projection value without scatter
- \( I_o \): number of photons generated by the source
\( I \): number of photons penetrating object

\( \mu_1 \): calculated linear attenuation coefficient without scatter

\( d \): thickness of the material.

Case 2: With scatter \( I_s \)

\[
P_2 = \ln \frac{I_o}{I + I_s} = \mu_2 d
\]  

(2.9)

\( P_2 \): projection value with scatter

\( \mu_2 \): calculated linear attenuation coefficient with scatter

From the above equations, we know that \( P_2 < P_1 \) and, consequently, \( \mu_2 < \mu_1 \) since the thickness is constant. Therefore, we can conclude that the addition of scatter causes both projection and reconstructed attenuation values to appear lower than they should be, causing errors in the CT data. The opposite effect is observed for neighborhoods of high-density objects, because smaller values are subtracted in the case of image reconstruction with convolution/backprojection, resulting in higher reconstructed values (T.N.Hangartner 1978).

2.5 Beam-Hardening

In any medium the probability of x-rays interacting photoelectrically varies roughly with \( 1/E^3 \), where \( E \) is the energy of the incident x-rays. When incident x-rays pass through a material, the low-energy photons are more attenuated compared to the high-energy photons. This causes the average energy of the beam to increase, thus hardening the beam. The use of filters helps in decreasing the effect of beam hardening, because a filtered beam is more energetic to start with. (W.R.Hendee 2002) (J.E.Cunnigham 1983).
The amount of attenuation produced by a material for a monoenergetic beam follows the Beer-Lambert’s equation (1); however, the x-ray spectrum produced by an x-ray tube, as used in most CT scanners, is polyenergetic. Therefore, the transmitted intensity produced for a polyenergetic beam is the summation of intensities produced over all energies. The attenuation is different for different energies because of the energy-dependent attenuation properties. The measured attenuated beam intensity $I$ can be expressed by the following equation (R.A.Brooks 1976) (G.T.Herman 1979) (P.Ruegsegger 1978),

$$ I = \int I(E)dE, \quad (2.10) $$

where $I(E)$ is the beam intensity at each energy $E$. The attenuation law applies to each energy separately

$$ I(E) = I_0(E)e^{-\mu(E)x}, \quad (2.11) $$

where

$\mu(E)$: linear attenuation coefficient at energy $E$

$I_0(E)$: incident photons at each energy.

The projection value $P$, defined as the logarithmic ratio of incident to transmitted photons, is given by

$$ P = \ln \frac{I_0}{I} = \ln \left( \frac{I_0}{\int I(E)dE} \right) = \frac{I_0}{\int I_0(E)e^{-\mu(E)x} dE} \quad (2.12) $$

This equation demonstrates that the projection $P$ is no longer linear relative to the thickness $x$. This non-linearity leads to cupping and streaking artifacts in the CT image.
2.6 Why Characterize a Cone-Beam CT Scanner?

Many parameters affect the performance of the CT scanner. These parameters have to be taken into account before reaching any conclusion pertaining to a given experiment. The performance of a CT scanner is dependent on parameters like the distances between the source, object and detector; the shape of the x-ray spectrum, and the energy absorbing properties, material composition and dimensions of the object to be measured. Properties of the detector like signal-to-noise ratio, resolution and dynamic range also affect the performance of the CT scanner. Reconstruction algorithms have important properties that have to be considered because the convolution kernel can influence reconstruction images in terms of noise.

The following aspects of a CT scanner should be characterized in order to better understand the accuracy of the reconstructed values.

Linearity of detector: Dead or non-linear pixels in the detector can cause streaks in the reconstructed image.

Scatter: Scatter causes change (increase or decrease) in reconstructed CT values.

Beam-hardening effect: Non-linear behavior of CT values with increasing object thickness.
3. LINEARIZATION

3.1 Introduction

For our research, we want to correct the non-linearity associated with the beam-hardening effect; therefore, it is very important to understand the response of the detector. This will help us eliminate other non-linearities in the data.

An easy way of testing linearity is to check the response of the detector against increasing x-ray tube current below the saturation limit. For any voltage setting, this response should be linear. If this response is non linear, then the detector response has to be linearized before we correct for beam-hardening. The voltage range, as mentioned before, is 0-225 kVp. We are interested in the settings mostly used by the Procter and Gamble scientists. Therefore, 50-150 kVp settings at an interval of 25 kVp were used for this experiment. The current limit was determined by establishing the saturation point at each voltage. At every voltage mark, the current was increased, and the mean of the image was inspected. When the mean stopped changing, the limit was established as the saturation limit by making a few more measurements to determine the exact point where the maximum was reached. Ten equal current intervals between zero and the saturation limit were used for our experiment. All points below this limit ideally should give a linear response. No phantom was needed for this experiment, and the raw response of the detector was recorded in 16 bit .tif format.
3.2 Method

The following procedure was used for this experiment:

1) At each voltage, images were stored for currents under the saturation limit. For example, when we used 50 kVp for our analysis, the pixels saturated at 1500 µA. So images were stored from 0-1500 µA at intervals of 150 µA.

2) These images were read in Matlab and stacked in ascending order with the last image being the point where the pixels saturate at that particular voltage and current setting.

3) After the images were stacked, pixels were plotted across images. For example, pixel (1, 1) was plotted from 0 to 1500 µA for the 50 kVp setting (Figure 3.1). The linearity was tested by performing an $R^2$ test of pixel value versus x-ray tube current. This process was then repeated for pixel (1, 2) and so on.

4) The $R^2$ values were stored at the location of the pixels. A threshold of 0.98 ($R^2$ value) was used to separate those pixels whose response was non-linear or that were dead (Figure 3.2). These identified pixels were then plotted on a pixel map (Figure 3.3).

5) This same procedure was repeated for the other voltage settings.

---

**Figure 3.1**: Typical response of a pixel value versus current for the 50 kVp voltage setting.

**Figure 3.2**: Example image of $R^2$ values for all pixel locations. High $R^2$ (above 0.98) values correspond to gray, low $R^2$ (below 0.98) values correspond to black.
3.3 Results

We see that the number and location of irregular pixels are almost the same for all tube voltages. A total of 5662 (0.18% of total) pixels (Table 3) were found to be bad. (Figure 3.3). The irregular pixels identified by the color blue (Figure 3.3) are not dead but behave erratically. (Figure 3.4). The identification of 2 additional pixels by the color red and green (Figure 3.3) each at 50 kVp and 75 kVp is attributed to errors in reading out the frames or differences in sensitivity compared to the other pixels (Figure 3.5). Based on the specification sheet provided by Varian, Inc, we know that linear or bilinear interpolation is being performed on the .cal and .att files to replace the erroneous pixels. Therefore, no correction of the pixels is required by the user.

Figure 3.3 Location of pixels that fall below the threshold of $R^2=0.98$. All pixels below the threshold for all voltage settings are combined in this plot, and most of them overlap. The arrows mark the location of pixels that are flagged only at 50 kVp and 75 kVp. The columns and rows denoted by the color blue are only one pixel wide.
Table 3: Number of pixels that lie above the threshold (good) and pixels that lie below the threshold (bad) of 0.98.

<table>
<thead>
<tr>
<th></th>
<th>50 kVp</th>
<th>75 kVp</th>
<th>100 kVp</th>
<th>125 kVp</th>
<th>150 kVp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad Pixels</td>
<td>5662 (0.18%)</td>
<td>5662 (0.18%)</td>
<td>5660 (0.18%)</td>
<td>5660 (0.18%)</td>
<td>5660 (0.18%)</td>
</tr>
<tr>
<td>Good Pixels</td>
<td>3,140,066</td>
<td>3,140,066</td>
<td>3,140,068</td>
<td>3,140,068</td>
<td>3,140,068</td>
</tr>
</tbody>
</table>

Figure 3.4: Pixels are not completely dead but have reduced and irregular response.

Figure 3.5: The 2 additional pixels, identified at 50 kVp and 75 kVp, have reduced response compared to a more standard pixel response.
4. MODULATION TRANSFER FUNCTION

4.1 Introduction

Four contributions affect the performance in terms of resolution of a cone-beam CT scanner: x-ray tube focal spot size, detector element size, magnification and reconstruction algorithm. The measurement of the modulation transfer function (MTF) takes all of these contributions into account. The MTF is conventionally used to measure the resolving power of an imaging system. It is a measure of the imaging system’s ability to record or reproduce a given spatial frequency. Therefore, it is important to measure the MTF as part of characterizing a system, since it gives us a quantitative assessment of the system’s performance. The MTF is the magnitude of the Fourier transformed edge spread function (ESF) of the imaging device. The normalized $MTF(f)_N$ is given as:

$$MTF(f)_N = \frac{MTF(f)}{MTF(0)} \tag{4.1}$$

where

$MTF (f)$: MTF at a given frequency $f$

$MTF (0)$: MTF of the DC component

The plot of $MTF(f)_N$ is the resolving power of the imaging system as a function of the frequency $f$. There are various methods used to measure the MTF of a system. For our research project, we are using a fitting method based on the edge spread function, since this allows the use of cylindrical objects, with which streaks generated from sharp long edges can be avoided.
Nevertheless, slight streaks from the aluminum rings can be seen in our phantom images (Figure 4.1).

![Figure 4.1: Cross-section of forearm phantom used for measuring MTF. Five boundaries as numbered above are used for our analyses.](image1)

![Figure 4.2 Using the canny edge-detection method, we create a binary image of the circular boundaries. Edges of the cylinder seem broken; this is due to resizing the image. Numbers marked are the boundaries, from which the profiles are derived.](image2)

4.2 Method

The following steps are used to measure the MTF from a reconstructed image of a cylindrical phantom. (Figure 4.1) (E.Casteele 2004):

1) We calculate the center of mass (COM) by using the canny edge detection method to create binary edge boundaries for our phantom image (Figure 4.2). Out of the five boundaries we label each boundary differently so as to make it easy to calculate the center of mass for each boundary separately. For example, boundary 1 is labeled with a value 1, boundary 2 is labeled with a value 2 and so on. Each COM is calculated using the following formulae applied to the binary image of the respective boundary.

\[
x_{\text{com}} = \frac{1}{k} \sum_{i=1}^{N} m_i x_i, \quad y_{\text{com}} = \frac{1}{k} \sum_{i=1}^{N} m_i y_i
\]  

(4.2)

and

\[
k = \sum m_i.
\]  

(4.3)
where \( m_i \) is the weight of point \( i \), \( k \) is the total mass of the boundary, \( N \) is the total number of pixels in the boundary region, \( x_{\text{com}} \) and \( y_{\text{com}} \) are the coordinates of the center of mass.

2) An annular region of interest (ROI) encompassing one of the boundaries is created. This is done manually using Matlab’s `imellipse` command; one circle is created outside the edge boundary and the other is created inside the edge boundary.

3) We compute the distance of all pixels within the ROI with respect to the COM of that boundary.

4) Subsequently, two vectors are created; one contains all the distances in ascending order and the other contains their corresponding intensity values. This process gives us five profiles of intensity versus distance (in pixel units) (Figure 4.3). This procedure is advantageous to use because it allows multiple radial profiles to be combined into one, allowing more sampled points at finer distance resolution to be plotted for the ESF.

5) These vectors are then sampled at an interval of 0.5 pixel to obtain a sufficient number of sampled points per bin. This process is repeated for all five profiles.

6) The profiles that are ‘Z’ shaped are flipped, and pseudo points are added to all five profiles to create level data at the top and bottom. Due to various effects (beam hardening, scatter, EEG), the values away from the actual edge go higher or lower (Figure 4.3 a and b).

7) Finally, an analytical fit consisting of a sum of cumulative error functions (Figure 4.3 (c)) is used to fit the edge profiles:

\[
ESF = c_1 + \sum_{p=1}^{j} \frac{b_p}{2} \left( 1 + \left( \text{erf} \left( \frac{x - x_{op}}{\sqrt{2} a_p} \right) \right) \right)
\]

(4.4)

with

\[
b_j = c_1 - c_2 - \sum_{p=1}^{j-1} b_p
\]

(4.5)

and

\[
\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt
\]

(4.6)
where

\( a_p \): the spread of the \( p \)th error function

\( x_{op} \): the position of the edge of the \( p \)th error function

\( b_p \): the weight given to the \( p \)th error function

The parameters \( a_p, b_p, x_{op} \) are optimized by a non-linear least-squares fit with an increasing number of error functions using Matlab’s \textit{lsqnonlin} (Figure 4.4). Equation (4.5) states that the sum of amplitudes must always be equal to the difference between the maximum and minimum grey value of the edge.

![Graphs showing edge profiles and error function](image)

Figure 4.3 We obtain two types of edge profiles, one that is ‘S’ shaped (a) and another that is ‘Z’ shaped (b). Pseudo points are added (dashed lines) to take care of points going lower or higher beyond the edge region, so that the fitting procedure is more effective. (c) shows a typical error function when plotted as a function of \( z \) (equation 4.6 below). In our case \( z \) is the distance between pixels.
8) The line spread function (LSF) (Figure 4.5) is calculated by taking the derivative of the ESF analytical function (equation (4.7)):

$$\text{LSF}(x) \sum_{p=1}^{I} b_p \frac{1}{\sqrt{2\pi} a_p} \exp \left( -\frac{(x - x_{op})^2}{2a_p^2} \right) \quad (4.7)$$

The parameters $a_p, b_p, x_{op}$ have already been calculated from the analytical fit using the sum of error functions in equation (4.4).

9) MTF $M(u)$, with $u$ the spatial frequency, is calculated by taking the modulus (Matlab: abs) of the Fourier transform (Matlab: fft) of the LSF and by normalizing the magnitude at zero frequency to unity (Figure 4.6) (Equation 4.8). The pixel size of the reconstructed image was
calculated by dividing the actual image diameter (mm) with the matrix size (pixels). To obtain the calibrated frequencies in cycles/mm, the profile sampling interval (Step 5), was included in the normalization process.

\[
M(u) = \sum_{p=1}^{j} b_p e^{-2\pi^2 a_p^2 u^2} e^{-i2\pi ux_{op}}
\]

(4.8)

10) The 10% value of the MTF was obtained by interpolation.

The number of error functions to be used was defined by the change in the observed 10% MTF value. The point where the 10% MTF value stopped changing (depending upon the number of error functions used) was the point where we stopped adding error functions. Adding one or more error functions helps fit the data better and thus improve our 10% cut-off value. In order to examine the method, we used the phantom measured at a distance of 550 mm from the source and applied six error functions to the fitting procedure (Figure 4.7). The plot demonstrates that, even with an increasing number of error functions beyond 2, the cut-off values of the MTF do not change. This process was repeated for multiple profiles from different stage positions from the source and different profiles. A sum of three error functions was used to calculate the 10% MTF for our experiments as a result of these analyses.

We also ran a test on a reconstructed image of the phantom at stage position 550 mm from the source after applying an averaging filter (in Matlab) of varying kernel sizes from 3 to 7 pixels to confirm that an increase in kernel size decreases the value of the MTF due to the amount of blurring in the image (Figure 4.8).
4.3 Results

Figure 4.8: The 10% MTF values for all five profiles fall with increasing size of the smoothing kernel effect.

We can observe that the error-function-based method provides an appropriate assessment of the reconstructed image of the phantom, because with increasing smoothing the value of the MTF decreases for all five profiles. Some differences among the profiles are observed, which are associated with the fact that the edge profiles are not symmetric. This is because the top and tail of the ESF, at which the pseudo points are added, are somewhat arbitrary.

An object closer to the source gives a higher 10% MTF cut-off value (Figure 4.9) compared to an object closer to the detector. Higher cut-off values will indicate superior contrast and resolution. The large improvement in the MTF cut-off value for position 300 mm can be attributed to the beam profile (Figure 4.10 (a)), which may be used to estimate the FWHM for different stage positions from the source. The width of the black area at a given distance from the source represents \( a \) in the beam profile (Figure 4.10 (b)), and the width of the white area represents \( b \).
The FWHM represents the width of the intensity profile at 50% of its maximum intensity value (Table 4). Increasing width of the intensity profile decreases the 10% cut-off frequency for the MTF because of the increased blurring effect.

Figure 4.9: The 10% MTF value at the 300 mm stage position from the source for all profiles is almost twice that of the MTF at the 550 mm stage position from the source and almost four times that of the MTF at 700 mm.

Table 4.1: The FWHM calculated at the given stage positions from the source. The values justify the difference in MTF values obtained for different stage positions from the source (Figure 4.9)

<table>
<thead>
<tr>
<th>Stage position from the source</th>
<th>300 mm</th>
<th>550 mm</th>
<th>700 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>FWHM</td>
<td>0.06885 mm</td>
<td>0.1242 mm</td>
<td>0.1575 mm</td>
</tr>
</tbody>
</table>

Table 4.2: 10% MTF cut-off values converted to microns.

<table>
<thead>
<tr>
<th>Profiles</th>
<th>Stage distance from the source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>300 mm</td>
</tr>
<tr>
<td>1</td>
<td>58 µm</td>
</tr>
<tr>
<td>2</td>
<td>65 µm</td>
</tr>
<tr>
<td>3</td>
<td>64 µm</td>
</tr>
<tr>
<td>4</td>
<td>69 µm</td>
</tr>
<tr>
<td>5</td>
<td>63 µm</td>
</tr>
</tbody>
</table>
Figure 4.10 The beam profile demonstrates how a detector pixel receives radiation from the source (top view). The trapezoid shown is the cross-section taken across the beam. 300 mm, 550 mm and 700 mm represent the stage positions from the source used in our experiments.
5. SCATTER

5.1 Introduction

Before correcting beam-hardening based non-linearities, it is essential that we first minimize the effect of scatter. The major source of scatter in our cone-beam CT scanner is the rotating stage. Scatter originating from the surface of the stage is influenced by three factors (Figure 5.1):

- Source to stage distance \( ss \)
- Angle at which the beam is incident on the stage \( \alpha \)
- Stage to detector distance \( sd \)

The amount of scatter \( I_s \) contributed to a point on the detector can be quantified as follows:

\[
I_s \sim \frac{1}{ss^2} \cdot \sin \alpha \cdot \frac{1}{sd^2}
\]  

(5.1)

The intensity measured at a distance \( ss \) or \( sd \) from a point source is subjected to the inverse square law. A point source emits radiation radially over the surface area of a sphere \( 4\pi r^2 \), where \( r \) is the radius. As the distance increases the radiation spreads over an area proportional to the square of the distance. Therefore, being further away from the source decreases the scatter a given point on the detector receives. The angle \( \alpha \) determines the intensity by which the beam illuminates a unit surface element. The steeper the angle, the higher the intensity (Figure 5.2 (a)), and as the angle decreases, the intensity decreases (Figure 5.2 (b)). X ray beam B hits the surface at maximum intensity when the projected beam area is minimized, i.e. \( c \times d \). If the same beam intensity is
Figure 5.1: Schematic view of the interaction of a single ray A with a point on the rotating stage and the scattered ray S hitting the detector. The relevant geometric parameters are also indicated.

Figure 5.2: The area illuminated by the beam on the surface is dependent upon angle $\alpha$. This change in the area due to the angle decreases the intensity with which the beam is incident on the surface.

distributed over a larger area, $c \times a$, due to a smaller incident angle $\alpha$, the beam intensity measured at a unit surface area is reduced. The relationship between incident beam angle $\alpha$ and
illuminated surface area is $a = \frac{d}{\sin \alpha}$. As the beam intensity decreases with increasing area, the relationship $I = \frac{1}{a} = \sin \alpha$ holds.

Computer-generated simulations were run in Matlab to map the effect of scatter from different positions of the stage between source and detector and at different heights of the stage, assuming a wide open beam without any restrictions and collimations. Figure 5.3 shows the simulation results. An individual line in the plots pertains to a specific point in the detector that receives scatter from the various stage height increments. The lowest curve represents the scatter received by the point 140 mm above the center of the detector, whereas the top curve represents the scatter received by the point in the center of the detector.

Figure 5.3: Computer simulations of scatter for three source-stage distances 300 mm (a), 550 mm (b) and 700 mm (c). The graph displays multiple lines, representing the amount of scatter observed at locations between the center (top curve) to the top most point (140 mm above the center, bottom curve) of the detector versus stage-height increment. The stage height $H$ represents the distance between the surface of the stage and the center of the detector.
Curves were generated from the center to the top at intervals of 20 mm. The difference in scatter arises from the change in the length of ray $sd$, which is shorter for the point in the center and longer for the point 140 mm above the center of the detector (Equation 5.1).

When the stage is positioned at a distance of 300 mm from the source, the increased angle of incidence of the beam on the surface is most dominant, whereas at 550 mm and 700 mm distance from the source, the decreased angle of incidence reduces the amount of scatter observed by the detector. The maximum scatter is observed at stage heights in the range of -200 mm to -150 mm for stage position 300 mm from the source, at stage heights in the range of -225 mm to -175 mm for stage position 550 mm from the source and in the range of -175 mm to -125 mm for stage position 700 mm from the source.

For our physical scatter experiments it is important to take into consideration the limited cone size of the beam, which can be further reduced by adding a collimator. At the initial heights of the stage, the stage is not in the field of view. At higher stage heights, the surface of the stage comes into the beam by different amounts and produces scatter (Figure 5.3).

5.2 Designing the collimator

To prevent scatter from emanating from the stage, we propose the use of a collimator to be placed in front of the source to limit the beam cone to the detector size (Figure 5.3). A collimator, the position of which can be adjusted in all three directions, was used for the scatter experiments (Figure 5.5 and 5.6). The Flash-CT ® scanner has a source-to-detector distance of 880 mm. This requires an accurate design of the collimator because of the large magnification factor. Therefore, we first estimated the actual source position (Figure 5.7). Two collimators were designed based on this information, one closer to and one further away from the source, to have two choices for the actual implementations.
Figure 5.4: This drawing shows a proportionately accurate representation (using a scale of 1:10) of the stage at different heights and stage distances from the source used for the step phantom measurements in our experiments.

Brackets were designed, one to hold the collimator (collimator bracket) on the adjustable x-y-z stage and the other to mount the stage with the collimator in front of the x-ray tube (tube bracket) (Figure 5.5 & Figure 5.6).

An estimate of the source position was used to calculate the dimensions of the collimator. Figure 5.7 is a schematic diagram of the source and detector seen from the top. Distances 1 and 2 were measured by placing the detector at distances 3 and 4, respectively, from the source and capturing images of the x-ray cone. Matlab was used to measure the horizontal diameter of the respective cones in the images, and a tape measure was used to measure distances 3 and 4.
Figure 5.5: The collimator device is attached to the x-ray tube by the tube bracket. The two collimators can be mounted on the x-y-z stage by reversing the collimator bracket.

Figure 5.6: Full assembly of the collimator device with the plate and the brackets.
Using similar triangles we get,

\[
\frac{x + 170}{x + 360} = \frac{185.46}{363.75}.
\]

Solving for \(x\), we obtain \(x = 27.20\ mm\).

Our collimator design allows us to attach the collimator bracket to the stage in two positions, giving us two collimator distances from the source. Based on these positions, we designed two separate collimators. The width and height of the collimator (Figure 5.8) can be defined using the following expressions,

\[
\text{Width of collimator (} W_o \text{)} = (\text{width of detector}) \times (1/\text{magnification}) \tag{5.2}
\]

\[
\text{Height of collimator (} H_o \text{)} = (\text{height of detector}) \times (1/\text{magnification}) \tag{5.3}
\]

where

\[
\text{magnification} = \frac{\text{Source to detector distance}}{\text{Source to collimator distance}} \tag{5.4}
\]

The resulting dimensions are shown in Table 5.1. The collimator plates were made of 1 mm thick lead sheets and attached to 0.635 mm thick aluminum brackets.

Collimator 2 was finally used for our purpose because it created a more accurate beam which nicely fitted the detector, whereas collimator 1 produced a beam larger in size than collimator 2.
Figure 5.7: Using similar triangles, position $x$ of the source was estimated. All dimensions are in mm.

Table 5.1: Collimator 1 and collimator 2 (Figure 5.5) are located 10 mm and 65 mm from the front plate of the x-ray tube, respectively.

<table>
<thead>
<tr>
<th>Collimator</th>
<th>$W_o$ (mm)</th>
<th>$H_o$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.4</td>
<td>12.3</td>
</tr>
<tr>
<td>2</td>
<td>40.6</td>
<td>30.3</td>
</tr>
</tbody>
</table>

Figure 5.8: Notations of width and height of the collimator.
5.3 Method

Experiments were conducted with and without the scatter collimator mounted. A foam block, placed on the rotating stage, was used to reduce the effect of scatter originating from the rotating stage by keeping the stage at a lower height. An additional reason to keep the stage low has to do with ongoing source calibration. When the stage is in the field of view, it blocks some rows at the bottom of the detector, which are used to calculate the calibration values.

Two types of phantoms (step phantom (Figure 5.9 (a) and cylindrical forearm phantom Figure 5.9(b)) were used to demonstrate the effect of scatter and how the collimator reduces scatter in the projections and the reconstructed images.

![Step Phantom](image1)

![Forearm Phantom](image2)

Figure 5.9: Schematic drawing of the (a) step phantom and (b) the forearm phantom.

Projection images of the step phantom were obtained with vertical translations of the stage at increments of 23 mm. The projection images were analyzed in three regions (Figure 5.10), in which the same thicknesses from step 1 to step 10 of the phantom could be measured at three different detector positions.
The forearm phantom was used to demonstrate the effect of the collimator in the reconstructed image and was also measured at all three stage distances from the source.

5.4 Results

Tables 5.2 and 5.3 show the effectiveness of using a collimator for projections and reconstructed images, respectively. The projections show a flattening of the curves towards the thicker steps, which is more pronounced at the 550 mm and 700 mm stage positions from the source because of an increased area of the stage in the beam. In the reconstructed image the higher peaks are an effect of reduced scatter when using the collimator.

Figure 5.10: Regions 1, 2 and 3 were used for our analyses. Vertical translation of the step phantom causes almost all the regions to be exposed by all ten step phantom thicknesses. However, because we want to avoid the rotating stage in the field of view, region 3 is not exposed by the thicker steps of the phantom. The position of the pixels used in all three regions was not consistent because the difference in magnification between the thinner and the thicker steps does not allow us to do so.

5.5 Discussion

Let’s first consider the scatter measurements of the step wedge without the collimator. Based on Figure 5.1, Figure 5.3 and Equation 5.1, we can deduce that, for the stage at 300 mm from the source, the most scatter is produced at increment 11 because the surface of the stage is fully in the beam only at this level, whereas at the 550 mm position, we expect the most scatter for the 7th
increment (highest $\alpha$ with full stage in beam). As the stage height increases, scatter decreases because the angle of the beam incident on the stage decreases and the attenuation values as a result go up. At the 7th stage height increment and above, the entire surface of the stage is exposed to the beam, whereas, at the 6th stage height increment only part of the stage is exposed (Table 5.5 and 5.6).

For the 700 mm stage position from the source, there is only one stage height, because the whole step phantom fits in the image. At this position, the stage is well within the uncollimated beam and contributes to the scatter. Any increase in the stage height would result in the stage obstructing a part of the detector that is used for calibration. From Table (5.5) we see that at steps 7, 8 and 9 at the 7th stage-height increment have lower attenuation values without collimator then the other increments. This happens because the entire surface of the stage is exposed to the beam, and the angle of the beam is steepest at this increment. When the collimator is in place, we do not see the 7th stage-height increment (highest $\alpha$ with the stage in the beam) to produce the lowest projection value, except for step 8. This is not an effect of scatter but a slight change in the length of the attenuation path due to the angle by which the beam penetrates the step phantom (Figure 5.11).

Since region 2 is in the center of the detector it will always have slightly shorter rays passing through a plain attenuator compared to region 1 and region 3; therefore, we expect the values in region 2 to be slightly lower (by 0.3%). This is detectable both in steps 7 and 8 with collimator (Table 5.6), as the middle columns represent region 2 in both plots, and they show lower values as compared to the other regions. Step 9 with collimator would justify this theory for region 2 compared to 1, but not for region 3 (Table 5.6, with collimator, step 9 at stage height increment 9), which shows the lowest value. Again this could be attributed to the step phantom itself, which
acts like an additional source of scatter; however, uncertainty in the measurements is more likely the reason for this discrepancy.

Based on these results we can now proceed to correct beam hardening, having placed a collimator in front of the x-ray tube to prevent scatter from the stage.
Table 5.2: Projection values obtained for three different stage distances from the source. The images produced at stage positions 300 and 550 mm from the source contain three regions, whereas those produced at 700 mm from the source contain ten due to the reduced magnification of the image. 1 represents data collected without collimator and 2 represents data collected with collimator.

<table>
<thead>
<tr>
<th>Stage Distances (mm)</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>550</td>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>700</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
<td><img src="image9" alt="Graph" /></td>
</tr>
</tbody>
</table>
Table 5.3: Reconstructed images with and without collimator were used to plot profiles across the diameter of the phantom on top of each other. 1 represents data collected without collimator, and 2 represents data collected with collimator.

<table>
<thead>
<tr>
<th>Stage Distances (mm)</th>
<th>300</th>
<th>550</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of profiles</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>Profiles</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>Magnified Profiles</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Table 5.4: Locations of the projected steps of the step phantom when vertically translated through the stage height increments at stage position 550 mm from the source. Bottom to top in each figure are regions 1 to 3. Step 7, 8, and 9 translated through each region are shown for illustration purposes.

<table>
<thead>
<tr>
<th>Stage-Height Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>

| 6                      |
| 7                      |
| 8                      |

| 7                      |
| 8                      |
| 9                      |

| 8                      |
| 9                      |
| 10                     |

| 9                      |
| 10                     |
Table 5.5: Projection values of the step phantom from the 5th to 9th stage-height increment (Table 5.4) at 550 mm without (1) and with (2) collimator. The three regions and their corresponding attenuation values for the respective increment are listed. The highlighted blocks show steps 7, 8 and 9 at different stage heights for the uncollimated case. It can be seen that stage-height increment 7 is always the lowest compared to the other stage-height increments. Projections values are dimensionless and scaled arbitrarily.

<table>
<thead>
<tr>
<th>Stage Height Increments</th>
<th>Scatter Presence</th>
<th>Step</th>
<th>Region 1</th>
<th>Step</th>
<th>Region 2</th>
<th>Step</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5 5</td>
<td>No</td>
<td>7</td>
<td>3.01</td>
<td>3.14</td>
<td>6</td>
<td>2.69</td>
<td>2.78</td>
</tr>
<tr>
<td>5 6</td>
<td>Partial</td>
<td>8</td>
<td>3.22</td>
<td>3.47</td>
<td>7</td>
<td>2.99</td>
<td>3.12</td>
</tr>
<tr>
<td>5 7</td>
<td>Maximum</td>
<td>9</td>
<td>3.37</td>
<td>3.79</td>
<td>8</td>
<td>3.22</td>
<td>3.46</td>
</tr>
<tr>
<td>5 8</td>
<td>Present</td>
<td>10</td>
<td>3.50</td>
<td>3.94</td>
<td>9</td>
<td>3.47</td>
<td>3.78</td>
</tr>
<tr>
<td>5 9</td>
<td>Present</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>10</td>
<td>3.41</td>
<td>3.89</td>
</tr>
</tbody>
</table>

Figure 5.11: The attenuation length \(a\) is increased in regions 1 and 3 compared to region 2 of a plane attenuator, this causes the attenuation value to be lower in region 2.
Table 5.6: Bar graph representation of values in Table 5.5.

<table>
<thead>
<tr>
<th>Step</th>
<th>Without Collimator</th>
<th>With Collimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td><img src="image1" alt="Bar graph" /></td>
<td><img src="image2" alt="Bar graph" /></td>
</tr>
<tr>
<td>8</td>
<td><img src="image3" alt="Bar graph" /></td>
<td><img src="image4" alt="Bar graph" /></td>
</tr>
<tr>
<td>9</td>
<td><img src="image5" alt="Bar graph" /></td>
<td><img src="image6" alt="Bar graph" /></td>
</tr>
</tbody>
</table>
6. BEAM HARDENING

6.1 Introduction

In this chapter we discuss our approach to correct the beam-hardening effect (Background Section) and the results we obtained by correcting it on the CT data. We already know, according to the Beer-Lambert law, that

\[ \ln \frac{I_0}{I} = \mu x, \quad (6.1) \]

and the projection value \( P \) is given by,

\[ P = \ln \frac{I_0}{I}. \quad (6.2) \]

From equations 6.1 and 6.2 we obtain the following expression:

\[ P = \mu x. \quad (6.3) \]

Data obtained from translating the step phantom were used to create coefficients for the correction (Scatter Section). Projection values were plotted against step-phantom thickness for each of the three detector regions (Figure 6.1) and a polynomial was fitted through these data. A polynomial was used because it is easy to evaluate. (G.T.Herman 1979).

\[ P_{\text{measured}} = A x^n + B x^{n-1} + \ldots + E x + F \quad (6.4) \]

The correction to these data can be applied by linearizing them to the following equation:
where $\mu$ : slope of the first few points in the projection-versus-thickness plot.

Substituting $x$ from Equation 6.5 in Equation 6.4 we get,

$$P_{\text{corrected}} = \mu x$$  \hspace{1cm} (6.5)$$

Once we decide the degree of the polynomial to be used, we calculate the slope from the first few points of the plot because these points are fairly linear and give us the best estimate of what monoenergetic projection values would be at the corresponding step thicknesses.

Depending on the order of the polynomial, we can calculate a number of roots. These roots need to be tested in equation 6.6, and the root yielding a physically meaningful result is selected. The resulting equation give us an expression, whereby $P_{\text{corrected}}$ is a function of $P_{\text{measured}}$, our correction for the beam-hardening effect.
6.2 Applying the Correction

Data collected using the step phantom (with collimator) for the scatter experiments, were used to calculate the coefficients for the beam-hardening correction. Second, third and fourth order polynomials were tested to fit the data and create the correction. The selection of the final polynomial was based on the residuals, created by fitting the corrected data to a straight line. Polynomials of order two and three showed a systematic pattern (cyclic pattern) in their residuals and were therefore not used in our correction. The fourth order polynomial did not show any systematic pattern (cyclic pattern) in its residuals and was selected as the best choice.

There are three data sets based on the different stage positions from the source. Each data set has 3 regions on the detector screen, which we use for analysis. Therefore, we can create 3 different

\[ \text{Figure 6.2: The plots for regions 1, 2 and 3 at stage position 300 mm from the source are shown in (a), (b) and (c), respectively. From the above plots we see that regions 1 and 3 have missing steps.} \]
sets of polynomials for each stage position from the source, except for 700 mm, where the stage blocks the rows at the bottom of the detector. While correcting for beam hardening, we have to choose polynomials that do not correct for scatter also. Although, the collimator prevents scatter, it does not entirely remove its effect. The stage position at 300 mm from the source is our best bet in this case, because this stage position produces minimum scatter, as we learned in the previous chapter. So the choice is between the three detector regions at the 300 mm stage position from the source (Figure 6.2). From our three choices, we select the coefficients from region 2 (Figure 6.2 (b)) because it is the only region that is exposed to all ten step thicknesses, whereas regions 1 and 3 (Figure 6.2 (a) and Figure 6.2 (b) respectively) miss one step because in region 1 (Figure 6.2(a)) the stage does not go below a certain height, which causes the second step to overlap with region 1, and in region 3 (Figure 6.2(c)) the stage is in the field of view, blocking the bottom rows of the detector. We now use equation 6.6 and calculate roots of the fourth order polynomial based on Ferrari’s solution (Appendix).

The same forearm phantom data (with collimator) measured for the scatter experiments were used to test the beam-hardening correction. A program was created in Matlab that reads, corrects, and writes the data out so that all the corrected projection files can be reconstructed. The files were read based on a program provided by the scanner company. The data, as read, are in 16-bit, scaled format and need to be converted into floating point numbers representing the actual attenuation values:

$$\text{Attenuation Values} = \frac{\text{Scaled Values}}{65536} (\text{Attmax} - \text{Attin}) + \text{Attin} \quad (6.7)$$

where

$\text{Attnax}$: Maximum floating point value obtained from the header.

$\text{Attin}$: Minimum floating point value obtained from the header.
Once the data are corrected for beam hardening, they are written back to their original format using the reverse formula:

\[
\text{Scaled Values} = \frac{\text{Attenuation Values} - \text{Attmin}}{\text{Attmax} - \text{Attmin}} \cdot 65536 \tag{6.8}
\]

The corrected projection data are then reconstructed using the manufacturer-supplied reconstruction programs.

### 6.3 Results

Two profiles were extracted and plotted from the reconstructed images (Figure 6.3(a) and (b)). It is evident that the cupping effect associated with beam hardening is eliminated or slightly overcorrected (Figures 6.4 and 6.5).

![Figure 6.3](image.png)

(a) (b)

(c)

Figure 6.3: Location of the two profiles extracted from the reconstructed images (a) and (b); (c) shows the reconstruction artifacts between the two aluminum cylinders at enhanced contrast.
This slight overcorrection is due to the difference between the material of the step wedge used to derive the correction coefficients and the material present in the phantom (aluminum and Plexiglas). In addition, corrections based on analytical correction equations are only accurate for single material-objects. All three profiles (Figure 6.4), corrected and uncorrected, show irregularities in the central region. These are due to tangential streaks emanating from aligned edges of the aluminum cylinders (Figure 6.3 (c)).

Figure 6.4: Horizontal image profiles (Figure 6.3 (a)) for the three stage positions from the source: 300 mm (a), 550 mm (b) and 700 mm (c). Solid lines represent the uncorrected data, and broken lines represent the corrected data. The irregularities in the center regions of all profiles are due to the artifacts generated by the aluminum rings.
Figure 6.5: Vertical image profiles (Figure 6.3(b)) taken across the reconstructed image for each stage positions: 300 mm (a), 550 mm (b) and 700 mm (c). Solid lines represent the uncorrected data, and broken lines represent the corrected data.
7. DISCUSSION

To achieve our objective of characterizing the cone-beam CT scanner, we used two types of phantoms, the step phantom and the forearm phantom. The step phantom was designed as large as possible but would still fit in the field of view. A phantom larger than ours (base = 150 mm, height = 260 mm) would create several problems, such as scatter, insufficient penetration of the x-ray beam or difficulty in moving the phantom through the various parts of the x-ray cone. The present phantom size gives us the worst-case scenario for most of our experiments, thus allowing us to take into account the various effects while investigating solutions. We did not design the forearm phantom; this was provided to us by the BioMedical Imaging Laboratory.

The step phantom was used when analysis needed to be done in the projection files, whereas the forearm phantom was used when the reconstruction files needed to be analyzed. This is because, the step phantom, when reconstructed, would create streaks from its sharp edges, whereas, the forearm phantom would have no such problems, being cylindrical in shape. In addition, the step phantom was too large for most stage positions from the source to fully fit in the field of view. The x-ray beam used was 65 kVp at 250 µA for all our experiments, because this is the energy mostly used by scientists at P&G to do their investigations. Consequently, the results presented here apply only to this beam energy, and the experiments would need to be repeated for other Kilovolt settings.

Three stage positions were used for our experiments (300 mm, 550 mm and 700 mm from the source). 550 mm was a natural choice since most of the investigations conducted by scientists at
P&G were at this stage position 300 mm and 700 mm were used to have one position closer to the source and the other closer to the detector. This allowed us to have a range of positions for all our experiments, to check if the theoretical calculations actually matched the experimental data. For example, based on the size of the focal-spot and the detector element, the 10% MTF cut-off value, in theory, should be higher when the stage is closer to the source and decrease as we move closer to the detector. For the scatter experiments, we would expect maximum scatter to be produced when the stage is closer to the detector and relatively less as the stage moves closer to the source due to the cone-beam size and the interaction of the x-rays with the stage. These facts can be confirmed with our results (MTF, Scatter section).

**Linearity**

To test the response of the detector, we created a 3D volume of images taken at intervals of currents for voltages in the range of 50 kVp to 150 kVp. The current limit was decided by keeping in mind that the pixels did not saturate. This allowed us to test the response of each individual pixel using the R-squared value test of pixel response versus tube current. A threshold ($R^2=0.98$) was set to differentiate between the bad and good pixels. The R-squared value test showed that 5660 pixels were bad in the flat panel detector, but this number was not consistent for all the voltages. At 50 kVp and 75 kVp two more pixels below the threshold were identified as compared to all other voltages. Further investigation showed that these pixels were not entirely bad or dead. They did respond to change in x-ray flux, but they behaved erratically in some intensity regions. Based on information provided on the detector manufacturer’s specification sheet, linear or bilinear interpolation between neighboring good pixels is carried out to correct for the bad pixels but the exact process of correction cannot be determined. No correction was needed on our part for any bad pixels.
MTF

For our MTF measurements, a cylindrical phantom was necessary. The forearm phantom made of Plexiglas and aluminum was a natural choice, because it provided a total of 5 cylindrical boundaries to evaluate. The sampling rate we used in our experiments was 0.5 pixels/bin, because a lower sampling rate generated profiles that were too noisy. In contrast, when we tried a sampling rate higher than 0.5 pixels/bin, not enough points were plotted, resulting in loss of information about the shape of the edge-response curve.

For the fitting process it is imperative to optimize all three parameters, spread, amplitude and center, simultaneously, using the non-linear least-squares method, otherwise the process fails. Trials of optimizing any two of the parameters did not yield good results, also, the number of error functions to be used depends on when the 10% MTF value stops changing. The MTF generally did not change further after three iterations of optimizing the ESF functions (three error functions were optimized at the same time, whose sum was fitted to the ESF profile from the data; MTF section) for the various profiles that were analyzed. Three ESFs were finally used in the measurement of the 10% MTF.

Generally, one would expect an MTF to be the same across the whole image. In our experiments, 5 ESF profiles were analyzed, and the 10% MTF value ranged from 7.2 cycles/mm to 8.5 cycles/mm for stage position 300 mm from the source, 2.9 cycles/mm to 3.5 cycles/mm for stage position 550 mm from the source and 2.2 cycles/mm to 2.76 cycles/mm for stage position 700 mm from the source. Differences in the 10% MTF value of the five profiles for each stage position may be a result of inconsistently choosing the parts of the profiles to be leveled resulting in the ESF not being symmetric. There may also be slight variations in the MTF depending on the position of the cylinder in the image.
Scatter

The exposures using the step phantom were analyzed by dividing the detector into three regions, which each step exposed after translating the phantom. This was done to assess if the detector regions responded the same way. The curves (attenuation values versus step-phantom thickness) for each region were comparable in terms of attenuation values and produced similar results within 1.12 % when exposed to the step phantom. This confirmed that the detector response was consistent in different regions of the detector.

Equation 5.1, which expresses the amount of scatter produced by the stage, as a function of the relevant geometric parameters, identifies the angle of incidence on the surface as the dominating factor. Based on the experimental results, we have confirmed that this equation does estimate where scatter would be maximum. The detailed data were shown for stage position 550 mm from the source (Table 5.5) only, because stage position 300 mm from the source has only the last two stage-height increments in the cone beam, and stage position 700 mm from the source is entirely inside the cone beam and has only one stage-height increment.

Equation 5.1 provides a good way to estimate the relative amount of scatter without an actual experiment. However, the size of the beam, which dictates by how much the surface of the stage is exposed to the beam, is also very important to predict scatter, and this information is not included in equation 5.1

Beam Hardening

Data obtained in the scatter experiments with both the phantoms were also used in our beam-hardening correction. The step phantom served to establish the beam-hardening equations, and the forearm phantom demonstrated how well the correction worked. Polynomials that correct for
beam hardening should not be obtained in the presence of scatter, and our collimator ensures that the effect of scatter is minimal. The polynomial coefficients were created from measurements with the stage at 300 mm from the source because, at this stage position from the source, scatter is minimal. This is because the cone is narrower closer to the source, resulting in no interaction of the x-rays with the rotating stage. Detector region 2 was eventually selected for the calculation of the beam-hardening correction coefficients, because it contained all ten steps of the phantom as opposed to the other regions. Second and third order polynomials produced cyclic systematic patterns in the residuals; therefore, a fourth-order polynomial was selected. The expression obtained from the fourth-order polynomial allowed us to apply the correction to the projection files.
8. CONCLUSION

Testing the linearity of the flat-panel detector and preventing scatter were important steps in developing an appropriate beam-hardening correction. Non-linearities associated with the detector response were non-existent in our case, because appropriate corrections were already implemented by the manufacturer.

The MTF is influenced by the focal-spot size, detector size, magnification and reconstruction algorithm. The focal-spot size and the detector size determine the beam profile. Position of the stage determines the magnification and the profile of the beam at that position. This causes the 10% MTF cut-off value to increase or decrease. Based on the geometries of the investigated system, higher MTF values or better contrast/resolution can be obtained with the rotating stage closer to the source, because the focal-spot diameter of 5 µm is smaller than the size of a detector element (194 µm).

From our results, we can deduce that the scatter effect is most dominant when the rotating stage is placed close to the detector with no collimation in place. This is because the cone size is larger closer to the detector, and the interaction of the x-rays with the rotating stage causes more scatter to emanate from its surface, whereas closer to the source, the cone size is smaller and, as a result, the stage is mostly not in the radiation beam. The stage-height also affects the amount of the scatter being produced because of the angle of incidence of the x-rays on the stage. However, minimal scatter is produced only if the stage is outside the radiation cone. Collimator 2 placed in front of the x-ray source adequately limits the beam to the size of the detector. This allows the
highest possible stage position without any part of the stage being inside the radiation cone. For a smaller object that would not be adequately centered in the field of the detector, it is recommended to use a low-density material (styrofoam) to prop up the object, making sure that the stage stays outside the radiation cone.

Although the best prevention of scatter is by physical use of collimators, there are software-based corrections of scatter, which can further reduce the effect of scatter on the reconstructed image values. One such correction method compensates for first-order scatter and takes into account the material distribution of the object (T.N.Hangartner 1978).

The proposed beam hardening correction is accurate only for Delrin or plastic materials of similar composition and density because the correction is based on the step phantom made from Delrin. Other materials may produce over or under corrections, and correction coefficients should be calculated based on step wedges of such other materials.

Beam-hardening correction is dependent on the material and beam energy. The present correction is accurate only for materials similar to Delrin measured at 65 kVp. A set of step phantoms could be made from different materials that are used on a regular basis by the scientists at P&G and these phantoms could be measured at all desired tube voltage.

A graphical user interface (GUI) could be designed, where experimental data can be inputted, and the appropriate polynomial and its coefficients based on experimental data with the same material as that in the scan to be corrected and at the same beam energy would be manually selected. This polynomial would then be fed to the beam-hardening correction program. If there are more than two materials present in the object, a more sophisticated correction would need to be applied like a material-selective iterative beam-hardening correction (P.Ruegsegger 1978).
REFERENCES


APPENDIX A-1

This program reads the .att file, converts this image from 16 bit data to floating point values and stores them in a .mat file. The beam-hardening correction (A-2) is then applied to this file.

function orig_image =readfct_native(filename)

% Reads the .att file and converts this image from 16 bit data to floating point values. These floating point values are then stored in the matfile. Beam hardening correction is then applied to these files.

% Anthony Davis
% 7-12-99
% Reads the image from FlashCT 16 bit float format function [outimage, imagenumber]
% =readfctraw(filename,max_attenuation(e.g. 4.0));
% Modified 4/4/08 by Tom Dufresne to convert to 8 bit based on maximum attenuation value.
% Author: Tom Dufresne
% Copyright 2008 The Procter & Gamble Company
% $Revision: 1.0 $ $Date: 2008/04/07 17:37:00 $

centershift = double(0.0);
[pathstr, name] = fileparts(filename);
fid=fopen(['C:\Documents and Settings\bmil\My Documents\jimishjoshi\seagatedatatill20thmarch\seagatemay21stupdate\beamhardeningfunctions\' name '.att' ], 'r', 'b');
typestring=fread(fid, 3, 'uchar'); % image type string
fprintf('File Type is .%s\n', typestring); %Prints the type of string
imagenumber=fread(fid, 1, 'uint32'); % Reads the image number

extradata=fread(fid, 1, 'uchar');
size2=fread(fid, 1, 'uint32'); % Reads the number of rows
size1=fread(fid, 1, 'uint32'); % Reads the number of columns
skip = fread(fid, 1, 'int8');
maxval=fread(fid, 1, 'float32'); % Reads the maximum value
minval=fread(fid, 1, 'float32'); % Reads the minimum value
% Each slice has a min and max value associated with it. It is
% therefore necessary to convert each slice back to Hounsfield units
% and
% then convert these floating point values back to byte data.
% timage=transpose(fread(fid, [size2, size1], 'uint16'));

hdr.typestring = typestring;
hdr.imagenumber = imagenumber;
hdr.extradata = extradata;
hdr.size2 = size2;
hdr.size1 = size1;
hdr.skip = skip;
    hdr.maxval = maxval;
    hdr.minval = minval;

orig_image = minval + ((timage/65536)*(maxval
    - minval));

fclose(fid);
% Saves the converted file and necessary header information.
    save([name '.mat'], 'orig_image', 'hdr', 'centershift');
APPENDIX A-2

A function is created that applies the beam-hardening correction to the projection files using Ferrari's solution to a quartic function. A polynomial is converted to a depressed quartic. After this calculation is done, the equation is converted to a perfect square by adding a valid identity to the polynomial. The roots of this equation are calculated. For our benefit, the roots yielding the appropriate correction has already been identified, and only this root is calculated.

```matlab
function beamhardeningcorrection(matfile)

% Function is created in order to apply beam-hardening correction to
% the projections files using Ferrari's solution to a quartic function.
% Polynomial is converted to a depressed quartic. After this
% calculation is done, the equation is converted to a perfect square by
% adding a valid identity to the polynomial. Roots of this equation are
% calculated. For our benefit, the roots yielding the correction has
% already been identified and only this root is calculated.

[pathstr, name] = fileparts(matfile);
load(matfile);

% Polynomials created from stage position 300 mm from the source
[p2 p1 p3]=creatincoeff;

% Projection file is subtracted from the constant in the polynomial
outnew1=p1(5)-orig_image;

% Converting to a depressed quartic
coeff1=p1(1);
coeff5=outnew1;
coeff2=((3*p1(2)^2)/(8*p1(1)))+(p1(3));
coeff3=((p1(2)^3/(8*p1(1)^2))-(p1(2)*p1(3)/(2*p1(1)))+(p1(4));
coeff4=((3*p1(2)^4/(256*p1(1)^3))+(p1(2)^2*p1(3)/(16*p1(1)^2))-
   (p1(2)*p1(4)/(4*p1(1))));

alpha=coeff2/coeff1;
beta=coeff3/coeff1;
gamma=(coeff4+coeff5)/coeff1;

% Calculating coefficients to convert the equation obtained into a perfect
% square.'
P = -((alpha^2/12)+gamma);
Q = ((-(alpha^3)/108)+((alpha*gamma)/3)+(-beta^2)/8));
R = (-Q/2) - (sqrt((Q.^2/4)+(P.^3/27)));
U = nthroot(R, 3);
% if U == 0
   y = (-0.833*alpha)+(U)-(nthroot(Q, 3));
else
   y = (-0.833*alpha)+(U)-(P./(3.*U));
end

W = sqrt(alpha+(2*y));

% Solution to the perfect square equation. Calculating only the root which
% has been already tested. x2 is the corrected projection file.

x2 = (-p1(2)/(4*p1(1))+(((W)-sqrt(-(3*alpha)+(2*y)+(2*beta./W))))/2);
% Saving the corrected projection values to matfile with the necessary
% header information

save([name '.mat'],'x2','hdr','centershift');
APPENDIX A-3

Loads (Matlab command `load`) the corrected matfile, inverses the readfct program procedure and writes the floating point file to a 16 bit file in the .att file format. The matfile is then deleted so space is not wasted.

```matlab
function writefct_native(matfile)

% Creates parts of the matfile to which correction has been applied
[pathstr, name] = fileparts(matfile);
load(matfile); % loads matfile

% Create a file to which data can be written.
fh = fopen(["C:\Users\bmil\Desktop\700mm\" name '_new.att'],"w","b");

% Finding minimum and maximum of the floating point file.
maxvalnew=max(max(x2));
minvalnew=min(min(x2));

% Inversing the read file and writing information into the header file.
fwrite(fh,hdr.typestring,'uchar'); % Writing the 3 byte file format
fwrite(fh,hdr.imagenumber,'uint32'); % Writing the file number
fwrite(fh,hdr.extradata,'uchar');
fwrite(fh,hdr.size2,'uint32'); % Writing the number of rows
fwrite(fh,hdr.size1,'uint32'); % Writing the number of columns
fwrite(fh,hdr.skip,'int8');
fwrite(fh,maxvalnew,'float32'); % Writing the maximum floating point value
fwrite(fh,minvalnew,'float32'); % Writing the minimum floating point value

% Converting the floating point corrected values back to 16 bit data type.
out_image=(x2-minvalnew)/(maxvalnew-minvalnew);
out_image=out_image*65536;
out_image=uint16(out_image);
fwrite(fh,out_image,'uint16');
if (centershift > 0)

```

A5
fwrite(fh,centershift);
end
fclose(fh);
% Deleting the matfile to save space.
delete([name '.*.mat'])