Design of HF Forward Transformer Including Harmonic Eddy Current Losses

Dhivya Ammanambakkam Nagarajan

Wright State University

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DESIGN OF HF FORWARD TRANSFORMER INCLUDING HARMONIC EDDY CURRENT LOSSES

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

DHIVYA AMMANAMBakkAM nAGARAJAN
B. E., Anna University, Tamil Nadu, India, 2008

2010
Wright State University

Marian K. Kazimierczuk, Ph.D.
Thesis Director

Kefu Xue, Ph.D.
Chair
Department of Electrical Engineering
College of Engineering and Computer Science

Committee on Final Examination

Marian K. Kazimierczuk, Ph.D.

Ronald G. Riechers, Ph.D.

Saiyu Ren, Ph.D.

Andrew Hsu, Ph.D.
Dean, School of Graduate Studies
Abstract


Pulse Width Modulated (PWM) Forward DC-DC converter is a buck-derived isolated power converter which is used extensively in low power to medium power applications. Satisfactory operation of the transformer utilized in forward converter plays a crucial role in the overall operation of the forward converter. Hence detailed analysis pertaining to design of forward transformer is important. The forward transformer is unique as the magnetizing inductance is not required to store magnetic energy. Additionally, the forward transformer has a tertiary winding, which is required to reset the core and to prevent core saturation. This adds to the complexity of design and analysis as compared to a flyback transformer. The effect of winding losses due to High-Frequency (HF) eddy currents caused by harmonics is also considered in this work. Dowell’s equation was extended to determine the winding resistances for forward transformer in Continuous Conduction Mode (CCM). The Fourier series of the transformer winding current waveforms are derived. The winding resistances derived based on the Dowell’s expression and the current expressions derived based on spectral analysis are employed in evaluating the winding losses. The procedure to design a HF forward transformer in CCM is presented. The effect of harmonics was computed using MATLAB, and verified by circuit simulation with Saber Sketch. The results were found to be in good agreement.
## Contents

1 Introduction 1
   1.1 Pulse Width Modulated (PWM) Converters 1
   1.2 Inductors and Transformers 2
   1.3 Motivation 2
   1.4 Thesis Objectives 3
   1.5 Thesis Outline 4

2 Forward PWM DC-DC Converter 5
   2.1 Description of the Forward Converter Circuit 5
      2.1.1 Time Interval: \([0 < t \leq DT]\) 9
      2.1.2 Time Interval: \([DT < t \leq (D + D1)T]\) 10
      2.1.3 Time Interval \([(D + D1)T < t \leq T]\) 11
   2.2 Duty Cycle Limitation and DC Voltage Transfer Function 11

3 Effect of Eddy Currents at High-Frequency and Harmonic Losses in Forward Transformer in CCM 13
   3.1 Introduction 13
   3.2 Losses in Forward Transformer 13
      3.2.1 Core Loss 13
      3.2.2 Winding or Copper Loss in a Conductor 14
   3.3 Fourier Series of Forward Transformer Winding Currents and Winding Power Loss Expressions 16
      3.3.1 Fourier Series and Power Loss Factor Expression of Primary Current 18
      3.3.2 Fourier Series and Power Loss Factor Expression of Secondary Current 21
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.3 Fourier Series and Power Loss Factor Expression of Tertiary Current</td>
<td>25</td>
</tr>
<tr>
<td>3.4 Expression of Losses in Forward Transformer</td>
<td>27</td>
</tr>
<tr>
<td>4 Design of Forward Transformer using Area Product ($A_p$) and Core Geometry ($K_g$) Methods</td>
<td>29</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>29</td>
</tr>
<tr>
<td>4.1.1 Materials</td>
<td>29</td>
</tr>
<tr>
<td>4.1.2 Core</td>
<td>30</td>
</tr>
<tr>
<td>4.1.3 Conductors</td>
<td>31</td>
</tr>
<tr>
<td>4.2 Assumptions and Area Product ($A_p$) method for Forward Transformer</td>
<td>32</td>
</tr>
<tr>
<td>4.3 Design Procedure of High-Frequency Forward Transformer in CCM</td>
<td>34</td>
</tr>
<tr>
<td>4.3.1 Design Example</td>
<td>42</td>
</tr>
<tr>
<td>4.4 Core Geometry ($K_g$) Method</td>
<td>60</td>
</tr>
<tr>
<td>4.4.1 Design Example</td>
<td>62</td>
</tr>
<tr>
<td>5 Conclusion</td>
<td>80</td>
</tr>
<tr>
<td>5.1 Summary</td>
<td>80</td>
</tr>
<tr>
<td>5.2 Matlab Results</td>
<td>80</td>
</tr>
<tr>
<td>5.3 Saber Results</td>
<td>81</td>
</tr>
<tr>
<td>5.4 Comparison of Results</td>
<td>86</td>
</tr>
<tr>
<td>5.5 Contributions</td>
<td>87</td>
</tr>
<tr>
<td>5.6 Future Work</td>
<td>89</td>
</tr>
<tr>
<td>6 Bibliography</td>
<td>90</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Single-switch three-winding forward converter. . . . . . . . . . . . . . 6
2.2 Three stages of single-switch three-winding forward PWM DC-DC con-
verter. (a) Stage 1. (b) Stage 2. (c) Stage 3. . . . . . . . . . . . . . . 7
2.3 Single-switch three-winding forward PWM DC-DC converter current
and voltage waveforms. . . . . . . . . . . . . . . . . . . . . . . . . . . 8
3.1 Flow of current at high-frequencies. . . . . . . . . . . . . . . . . . . . 15
3.2 Ideal switch voltage and winding currents waveforms of single-switch
forward converter. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 17
3.3 Construction of primary current waveform using the derived Fourier
series expression. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 20
3.4 Primary current waveform obtained from Saber simulation. . . . . . . 21
3.5 Construction of secondary current waveform using the derived Fourier
series expression. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 23
3.6 Secondary current waveform obtained from Saber simulation. . . . . . 24
3.7 Construction of tertiary current waveform using the derived Fourier
series expression. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
3.8 Tertiary current waveform obtained from Saber simulation. . . . . . . 28
4.1 Illustration of current through magnetizing inductance. . . . . . . . . 32
4.2 Core loss $P_C$ with respect to input DC voltage $V_I$ at specific output
powers $P_O$ for forward converter in CCM. . . . . . . . . . . . . . . . . 47
4.3 Harmonic primary power loss factor $F_{Rph}$ with respect to input DC
voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 48
4.4 Primary winding power loss $P_{wp}$ with respect to input DC voltage $V_I$
at specific output powers $P_O$ for forward converter in CCM. . . . . . . 49
4.5 Primary winding power loss $P_{wp}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. . . . . . . . 49

4.6 Harmonic secondary power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 51

4.7 Secondary winding power loss $P_{ws}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. . . . . . . 51

4.8 Secondary winding power loss $P_{ws}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. . . . . . 52

4.9 Harmonic tertiary power loss factor $F_{Rth}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 54

4.10 Tertiary winding power loss $P_{wt}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. . . . . . 54

4.11 Tertiary winding power loss $P_{wt}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. . . . . . 55

4.12 Total winding loss $P_w$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. . . . . . . . . . . . 56

4.13 Total winding loss $P_w$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. . . . . . . . . . . 56

4.14 Total loss in the transformer $P_{cw}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. . . . . . . 57

4.15 Total loss in the transformer $P_{cw}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. . . . . . . 57

4.16 Efficiency of transformer $\eta_t$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. . . . . . . . . . 58

4.17 Efficiency of transformer $\eta_t$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. . . . . . . . . 59
4.18 Transformer temperature rise $\Delta T$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. 

4.19 Core loss $P_C$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.20 Harmonic primary power loss factor $F_{Rph}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.21 Primary winding power loss $P_{wp}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.22 Primary winding power loss $P_{wp}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. 

4.23 Harmonic secondary power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.24 Secondary winding power loss $P_{ws}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.25 Secondary winding power loss $P_{ws}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. 

4.26 Harmonic tertiary power loss factor $F_{Rth}$ with respect to input DC voltage $V_I$ at specific of output powers $P_O$ for forward converter in CCM. 

4.27 Tertiary winding power loss $P_{wt}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.28 Tertiary winding power loss $P_{wt}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM. 

4.29 Total winding loss $P_w$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. 

4.30 Total winding loss $P_w$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
4.31 Total loss in the transformer $P_{cw}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

4.32 Total loss in the transformer $P_{cw}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.

4.33 Efficiency of a transformer $\eta_t$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

4.34 Efficiency of a transformer $\eta_t$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.

4.35 Transformer temperature rise $\Delta T$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.

5.1 Construction of primary current waveform for forward converter in CCM using Fourier series.

5.2 Construction of secondary current waveform for forward converter in CCM using Fourier series.

5.3 Construction of tertiary current waveform for forward converter in CCM using Fourier series.

5.4 Primary current spectrum of forward converter operating in CCM obtained from Matlab for maximum input voltage and output power.

5.5 Secondary current spectrum of forward converter operating in CCM obtained from Matlab for maximum input voltage and output power.

5.6 Tertiary current spectrum of forward converter operating in CCM obtained from Matlab for maximum input voltage and output power.

5.7 Primary current spectrum of forward converter operating in CCM obtained from Matlab for minimum input voltage and maximum output power.
5.8 Secondary current spectrum of forward converter operating in CCM obtained from Matlab for minimum input voltage and maximum output power. .............................................................. 85

5.9 Tertiary current spectrum of forward converter operating in CCM obtained from Matlab for minimum input voltage and maximum output power. .............................................................. 85

5.10 Saber schematic of forward converter operating in CCM. ................................. 86

5.11 Saber output DC voltage $V_O$ of forward converter operating in CCM. ........... 87

5.12 Forward converter winding current waveforms operating in CCM obtained from Saber for maximum input voltage and output power. .............................................................. 88

5.13 Forward converter winding currents spectrum operating in CCM obtained from Saber for maximum input voltage and output power. .............................................................. 88
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1 Introduction

1.1 Pulse Width Modulated (PWM) Converters

A switching converter is broadly used in the power engineering applications such as industrial electronics, consumer electronics, etc. It is used as an interface to provide a regulated output voltage. The converter can be divided into voltage step-up converter and voltage step-down converter [1]. The Pulse Width Modulated (PWM) converter can be classified as an isolated and non-isolated converter depending on whether the converter has an inductor or a transformer in its configuration. Magnetic components and capacitors are building blocks of a converter [1]-[4]. A filter circuit (Low Pass Filter) is provided to eliminate the harmonics, thereby improving the efficiency of the converter. At Low-Frequency (LF) the presence of magnetic components makes the converter bulky but easier to analyse [2]. However, at High-Frequency (HF), the converter is scaled down causing further complexity in the analysis due to eddy current losses in addition to pulsating waveforms of the converter which are rich in harmonics [1], [2]. Forward PWM power converter is a buck-derived isolated converter and can be used in low to medium power applications [1] -[4]. Other versions of forward converters [1], [5] are:

- push-pull converter
- half-bridge converter
- full-bridge converter.

Specific field of applications include wireless technologies, aerospace, medical equipments and laptop chargers [1], [2], [6].
1.2 Inductors and Transformers

The use of energy storage elements i.e. inductors and transformers in power converters is essential. The inductors are used as a part of the filter circuit and to store energy. The principal usage of a transformer in the converter is to provide galvanic isolation between input and output [1]. The two widely used methods to design a magnetic component are area product \( (A_p) \) and core geometry \( (K_g) \) methods [2], [5], [8]. Design procedures of inductors and transformers are presented in [5]. The design methodology proposed in [8]-[12] incorporates the winding losses due to harmonics and the winding resistances are calculated using Dowell’s equation. The derived expressions for Fourier series of current waveforms and winding resistances are used in evaluating the winding losses of the transformer and inductor in [8]-[12].

1.3 Motivation

This thesis deals with single-switch three-winding forward converter. It is widely used in the industry for low dc-to-dc power conversions [7]. The efficient working of this power converter is based on the performance of the transformer. The inherent complexity of the transformer due to the core reset circuit presents a greater challenge in the design and analysis, which was a driving force behind this work.

The procedure to calculate the harmonic winding losses of non-isolated PWM DC-DC converter in Continuous Conduction Mode (CCM) and buck DC-DC converter in Discontinuous Conduction Mode (DCM) is presented in [12], [9], [8]. The design procedure of two-winding flyback transformer uses area product \( (A_p) \) method taking into account the harmonic winding losses in HF transformer is proposed in [10]-[11] and in [8] both area product \( (A_p) \) and core geometry \( (K_g) \) methods are discussed. This thesis extends the theory developed in [8]-[12] to design a single-switch three-winding Forward transformer operated in CCM using the two popular methods area...
product \((A_p)\) and core geometry \((K_g)\) methods [5].

Some of the different features between single-switch three-winding forward and flyback transformer are as follows:

- forward transformer has three windings, compared to flyback which has two windings
- can be used in higher output power applications
- not required to have an air gap since magnetizing inductance does not store energy [2], [5]
- has less leakage inductances.

1.4 Thesis Objectives

The objectives of this thesis are highlighted below:

- to derive the Fourier series of non-sinusoidal winding currents for a forward transformer in CCM
- to present the expressions for winding power loss by implementing the derived expressions of current waveforms
- to present a procedure for the design of forward transformer in CCM using the well known area product \((A_p)\) and core geometry \((K_g)\) methods [5], [8]-[12]
- to plot characteristics of the transformer using the derived expressions in MATLAB and to verify it by Saber Sketch simulation.
1.5 Thesis Outline

The contents of this thesis are:

- Brief description of single-switch forward converter presented in Chapter 2.

- The effects of eddy currents at high-frequency and harmonic losses in forward transformer in CCM is dealt in Chapter 3.

- The procedure to design forward transformer in CCM using area product ($A_p$) and core geometry ($K_g$) methods are discussed in Chapter 4, and

- Finally, Chapter 5 deals with simulation results and future work.
2 Forward PWM DC-DC Converter

A forward PWM DC-DC power converter is widely used for applications under 500 W. It is a buck-derived isolated converter. The conventional forward converter has three windings [1]. The purpose of the tertiary winding is to reset the core and to prevent core saturation [1]-[4] since the magnetizing inductance is not required to store the magnetic energy. The current through the tertiary winding is small as it carries only the demagnetizing winding current [1], [4]. Similar to other converters the forward converter can also be operated in both Continuous Conduction Mode (CCM) and Discontinuous Conduction Mode (DCM) [1]. In most applications, the number of primary and tertiary turns are assumed to be same [1], [2], and if the number of turns are the same, then there is a limitation on the duty cycle of 50 % i.e. 0.5. The forward converter is complex when compared to other converters because of the demagnetizing core reset circuit. This chapter presents the three-winding forward PWM DC-DC converter circuit and its operating stages in CCM.

2.1 Description of the Forward Converter Circuit

The three-winding single-switch forward PWM DC-DC converter is drawn from a buck converter by adding a diode and a transformer to it [1]. Accordingly, the forward converter belongs to the category of buck-derived converter, called the transformer version of buck converter [1]. The single-switch three-winding forward converter is shown in Figure 2.1 and its equivalent waveforms are shown in Figure 2.3. The converter comprises of a switch represented as $S$, three diodes denoted by $D1$, $D2$, and $D3$, where $D2$ is the freewheeling diode and $D3$ is the core reset diode. The converter also consists of passive elements, a resistor, inductor and a capacitor denoted as $R$, $L$ and $C$ respectively. In addition to these components it has a DC input supply voltage denoted by $V_I$ [1].
Figure 2.1: Single-switch three-winding forward converter.
Figure 2.2: Three stages of single-switch three-winding forward PWM DC-DC converter. (a) Stage 1. (b) Stage 2. (c) Stage 3.
Figure 2.3: Single-switch three-winding forward PWM DC-DC converter current and voltage waveforms.
Some assumptions are made for the analysis of forward PWM DC-DC converter [1], they are:

- passive components are considered to be linear and independent of frequency
- transformer is considered ideal
- the ac and dc output impedance of the input voltage source are assumed to be zero
- the switch and diodes are assumed to be ideal and
- the switching losses are assumed to be zero.

The transformer turns ratio and voltages are related by the following expression

\[ v_1 : v_2 : v_3 = N_1 : N_2 : -(N_3) \]  \hspace{1cm} (2.1)

where \( N_1, N_2, \) and \( N_3 \) are the number of primary, secondary and tertiary turns, respectively [1]. Dividing the equation (2.1) by \( N_2 \) it yields

\[ v_1 : v_2 : v_3 = n_1 : 1 : -(n_3) \]  \hspace{1cm} (2.2)

where \( n_1 = \frac{N_1}{N_2} \) and \( n_3 = \frac{N_3}{N_2} \) are the transformer ratio of primary to secondary and tertiary to secondary, respectively [1].

There are three stages of operation for a three-winding forward converter. They are as follows:

2.1.1 Time Interval: \([0 < t \leq DT]\)

During this stage the switch \( S \) and diode \( D1 \) are active. Other diodes \( D2 \) and \( D3 \) are OFF. The circuit for this stage is shown in Figure 2.2(a). The voltage across the
primary of the transformer and the magnetizing inductance remains the same when the switch is ON [1], [2] and is given by

\[ v_1 = v_{Lm} = V_I. \] (2.3)

The current through the secondary winding, diode D1 and inductor L remains the same [1] and is given by

\[ i_2 = i_{D1} = i_L = \frac{V_i}{n_1} - \frac{V_O}{L} + i_L(0). \] (2.4)

From equation (2.4), the current through the primary is given by

\[ i_1 = \frac{i_2}{n_1} = \frac{V_i}{n_1} - \frac{V_O}{n_1 L} + \frac{i_L(0)}{n_1}. \] (2.5)

The maximum magnetizing current [1] is given by

\[ \Delta i_{L_{max}} = \frac{D_{\text{min}} V_{I_{\text{max}}}}{f_s L_{m(\text{min})}}. \] (2.6)

The variation of magnetizing current \( \Delta i_{L_{\text{max}}} \) is usually from 5-10\% of the maximum primary current of the transformer [1].

2.1.2 Time Interval: \([DT < t \leq (D + D1)T]\)

During the second stage of operation the diodes D2 and D3 are active. The diode D2 will be active when both the switch S and diode D1 are OFF. The circuit for this stage of operation is shown in Figure 2.2(b). During this stage, the current through the inductor is circulated on the secondary side, as a result making diode D2 forward biased. This causes the inductor current to decrease [1]. The reset circuit is utilized in this stage by transferring the energy from the magnetizing inductance to the input source thereby preventing core saturation [1], [2]. This results in decrease in current through magnetizing inductance \( L_m \). To end this stage, the currents through the magnetizing inductance and reset diode should reach zero [1], [2]. The current
through the diode $D2$ and inductor $L$ are same [1] and is given by

$$i_{D2} = \frac{-V_O}{L}(t - DT) + i_L(DT). \tag{2.7}$$

The current through the diode $D3$ and the tertiary winding are same [1] and is given by

$$i_{D3} = -\frac{n_1}{n_3}\frac{n_1V_I}{n_3L_m}(t - DT) - \frac{DV_I}{f_sL_m}$$

$$i_{D3} = -\frac{n_1}{n_3}i_1 = \frac{n_1}{n_3}i_{Lm}. \tag{2.8}$$

The voltage across the switch is given by

$$v_s = \left(\frac{n_1}{n_3} + 1\right)V_I. \tag{2.9}$$

When the number of primary and tertiary turns are same, then the voltage across the switch will be twice the input voltage $V_I$ [1].

### 2.1.3 Time Interval $[(D + D1)T < t \leq T]$

During this stage only diode $D2$ is active. The circuit for this stage is shown in Figure 2.2(c). The voltage across the windings of the transformer and magnetizing inductance are zero [1]. The voltage across the switch is

$$v_S = V_I. \tag{2.10}$$

The current through the diode $D2$ will remain the same as in stage 2 and is given by

$$i_{D2} = \frac{-V_O}{L}(t - DT) + i_L(DT). \tag{2.11}$$

### 2.2 Duty Cycle Limitation and DC Voltage Transfer Function

The magnetizing inductance is made to operate in DCM i.e. current must reach zero before the switching period [2]. Otherwise the current will increase if operated in CCM and might lead to core saturation [2]. The magnetizing inductance is not
required to store magnetic energy [1]. Hence the core will be reset, thereby preventing it from saturation. But this has a limitation on the maximum value of the duty cycle for forward converter [1], [2].

For the worst case [1],

\[ D_{MAX}T + t_m = T. \]  \hspace{1cm} (2.12)

Simplifying equation (2.12) we get

\[ t_m = T - D_{MAX}T. \]  \hspace{1cm} (2.13)

Applying volt-second balance to the magnetizing inductance voltage, the value of maximum duty cycle of the forward converter is given by [1]

\[ D_{MAX} = \frac{1}{\frac{n_2}{n_1} + 1}. \]  \hspace{1cm} (2.14)

If the transformer turns ratio are same then \( D_{MAX} = 0.5 \) [1], [2].

Applying volt-second balance on the inductor voltage waveform and on simplifying, the dc voltage transfer function can be obtained as [1]

\[ M_{V_{DC}} \equiv \frac{V_O}{V_I} = \frac{I_I}{I_O} = \frac{D}{n_1}, \quad D \leq D_{MAX}. \]  \hspace{1cm} (2.15)

And current transfer function is given by,

\[ M_{I_{DC}} \equiv \frac{I_O}{I_I} = \frac{n_1}{D}, \quad D \leq D_{MAX}. \]  \hspace{1cm} (2.16)

From equation (2.15) we can obtain the relation between output voltage in-terms of input voltage [1]

\[ V_O = \frac{D V_I}{n_1}. \]  \hspace{1cm} (2.17)
3 Effect of Eddy Currents at High-Frequency and Harmonic Losses in Forward Transformer in CCM

3.1 Introduction

The Pulse Width Modulated (PWM) converters generate periodic rectangular voltage and current waveforms. These waveforms are rich in harmonics because of hard switching action of the converter [1], [2]. Generally the source of excitation is considered to be sinusoidal at low-frequency (LF) for the design of transformers [13]. There is an added intricacy in the evaluation of losses at high-frequency due to the hard switching behaviour of converter which gives rise to non-sinusoidal waveforms [1], [2], [13].

When converters are operated at HF, the size of the magnetic components reduce, which considerably saves the space occupied by the transformer [8]. However the adverse effects of this is an increase in losses caused by eddy currents [2], [8]-[12].

3.2 Losses in Forward Transformer

As mentioned previously, the non sinusoidal periodic voltage and current waveforms of a transformer at HF engenders eddy current losses in both, the core and conductor. The losses of transformer can be classified into core and winding or copper loss [2], [8].

3.2.1 Core Loss

The core loss is further subdivided into hysteresis and eddy current core loss [8].

Hysteresis Loss

The hysteresis loss is the energy consumed by the core to get magnetized, and is dependent on the properties of the core such as material composition, size and shape [8]. This loss accounts for the energy required for the magnetic domains of a core to
get aligned. Some energy is lost as heat which leads to an increase in temperature of the core [8]. The hysteresis loss is examined by plotting a graph of flux density and field intensity of the core. The area traversed by the core corresponds to the hysteresis loss per unit core volume [8].

**Eddy Current Core Loss**

According to Amperes law, a magnetic field is produced by a time varying current. This magnetic field induces eddy currents by Lenz’s law. These induced eddy currents produce a secondary magnetic field which in turn induces another group of eddy currents. Because of this continuous chain reaction, eddy currents are built up in a core [8].

The core loss is expressed by an empirical formula [8]

\[
P_c = P_h + P_{ev} = P_v V_c = k(f_s \text{ in kHz})^a(10B_m(\text{max})\text{ in T})^bV_c \quad (\text{mW/cm}^3). \quad (3.1)
\]

where, \(f_s\) is the frequency in kHz, \(B_m\) is maximum core flux density in T, and \(V_c\) is core volume in cm\(^3\). The co-efficients \(k, a,\) and \(b\) can be obtained from core manufactures data sheet [17].

**3.2.2 Winding or Copper Loss in a Conductor**

The winding losses of a conductor due to eddy currents at HF can be divided into two, the skin and proximity effects [8].

**Skin and Proximity Effects**

Skin effect arises in a single conductor carrying a current at HF [2], [8], [13]. According to Amperes law, a time varying magnetic field is produced by a time varying current. Eddy currents are induced by Lenz law. These eddy currents consequentially produce another magnetic field which in turn produce eddy currents [8]. Since the eddy currents are build up in the conductor there is an additional flow of current, and it
flows towards the surface of the conductor [2], [8]. The Figure 3.1 shows the flow of current at HF. Subsequently the current flow throughout the conductor is inconsistent, the area occupied by the current is less, therefore ac resistance is high. The current density is higher towards the conductor surface and decreases as it approaches the center [2], [8].

Proximity effect arises when there are more than two conductors in close proximity [2], [8], [13]. It is similar to skin effect. The influence of the magnetic field of the first conductor on the second conductor will induce eddy currents in the second conductor and skin effect will also be produced, if the second conductor carries current, otherwise only proximity effect is present [8]. This effect is more pronounced when there is more than one layer [8]. This effect also has non-uniform current flowing through the conductor and therefore the conductor area usage is less [8].

**Dowell’s Equation for Round Conductor**

The expression of conductor ac winding resistance at HF presented by Dowell is used [15], [8]. The ac to dc resistance ratio equation for round conductor is given by [15],
where $F_S$ is the skin effect factor

$$F_S = \frac{\sinh(2A) + \sin(2A)}{\cosh(2A) - \cos(2A)}$$  \hspace{1cm} (3.4)$$

and $F_P$ is the proximity effect factor

$$F_P = \frac{2(N_l^2 - 1) \sinh(2A) - \sin(2A)}{3 \cosh(2A) + \cos(2A)}.$$  \hspace{1cm} (3.5)$$

The thickness of a round conductor with respect to skin depth is given by

$$A = \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \frac{d}{\delta_w} \sqrt{\eta}.$$  \hspace{1cm} (3.6)$$

where $N_l$ is the number of layers, $\delta_w$ is the skin depth, $\eta$ is the porosity factor and $d$ is the bare wire diameter [8].

At low frequency (LF) these effects is neglected as the current flow is consistent, whereas at HF these effects cannot be neglected as the ac resistance is substantially increased. The ac resistance at each harmonic can be obtained using Dowell’s equation [8]-[12].

### 3.3 Fourier Series of Forward Transformer Winding Currents and Winding Power Loss Expressions

To calculate the losses due to harmonics, the current at each harmonic can be obtained by deriving the Fourier series of the winding currents as demonstrated in [8]-[12], [2], [14]. The PWM forward converter carries pulsating non sinusoidal waveforms which has intense harmonics [1], [2]. The winding losses can be evaluated by summing the product of current and resistance at dc and the summation of power loss at each harmonics [8]-[12].
Figure 3.2: Ideal switch voltage and winding currents waveforms of single-switch forward converter.
The Fourier series of the winding current waveforms of a forward transformer are determined using [16]. The magnetizing current generally varies from 5-10% of maximum primary current [1]. This magnetizing current percentage is expressed as $y$. The waveforms of the forward transformer are shown in Figure 3.2, they are not modified and the Fourier series of current waveforms are as follows:

### 3.3.1 Fourier Series and Power Loss Factor Expression of Primary Current

The forward transformer primary current for CCM operation is given by

$$i_p = \begin{cases} 
\frac{1}{n_1} \left( \frac{\Delta i_{L\text{max}}}{DT} t + I_{O\text{max}} - \frac{\Delta i_{L\text{max}}}{2} \right), & \text{for } 0 < t \leq DT \\
\frac{y}{n_1} \left( I_{O\text{max}} + \frac{\Delta i_{L\text{max}}}{2} \right) \left[ \frac{(t-DT)}{D1T} - 1 \right], & \text{for } DT < t \leq (D + D1)T \\
0, & \text{for } (D + D1)T < t \leq T 
\end{cases}$$

(3.7)

The DC component is given by

$$I_{pdc} = \frac{1}{T} \int_0^T i_p dt$$

$$= \int_0^{DT} \frac{1}{n_1} \left( \frac{\Delta i_{L\text{max}}}{DT} t + I_{O\text{max}} - \frac{\Delta i_{L\text{max}}}{2} \right) dt$$

$$+ \int_{DT}^{(D+D1)T} \frac{y}{n_1} \left( I_{O\text{max}} + \frac{\Delta i_{L\text{max}}}{2} \right) \left[ \frac{(t-DT)}{D1T} - 1 \right] dt$$

$$= \frac{1}{n_1} \left\{ I_{O\text{max}} D + y \left( I_{O\text{max}} + \frac{\Delta i_{L\text{max}}}{2} \right) \left[ \frac{(D+D1)^2}{2D1} - \frac{D^2}{2D1} - D - D1 \right] \right\}.$$  

(3.8)

The primary winding current waveform Fourier series coefficients are:

$$a_{np} = \frac{2}{T} \int_0^T i_p \cos(n \omega t) dt$$

$$= \int_0^{DT} \frac{1}{n_1} \left( \frac{\Delta i_{L\text{max}}}{DT} t + I_{O\text{max}} - \frac{\Delta i_{L\text{max}}}{2} \right) \cos(n \omega t) dt$$

18
\[
+ \int_{DT}^{(D+D1)T} \frac{y}{n_1} \left( I_{Omax} + \frac{\Delta i_{Lmax}}{2} \right) \left[ \frac{(t - DT)}{D1T} - 1 \right] \cos(n\omega t) dt = \frac{I_{Omax}}{2n_1n^2\pi^2} X_{np} (3.9)
\]

and

\[
b_{np} = \frac{2}{T} \int_0^T i_p \sin(n\omega t) dt = \int_0^{DT} \frac{1}{n_1} \left( \frac{\Delta i_{Lmax}}{D1T} t + I_{Omax} - \frac{\Delta i_{Lmax}}{2} \right) \sin(n\omega t) dt + \int_{DT}^{(D+D1)T} \frac{y}{n_1} \left( I_{Omax} + \frac{\Delta i_{Lmax}}{2} \right) \left[ \frac{(t - DT)}{D1T} - 1 \right] \sin(n\omega t) dt = \frac{I_{Omax}}{2n_1n^2\pi^2} Y_{np} (3.10)
\]

where

\[
X_{np} = \left( 1 + \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) 2n\pi \sin(2n\pi D) + \frac{\Delta i_{Lmax}}{D1I_{Omax}} (\cos(2n\pi D) - 1) + y \left( 1 + \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) \left[ \frac{1}{D1} (\cos(2n\pi (D + D1)) - \cos(2n\pi D)) + 2n\pi \sin(2n\pi D) \right] (3.11)
\]

and

\[
Y_{np} = \frac{\Delta i_{Lmax}}{D1I_{Omax}} \sin(2n\pi D) - \left( 1 + \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) 2n\pi \cos(2n\pi D) + 2n\pi \left( 1 - \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) + y \left( 1 + \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) \left[ \frac{1}{D1} (\sin(2n\pi (D + D1)) - \sin(2n\pi D)) - 2n\pi \cos(2n\pi D) \right] . (3.12)
\]

The primary winding current harmonics is given by

\[
I_{pn} = \sqrt{a_{np}^2 + b_{np}^2} = \frac{I_{Omax}}{2n_1n^2\pi^2} \sqrt{X_{np}^2 + Y_{np}^2}. (3.13)
\]

The Fourier series of primary current waveform is expressed as

\[
i_p = I_{pdc} + \frac{I_{Omax}}{2n_1n^2} \sum_{n=1}^{\infty} \frac{X_{np}}{n^2} \cos(n\omega t) + \frac{Y_{np}}{n^2} \sin(n\omega t)
\]
Figure 3.3: Construction of primary current waveform using the derived Fourier series expression.

\[
I_p = I_{pdc} \left[ 1 + \frac{I_{Omax}}{I_{pdc}} \frac{1}{2n_1 \pi^2} \sum_{n=1}^{\infty} \frac{X_{np}}{n^2} \cos(n \omega t) + \frac{Y_{np}}{n^2} \sin(n \omega t) \right]. \tag{3.14}
\]

Figures 3.3 and 3.4 show the primary current waveform constructed using Fourier series and primary current obtained from Saber simulation respectively. The primary winding power loss is expressed as a sum of DC power loss and summation of harmonic power loss,

\[
P_{wp} = R_{wpdc} I_{pdc}^2 + \frac{1}{2} \sum_{n=1}^{\infty} R_{wpi} I_{pn}^2. \tag{3.15}
\]

Simplifying equation (3.15) we get

\[
P_{wp} = R_{wpdc} I_{pdc}^2 \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} \left( \frac{R_{wpn}}{R_{wpdc}} \right) \left( \frac{I_{pn}}{I_{pdc}} \right)^2 \right] \tag{3.16}
\]

= \[ R_{wpdc} I_{pdc}^2 \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} F_{Rpm} \left( \frac{I_{pn}}{I_{pdc}} \right)^2 \right] \]
\[ P_{wpdc} \left[ 1 + \frac{1}{8\pi^4 n_1^2} \left( \frac{I_{Omax}}{I_{pdc}} \right)^2 \sum_{n=1}^{\infty} \frac{F_{Rpm}(X_{np}^2 + Y_{np}^2)}{n^4} \right] = P_{wpdc} F_{Rph}. \]  

(3.17)

The ratio of ac to dc resistance is replaced by Dowell’s equation in [8]-[12]. Similarly the resistance ratio is replaced in the above expression and \( F_{Rpm} \) is the primary winding ac to dc resistance ratio. The ratio of ac to dc primary winding power loss and is expressed by a factor \( F_{Rph} \)

\[ F_{Rph} = \frac{P_{wp}}{P_{wpdc}} = 1 + \frac{1}{8\pi^4 n_1^2} \left( \frac{I_{Omax}}{I_{pdc}} \right)^2 \sum_{n=1}^{\infty} \frac{F_{Rpm}(X_{np}^2 + Y_{np}^2)}{n^4}. \]  

(3.18)

3.3.2 Fourier Series and Power Loss Factor Expression of Secondary Current

The forward transformer secondary current for CCM operation is given by

\[ i_s = \begin{cases} 
I_{Omax} + \Delta i_{Lmax} \left( \frac{t}{DT} - \frac{1}{2} \right), & \text{for } 0 < t \leq DT \\
0, & \text{for } DT < t \leq T 
\end{cases}. \]  

(3.19)
The DC component is given by

\[ I_{sdc} = \frac{1}{T} \int_0^T i_s dt \]

\[ = \frac{DT}{T} \int_0^{DT} \left( I_{Omax} + \Delta i_{Lmax} \left( \frac{t}{DT} - \frac{1}{2} \right) \right) dt = DI_{Omax}. \quad (3.20) \]

The secondary winding current waveform Fourier series coefficients are:

\[ a_{ns} = \frac{2}{T} \int_0^T i_s \cos(n\omega t) dt \]

\[ = \frac{DT}{T} \int_0^{DT} \left( I_{Omax} + \Delta i_{Lmax} \left( \frac{t}{DT} - \frac{1}{2} \right) \right) \cos(n\omega t) dt \]

\[ = \frac{I_{Omax}}{2n^2 \pi^2} X_{ns} \quad (3.21) \]

and

\[ b_{ns} = \frac{2}{T} \int_0^T i_s \sin(n\omega t) dt \]

\[ = \frac{DT}{T} \int_0^{DT} \left( I_{Omax} + \Delta i_{Lmax} \left( \frac{t}{DT} - \frac{1}{2} \right) \right) \sin(n\omega t) dt \]

\[ = \frac{I_{Omax}}{2n^2 \pi^2} Y_{ns} \quad (3.22) \]

where

\[ X_{ns} = \left( 1 + \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) 2n\pi \sin(2n\pi D) + \frac{\Delta i_{Lmax}}{DI_{Omax}} (\cos(2n\pi D) - 1) \quad (3.23) \]

and

\[ Y_{ns} = \frac{\Delta i_{Lmax}}{DI_{Omax}} \sin(2n\pi D) - \left( 1 + \frac{\Delta i_{Lmax}}{2I_{Omax}} \right) 2n\pi \cos(2n\pi D) \]

\[ + 2n\pi \left( 1 - \frac{\Delta i_{Lmax}}{2I_{Omax}} \right). \quad (3.24) \]

The secondary winding current harmonics is given by

\[ I_{sn} = \sqrt{a_{ns}^2 + b_{ns}^2} = \frac{I_{Omax}}{2n^2 \pi^2} \sqrt{X_{ns}^2 + Y_{ns}^2} \]
\[ i_s = I_{sdc} + \frac{I_{O_{\text{max}}}}{2\pi^2} \sum_{n=1}^{\infty} \frac{X_{ns}}{n^2} \cos(n\omega t) + \frac{Y_{ns}}{n^2} \sin(n\omega t) \]

\[ = I_{sdc} \left[ 1 + \frac{I_{O_{\text{max}}}}{I_{sdc}} \frac{1}{2\pi^2} \sum_{n=1}^{\infty} \frac{X_{ns}}{n^2} \cos(n\omega t) + \frac{Y_{ns}}{n^2} \sin(n\omega t) \right]. \]  

(3.26)

The Fourier series of secondary current waveform is expressed as

\[ = \frac{I_{sdc}}{2n^2\pi^2 D} \sqrt{X_{ns}^2 + Y_{ns}^2} \]  

(3.25)

Figures 3.5 and 3.6 show the secondary current waveform constructed using Fourier series and secondary current obtained from Saber simulation respectively. The secondary winding power loss is expressed as a sum of DC power loss and summation of harmonic power loss,

\[ P_{ws} = R_{wsdc}I_{sdc}^2 + \frac{1}{2} \sum_{n=1}^{\infty} R_{wns}I_{sn}^2. \]  

(3.27)
Figure 3.6: Secondary current waveform obtained from Saber simulation.

Simplifying equation (3.27) we get

\[ P_{ws} = R_{wsdc}I_{sdc}^2 \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} \left( \frac{R_{wsn}}{R_{wsdc}} \right) \left( \frac{I_{sn}}{I_{sdc}} \right)^2 \right] \] (3.28)

\[ = R_{wsdc}I_{sdc}^2 \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} F_{Rsn} \left( \frac{I_{sn}}{I_{sdc}} \right)^2 \right] \]

\[ = P_{wsdc} \left[ 1 + \frac{1}{8\pi^4D^2} \sum_{n=1}^{\infty} F_{Rsn} \left( \frac{X_{ns}^2 + Y_{ns}^2}{n^4} \right) \right] \]

\[ = P_{wsdc} F_{Rsh}. \] (3.29)

The ratio of ac to dc resistance is replaced by Dowell’s equation in [8]-[12]. Similarly the resistance ratio is replaced in the above expression and \( F_{Rsn} \) is the secondary winding ac to dc resistance ratio. The ratio of ac to dc secondary winding power loss is expressed by a factor \( F_{Rsh} \)

\[ F_{Rsh} = \frac{P_{ws}}{P_{wsdc}} = 1 + \frac{1}{8\pi^4D^2} \sum_{n=1}^{\infty} F_{Rsn} \left( \frac{X_{ns}^2 + Y_{ns}^2}{n^4} \right). \] (3.30)
3.3.3 Fourier Series and Power Loss Factor Expression of Tertiary Current

The forward transformer tertiary current for CCM operation is given by

\[ i_t = \begin{cases} 
\frac{y}{n_1} (I_{Omax} + \frac{\Delta i_{Lmax}}{2}) \left[ 1 - \frac{(t-DT)}{D1T} \right], & \text{for } 0 < t \leq DT \\
0, & \text{for } DT < t \leq (D + D1)T \\
0, & \text{for } (D + D1)T < t \leq T 
\end{cases} \]  
(3.31)

The DC component is given by

\[ I_{t_{dc}} = \frac{1}{T} \int_0^T i_t dt \]

\[ = \int_{DT}^{(D+D1)T} \frac{y}{n_1} \left( I_{Omax} + \frac{\Delta i_{Lmax}}{2} \right) \left[ 1 - \frac{(t-DT)}{D1T} \right] dt \]

\[ = \frac{y}{n_1} \left( I_{Omax} + \frac{\Delta i_{Lmax}}{2} \right) \left[ \frac{D^2}{2D1} - \frac{(D + D1)^2}{2D1} + D + D1 \right]. \]  
(3.32)

The tertiary winding current Fourier series coefficients are:

\[ a_{nt} = \frac{2}{T} \int_0^T i_t \cos(n \omega t) dt \]

\[ = \int_{DT}^{(D+D1)T} \frac{y}{n_1} \left( I_{Omax} + \frac{\Delta i_{Lmax}}{2} \right) \left[ 1 - \frac{(t-DT)}{D1T} \right] \cos(n \omega t) dt \]

\[ = \frac{I_{Omax}}{2n_1 n_2^2 \pi^2} X_{nt} \]  
(3.33)

and

\[ b_{nt} = \frac{2}{T} \int_0^T i_t \sin(n \omega t) dt \]

\[ = \int_{DT}^{(D+D1)T} \frac{y}{n_1} \left( I_{Omax} + \frac{\Delta i_{Lmax}}{2} \right) \left[ 1 - \frac{(t-DT)}{D1T} \right] \sin(n \omega t) dt \]

\[ = \frac{I_{Omax}}{2n_1 n_2^2 \pi^2} Y_{nt} \]  
(3.34)
where

\[ X_{nt} = y \left(1 + \frac{\Delta i_{L_{\text{max}}}}{2I_{O_{\text{max}}}}\right) \]

\[ \left[ \frac{1}{D_1} (\cos(2n\pi D) - \cos(2n\pi(D + D1))) - 2n\pi \sin(2n\pi D) \right] \]

and

\[ Y_{nt} = y \left(1 + \frac{\Delta i_{L_{\text{max}}}}{2I_{O_{\text{max}}}}\right) \]

\[ \left[ \frac{1}{D_1} (\sin(2n\pi D) - \sin(2n\pi(D + D1))) + 2n\pi \cos(2n\pi D) \right] . \]  

(3.35)

The tertiary winding current harmonics is given by

\[ I_{tn} = \sqrt{a_{nt}^2 + b_{nt}^2} = \frac{I_{O_{\text{max}}}}{2n_1^2} \sqrt{X_{nt}^2 + Y_{nt}^2}. \]

(3.37)

The Fourier series of tertiary current waveform is expressed as

\[ i_t = I_{tdc} + \frac{I_{O_{\text{max}}}}{2n_1^2} \sum_{n=1}^{\infty} \frac{X_{nt}}{n^2} \cos(n\omega t) + \frac{Y_{nt}}{n^2} \sin(n\omega t) \]

\[ = I_{tdc} \left[ 1 + \frac{1}{I_{tdc}} \frac{1}{2n_1^2} \sum_{n=1}^{\infty} \frac{X_{nt}}{n^2} \cos(n\omega t) + \frac{Y_{nt}}{n^2} \sin(n\omega t) \right]. \]

(3.38)

Figures 3.7 and 3.8 show the tertiary current waveform constructed using Fourier series and tertiary current obtained from Saber simulation respectively. The tertiary winding power loss is expressed as a sum of DC power loss and summation of harmonic power loss,

\[ P_{wt} = R_{wt_{dc}}I_{tdc}^2 + \frac{1}{2} \sum_{n=1}^{\infty} R_{wn}I_{tn}^2. \]

(3.39)

Simplifying equation (3.39) we get

\[ P_{wt} = R_{wt_{dc}}I_{tdc}^2 \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} \left( \frac{R_{wn}}{R_{wt_{dc}}} \right) \left( \frac{I_{tn}}{I_{tdc}} \right)^2 \right] \]

(3.40)

\[ = R_{wt_{dc}}I_{tdc}^2 \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} F_{Rtn} \left( \frac{I_{tn}}{I_{tdc}} \right)^2 \right] \]

\[ = P_{wt_{dc}} \left[ 1 + \frac{1}{8\pi^4 n_1^2} \left( \frac{I_{O_{\text{max}}}}{I_{tdc}} \right)^2 \sum_{n=1}^{\infty} F_{Rtn} \left( \frac{X_{nt}^2 + Y_{nt}^2}{n^4} \right) \right] \]

26
Figure 3.7: Construction of tertiary current waveform using the derived Fourier series expression.

\[ = P_{wt} F_{Rth}. \] (3.41)

The ratio of ac to dc resistance is replaced by Dowell’s equation in [8]-[12]. Similarly the resistance ratio is replaced in the above expression and \( F_{Rth} \) is the tertiary winding ac to dc resistance ratio. The ratio of ac to dc tertiary winding power loss is expressed by a factor \( F_{Rth} \)

\[
F_{Rth} = \frac{P_{wt}}{P_{wt} dc} = 1 + \frac{1}{8\pi^4 n_1^2} \left( \frac{I_{O_{max}}}{I_{tdc}} \right)^2 \sum_{n=1}^{\infty} \frac{F_{Rtn}(X_{nt}^2 + Y_{nt}^2)}{n^4}. \] (3.42)

### 3.4 Expression of Losses in Forward Transformer

The forward transformer core loss is given by [8]

\[
P_c = k(f_s \text{ in kHz})^a (10B_{m_{(max)}} \text{inT})^b V_c \quad \text{mW/cm}^3. \] (3.43)
The total winding or copper loss of a forward transformer is given by [8]

\[ P_w = P_{wp} + P_{ws} + P_{wt}. \]  \hspace{1cm} (3.44)

The total loss in forward transformer is given by [8]

\[ P_{cw} = P_c + P_w. \]  \hspace{1cm} (3.45)
4 Design of Forward Transformer using Area Product ($A_p$) and Core Geometry ($K_g$) Methods

4.1 Introduction

To transfer energy in a transformer through magnetic field medium without loss, the transformer design should be efficient [8]. This chapter deals with the design of forward transformer using area product ($A_p$) and core geometry ($K_g$) methods taking into account the effects of losses due to eddy currents at HF. The design equations are illustrated using an example and plots of winding losses are also presented. There are three important factors to be considered in designing a HF transformer [8], they are:

- material to conduct the flux

- core to provide a definite magnetic path

- conductors.

4.1.1 Materials

Since the transformer transfers the energy by magnetic coupling the core material is one the most important factors because it supports the magnetic flux and decreases the reluctance [8]. The core material can be classified as hard or soft magnetic materials. The hard magnetic material holds the magnetic domains for a long time, so it is difficult to magnetize and demagnetize [8]. Contrarily, the soft magnetic material acts as a temporary magnet, thus it is easy to magnetize and demagnetize [8]. Therefore soft magnetic materials are widely used in designing inductor and transformers at HF [8].

The various types of magnetic materials are iron powder, ferrites, cobalt iron, nickel iron, etc [8]. A perfect core is expected to have high permeability to conduct
more flux and thus ferromagnetic material is used for HF frequency applications [8].

Some of its important parameters are:

- relative permeability $\mu_{rc}$, it ranges from 1 to $10^5$

- saturation flux density $B_s$, 0.5 at $T = 25^\circ C$

- Curie temperature, above this temperature the material loses its ferromagnetic property [8].

### 4.1.2 Core

Soft ferromagnetic cores used in the design of HF magnetic components are discussed. For efficient transfer of energy a well defined magnetic path is provided by a core thereby reducing the leakage inductances [8]. The magnetic path is dependent on the core’s shape. Various shapes like POT, EE, EI, toroidal, etc are available. They are made up of two halves and can be gapped or un-gapped depending upon the requirement [8]. However there is not much provision for the heat to escape and this leads to temperature rise in the core. Ferrite cores are preferred because of the following advantages [8]:

- wide frequency range

- low cost

- high permeability

- acts as an insulator at HF.

While choosing a core some assumptions are made. They are

- Core utilization factor $K_u$: This is a ratio between the copper area $A_{cu}$ and available window area $W_a$. Usually this takes a value of 0.3 to 0.4 for inductors and transformers [8].
• Current density $J_m$ is also related to the core, since current density is related to copper area. The value of $J_m$ varies from 0.1 to 6 A/mm$^2$ [8].

The dimensions of a core are represented as follows:

• Area product: $A_p$ (cm$^4$)

• Cross-sectional area of core: $A_c$ (cm$^2$)

• Mean magnetic path length (MPL): $l_c$ (cm)

• Core volume: $V_c$ (cm$^3$)

The other parameter necessary for the design procedure is calculated. The core window area,

$$W_a = \frac{A_p}{A_c} \text{ (cm$^2$).}$$  \hspace{1cm} (4.1)

The mean length turn (MLT) $l_T$ (cm), surface area $A_s$ (cm$^2$) is calculated by selecting the appropriate parameters from manufacturers data sheet [17].

4.1.3 Conductors

Conductors are available in various shapes like square, round, rectangular or litz wire [8]. The windings can be single or multiple stranded. In single strand, depending on the calculated area of a winding a conductor is chosen from the datasheet. In multiple strands a wire is chosen depending upon skin depth [8]. For the design example in this chapter the single strand winding is implemented and American Wire Gauge (AWG) [8] wires are used. The windings are wound on the bobbin and it is placed inside a core. Some of the parameters of the wire are:

• Bare strand diameter: $d_{ss}$ (mm)

• Bare strand cross-sectional area: $A_{wss}$ (mm$^2$)
• Insulated strand diameter: $d_{os}$ (mm)

• DC resistance of the strand per unit length: $R_{wdc}/l_w$ (Ω/m).

4.2 Assumptions and Area Product ($A_p$) method for Forward Transformer

Figure 4.1: Illustration of current through magnetizing inductance.

**Assumptions**

There are two assumptions in the design of single-switch forward transformer. They are:

1. The magnetizing current is assumed to be 5-10% of the maximum primary winding current [1]. To provide a general expression, this factor is assumed to be $y$ ($y$ is in percentage) in the design procedure.

2. The winding allocation of the forward transformer is given by

$$N_p A_{wp} = N_s A_{ws} = x N_p A_{wp}$$

(4.2)
where \( N_t A_{wt} = x N_p A_{wp} \).

The winding allocation for primary and secondary winding is assumed to be equal as already stated in the above expression. The tertiary winding allocation is a fraction of the primary winding allocation as noticed above in the equation (4.2). This is due to the small current in it compared to primary or secondary winding. This factor is denoted by \( x \).

**Area Product \((A_p)\) Method**

The area product gives a rough estimate to select a core depending on the energy of the core [8]. The magnetic flux through a surface is given by [8]-[12]

\[
\phi_{pk} = A_c B_{pk}. \tag{4.3}
\]

The energy is transferred depending on the flux linking the number of turns wound and is given by [8]-[12]

\[
\lambda_{pk} = N_p A_c B_{pk} = L_p I_{pmax}. \tag{4.4}
\]

The maximum current density through a winding is given by [8]-[12]

\[
J_m = \frac{I_{pmax}}{A_{wp}}. \tag{4.5}
\]

The window area occupied by the windings of a forward transformer is expressed as [8]-[12]

\[
W_a = \frac{N_p A_{wp} + N_s A_{ws} + N_t A_{wt}}{K_u}. \tag{4.6}
\]

Using equation (4.2), the window area can be expressed in terms of primary allocation and is given by [8]

\[
W_a = \frac{(2 + x) N_p A_{wp}}{K_u}. \tag{4.7}
\]
The core utilization factor is the ratio of copper to window area and is obtained from the above expression

\[ K_u = \frac{A_{Cu}}{W_a} = \frac{(2 + x) N_p A_{wp}}{W_a}. \] \hspace{1cm} (4.8)

The energy stored in a transformer is given by [8]-[12]

\[ W_m = \frac{1}{2} L_m I_{Lm(max)}^2. \] \hspace{1cm} (4.9)

The area product of a core is [8]

\[ A_p = W_a A_c = \frac{(2 + x) A_{wp} L_p I_{pmax}}{K_u B_{pk}} = \frac{(2 + x) L_p I_{pmax}^2}{K_u J_m B_{pk}}. \] \hspace{1cm} (4.10)

The maximum primary current expressed in terms of magnetizing current is given by

\[ I_{pmax} = \frac{I_{Lm(max)}}{y}. \] \hspace{1cm} (4.11)

Substituting equation (4.11) in equation (4.10), area product can be expressed as

\[ A_p = \frac{2 (2 + x) W_m}{y^2 K_u J_m B_{pk}}. \] \hspace{1cm} (4.12)

### 4.3 Design Procedure of High-Frequency Forward Transformer in CCM

The specifications of the forward PWM DC-DC converter:

- Input DC voltage range: \( V_{I_{max}} \leq V_I \leq V_{I_{min}} \) (V)
- Output current range: \( I_{O_{max}} \leq I_O \leq I_{O_{min}} \) (A)
- Output DC voltage: \( V_O \) (V)
- Switching frequency: \( f_s \) (kHz)

The design procedure of a forward transformer using area product method \( A_p \) for CCM operation are as follows. The maximum and minimum output power are given by

\[ P_{O_{max}} = V_O I_{O_{max}} \] \hspace{1cm} (4.13)
and

\[ P_{O_{\text{min}}} = V_O I_{O_{\text{min}}} \text{ (W)}. \] \hfill (4.14)

The maximum and minimum DC voltage transfer functions are given by

\[ M_{V_{\text{DCmax}}} = \frac{V_O}{V_{I_{\text{min}}}} \] \hfill (4.15)

and

\[ M_{V_{\text{DCmin}}} = \frac{V_O}{V_{I_{\text{max}}}}. \] \hfill (4.16)

The maximum and minimum load resistances are given by

\[ R_{L_{\text{max}}} = \frac{V_O}{I_{O_{\text{min}}}} \text{ (Ω)} \] \hfill (4.17)

and

\[ R_{L_{\text{min}}} = \frac{V_O}{I_{O_{\text{max}}}} \text{ (Ω)}. \] \hfill (4.18)

The number of primary and tertiary turns are considered equal [1], [2], hence the primary-to-secondary and tertiary-to-secondary is same. In equation (4.19), the converter’s maximum duty cycle \(D_{\text{max}}\) and efficiency \(\eta_{\text{conv}}\) are assumed. The turns ratio of the transformer is given by

\[ n_1 = \frac{\eta_{\text{conv}} D_{\text{max}}}{M_{V_{\text{DCmax}}}}. \] \hfill (4.19)

The maximum and minimum duty cycle are given by

\[ D_{\text{max}} = \frac{n_1 M_{V_{\text{DCmax}}}}{\eta} \] \hfill (4.20)

and

\[ D_{\text{min}} = \frac{n_1 M_{V_{\text{DCmin}}}}{\eta}. \] \hfill (4.21)

The minimum inductance is given by

\[ L_{\text{min}} = \frac{R_{L_{\text{max}}}(1 - D_{\text{min}})}{2f_s} \text{ (H)}. \] \hfill (4.22)
Choose $L > L_{\text{min}}$. The maximum and minimum inductor ripple current expressions are given by

$$\Delta i_{L(\text{max})} = \frac{V_O(1 - D_{\text{min}})}{f_s L} \text{ (A)}$$

(4.23)

and

$$\Delta i_{L(\text{min})} = \frac{V_O(1 - D_{\text{max}})}{f_s L} \text{ (A).}$$

(4.24)

The diodes current stresses are given by

$$I_{D_{1\text{max}}} = I_{D_{2\text{max}}} = I_{O_{\text{max}}} + \frac{\Delta i_{L(\text{max})}}{2} \text{ (A).}$$

(4.25)

The maximum primary winding current is given by

$$I_{p_{\text{max}}} = \frac{I_{D_{1\text{max}}}}{n_1} \text{ (A).}$$

(4.26)

The maximum current through the magnetizing inductance is assumed to be $y$ (in percentage) times the maximum primary current of the transformer [1]

$$\Delta i_{L_{m(\text{max})}} = y I_{p_{\text{max}}} \text{ (A).}$$

(4.27)

As mentioned previously the factor $y$ varies from 5 – 10%.

The maximum input DC current is given by

$$I_{I_{\text{max}}} = \frac{M_{VDC_{\text{max}}} I_{O_{\text{max}}}}{n_1 \eta_{\text{conv}}} \text{ (A).}$$

(4.28)

The expression for minimum magnetizing inductance is given by

$$L_{p_{(\text{min})}} = L_{m_{(\text{min})}} = \frac{D_{\text{min}} V_{I_{\text{max}}}}{f_s \Delta i_{L_{m(\text{max})}}} \text{ (H).}$$

(4.29)

Choose $L_p > L_{p_{(\text{min})}}$, let $L_p = L_m$. The secondary winding inductance is given by

$$L_s = \frac{L_p}{n_1^2} \text{ (H).}$$

(4.30)

The maximum energy stored in the transformer is given by

$$W_m = \frac{1}{2} L_m \Delta i_{L_{m(\text{max})}}^2 \text{ (J).}$$

(4.31)
From equation (4.12) the general expression for core area product is given by

$$A_p = \frac{2(2 + x) W_m}{y^2 K_u J_m B_{pk}} \text{ (cm}^4\text{)}.$$  \hfill (4.32)

**Core Selection** The core is chosen from the manufacturers datasheet [17] by comparing the calculated value from equation (4.32) with the core catalogue. The parameters of the selected core are considered for further calculations, they are:

- Core selected: Core Number
- Area product: \(A_p\) (cm\(^4\))
- Cross-sectional area of core: \(A_c\) (cm\(^2\))
- Mean magnetic path length (MPL): \(l_c\) (cm)
- Core volume: \(V_c\) (cm\(^3\))

The core window area is calculated by

$$W_a = \frac{A_p}{A_c} \text{ (cm}^2\text{)}. \hfill (4.33)$$

**Single Strand**

**Primary Winding Wire Selection** The calculated cross-sectional area of the primary winding is given by

$$A_{wp} = \frac{I_{pmax}}{J_m} \text{ (mm}^2\text{)}. \hfill (4.34)$$

Depending upon the calculated primary winding cross-sectional area, an appropriate AWG copper wire is chosen by comparing it with non-insulated cross-sectional area of wire and similarly the wires for secondary and tertiary are also selected. The parameters of the chosen wire are obtained from wire datasheet [8]:

- Wire selected: Wire Number
• Bare strand diameter: \( d_{\text{is}} \) (mm)

• Bare strand cross-sectional area: \( A_{\text{wsi}} \) (mm\(^2\))

• Insulated strand diameter: \( d_{\text{os}} \) (mm)

• DC resistance of the strand per unit length: \( R_{\text{wdcs}}/l_w \) (\( \Omega \)/m)

The primary winding allocation is given by

\[
W_{\text{ap}} = \frac{W_a}{(2 + x)} \text{ (cm}^2\text{).} \tag{4.35}
\]

Primary winding number of turns is given by

\[
N_p = \frac{K_u W_{\text{ap}}}{A_{\text{wsi}}} \text{.} \tag{4.36}
\]

Tertiary winding number of turns is assumed to be equal to the primary turns

\[
N_p = N_t \tag{4.37}
\]

Secondary winding number of turns is given by

\[
N_s = \frac{N_p}{n_1} \tag{4.38}
\]

In single strand method the number of layers have to be calculated which is useful in determining the harmonic losses of each windings. The length of the air gap is given by

\[
l_g = \frac{\mu_0 A_c N_p^2}{L_p} - \frac{l_c}{\mu_{rc}} \text{ (mm).} \tag{4.39}
\]

The peak value of magnetic flux density is given by

\[
B_{pk} = \frac{\mu_0 N_p I_{\text{pmax}}}{l_g + \frac{l_c}{\mu_{rc}}} \text{ (T).} \tag{4.40}
\]

The maximum ac component of flux density is given by

\[
B_{m(\text{max})} = \frac{\mu_0 N_p}{l_g + \frac{l_c}{\mu_{rc}}} \left[ \frac{\Delta i_{Lm(\text{max})}}{2} \right] \text{ (T).} \tag{4.41}
\]
The expression for core power loss density is given by

\[ P_v = k (f_s \text{ in kHz})^a (10B_p(\text{max}) \text{ in T})^b \text{ (mW/cm}^3) \]. \hspace{1cm} (4.42)

The coefficients \( k, a, \) and \( b \) are obtained from manufacturers datasheet [17] depending upon the material and frequency of operation. The core loss is given by

\[ P_C = V_c P_v \text{ (mW)}. \] \hspace{1cm} (4.43)

The length of the primary winding wire is given by

\[ l_{wp} = N_p l_T \text{ (cm)}. \] \hspace{1cm} (4.44)

The primary DC and low-frequency resistance is given by

\[ R_{wpdc} = \left( \frac{R_{wdc1}}{l_w} \right) l_{wp} \text{ (}\Omega\).} \hspace{1cm} (4.45)

The primary DC and low-frequency power loss is given by

\[ P_{wpdc} = R_{wpdc} I_{Imax}^2 \text{ (W)}. \] \hspace{1cm} (4.46)

The primary power loss factor \( F_{Rph} \) for CCM is calculated using equation (3.18) from which, the winding power loss is obtained as

\[ P_{wp} = F_{Rph} P_{wpdc} \text{ (W)}. \] \hspace{1cm} (4.47)

The maximum secondary winding current is given by

\[ I_{smax} = I_{Omax} + \frac{\Delta i_{L(\text{max})}}{2} \text{ (A)}. \] \hspace{1cm} (4.48)

**Secondary Winding Wire Selection** The calculated cross-sectional area of the secondary winding wire is given by

\[ A_{ws} = \frac{I_{smax}}{J_m} \text{ (mm}^2) \]. \hspace{1cm} (4.49)
Depending upon the calculated secondary winding cross-sectional area a wire is chosen. The length of the secondary winding wire is given by

\[ l_{ws} = N_s l_T \text{ (cm)}. \quad (4.50) \]

The secondary DC and low-frequency resistance is given by

\[ R_{w_{sd}} = \left( \frac{R_{w_{dc}2}}{l_w} \right) l_{ws} \text{ (Ω)}. \quad (4.51) \]

The secondary DC and low-frequency power loss is given by

\[ P_{w_{sd}} = R_{w_{sd}} (D_{max} I_{omax})^2 \text{ (W)}. \quad (4.52) \]

The secondary power loss factor \( F_{Rsh} \) for CCM is calculated using equation (3.30) from which, the winding power loss is obtained as

\[ P_{ws} = F_{Rsh} P_{w_{sd}} \text{ (W)}. \quad (4.53) \]

The maximum tertiary winding current is given by

\[ I_{t_{max}} = \left( \frac{n_1}{n_3} \right) \frac{D_{min} V_{l_{max}}}{f_s L_m} \text{ (A)}. \quad (4.54) \]

where \( n_3 \) is the ratio of tertiary to secondary turns. Since the primary and tertiary have same number of turns \( n_1 = n_3 \) [1].

**Tertiary Winding Wire Selection** The calculated cross-sectional area of the tertiary winding wire is given by

\[ A_{wt} = \frac{I_{t_{max}}}{J_m} \text{ (mm}^2). \quad (4.55) \]

Depending upon the calculated cross-sectional area of the tertiary winding a wire is chosen. The length of the tertiary(assuming the number of turns of primary and tertiary are equal i.e \( N_p = N_t \) winding wire is given by

\[ l_{wt} = N_t l_T \text{ (cm)}. \quad (4.56) \]
The tertiary DC and low-frequency resistance is given by

\[ R_{wtdc} = \left( \frac{R_{wdc}}{l_w} \right) l_w \, (\Omega). \quad (4.57) \]

The tertiary DC and low-frequency power loss is given by

\[ P_{wtdc} = R_{wtdc} \left( \frac{(1 - D_{max})}{2 y l_{omax}} \right)^2 \, (W). \quad (4.58) \]

The tertiary power loss factor \( F_{Rth} \) for CCM is calculated using equation (3.42) from which, the winding power loss is obtained as

\[ P_w = F_{Rth} P_{wtdc} \, (W). \quad (4.59) \]

The total DC windings loss is given by

\[ P_{wdc} = P_{wpdc} + P_{wsdc} + P_{wtdc} \, (W). \quad (4.60) \]

The total windings loss is given by

\[ P_w = P_{wp} + P_{ws} + P_w \, (W). \quad (4.61) \]

Total loss in the transformer is given by

\[ P_cw = P_C + P_w \, (W). \quad (4.62) \]

The efficiency of the transformer is given by

\[ \eta_t = \frac{P_O}{P_O + P_{cw}}. \quad (4.63) \]

The surface power loss density is given by

\[ \psi = \frac{P_{cw}}{A_t} \, (W/cm^2). \quad (4.64) \]

The surface area is calculated depending upon the geometry of the core. The temperature rise of the transformer is given by

\[ \Delta T = 450 \psi^{0.826} \, (^\circ C). \quad (4.65) \]

The recalculated core window utilization is given by

\[ K_u = \frac{N_p A_{wsp} + N_s A_{wsis} + N_t A_{wst}}{W_a}. \quad (4.66) \]
4.3.1 Design Example

For the given specifications design a single-switch forward transformer using area product \((A_p)\) method operating in CCM \(V_{I_{\text{max}}} = 60\ \text{V},\ V_{I_{\text{min}}} = 40\ \text{V},\ I_{O_{\text{max}}} = 2\ \text{A},\ I_{O_{\text{min}}} = 0.2\ \text{A},\ V_O = 18\ \text{V},\ \text{and}\ F_s = 100\ \text{kHz} .\)

**Solution:**

The maximum and minimum values of output power are given by

\[
P_{O_{\text{max}}} = V_O I_{O_{\text{max}}} = 18 \times 2 = 36\ \text{W} \quad (4.67)
\]

and

\[
P_{O_{\text{min}}} = V_O I_{O_{\text{min}}} = 18 \times 0.2 = 3.6\ \text{W}. \quad (4.68)
\]

The maximum and minimum values of DC voltage transfer functions are given by

\[
M_{V_{DC_{\text{max}}}} = \frac{V_O}{V_{I_{\text{min}}}} = \frac{18}{40} = 0.45 \quad (4.69)
\]

and

\[
M_{V_{DC_{\text{min}}}} = \frac{V_O}{V_{I_{\text{max}}}} = \frac{18}{60} = 0.3. \quad (4.70)
\]

The maximum and minimum load resistances are given by

\[
R_{L_{\text{max}}} = \frac{V_O}{I_{O_{\text{min}}}} = \frac{18}{0.2} = 90\ \Omega \quad (4.71)
\]

and

\[
R_{L_{\text{min}}} = \frac{V_O}{I_{O_{\text{max}}}} = \frac{18}{2} = 9\ \Omega. \quad (4.72)
\]

Let us assume the converter’s maximum duty cycle as \(D_{\text{max}} = 0.4\) and efficiency as \(\eta = 0.9\). The turns ratio of the transformer is given by

\[
n_1 = \frac{\eta_{\text{conv}} D_{\text{max}}}{M_{V_{DC_{\text{max}}}}} = \frac{0.9 \times 0.4}{0.45} = 0.8. \quad (4.73)
\]

Pick \(n_1 = n_3 = 1\). The maximum and minimum values of duty cycle are given by

\[
D_{\text{max}} = \frac{n_1 M_{V_{DC_{\text{max}}}}}{\eta} = \frac{1 \times 0.45}{0.9} = 0.5 \quad (4.74)
\]
and
\[ D_{\text{min}} = \frac{n_1 M V_{DC\text{min}}}{\eta} = \frac{1 \times 0.3}{0.9} = 0.3333. \]  \hspace{1cm} (4.75)

The minimum inductance value is given by
\[ L_{\text{min}} = \frac{R_{L\text{max}} (1 - D_{\text{min}})}{2 f_s} = \frac{90 \times (1 - 0.3333)}{2 \times 100 \times 10^3} = 0.3 \text{ mH}. \]  \hspace{1cm} (4.76)

Choose \( L > L_{\text{min}} \). Let \( L = 0.4 \) mH. The maximum value of the inductor ripple current is given by
\[ \Delta i_{L(\text{max})} = \frac{V_o (1 - D_{\text{min}})}{f_s L} = \frac{18 \times (1 - 0.3333)}{100 \times 10^3 \times 0.4 \times 10^{-3}} = 0.300 \text{ A}. \]  \hspace{1cm} (4.77)

The diodes current stresses are given by
\[ I_{D1\text{max}} = I_{D2\text{max}} = I_{O\text{max}} + \frac{\Delta i_{L(\text{max})}}{2} = 2 + \frac{0.300}{2} = 2.15 \text{ A}. \]  \hspace{1cm} (4.78)

The maximum primary winding current is given by
\[ I_{p\text{max}} = \frac{I_{D1\text{max}}}{n_1} = \frac{2.15}{1} = 2.15 \text{ A}. \]  \hspace{1cm} (4.79)

The general equation for maximum magnetizing current is given by
\[ \Delta i_{Lm(\text{max})} = y I_{p\text{max}} \text{ (A)}. \]  \hspace{1cm} (4.80)

The maximum current through the magnetizing inductance is assumed to be 10% (i.e. \( y = 0.1 \)) of the maximum primary current of the transformer [1]
\[ \Delta i_{Lm(\text{max})} = 0.1 I_{p\text{max}} = 0.1 \times 2.15 = 0.215 \text{ A}. \]  \hspace{1cm} (4.81)

The maximum primary DC current is given by
\[ I_{1\text{max}} = \frac{M V_{D\text{Cmax}} I_{O\text{max}}}{n_1 \eta_{\text{conv}}} = \frac{0.45 \times 2}{1 \times 0.9} = 1 \text{ A}. \]  \hspace{1cm} (4.82)

The minimum magnetizing inductance value is given by
\[ L_{p\text{(min)}} = \frac{D_{\text{min}} V_{I\text{max}}}{f_s \Delta i_{Lm(\text{max})}} = \frac{0.3333 \times 60}{100 \times 10^3 \times 0.215} = 0.93023 \text{ mH}. \]  \hspace{1cm} (4.83)
Choose $L_p > L_{p(min)}$. Let $L_p = L_m = 1.1$ mH. The secondary winding inductance is given by

$$L_s = \frac{L_p}{n_1^2} = \frac{1.1 \times 10^{-3}}{1^2} = 1.1 \text{ mH.} \quad (4.84)$$

The maximum energy stored by the transformer is given by

$$W_m = \frac{1}{2} L_m \Delta i_{Lm(max)}^2 = \frac{1}{2} \times 1.1 \times 10^{-3} \times 0.215^2 = 0.0254 \text{ mJ.} \quad (4.85)$$

**Core Selection** To select a core some factors need to be assumed as demonstrated in [8]-[12]. Assuming $K_u = 0.3$, $J_m = 6$ A/mm², and $B_{pk} = 0.25$ T. The general core area product expression for forward transformer is given by

$$A_p = \frac{2 \left(2 + x \right) W_m}{y^2 K_u J_m B_{pk}}. \quad (4.86)$$

Having assumed the tertiary winding occupies one-fourth of the space occupied by the primary winding, so $x = \frac{1}{4}$ and magnetizing current to be 10% of the maximum primary current, so $y = 0.1$. The area product is given by

$$A_p = \frac{2 \left(2 + \frac{1}{4} \right) W_m}{0.1^2 K_u J_m B_{pk}} = \frac{4.50 \times 0.0254 \times 10^{-3}}{0.1^2 \times 0.3 \times 6 \times 10^6 \times 0.25} = 2.54327 \text{ cm}^4. \quad (4.87)$$

A DS44229 core with central hole is chosen. The parameters of the core are $A_p = 2.91 \text{ cm}^4$, $A_c = 2.03 \text{ cm}^2$, $l_c = 7.11 \text{ cm}$, $V_c = 14.56 \text{ cm}^3$, $A = 42.4 \text{ mm}$, $B = 14.8 \text{ mm}$ and P-type material is used. The window area is given by

$$W_a = \frac{A_p}{A_c} = \frac{2.91}{2.03} = 1.4334 \text{ cm}^2. \quad (4.88)$$

The primary winding allocation is given by

$$W_{ap} = \frac{W_a}{(2 + x)} = \frac{1.4334}{2.25} = 0.6370 \text{ cm}^2. \quad (4.89)$$

**Primary Winding Wire Selection** The calculated cross-sectional area of the primary winding is given by

$$A_{wp} = \frac{I_{pmax}}{J_m} = \frac{2.15}{6} = 0.3583 \text{ mm}^2. \quad (4.90)$$
Depending upon the calculated primary winding cross-sectional area, AWG 21 wire is chosen. The parameters of the wire are $d_{i1} = 0.7229$ mm, $A_{wsi1} = 0.4116$ mm$^2$, $d_{o1} = 0.785$ mm and $R_{dc} = 0.04189$ Ω/m. The primary winding number of turns is given by

$$N_p = \frac{K_u W_{ap}}{A_{wsi}} = \frac{0.3 \times 0.6370}{0.4116 \times 10^{-2}} = 46.42. \quad (4.91)$$

Pick $N_p = N_t = 46$. The secondary winding number of turns is given by

$$N_s = \frac{N_p}{n_1} = \frac{46}{1} = 46. \quad (4.92)$$

Pick $N_s = 46$.

To Calculate the Number of Layers for Primary Winding

The height of the core is given by

$$H_w = 2D = 2 \times 10.21 = 20.42 \text{ mm}. \quad (4.93)$$

The primary maximum number of turns per layer is given by

$$N_{l1} = \frac{H_w}{d_{o1}} = \frac{20.42}{0.785} = 26.01. \quad (4.94)$$

Pick $N_{l1} = 26$. The primary number of layers is given by

$$N_{lp} = \frac{N_p}{N_{l1}} = \frac{46}{26} = 1.7. \quad (4.95)$$

Pick $N_{lp} = 2$. The primary number of turns per layer is given by

$$N_{l1} = \frac{N_p}{N_{lp}} = \frac{46}{2} = 23. \quad (4.96)$$

The length of the air gap is given by

$$l_g = \frac{\mu_0 A_{c} N_p^2}{L_p} - \frac{l_c}{\mu_{rc}} = \frac{4\pi \times 10^{-7} \times 2.03 \times 10^{-4} \times 46^2}{1.1 \times 10^{-3}} - \frac{7.11 \times 10^{-2}}{2500} = 0.4622 \text{ mm}. \quad (4.97)$$

45
The peak value of magnetic flux density is given by

\[ B_{pk} = \frac{\mu_0 N_p I_{pmax}}{l_g + \frac{l_c}{\mu_{rc}}} = \frac{4\pi \times 10^{-7} \times 46 \times 2.15}{(0.4622 \times 10^{-3}) + \frac{7.11 \times 10^{-2}}{2500}} = 0.2533 \text{ T.} \]  

(4.98)

The maximum ac component of flux density is given by

\[ B_{m(max)} = \frac{\mu_0 N_p}{l_g + \frac{l_c}{\mu_{rc}}} \left( \frac{\Delta I_{Lm(max)}}{2} \right) = \frac{4\pi \times 10^{-7} \times 46}{(0.4622 \times 10^{-3}) + \frac{7.11 \times 10^{-2}}{2500}} \left( \frac{0.215}{2} \right) = 0.0127 \text{ T.} \]  

(4.99)

The constants \( k = 0.0434, a = 1.63 \) and \( b = 2.62 \) are obtained from manufacturers datasheet [17] for p-type material and frequency of 100 kHz. The core power loss density is given by

\[ P_v = k f_s \text{ in kHz}^a (10 B_{m(max)} \text{ in T})^b \]

\[ = 0.0434 \times (100)^{1.63} \times (10 \times 0.0127)^{2.62} = 0.3514 \text{ mW/cm}^3. \]  

(4.100)

The core loss is given by

\[ P_C = V_c P_v = 14.56 \times 0.3543 = 5.116 \text{ mW.} \]  

(4.101)

Figure 4.2 shows a plot of core loss \( P_C \) with respect to input DC voltage \( V_I \) at specific output powers \( P_O \) for forward converter in CCM.
The calculated length of the primary winding wire is given by

\[ l_{wp} = N_p l_T = 46 \times 8.3739 = 385.19 \text{ cm.} \]  \hspace{1cm} (4.102)

Pick \( l_{wp} = 387 \text{ cm.} \) The primary DC and low-frequency resistance is given by

\[ R_{wpdc} = \left( \frac{R_{wdc1}}{l_w} \right) l_{wp} = 0.04189 \times 387 \times 10^{-2} = 0.1621 \Omega. \]  \hspace{1cm} (4.103)

The primary DC and low-frequency power loss is given by

\[ P_{wpdc} = R_{wpdc} I_{Imax}^2 = 0.1621 \times 1^2 = 0.1621 \text{ W.} \]  \hspace{1cm} (4.104)

The primary power loss factor \( F_{Rph} \) is calculated using equation (3.18), with two layers \( N_{lp} = 2 \) for CCM is found to be 13.74 from Figure 4.3. This figure shows a plot of harmonic primary power loss factor \( F_{Rph} \) with respect to input DC voltage \( V_I \) at specific output powers \( P_O \) for forward converter in CCM. The primary winding power loss is obtained as

\[ P_{wp} = F_{Rph} P_{wpdc} = 13.74 \times 0.1621 = 2.227 \text{ W.} \]  \hspace{1cm} (4.105)
Figures 4.4 and 4.5 show plots of primary winding power loss $P_{wp}$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The maximum secondary winding current is given by

$$I_{smax} = I_{Omax} + \Delta i_{L(max)} = 2 + \frac{0.3}{2} = 2.15 \text{ A.} \quad (4.106)$$

**Secondary Winding Wire Selection** The calculated cross-sectional area of the secondary winding is given by

$$A_{ws} = \frac{I_{smax}}{J_m} = \frac{2.15}{6} = 0.358 \text{ mm}^2. \quad (4.107)$$

Depending upon the calculated secondary winding cross-sectional area, AWG 20 wire is chosen. The parameters of the wire are $d_{i2} = 0.7229 \text{ mm}$, $A_{wsi2} = 0.4116 \text{ mm}^2$, $d_{o2} = 0.785 \text{ mm}$ and $\frac{R_{wdc2}}{l_w} = 0.04189 \Omega/\text{m}$. The calculated length of the secondary winding wire is given by

$$l_{ws} = N_w l_T = 46 \times 8.3739 = 385.199 \text{ cm.} \quad (4.108)$$
Figure 4.4: Primary winding power loss $P_{wp}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.5: Primary winding power loss $P_{wp}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Pick $l_{ws} = 389 \text{ cm}$.

**To Calculate the Number of Layers for Secondary Winding**

The secondary maximum number of turns per layer is given by

$$N_{l2} = \frac{H_w}{d_{02}} = \frac{20.42}{0.785} = 26.01. \quad (4.109)$$

Pick $N_{l2} = 26$. The secondary number of layers is given by

$$N_{ls} = \frac{N_s}{N_{l2}} = \frac{46}{26} = 1.76. \quad (4.110)$$

Pick $N_{ls} = 2$. The secondary number of turns per layer is given by

$$N_{l2} = \frac{N_s}{N_{ls}} = \frac{46}{2} = 23. \quad (4.111)$$

The secondary DC and low-frequency resistance is given by

$$R_{wsdc} = \left( \frac{R_{wdc}}{l_w} \right) l_{ws} = 0.04189 \times 389 \times 10^{-2} = 0.1629 \text{ }\Omega. \quad (4.112)$$

The secondary DC and low-frequency power loss is given by

$$P_{wsdc} = R_{wsdc} (D_{max} I_{Omax})^2 = 0.1629 \times (0.5 \times 2)^2 = 0.1629 \text{ W}. \quad (4.113)$$

The secondary power loss factor $F_{Rsh}$ is calculated using equation (3.30), with $N_{ls} = 2$ for CCM is found to be 11.27 from Figure 4.6. This figure shows a plot of harmonic secondary power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. The secondary winding power loss is obtained as

$$P_{ws} = F_{Rsh} P_{wsdc} = 11.27 \times 0.1629 = 1.835 \text{ W}. \quad (4.114)$$

Figures 4.7 and 4.8 show plots secondary winding power loss $P_{ws}$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The maximum tertiary
Figure 4.6: Harmonic secondary power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.7: Secondary winding power loss $P_{ws}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.
winding current is given by

\[ I_{t_{\text{max}}} = \left( \frac{n_1}{n_3} \right) \frac{D_{\text{min}} V_{I_{\text{max}}}}{f_s L_m} = \left( \frac{1}{1} \right) \frac{0.333 \times 60}{10^5 \times 1.1 \times 10^{-3}} = 0.1818 \text{ A.} \] (4.115)

**Tertiary Winding Wire Selection** The calculated cross-sectional area of the tertiary winding is given by

\[ A_{wt} = \frac{I_{t_{\text{max}}}}{J_m} = \frac{0.1818}{6} = 0.0303 \text{ mm}^2. \] (4.116)

Depending upon the calculated tertiary winding cross-sectional area, AWG 32 wire is chosen. The parameters of the wire are \( d_{i3} = 0.2019 \text{ mm}, A_{wsi3} = 0.03242 \text{ mm}^2, \)
\( d_{o3} = 0.2417 \text{ mm} \) and \( \frac{R_{\text{w3}}}{l_w} = 0.53149 \text{ \Omega/m}. \) The calculated length of the tertiary winding wire is given by

\[ l_{wt} = N_t l_T = 46 \times 8.3739 = 385.199 \text{ cm}. \] (4.117)

Pick \( l_{wt} = 391 \text{ cm}. \)
To Calculate the Number of Layers for Tertiary Winding

The tertiary maximum number of turns per layer is given by

\[ N_{l3} = \frac{H_w}{d_{03}} = \frac{20.42}{0.2417} = 84.48 \]  

(4.118)

Pick \( N_{l3} = 84 \). The tertiary number of layers is given by

\[ N_{lt} = \frac{N_l}{N_{l3}} = \frac{46}{84} = 0.5. \]  

(4.119)

Pick \( N_{lt} = 1 \). The tertiary number of turns per layer is given by

\[ N_{l3} = \frac{N_l}{N_{lt}} = \frac{46}{1} = 46. \]  

(4.120)

The tertiary DC and low-frequency resistance is given by

\[ R_{wtdc} = \left( \frac{R_{wdc3}}{l_w} \right) l_{wt} = 0.53149 \times 391 \times 10^{-2} = 2.078 \Omega. \]  

(4.121)

The tertiary DC and low-frequency power loss is given by

\[ P_{wtdc} = R_{wtdc} \left( \frac{(1 - D_{max}) y I_{omax}}{2} \right)^2 = 2.078 \left( \frac{(1 - 0.5)}{2} \times 0.1 \times 2 \right)^2 = 5.195 \text{ mW}. \]  

(4.122)

The tertiary power loss factor \( F_{Rth} \) is calculated using equation(3.42), with \( N_{lt} = 1 \) for CCM is found to be 2.976 from Figure 4.9. This figure shows a plot of harmonic tertiary power loss facto \( F_{Rth} \) with respect to input DC voltage \( V_I \) at specific output powers \( P_O \) for forward converter in CCM. The tertiary winding power loss is obtained as

\[ P_{wt} = F_{Rth} P_{wtdc} = 2.976 \times 5.195 \times 10^{-3} = 15.46 \text{ mW}. \]  

(4.123)

Figures 4.10 and 4.11 show plots of tertiary winding power loss \( P_{wt} \) with respect to input DC voltage \( V_I \) and output power \( P_O \), at specific output powers \( P_O \) and input DC voltages \( V_I \) for forward converter in CCM respectively. The total DC windings loss is given by

\[ P_{wdc} = P_{wpdc} + P_{wsdc} + P_{wtdc} = 0.1621 + 0.1629 + 0.005195 = 0.33423 \text{ W}. \]  

(4.124)
Figure 4.9: Harmonic tertiary power loss factor $F_{Rth}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.10: Tertiary winding power loss $P_{wt}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.
The total windings loss is given by

$$P_w = P_{wp} + P_{ws} + P_{wt} = 2.227 + 1.835 + 0.01546 = 4.077 \text{ W}. \quad (4.125)$$

Figures 4.12 and 4.13 show plots total power windings loss $P_w$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. Total loss in the transformer is given by

$$P_{cw} = P_C + P_w = 0.005116 + 4.077 = 4.0821 \text{ W}. \quad (4.126)$$

Figures 4.14 and 4.15 show plots of total loss in the transformer $P_{cw}$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The efficiency of the transformer is given by

$$\eta_t = \frac{P_O}{P_O + P_{cw}} = \frac{36}{36 + 4.0821} = 89.815 \%. \quad (4.127)$$
Figure 4.12: Total winding loss $P_w$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.13: Total winding loss $P_w$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Figure 4.14: Total loss in the transformer $P_{cw}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.15: Total loss in the transformer $P_{cw}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Figures 4.16 and 4.17 show plots of efficiency of transformer $\eta_t$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The surface area for a DS core is given by

$$A_t = \frac{\pi A^2}{2} + 2\pi AB = \frac{\pi \times 42.4^2}{2} + 2\pi \times 42.4 \times 14.8 = 67.667 \text{ cm}^2. \quad (4.128)$$

The surface power loss density using equation (4.128) is given by

$$\psi = \frac{P_{cw}}{A_t} = \frac{4.0821}{67.667} = 0.06032 \text{ W/cm}^2. \quad (4.129)$$

The temperature rise of a transformer is given by

$$\Delta T = 450\psi^{0.826} = 450 \times 0.06032^{0.826} = 44.245 \degree C. \quad (4.130)$$

Figure 4.16: Efficiency of transformer $\eta_t$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.
Figure 4.17: Efficiency of transformer $\eta_t$ with respect to output power $P_o$ at specific input DC voltages $V_I$ for forward converter in CCM.

Figure 4.18: Transformer temperature rise $\Delta T$ with respect to output power $P_o$ at specific input DC voltages $V_I$ for forward converter in CCM.
calculated window utilization factor is given by

\[ K_u = \frac{N_p A_{wsi1} + N_s A_{wsi2} + N_t A_{wsi3}}{W_a} \]

\[ = \frac{46(0.4116 + 0.4116 + 0.03242)}{1.4334} = 0.2745. \quad (4.131) \]

### 4.4 Core Geometry \((K_g)\) Method

The core geometry \((K_g)\) method is another approach to choose a core. This method gives a rough estimate based on the geometrical parameters of the core such as, cross sectional area of core \((A_c)\), available window area \((W_a)\) and mean turn length (MLT) \([5], [8]\). The core geometry \((K_g)\) coefficient for flyback transformer was derived in \([8]\), the procedure was extended for forward transformer which has three windings. The core geometrical coefficient is not provided by the manufacturers and it has to be calculated using the above stated parameters of the core from datasheet \([17]\). The core geometry coefficient using the mechanical parameters of a core is given by \([8]\)

\[ K_g = \frac{W_a A_c^2 K_u}{l_T}. \quad (4.132) \]

Using equation (4.4) the primary winding number of turns is obtained as \([8]\)

\[ N_p = \frac{L_p I_{pmax}}{A_c B_{pk}}. \quad (4.133) \]

The primary winding dc resistance is given by \([8]\)

\[ R_{wpdc} = \rho_w \frac{N_p l_T}{A_{wp}}. \quad (4.134) \]

The primary number of turns from (4.134) is obtained as

\[ N_p = \frac{R_{wpdc} A_{wp}}{\rho_w l_T}. \quad (4.135) \]

Equating equations (4.133) and (4.135), we get area occupied by primary winding as

\[ A_{wp} = \frac{L_p I_{pmax} \rho_w l_T}{R_{wpdc} A_c B_{pk}}. \quad (4.136) \]
Substituting equations (4.133) and (4.136) in equation (4.7), the window area is expressed as

$$W_a = \frac{(2 + x) L_p^2 I_{p max}^2 \rho_w l_i}{K_u R_{wpdc} A_e^2 B_{pk}^2}. \quad (4.137)$$

Substituting equation (4.137) in (4.132), we get

$$K_g = \frac{(2 + x) L_p^2 I_{p max}^2 \rho_w}{R_{wpdc} B_{pk}^2}. \quad (4.138)$$

The primary winding dc and low frequency resistance is given by [8]

$$R_{wpdc} = \frac{P_{wpdc}}{I_{prms}^2}. \quad (4.139)$$

Substituting equation (4.139) in (4.138), we get

$$K_g = \frac{(2 + x) L_p^2 I_{p max}^2 I_{prms}^2 \rho_w}{P_{wpdc} B_{pk}^2}. \quad (4.140)$$

The transformer load regulation is the ratio between total winding power loss and output power and is given by [8], [5]

$$\alpha = \frac{P_{wdc}}{P_O} = \frac{P_{wpdc} + P_{wsdc} + P_{wtdc}}{P_O}, \quad (4.141)$$

$$P_{wpdc} = P_{wsdc} = P_{wtdc}, \text{ where } P_{wtdc} = y P_{wpdc}. \text{ Simplifying, we get}$$

$$\alpha = \frac{P_{wdc}}{P_O} = \frac{(2 + y) P_{wpdc}}{P_O}. \quad (4.142)$$

The efficiency of the transformer at low frequency is given by [8]

$$\eta_{tdc} = \frac{P_O}{P_O + P_{wdc}} = \frac{1}{1 + \alpha}. \quad (4.143)$$

which yield

$$\alpha = \frac{1}{\eta_{tdc}} - 1. \quad (4.144)$$

From equation (4.142) the dc primary winding loss is given by

$$P_{wpdc} = \frac{\alpha P_O}{(2 + y)}. \quad (4.145)$$
Substituting equation (4.145) in (4.140) we get

\[ K_g = \frac{(2 + x)(2 + y)L_p^2 I_{pmax}^2 I_{prms}^2 \rho_w}{\alpha P_O B_{pk}^2}. \]  

(4.146)

The primary rms current is given by

\[ I_{prms} = I_{pmax} \sqrt{D_{max}}. \]  

(4.147)

As mentioned previously in the assumptions the magnetizing current is \( y \) times the maximum primary winding current i.e \( I_{Lm(max)} = yI_{pmax} \). \( y \) is expressed in percentage. Rearranging equation (4.146) in terms of magnetizing current and substituting equation (4.9) in the modified equation we get

\[ K_g = \frac{4(2 + x)(2 + y)D_{max} W_m^2 \rho_w}{y^4 \alpha P_O B_{pk}^2}. \]  

(4.148)

Substituting \( \rho_{cu} = 1.724 \times 10^{-8}\Omega m \), we get the general core geometry coefficient \((K_g)\) for forward transformer

\[ K_g = \frac{(2 + x)(2 + y)D_{max} W_m^2}{0.145 y^4 \alpha P_O B_{pk}^2 \times 10^{-2}} \text{ (cm}^5). \]  

(4.149)

### 4.4.1 Design Example

For the given specifications design a single-switch forward transformer using core geometry \((K_g)\) method operating in CCM

- \( V_{Imax} = 60 \text{ V}, \) \( V_{Imin} = 40 \text{ V}, \) \( I_{Omax} = 2 \text{ A}, \)
- \( I_{Omin} = 0.2 \text{ A}, \) \( V_O = 18 \text{ V}, \) and \( F_s = 100 \text{ kHz} \).

**Solution:**

The maximum and minimum values of output power are given by

\[ P_{Omax} = V_O I_{Omax} = 18 \times 2 = 36 \text{ W} \]  

(4.150)

and

\[ P_{Omin} = V_O I_{Omin} = 18 \times 0.2 = 3.6 \text{ W}. \]  

(4.151)
The maximum and minimum values of DC voltage transfer functions are given by

\[ M_{V\text{DCmax}} = \frac{V_O}{V_{I\text{min}}} = \frac{18}{40} = 0.45 \]  

(4.152)

and

\[ M_{V\text{DCmin}} = \frac{V_O}{V_{I\text{max}}} = \frac{18}{60} = 0.3. \]  

(4.153)

The maximum and minimum load resistances are given by

\[ R_{L\text{max}} = \frac{V_O}{I_{O\text{min}}} = \frac{18}{0.2} = 90 \, \Omega \]  

(4.154)

and

\[ R_{L\text{min}} = \frac{V_O}{I_{O\text{max}}} = \frac{18}{2} = 9 \, \Omega. \]  

(4.155)

Let us assume the converter’s maximum duty cycle as \( D_{\text{max}} = 0.4 \) and efficiency as \( \eta = 0.9 \). The turn ratio of the transformer is given by

\[ n_1 = \frac{\eta_{\text{conv}} D_{\text{max}}}{M_{V\text{DCmax}}} = \frac{0.9 \times 0.4}{0.45} = 0.8. \]  

(4.156)

Pick \( n_1 = n_3 = 1 \). The maximum and minimum values of duty cycle are given by

\[ D_{\text{max}} = \frac{n_1 M_{V\text{DCmax}}}{\eta} = \frac{1 \times 0.45}{0.9} = 0.5 \]  

(4.157)

and

\[ D_{\text{min}} = \frac{n_1 M_{V\text{DCmin}}}{\eta} = \frac{1 \times 0.3}{0.9} = 0.3333. \]  

(4.158)

The minimum inductance value is given by

\[ L_{\text{min}} = \frac{R_{L\text{max}} (1 - D_{\text{min}})}{2 f_s} = \frac{90 \times (1 - 0.3333)}{2 \times 100 \times 10^3} = 0.3 \, \text{mH}. \]  

(4.159)

Choose \( L > L_{\text{min}} \). Let \( L = 0.4 \, \text{mH} \). The maximum value of the inductor ripple current is given by

\[ \Delta i_{L(\text{max})} = \frac{V_O (1 - D_{\text{min}})}{f_s L} = \frac{18 \times (1 - 0.3333)}{100 \times 10^3 \times 0.4 \times 10^{-3}} = 0.300 \, \text{A}. \]  

(4.160)
The diodes current stresses are given by

\[ I_{D1\text{max}} = I_{D2\text{max}} = I_{O\text{max}} + \frac{\Delta i_{L(max)}}{2} = 2 + \frac{0.300}{2} = 2.15 \text{ A.} \quad (4.161) \]

The maximum primary winding current is given by

\[ I_{p\text{max}} = \frac{I_{D1\text{max}}}{n_1} = \frac{2.15}{1} = 2.15 \text{ A.} \quad (4.162) \]

The general equation for maximum magnetizing current is given by

\[ \Delta i_{Lm(max)} = y I_{p\text{max}} \text{ (A).} \quad (4.163) \]

The maximum current through the magnetizing inductance is assumed to be 10% (i.e. \( y = 0.1 \)) of the maximum primary current of the transformer [1]

\[ \Delta i_{Lm(max)} = 0.1 I_{p\text{max}} = 0.1 \times 2.15 = 0.215 \text{ A.} \quad (4.164) \]

The maximum primary DC current is given by

\[ I_{I\text{max}} = \frac{M_{VDC\text{max}} I_{O\text{max}}}{n_1 \eta_{\text{conv}}} = \frac{0.45 \times 2}{1 \times 0.9} = 1 \text{ A.} \quad (4.165) \]

The minimum magnetizing inductance value is given by

\[ L_{p(min)} = L_{m(min)} = \frac{D_{\text{min}} V_{I\text{max}}}{f_s \Delta i_{Lm(max)}} = \frac{0.3333 \times 60}{100 \times 10^4 \times 0.215} = 0.93023 \text{ mH.} \quad (4.166) \]

Choose \( L_p > L_{p(min)}. \) Let \( L_p = L_m = 1.1 \text{ mH.} \) The secondary winding inductance is given by

\[ L_s = \frac{L_p}{n_1^2} = \frac{1.1 \times 10^{-3}}{1^2} = 1.1 \text{ mH.} \quad (4.167) \]

The maximum energy stored by the transformer is given by

\[ W_m = \frac{1}{2} L_m \Delta i_{Lm(max)}^2 = \frac{1}{2} \times 1.1 \times 10^{-3} \times 0.215^2 = 0.0254 \text{ mJ.} \quad (4.168) \]

**Core Selection** To select a core some factors need to be assumed as demonstrated in [8], [5]. Assuming \( K_u = 0.3, \alpha = 0.05, \) and \( B_{pk} = 0.25 \text{T}. \) The general core geometry expression for forward transformer is given by

\[ K_g = \frac{(2 + x) (2 + y) D_{\text{max}} W_m^2}{0.145 y^4 \alpha P_0 B_{pk}^2 \times 10^{-2}} \text{ (cm}^5). \quad (4.169) \]
Substituting the assumed values of $x$ and $y$, $x = \frac{1}{4}$ and $y = 0.1$. The maximum duty cycle for a forward transformer is 0.5. The core geometry ($K_g$) coefficient is given by

$$K_g = \frac{(2 + (\frac{1}{4})) (2 + 0.1) \times 0.5(0.0254 \times (10^{-3}))^2}{0.145 \times 0.1^4 \times 0.05 \times 36 \times 0.25^2 \times 10^{-2}} = 0.094 \text{ (cm}^5\text{).}$$

A PC43622 core is chosen. The parameters of the core are $K_g = 0.1743 \text{cm}^5$, $A_p = 1.530 \text{ cm}^2$, $A_c = 2.020 \text{ cm}^2$, $l_c = 5.320 \text{ cm}$, $V_c = 10.7 \text{ cm}^3$ and P-type material is used.

The window area is given by

$$W_a = \frac{A_p}{A_c} = \frac{1.530}{2.020} = 0.7574 \text{ cm}^2.$$  

The primary winding allocation is given by

$$W_{ap} = \frac{W_a}{(2 + x)} = \frac{0.7574}{2.25} = 0.3366 \text{ cm}^2.$$  

The maximum current density is given by

$$J_m = \frac{2 \left(2 + \frac{1}{4}\right) W_m}{K_u A_p B_{pk}} = \frac{2 \times 2.25 \times 0.0254 \times 10^{-3}}{0.3 \times 2.020 \times 0.25 \times 10^{-8}} = 0.099 \text{ A/mm}^2.$$  

**Primary Winding Wire Selection** The calculated cross-sectional area of the primary winding is given by

$$A_{wp} = \frac{I_{pmax}}{J_m} = \frac{2.15}{0.099} = 21.71 \text{ mm}^2.$$  

Depending upon the calculated primary winding cross-sectional area, AWG 3 wire is chosen. The parameters of the wire are $d_{i1} = 5.827 \text{ mm}$, $A_{wsi1} = 26.67 \text{ mm}^2$, $d_{o1} = 6.007 \text{ mm}$ and $\frac{R_{wsi1}}{l_w} = 0.0006465 \Omega/\text{m}$. The primary winding number of turns is given by

$$N_p = \frac{K_u W_{ap}}{A_{wsi}} = \frac{0.3 \times 0.3366}{26.67 \times 10^{-2}} = 0.378.$$  

Pick $N_p = N_t = 1$. The secondary winding number of turns is given by

$$N_s = \frac{N_p}{n_1} = \frac{1}{1} = 1.$$  

65
Pick $N_s = 1$. The length of the air gap is assumed to be zero. The peak value of magnetic flux density is given by

$$ B_{pk} = \frac{\mu_0 N_p I_{pmax}}{\mu_{rc}} = \frac{4\pi \times 1 \times 10^{-7} \times 2.15}{5.32 \times 10^{-2}} = 0.12696 \, T. \quad (4.177) $$

The maximum ac component of flux density is given by

$$ B_{m(max)} = \frac{\mu_0 N_p}{\mu_{rc}} \left( \frac{\Delta i_{Lm(max)}}{2} \right) = \frac{4\pi \times 10^{-7} \times 1}{5.32 \times 10^{-2}} \left( \frac{0.215}{2} \right) = 0.00635 \, T. \quad (4.178) $$

The constants $k = 0.0434$, $a = 1.63$ and $b = 2.62$ are obtained from manufacturers datasheet [17] for p-type material and frequency of 100 kHz. The core power loss density is given by

$$ P_v = k(f_s \text{ in kHz})^a(10B_{m(max)} \text{ in T})^b = 0.0434 \times (100)^{1.63} \times (10 \times 0.00635)^{2.62} = 0.0576 \, \text{mW/cm}^3. \quad (4.179) $$

The core loss is given by

$$ P_C = V_c P_v = 10.7 \times 0.0576 = 0.61632 \, \text{mW}. \quad (4.180) $$

Figure 4.19 shows a plot of core loss $P_C$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. The calculated length of the primary winding wire is given by

$$ l_{wp} = N_p l_T = 1 \times 7.327 = 7.327 \, \text{cm}. \quad (4.181) $$

Pick $l_{wp} = 9 \, \text{cm}$. The primary DC and low-frequency resistance is given by

$$ R_{wpdc} = \left( \frac{R_{wpdc}}{l_w} \right) l_w = 0.0006465 \times 9 \times 10^{-2} = 0.05815 \, \text{m}\Omega. \quad (4.182) $$

The primary DC and low-frequency power loss is given by

$$ P_{wpdc} = R_{wpdc} I_{lmax}^2 = 0.05815 \times 10^{-3} \times 1^2 = 0.05815 \, \text{mW}. \quad (4.183) $$

The primary power loss factor $F_{Rph}$ is calculated using equation (3.18) for CCM and is found to be 35.92 from Figure 4.20. This figure shows a plot of harmonic primary
Figure 4.19: Core loss $P_C$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

The primary winding power loss is obtained as

$$P_{wp} = F_{Rph} P_{wpdc} = 35.92 \times 0.05815 \times 10^{-3} = 2.0898 \text{ mW}. \quad (4.184)$$

Figures 4.21 and 4.22 show plots of primary winding power loss $P_{wp}$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The maximum secondary winding current is given by

$$I_{smax} = I_{Omax} + \frac{\Delta I_{L(max)}}{2} = 2 + \frac{0.3}{2} = 2.15 \text{ A}. \quad (4.185)$$

**Secondary Winding Wire Selection** The calculated cross-sectional area of the secondary winding is given by

$$A_{ws} \frac{I_{smax}}{J_m} = \frac{2.15}{0.099} = 21.717 \text{ mm}^2. \quad (4.186)$$
Figure 4.20: Harmonic primary power loss factor $F_{Rph}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.21: Primary winding power loss $P_{wp}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.
Depending upon the calculated secondary winding cross-sectional area, AWG 3 wire is chosen. The parameters of the wire are $d_{i2} = 5.827$ mm, $A_{wsi2} = 26.67$ mm$^2$, $d_{o2} = 6.007$ mm and $\frac{R_{wdc}}{l_w} = 0.0006465$ $\Omega$/m. The calculated length of the secondary winding wire is given by

$$l_{ws} = N_s l_T = 1 \times 7.273 = 7.273 \text{ cm.} \quad (4.187)$$

Pick $l_{ws} = 11$ cm. The secondary DC and low-frequency resistance is given by

$$R_{wsdc} = \left( \frac{R_{wdc2}}{l_w} \right) l_{ws} = 0.0006456 \times 11 \times 10^{-2} = 0.0710 \text{ m}\Omega. \quad (4.188)$$

The secondary DC and low-frequency power loss is given by

$$P_{wsdc} = R_{wsdc} \left( D_{max} I_{Omax} \right)^2 = 0.0710 \times 10^{-3} \times (0.5 \times 2)^2 = 0.07111 \text{ mW.} \quad (4.189)$$

The secondary power loss factor $F_{Rsh}$ is calculated using equation (3.30) for CCM and is found to be 29.03 from Figure 4.23. This figure shows a plot of harmonic secondary

Figure 4.22: Primary winding power loss $P_{wp}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. The secondary winding power loss is obtained as

$$P_{ws} = F_{Rsh}P_{wsdc} = 29.03 \times 0.07111 \times 10^{-3} = 2.064 \text{ mW}. \quad (4.190)$$

Figures 4.24 and 4.25 show plots secondary winding power loss $P_{ws}$ with respect to

![Graph showing harmonic secondary power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$.]

Figure 4.23: Harmonic secondary power loss factor $F_{Rsh}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The maximum tertiary winding current is given by

$$I_{tmax} = \left( \frac{n_1}{n_3} \right) \frac{D_{min}V_{imax}}{f_sL_m} = \left( \frac{1}{1} \right) \frac{0.333 \times 60}{10^5 \times 1.1 \times 10^{-3}} = 0.1818 \text{ A}. \quad (4.191)$$

**Tertiary Winding Wire Selection** The calculated cross-sectional area of the tertiary winding is given by

$$A_{wt} = \frac{I_{tmax}}{J_m} = \frac{0.1818}{0.099} = 1.837 \text{ mm}^2. \quad (4.192)$$
Figure 4.24: Secondary winding power loss $P_{ws}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.25: Secondary winding power loss $P_{ws}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Depending upon the calculated tertiary winding cross-sectional area, AWG 14 wire is chosen. The parameters of the wire are $d_{i3} = 1.63$ mm, $A_{wsi3} = 2.0822$ mm$^2$, $d_{o3} = 1.71$ mm and $\frac{R_{wdc}}{l_w} = 0.00828$ Ω/m. The calculated length of the tertiary winding wire is given by

$$l_{wt} = N_I l_T = 1 \times 7.273 = 7.273 \text{ cm.} \quad (4.193)$$

Pick $l_{wt} = 13$ cm. The tertiary DC and low-frequency resistance is given by

$$R_{wtdc} = \left(\frac{R_{wdc3}}{l_w}\right) l_{wt} = 0.00828 \times 13 \times 10^{-2} = 1.0764 \text{ mΩ.} \quad (4.194)$$

The tertiary DC and low-frequency power loss is given by

$$P_{wtdc} = R_{wtdc} \left(\frac{1 - D_{max}}{2} y I_{omax}\right)^2$$

$$= 1.0764 \times 10^{-3} \left(\frac{1 - 0.5}{2} \times 0.1 \times 2\right)^2 = 0.00269 \text{ mW.} \quad (4.195)$$

The tertiary power loss factor $F_{Rth}$ is calculated using equation (3.42) for CCM and is found to be 13.86 from Figure 4.26. This figure shows a plot of harmonic tertiary power loss factor $F_{Rth}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM. The tertiary winding power loss is obtained as

$$P_{wt} = F_{Rth} P_{wtdc} = 13.86 \times 0.00269 \times 10^{-3} = 0.0372 \text{ mW.} \quad (4.196)$$

Figures 4.27 and 4.28 show plots of tertiary winding power loss $P_{wt}$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM respectively. The total DC windings loss is given by

$$P_{wdc} = P_{wpdc} + P_{wsdc} + P_{wtdc}$$

$$= (0.05815 \times 10^{-3}) + (0.0711 \times 10^{-3}) + (0.00269 \times 10^{-3}) = 0.13194 \text{ mW.} \quad (4.197)$$

The total windings loss is given by

$$P_w = P_{wp} + P_{ws} + P_{wt}$$
Figure 4.26: Harmonic tertiary power loss factor $F_{Rth}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.27: Tertiary winding power loss $P_{wt}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.
Figure 4.28: Tertiary winding power loss $P_{wt}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.

$$= (2.0898 \times 10^{-3}) + (2.064 \times 10^{-3}) + (0.0372 \times 10^{-3}) = 4.191 \text{ mW.} \quad (4.198)$$

Figures 4.29 and 4.30 show plots total winding loss $P_w$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM, respectively. Total loss in the transformer is given by

$$P_{cw} = P_C + P_w$$

$$= (0.61632 \times 10^{-3}) + (4.191 \times 10^{-3}) = 4.807 \text{ mW.} \quad (4.199)$$

Figures 4.31 and 4.32 show plots of total loss in the transformer $P_{cw}$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM, respectively. The efficiency of the transformer is given by

$$\eta_t = \frac{P_O}{P_O + P_{cw}} = \frac{36}{36 + (4.807 \times 10^{-3})} = 99.96 \%.$$  

(4.200)
Figure 4.29: Total winding loss $P_w$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.30: Total winding loss $P_w$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Figure 4.31: Total loss in the transformer $P_{cw}$ with respect to input DC voltage $V_I$ at specific output powers $P_O$ for forward converter in CCM.

Figure 4.32: Total loss in the transformer $P_{cw}$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Figures 4.33 and 4.34 show plots of efficiency of a transformer $\eta_t$ with respect to input DC voltage $V_I$ and output power $P_O$, at specific output powers $P_O$ and input DC voltages $V_I$ for forward converter in CCM, respectively. The surface area for a

\[
A_t = \frac{\pi A^2}{2} + 2\pi AB = \frac{\pi \times 35.6^2}{2} + 2\pi \times 35.6 \times 10.95 = 44.40 \text{ cm}^2.
\]

(4.201)

The surface power loss density using equation (4.201) is given by

\[
\psi = \frac{P_{cw}}{A_t} = \frac{4.807 \times 10^{-3}}{44.40} = 0.1082 \text{ mW/cm}^2.
\]

(4.202)

The temperature rise of a transformer is given by

\[
\Delta T = 450\psi^{0.826} = 450 \times (0.182 \times 10^{-3})^{0.826} = 0.239 \degree C.
\]

(4.203)

Figure 4.35 shows a plot of transformer temperature rise $\Delta T$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
Figure 4.34: Efficiency of a transformer $\eta_t$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.

Figure 4.35: Transformer temperature rise $\Delta T$ with respect to output power $P_O$ at specific input DC voltages $V_I$ for forward converter in CCM.
The calculated window utilization factor is given by

\[
K_u = \frac{N_p A_{wisi1} + N_s A_{wisi2} + N_t A_{wisi3}}{W_a}
\]

\[
= \frac{1 (26.67 + 26.67 + 2.082)}{0.7574} = 0.731. \tag{4.204}
\]
5 Conclusion

5.1 Summary

The transformer design plays a major role in the single-switch forward converter, as it improves the working of the converter. The content of this thesis is summarized below:

- The Fourier series of the winding currents operated in CCM were derived to determine the harmonic losses.
- The design procedure for area product ($A_p$) is presented incorporating the derived Fourier series expression to develop general power loss expressions for the forward transformer.
- The core geometry ($K_g$) design procedure is presented taking into consideration the harmonic losses.
- The characteristics of the transformer were studied by Matlab plots and Saber sketch simulation.

5.2 Matlab Results

To calculate the winding losses due to harmonics at HF, the Fourier series of the winding currents waveforms were derived as presented in [8]-[12]. Dowell’s equation was used for ac resistances for a round conductor [8]. Thus the winding power loss factors for primary, secondary and tertiary was developed extending the theory demonstrated in [8]-[12]. These factors were plotted for the entire range of the converter, which in turn were utilized to find the winding power loss for all the three windings of the transformer.

From the Matlab plots the harmonic factors for primary $F_{Rph}$ and secondary $F_{Rsh}$ are found to be maximum at maximum input DC voltage $V_I$, and for tertiary
harmonic loss factor $F_{Rth}$ is found to be maximum at minimum input DC voltage $V_I$ having number of harmonics to be 100. The total loss of a transformer is found to be maximum, at minimum input DC voltage $V_I$ and maximum output power $P_O$. The efficiency is found to be minimum at minimum input DC voltage $V_I$ and maximum output power $P_O$. The spectrum of currents at maximum input voltage $V_I$ and maximum output power $P_O$ were obtained. Figures 5.1, 5.2 and 5.3 show the transformer current waveform constructed using derived Fourier series expressions.

5.3 Saber Results

The single-switch forward transformer operating in CCM was simulated in Saber Sketch and steady state analysis was performed. Figure 5.10 shows the Saber schematic forward transformer in CCM. The dimensions of the selected Double Slab (DS) with central hole core for area product ($A_p$) method was assigned in the property editor.
Figure 5.2: Construction of secondary current waveform for forward converter in CCM using Fourier series.

Figure 5.3: Construction of tertiary current waveform for forward converter in CCM using Fourier series.
Figure 5.4: Primary current spectrum of forward converter operating in CCM obtained from Matlab for maximum input voltage and output power.

Figure 5.5: Secondary current spectrum of forward converter operating in CCM obtained from Matlab for maximum input voltage and output power.
Figure 5.6: Tertiary current spectrum of forward converter operating in CCM obtained from Matlab for maximum input voltage and output power.

Figure 5.7: Primary current spectrum of forward converter operating in CCM obtained from Matlab for minimum input voltage and maximum output power.
Figure 5.8: Secondary current spectrum of forward converter operating in CCM obtained from Matlab for minimum input voltage and maximum output power.

Figure 5.9: Tertiary current spectrum of forward converter operating in CCM obtained from Matlab for minimum input voltage and maximum output power.
of the transformer. The spectrum of the transformer currents were obtained from Fourier analyses in Saber Sketch. Figure 5.11 and 5.13 shows the output voltage waveform and the current spectrum of forward transformer.

Figure 5.10: Saber schematic of forward converter operating in CCM.

5.4 Comparison of Results

The transformer current waveforms were reconstructed in MATLAB using the derived Fourier series expressions, these waveform was similar to the simulated current waveforms in Saber Sketch. The spectrum current results of the three windings of the transformer shows good agreement with MATLAB and Saber Sketch. The recalculated core utilization factor is very close to the assumed one. From the results it
Figure 5.11: Saber output DC voltage $V_O$ of forward converter operating in CCM.

is evident that winding losses due to harmonics are significant and should be taken into consideration in the design procedure of the transformer.

5.5 Contributions

For satisfactory working of an isolated converter there is a need for efficient transformer design. The design of transformers are converter dependent. The transformer design at LF are less complicated as the eddy currents are absent. Whereas HF operation of transformer experiences harmonic losses as a result of skin and proximity effects, in addition with the pulsating waveforms of the converter.

The forward transformer working it is not required to store energy unlike flyback
Figure 5.12: Forward converter winding current waveforms operating in CCM obtained from Saber for maximum input voltage and output power.

Figure 5.13: Forward converter winding currents spectrum operating in CCM obtained from Saber for maximum input voltage and output power.
transformer, so there is no need of air gap. But to avoid saturation of the transformer it is safe to have a minimum gap. The addition of a third winding in the forward transformer makes the design procedure complicated. This thesis presents design procedure of forward transformer using area product ($A_p$) and core geometry ($K_g$) methods with the illustration of an example problem. The design procedure calculates the losses due to eddy currents at HF.

5.6 Future Work

The design procedure can be developed using area product ($A_p$) and core geometry ($K_g$) methods for forward converter in DCM. The theory can also be extended to determine the winding losses for other PWM converters like push-pull, half-bridge and full-bridge.
6 Bibliography

References


