Steady-State and Small-Signal Modeling of a PWM DC-DC Switched-Inductor Buck-Boost Converter in CCM

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STEADY-STATE AND SMALL-SIGNAL MODELING
OF A PWM DC-DC SWITCHED-INDUCTOR
BUCK-BOOST CONVERTER IN CCM

Dissertation submitted in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

By

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Abstract


Pulse-width modulated (PWM) buck-boost converters have a significant role in power electronic systems for renewable energy applications. A new hybrid, the switched-inductor buck-boost converter, is superior to the conventional buck-boost because it uses less energy in the magnetic field, has smaller component size of inductors, and produces less current stresses in the switching elements. Steady-state and dynamic modeling of the switched-inductor buck-boost converter is essential to design and implement of a feed-back network. The objective of this work is to present the steady-state analysis of a PWM switched-inductor buck-boost dc-dc converter operating in continuous conduction mode (CCM). The idealized voltage and current waveforms, and expressions for steady-state operations of the converter are presented. The minimum values to ensure CCM operation for an inductance and capacitance are derived. The filter capacitor and its ESR with the ripple voltage effects are derived. Expressions for power losses and the overall efficiency of the PWM switched-inductor dc-dc buck-boost converter are given. A PWM switched-inductor buck-boost is designed, and a laboratory prototype is built and tested per given specifications. The theoretical and simulated analysis was in accordance with the experimental results. Small-signal modeling of PWM switched-inductor dc-dc buck-boost converter operating in CCM is presented. The averaged large-signal, dc, and time-invariant linear small-signal circuit models of a PWM switched-inductor dc-dc buck-boost converter power stage operating in CCM are presented. The small-signal modeling focuses on the dynamics introduced by the switched-inductor dc-dc buck-boost converter. Using the small-signal model to derive the open-loop power stage transfer functions: the input-to-output voltage, inductor current-to-input voltage, control-to-output voltage, input impedance and output impedance are derived. These transfer functions and their
associated theoretical Bode plots are illustrated using MatLab. Using discrete point method, the transfer functions are also verified by circuit simulation. The laboratory prototype experimental validates the small-signal models. The theoretical, simulated and experimental results were in excellent accordance. The effects of the PWM frequency and its effects on the switching elements of the switched-inductor buck-boost converter, the size of inductor and capacitor, and switching losses are presented. Also, studied were the effects of raising the frequency of the PWM to determine the impact on the current and voltage waveforms for the switching elements using saber sketch circuit simulator. The prototype was used to validate the simulated current and voltage waveforms. Another expansion for a PWM switched-inductor buck-boost converter, is explored by deriving the digital open-loop transfer functions: control-to-output voltage, input-to-output voltage, input voltage-to-inductor current, input impedance, and output impedance. The theoretically predicted transfer functions with a step input are theoretically plotted in MatLab, and are in accordance with the experimental step responses.
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1 Introduction

1.1 Background

Power converters can be classified into four categories: DC-DC converters, DC-AC inverters, AC-DC rectifiers, and AC-AC converters. DC-DC converters are used in portable electronic devices, green energy, and military applications. There are three classes of DC-DC converters: buck (step down), boost (step up), and buck-boost (step-up-down). Power converters are an efficient way to deliver regulated voltage from a traditional power source. Power electronic systems regulate different parameters so the load requirements can be met safely. Power engineering and power electronics have resulted in a renewed interest due to photovoltaic and green renewable energy application.

PWM converters are nonlinear due to the presence of one or more transistors and diodes in the system. Linearization and power stage averaging are done so that linear control theory may be utilized. There are two common techniques employed to achieve averaging and linearization: circuit averaging technique and state-space averaging. Circuit averaging involves replacing the PWM switch model with an equivalent analog model, obtained by employing dependent current and voltage sources. Most PWM DC-DC converters consist of a power MOSFET and a diode connected so that the two devices have a common point. The switched-inductor buck-boost converter’s schematic does not include a common point for all the switching component, but does contain the traditional common point of the buck-boost converter. This present work extends the circuit averaging technique to
1 Introduction

averaging the new switching network of the inductance.

Faster switching frequency is another objective for DC-DC converters. Increasing the switching frequency will effect (decrease) the size of the inductor and capacitor, increase the bandwidth, and result in a faster response time. It also negatively effect the converter by increasing switching losses and thereby decreasing efficiency. The requirement of a high side driver for the switched inductor buck-boost also limits the increase in attainable frequency. The present work will show the limitations in frequency for high powered DC-DC converters.

Closed-current loop modeling in a PWM DC-DC converter with current-mode control (CMC) has been steadily growing since the late 70’s. CMC is faster and results in lower error than voltage-mode control (VMC). CMC has been analyzed for analog circuits but further modeling and testing is required for digital circuits. This thesis models the new switched-inductor buck-boost DC-DC converter using circuit modeling technique, converts it to an accurate digital model, and explores the effects of higher frequencies on the converter.
1 Introduction

1.2 Motivation

DC-DC converters play an important role in industrial applications with the advent of cellular phones, laptops, tablets, and the renewal of interest in green renewable energy and has resulted in an increased interest in power electronics. DC-DC converters for portable electronic devices must:

- regulate voltage and output current
- increase the efficiency, thereby using less energy
- decrease component sizes
- produce less stress on components.

The switched-inductor buck-boost is a step up/down converter that has a switching network for the inductor which allows the use of two smaller inductors to be used. This decreases the magnetic field and energy losses. Also, the switched-inductor buck-boost has less current stresses in the switching elements, and therefore, results in less circuit conduction losses. The switched-inductor buck-boost is advantageous over the conventional buck-boost and therefore, requires further examination. An understanding of the power stage transfer functions of the switched-inductor buck-boost converter is necessary to design a feedback system. The dc voltage transfer ratio is presented in [4] based on voltage balance equations. The transfer functions found will be extended and derived using circuit averaging techniques which will contribute to a greater topological understanding of the converter. The primary motivation for this work is to extend circuit averaging techniques to the PWM switched-inductor buck-boost converter. The dc and small-signal models will be derived, along with important transfer functions for understanding, and controlling the switched-inductor buck-boost.
1 Introduction

1.3 Objective

- Characterization of the switched-inductor buck-boost converter
  - DC analysis of PWM switched-inductor buck-boost converter for CCM power stage design
    * DC voltage transfer function
    * Device stresses
    * Minimum inductance for CCM operation
    * Ripple voltage
    * Component losses and converter efficiency
    * Design procedure
    * Experimental validation
  - Frequency limitations of power stage
    * Simulation to find power stage frequency limitations
  - Analysis of the power stage dynamics
    * Derivation of the large-signal model
    * Linearization of large-signal model
    * Extraction of dc and small-signal models
    * Derivation of power stage transfer functions:
      - DC voltage transfer function
      - Control-to-output voltage transfer function
      - Input-to-output voltage transfer function
      - Input voltage-to-inductor current transfer function
      - Input impedance
      - Output impedance
    * Small-signal frequency response using discrete point method with simulated real component circuit
1 Introduction

* Experimental validation

- Digital model of the CCM switched-inductor buck-boost converter
  
  * Digital open-loop power stage transfer functions:
    
    - Control-to-output voltage transfer function
    - Input-to-output voltage transfer function
    - Input voltage-to-inductor current transfer function
    - Input impedance
    - Output impedance

* Determine the poles and zeroes of the transfer function with real components

* Experimental validation
2 DC Analysis of Switched-Inductor Buck-Boost Converter in CCM

2.1 Introduction

This section presents the steady state analysis of a PWM switched-inductor buck-boost converter in continuous conduction mode (CCM). The voltage and current waveforms are derived, and the devices’ stresses are presented. The dc voltage transfer function, the expressions for minimum inductance, and minimum capacitance are found. Component power losses and converter efficiency are estimated. The power losses in the switched-inductor buck-boost are compared to the losses in the traditional buck-boost converter. The analysis assumes the switched-inductor buck-boost converter is operating in CCM.

2.2 Switch-Inductor Buck-Boost Converter Circuit

Description

A PWM switched-inductor buck-boost converter is shown in fig. 2.1. The converter consists of four diodes; \(D_0, D_1, D_2,\) and \(D_{12}\) which are considered to be an uncontrolled switch, a MOSFET \(S\) considered a controllable switch, two inductors \(L_1\) and \(L_2,\) a capacitor \(C,\) and a dc load resistance of \(R_L.\) The switch \(S\) is switched at a constant switching frequency \(f_s = \frac{1}{T}.\) The duty cycle is defined as \(D = \frac{t_{on}}{T} = \frac{t_{on}}{t_{on}+t_{off}}\) where \(t_{on}\) is the time duration when the switch \(S\) is on,
the leads to $t_{off}$ is the time duration when the switch $S$ is OFF. In most cases with the PWM converters, the switches $S$, $D_0$, $D_1$, $D_2$, and $D_{12}$ are on and off in a complimentary manner. For this converter, when the switch $S$ is ON so are diodes $D_1$, and $D_2$. When the switch $S$ is OFF, the diodes $D_0$, and $D_{12}$ are ON. The MOSFET source is not connected to ground; therefore, the switched-inductor buck-boost converter is floating similar to the traditional buck-boost converter.

2.3 Assumptions

The following assumptions have been made for the analysis of the switched-inductor buck-boost PWM converter:

- The output capacitance of the transistor and the junction capacitance of the diode are neglected.
- Inductors, capacitors and resistors are assumed to be linear, time-invariant and frequency independent.
- The transistors on-state resistance is linear and the off-state is infinite.
- The diodes are represented as a linear battery with a forward resistance while in the on-state. The diodes have an infinite resistance in the off-state.
- The voltage source $V_I$ has an output impedance equal to zero.

2.4 DC Analysis of PWM Switched-Inductor Buck-Boost Converter in CCM

2.4.1 Time Interval: $0 < t \leq DT$

During the time interval of $0 < t \leq DT$ the switch and diodes 1 and 2 are on, and diodes 0 and 12 are off. An ideal equivalent circuit for this time interval is shown in Fig. 2.2. When the switch is On, the voltage across the $D_0$, $D_{12}$, $D_1$, and $D_2$ respectively are:
Figure 2.1: PWM switched-inductor buck-boost converter and ideal equivalent circuits in CCM.

\[ v_{D_0} = -(V_I + V_O), \]  
\[ v_{D_{12}} = V_I, \]  
\[ v_{D_1} = v_{D_2} = 0. \]

The voltage across each inductor \( L_1 = L_2 = L \) is given by

\[ v_L = V_I. \]  

The current through the switch \( i_S \) is approximately equal to the averaged current of the source \( I_I \). The output current \( I_O \) is also averaged. The currents though the inductor \( L \) and the switch are given by a linear rise with a slope of \( \frac{2V_I}{L} \). By Kirchoff’s current law, the switch current becomes

\[ i_S = 2i_L = \frac{2}{L} \int_0^t V_I dt + i_L(0) = 2 \left[ \frac{V_I}{L} t + i_L(0) \right]. \]  

The peak-to-peak inductor current becomes

\[ \Delta i_L = i_L(DT) - i_L(0) = \frac{2V_I DT}{L} = \frac{2V_I D}{f_s L}. \]

The diodes currents are

\[ i_{D_0} = 0. \]
2.4.2 Time Interval: $DT < t \leq T$

During the time interval of $DT < t \leq T$, the switch, and diodes 1 and 2 are OFF, and diodes 0 and 12 are ON. An ideal equivalent circuit for this time interval is shown in Fig. 2.3. When the switch is off the energy stored in the inductors forces the diodes to turn on. The voltage across the inductor $L_1 = L_2 = L$ is presented

\[ v_L = \frac{1}{2} V_O = L \frac{di_L}{dt} \quad (2.10) \]

The voltage across the diodes $D_1$ and $D_2$ is

\[ v_L = v_{D_1} = v_{D_2} \quad (2.11) \]

The current through the inductor is equal to

\[ i_{D_{12}} = 0, \quad (2.8) \]

and

\[ i_L = i_{D_1} = i_{D_2} = \frac{1}{2} i_S \leq I_L = \frac{1}{2} I_I + I_O. \quad (2.9) \]
\[ i_L = i_{D_0} = \frac{1}{L} \int_{DT}^{t} v_L dt + i_L(DT) = -\frac{V_O}{2L} (t - DT) + i_L(DT). \] (2.12)

The current through the inductor \( L \) and \( D_0 \) shows a linear fall with slope of \( \frac{V_O}{2L} \). The current through the switch is zero. The idealized waveforms for the switched-inductor buck-boost DC-DC converter operating in CCM is shown in Figs. 2.4 and 2.5. The peak-to-peak ripple current through the inductor \( L \) is

\[ \Delta i_L = i_L(DT) - i_L(T) = -\frac{1}{2} V_O (T - DT) = -\frac{V_O (1 - D)}{2f_s L}. \] (2.13)

### 2.4.3 DC Voltage Transfer Function for CCM

When the switch \( S \) and diodes \( D_1 \) and \( D_2 \) are ON, as shown in Fig. 2.2.

\[ V_I = v_L. \] (2.14)

When the switch \( S \) is OFF, and diodes \( D_0 \) and \( D_{12} \) are ON, as shown in Fig. 2.3.

\[ -\frac{1}{2} V_O = v_L. \] (2.15)

Using the volt-second balance equation

\[ DV_I = -\frac{1}{2} V_O (1 - D). \]
Figure 2.4: Ideal current and voltage waveforms for the PWM switched-inductor buck-boost converter in CCM.

Figure 2.5: Ideal current and voltage waveforms for the PWM switched-inductor buck-boost converter in CCM.
Hence, equating (2.14) and (2.15), the dc input-to-output voltage transfer function for a lossless switched-inductor buck-boost converter becomes

\[ M_{VDC} = \frac{V_O}{V_I} = -\frac{2D}{1 - D}. \]  

(2.16)

Using the principle of energy conservation, \( V_I I_I = V_O I_O \), and (2.16), one obtains dc current transfer function for a lossless switched-inductor buck boost converter as

\[ M_{IDC} = \frac{I_O}{I_I} = -\frac{1 - D}{2D}. \]  

(2.17)

The efficiency of the converter is

\[ \eta = \frac{P_O}{P_I} = \frac{V_O I_O}{V_I I_I} = M_{VDC} M_{IDC} = \frac{(1 - D)}{2D} M_{VDC} \]

Therefore, a dc input-to-output voltage transfer function for a lossy switched-inductor buck-boost converter with an efficiency of \( \eta \) becomes

\[ M_{VDC_{lossy}} = \frac{V_O}{V_I} = -\frac{2\eta D}{1 - D}. \]  

(2.18)

The dc current transfer function for a lossy PWM switched-inductor buck-boost converter is

\[ M_{IDC_{lossy}} = \frac{I_O}{I_I} = -\frac{1 - D}{2\eta D}. \]  

(2.19)

2.4.4 Device Stresses for CCM

As shown above in (2.16) the DC voltage transfer function is \( M_{VDC} = V_O/V_I = 2D/(1 - D) \). The maximum input voltage is

\[ V_{I_{max}} = V_O \frac{1 - D_{min}}{2D_{min}}. \]

Therefore, the maximum voltage stress across the MOSFET \( S \) and the rectifier diode \( D_0 \) is

\[ V_{SM} = V_{DM} = V_{I_{max}} + V_O = \frac{(1 + D_{min})}{2D_{min}} V_O. \]  

(2.20)
The average value of the inductor current $I_L$ is equal to sum of the half the dc input current $I_I$ and the dc output current $I_O$. Therefore, the peak value of the switch current due to the switched-inductor network becomes

$$I_{SM} = I_{Lpeak} = 2I_L = 2 \left( \frac{1}{2} I_I + I_O + \frac{\Delta i_L}{2} \right) = \frac{2}{1 - D_{min}} I_{Omax} + \Delta i_L. \quad (2.21)$$

The peak current through diode $D_0$ is

$$I_{DM} = I_{D_{peak}} = I_L + \frac{\Delta i_L}{2} = \frac{1}{1 - D_{min}} I_{Omax} + \frac{\Delta i_L}{2}. \quad (2.22)$$

### 2.4.5 Boundary Between CCM and DCM

Fig.2.6 shows the idealized inductor current waveform at the boundary condition between CCM and DCM. The current must exceed the $\Delta i_{L_{max}}$ level to remain in CCM mode. The equation for $\Delta i_{L_{max}}$ is

$$\Delta i_{L_{max}} = \frac{V_O (1 - D_{min})}{2f_s L_{min}}.$$  

Hence, the dc inductor current boundary between CCM and DCM is

$$I_{LB} = \frac{\Delta i_{L_{max}}}{2} = \frac{V_O (1 - D_{min})}{4f_s L_{min}}. \quad (2.23)$$

The dc output current is identified using the relationship between $I_L$ and $I_O$

$$I_L = \frac{1}{1 - D} I_O. \quad (2.24)$$

Substitution of (2.24) into the inductor current boundary condition in (2.23) yields the output current boundary condition.
Figure 2.7: Normalized load current $I_{OB}/(V_O/4f_sL)$ as a function of $D$ at the CCM/DCM boundary for the switched-inductor buck-boost converter.

\[ I_{OB} = \frac{V_O (1 - D_{\text{min}})^2}{4f_sL}. \]  

(2.25)

This leads to the load resistance boundary

\[ R_{LB} = \frac{V_O}{I_{OB}} = \frac{4f_sL}{(1 - D_{\text{min}})^2}. \]  

(2.26)

The minimum value of inductance $L$ to operate the converter in CCM is

\[ L_{\text{min}} = \frac{R_{L_{\text{max}}}(1 - D_{\text{min}})^2}{4f_s}. \]  

(2.27)

Therefore, the minimum inductance of each inductor is

\[ L_{\text{min}} = \frac{R_{L_{\text{max}}}(1 - D_{\text{min}})^2}{4f_s}. \]
Figure 2.8: Normalized load resistance $R_{LB}/(4f_s L)$ as a function of $D$ at the CCM/DCM boundary for the switched-inductor buck-boost converter.

Figure 2.9: The load resistor, filter capacitor with ESR, and diode $D_0$ of the switched-inductor buck-boost for finding the output ripple voltage.
2.4.6 Ripple Voltage in Switched-Inductor Buck-Boost Converter for CCM

Fig. 2.9 shows the switched inductor buck-boost converter with only the diode and output impedance. The peak-to-peak value of the capacitance current is

\[ I_{C_{pp}} = I_{DM} \approx \frac{1}{2} I_I + I_O = \frac{1}{1-D} I_O. \]

The peak-to-peak voltage across the ESR of the capacitor can be derived

\[ V_{r_{cpp}} = r_C I_{C_{pp}} \approx \frac{r_C I_{O_{max}}}{1-D_{max}}. \]

The peak-to-peak value of the output ripple voltage \( V_r \) is typically provided as a standard value given. Therefore, the ac component of the voltage across the capacitor \( C \) is

\[ V_{C_{pp}} \approx V_r - V_{r_{cpp}}. \] (2.28)

The ac component of the output voltage across the capacitance is also given by

\[ V_{C_{pp}} = \frac{I_{O_{max}} D_{max} T}{C_{min}} = \frac{V_O D_{max}}{f_s R_{L_{min}} C_{min}}. \] (2.29)

Rearranging equation 2.29 and substituting equation 2.28 one achieves an equation for the minimum capacitance needed

\[ C_{min} = \frac{V_O D_{max}}{f_s R_{L_{min}} V_{C_{pp}}}. \] (2.30)

2.4.7 Power Losses and Efficiency of the Switched-Inductor Buck-Boost Converter for CCM

A circuit for the switched-inductor buck-boost with parasitic resistances is shown in Fig. 2.11. In the figure, \( r_{DS} \) is the MOSFET on-resistance; \( R_{F0}, R_{F1}, R_{F2} \), and
Figure 2.10: Current and voltage waveforms for the ripple voltage for the PWM switched-inductor buck-boost converter.

Figure 2.11: Equivalent circuit of the switched-inductor buck-boost converter with parasitic resistances and diode voltage to determine power losses.
$R_{F12}$ are the diodes forward resistances; $V_{F0}$, $V_{F1}$, $V_{F2}$, and $V_{F12}$ are the junction voltages; $r_L$ is the equivalent series resistance of the inductors, and $r_C$ is the ESR of the capacitor $C$. It is assumed that the inductor current $i_L$ does not have a ripple. The switch current waveform is

$$i_S = \begin{cases} \frac{I_I + I_O}{1 - D}, & \text{for } 0 < t \leq D T \\ 0, & \text{for } D T < t \leq T \end{cases}. \quad (2.31)$$

This leads to an RMS value of the switch current

$$I_{Srms} = \sqrt{\frac{1}{T} \int_0^T i_S^2 dt} = \frac{2 I_O}{1 - D} \sqrt{\frac{1}{T} \int_0^{DT} dt} = \frac{I_O D^\frac{1}{2}}{(1 - D)}. \quad (2.32)$$

The MOSFET $S$ conduction loss becomes

$$P_{rDS} = r_{DS} I_{Srms}^2 = \frac{4 D r_{DS} I_O^2}{(1 - D)^2}. \quad (2.33)$$

Assuming the transistor output capacitance $C_O$ is linear, the switching loss is

$$P_{sw} = f_s C_o (V_I + V_O)^2. \quad (2.34)$$

The total power dissipated in the MOSFET, not including the drive power, is expressed by

$$P_{FET} = P_{rDS} + \frac{P_{sw}}{2} = \frac{4 D r_{DS} I_O^2}{(1 - D)^2} + \frac{f_s C_o (V_I + V_O)^2}{2}. \quad (2.35)$$

The current waveform of diode $D_0$ may be approximated by

$$i_{D0} = \begin{cases} 0, & \text{for } 0 < t \leq D T \\ \frac{1}{2} I_I + I_O, & \text{for } D T < t \leq T \end{cases}, \quad (2.36)$$

resulting in the RMS of the diode current

$$I_{D0rms} = \sqrt{\frac{1}{T} \int_0^T i_D^2 dt} = \frac{I_O}{1 - D} \sqrt{\frac{1}{T} \int_{DT}^T dt} = \frac{I_O}{\sqrt{1 - D}}. \quad (2.37)$$

The power loss in the resistance of the diode $R_{F0}$ is

$$P_{RF0} = R_{F0} I_{D0rms}^2 = \frac{R_F I_O^2}{1 - D}. \quad (2.38)$$

The average current through the diode is
2 DC Analysis of Switched-Inductor Buck-Boost Converter in CCM

\[ I_{D_0} = \sqrt{\frac{1}{T} \int_0^T i_D dt} = \frac{I_O}{1-D} \sqrt{\frac{1}{T} \int_{DT}^T dt} = I_O. \quad (2.39) \]

The power loss in the diode due to the junction voltage is

\[ P_{VF} = V_F I_{D_0} = V_F I_O. \quad (2.40) \]

Therefore, the total power loss in diode \( D_0 \) is

\[ P_{D_0} = P_{VF_0} + P_{RF_0}. \quad (2.41) \]

The current waveforms of diodes \( D_1 \) and \( D_2 \) may be approximated by

\[ i_{D_1} = \begin{cases} I_L = \frac{1}{2} I_I + I_O, & \text{for } 0 < t \leq DT, \\ 0, & \text{for } DT < t \leq T, \end{cases} \quad (2.42) \]

yielding

\[ I_{D_{1rms}} = \sqrt{\frac{1}{T} \int_0^T i_{D_1}^2 dt} = I_O \sqrt{\frac{1}{T} \int_0^{DT} dt} = I_O \sqrt{1 - D}. \quad (2.43) \]

The power loss in the forward resistance of the diode \( R_F \) is

\[ P_{RF_1} = R_F I_{D_{1rms}}^2 = \frac{R_F I_O^2 D}{(1-D)^2}. \quad (2.44) \]

The average current through the diode is

\[ I_{D_1} = \sqrt{\frac{1}{T} \int_0^T i_D dt} = \frac{I_O}{1-D} \sqrt{\frac{1}{T} \int_0^D dt} = \frac{D}{1-D} I_O. \quad (2.45) \]

The power loss in the diode due to the junction voltage is

\[ P_{VF_1} = V_F I_{D_1} = \frac{V_F I_O D}{1-D}. \quad (2.46) \]

Therefore, the total power loss in diode \( D_1 \) is

\[ P_{D_1} = P_{VF_1} + P_{RF_1}. \quad (2.47) \]
The current waveform of diode $D_{12}$ may be approximated by

$$i_{D_{12}} = \begin{cases} 0, & \text{for } 0<t<DT \leq T, \\ \frac{1}{2}I_l + I_o, & \text{for } DT<t \leq T, \end{cases}$$

resulting in

$$I_{D_{12}rms} = \sqrt{\frac{1}{T} \int_0^T i_{D_{12}}^2 dt} = \frac{I_o}{1-D} \sqrt{\frac{1}{T} \int_{DT}^T dt} = \frac{I_o}{\sqrt{1-D}}. \quad (2.49)$$

The power loss in the diode forward resistance $R_{F_{12}}$ is

$$P_{RF_{12}} = R_{F_{12}} I_{D_{12}rms}^2 = \frac{R_F I_o^2}{1-D}. \quad (2.50)$$

The average current through the diode is

$$I_{D_{12}} = \sqrt{\frac{1}{T} \int_0^T i_{D_{12}} dt} = \frac{I_o}{1-D} \sqrt{\frac{1}{T} \int_{DT}^T dt} = I_o. \quad (2.51)$$

The power loss in the diode due to the junction voltage is

$$P_{VF_{12}} = V_F I_{D_{12}} = V_F I_o. \quad (2.52)$$

Therefore, the total power loss in diode $D_{12}$ is

$$P_{D_{12}} = P_{VF_{12}} + P_{RF_{12}}. \quad (2.53)$$

The inductor current waveform can be approximated by

$$i_L \approx \frac{1}{2}I_l + I_o = \frac{I_o}{1-D}, \quad (2.54)$$

$$I_{Lrms} = \frac{I_o}{1-D}. \quad (2.55)$$

The power loss in the inductor due to its series resistance is

$$P_{rL} = r_L I_{Lrms}^2 = \frac{r_L I_o^2}{(1-D)^2}. \quad (2.56)$$
Figure 2.12: Comparison of the efficiency of the traditional buck-boost converter and the switched-inductor buck-boost converter as a function of duty cycle $D$ for $V_O = 28$ V, $r_{DS} = 0.27 \Omega$, $C_O = 150$ pF, $V_F = 0.65$ V, $R_F = 0.2 \Omega$, $r_L = 0.5 \Omega$, $r_C = 0.155 \Omega$, and $f_s = 100$ kHz.

The current waveform through the capacitor is approximated by

$$i_C = \begin{cases} -I_O, & \text{for } 0 < t < DT, \\ \frac{1}{2}I_1, & \text{for } DT < t < T, \end{cases}$$

leading to

$$I_{Crms} = \sqrt{\frac{1}{T} \int_0^T i_C^2 dt} = I_O \sqrt{\frac{D}{1 - D}}. \quad (2.58)$$

$$P_{rC} = r_CI_{Crms}^2 = r_CI_O^2 \frac{D}{1 - D}. \quad (2.59)$$

$$P_{LS} = P_{rC} + 2P_{rL} + P_{sw} + P_{rDS} + P_{D0} + P_{D1} + P_{D2} + P_{D12}. \quad (2.60)$$

The converter efficiency is

$$\eta = \frac{P_O}{P_O + P_{LS}} = \frac{1}{1 + \frac{P_{LS}}{P_O}}. \quad (2.61)$$

Fig. 2.12 compares the efficiency of the switched-inductor buck boost converter $\eta$. 
as a function of the $D$ with the efficiency of the traditional buck boost converter. As shown in the fig. 2.12, the efficiency of the converters are in congruence.

### 2.4.8 DC Voltage Transfer Function of Lossy Switched-Inductor Buck-Boost Converter for CCM

The dc component of the input current is

$$I_I = \frac{1}{T} \int_0^T i_S dt = \frac{1}{T} \int_0^{DT} 2I_L dt = 2DI_L = 2D \left( \frac{1}{2} I_I + I_O \right) = 2D \left( \frac{I_O}{1 - D} \right).$$

The dc component of the output current is

$$I_O = \frac{1}{T} \int_0^T i_D dt = \frac{1}{T} \int_{DT}^T I_L dt = (1-D)I_L = (1-D) \left( \frac{1}{2} I_I + I_O \right) = (1-D) \left( \frac{I_O}{1 - D} \right).$$

Therefore, the dc current transfer function for the switched-inductor buck-boost converter

$$M_{IDC} = \frac{I_O}{I_I} = \frac{1 - D}{2D}.$$

The above expression is valid for both a lossy and lossless converter, therefore the converter efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{V_O I_O}{V_I I_I} = \frac{1 - D}{2D} M_{VDC}. \quad \text{(2.62)}$$

This leads to a dc voltage transfer function for a lossy switched-inductor buck-boost converter as

$$M_{VDC} = \frac{\eta}{M_{IDC}} = \frac{2\eta D}{(1 - D)}. \quad \text{(2.63)}$$
2.4.9 Switched-Inductor Buck-Boost for CCM Design

Designing a PWM switched-inductor buck-boost DC-DC converter for the following specifications: $V_I = 48 \pm 4$ V, $V_O = 28$ V, $I_O = 0.5$ to $5$ A, $\eta = 90\%$, $f_s = 100$ kHz, and $\frac{V}{V_O} = 1\%$, requires the maximum and minimum values of the output power are, respectively,

$$P_{O_{\text{max}}} = V_O I_{O_{\text{max}}} = 56 \text{ W}$$

and

$$P_{O_{\text{min}}} = V_O I_{O_{\text{min}}} = 14 \text{ W}.$$ 

The maximum and minimum values of the load resistance are, respectively

$$R_{L_{\text{max}}} = \frac{V_O}{I_{O_{\text{min}}}} = 56 \Omega$$

and

$$R_{L_{\text{min}}} = \frac{V_O}{I_{O_{\text{max}}}} = 14 \Omega.$$ 

The minimum, nominal, and maximum values of the dc voltage transfer function are respectively

$$M_{V_{DC_{\text{min}}}} = \frac{V_O}{V_{I_{\text{max}}}} = 0.54,$$

$$M_{V_{DC_{\text{nom}}}} = \frac{V_O}{V_{I_{\text{nom}}}} = 0.58,$$

and

$$M_{V_{DC_{\text{max}}}} = \frac{V_O}{V_{I_{\text{min}}}} = 0.64.$$
The minimum, nominal, and maximum values of the duty cycle are respectively

\[ D_{\text{min}} = \frac{M_{V\text{DC}\text{min}}}{M_{V\text{DC}\text{min}} + 2\eta} = 0.212, \]

\[ D_{\text{nom}} = \frac{M_{V\text{DC}\text{nom}}}{M_{V\text{DC}\text{nom}} + 2\eta} = 0.225, \]

and

\[ D_{\text{max}} = \frac{M_{V\text{DC}\text{max}}}{M_{V\text{DC}\text{max}} + 2\eta} = 0.24. \]

The minimum inductance needed is

\[ L_{\text{min}} = \frac{R_{L\text{max}}(1 - D_{\text{min}})^2}{4f_s} = 87 \, \mu\text{H}. \]

Therefore a value of 330 \( \mu\)H was chosen is the standard value. The peak-to-peak value of the ac component of the inductor current is

\[ \Delta i_{L\text{min}} = \frac{V_O(1 - D_{\text{max}})}{2f_sL} = 1.22 \, \text{A}. \]

The current and voltage stresses of the semiconductor devices are:

\[ I_{\text{SM}} = \frac{2}{1 - D_{\text{min}}} I_{O\text{max}} + \Delta i_L = 4.5 \, \text{A}. \]

\[ I_{DM} = \frac{1}{1 - D_{\text{min}}} I_{O\text{max}} + \frac{\Delta i_L}{2} = 3.2 \, \text{A}. \]

and

\[ V_{\text{SM}} = V_{DM} = V_{i\text{max}} + V_O = 80 \, \text{V}. \]

The ripple voltage is
\[ V_r = \frac{V_O}{100} = 0.28 \text{ V}. \]

Assuming \( V_{\text{rcpp}} = 100 \text{ mV} \), leads to determining the ESR maximum value of the filter capacitor as

\[ r_{C_{\text{max}}} = \frac{V_{\text{rcpp}}}{I_{D_{\text{Mmax}}}} = 22.2 \text{ mΩ}. \]

The ripple voltage across the capacitor is

\[ V_{\text{Cpp}} = V_r - V_{\text{rcpp}} = 0.1 \text{ V}. \]

The minimum value of capacitance required is

\[ C_{\text{min}} = \frac{D_{\text{max}}V_O}{f_sR_{\text{Lmin}}V_{\text{Cpp}}} = 53 \mu \text{F}. \]

Therefore, 100 \( \mu \text{F} \) was chosen from the standard values.

Power losses and efficiency will be calculated for a full load. The switch RMS current is

\[ I_{\text{Srms}} = \frac{I_OD^2}{1 - D} = 2.455 \text{ A}. \]

The MOSFET \( S \) conduction loss is then

\[ P_{\text{rDS}} = r_{\text{DS}}I_{\text{Srms}}^2 = \frac{4D_{\text{rDS}}I_O^2}{(1 - D)^2} = 1.6 \text{ W}. \]

Assuming that the transistor output capacitance \( C_O \) is linear, the switching loss is

\[ P_{\text{sw}} = f_sC_o(V_I + V_O)^2 = 0.0866 \text{ W}. \]

The total power dissipated in the MOSFET, not including the drive power, is expressed by

\[ P_{\text{FET}} = P_{\text{rDS}} + \frac{P_{\text{sw}}}{2} = \frac{4D_{\text{rDS}}I_O^2}{(1 - D)^2} + \frac{f_sC_o(V_I + V_O)^2}{2} = 2.63 \text{ W}. \]
The RMS current of diode $D_0$ is

$$I_{D_{0 \text{rms}}} = \frac{I_O}{\sqrt{1 - D}} = 2.3\ \text{A}.$$ 

The power loss in the resistance of the diode $R_{F0}$ is

$$P_{R_{F0}} = R_{F0}I_{D_{0 \text{rms}}}^2 = \frac{R_FI_O^2}{1 - D} = 1.033\ \text{W}.$$ 

The power loss in the diode due to the junction voltage is

$$P_{V_F} = VFI_O = 1.3\ \text{W}.$$ 

Therefore, the total power loss in diode $D_0$ is

$$P_{D_0} = P_{V_{F0}} + P_{R_{F0}} = 2.33\ \text{W}.$$ 

The RMS current of diode $D_1$ is

$$I_{D_{1 \text{rms}}} = \frac{I_OD_1^2}{1 - D} = 1.22\ \text{A}.$$ 

The power loss in the diode due to the junction voltage is

$$P_{R_{F1}} = R_{F1}I_{D_{1 \text{rms}}}^2 = \frac{R_FI_O^2D}{(1 - D)^2} = 0.15\ \text{W}.$$ 

The power loss in the dc voltage source of the diode $V_F$ is

$$P_{V_{F1}} = VFID_1 = \frac{VFI_OD}{1 - D} = 0.38\ \text{W}.$$ 

Therefore, the total power loss in diode $D_1$ is

$$P_{D_1} = P_{V_{F1}} + P_{R_{F1}} = 0.53\ \text{W}.$$ 

The RMS current of diode $D_{12}$ is

$$I_{D_{12 \text{rms}}} = \frac{I_O}{\sqrt{1 - D}} = 2.3\ \text{A}.$$ 

The power loss in the diode forward resistance $R_F$ is
\[ P_{RF_{12}} = R_{F_{12}} I_{D_{rms}}^2 = \frac{R_F I_O^2}{(1-D)} = 1.03 \text{ W}. \]

The power loss in the diode due to the junction voltage is

\[ P_{VF_{12}} = V_F I_{D_{12}} = V_F I_O = 1.3 \text{ W}. \]

Therefore, the total power loss in diode \( D_{12} \) is

\[ P_{D_{12}} = P_{VF_{12}} + P_{RF_{12}} = 2.33 \text{ W}. \]

The inductor current RMS value is

\[ I_{L_{rms}} = \frac{I_O}{1-D} = 2.58 \text{ A}. \]

The inductor power loss due to series resistance is

\[ P_{rL} = r_L I_{L_{rms}}^2 = \frac{r_L I_O^2}{(1-D)^2} = 3.33 \text{ W}. \]

The capacitor current RMS value is

\[ I_{C_{rms}} = I_O \sqrt{\frac{D}{1-D}} = 0.19 \text{ A}. \]

\[ P_{rC} = r_C I_{C_{rms}}^2 = r_C I_O^2 \frac{D}{1-D} = 0.1808 \text{ W}. \]

\[ P_{LS} = P_{rC} + P_{rL} + P_{sw} + P_{rDS} + P_{D_0} + P_{D_1} + P_{D_2} + P_{D_{12}} = 10.96 \text{ W}. \]

The converter efficiency is

\[ \eta = \frac{P_O}{P_O + P_{LS}} = 84\%. \]

Table 2.1 shows a comparison of current stresses, inductor current RMS, minimum inductance, and efficiency values between the traditional buck-boost and the
Table 2.1: Power Loss Comparison of Traditional Buck-Boost Converter Versus the Switched-Inductor Buck-Boost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Traditional</th>
<th>Switched-Inductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>0.39</td>
<td>0.24</td>
</tr>
<tr>
<td>$I_{SM}$</td>
<td>4.0 A</td>
<td>4.5 A</td>
</tr>
<tr>
<td>$I_{DM}$</td>
<td>4.0 A</td>
<td>3.2 A</td>
</tr>
<tr>
<td>$I_{\text{Lrms}}$</td>
<td>3.2 A</td>
<td>2.58 A</td>
</tr>
<tr>
<td>$L$</td>
<td>118 $\mu$H</td>
<td>87 $\mu$H</td>
</tr>
<tr>
<td>$\eta$</td>
<td>86.1%</td>
<td>84%</td>
</tr>
</tbody>
</table>

Switched-inductor buck-boost. As shown in the table, the inductor current RMS value is always less than the switched-inductor buck-boost, which leads to smaller conduction losses in the inductors even though the circuit has two inductors. The efficiency of the switched-inductor buck-boost is comparable to the traditional buck-boost efficiency despite the addition of three diodes and an inductor. Also, the switched-inductor buck-boost duty ratio will always be smaller than that of the traditional buck-boost. Therefore, a switched-inductor buck-boost should be considered when the duty ratio is a higher value.

2.4.10 Simulation Results

An example PWM switched-inductor buck-boost converter was designed with specifications of $V_I = 48$ V, $V_O = 28$ V, $L_1 = L_2 = 330 \mu$H, $C = 100 \mu$F, $R_L = 50 \Omega$, $f_s = 100$ kHz, and $D = 0.24$. Inductors $L_1$ and $L_2$ are manufactured by Murata Power Solutions with a measured dc resistance $r_L = 0.42 \Omega$. The capacitor $C$ was electrolytic and had a measured dc resistance of $r_C = 0.155 \Omega$. An International Rectifier power MOSFET IRF520, rated 9.2 A/100 V and with a maximum $r_{DS} = 0.27 \Omega$ and $C_o = 150$ pF, and an ON-Semiconductor SWITCH-MODE power rectifier MBR10100 rated 10 A/100 V and having $V_F = 0.65$ V and $R_F = 0.2 \Omega$ were selected. For the example considered, the minimum value of inductance to ensure CCM operation was $L > L_{\text{min}} = 87 \mu$H. The capacitance
minimum value necessary to ensure the ripple voltage is only dependent on the capacitor ESR is shown as $C > C_{\text{min}} = 53 \mu F$. Selected values of $L = 330 \mu H$ and $C = 100 \mu F$ are in agreement with (2.27) and (2.30), respectively.

During the time interval $0 < t \leq DT$, the calculated voltage across the diode was

$$v_{D_0} = V_I - V_O = 76 \text{ V}$$  

(2.64)

and the calculated voltage across the inductor $L$ was

$$v_L = V_I = 48 \text{ V}.$$  

(2.65)

During the time interval $DT < t \leq T$, the current through and the voltage across MOSFET $S$ and diode $D_0$ are zero, respectively. This is only true for ideal semiconductor devices. When the switch is OFF, the calculated voltage across the inductor, is

$$v_L = \frac{1}{2}V_O = -14 \text{ V}.$$  

(2.66)

The designed PWM switched-inductor buck-boost DC-DC converter was sim-
Figure 2.14: Simulated current and voltage waveforms for the PWM switched-inductor buck-boost converter in CCM.

ulated with Saber circuit simulator. The simulation models of the IRF520 power MOSFET and the MBR10100 ultra-fast recovery diode were selected from the Saber library. The passive component parasitic were also included was stated above. A transient simulation was carried out for the designed converter. The selected key voltage and current waveforms for one time period are shown in Figs. 2.13 and 2.14. The values predicted by equations (2.64), (2.65), and (2.66) were in accordance with the measured voltages and current.

The calculated value of duty ratio for the designed converter was \( D = 0.253 \) at \( \eta = 86\% \). A duty ratio of \( D = 0.254 \) was used in the simulated circuit to achieve the design specified output voltage \( V_O = 28.1 \text{V} \). Fig. 2.15 shows the simulated transient output voltage waveform when the input voltage \( V_I \) is turned on. The steady-state dc output voltage was \( V_O = 28.1 \text{V} \) at \( D = 0.254 \), which approximates the specified dc output voltage. This validates the steady-state analysis presented in the previous section for the switched-inductor buck-boost DC-DC converter.
2.4.11 Experimental Results

A laboratory prototype was built in accordance with the design example. The PWM switched-inductor buck-boost DC-DC converter shown in Fig. 2.1 was setup. An IRF2110 driver was employed to drive the high-side MOSFET. Table 2.2 presents the theoretically predicted and experimentally measured values of key parameters corresponding to the steady-state analysis. The peak-to-peak ripple voltage across the filter capacitor $v_{C_{i-p}}$ was obtained by measuring capacitor current waveform $i_C$ to obtain $v_{C_{i-p}} = i_C r_C$. A Tektronix P6021 AC current probe with a conversion factor of 2 mA/mV was employed to perform current measurements. Fig. 2.16 shows the voltage and current waveforms of switches $S$ and $D_0$. Differential Probe Master 4231 was employed to perform the pulsating voltage measurements. The same differential probe and a 1 ohm resistor were employed to capture the current waveform. The output current $I_O$ varied from $I_O = 0.1\,\text{A}$ to $I_O = 0.5\,\text{A}$ in steps of $\Delta I_O = 0.1\,\text{A}$. The duty ratio $D$ varied in
increments between $D = 0.1$ to $D = 0.8$. Fig. 2.17 presents the theoretically predicted and experimentally measured $\eta$ as a function of $I_O$. Fig. 2.18 shows the theoretically predicted and experimentally measured $M_{VDC}$ as a function of $D$. The predicted and measured efficiency values of the switched-inductor buck-boost DC-DC converter were in congruence. The difference between the measured and predicted values for $M_{VDC}$ at duty ratios higher than 0.6 can be attributed to the losses in the converter. These losses were due to ringing in the MOSFET and other non-ideal aspects in the setup, such as stray inductance and capacitance, which were not included in the analysis.
Figure 2.17: Efficiency $\eta$ of the switched-inductor buck-boost converter as a function of $I_O$.

Figure 2.18: DC voltage ratio $M_{V_{DC}}$ of the switched-inductor buck-boost converter as a function of $D$. 

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Table 2.2: Theoretical and Experimental Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_O$</td>
<td>-28 V</td>
<td>-28.1 V</td>
</tr>
<tr>
<td>$v_L$</td>
<td>48 and 14 V</td>
<td>47.5 and 14.1 V</td>
</tr>
<tr>
<td>$\Delta i_{L_{max}}$</td>
<td>0.327 mA</td>
<td>0.34 mA</td>
</tr>
<tr>
<td>$v_{C_{(p-p)}}$</td>
<td>137 mV</td>
<td>200 mV</td>
</tr>
</tbody>
</table>

2.5 Conclusion

A detailed steady-state analysis of the PWM switched-inductor buck-boost DC-DC converter operating in CCM was presented. The dc input-to-output voltage transfer function for an ideal PWM switched-inductor buck-boost DC-DC converter were derived. Equations for power loss in each of the PWM switched-inductor buck-boost DC-DC converter components was also derived. An efficiency expression was derived using the power loss expressions. The minimum inductance $L$ required to ensure CCM operation, an expression for output voltage ripple, and the minimum capacitance $C$ were derived. A laboratory prototype of a switched-inductor buck-boost DC-DC converter was designed, built, and tested to verify the theoretical analysis with simulated results. The theoretically predicted and simulated values were in good agreement with the experimental results. The predicted output voltage $V_O$ is in agreement with the experimental results as shown in Table 2.2. The predicted efficiency was paralleled with the experimental results for the discrete points of output current $I_O$ as shown in Fig. 2.17. The experimental results of the input-to-output voltage ratio $M_{V_{DC}}$ were in good agreement with the predicted results from $D = 0.05$ to $D = 0.6$ at selected values of duty cycle. For the higher duty ratio values (0.6 to 0.8), a greater difference resulted due to higher frequency ringing losses in the MOSFET, which was not included in the analysis. The predicted output voltage ripple, the inductor voltage waveform range, and change in inductor current waveform were in accordance with the experimental measured results.
The disadvantage of the PWM switched-inductor buck-boost DC-DC converter as compared to the traditional buck-boost DC-DC converter is more components are necessary (an inductor and three diodes). The advantages of the PWM switched-inductor buck-boost include:

1. Less current stresses in the inductor, therefore, a reduction in conduction losses at a given duty cycle.
2. Reduction in inductor sizes and cost.
3. The transfer function of the switched-inductor buck-boost has twice the gain of the traditional buck-boost converter.
3 Small-Signal Model of PWM
Switched-Inductor Buck-Boost in CCM

3.1 Introduction

Circuit averaging technique involves the transformation of the topology rather than using state equations of the converter. Circuit averaging over state space analysis provides a better insight into the actual circuit. The averaged circuits presented in this paper using the circuit averaging technique are simulated using modern circuit simulators. This section covers the derivation of the averaged large-signal, dc and small-signal linear time-invariant models of PWM switched-inductor buck-boost using circuit averaging technique. The non-linear switching network is replaced by dependent voltage and current sources; however, the linear passive components remain untransformed. The averaged low frequency voltage across, and the averaged current through the switch must remain identical to the PWM switch’s voltages and currents. The independent and dependent sources current and voltages remain independent variables.

The following assumptions were used for the derived model:

- The transistor’s output capacitance and diode’s capacitance are neglected.
- All passive components (i.e. inductors, resistors, capacitors) are assumed to be linear, time-invariant, and frequency independent.
- Storage-time modulation of bipolar transistors is neglected.
3 Small-Signal Model of PWM Switched-Inductor Buck-Boost in CCM

- The diode is represented by a linear battery in the on-state and by an infinite resistance in the off-state.
- The transistor has a linear on-state resistance and the off-state resistance is infinite.
- The converters time constant is much larger than the switching period.
3.2 Averaged Model of the Non-linear Switching Network of the PWM Switched-Inductor Buck-Boost

The PWM switched-inductor buck-boost converter with the nonlinear switch and diodes is shown in Fig. 2.1. Fig. 3.1 shows the PWM with the non-linear components boxed. Compared to other DC-DC converters such as the buck, boost, and traditional buck-boost, the PWM switched-inductor buck-boost converter has more than one diode. The non-linear blocks depicted in Fig. 3.1 show the switching network of the PWM switched-inductor buck-boost converter. Averaging and linearizing the switched-inductor buck-boost converter requires averaging and linearizing the switching network’s non-linear components. The voltages and currents associated with the switching network of the switched-inductor buck-boost converter are discontinuous functions for time. Therefore, average values must be obtained for one switching time period. Based on equation 2.5 yields

\[
i_s = i_i = \begin{cases} 
2I_L, & \text{for } 0 < t \leq DT \\
0, & \text{for } DT < t \leq T 
\end{cases}
\]  

(3.1)
Therefore, the steady state approximation for the switch current is

\[ I_s = \frac{1}{T} \int_0^T i_s dt = \frac{1}{T} \int_0^{DT} 2I_L dt = 2DI_L. \]  (3.2)

This expression describes the ideal dc current-dependent current source controlled by the inductor current \( I_L \).

Based on equation 2.1 it is

\[ v_{SD} = v_{DO} = \begin{cases} (V_I - V_O), & \text{for } 0 < t \leq DT, \\ 0, & \text{for } DT < t \leq T \end{cases}. \]  (3.3)

The steady state approximation for the voltage across the ideal diode zero is

\[ V_{D0} = \frac{1}{T} \int_0^{DT} -(V_I + V_O) dt = (V_I - V_O)D. \]  (3.4)

Based on the equation 2.12 it is

\[ i_{D0} = -i_O = \begin{cases} I_L, & \text{for } 0 < t \leq DT, \\ 0, & \text{for } DT < t \leq T \end{cases}. \]  (3.5)

The steady state approximation for the current through the ideal diode zero is

\[ I_{D0} = \frac{1}{T} \int_D^{T} I_L dt = I_L(1 - D). \]  (3.6)

Based on equation 2.11 it is

\[ v_{D1} = v_{D2} = -\frac{1}{2}V_O. \]  (3.7)

The steady state approximation for the voltage across the ideal diode zero is

\[ V_{D1} = \frac{1}{T} \int_D^{T} -\frac{1}{2}V_O dt = \left(-\frac{1}{2}V_O\right)(1 - D). \]  (3.8)

The current passing through the inductor is always \( I_L \) therefore the current passing through diodes \( D_1, D_2, \) and \( D_{12} \) is \( I_L \). Based on equation it is given
Figure 3.2: Averaged model of the PWM switched-inductor buck-boost converter.

\[ v_{D_{12}} = \begin{cases} V_I, & \text{for } 0 < t \leq DT \\ 0, & \text{for } DT < t \leq T \end{cases} \]  

(3.9)

The steady state approximation for the voltage across the ideal diode zero is

\[ V_{D_{12}} = \frac{1}{T} \int_0^{DT} V_I dt = V_I D. \]  

(3.10)

3.3 Large-Signal Model for CCM

The actual switching network is shown in Fig. 3.1. The averaged dc model of the PWM switched-inductor buck-boost converter can be obtained by replacing the switch \( S \) and the diodes \( D \) in Fig. 3.1 by their average values provided in equations 3.2, 3.4, 3.8, and 3.10 respectively. The PWM switched-inductor buck-boost converter with the non-linear network is replaced by the averaged dc model shown in Fig. 3.2. Replacing the dc quantities such as \( I_s, I_I, V_I, V_O, \) and \( D \) in the averaged model by low-frequency, time-dependent, large-signal quantities such as \( i_S, i_I, v_I, v_O, \) and \( d_T \), provides the large signal averaged model of the
Figure 3.3: Low-frequency large signal model of the PWM switched-inductor buck-boost converter.

The low-frequency large signal model of PWM switched-inductor buck-boost converter is shown in Fig. 3.3. The low-frequency large signal variables can be approximated as

\[ i_s = i_Id_T, \quad (3.11) \]

\[ v_{D0} = (v_I - v_O)d_T, \quad (3.12) \]

\[ v_{D1} = v_{D2} = -\frac{1}{2}(1 - d_T)v_O, \quad (3.13) \]

and

\[ v_{D12} = v_Id_T. \quad (3.14) \]
3.4 DC and Small-Signal Linear Circuit Model of PWM Switched-Inductor Buck-Boost Converter

Obtaining the dc and small-signal linear circuit models, the averaged model of the PWM switched-inductor buck-boost converter shown in Fig. 3.4 will be perturbed and linearized. Consider the PWM switched-inductor buck-boost converter excited by a low-frequency perturbation riding over the dc component. The waveforms associated with the switched-inductor buck-boost converter will include the following types of signals

- dc component
- fundamental component of the low-frequency perturbation and its harmonics
- fundamental component of the switching frequency and its harmonics.

DC and low-frequency signals are significant because of the role played in closed-loop control of PWM converters, where the high frequency switching components can be neglected. The perturbation and linearization method will only be valid
for frequencies less than or equal to half switching frequency

$$f_p \leq \frac{f_s}{2}.$$  \hspace{1cm} (3.15)

The average voltages, currents, and duty cycle expressed as a sum of the dc components and the low-frequency perturbation components are

$$v_I = V_I + v_i,$$  \hspace{1cm} (3.16)

$$i_S = I_S + i_s,$$  \hspace{1cm} (3.17)

$$i_I = I_I + i_i,$$  \hspace{1cm} (3.18)

$$v_{SD} = V_{SD} + v_{sd},$$  \hspace{1cm} (3.19)

$$v_{D_1} = V_{D_1} + v_{d_1},$$  \hspace{1cm} (3.20)

$$v_O = V_O + v_o,$$  \hspace{1cm} (3.21)

and

$$d_T = D + d.$$  \hspace{1cm} (3.22)

In equations 3.16 through 3.22, the left hand side of the equation expresses the sum of the dc and the ac low-frequency component. The right hand side of the equation in upper-case letters represent the dc components while the right hand side equations in lower-case letters represent the ac low-frequency components.
Substituting equations 3.16-3.22 into (3.11)-(3.14) achieves:

\[ I_S + i_s = (D + d) (I_L + i_l) = D I_L + d I_L + D i_l + di_l, \]  
\hspace{1cm} (3.23)  

\[ V_{D0} + v_{d0} = (V_I + v_i + V_O + v_o) (D + d) = V_I D + v_i D + V_O D + V_O d + V_O d + v_o d, \]  
\hspace{1cm} (3.24)  

\[ V_{D1} + v_{d1} = -\frac{1}{2} (V_O + v_o) (1 - D + d) = -\frac{1}{2} (V_O (1 - D) + v_o (1 - D) - V_O d - v_o d), \]  
\hspace{1cm} (3.25)  

and

\[ V_{D12} + v_{d12} = (V_I + v_i) (D + d) = V_I D + v_i D + d V_I + d v_i. \]  
\hspace{1cm} (3.26)  

Equations 3.23 through 3.26 mathematically represent the non-linear model PWM switched-inductor buck-boost converter switching components. Small-signality condition is employed to realize the dc and linear models from the non-linear model. Small-signal conditions are

\[ d \ll D, \]  
\hspace{1cm} (3.27)  

\[ i_l \ll I_L, \]  
\hspace{1cm} (3.28)  

\[ v_O \ll V_O, \]  
\hspace{1cm} (3.29)  

\[ i_O \ll I_O. \]  
\hspace{1cm} (3.30)
3 Small-Signal Model of PWM Switched-Inductor Buck-Boost in CCM

and

\[ v_i << V_f. \]  \hspace{1cm} (3.31) \]

Using small-signality condition above the products of two or more small-signal components are neglected. Thus the equations (3.23) through (3.26) equal

\[ I_s + i_s = DI_L + D_i + dI_L, \]  \hspace{1cm} (3.32) \]

\[ V_d + v_d = DV_{SD} + Dv_{sd} + dV_{SD} = D(V_f + V_o) + D(v_i + v_o) + d(V_f + V_o), \]  \hspace{1cm} (3.33) \]

\[ V_{d1} + v_{d1} = -\frac{1}{2}V_o(1 - D) - \frac{1}{2}v_o(1 - D) + \frac{1}{2}V_od, \]  \hspace{1cm} (3.34) \]

and

\[ V_{d12} + v_{d12} = V_f D + v_i D + dV_f. \]  \hspace{1cm} (3.35) \]

Equations 3.32 through 3.35 represent the DC and linear small-signal model of the PWM switched-inductor buck-boost converter. The dc and linear small-signal model is shown in Fig. 3.5. Using the principle of superposition, the model can be broken into a dc model and a linear small-signal model. The dc model replaces the inductor and capacitor by a short and open circuit respectively as shown in Fig. 3.6. The linear small-signal time-invariant model of the switched-inductor buck-boost converter is shown in Fig. 3.7.

3.5 Conclusions

The averaged low-frequency large-signal, dc, and small-signal linear time-invariant models for the PWM switched-inductor buck-boost converter have been derived.
Figure 3.5: DC and small-signal model of PWM switched-inductor buck-boost converter.

Figure 3.6: DC model of PWM switched-inductor buck-boost converter.
Figure 3.7: Small-Signal linear time-invariant model of PWM switched-inductor buck-boost converter.

based on circuit averaging. The ideal switch \( S \) is replaced by an ideal current-controlled current source of magnitude equating to the average steady state current through the non-linear switch in the PWM switching network. The ideal diode is replaced by an ideal voltage-controlled voltage source with a voltage equating to the average steady state voltage across the non-linear diode in the PWM switching network. The averaged model is approximated by a low-frequency large signal model. Finally, using the small-signality condition, dc and small-signal linear models are derived from the bi-linear model.
4 Open-Loop Small-Signal Characteristics of PWM Switched-Inductor Buck-Boost Converter for CCM

4.1 Introduction

This chapter presents the dc and small-signal characteristics based on the dc and small-signal models derived in Chapter 3. The dc voltage ratio based on the dc model will be derived and presented. The small-signal model of the open-loop control-to-output voltage transfer function, open-loop input-to-output voltage transfer function, open-loop input voltage-to-inductor current transfer function, open-loop input impedance, open-loop output impedance of the PWM switched-inductor buck-boost converter in CCM will be derived. The theoretically predicted transfer functions plotted by MatLab will be verified by Saber Sketch simulation package.
4 Open-Loop Small-Signal Characteristics of PWM Switched-Inductor Buck-Boost Converter for CCM

Figure 4.1: Small-Signal model of PWM switched-inductor buck-boost with disturbances $d$, $v_i$, and $i_o$.

4.2 Open-Loop Duty Cycle-to-Output Voltage Transfer Function

Fig. 4.2 shows the small-signal model of a PWM switched-inductor buck-boost converter in CCM. It replaces the switch with a dependent current source and the diodes with dependent voltage sources. This model does not account for switching losses or capacitance in the devices. The disturbances are independent variables and can be used to evaluate the effect of each variable independently. The disturbances not evaluated are set to zero. Therefore, in the case of open-loop control-to-output voltage transfer function, $v_i = i_o = 0$. This leaves only the input $d$ in the small-signal switched-inductor buck-boost model, which is required to derive the control-to-output voltage transfer function. The control-to-output voltage transfer function is found using Kirchoff’s voltage and current laws and Ohm’s Law. First, the current is given by
Figure 4.2: Small-Signal model of the PWM switched-inductor buck-boost converter for determining the control-to-output voltage transfer function $T_p$. 

\[ 2i_l - 2Di_l + i_{Zo} - 2I_Ld = 0 \]  
\( (4.1) \)

Next, the voltage loop for the load and inductor branches is

\[ Z_1i_l - dV_l - Dv_i + Z_1i_l + Dv_{sd} + (V_l - V_o)d - v_o = 0. \]  
\( (4.2) \)

Rearrangement of (4.2) produces the current through the inductor

\[ i_l = \frac{-v_o(1 - D) - dV_l + d(V_l - V_o)}{2Z_1}. \]  
\( (4.3) \)

Substitution of (4.3) into (4.2) yields

\[ 2(1 - D) \left[ -\frac{v_o(1 - D) - dV_l + d(V_l - V_o)}{2Z_1} \right] - \frac{v_o}{Z_o} - 2I_Ld = 0, \]  
\( (4.4) \)

yielding

\[ v_o \left[ \frac{1}{Z_o} + \frac{(1 - D)^2}{Z_1} \right] = d \left[ \frac{(1 - D)(V_l - V_o)}{2Z_1} - 2I_L - \frac{(1 - D)V_l}{Z_1} \right]. \]  
\( (4.5) \)

Using (4.5), the control-to-output voltage transfer function is
\[ T_p = \frac{v_o}{d} = \frac{(1-D)(V_I-V_O)}{I_L} - \frac{(1-D)V_i}{I_L} - 2Z_1 \left( \frac{d}{Z_o} + (1-D)^2 \right). \] (4.6)

The impedances \( Z_1 \) and \( Z_o \) are
\[ Z_1 = sL + r, \] (4.7)
and
\[ Z_0 = \frac{sC_rC_rL + R_L}{sC(r_C + R_L)}. \] (4.8)

Substitution of (4.7) and (4.8) into (4.6) leads to the final equation for the control-to-output voltage transfer function being
\[ T_p = \frac{v_o}{d} = T_{px} \frac{(s + \omega_n)(s - \omega_p)}{s^2 + 2\zeta \omega_o s + \omega_o^2}, \] (4.9)
where
\[ T_{px} = -\frac{2V_0r_c}{(1-D)(R_L + r_C)}, \] (4.10)
\[ \omega_n = \frac{1}{C r_C}, \] (4.11)
\[ \omega_p = \frac{1}{2DL} \left[ R_L(1-D)^2 \left( 1 + \frac{V_F}{|V_O|} \right) + r(1-3D) \right], \] (4.12)
\[ \zeta = \frac{C \left[ r( R_L + r_C) + (1-D)^2 R_L r_C \right] + L}{2\sqrt{LC} \left( R_L + r_C \right) \left[ r + (1-D)^2 R_L \right]}, \] (4.13)
and
\[ \omega_o = \sqrt{\frac{r + (1-D)^2 R_L}{LC(R_L + r_C)}}. \] (4.14)

The transfer functions derived from the small-signal models using designed component values found in Chapter 2 are validated; these values will be used as an example to present the following work to obtain theoretical and simulated results.
4 Open-Loop Small-Signal Characteristics of PWM Switched-Inductor Buck-Boost Converter for CCM

As stated in Chapter 2, the following values and components will be used: input voltage $V_I = 48$ V, duty ratio $D = 0.24$, output impedance $R_L = 14 \, \Omega$, converter capacitance $C = 100 \, \mu\text{F}$, and converter inductance $L_1 = L_2 = 330 \, \mu\text{H}$. The plots of magnitude and phase, respectively, of the control-to-output voltage transfer function 4.9 are shown in Figs. 4.3 and 4.4.

Fig. 4.5 is the simulated small-signal model using Saber Sketch circuit simulation. The simulated circuit parameters are identical to the values used to generate the MatLab theoretical Bode plots shown in Figs. 4.3 and 4.4. The circuit simulation has a non-linear switching network which requires using the discrete point Bode plot method. The small-signal disturbance $d$ is simulated by a variable frequency sinusoidal voltage source with a dc offset which feeds the negative terminal of the difference amplifier. The positive terminal is given a triangle wave. The difference amplifier output gives the square pulse of the PWM with the disturbance. Figs. 4.6 and 4.7 show the Saber Sketch circuit Simulator plots of magnitude and phase. Figs. 4.6 and 4.7 show the obtained points imposed on the theoretical Bode plots.
4 Open-Loop Small-Signal Characteristics of PWM Switched-Inductor Buck-Boost Converter for CCM

4.3 Open-Loop Input-to-Output Voltage Transfer Function

Considering open-loop audio susceptibility, \( d = i_o = 0 \), remains \( v_i \) as the only circuit disturbance. This allows the small-signal model of the PWM switched-inductor buck-boost converter in CCM to find the open-loop input-to-output voltage transfer function \( M_v \) (open-loop audio susceptibility), input voltage-to-inductor current transfer function \( M_{vi} \), and the input impedance \( Z_i \). The small-signal model for deriving \( M_v \) is shown in Fig. 4.8.

The open-loop input-to-output voltage transfer function was found applying Kirchhoff’s current law

\[
2Di_i - 2i_l + i_o = 0. \tag{4.15}
\]

Solving for the inductor current yields

\[
i_l = \frac{v_o}{2Z_o(1 - D)}. \tag{4.16}
\]
Figure 4.5: Simulated circuit for obtaining Bode plot for control-to-output voltage.
Figure 4.6: Simulated Bode plot of the magnitude of open-loop control-to-output voltage.

Figure 4.7: Simulated Bode plot of the phase of open-loop control-to-output voltage.
Applying Kirchhoff’s voltage law for the load and inductor branches

\[-2v_L + Dv_i + Dv_{sd} - v_o = 0.\]  \hspace{1cm} (4.17)

Substituting known values for \(v_{sd} = v_i - v_o\) and \(v_L = Z_1 i_l\) yields

\[-2Z_1 i_l + 2Dv_i - v_o(1 - D) = 0.\]  \hspace{1cm} (4.18)

Substituting equation 4.16 to equation 4.18 yields

\[2Dv_i = v_o(1 - D) \left[1 + \frac{Z_1}{Z_o(1 - D)^2}\right].\]  \hspace{1cm} (4.19)

Finally, the input-to-output transfer function is

\[M_v = \frac{v_o}{v_i} = \frac{2D}{(1 - D)} \left[\frac{Z_o}{Z_1^2} + Z_o\right].\]  \hspace{1cm} (4.20)

Substituting the impedances (4.7) and (4.8) into (4.20), provides the equation for the audio susceptibility.
Figure 4.9: Theoretical Bode plot of the magnitude of open-loop input-to-output voltage.

\[ M_v = \frac{v_o}{v_i} = M_{vx} \frac{(s + \omega_n)}{s^2 + 2\zeta \omega_o s + \omega_o^2}, \]

(4.21)

where

\[ M_{vx} = \frac{2Dr_CR_L(1-D)}{L(r_C + R_L)}. \]

(4.22)

Chapter 2 component values are used to validate the transfer functions derived from the small-signal models, and obtain theoretical and simulated results: input voltage \( V_I = 48V \), duty ratio \( D = 0.24 \), output impedance \( R_L = 14 \Omega \), converter capacitance \( C = 100 \mu F \), and converter inductance \( L_1 = L_2 = 330 \mu H \). Magnitude and phase Bode plots for the input-to-output voltage transfer function in (4.20) are shown in Figs. 4.9 and 4.10.

Fig. 4.11 is the simulated small-signal model using Saber Sketch circuit simulation. The parameters considered for the simulated circuit are identical to the values used to generate the MatLab theoretical Bode plots shown in Figs. 4.9 and 4.10. The circuit simulation has a non-linear switching network which requires using the
Figure 4.10: Theoretical Bode plot of the phase of open-loop input-to-output voltage.

Figure 4.11: Simulated circuit for obtaining Bode plots for input-to-output voltage, input voltage-to-inductor current, and input impedance.
Figure 4.12: Simulated Bode plot of the magnitude of open-loop input-to-output voltage.

discrete point Bode plot method. The small-signal disturbance \( v_i \) was simulated by a variable frequency sinusoidal voltage source with a dc offset which is in series with the ideal voltage source of the circuit. Figs. 4.12 and 4.13 show the plots of magnitude and phase respectively, for the simulated plots generated by Saber Sketch circuit simulator. The obtained points are imposed on the theoretical Bode plot shown in Figs. 4.6 and 4.7.
Figure 4.13: Simulated Bode plot of the phase of open-loop input-to-output voltage.

4.4 Open-Loop Input Voltage-to-Inductor Current Transfer Function

The small-signal model obtained to derive the input-to-output voltage transfer function can be employed to obtain the input voltage-to-inductor current transfer function $M_{vi}$. The small-signal model is shown in Fig. 4.8. Equation 4.16 gives the the inductor current in terms of output voltage. Expressing equation 4.16 in terms of output voltage and inductor current, and inserting terms from equation 4.19 achieves the equation

$$2Dv_i = 2Z_0i_l(1 - D)^2 \left(1 + \frac{Z_1}{Z_0(1 - D)^2}\right). \quad (4.23)$$

Rearranging the equation yields

$$M_{vi} = \frac{i_l}{v_i} = \frac{2D}{(1 - D)^2} \left(\frac{1}{Z_0 + \frac{Z_1}{(1 - D)^2}}\right). \quad (4.24)$$
Substituting the impedances (4.7) and (4.8) into (4.24), provides

\[ M_{vi} = \frac{i_l}{v_i} = M_{vix} \frac{s + \omega_c}{s^2 + 2\zeta\omega_o s + \omega_o^2}, \tag{4.25} \]

where

\[ M_{vix} = \frac{2D}{L}, \tag{4.26} \]

and

\[ \omega_c = \frac{1}{C(r_C + R_L)}. \tag{4.27} \]

Chapter 2 component values are used to validate the transfer functions derived from the small-signal models, and will be used to obtain theoretical and simulated results. The following values and components from chapter 2 will be used: input voltage \( V_I = 48 \, \text{V} \), duty ratio \( D = 0.24 \), output impedance \( R_L = 14 \, \Omega \), converter capacitance \( C = 100 \, \mu\text{F} \), and converter inductance \( L_1 = L_2 = 330 \, \mu\text{H} \). The plots of magnitude and phase respectively of the input voltage-to-inductor current.
transfer function (4.20) are shown in Figs. 4.14 and 4.15.

Fig. 4.11 shows the simulated small-signal model using Saber Sketch circuit simulation. The parameters considered for the simulated circuit are identical to the values used to generate the MatLab theoretical Bode plots, shown in Figs. 4.14 and 4.15. The circuit simulation has a non-linear switching network which required using discrete point method to obtain the Bode plot. The small-signal disturbance \( v_i \) is simulated by a variable frequency sinusoidal voltage source with a dc offset, which is in series with the ideal voltage source of the circuit. Figs. 4.16 and 4.17 show the plots of magnitude and phase respectively for the simulated plots using Saber Sketch circuit simulator. The obtained points are imposed on the theoretical Bode plot and shown for comparison.
Figure 4.16: Simulated Bode plot of the magnitude of open-loop input voltage-to-inductor current.

Figure 4.17: Simulated Bode plot of the phase of open-loop input voltage-to-inductor current.
4.5 Open-Loop Input Impedance Transfer Function

The small-signal model shown in Fig. 4.8 was used to derive the open-loop input impedance for the PWM switched-inductor buck-boost in CCM. The input impedance transfer function was found by reducing the change in duty cycle and output current to 0. Once those changes were accomplished, the input impedance could be found by applying Kirchoff’s voltage and current laws and Ohm’s Law. First, the current equals

\[ i_i = 2D i_l. \]  

(4.28)

Using the equations 4.16 and 4.19 it is given that

\[ 2Dv_i = \frac{i_i}{D} \left( Z_o (1 - D)^2 + Z_1 \right). \]  

(4.29)

Solving for input impedance the equation becomes

\[ Z_i = \frac{v_i}{i_i} = \frac{1}{2D^2} \left( Z_o (1 - D)^2 + Z_1 \right) \]  

(4.30)

Substituting the impedances (4.7) and (4.8) into (4.24), yields

\[ Z_i = \frac{v_i}{i_i} = Z_{ix} \frac{s^2 + 2\zeta \omega_o s + \omega_o^2}{s + \omega_c}, \]  

(4.31)

where

\[ Z_{ix} = \frac{L}{2D^2}. \]  

(4.32)

Chapter 2 component values are used to validate the transfer functions derived from the small-signal models and obtain the following theoretical and simulated results. Chapter 2 values and components include: input voltage \( V_I = 48 \) V, duty ratio \( D = 0.24 \), output impedance \( R_L = 14 \) Ω, converter capacitance \( C = 100 \mu F \), and converter inductance \( L_1 = L_2 = 330 \mu H \). The plots of magnitude and phase, respectively, of the input impedance transfer function (4.30) are shown in Figs.
Figure 4.18: Theoretical Bode plot of the magnitude of open-loop input impedance.

Figure 4.19: Theoretical Bode plot of the phase of open-loop input impedance.
Figure 4.20: Simulated Bode plot of the magnitude of open-loop input impedance.

4.18 and 4.19.

Fig.4.11 is the simulated small-signal model using Saber Sketch circuit simulation. The parameters considered for the simulated circuit are identical to the values used to generate the MatLab theoretical Bode plots which are shown in Figs. 4.18 and 4.19. The circuit simulation has a non-linear switching network which required the use of a discrete point method to obtain the Bode plot. The small-signal disturbance $v_i$ is simulated by a variable frequency sinusoidal voltage source with a dc offset which is in series with the ideal voltage source of the circuit. Figs.4.20 and 4.21 show the plots of magnitude and phase respectively, for the simulated plots using Saber Sketch circuit simulator. The obtained points are imposed on the theoretical Bode plot and shown for comparison.
4.6 Open-Loop Output Impedance Transfer Function

Fig. 4.22 shows a small-signal model of the PWM switched-inductor buck-boost converter in CCM. Using Ohm’s law and Kirchhoff’s current and voltage laws, the open-loop output impedance transfer function was found. Applying Kirchhoff’s current law, the sum of the currents is

\[ i_t = i_{Z_o} + i_D. \]  \hfill (4.33)

Again using Kirchhoff’s current law with the diode and inductor currents yields

\[ i_D = -2Di_t + 2i_l. \]  \hfill (4.34)

Kirchhoff’s voltage law applied to the inductor current branches and output current branches which gives

\[ 2Z_1i_t = -Dv_{sd} + v_t. \]
The inductor current is

\[ i_l = \frac{v_t (1 - D)}{2Z_1}. \]  

(4.35)

Rearranging and substituting (4.35) and (4.34) into equation 4.33 yields the output impedance transfer function

\[ Z_{\text{out}} = \frac{v_t}{i_t} = \frac{1}{\frac{1}{Z_o} + \frac{(1-D)^2}{Z_1}} = \frac{Z_o Z_1}{Z_1 + Z_o (1 - D)^2}. \]  

(4.36)

Substituting the impedances (4.7) and (4.8) into (4.36), the output impedance transfer function is

\[ Z_{\text{out}} = \frac{v_t}{i_t} = Z_{\text{outx}} \frac{(s + \omega_n) (s + \omega_t)}{s^2 + 2\zeta \omega_o s + \omega_o^2}, \]  

(4.37)
Figure 4.23: Theoretical Bode plot of the magnitude of open-loop output impedance.

where
\[ Z_{outx} = \frac{r_C R_L}{(r_C + R_L)}, \]  
\[ \omega_L = \frac{r}{L}. \]  

As previously stated, Chapter 2 component values are used to validate the transfer functions derived from the small-signal models and obtain theoretical and simulated results. Chapter 2 values and components include: input voltage \( V_I = 48\, \text{V} \), duty ratio \( D = 0.24 \), output impedance \( R_L = 14\, \Omega \), converter capacitance \( C = 100\, \mu\text{F} \), and converter inductance \( L_1 = L_2 = 330\, \mu\text{H} \). The plots of magnitude and phase respectively of the output impedance transfer function (4.36) are shown in Figs. 4.23 and 4.24.

Fig. 4.25 is the simulated small-signal model using Saber Sketch circuit simulation. The parameters considered for the simulated circuit are identical to the values used to generate the MatLab theoretical Bode plots shown in Figs. 4.23 and 4.24. The
Figure 4.24: Theoretical Bode plot of the phase of open-loop output impedance.

Figure 4.25: Simulated Circuit to obtain output impedance
Figure 4.26: Simulated Bode plot of the magnitude of open-loop output impedance

Figure 4.27: Simulated Bode plot of the phase of open-loop output impedance
circuit simulation has a non-linear switching network which required the use of a
discrete point method to obtain the Bode plot. The small-signal disturbance $v_i$ is
simulated by a variable frequency sinusoidal voltage source with a dc offset which
is in series with the ideal voltage source of the circuit. Figs. 4.26 and 4.27 show
the plots of magnitude and phase, respectively, for the simulated plots using Saber
Sketch circuit simulator. The obtained points are imposed on the theoretical Bode
plot and shown for comparison.
4.7 Open-Loop Step Responses

4.7.1 Open-Loop Response of Output Voltage to Step Change in Input Voltage

Consider a step change in the input voltage magnitude $\Delta V_I$ at an arbitrary time $t = 0$. The step change of the input voltage in the s-domain is

$$v_i(s) = \frac{\Delta V_I}{s}.$$  \hfill (4.40)

The output voltage due to input voltage step change of the open-loop switched inductor buck-boost total input voltage is

$$v_o(s) = \frac{\Delta V_I}{s} M_o(s) = \Delta V_I M_{ex} \frac{(s + \omega_n)}{s \left( s^2 + 2\zeta \omega_o s + \omega_o^2 \right)}.$$  \hfill (4.41)

Fig. 4.28 shows the output voltage response to a step change in input voltage from 48V to 53V for the open-loop PWM switched-inductor buck-boost converter. As stated previously, the component values were designed in chapter 2: $D = 0.24$, $R_L = 14 \Omega$, $C = 100 \mu F$, $L_1 = L_2 = 330 \mu H$, $r_L = 0.42 \Omega$, $r_C = 0.155 \Omega$, $C_o = 150 pF$, $V_F = 0.65 V$ and $R_F = 0.2 \Omega$. 
4.7.2 Open-Loop Response of Output Voltage to Step Change in Duty Cycle

Consider a step change in the input voltage magnitude $\Delta d_T$ at an arbitrary time $t = 0$. The step change of the input voltage in the s-domain is

$$d(s) = \frac{\Delta d_T}{s}.$$  \hspace{1cm} (4.42)

The output voltage due to input voltage step change of the open-loop switched inductor buck-boost total input voltage is

$$v_o(s) = \frac{\Delta d_T}{s} T_p(s) = \frac{\Delta d_T T_{px}}{s} \frac{(s + \omega_n)(s - \omega_p)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$  \hspace{1cm} (4.43)

Fig. 4.29 shows the output voltage response to a duty cycle step change from 0.24 to 0.29 for the open-loop PWM switched-inductor buck-boost converter.
4.7.3 Open-Loop Response of Output Voltage to Step Change in Output Current

Consider a step change in the input voltage magnitude $\Delta I_O$ at an arbitrary time $t = 0$. The step change of the input voltage in the s-domain is

$$i_O(s) = \frac{\Delta I_O}{s}.$$  \hspace{1cm} (4.44)

The output voltage due to input voltage step change of the open-loop switched inductor buck-boost total input voltage is

$$v_o(s) = \frac{\Delta i_O}{s} Z_{out}(s) = \Delta i_O Z_{out} \frac{(s + \omega_i)(s + \omega_n)}{s^2 + 2\zeta\omega_o s + \omega_o^2}.$$  \hspace{1cm} (4.45)

Fig. 4.30 shows the output voltage response to a duty cycle step change from 0.56 to 1.12 for the open-loop PWM switched-inductor buck-boost converter.
Figure 4.30: Output Voltage $v_O$ response to a step change in $i_O$ from 0.56 to 1.12 A for an open-loop switched-inductor buck-boost converter.

4.8 Experimental Results

4.8.1 Open-Loop Transfer Functions Bode Plots

The experimental circuit was built and tested using the example design specifications stated in Chapter 2 with an IR2110, a high side MOSFET driver used to trigger the MOSFET. A Colikraft SD250-IL gate drive transformer connected the IR2110 and the IRF520. The parasitic resistances of the inductors and capacitor are included when predicting the magnitude and phase of the different transfer functions. To verify the model, a LM357N (as a comparator) was used to obtain the required duty-cycle modulation. The reference ramp voltage was a saw-tooth with a peak voltage of 6 V. Therefore, the duty cycle modulator had a gain of $20 \log(1/6) = -16$ dB. This was also considered in the predicted MatLab Bode plot. Experimental Bode plots shown in Fig. 4.31 for the control-to-output voltage transfer function were obtained with a Hewlett Packard 4194A Impedance/Gain-Phase Analyzer.
Figure 4.31: Experimental Bode plot of the magnitude and phase of open-loop control-to-output voltage.
Fig. 4.32 presents the experimental Bode plots of the input-to-output voltage transfer function. The Bode plots of input-to-output voltage transfer function were measured using Hewlett-Packard 4194A Impedance/Gain-Phase Analyzer. The voltages were measured with and Pearson model 411 wide-bandwidth current probe. The experimental Bode plots shows congruence with the theoretical and simulated Bode plots shown in Figs. 4.3, 4.4, and 4.7.

Fig. 4.33 presents the experimental Bode plots of the input voltage-to-inductor current transfer function. The Bode plots of control voltage-to-inductor current transfer function were measured using a Hewlett-Packard 4194A Impedance/Gain-Phase Analyzer. A simple sense resistor of 1 ohm was used in series with the inductor to obtain the Bode plots $|M_{vi}|$ and $\phi_{M_{vi}}$. The experimental Bode plots shows accordance with the theoretical and simulated Bode plots shown in Figs. 4.9, 4.10, 4.12, and 4.13.

The experimental circuit was built and tested using the example design specifications stated in Chapter 2. An IR2110, a high side MOSFET driver was used to
trigger the MOSFET. A Colikraft SD250-IL gate drive transformer connected the IR2110 and the IRF520. The Bode plots of output impedance transfer function was measured using a Hewlett-Packard 4194A Impedance/Gain-Phase Analyzer. Fig. 4.34 presents the experimental Bode plots of the input impedance transfer function. The input current was measured using a simple sense resistive load of one ohm in series with the applied test voltage to obtain the Bode plots $|Z_i|$ and $\phi_Z$. The experimental Bode plots shows accordance with the theoretical and simulated Bode plots shown in Figs. 4.18, 4.19, 4.20, and 4.21.

Finally, Fig. 4.35 presents the experimental Bode plots of the output impedance transfer function. The same sense resister used to sense the currents previously is again used in the output impedance transfer function. The experimental Bode plots shows accordance with the theoretical and simulated Bode plots shown in Figs. 4.23, 4.24, 4.26, and 4.27.
Figure 4.34: Experimental Bode plot of the magnitude and phase of open-loop input impedance.

Figure 4.35: Experimental Bode plot of the magnitude and phase of open-loop output impedance.
4.8.2 Open-Loop Step Changes

The experimental circuit was built and tested using the example design specifications stated in Chapter 2. An IR2110, a high side MOSFET driver, was used to trigger the MOSFET. The parasitic resistances of the inductors and capacitor are accounted for when predicting the magnitude and phase of the different transfer functions. In the experiment an extra voltage source set to the step change was placed in series with the input voltage source and manually turned on and off. This action produced the required input voltage step change. The experimental results are in accordance with the theoretical results shown in Fig. 4.28.

A LM357N (used as a comparator) was used to obtain the required step response change in duty-cycle modulation in the experimental model verification. The experimental results are in accordance with the theoretical results shown in Fig. 4.29.

In the experiment for verification of the model for the output voltage step response with a step change in output current was obtained by changing the load resistance.
4 Op en-Lo op Small-Signal Characteristics of PWM Switc hed-Inductor Buck-Boost Con v erter for CCM

Figure 4.37: Output Voltage $v_O$ response to a step change in $d_T$ from 0.24 to 0.29 for an open-loop switched-inductor buck-boost converter.

in the switched-inductor buck-boost converter. The experimental results are in accordance with the theoretical results shown in Fig. 4.30.

4.9 Conclusions

This section presented a small-signal model of the PWM switched-inductor buck-boost DC-DC converter operating in CCM, and derived the power stage control-to-output voltage, input-to-output voltage, and the input voltage-to-inductor current transfer functions. Transfer functions in impedance form, and transfer functions with the parasitics were presented. An example PWM switched-inductor buck-boost converter was considered, and the control-to-output voltage, input-to-output voltage, and input voltage-to-inductor current transfer functions were predicted using MatLab. The measured ESR values of the inductors and the capacitor were also provided. A simulated circuit using Saber Sketch circuit simulator was built, and Bode plots for all five transfer functions were found using a discreet
Figure 4.38: Output Voltage $v_O$ response to a step change in $i_O$ from 0.56 to 1.12 A for an open-loop switched-inductor buck-boost converter.

point method. A laboratory prototype corresponding to the Chapter 2 values was built, and the control-to-output voltage, input-to-output voltage, input voltage-to-inductor current, and output impedance transfer functions were measured using a HP4194A Gain-Phase Analyzer. The theoretically predicted, simulated values were in accordance with the experimentally measured Bode plots. Therefore, this validates the derived small-signal models and the transfer functions.
5 Frequency for PWM

5.1 Introduction

This section presents the effects of the PWM frequency and its effects on the switching elements of the switched-inductor buck-boost converter, the size of inductor and capacitor, and switching losses. Also studied were the effects of raising the frequency of the PWM to determine the impact on the current and voltage waveforms for the switching elements. Steady state analysis was employed to obtain the necessary efficiencies and waveforms with Saber Sketch circuit simulator. Parameters of interest were efficiency of the converter, and the currents and voltages through and across the switch $S$, and diodes $D_0$, $D_1$, $D_2$, and $D_{12}$. The analysis assumed the switched-inductor buck-boost converter was operating in continuous conduction mode.

5.2 Higher Frequency Analysis for PWM

5.2.1 Background

Silicon MOSFETs are widely used in both digital and analog circuits. Reducing size has led to a increase in speed and allows more devices to be inserted in the chip area. Reduction also caused negative effects, including but not limited to: higher sub-threshold conduction (increased losses), increased gate-oxide leakage (increased power consumption), increase in junction leakage (current leak-
Frequency for PWM

(average), interconnect capacitance (delays and lower performance), and lower output resistance (decreases gain). These unwanted effects have led to the use of new materials, Silicon Carbide (SiC) and Gallium Nitride (GaN), which increase the drain to source voltage and drain current; however, they also increase speed and performance.

Electromechanical switches could also be employed for the diodes. The main concerns about the use of electromechanical switches, instead of diodes, are chatter, size, driver circuit for the switch, and electromagnetic interaction with other components. The electromechanical switch will be the slower compared to the diode and mosfet. The benefits are that there are no switching losses, and higher reliability. Certain disadvantages of electromechanical switches such chatter and size can be limited. Great achievements in size reduction has helped immensely in this area. There are continual efforts to reduce chatter as well.

Silicon devices are only one limiting factor for increasing speeds on DC-DC converters; driving the MOSFETs can be problematic as well. Switching delays can create losses in devices and affect overall performance. Further, the speed of the driver limits the speed of the MOSFET regardless of the material used to manufacture the MOSFET. Tables 5.1 - 5.3 show the available commercial devices, along with associated speeds and other important characteristics. As shown in the tables, the current maximum frequency of other DC-DC converters is around 1-2 MHz. The speed of the driver seems to be a dominant limiting factor in speed.
5 Frequency for PWM

### Table 5.1: Commercial available drivers

<table>
<thead>
<tr>
<th>Base Part</th>
<th>Output Rise Time $t_r$ (ns)</th>
<th>Output Fall Time $t_f$ (ns)</th>
<th>Pulse Width Min (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM5113</td>
<td>4</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>LM5101A</td>
<td>10</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>IR2110</td>
<td>25</td>
<td>17</td>
<td>70</td>
</tr>
</tbody>
</table>

### Table 5.2: Commercial available switching controllers

<table>
<thead>
<tr>
<th>Base Part</th>
<th>Input Min Voltage (V)</th>
<th>Input Max Voltage (V)</th>
<th>Min Freq (kHz)</th>
<th>Max Freq (kHz)</th>
<th>$\eta$ peak (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM3578A</td>
<td>40</td>
<td>100</td>
<td>1</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>LM5032</td>
<td>13</td>
<td>100</td>
<td>100</td>
<td>1000</td>
<td>85</td>
</tr>
<tr>
<td>LM5046</td>
<td>14</td>
<td>100</td>
<td>100</td>
<td>2000</td>
<td>92</td>
</tr>
<tr>
<td>LM5116</td>
<td>6</td>
<td>100</td>
<td>50</td>
<td>1000</td>
<td>95</td>
</tr>
</tbody>
</table>

### Table 5.3: Commercial available MOSFETs

<table>
<thead>
<tr>
<th>Base Part</th>
<th>Material</th>
<th>Max Voltage $V_{DS}$ (V)</th>
<th>Max Current $I_D$ (A)</th>
<th>$R_{DS_{max}}$ (m$\Omega$)</th>
<th>$Q_g$ (nC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRF520</td>
<td>Si</td>
<td>100</td>
<td>9.2</td>
<td>270</td>
<td>10</td>
</tr>
<tr>
<td>FDMC86102</td>
<td>Si</td>
<td>100</td>
<td>20</td>
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<tr>
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<td>-</td>
<td>525</td>
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5.3 Effect of Higher Frequency on Passive Components

As stated in chapter 2, the size of the inductor and capacitor are dependent on the converter switching frequency shown in (2.27) and (2.30). Figs. 5.1 and 5.2 show the size of the capacitor and inductor are inversely proportional to the switching frequency. However, process specifications in regards to voltage and current often dictate the size of the inductor and capacitor.

5.4 Effect of Higher Frequency on Current and Voltage Waveforms

5.4.1 Simulated Results

The effects of a higher switching frequency on the current and voltage waveforms for the MOSFET and diodes are shown in this section. The frequencies chosen for study are $f = 100 \text{ kHz}, 300 \text{ kHz}, 500 \text{ kHz}, \text{ and } 1 \text{ MHz}$. A model of the switched-
Figure 5.2: Minimum Inductance as a function of switching frequency.

The previous design used values for the 100 kHz case: International Rectifier power MOSFET IRF520, rated 9.2 A/100 V with a maximum \( r_{DS} = 0.27 \Omega \) and \( C_o = 150 \) pF, ON-Semiconductor SWITCHMODE power rectifier MBR10100 rated 10 A/100 V with \( V_F = 0.65 \) V and \( R_F = 0.2 \Omega \), \( L_1 = L_2 = 330 \mu H \), \( C = 100 \mu F \), and \( R_L = 50 \Omega \). A pulse source was used to model the high side driver for the MOSFET. Figs. 2.4 and 2.5 show the MOSFET and Diodes current and voltage waveforms compared with the ideal case. Figs. 5.3 and 5.4 show the switches and inductors voltages and currents waveforms for \( f = 100 \) kHz. These figures display a slight ringing in the currents and voltages waveforms. Fig. 5.3 shows a noticeable ringing spike in the switch current upon commencement of the pulse. The ringing is unprolonged, quickly returns to the intended value and proceeding on its intended trajectory. The ringing is present in all current waveforms of the four diodes. The voltages for the MOSFET and diodes, as expected, show a rounding at the edges as well as a slight rise and fall time. The current through the inductors remains the same as the ideal case, but shows a sharp ringing at the
Figure 5.3: Simulated current and voltage waveforms for a 100 kHz frequency PWM Switch-Inductor buck-boost converter in CCM.

Figure 5.4: Simulated current and voltage waveforms for a 100 kHz frequency PWM switched-inductor buck-boost converter in CCM.
Figure 5.5: Simulated current and voltage waveforms for a 300 kHz frequency PWM Switch-Inductor buck-boost converter in CCM.

peak and valley of the dc offset triangular pulse. The voltage across the inductor has a slight ringing and a slight rise and fall time.
Figure 5.6: Simulated current and voltage waveforms for a 300 kHz frequency PWM switched-inductor buck-boost converter in CCM.

The voltages and currents waveforms for $f = 300$ kHz of the $S$ and $D_0$ are shown in Figs. 5.5 and 5.6. The inductors and capacitor were redesigned for the new frequency of 300 kHz and reduced to 160 uH and 68 uF, respectively. The other components remain unchanged for the simulation. Figs. 5.3 and 5.4 still display a slight ringing present in the currents and voltages waveforms. The switch and inductor settling time is finite and quickly returns to its intended trajectory. The ringing is present in all current waveforms of the four diodes. As expected, the voltages for the MOSFET and diodes show a rounding at the edges as well as a slight rise and fall time. The current through the inductors is unchanged from the ideal case, but slightly longer sharp ringing at the peak and valley of the dc offset triangular pulse. The voltage across the inductor has a damped ringing and a slight rise and fall time.
Figure 5.7: Simulated current and voltage waveforms for a 500 kHz frequency PWM Switch-Inductor buck-boost converter in CCM.

Figure 5.8: Simulated current and voltage waveforms for a 500 kHz frequency PWM switched-inductor buck-boost converter in CCM.
The voltages and currents waveforms for $f = 500\,\text{kHz}$ of the $S$ and $D_0$ are shown in Figs. 5.7 and 5.8. The inductor and capacitor were redesigned for the new frequency of 500 kHz and reduced to 100 uH and 48 uF, respectively. The other components remain unchanged for the simulation. The figure shows ringing in the current and voltage waveforms. Fig. 5.7 shows the inductor current peaks with a pronounced rounding and longer rise and fall time. The voltage across the inductor has a longer ringing, more rounding, and a more pronounced rise and fall time. The voltages for the MOSFET and diodes show a significant amount of rounding at the edges as well as a increased rise and fall time.
5 Frequency for PWM

Figure 5.10: Simulated current and voltage waveforms for a 1 MHz frequency PWM switched-inductor buck-boost converter in CCM.

Figs. 5.9 and 5.10 show the voltages and currents waveforms for $f = 1$ MHz of the $S$ and $D_0$. The inductor and capacitor were redesigned for the new frequency of 1 MHz and reduced to 16 uH and 8 uF, respectively. The figures show ringing present in the current and voltage waveforms. The switch current has a ringing spike but returns to its intended trajectory. The ringing is present in all diodes. The maximum peak ringing of the current through the diodes $D_0$, $D_1$ and $D_2$, and $D_{12}$ is greater than the previous cases. Fig. 5.9 shows rounding in the inductor current peaks. The voltage across the inductor has a slight ringing, significant rounding, and a pronounced rise and fall time as compared to previous cases. Similarly, the voltages for the MOSFET and diodes show a significant amount of rounding at the edges and an increased rise and fall time. The settling time for the ringing is dramatically increased from the 100 kHz case and the rounding of the voltage waveforms is more prominent than in the previous cases.
Figure 5.11: Saber Sketch circuit simulator model of PWM switched-inductor buck-boost converter with SiC Mosfet.

Fig. 5.11 replaces the silicon MOSFET with a silicon carbide MOSFET in Saber sketch circuit simulator. The other components are unchanged from the designed component values for 1 MHz frequency. Figs. 5.9 and 5.10 show the voltages and currents waveforms for $f = 1$ MHz of the silicon carbide MOSFET $S$ and a silicon diode $D_0$. The remaining components of the converter are unchanged. The ringing is reduced from the 1 MHz silicon MOSFET case. The voltages for the MOSFET and diodes still show rounding at the edges and an apparent rise and fall time. The ringing settling time is more prominent than the 100 kHz silicon case but is not unexpected. The rounding of the voltage waveforms is less prominent than the 1 MHz silicon case.
Figure 5.12: Simulated current and voltage waveforms for a 1 MHz frequency Silicon Carbide MOSFET PWM Switch-Inductor buck-boost converter in CCM.

Figure 5.13: Simulated current and voltage waveforms for a 1 MHz frequency SiC PWM switched-inductor buck-boost converter in CCM.
Figure 5.14: Saber Sketch circuit simulator model of PWM switched-inductor buck-boost converter with GaN Mosfet.

Fig. 5.14 shows the replacement of the silicon MOSFET with a Galium Nitride (GaN) MOSFET in Saber sketch circuit simulator. The other components are unchanged from the designed component values for 1 MHz frequency. Figs. 5.9 and 5.10 show the voltages and currents waveforms for $f = 1$ MHz case of the silicon carbide MOSFET $S$ and a silicon diode $D_0$. The ringing is reduced from silicon and SiC case. The voltages for the MOSFET and diodes show rounding at the edges, though less prominent than the silicon and SiC. The rise and fall time are apparent, though again less prominent. The settling time for the ringing is more prominent than the 100 kHz silicon case but is not unexpected.

In conclusion, increasing the switching frequency of the PWM causes some unwanted effects; as the frequency is increased, the ringing and settling time also increase although the waveform does return to its intended target. The rounding of the voltage waveforms is more pronounced as the increase of frequency.
5 Frequency for PWM

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<td>4.276</td>
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Figure 5.15: Simulated current and voltage waveforms for a 1 MHz frequency Silicon Carbide MOSFET PWM Switch-Inductor buck-boost converter in CCM.

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<tr>
<th>t(s)</th>
<th>V</th>
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</table>

Figure 5.16: Simulated current and voltage waveforms for a 1 MHz frequency SiC PWM switched-inductor buck-boost converter in CCM.
5.4.2 Experimental Results

The effects of a higher switching frequency on the current and voltage waveforms for the MOSFET and diode $D_0$ are shown in this section. The frequencies chosen for study are $f = 100$ kHz, 300 kHz, 500 kHz, and 1 MHz. The current and voltage waveforms of the MOSFET and diode $D_0$ will be compared with the ideal case as shown in Figs. 2.4 and 2.5. The values expected by equations 2.64, 2.65, and 2.66 are in accordance with the ideal voltage and current waveforms for frequencies of 100-500 kHz. Frequencies above 500 kHz distort the values of output voltage $V_O$, diode $D_0$ voltage and current, and the switch voltage and current.

Figs. 5.17, and 5.18 show the voltages and currents waveforms for $f = 100$ kHz of the $S$ and $D_0$ with ringing present in the current and voltage waveforms. A ringing spike in the current waveform is more pronounced than in the voltage waveform upon commencement of the pulse. The ringing quickly returns to the intended value and proceeds on its intended trajectory. The ringing is also present in the diode current and voltage waveforms. The maximum peak ringing of the current
Figure 5.18: Experimental diode $D_0$ current and voltage waveforms for a 100 kHz frequency PWM switched-inductor buck-boost converter in CCM.

through the diode $D_0$ is 4 times desired value. Also, the ringing in the current waveforms and the voltage $V_{ds}$ settles at approximately one third of the period. The voltages for the MOSFET and diode show a rounding at the edges as well as a slight rise and fall time.
Figure 5.19: Experimental switch current and voltage waveforms for a 300 kHz frequency PWM switched-inductor buck-boost converter in CCM.

Figs. 5.19, and 5.20 show the voltages and currents waveforms for $f = 300$ kHz of the $S$ and $D_0$ with ringing present. A ringing spike in the current waveform is more pronounced than in the voltage waveform upon commencement of the pulse. The ringing quickly returns to the intended value and proceeds on its intended trajectory. The ringing is also present in the diode current and voltage waveform. The maximum peak ringing of the current through the diode $D_0$ is 5 times desired value. As expected, the voltages for the MOSFET and diode as expected show a rounding at the edges and an associated slight rise and fall time. The overall waveforms shape approximate the ideal waveforms.
Figure 5.20: Experimental diode $D_0$ current and voltage waveforms for a 300 kHz frequency PWM switched-inductor buck-boost converter in CCM.

Figs. 5.21, and 5.22 show the voltages and currents waveforms for $f = 500$ kHz of the $S$ and $D_0$ with ringing present. As shown in the figures, ringing is present in the current and voltage waveforms. A ringing spike in the current waveform is more pronounced than in the voltage waveform upon commencement of the pulse. The ringing eventually returns to the intended value and proceeds on its intended trajectory. The switch voltage has a noticeable fall time and appears trapezoidal than square. The diode voltage has a noticeable rise time and a pronounced ringing in the current. The maximum peak ringing of the current through the diode $D_0$ is 6 times desired value. The shape of the voltages are more trapezoidal than square and the current waveforms the high frequency ringing is present for almost the entire ON or OFF duration. The duty cycle did not need to be adjusted to achieve the desired output of 28 V.
Figure 5.21: Experimental switch current and voltage waveforms for a 500 kHz frequency PWM switched-inductor buck-boost converter in CCM.

Figure 5.22: Experimental diode $D_0$ current and voltage waveforms for a 500 kHz frequency PWM switched-inductor buck-boost converter in CCM.
Figure 5.23: Experimental switch current and voltage waveforms for a 1 MHz frequency PWM switched-inductor buck-boost converter in CCM.

Figs. 5.23, and 5.24 show the voltages and currents waveforms for $f = 1$ MHz of the $S$ and $D_0$ with ringing present. A ringing spike in the current waveform is more pronounced, and doesn’t reach its intended value. The ringing is continuous for both current and voltage throughout the entire waveform. The current’s maximum peak ringing through the diode $D_0$ is 6 times the desired value. The high frequency ringing is present in both the ON and OFF durations of the diode. The voltages for the MOSFET and diode are triangular in nature and do not resemble the ideal waveforms. Also, the desired output voltage could not be achieved by changing the duty cycle.

In conclusion, increasing the frequency of the PWM causes some serious unwanted effects. As frequency increases the maximum of the ringing and settling time increases. Also as frequency increases the voltage waveform rounding is more pronounced, and eventually reaches an unrecognizable state. 500 kHz is the maximum frequency to achieve the desired current and voltage waveforms.
5.4.3 Switching losses

The MOSFET switching loss is directly proportional to the increase in frequency as shown in (2.34). The overall efficiency of the switched-inductor buck-boost converter is shown in (2.61). Using this frequency dependent equation, the overall efficiency of the converter was plotted using MatLab. A laboratory prototype was built according to the specifications in the design example. The PWM switched-inductor buck-boost DC-DC converter is shown in Fig. 2.1. An IRF2110 driver was employed to drive the high-side MOSFET with a square input to control the duty cycle and frequency of the pulse. The inductors $L_1$ and $L_2$ are manufactured by Murata Power Solutions with a measured dc resistance $r_L = 0.42\,\Omega$. The capacitor $C$ was electrolytic and had a measured dc resistance of $r_C = 0.155\,\Omega$. An International Rectifier power MOSFET IRF520, rated 9.2 A/100 V with a maximum $r_{DS} = 0.27\,\Omega$ and $C_o = 150\,\text{pF}$, and an ON-Semiconductor SWITCH-MODE power rectifier MBR10100 rated 10 A/100 V and having $V_F = 0.65\,\text{V}$ and $R_F = 0.2\,\Omega$ were selected. Note the inductor was not re-sized in the experiment,
Figure 5.25: Converter efficiency as a function of switching frequency.

Table 5.4: Experimental values for different switching frequencies

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<th>$f_s$ (kHz)</th>
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<th>$V_o$ (V)</th>
<th>$\eta$ (%)</th>
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<td>-28</td>
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<tr>
<td>300</td>
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<td>700</td>
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<td>-17.1</td>
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<tr>
<td>1MHz</td>
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<td>-15.7</td>
<td>33</td>
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however the capacitor was re-sized with each new frequency. Table 5.4 presents the duty cycle, output voltage and efficiency of the experimentally built switched-inductor buck-boost. The values are in accordance with the predicted value up to a frequency of 500 kHz; frequencies exceeding 500 kHz show a dramatic difference in the predicted and experimental results.

The voltages and currents waveforms for $f = 100$ kHz of the $S$ and $D_0$ are shown in Figs. 5.26 and 5.27. The switch voltage has a noticeable fall time as well as an apparent ringing in the current, causing greater losses than at zero switching. The diode voltage has a noticeable rise time, as well as a pronounced ringing in the current, causing greater losses because it is not zero switching. The intersection
Figure 5.26: Experimental gate current and gate to source voltage waveforms to show switching losses for a 100 kHz frequency PWM Switch-Inductor buck-boost converter in CCM.

Figure 5.27: Experimental switch current and voltage waveforms to show switching losses for a 100 kHz frequency PWM switched-inductor buck-boost converter in CCM.
of the two waveforms cause switching losses greater than expected.
Figure 5.28: Experimental gate current and gate to source voltage waveforms to show switching losses for a 500 kHz frequency PWM Switch-Inductor buck-boost converter in CCM.

The voltages and currents waveforms for \( f = 500 \text{ kHz} \) of the \( S \) and \( D_0 \) are shown in Figs. 5.28 and 5.29. The switch voltage has a noticeable fall time; it appears more trapezoidal in nature rather than the desired square. The switch current ringing is more pronounced, resulting in greater losses and decreases in the overall efficiency of the switched-inductor buck-boost. The diode voltage has a noticeable rise time and a pronounced ringing in the current. This causes greater losses than at zero switching and decreases the efficiency, compared to the predicted values. The desired output voltage can be reached by adjusting the duty cycle shown in table 5.4.
Figure 5.29: Experimental switch current and voltage waveforms to show switching losses for a 500 kHz frequency PWM switched-inductor buck-boost converter in CCM.

Figs. 5.30 and 5.31 show the voltages and currents waveforms for \( f = 1 \text{ MHz} \) of the \( S \) and \( D_0 \). The switch voltage has a noticeable fall time, it appears triangular in nature rather than a square. The switch current has a greater settling time and does not completely damp out the higher order ringing. The diode voltage has a noticeable fall time, which makes it appear trapezoidal in nature rather than square. The diode current ringing does not damp to a constant value. The switch and diodes’ greater rise and fall time cause greater efficiency losses compared to the predicted value. Also, adjusting the duty cycle did not achieve the desired output voltage. The ringing causes a false detection and turns the MOSFET on or off prematurely. Table 5.4 presents the maximum output voltage achievable, associated duty cycle and efficiency of the switched-inductor buck-boost.
Figure 5.30: Experimental gate current and gate to source voltage waveforms to show switching losses for a 1000 kHz frequency PWM Switch-Inductor buck-boost converter in CCM.

Figure 5.31: Experimental switch current and voltage waveforms to show switching losses for a 1000 kHz frequency PWM switched-inductor buck-boost converter in CCM.
5 Frequency for PWM

5.5 Conclusions

The higher frequencies and related current and voltage effects on the size of the inductor and capacitor were detailed. Switching frequency was directly proportional to switching losses in the MOSFET and also produces negative effects on the converter efficiency. Increasing frequencies and related effects on PWM with respect to the switching network were observed. The results and conclusions drawn for the analysis by Saber Sketch circuit simulation and experimental waveforms were presented. The parameters of interest include the efficiency of the converter, and the currents and voltages through and across the switch $S$, and diodes $D_0, D_1, D_2$, and $D_{12}$. A damaging ringing effect is seen in the voltage and current waveforms of the switching devices. The greatest value for frequency while still approximating the ideal waveform and desired output voltages was 500 kHz. The analysis assumed the switched-inductor buck-boost converter to be operating in continuous conduction mode.
6 Control

6.1 Introduction

Digital transfer function are explored using the previous transfer functions found in chapter 4. The open-loop small-signal digital transfer functions: control-to-output voltage, input-to-output voltage, input voltage-to-inductor current, input impedance, and output impedance of the PWM switched-inductor buck-boost converter in CCM are derived. The theoretically predicted transfer functions with a step input are plotted by MatLab are verified by circuit experimentation.

6.2 Digital Open-Loop Duty Cycle-to-Output Voltage Transfer Function

Fig. 4.2 shows the small-signal model of a PWM switched-inductor buck-boost converter in CCM. The model does not account for switching losses or stray inductances/capacitance in the devices. Also, recalling open-loop control-to-output voltage transfer function derived in chapter 4

\[ T_p = \frac{v_o}{d} = T_{px} \frac{(s + \omega_n)(s - \omega_p)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}. \]

Using the bi-linear transform where \( s = \frac{2}{T} \frac{z - 1}{z + 1} \), the digital duty cycle-to-output voltage transfer function is
6 Control

\[ T_p(z) = T_{px} \left( \frac{\left( \frac{2}{T} \frac{T_n}{z+1} \right) + \omega_n \left( \frac{2}{T} \frac{T_n}{z+1} \right) - \omega_p}{\left( \frac{2}{T} \frac{T_n}{z+1} \right)^2 + 2\zeta\omega_o \left( \frac{2}{T} \frac{T_n}{z+1} \right) + \omega_o^2} \right). \] (6.1)

Rearranging the equation yields

\[ T_p(z) = T_{px}(\omega_nT+2)(2-\omega_pT)p \left( \frac{z + \omega_n T - \frac{2}{T} \omega_p T}{\omega_n T + \frac{2}{T} \omega_p T} \right) \left( \frac{z - \omega_p T + \frac{2}{T} \omega_p T}{z - 2 \omega_p T} \right) \left( z - 4 + 2\zeta\omega_o T^2 + \omega_o^2 T^2 \right). \] (6.2)

This equation written in pole/zero format renders

\[ T_p(z) = T_{pxz} \frac{(z + \omega_n z)(z - \omega_p z)}{(z + \omega_{z1})(z + \omega_{z2})}. \] (6.3)

Where the gain is

\[ T_{pxz} = -\frac{V_O (T + 2Cr_C) \left( 4DL - \left[ R_L(1 - D)^2 \left( 1 + \frac{V_F}{V_O} \right) + r(1 - 3D) \right] T \right)}{LCD(1 - D)(R_L + r_C)}. \] (6.4)

The zeros of the system are

\[ \omega_{nz} = \frac{T - 2Cr_C}{T + 2Cr_C} \] (6.5)

and

\[ \omega_{pz} = \frac{4DL + \left[ R_L(1 - D)^2 \left( 1 + \frac{V_F}{V_O} \right) + r(1 - 3D) \right] T}{4DL - \left[ R_L(1 - D)^2 \left( 1 + \frac{V_F}{V_O} \right) + r(1 - 3D) \right] T}. \] (6.6)

Using the quadratic equation to find the poles of the system. The poles are

\[ \omega_{z1,2} = \frac{8 - 2\omega_o^2 T^2 \pm \sqrt{(-8 + 2\omega_o^2 T^2)^2 - 4 \left( 4 + 4T\zeta\omega_o + \omega_o^2 T^2 \right) \left( 4 - 4T\zeta\omega_o + \omega_o^2 T^2 \right)}}{2 \left( 4 + 4T\zeta\omega_o + \omega_o^2 T^2 \right)}. \] (6.7)
Rearranging and reducing the equation, the digital open-loop control-to-output voltage transfer function poles are

\[
\omega_{z1}, \omega_{z2} = \frac{4 - \omega_o^2 T^2 \pm 4 \omega_o T \sqrt{(\zeta^2 - 1)}}{4 + 4 T \zeta \omega_o + \omega_o^2 T^2}.
\]

(6.8)

### 6.3 Digital Open-Loop Input-to-Output Voltage Transfer Function

The transfer function equation for the audio susceptibility is

\[
M_v = \frac{v_o}{v_i} = M_{vx} \frac{(s + \omega_n)}{s^2 + 2 \zeta \omega_o s + \omega_o^2}.
\]

The bi-linear transform is applied

\[
M_v(z) = M_{vx} \frac{\left(\frac{2}{T} \frac{z - 1}{z + 1} + \omega_n\right)}{\left(\frac{2}{T} \frac{z - 1}{z + 1}\right)^2 + 2 \zeta \omega_o \left(\frac{2}{T} \frac{z - 1}{z + 1}\right) + \omega_o^2}.
\]

(6.9)

Reduction and rearrangement of the equation renders

\[
M_v(z) = M_{vxz} \frac{(z + 1)(z + \omega_{nz})}{(z + \omega_{z1})(z + \omega_{z2})}.
\]

(6.10)

Where the digital audio susceptibility gain is

\[
M_{vxx} = \frac{2 T D r_C R_L (1 - D) (T + 2 C r_C)}{LC (r_C + R_L)}.
\]

(6.11)
6.4 Digital Open-Loop Input Voltage-to-Inductor Current Transfer Function

Recalling the equation for the input voltage-to-inductor current transfer function is

$$M_{vi} = \frac{i_l}{v_i} = M_{vix} \frac{s + \omega_c}{s^2 + 2\zeta\omega_0 s + \omega_0^2}.$$  

Transforming the equation using $s = \frac{2}{T} z^{-1}$, the transfer function becomes

$$M_{vi}(z) = M_{vix} \frac{(\frac{2}{T} z^{-1} + \omega_c) + \omega_c}{(\frac{2}{T} z^{-1} + \omega_c)^2 + 2\zeta\omega_0 (\frac{2}{T} z^{-1} + \omega_c) + \omega_0^2}.$$  

Rearranging and reducing the equations yields

$$M_{vi}(z) = M_{vix} \frac{(z + 1) (z + \omega_{cz})}{(z + \omega_{z1})(z + \omega_{z2})}. \quad (6.12)$$

Where the zero of the inductor-to-output voltage transfer function is

$$\omega_c = \frac{1}{C(r_C + R_L)}.$$  

Using $\omega_c$, the digital domain zero of the inductor-to-output voltage transfer function yields

$$\omega_{cz} = \frac{(\omega_c T - 2)}{(\omega_c T + 2)} = \left(\frac{\frac{1}{C(r_C + R_L)}}{\frac{1}{C(r_C + R_L)}}\right) T - 2 = \frac{T - 2C(r_C + R_L)}{T + 2C(r_C + R_L)}. \quad (6.13)$$

Recalling the gain of the inductor-to-output voltage transfer function and

$$M_{vix} = \frac{2D}{L},$$

thereby

$$M_{vixz} = \frac{2DT}{L} \left(\frac{1}{C(r_C + R_L)} T + 2\right) = \frac{2DT (T + 2C(r_C + R_L))}{LC(r_C + R_L)}. \quad (6.14)$$
6 Control

6.5 Digital Open-Loop Input Impedance Transfer Function

Input impedance in the s-domain is

\[ Z_i = \frac{v_i}{i_i} = Z_{ix} \frac{s^2 + 2\zeta\omega_0 s + \omega_0^2}{s + \omega_c}. \]

Using the bi-linear transform where \( s = \frac{2z - 1}{\tau z + 1} \), the digital input impedance is

\[ Z_i(z) = Z_{ixz} \frac{(z + \omega_{z1})(z + \omega_{z2})}{(z + 1)(z + \omega_{zz})}. \]  
(6.15)

Recalling the input impedance gain being

\[ Z_{ix} = \frac{L}{2D^2}. \]

The digital input impedance gain is

\[ Z_{ixz} = \frac{L}{2D^2} \frac{T(\omega_c T + 2)}{(\omega_c T + 2)} = \frac{LC(r_C + R_L)}{2TD^2(T + 2C(r_C + R_L))}. \]  
(6.16)

6.6 Open-Loop Output Impedance Transfer Function

Output impedance in the s-domain is

\[ Z_{out} = \frac{v_t}{i_t} = Z_{outx} \frac{(s + \omega_n)(s + \omega_l)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}. \]

Substituting the bi-linear transform, and rearranging and reducing the equation the digital domain output impedance is

\[ Z_{out}(z) = Z_{oxz} \frac{(z + \omega_{nz})(z + \omega_{lz})}{(z + \omega_{z1})(z + \omega_{z2})}. \]  
(6.17)

Where

\[ \omega_{lz} = \frac{\omega_l T - 2}{\omega_l T + 2} = \frac{T - 2L}{rT + 2L}. \]  
(6.18)
and the digital gain of the output impedance is

\[ Z_{oxz} = Z_{ox}(\omega_n T + 2)(\omega_l T + 2) = \frac{R_L (T + 2 r_C)}{LC (R_L + r_C)}. \] (6.19)

6.7 Digital Open-Loop Step Responses

6.7.1 Open-Loop Response of Output Voltage to Step Change in Input Voltage

Verification of the open-loop switched-inductor digital transfer functions as an accurate representation of the analog circuit is accomplished by the step response for each or the control or perturbations (d, vi, and iO). Consider a step change in the input voltage magnitude \( \Delta V_I \) at an arbitrary time \( t = 0 \). The step change of the input voltage in the digital domain is

\[ v_i(n) = \frac{\Delta V_I}{1 - z^{-1}}. \] (6.20)

The output voltage due to input voltage step change of the open-loop switched inductor buck-boost total input voltage in the digital domain is

\[ v_o(s) = \frac{\Delta V_I}{1 - z^{-1}} M_V(z) = \Delta V_I M_{Vxz} \frac{z (z + 1)(z + \omega_nz)}{(z - 1)(z + \omega_{z1})(z + \omega_{z2})}. \] (6.21)

The chapter 4 component values are used to obtain the step responses: \( D = 0.24 \), \( R_L = 14 \Omega \), \( C = 100 \mu F \), \( L_1 = L_2 = 330 \mu H \), \( r_L = 0.42 \Omega \), \( r_C = 0.155 \Omega \), \( C_o = 150 \) pF, \( V_F = 0.65 \) V and \( R_F = 0.2 \) \( \Omega \). Fig. 6.1 shows the output voltage response to a step change in input voltage from 48V to 53V using the digital input-to-output voltage transfer function.
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Figure 6.1: Output Voltage $v_O$ response to a step change in $v_I$ from 48 to 53 V using the digital input-to-voltage output transfer function.

6.7.2 Open-Loop Response of Output Voltage to Step Change in Duty Cycle

Considering next a step change in the input voltage magnitude $\Delta d_T$ at an arbitrary time $t = 0$. The step change of the input voltage in the digital domain is

$$d(n) = \frac{\Delta d_T}{1 - z^{-1}}.$$  \hfill (6.22)

The output voltage due to input voltage step change of the open-loop switched inductor buck-boost total input voltage in the digital domain is

$$v_o(s) = \frac{\Delta d_T}{1 - z^{-1}} T_p(z) = \Delta d_T T_{pz} \frac{z (z + \omega_{n2}) (z - \omega_{p2})}{(z - 1) (z + \omega_{z1}) (z + \omega_{z2})}.$$  \hfill (6.23)

Fig. 6.2 shows the output voltage response to a duty cycle step change from 0.24 to 0.29 for the open-loop PWM switched-inductor buck-boost converter.
6.7.3 Open-Loop Response of Output Voltage to Step Change in Output Current

Consider a step change in the input voltage magnitude $\Delta I_O$ at an arbitrary time $t = 0$. The step change of the input voltage in the digital domain is

$$i_O(n) = \Delta I_O \frac{1}{1 - z^{-1}}. \quad (6.24)$$

The output voltage due to input voltage step change of the open-loop switched inductor buck-boost total input voltage is

$$v_o(s) = \Delta i_O \frac{z \left( z + \omega_{nz} \right) \left( z + \omega_{lz} \right)}{(z - 1) (z + \omega_{s1}) (z + \omega_{s2})}. \quad (6.25)$$

Fig. 6.3 shows the output voltage response to a duty cycle step change from 0.56 to 1.12 for the open-loop PWM switched-inductor buck-boost converter.
6.8 Conclusions and Possible Future Work

The digital transfer functions of the switched-inductor buck-boost converter in CCM were presented; the bi-linear transform was used to convert the analog transfer functions into digital transfer functions. The accuracy of the model was proven by the analog and digital step responses being in accordance. This provides an accurate model for further development of a robust digital control system for the switched-inductor buck-boost.

There is no single control topology for all applications, as control topologies are designed for a specific application and depend on design specifications. Voltage mode control (VMC) and current mode control (CMC) present their own advantages and disadvantages. VMC was the first control used in switching power supply design which included a constant ramp waveform compared to a error signal which created the PWM pulse. Current limiting is done separately. The benefits of the VMC are:
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- A single feedback loop
- A stable modulation process with good noise margin when using a large amplitude ramp
- A low-impedance power output

The disadvantages of VMC are:

- Slower response times because the output must be sensed first and then corrected by the feedback loop.
- Addition of at least two poles or the addition of a zero in the control loop
- Complications result when input voltage varies significantly

CMC fixes the weaknesses inherent in VMC. The CMC topology has three types: peak, valley, and average. Each has its own advantages and disadvantages, but in general, the advantages of CMC over VMC include:

- A faster response time due to the inductor slope compensator which decreases or may eliminate delaying response.
- Attenuation or possible elimination of input noise
- Potential for a single pole in the feedback loop which allows simpler compensation methods. Although RHP is not eliminated, the compensating frequency doesn’t have to be much greater than the resonant frequency.
- Potential to design acceptable compensation in both CCM and DCM.

Disadvantages to CMC:

- Analysis becomes more challenging with two feedback loops.
- Slope compensation must be added to control loop if the duty cycle is greater than 50%.
- Noisy current sense signal. The combination of noise and ringing may be too great to control over the full range of operation.

The advantages of CMC outweigh the disadvantages. An accurate digital model is needed before the circuit can be fully understood and therefore controlled.
7 Summary and Contributions of the Dissertation

7.1 Summary

The current status of the present work shown in this dissertation follows:

- Overview of operation of DC-DC converters.
- Design of the switched-inductor buck-boost DC-DC converter in CCM given a set of specifications.
- Steady-state analysis of the PWM switched inductor buck-boost converter in CCM. The designed switched-inductor buck-boost was demonstrated in both theoretical, simulated, and experimental results. The results are presented and found to correspond well.
- Small-signal models for the PWM switched-inductor buck-boost converter are derived based on circuit averaging technique. The derivation is explained in detail in Chapter 3. The small-signal model derivation included the derivations of averaged model, large-signal model, DC model and small-signal model of the PWM switched-inductor buck-boost converter.
- The transfer functions for control-to-output voltage, input-to-output voltage, input voltage-to-inductor current, input impedance, and output impedance were derived and presented using the small-signal models. All the Bode plots for theoretical, simulated, and experimental results are in accordance.
7 Summary and Contributions of the Dissertation

- The effects of higher frequencies and related effects on the size of the inductor and capacitor were detailed. Switching frequency and its effect on switching losses in the MOSFET was presented in both simulated and experimental results. The current and voltage waveforms of the PWM switched-inductor buck-boost converter and the effects of higher frequency was shown in simulated and experimental values.

- Digital transfer functions control-to-output voltage, input-to-output voltage, input voltage-to-inductor current, input impedance, and output impedance were presented. The step responses for the theoretical, and experimental results are in accordance.

7.2 Contributions

- Characterization of the switched-inductor buck-boost converter
  - DC analysis of PWM switched-inductor buck-boost converter for CCM design of power stage
    * DC voltage transfer function
    * Device stresses
    * Minimum inductance for CCM operation
    * Ripple voltage
    * Component losses and converter efficiency
    * Design procedure
      - Real components
    * Experimental validation

The importance of the contributions are that the performance of the switched-inductor buck-boost converter has never been analyzed. Also the steady-state analysis has never been reported. This allows the converter to be compared with more traditional converters. This com-
Summary and Contributions of the Dissertation

- Frequency limitations of power stage
  * Simulation of different frequency to identify limitations
  * Experimental results

The limitations of the higher frequency and the effects of higher frequency on the current and voltage waveforms are discussed. It is determined that the high-side driver, and the switching losses contribute to the frequency limitations. The high-side driver rise and fall time, as well as, the current driving ability limits how fast the MOSFET can be switched. Switching losses are also a factor for frequency limitations, when the frequency is increased, the switching losses increase, which causes an inversely proportional reaction to the efficiency.

- Analysis of the dynamics of the power stage
  * Derivation of the large-signal model
  * Linearization of large-signal model
  * Extraction of dc and small-signal models
  * Derivation of power stage transfer functions:
    - DC voltage transfer function
    - Control-to-output voltage transfer function
    - Input-to-output voltage transfer function
    - Input voltage-to-inductor current transfer function
    - Input impedance
    - Output impedance
  * Determined the poles and zeros of transfer function
  * Experimental validation
  * Small-Signal frequency response using discrete point method with simulated real component circuit
Summary and Contributions of the Dissertation

* Poles and zeroes of converter determined with real components of transfer functions
* Simulation of the power stage dynamics
* Experimental validation

Linearizing the switches is a very complex and difficult undertaking. There has been no research in the linearization of five non-linear switches into a linearized model. Determining which variables are the independent and dependent is important to create an accurate model of the circuit. Small-signal modeling of the switched-inductor buck-boost has not been published or reported.

- Digital model of switched-inductor buck-boost for CCM
  * Transform of open-loop power stage transfer functions:
    - Control-to-output voltage transfer function
    - Input-to-output voltage transfer function
    - Input voltage-to-inductor current transfer function
    - Input impedance
    - Output impedance
  * Determining the poles and zeroes with real components of transfer functions
  * Experimental validation

Digital controls is mainstream and future of all technology. An accurate digital model for the switched-inductor buck-boost dc-dc converter will allow for robust precise control schemes to be implemented. Advancements in this technology will be able to be utilized in multiple markets and applications.
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