Distributed Sensor Fault Diagnosis for Automated Highway Systems

Hui Chen
Wright State University

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Distributed Sensor Fault Diagnosis for
Automated Highway Systems

A thesis submitted in partial fulfillment of the
requirements for the degree of
Master of Science in Engineering

By

Hui Chen
B.S., Ludong University, 2009

2013
Wright State University
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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Hui Chen ENTITLED Distributed Sensor Fault Diagnosis for Automated Highway Systems BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

_________________________________________________________
Xiaodong Zhang, Ph.D.
Thesis Director

_________________________________________________________
Kefu Xue, Ph.D., Chair
Department of Electrical Engineering

Committee on
Final Examination

_________________________________________________________
Xiaodong Zhang, Ph.D.

_________________________________________________________
Kefu Xue, Ph.D.

_________________________________________________________
Zhiqiang Wu, Ph.D.

_________________________________________________________
R. William Ayres, Ph.D.
Interim Dean, Graduate School
ABSTRACT


Fault diagnosis problems for large-scale nonlinear systems have attracted significant attentions from researchers in recent years. Most fault detection and isolation (FDI) methods have been proposed based on a centralized architecture. However, due to the complexity of the system, most of these centralized fault detection and diagnosis schemes are not able to delivery effective fault detection and isolation performance for a large-scale nonlinear system, which contains subsystems interacting with neighboring subsystems.

In this thesis, a distributed fault detection and isolation method is developed for the automated highway systems (AHS). For each subsystem of AHS, a distributed fault detection and isolation component is designed to detect and isolate a sensor fault in the system. Each component uses the local measurements and communicated information from other neighboring fault detection and isolation components. In each local subsystem of AHS, adaptive thresholds for fault detection and isolation are derived based on the
distributed fault diagnosis decision scheme. Simulation results for two case studies show the effectiveness of the distributed FDI method.
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1. Introduction

The scale and complexity of modern control systems are increasing at a fast pace. Modern control systems must meet higher safety and stability standards to ensure that when faults occur in a system, personnel and property losses are kept at a minimum. To reduce the complexity, cost, and weight of control systems, traditional approaches such as built-in tests and multiple hardware redundancies are no longer sufficient in many applications. Therefore, it is vital to build a system with comprehensive and effective fault diagnosis and accommodation capabilities to realize these design objectives.

As described in literature, fault diagnosis can include any combination of three parts: detection, isolation, and then identification of a fault [1]. Fault detection is designed to determine whether a fault has occurred in the system; fault isolation is used to obtain the type or location of the fault in the system; the fault identification process estimates the size of the fault. In the recent decade, researchers are showing greater interest in fault diagnosis of large scale distributed systems because they are increasingly used in various industrial and military applications.

In this chapter, some background information of fault diagnosis is reviewed, including the significance of fault diagnosis and various fault diagnosis methods. Then, the concept of distributed systems is introduced. Finally, the research objectives of this thesis are given.
1.1 Background of Fault Diagnosis

1.1.1 Significance of fault diagnosis

With the demand for high productivity in industrial systems, there is an increasing awareness about the risks associated with system malfunctions. It is important to design a fault diagnosis scheme with quick detection and diagnosis capabilities, high robustness, and great adaptability. In addition, fault diagnosis helps to provide the operator of the system with fault information so that certain actions can be done to maintain system safety.

1.1.2 Classification of fault diagnosis approaches

Because of the broad scope of the fault diagnosis problem and the difficulties in its real-time solutions, various approaches have been developed over the years. Generally, there are two types of methods for fault diagnosis: hardware redundancy and analytical (software) redundancy [2]. Because the hardware redundancy method requires extra equipment, which means higher cost and weight for the system, the analytical redundancy based methods are preferred in many applications. In this thesis, we focus on the analytical redundancy based methods.

In the literature, various analytical redundancy based fault diagnostic methods have been developed. These diagnostic methods differ not only in the way the process knowledge is used but also in the form of knowledge required. Fault diagnosis methods utilizing analytical redundancy are classified into two general groups: model-based methods and data-driven based methods [2].
1.1.2.1 Model-Based Techniques

In model-based fault diagnosis method, a system model is used to provide an estimate of the system variables under monitoring. When a measured system variable is different from its estimation generated by the model, it can be concluded that a fault has occurred. A general scheme of a model-based fault diagnosis system is shown in Figure 1.

![General scheme for the model-based fault diagnosis](image)

Figure 1.1: General scheme for the model-based fault diagnosis.

- Quantitative models

Quantitative methods mainly utilize the mathematical relations that exist between system variables. For instance, consider a linear time-invariant process represented as:

\begin{align}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align}

(1.1) (1.2)

where \(x(t)\) is the system states, \(u(t)\) the is input, \(y(t)\) is the output, \(A, B, C,\) and \(D\) are constant matrices of appropriate dimensions. Based on the model above, we can design the following observer to generate an estimated output and the residual \(e(t)\), which is used for fault diagnosis:
\[ \hat{x}(t) = A\hat{x}(t) + Bu(t) + L(y - \hat{y}) \]  
\[ e(t) = y(t) - C\hat{x}(t) \]

where \( \hat{x}(t) \) is the system estimated states, \( \hat{y} \) is the estimated output, and \( L \) is a designed gain matrix.

Under nominal conditions, the residual is designed to be zero or very small due to system noise and the modeling uncertainty. When there is a fault in the system, the residual deviates from its small nominal value significantly. Then, we can set a threshold to compare with the value of a residual. If it exceeds the threshold, we can draw the conclusion that a fault has occurred.

- **Qualitative models**

The strategy employed by qualitative models is the cause-effect reasoning of system behaviors. For instance, diagnostic methods based on fault-trees and signed digraphs have been proposed. Fault Trees uses backward chaining until a primary event is found that presents a possible root cause for observed process deviation. Another representation of the causal information is Signed Digraphs where the process variables are represented as causal relations and graph nodes by directed arcs \([3]\).

Many theories have been developed based on model-based fault diagnosis. However, many methods have been restricted to linear systems. For nonlinear systems, it is very difficult to build an accurate analytical model to monitor all the states and parameters for a complex system.
1.1.2.2 Data-Driven Techniques

In model-based approaches, a system model (either quantitative or qualitative) is needed. If such a model is not available, then data-driven methods are more suitable, in which a large amount of historical data is required instead to generate the analytical redundancy. General procedures for data-driven techniques consist of three steps: system data collection, data processing, and fault detection and isolation. Below, we describe some examples of data-driven techniques.

- **Multivariate statistical approaches**

  Principal Component Analysis (PCA) and Partial Least Squares (PLS) have been successfully applied for fault diagnosis. Overview of these two methods can be found in the fault diagnosis literatures (see, for instance, [4], [5], [6]). The PCA-based fault diagnosis process includes three steps: 1) data pre-processing; 2) building the healthy data set and obtaining thresholds; 3) comparing the testing data with threshold and making fault diagnosis decisions.

- **Neural-networks-based approaches:**

  Neural-networks-based methods for fault diagnosis have received considerable attention over the last two decades [7], [8], [9]. Learning and interpolation capabilities of neural networks have led to several successful implementations for various control systems. In general, neural networks used for fault diagnosis can be classified along two dimensions: the architecture of the network and the learning strategy, such as supervised and unsupervised learning.

- **Knowledge based approaches:**
Knowledge based fault diagnosis has also been applied to perform online monitoring for industrial processes [10]. Two of the major methods are expert systems and qualitative trend analysis (QTA). An expert system is generally a very specialized system that solves problems in a narrow domain of expertise. For instance, the application of expert systems for fault diagnosis can be found in Henley [11]. Trend analysis can be used to explain the various important events that happen in a process, to diagnose malfunctions, and to predict future states.

For many practical applications, it is impossible that one method listed above will solve all issues involved in the fault diagnosis. A hybrid architecture is an effective way to use all available system information to realize the objectives of fault diagnosis.

1.2 Distributed Control System

In recent years, there has been a growing interest in distributed control systems, in which distributed sensors, controllers, and plants are connected over a network media [12], [13], [14], [15]. Distributed systems have been widely used in many different application fields because of their advantages, such as easy maintenance, low cost, and reliability. For a distributed system, we can easily change the control mode to achieve a variety of complex control functions, and improve the reliability of the control system. At the same time, the risk of causing system failure is dispersed. Due to the advantage of the distributed systems, the area of large-scale distributed systems has drawn a significant amount of attention from researchers. These efforts include improving the reliability, flexibility and coordination of the system components.
A distributed control structure is shown in Figure 1.2. In this figure, each block labeled ‘actuator,’ such as ‘actuator1,’ actually represents a group of actuators. The same also applies to the sensors in the figure. The controller provides the control decision for the system.

![Diagram](image1)

**Figure 1.2: Typical structure of a distributed control system**

Most distributed systems can be mapped into sets of interconnected subsystems as illustrated in Figure 1.3.

![Diagram](image2)

**Figure 1.3: Example of a distributed system consist of three subsystems**
Based on the figure above, we know that the signals of each subsystem are transmitted from a local subsystem to its neighboring subsystems. Compared with a centralized system, when a fault occurs to one of the subsystems, the fault effect could be propagated between different subsystems through the interconnection among subsystems. Therefore, how to detect the fault and localize the fault become vital for designing distributed control systems.

Distributed fault diagnosis methods have been investigated by some researchers in the recent decades. However, for general nonlinear large-scale distributed systems, it is very challenging due to the uncertainty and complexity of the system.

### 1.3 Research Motivation

In literature, most fault detection and isolation (FDI) methods employ a centralized architecture. However, these centralized methods cannot provide effective fault detection and isolation for distributed systems because of the limitations of computational resources and communication bandwidth. A distributed large-scale system contains many subsystems, and there is a large amount of sensor data needed to be processed and transmitted. Such characteristics of the system require intensive computation and communication, which is not suitable for FDI employing a centralized architecture.

In order to overcome the limitations of centralized FDI methods for large-scale systems, distributed FDI architecture is proposed by some researchers [16], [17], [18]. Distributed fault diagnosis is an attempt to conduct effective fault diagnosis by distributing the computation across different nodes in the network and by only requiring limited communication among subsystems.
In this thesis, our objective is to design and implement a distributed FDI method for sensor fault in the automated highway system (AHS), which is a part of the intelligent highway system effort. AHS was originally introduced in the General Motors Pavilion at the 1939 World’s Fair. Radio Corporation of America and General Motors Corporation conducted initial research work [19], [20]. Since then, AHS has been studied by many researchers and organizations.

A theoretical FDI method for a class of distributed large-scale nonlinear systems has been proposed in [22]. This thesis applies the general theory to the AHS application. The main research objectives of this thesis are as follows: First, develop a distributed FDI scheme for AHS in the presence of unstructured modeling uncertainty. Second, verify the effectiveness of the distributed FDI scheme in AHS using Matlab simulation.

This thesis is organized as follows. Chapter 1 gives some background information on fault diagnosis, the research motivation, and research objectives of this thesis. In Chapter 2, the problem of distributed fault detection and isolation for the automated highway system is formulated. In Chapter 3, the details of the distributed fault detection and isolation scheme for the AHS system are given, which includes the design of adaptive thresholds for distributed FDI. Chapter 4 shows some simulation results of the distributed FDI scheme to demonstrate its effectiveness. Finally, Chapter 5 describes the conclusions and future work.
2. Problem Formulation

As described in [21], the dynamics of the nonlinear automated highway systems (AHS) composed of M interconnected subsystem can be described as:

\[ \dot{\psi}_i = v_i - v_{(i-1)} \]  \hspace{1cm} (2.1)

\[ \dot{v}_i = \frac{1}{m_i} (-D_i v_i^2 - d_i + \xi_i) \]

\[ \dot{\xi}_i = \frac{1}{\tau_i} (-\xi_i + u_i) \]

\[ y_i = \begin{bmatrix} \psi_i + \sigma v_i \\ \xi_i \end{bmatrix} \]

where, for \( i = 1, \ldots, M \), \( \psi_i \) is the distance between the \( i \)th and the \((i - 1)\)th vehicle, \( v_i \) is the \( i \)th vehicle’s speed, \( \xi_i \) is the driving/breaking force of the \( i \)th vehicle, and \( u_i \) is the control input. The system output \( y_i \) allows for a velocity-dependent inter-vehicle spacing due to \( \sigma v_i \), where \( \sigma \) is a positive constant. Additionally, \( m_i \) is the mass of the vehicle, \( D_i \) is the aerodynamic drag, \( d_i \) and \( \tau_i \) are the constant frictional force and engine break time constant, respectively.
In this thesis, we consider the case of 3 cars in the system, as shown in the Figure 2.1. Moreover, the following simulation parameters are used: $m_i=1300\,\text{kg}$, $D_i=0.3\,\text{Ns}^2/\text{m}^2$, $d_i=100\,\text{N}$, $\tau_i=0.2\,\text{s}$, and $\varpi=0.4$. Thus, we can rewrite the system model given in (2.1) as follows:

$$
\begin{bmatrix}
\dot{\psi}_i \\
\dot{v}_i \\
\dot{\xi}_i
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{1}{1300} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
\psi_i \\
v_i \\
\xi_i
\end{bmatrix} +
\begin{bmatrix}
-\frac{1}{13} \\
-0.3 \\
5u_i
\end{bmatrix} \left[ v_{(i-1)} \right]
$$

(2.2)

$$
y_i =
\begin{bmatrix}
1 & 0.4 & 0 \\
0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
\psi_i \\
v_i \\
\xi_i
\end{bmatrix}.
$$

For the AHS dynamic system described above, a linear transformation of coordinates in the form of $z_i = T[\psi_i, v_i, \xi_i]^T = [z_{i1}, z_{i2}, z_{i3}]^T$ with $T= [1 \ 0 \ 0; \ 0.1 \ 0.04 \ 0; \ 0 \ 0 \ 1]$ can be applied. Thus, the state space model in the new coordinate system is:
\[
\begin{bmatrix}
\dot{z}_{i1} \\
\dot{z}_{i2} \\
\dot{z}_{i3}
\end{bmatrix} =
\begin{bmatrix}
-2.5 & 25 & 0 \\
-0.25 & 2.5 & 3.077 \times 10^{-7} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
z_{i1} \\
z_{i2} \\
z_{i3}
\end{bmatrix} +
\begin{bmatrix}
0 \\
-0.0031 - 0.0058(z_{i2} - 0.1z_{i1}) \\
5u_i
\end{bmatrix} +
\begin{bmatrix}
-25(z_{i-1,2} - 0.1z_{i-1,1}) \\
-25(z_{i-1,2} - 0.1z_{i-1,1}) \\
0
\end{bmatrix} + \eta_i 
\]
\[
y_i = \begin{bmatrix} 0 & 10 & 0 \end{bmatrix} z_i + \beta_i(t_i - T_0)\theta_i.
\]

Note that in the new state model described by (2.3), the effect of the modeling uncertainty and sensor faults have been added to the system model. Specifically, $\eta_i$ represents the modeling uncertainty, and $\beta_i(t_i - T_0)\theta_i$ represents sensor faults. The function $\beta_i(t_i - T_0)$ is a step function describing the fault time profile with unknown fault occurrence time $T_0$, and $\theta_i \in R^2$ represents a constant sensor bias vector.

By defining $x_{i1} = z_{i1}, x_{i2} = [z_{i2} \ z_{i3}]^T$, the system model (2.3) can be rewritten as:

\[
\begin{align*}
\dot{x}_{i1} &= A_{i1}x_{i1} + A_{i2}x_{i2} + \eta_{i1}(x_{i1}, u_{i1}, t) + \sum_{j=1}^{3} H_{i1}^j(x_j, u_j) \\
\dot{x}_{i2} &= A_{i3}x_{i1} + A_{i4}x_{i2} + \rho_{i2}(x_{i1}, u_{i1}) + \eta_{i2}(x_{i1}, u_{i1}, t) + \sum_{j=1}^{3} H_{i2}^j(x_j, u_j) \\
y_i &= C_i x_{i2} + \beta_i(t_i - T_0)\theta_i(t)
\end{align*}
\]

where

- $\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} = \begin{bmatrix} -2.5 & 25 & 0 \\ -0.25 & 2.5 & 3.077 \times 10^{-7} \\ 0 & 0 & -5 \end{bmatrix}$, $C_i = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$
- $\rho_{i2}(x_{i1}, u_{i1}) = \begin{bmatrix} -0.0031 - 0.0058(z_{i2} - 0.1z_{i1}) \end{bmatrix}$
- $H_{i1}^j(x_j, u_j) = -25(z_{i-1,2} - 0.1z_{i-1,1})$
\[ H_{ij}^2(x_j, u_j) = \begin{bmatrix} -2.5(z_{(i-1)2} - 0.1z_{(i-1)1}) \\ 0 \end{bmatrix}. \]

Specifically, \( H_{ij}^1 \) and \( H_{ij}^2 \) represents the interconnections between neighboring subsystems, \( \eta_{i1} \) and \( \eta_{i2} \) represent the unstructured system modeling uncertainty (\( \eta_{i1} = 0 \) in our system), and \( \rho_{i2} \) represents the nonlinearity of the nominal system model. In this thesis, it is assumed that the sensor bias magnitude \( \theta_i \) is bounded, i.e., \( |\theta_i| \leq \bar{\theta}_i \), where \( \bar{\theta}_i \) is a known constant. Additionally, we assume the model uncertainty \( \eta_{i2} \) is bounded, i.e., \( |\eta_{i2}| \leq \bar{\eta}_{i2} \), where \( \bar{\eta}_{i2} \) is a known constant vector.

The objectives of this research include: 1) detect the occurrence of any sensor fault in the interconnected vehicle system; 2) determine the specific vehicle with faulty sensors; 3) verify the effectiveness of distributed FDI algorithm using simulation results.
3. Distributed Fault Detection and Isolation Method

3.1 Distributed Fault Detection and Isolation Architecture

The distributed FDI architecture is comprised of three local FDI components for AHS, as illustrated in Figure 3.1. For each subsystem, one FDI component is designed, which consists of one fault detection estimator (FDE) and one fault isolation estimator (FIE). Under normal conditions, the local FDE component observers the corresponding subsystem to detect any sensor fault’s occurrence. The FDE is designed using nonlinear estimation techniques that generate the residuals, which are compared to adaptive thresholds to make local diagnostic decisions. If a fault is detected, then the FIEs are activated to determine the location of fault that has occurred in the AHS system.
Figure 3.1: Distributed architecture for fault detection and isolation

3.2 Distributed Fault Detection Method

Based on the system model (2.4), the distributed FDE for each local subsystem is designed as follows:

\[
\begin{align*}
\dot{x}_{l1} &= A_{l1} \hat{x}_{l1} + A_{l2} C_l^{-1} y_l + \sum_{j=1}^{3} H_{ij}^1 (\hat{x}_j, u_j) \\
\dot{x}_{l2} &= A_{l3} \hat{x}_{l1} + A_{l4} \hat{x}_{l2} + \rho_{l2} (\hat{x}_l, u_l) + L_l (y_l - \hat{y}_l) + \sum_{j=1}^{3} H_{ij}^2 (\hat{x}_j, u_j) \\
\hat{y}_l &= C_l \hat{x}_{l2},
\end{align*}
\] (3.1)
where, $\hat{x}_{l1}, \hat{x}_{l2}$ represent the estimated state of the local subsystem and $\hat{y}_l$ represents the output of the $i$th local subsystem, $L_i$ is a designed gain matrix chosen such that $\tilde{A}_{l4} = A_{l4} - L_i C_i$ is Hurwitz, and $\tilde{x}_i = [(\hat{x}_{l1})^T (C_i^{-1} y_i)^T]^T$, $\hat{x}_j = [(\hat{x}_{j1})^T (C_j^{-1} y_j)^T]^T$ (here $\hat{x}_{j1}$ is the estimate of state $x_{j1}$ of the $j$th interconnected subsystem).

For each local FDE, the state estimation error can be defined as:

$$\tilde{x}_{l1} = x_{l1} - \hat{x}_{l1}$$

$$\tilde{x}_{l2} = x_{l2} - \hat{x}_{l2}$$

Thus, based on (2.4) and (3.1), the state error dynamics is given by:

$$\dot{\tilde{x}}_{l1} = A_{l1} \tilde{x}_{l1} + \sum_{j=1}^{3} [H_{ij}^1(x_j, u_j) - H_{ij}^1(\hat{x}_j, u_j)]$$

$$\dot{\tilde{x}}_{l2} = A_{l3} \tilde{x}_{l1} + \tilde{A}_{l4} \tilde{x}_{l2} + \rho_{l2}(x_i, u_i) - \rho_{l2}(\hat{x}_i, u_i) + \eta_{l2} + \sum_{j=1}^{3} [H_{ij}^2(x_j, u_j) - H_{ij}^2(\hat{x}_j, u_j)]$$

$$\tilde{y}_l = C_i \tilde{x}_{l2},$$

where $\tilde{y}_l$ is the output estimation error used as the residual for FDI.

Now, we will investigate the design of adaptive thresholds for distributed residual evaluation in each subsystem. It consists of the following two steps.

- First, let us define a state estimation error vector as $\tilde{x}_1(t) = [\tilde{x}_{11}, \tilde{x}_{21}, \tilde{x}_{31}]^T$ and derive a bounding function on $\tilde{x}_1(t)$ before a fault occurs to the system (i.e., for $t < T_0$).

Note that in the system model described by (2.4), the interaction between the subsystems terms satisfies the following condition: for $i=1, 2, 3$ and $j=1, 2, 3$,

$$|H_{ij}^1(x_j, u_j) - H_{ij}^1(\hat{x}_j, u_j)| \leq \gamma_{ij}^1 |x_j - \hat{x}_j|,$$  

(3.4)
where $\gamma_{ij}^1$ is the known Lipschitz constant for the interconnection terms. Specifically, for vehicle 1, there is no interaction effect coming from other parts of the system, hence $\gamma_{12}^1 = \gamma_{13}^1 = 0$. For vehicle 2, the interconnection effect only from vehicle 1, we have

$$H_{21}^1(x_1, u_1) - H_{21}^1(\hat{x}_1, u_1) = -25(z_{12} - 0.1z_{11}) + 25(z_{12} - 0.1\hat{z}_{11})$$

$$= -2.5(z_{11} - \hat{z}_{11}).$$

Thus, $\gamma_{21}^1 = 2.5$ and $\gamma_{23}^1 = 0$. Additionally, for vehicle 3, the interconnection due to the effect only from vehicle 2, we have

$$H_{32}^1(x_1, u_1) - H_{32}^1(\hat{x}_1, u_1) = -25(z_{22} - 0.1z_{21}) + 25(z_{22} - 0.1\hat{z}_{21})$$

$$= -2.5(z_{21} - \hat{z}_{21}).$$

Thus, we can find $\gamma_{32}^1 = 2.5$ and $\gamma_{31}^1 = 0$. Now, we need the following result from [22].

**Lemma 1.** Consider the interconnected systems described by (2.4) and the fault detection estimators described by (3.3). Assume that there exists a symmetric positive definite matrix $\bar{P}_i$, for $i=1,2...M$, such that:

1. The symmetric matrix $\bar{R}_i \triangleq -A_i^T \bar{P}_i - \bar{P}_i A_i > 0$,

2. The matrix $\bar{Q} \in \mathbb{R}^{M \times M}$, whose entries are given by

$$\bar{Q}_{ij} = \begin{cases} 
\lambda_{\min}(\bar{R}_i), & i = j \\
-\|\bar{P}_i\| \gamma_{ij}^1 - \|\bar{P}_j\| \gamma_{ji}^1, & i \neq j, j = 1, \cdots, M,
\end{cases}$$

is a positive definite, where $\gamma_{ij}^1$ and $\gamma_{ji}^1$ are the Lipschitz constants.

Then, for $0 \leq t < T_0$, the state estimation error vector $\tilde{x}_i(t)$ satisfies the following inequality:

$$|\tilde{x}_i(t)| \leq \chi(t)$$

where $\chi = \frac{\bar{V}_0 e^{-ct}}{\lambda_{\min}(\bar{P})}$, the matrix $\bar{P} \triangleq \text{diag}\{\bar{P}_i, \cdots, \bar{P}_M\}$, the constant $c = \frac{\lambda_{\min}(\bar{Q})}{\lambda_{\min}(\bar{P})}$, and $\bar{V}_0$ is a positive constant.
Therefore, according to the system model (2.4), let us choose define a symmetric and positive matrix \( \overline{P}_i = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \), for \( i = 1, 2, 3 \). Then \( \overline{R}_i = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix} \), and the matrix \( \overline{Q} = \begin{bmatrix} 2.5 & -1.25 & 0 \\ -1.25 & 2.5 & -1.25 \\ 0 & -1.25 & 2.5 \end{bmatrix} \). Then, we have

\[
|\overline{x}_i(t)| \leq \chi(t) = \frac{\overline{V}_0 e^{-ct}}{\lambda_{\min}(\overline{P})} = 2\overline{V}_0 e^{-1.46t},
\]

where the constant \( c = \frac{\lambda_{\min}(\overline{Q})}{\lambda_{\min}(\overline{P})} = 1.46 \). \( \overline{V}_0 \) is a positive constant, \( \lambda_{\min} \) means minimum eigenvalue of the matrix.

- Second, we design the threshold for the error dynamics for AHS. Let us consider every component of the output estimation error, i.e., \( \overline{y}_{ip} = C_{ip}\overline{x}_{i2}(t) \), \( p = 1, 2, 3 \), where \( C_{ip} \) is the \( p \)th row vector of matrix \( C_i \). When there is no fault occurrence, the estimation error should satisfy [22],

\[
|\overline{y}_{ip}| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ |\overline{q}_i| \chi(t) + |\overline{\eta}_i| \right] dt,
\]

(3.6)

where \( k_{ip} \), \( \lambda_{ip} \) are positive constants chosen such that \( |C_{ip}e^{\overline{A}_{i4}t}| \leq k_{ip}e^{-\lambda_{ip}t} \) (since \( \overline{A}_{i4} \) is stable, constants \( k_{ip}, \lambda_{ip} \) and satisfying the above inequality always exist.

Additionally, in (3.6),

\[
\overline{q}_i = [\gamma_{i1}^2, \ldots, \gamma_{i(l-1)}^2, \|A_{i2}\| + \sigma_{i2}, \gamma_{i(l+1)}^2, \ldots, \gamma_{iM_i}^2]^T,
\]

(3.7)

where \( \sigma_{i2} \) is a known Lipschitz constant satisfying the following inequality [22],

\[
|\rho_{i2}(x_i, u_i) - \rho_{i2}(\hat{x}_i, u_i)| \leq \sigma_{i2} |x_i - \hat{x}_i|.
\]

(3.8)

The detailed design is described below

- We choose \( k_{ip} = 1 \), \( L_i = [0.4 \ 0; 0 \ -3.3] \), which result in \( \lambda_{i2} = 1.5 \) and \( \lambda_{i3} = 1.7 \).

- The interaction between the subsystems satisfies the following condition

\[
|H^2_{ij}(x_j, u_j) - H^2_{ij}(\hat{x}_j, u_j)| \leq \gamma^2_{ij} |x_j - \hat{x}_j|.
\]

(3.9)

Specifically, for vehicle1, there is no effect coming from other parts of the system, hence \( \gamma^2_{12} = \gamma^2_{13} = 0 \). For vehicle2, due to the interconnection effect only from
vehicle1, we have $\gamma_{21}^2 = 0.25$, $\gamma_{23}^2 = 0$. Additionally, for vehicle 3, the interconnection is only from vehicle 2, and we have $\gamma_{32}^2 = 0.25$ and $\gamma_{31}^2 = 0$.

- Based on (2.4) and (3.8), the nonlinearity of the nominal system model $\rho_{l2}$ follows

$$
\rho_{l2}(x_i, u_i) - \rho_{l2}(\hat{x}_i, u_i) = -0.0058[(z_{i2} - 0.1z_{i1})^2 - (z_{i2} - 0.1\hat{z}_{i1})^2]
$$

$$
= -0.0058[-0.2z_{i2}\hat{z}_{i1} + 0.01(z_{i1} - \hat{z}_{i1})(z_{i1} + \hat{z}_{i1})]
$$

$$
= -0.0058(-0.2z_{i2} + 0.01\hat{z}_{i1} + 0.02\hat{z}_{i1})\hat{z}_{i1}. \quad (3.10)
$$

Thus, based on (3.8), we have

$$
\sigma_{l2} = (| - 0.2z_{i2}| + 0.02|\hat{z}_{i1}| + 0.01\chi(t))0.0058. \quad (3.11)
$$

Based on above discussion and (3.7), we design the following adaptive threshold for fault detection

$$
v_{ip}(t) \triangleq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)}[|q_i|\chi(\tau) + |\tilde{q}_i|]dt. \quad (3.12)
$$

Specifically, for vehicle1, the thresholds for the second and third outputs of the vehicle1 base on the (3.12) are:

$$
v_{12}(t) \triangleq k_{12} \int_0^t e^{-\lambda_{12}(t-\tau)}[|q_1|\chi(\tau) + |\tilde{q}_{12}|]dt \quad (3.13)
$$

$$
v_{13}(t) \triangleq k_{13} \int_0^t e^{-\lambda_{13}(t-\tau)}|\tilde{q}_{12}|^2 dt,
$$

where $\lambda_{12} = 1.5, \lambda_{13} = 1.7$, $|q_1| = [(||A_{13}|| + \sigma_{12})^2 + \gamma_{21}^2]^{1/2} = [(0.25 + \sigma_{12})^2]^{1/2}$, where $\sigma_{12}$ is given in (3.11). Note that $\tilde{q}_{12} = [\tilde{q}_{12}^1 \tilde{q}_{12}^2]^T$ represents the bounding function for the model uncertainty $q_{12}$ in (2.4).

For vehicle 2, in order to develop thresholds for the second and third output of the second subsystem we use (3.12) which yields:

$$
\tilde{v}_{22}(t) \triangleq k_{12} \int_0^t e^{-\lambda_{12}(t-\tau)}[|q_2|\chi(\tau) + \tilde{q}_{22}]^1 dt \quad (3.14)
$$

$$
\tilde{v}_{23}(t) \triangleq k_{23} \int_0^t e^{-\lambda_{23}(t-\tau)}\tilde{q}_{22}^2 \tilde{q}_{23} dt,
$$
where $\lambda_{22} = 1.5, \lambda_{23} = 1.7, |q_2| = \left(\left(\|A_{23}\| + \sigma_{22}\right)^2 + \gamma_{21}^2\right)^{\frac{1}{2}} = \left(0.25 + \sigma_{22}\right)^2 + 0.0625\right)^{\frac{1}{2}}$, where $\sigma_{22}$ is given in (3.11). Note that $\tilde{\eta}_{22} = \begin{bmatrix} \tilde{\eta}_{22}^1 \\ \tilde{\eta}_{22}^2 \end{bmatrix}^T$ represents the bounding function for the model uncertainty $\eta_{22}$ in (2.4).

Analogously, for vehicle 3, the thresholds for the second and third outputs are

$$
\tilde{v}_{32}(t) = k_{32} \int_0^t e^{-\lambda_{32}(t-\tau)} \left(|q_3| + \tilde{\eta}_{32}\right) d\tau
$$

(3.15)

$$
\tilde{v}_{33}(t) = k_{33} \int_0^t e^{-\lambda_{33}(t-\tau)} \tilde{\eta}_{32}^2 d\tau,
$$

where $\lambda_{32} = 1.5, \lambda_{33} = 1.7, |q_3| = \left(\left(\|A_{33}\| + \sigma_{32}\right)^2 + \gamma_{32}^2\right)^{\frac{1}{2}} = \left(0.25 + \sigma_{32}\right)^2 + 0.0625\right)^{\frac{1}{2}}$.

The distributed fault detection decision scheme is as follows. If at least one component of the output estimation error generated by one of local FDE, exceed the corresponding threshold, then the occurrence of a fault can be concluded.

### 3.3 Distributed Fault Isolation Method

After a fault is detected at the time $T_d$, local fault isolation estimators are activated to determine the particular subsystem with the fault sensors. Each fault isolation estimator generates the residual based on the input and output of each subsystem and certain information of directly neighboring subsystems. Then, the FIE gives local fault isolation information.

Based on the system model (2.4), in the presence of a fault in the $s$th subsystem, $s=1, 2, 3$, we have
\[ \dot{x}_{s1} = A_{s1}x_{s1} + A_{s2}x_{s2} + \sum_{j=1}^{3} H_{sj}^{1}(x_j, u_j) \] (3.16)

\[ \dot{x}_{s2} = A_{s3}x_{s1} + A_{s4}x_{s2} + \rho_{s2}(x_s, u_s) + \eta_{s2}(x_s, u_s, t) + \sum_{j=1}^{3} H_{sj}^{2}(x_j, u_j) \]

\[ y_s = C_s x_{s2} + \beta_s(t_s - T_0)\theta_s. \]

Based on the above system model, the distributed fault isolation estimator for each local subsystem is designed as follows:

\[ \dot{x}_{s1} = A_{s1}\hat{x}_{s1} + A_{s2}C_s^{-1}(y_s - \hat{\theta}_s) + \sum_{j=1}^{3} H_{sj}^{1}(\hat{x}_j, u_j) + \Omega_{s1}\dot{\theta}_s \] (3.17)

\[ \dot{x}_{s2} = A_{s3}\hat{x}_{s1} + A_{s4}\hat{x}_{s2} + \rho_{s2}(\hat{x}_s, u_s) + L_s(y_s - \hat{y}_s) + \Omega_{s2}\dot{\theta}_s + \sum_{j=1}^{3} H_{sj}^{2}(\hat{x}_j, u_j) \]

\[ \hat{y}_s = C_s\hat{x}_{s2} + \hat{\theta}_s \]

\[ \dot{\Omega}_{s1} = A_{s1}\Omega_{s1} - A_{s2}C_s^{-1} \]

\[ \dot{\Omega}_{s2} = \tilde{A}_{s4}\Omega_{s2} - L_s, \]

where \( \hat{x}_{s1}, \hat{x}_{s2} \) and \( \hat{y}_s \) represents the estimated state and output of the \( s \)th local subsystem, \( L_s \) is a designed gain matrix chosen such that \( \tilde{A}_{s4} = A_{s4} - L_sC_s \) is Hurwitz, and \( \hat{x}_s = \left[ (\hat{x}_{s1})^T \ (C_s^{-1}(y_s - \hat{\theta}_s))^T \right]^T, \hat{x}_j = \left[ (\hat{x}_{j1} - \Omega_{j1}\dot{\theta}_j)^T \ (C_j^{-1}y_j)^T \right]^T \) (here \( \hat{x}_{j1} \) is the estimation of \( x_{j1} \) of the \( j \)th interconnected subsystem).

The learning algorithm of the adaption law for adjusting \( \hat{\theta}_s \) is given by [22]

\[ \dot{\hat{\theta}}_s = \mathcal{P}_{\Theta_s}(C_s\Omega_{s2} + I)^T\hat{y}_s \].
where $\tilde{y}_s$ is the output estimation error, $\Gamma_s$ is the learning rate, $I$ is the identity matrix, and $\mathcal{P}_{\Theta_s}$ is a projection operator in which restricts the value of $\hat{\Theta}_s$ to the corresponding known set $\Theta_s$ to guarantee stability of the learning algorithm in the presence of modeling uncertainty [23] [24].

Based on the analytical results given in [22], for each local FIE, we can define the state estimation error as:

$$\tilde{x}_{s1} = x_{s1} - \hat{x}_{s1}$$  \hspace{1cm} (3.18)

$$\tilde{x}_{s2} = x_{s2} - \hat{x}_{s2}.$$  

Thus, based on (3.16) and (3.18), the state estimation error dynamics is given by:

$$\dot{\tilde{x}}_{s1} = A_{s1}\tilde{x}_{s1} + A_{s2}C_s^{-1}\hat{\Theta}_s + \sum_{j=1}^{3} [H_{sj}^1(x_j, u_j) - H_{sj}^1(\hat{x}_j, u_j)] - \Omega_{s1}\hat{\Theta}_s$$  \hspace{1cm} (3.19)

$$\dot{\tilde{x}}_{s2} = A_{s3}\tilde{x}_{s1} + \tilde{A}_{s4}\tilde{x}_{s2} + \rho_{s2}(x_s, u_s) - \rho_{s2}(\tilde{x}_s, u_s) + \eta_{s2}$$

$$+ \sum_{j=1}^{3} [H_{sj}^2(x_j, u_j) - H_{sj}^2(\hat{x}_j, u_j)] + L_s\bar{\Theta}_s - \Omega_{s2}\hat{\Theta}_s$$

$$\tilde{y}_{s2} = C_s\tilde{x}_{s2},$$

where $\tilde{y}_{s2}$ is output residual used for fault isolation, and fault parameter estimation error is defined as $\bar{\Theta}_s = \hat{\Theta}_s - \Theta_s$, and the sensor bias magnitude $\Theta_s$ is consistently bounded, i.e., $|\Theta_s| \leq \bar{\Theta}_s$.

By substituting $A_{s2}C_s^{-1} = -\hat{\Omega}_{s1} + A_{s1}\Omega_{s1}$ into (3.21) and by letting $\bar{x}_{s1} = \tilde{x}_{s1} + \Omega_{s1}\bar{\Theta}_s$, we have:

$$\dot{\bar{x}}_{s1} = A_{s1}\bar{x}_{s1} + \sum_{j=1}^{M} [H_{sj}^1(x_j, u_j) - H_{sj}^1(\hat{x}_j, u_j)].$$  \hspace{1cm} (3.20)
Now, let us investigate the design of adaptive thresholds for distributed FIE in each subsystem. It contains the following two steps.

- First, we can define a state estimation error vector \( \bar{x}_1(t) \) as follows:

\[
\bar{x}_1(t) = [(\bar{x}_{11})^T, (\bar{x}_{21})^T, (\bar{x}_{31})^T]^T.
\] (3.21)

Similarly to distributed fault detection method, in the system model described by (3.19), the interaction between the subsystems terms satisfies the following condition: for \( s=1, 2, 3 \) and \( j=1, 2, 3 \),

\[
|H_{sj}^1(x_j, u_j) - H_{sj}^1(\hat{x}_j, u_j)| \leq \gamma_{sj}^1 |x_j - \hat{x}_j|,
\] (3.22)

where \( \gamma_{sj}^1 \) is the known Lipschitz constant for the interconnection terms. The value of \( \gamma_{sj}^1 \) have been given in the section of FDE. Now, we use the following result from [22]:

**Lemma 2.** Consider the interconnected systems described by (2.4) and the fault isolation estimators described by (3.19). Assume that there exists a symmetric positive definite matrix \( P_s \), for \( s=1, 2, \ldots, M \), such that:

3. The symmetric matrix \( R_s = -A_s^T P_s - P_s A_s - 2P_s P_s > 0 \),

4. The matrix \( Q \in \mathbb{R}^{M \times M} \), whose entries are given by

\[
Q_{sj} = \begin{cases} 
\lambda_{\min}(R_s) & j = s \\
-\|P_s\| \gamma_{sj}^1 - \|P_j\| \gamma_{js}^1 & j \neq s, s = 1, \ldots, M,
\end{cases}
\]

is positive definite, where \( \gamma_{sj}^1 \) and \( \gamma_{js}^1 \) are the Lipschitz constants defined in (3.22).

Then, for \( t > T_d \), the state estimation error vector \( \bar{x}_1(t) \) defined by (3.21) satisfies the following inequality:

\[
|\bar{x}_1(t)| \leq \chi_s(t),
\] (3.23)

where
\[ X_s(t) = \left\{ \frac{\bar{V}_0 e^{-c_s(t-\tau_d)}}{\lambda_{\min}(P)} + \frac{1}{2\lambda_{\min}(P)} \int_{\tau_0}^{t} e^{-c_s(t-\tau)} \left[ \sum_{j=1}^{3} |y_j^1 \bar{\theta}_s(\|\Omega_{s1}\| + \|C_s^{-1}\|)^2 \right] d\tau \right\}^{1/2}, \]

the matrix \( P \triangleq \text{diag}\{P_1, \cdots, P_M\} \), the constant \( c_s = \lambda_{\min}(Q)/\lambda_{\min}(P) \), and \( \bar{V}_0 \) is a positive design constant.

Therefore, according to the above lemma, we choose a symmetric and positive definite matrix \( P_s = [0.5 \ 0 \ 0; 0 \ 0.5 \ 0; 0 \ 0 \ 0.5] \), for \( i = 1, 2, 3 \). Then, the matrix \( R_s = [2 \ 0 \ 0; 0 \ 2 \ 0; 0 \ 0 \ 2] \), and the matrix \( Q = [2 \ -1.25 \ 0; -1.25 \ 2 \ -1.25; 0 \ -1.25 \ 2] \). Then we have constant \( c_s = \frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)} = 0.464 \).

For the three vehicles, the value of \( \sum_{j=1}^{3} |y_j^1 \bar{\theta}_s(\|\Omega_{s1}\| + \|C_s^{-1}\|)^2 \) is different from each other. In vehicle 1, as described in (3.22), we have \( y_{21}^1 = 2.5, y_{11}^1 = y_{31}^1 = 0 \), hence

\[
\sum_{j=1}^{3} y_j^1 \bar{\theta}_1(\|\Omega_{11}\| + \|C_1^{-1}\|)^2 = 2.5 \bar{\theta}_1(\|\Omega_{11}\| + \|C_1^{-1}\|)^2.
\]  

(3.24)

In vehicle 2, we have \( y_{12}^1 = y_{22}^1 = 0, y_{32}^1 = 2.5 \), hence

\[
\sum_{j=1}^{3} y_j^1 \bar{\theta}_2(\|\Omega_{21}\| + \|C_2^{-1}\|)^2 = 2.5 \bar{\theta}_2(\|\Omega_{21}\| + \|C_2^{-1}\|)^2.
\]  

(3.25)

In vehicle 3, \( y_{13}^1 = y_{23}^1 = y_{33}^1 = 0 \), so we have:

\[
\sum_{j=1}^{3} y_j^1 \bar{\theta}_3(\|\Omega_{31}\| + \|C_3^{-1}\|)^2 = 0.
\]  

(3.26)

Thus, based on (3.23), we have

\[
X_1(t) = \left\{ 2\bar{V}_0 e^{-0.464(t-\tau_0)} + \int_{\tau_0}^{t} e^{-0.464(t-\tau)} 2.5 \bar{\theta}_1(\|\Omega_{11}\| + \|C_1^{-1}\|)^2 d\tau \right\}^{1/2},  
\]  

(3.27)

\[
X_2(t) = \left\{ 2\bar{V}_0 e^{-0.464(t-\tau_0)} + \int_{\tau_0}^{t} e^{-0.464(t-\tau)} 2.5 \bar{\theta}_2(\|\Omega_{21}\| + \|C_2^{-1}\|)^2 d\tau \right\}^{1/2},  
\]  

(3.28)
and
\[
\chi_3(t) = \left\{ 2 \overline{V}_0 e^{-0.464(t-\tau_0)} \right\}^{1/2},
\]  
(3.29)
where \( \overline{V}_0 \) is chosen as \( 1e^{-5} \) in this thesis.

- Second, we design the threshold for the error dynamics of FIEs. Based on the theory developed in [22], for all \( t > T_d \), the \( p \)th component of the output estimation error generated by the local fault isolation component for the \( s \)th subsystem satisfies
\[
|\tilde{y}_{sp}(t)| \leq k_{sp} \int_{T_d}^{t} e^{-\lambda_{sp}(t-\tau)} \left[ (\sigma_{s2} + \|A_{s3}\|)|\Omega_{s1}\tilde{\theta}_s| + |q_s|\chi_s(t) + \sigma_{s2}|C_{s}^{-1}\tilde{\theta}_s| + \tilde{\eta}_{s2} \right] d\tau + |(C_{sp}\Omega_{s2} + F_{sp})\tilde{\theta}_s| + \omega_{s2} e^{-\lambda_{sp}(t-T_d)},
\]  
(3.30)
where \( k_{ip} \) and \( \lambda_{sp} \) are positive constants chosen such that \( |C_{sp}e^{\tilde{\alpha}_{s4}t}| \leq k_{sp}e^{-\lambda_{sp}t} \). \( F_{sp} \) is defined as a constant row vector with all entries being 0 except the \( p \)th entry, \( \omega_{s2} \) is the upper bound of \( |x_{s2}| \). \( \tilde{\eta}_{s2} \) is the bounding function for the model uncertainty \( \eta_{s2} \) in (3.16), and
\[
q_s = [y_{s1}^2, \ldots, y_{s(s-1)}^2, A_{s3}|| + \sigma_{s2}, y_{s(s+1)}^2, \ldots, y_{SM}^2]^T,
\]  
(3.31)
where \( \sigma_{s2} = [\sigma_{s2}^1 \ldots \sigma_{s2}^2] \) is a known Lipschitz constant matrix satisfying the following inequality
\[
|\rho_{s2}(x_s, u_s) - \rho_{s2}(\tilde{x}_s, u_s)| \leq \alpha_{s2}^1|x_{s1}| + \alpha_{s2}^2|C_{s}^{-1}\tilde{\theta}_s|.
\]  
(3.32)
When a sensor fault occurs in subsystem \( s \), we have
\[
x_s - \tilde{x}_s = \begin{bmatrix} x_{s1} - \tilde{x}_{s1} \\ x_{s2} - C_{s}^{-1}(y_s - \tilde{\theta}_s) \end{bmatrix} = \begin{bmatrix} \tilde{x}_{s1} \\ C_{s}^{-1}\tilde{\theta}_s \end{bmatrix},
\]  
(3.33)
The detailed design is described below:
- We choose \( k_{sp} = 1, L_s = [0.4\ 0; \ 0\ -3.3] \), which result in \( \lambda_{s2} = 1.5 \) and \( \lambda_{s3} = 1.7 \).
Based on (2.4), (3.32) and (3.33), the nonlinearity of the nominal system model $\rho_{s2}$ satisfies:

$$\rho_{s2}(x_s, u_s) - \rho_{s2}(\hat{x}_s, u_s) = -0.0058[(z_{s2} - 0.1z_{s1})^2 - (0.1(y_{s2} - \hat{\theta}_{s1}) - 0.1\hat{z}_{s1})^2]$$

$$= -0.0058\left[(z_{s2} - 0.1(y_{s2} - \hat{\theta}_{s1}) + 0.2(y_{s2} - \hat{\theta}_{s1})) - 0.1(z_{s1} - \hat{z}_{s1} + 2\hat{z}_{s1})\right]\left[(z_{s2} - 0.1(y_{s2} - \hat{\theta}_{s1})) - 0.1(z_{s1} - \hat{z}_{s1})\right]$$

$$= -0.0058[0.1\bar{\theta}_{s1} + 0.2(y_{s2} - \hat{\theta}_{s1}) - 0.1\bar{z}_{s1} - 0.2\hat{z}_{s1}][0.1\bar{\theta}_{s1} - 0.1\bar{z}_{s1}].$$  (3.34)

Based on (3.17), letting $\bar{x}_{s1} = \bar{x}_{s1} + \Omega_{s1}\tilde{\bar{\theta}}$, we have:

$$\bar{x}_{s1} \leq \chi_s(t) - \Omega_{s1}\kappa_s,$$  (3.35)

where $\kappa_s$ is the bound function for $\tilde{\bar{\theta}}$.

Therefore, after some algebraic manipulation, we have:

$$|\rho_{s2}(x_s, u_s) - \rho_{s2}(\hat{x}_s, u_s)|$$

$$\leq 0.0058(0.1\kappa_s + 0.2|y_{s2} - \bar{\theta}_{s1}| + 0.1|\chi_s - \Omega_{s1}\kappa_s| + 0.2|\hat{z}_{s1}|)|0.1\tilde{\bar{\theta}}| + 0.0058(0.1\kappa_s + 0.2|y_{s2} - \bar{\theta}_{s1}| + 0.1|\chi_s - \Omega_{s1}\kappa_s| + 0.2|\hat{z}_{s1}|)(0.1)|\bar{x}_{s1}|.$$  (3.36)

Therefore, we can find $\sigma_{s2}^1$ and $\sigma_{s2}^2$ in (3.34):

$$\sigma_{s2}^1 = (0.1\kappa_s + 0.2|y_{s2} - \bar{\theta}_{s1}| + 0.1|\chi_s - \Omega_{s1}\kappa_s| + 0.2|\hat{z}_{s1}|)(0.0058)(0.1),$$

$$\sigma_{s2}^2 = (0.1\kappa_s + 0.2|y_{s2} - \bar{\theta}_{s1}| + 0.1|\chi_s - \Omega_{s1}\kappa_s| + 0.2|\hat{z}_{s1}|)(0.0058).$$  (3.37)

Based on the above description, the following threshold function for FIE can be chosen:

$$h_{sp}(t) \triangleq k_{sp} \int_{t-d}^{t} e^{-\lambda_{sp}(t-\tau)}\left[||A_{s3}|| + \sigma_{s2}\right]||\Omega_{s1}||\kappa_s + ||\varrho_{s}|\chi_s(\tau)| + \sigma_{s2}||C_{s}^{-1}||\kappa_s + \bar{\eta}_{s2}]d\tau + ||(C_{sp}\Omega_{s2} + F_{sp})\kappa_s| + k_{sp}\omega_{s2}e^{-\lambda_{sp}(t-\tau)}d\tau.$$  (3.38)

Thus, for vehicle 1, the thresholds for the second and third outputs based on the (3.42) are:
\[ h_{12}(t) \triangleq k_{12} \int_{t_d}^{t} e^{-\lambda_{12}(t-\tau)} \left[ \left( |A_{s3}| + \sigma_{12} \right) \| \Omega_{11} \| \kappa_1 + |q_1| \chi_1(\tau) + \sigma_{12} \| C_{1}^{-1} \| \kappa_1 + \right. \]
\[
\left. \tilde{\eta}_{12}^{1} \right] d\tau + \left( (C_{12}\Omega_{12} + F_{12}) \kappa_1 \right) + k_{12} \omega_{12} e^{-\lambda_{12}(t-t_d)} , \right. \]
\[ h_{13}(t) \triangleq k_{13} \int_{t_d}^{t} e^{-\lambda_{13}(t-\tau)} \tilde{\eta}_{12}^{2} d\tau + \left( (C_{13}\Omega_{12} + F_{13}) \kappa_1 \right) + k_{13} \omega_{12} e^{-\lambda_{12}(t-t_d)} , \right. \]
\[
\text{where, } k_{12} = 1, k_{13} = 1, \lambda_{12} = 1.5, \lambda_{13} = 1.7, |q_1| = \left( |A_{13}| + \sigma_{12} \right)^{2} + \gamma_{12}^{2} \right. \]
\[
\left. \left[ (0.25 + \sigma_{12})^{2} \right]^{\frac{1}{2}}, \sigma_{12} \text{ is given in (3.37), and } \chi_1(t) \text{ is given in (3.27). Note that } \tilde{\eta}_{12} = \left[ \tilde{\eta}_{12}^{1} \tilde{\eta}_{12}^{2} \right]^T \text{ represents the bounding function for the model uncertainty } \eta_{12} \text{ in (3.16).} \right. \]

Similarly, we can get the thresholds for the second and third outputs of the vehicle 2 and vehicle 3 for the local FIEs.

If a sensor fault in subsystem \( s \) is detected at time \( T_d \) by FDE, then the FIE scheme, characterized by the FIE estimators and the adaptive thresholds, will be activated.
4. Simulation Results

In this section, we present some simulation studies to illustrate the effectiveness of the distributed FDI method in [22]. In this thesis, we assume that only one sensor fault occurs in the AHS at any given time.

As shown in Figure 3.1, the FDE blocks generate estimated outputs to compare with actual outputs. The difference between estimated output and actual output will be used as residual to comparing with the corresponding threshold generated by each FDE components as discussed in Chapter 3. If any one of the FDE block’s residual exceeds its threshold, we can conclude that a fault occurred to the system. After a fault was detected by FDEs, the fault isolation blocks will be activated. Similarly, the FIEs also generate thresholds and residuals to isolate the fault. Based on fault isolation decision scheme in [22], a fault is isolated if all the output residual components generated by one fault isolation component do not exceed the corresponding threshold, and at least one component of the residuals generated by any other FIE exceeds its threshold at some finite time.

4.1 Fault Model

For the AHS model, we consider two fault cases in the simulation study, including a sensor fault in vehicle 1 and a sensor fault in vehicle 2, respectively. Upon the occurrence of the fault in the AHS, the goal of the FDI scheme implemented is to detect the
fault and determine the type/location of the fault. In the simulation, there are two sensors measuring the output for each vehicle as described in Chapter 2.

- **Fault case 1: Sensor fault as a result of a sensor bias in vehicle 1**
  In this case, the fault is considered to be a sensor bias in the sensor measuring the outputs of vehicle 1, which is represented by letting $y_{12} = \bar{y}_{12} + \theta_1$, where $\bar{y}_{12}$ is the nominal first output for vehicle 1, and $\theta_1$ represents the magnitude of the bias, which is bounded in this thesis. Thus, we assume the range of magnitude of the sensor bias is $\theta_1 \in [0, 20]$.

- **Fault case 2: Sensor fault as a result of a sensor bias in vehicle 2**
  In this case, the fault is considered to be a sensor bias in the sensor measuring the outputs of vehicle 2, which is represented by letting $y_{22} = \bar{y}_{22} + \theta_2$, where $\bar{y}_{22}$ is the nominal first output for vehicle 2, and $\theta_2$ represents the magnitude of the bias, which is bounded. Thus, we assume the range of magnitude of the sensor bias is $\theta_2 \in [0, 0.01]$.

For the AHS model, the modeling uncertainties are unstructured and unknown, but assumed to be bounded by certain known functions. The modeling uncertainty of AHS is 5% inaccuracy in the engine/brake time constant $\tau_i$. Therefore, the bounding function of $\bar{y}_{l2} = [0, 0.26[-y_{l2} + u_{l1}]$.

### 4.2 FDI Design

For the AHS, a local fault detection estimator and a fault isolation estimator for each vehicle are constructed by employing the method in Chapter 3. Based on the system model designed by (2.3), the FDE estimators can be constructed as:
\[
\begin{bmatrix}
\dot{\hat{z}}_{i1} \\
\dot{\hat{z}}_{i2} \\
\dot{\hat{z}}_{i3}
\end{bmatrix} = 
\begin{bmatrix}
-2.5 & 25 & 0 \\
-0.25 & 2.5 & 3.077 \times 10^{-7} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
\hat{z}_{i1} \\
\hat{z}_{i2} \\
\hat{z}_{i3}
\end{bmatrix} + 
\begin{bmatrix}
0 \\
-0.0031 - 0.0058(\hat{z}_{i2} - 0.1\hat{z}_{i1})^2 \\
5u_i
\end{bmatrix} + 
\begin{bmatrix}
-25(\hat{z}_{(i-1)2} - 0.1\hat{z}_{(i-1)1}) \\
-25(\hat{z}_{(i-1)2} - 0.1\hat{z}_{(i-1)1}) \\
0
\end{bmatrix} + 
\begin{bmatrix}
0 \\
L(\gamma_i - \hat{\gamma}_i)
\end{bmatrix}
\]

\[
\hat{\gamma}_i = \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} \hat{z}_{i2} \\ \hat{z}_{i3} \end{bmatrix},
\]

where the gain matrix \( L = \begin{bmatrix} 0.4 & 0 \\ 0 & -3.3 \end{bmatrix} \).

The FIE estimators can be constructed as:

\[
\begin{bmatrix}
\dot{\hat{z}}_{s1} \\
\dot{\hat{z}}_{s2} \\
\dot{\hat{z}}_{s3}
\end{bmatrix} = 
\begin{bmatrix}
-2.5 & 25 & 0 \\
-0.25 & 2.5 & 3.077 \times 10^{-7} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
\hat{z}_{s1} \\
\hat{z}_{s2} \\
\hat{z}_{s3}
\end{bmatrix} + 
\begin{bmatrix}
0 \\
-0.0031 - 0.0058(\hat{z}_{s2} - 0.1\hat{z}_{s1})^2 \\
5u_s
\end{bmatrix} + 
\begin{bmatrix}
-25(\hat{z}_{(s-1)2} - 0.1\hat{z}_{(s-1)1}) \\
-25(\hat{z}_{(s-1)2} - 0.1\hat{z}_{(s-1)1}) \\
0
\end{bmatrix} + 
\begin{bmatrix}
0 \\
L_s(\gamma_s - \hat{\gamma}_s) \\
\Omega_{s1}\dot{\hat{\theta}}_s
\end{bmatrix}
\]

\[
\hat{\Omega}_{s1} = -2.5\Omega_{s1} - 0.25
\]

\[
\hat{\Omega}_{s2} = \left[ \begin{array}{cc} -1.5 & 0 \\ 0 & -1.7 \end{array} \right] \Omega_{s2} - \begin{bmatrix} 0.4 \\ 0 \end{bmatrix}
\]

\[
\hat{\gamma}_s = \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} \hat{z}_{s2} \\ \hat{z}_{s3} \end{bmatrix} + \hat{\theta}_s,
\]

where the gain matrix \( L_s = \begin{bmatrix} 0.4 & 0 \\ 0 & -3.3 \end{bmatrix} \).

The learning algorithm of the adaption law for adjusting \( \hat{\theta}_s \) is given by [22]:

\[
\dot{\hat{\theta}}_s = P_{\theta_s}\{\Gamma_s(C_{s1}\Omega_{s2} + I)^T\hat{\gamma}_s\},
\]

The projection operator \( P_{\theta_s} \) is used as in [23]:

\[
\dot{\hat{\theta}}_s = 0 \text{ if } \dot{\hat{\theta}}_s > M_\theta \text{ and } \dot{\hat{\theta}}_s > 0 \text{ or } \dot{\hat{\theta}}_s < -M_\theta \text{ and } \dot{\hat{\theta}}_s < 0
\]

\[
\dot{\hat{\theta}}_s = \{\Gamma_s(C_{s1}\Omega_{s2} + I)^T\hat{\gamma}_s\} \text{ if } \dot{\hat{\theta}}_s > M_\theta,
\]
where $M_\theta$ is the boundary of compact set for $\hat{\theta}_s$. For two fault cases, values of $M_\theta$ are 5 and 0.01 respectively. The value of learning rate $\Gamma_s$ is 10.

4.3 Sensor fault in vehicle 1

4.3.1 Fault Detection simulation results

Figure 4.1-4.3, show the FDE simulation results when a sensor bias $\theta_1 = 0.01$ occurs in the sensor measuring the first output of vehicle 1 at $T_1=10$ second. In Figure 4.1, Figure 4.2 and Figure 4.3, all the residuals generated by local FDEs associated with $y_{12}$ exceeded its corresponding threshold after the fault happened. The FDEs detected the faults immediately after the fault has occurred.

![Figure 4.1: Fault case 1, fault detection residual and corresponding threshold associated with the vehicle 1’s output $y_{12}$](image)

Figure 4.1: Fault case 1, fault detection residual and corresponding threshold associated with the vehicle 1’s output $y_{12}$
Figure 4.2: Fault case 1, fault detection residual and corresponding threshold associated with the vehicle 2’s output $y_{22}$

Figure 4.3: Fault case 1, fault detection residual and corresponding threshold associated with the vehicle 3’s output $y_{32}$
4.3.2 Fault Isolation simulation results

At the moment of detection, the FIEs are activated to isolate the fault. Form the simulation result shown in Figure 4.4 and Figure 4.5, the residuals generated by the FIE associated with $y_{12}$ and $y_{13}$ for vehicle 1 always remain below their thresholds. From Figure 4.6 and Figure 4.7, the two residuals generated by local FIEs associated with $y_{22}$ and $y_{32}$ exceeded the threshold after the fault happened. Thus, we conclude the fault occurs to vehicle 1.

![FIE 1 (y12)](image)

Figure 4.4: Fault case 1, fault isolation residual and corresponding threshold associated with the vehicle 1’s output $y_{12}$
Figure 4.5: Fault case 1, fault isolation residual and corresponding threshold associated with the vehicle 1’s output $y_{13}$

Figure 4.6: Fault case 1, fault isolation residual and corresponding threshold associated with the vehicle 2’s output $y_{22}$
Figure 4.7: Fault case 1, fault isolation residual and corresponding threshold associated with the vehicle 3’s output $y_{32}$

4.4: Sensor fault occurs to vehicle 2

4.4.1 Fault Detection simulation results

Figure 4.8-4.13 show the simulation results of all local FDE when a sensor bias $\theta_2 = 0.004$ occurs to the sensor measuring the first output of vehicle 2 at $T_1=10$ second.

As shown in Figure 4.8, the residual generated by FDE1 for $y_{12}$ remains below the threshold after the fault occurred. From Figure 4.9 and Figure 4.10, the residuals generated by FDE2 and FDE3 associated with $y_{22}$ and $y_{32}$ exceeded its corresponding threshold after the fault happened at $t=10.04$ seconds. From Figure 4.11, Figure 4.12 and Figure 4.13, all the residuals generated by local FDEs for $y_{13}$ always remain below its corresponding threshold after the fault occurred. FDE2 and FDE3 detected the faults immediately after the fault occurred.
Figure 4.8: Fault case 2, fault detection residual and corresponding threshold associated with the vehicle 1’s output $y_{12}$

Figure 4.9: Fault case 2, fault detection residual and corresponding threshold associated with the vehicle 2’s output $y_{22}$
Figure 4.10: Fault case 2, fault detection residual and corresponding threshold associated with the vehicle 3’s output $y_{32}$

Figure 4.11: Fault case 2, fault detection residual and corresponding threshold associated with the vehicle 1’s output $y_{13}$
Figure 4.12: Fault case 2, fault detection residual and corresponding threshold associated with the vehicle 2’s output $y_{23}$.

Figure 4.13: Fault case 2, fault detection residual and corresponding threshold associated with the vehicle 3’s output $y_{33}$. 

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4.4.2 Fault Isolation simulation result

Figure 4.14, Figure 4.15 and Figure 4.16 show the simulation results for FIEs when the sensor fault occurs to vehicle 2. Since the fault only is detected by FDE2 and FDE3, only FIE2 and FIE3 are activated to isolate the fault. As shown in the Figure 4.16, the residual associated with $y_{32}$ generated by FIE3 exceeded its threshold at approximately $t=11.2$ second. Meanwhile, the residuals generated by the FIE2 associated with $y_{22}$ and $y_{23}$ for vehicle 2 always remain below its corresponding threshold, shown in Figure 4.14 and Figure 4.15. Thus, the sensor fault in the second vehicle is correctly isolated.

![Graph showing fault isolation simulation results](image)

Figure 4.14: Fault case 2, fault isolation residual and corresponding threshold associated with the vehicle 2’s output $y_{22}$
Figure 4.15: Fault case 2, fault isolation residual and corresponding threshold associated with the vehicle 2’s output $y_{23}$

Figure 4.16: Fault case 2, fault isolation residual and corresponding threshold associated with the vehicle 3’s output $y_{32}$
Conclusions

5.1 Conclusion

In this thesis, a distributed fault detection and isolation method in [22] is developed for the automated highway system. A distributed fault detection estimator, a fault isolation estimator, and the adaptive thresholds are designed for each vehicle in the interconnected dynamic system to achieve distributed fault detection and isolation. Furthermore, the simulation results of automated highway show the effectiveness of this distributed fault detection and isolation method.

5.2 Future work

One of the most challenging goals for future research is to improve the robustness of this fault detection and isolation method. In this thesis, the only uncertainty is the modeling uncertainty. However, for many other systems, sensor noise could affect the outputs of the system. Hence, one direction for future research is how to implement this method to the system with various modeling uncertainties. Another interesting direction for future research is to extend this method to other large-scale nonlinear systems with more general nonlinearities and interconnection terms.
References


