2015

Performance Prediction of Quantization Based Automatic Target Recognition Algorithms

Matthew Steven Horvath
Wright State University

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PERFORMANCE OF QUANTIZATION BASED AUTOMATIC TARGET RECOGNITION ALGORITHMS

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

by

MATTHEW S. HORVATH
B.S., The Ohio State University, 2007
M.S., Wright State University, 2011

2015
Wright State University
I HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER MY SUPERVISION BY Matthew S. Horvath ENTITLED Performance of Quantization Based Automatic Target Recognition Algorithms BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Doctor of Philosophy

________________________________________
Brian D. Rigling, Ph.D.
Dissertation Director

________________________________________
Ramana V. Grandhi, Ph.D.
Director, Ph.D. in Engineering Program

________________________________________
Robert E. W. Fyffe, Ph.D.
Vice President for Research and Dean of the Graduate School

Committee on Final Examination

________________________________________
Brian D. Rigling, Ph.D.

________________________________________
Fred D. Garber, Ph.D.

________________________________________
Mateen M. Rizki, Ph.D.

________________________________________
Michael L. Raymer, Ph.D.

________________________________________
Mark E. Oxley, Ph.D.
Performance of Quantization Based Automatic Target Recognition Algorithms

The investment in and subsequent development of sensor technology has led to a glut of sensor data burdening the typically human-centric analysis and exploitation process. It is now more important than ever to have robust and well-studied automatic target recognition (ATR) algorithms to alleviate some of the burden on the human operators. The difficulty of designing these systems is that there are many sources of potential variation in the data, often referred to as operating conditions (OCs) and designing algorithms robust to these OCs is difficult. Additionally, analytically determining algorithm performance, often referred to as performance prediction, as a function of these OCs is important as it provides insight into when the algorithms will fail.

Quantization based ATR algorithms have shown to be robust to certain OCs. These quantization based algorithms first discretize the pseudo-continuous data into \( N_q \) discrete bins. This discretization step is important as it hypothetically reduces the variation due to certain nuisance parameters in the data and errors resulting from approximated signatures. This research focused on three algorithms: multinomial pattern matching (MPM), quantized grayscale matching (QGM), and a quantized mean-squared error approach (QMSE). The first two are known as model-based ATR algorithms and assume that in-class images are the result of realizations of a statistical model with class-conditional parametrizations. The last is a template-based algorithm which assumes a deterministic “mean” image is available with which to compare candidate targets.

The goal of this research is to develop analytic solutions, or approximations, to the performance of these algorithms in a process known as performance prediction. This analysis shows the expected performance of these algorithms as a function of the parameters used to model the OCs, which is difficult to do with empirical simulations on even a large truthed
dataset. We focus on performance prediction approaches to a baseline AWGN noise case applicable to both SAR and EO/IR imagery, an degradation case again applicable to both SAR and EO/IR imagery, and an ideal point response case applicable to SAR imagery only, and assume that these variations are conditionally independent from other sources of variation in the data.
Abbreviations and Symbols

Throughout this dissertation numerous abbreviations and symbols are used. While the definitions can be found in surrounding text, this section provides a quick reference.

List of Abbreviations

AFRL Air Force Research Laboratory
ARL Army Research Laboratory
ATR Automatic Target Recognition
AWGN Additive White Gaussian Noise
CAD Computer Aided Design
CEM Computational Electromagnetic
CFAR Constant False Alarm Rate
CV Civilian Vehicle
DARPA Defense Advanced Research Projects Agency
DM Dirichlet Multinomial
DMM Dirichlet Multinomial Mixture
DSS Defense, Security, and Sensing
EM Electromagnetic
EO Electro-optical
EOC Extended Operating Condition
FOA Focus of Attention
GM Gaussian Mixture
IID Independent and Identically Distributed
IEEE Institute of Electrical and Electronics Engineers
INID Independent but Not Necessarily Identically Distributed
IPR Individual Point Response
IR Infrared
KLD Kullback-Liebler Divergence
MC Monte Carlo
MMSE Minimum Mean Square Error
MPM Multinomial Pattern Matching
MSE Mean Square Error
MSTAR Moving and Stationary Target Acquisition and Recognition
OC Operating Condition
PEMS Predict, Extract, Match, and Score
PTM Peaky Template Matching
QGM Quantized Grayscale Matching
QMSE Quantized Mean Square Error
RV Random Variable
SAIP Semi-Automated IMINT Processing
SAR Synthetic Aperture Radar
SPIE International Professional Society for Optics and Photonics
TAES Transactions for Aerospace and Electronic Systems
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Chapter 1

Introduction

1.1 Motivation

The investment in and subsequent development of sensor technology has led to a glut of sensor data burdening the typically human-centric analysis and exploitation process. Additionally, certain on-board sensors operate in time critical environments where the sensor operator is cognitively loaded with other tasks. Therefore, it is important to have robust and well-studied automatic target recognition (ATR) algorithms to alleviate some of the burden on the human operators operating in increasingly complex environments.

Designing of robust ATR systems is complicated by the many sources of potential variation in the data, often referred to as operating conditions (OCs). Additionally, analytically determining algorithm performance, often referred to as performance prediction, as a function of these OCs is important as it provides insight into when the algorithms will fail. While these problems are encountered in data from all modalities, our work focuses primarily on data acquired from SAR (synthetic aperture radar) sensors, with additional studies utilizing electro-optical (EO) and infrared (IR) data.

Despite long being recognized as a problem, the formalization of the operating condition or extended operating condition (EOC) concept was the result of work in SAR ATR. A
A non-comprehensive list of these OCs is shown in Figure 1.1. Certain of these OCs can be either controlled or are known a priori, typically sensor parameters and certain environmental conditions, allowing the ATR algorithm to potentially exploit this given information. Other OCs, like target pose/orientation, have to first be estimated before a classification decision is made. Still other OCs, such as configuration, occlusion, and battle damage are not known a priori, cannot typically be estimated and will degrade algorithm performance.

The Moving and Stationary Target Acquisition and Recognition (MSTAR) program was a joint program between the U.S. Defense Advanced Research Projects Agency (DARPA) and the U.S. Air Force Research Laboratory (AFRL) beginning in 1994, which proposed a solution to some of these difficulties [2]. While MSTAR produced a fully automated system, including target detection, constant false alarm rate (CFAR) processing, pre-screening, and classification, we focus on the classification portion that utilized a model-based prediction approach. This allowed signatures of likely target hypotheses be generated on the fly using a computational electromagnetic (CEM) software package. The benefits were two-fold. First, an intractably large database of signatures did not have to be maintained. Second, various OCs such as sensor resolution, imaging geometry, and background could be accounted for in the predicted signature, in addition to various target configurations. If
additional measured data was available, it could then also be used to augment the training dataset.

Despite the development of mature CEM software packages, the complexity of EM scattering still requires various approximations to be made in the predicted solution, not to mention having to rely on computer aided design (CAD) models which may not accurately model the target of interest. Additionally, radar scattering is highly sensitive to small variations in aspect angle, while viewing angles are generally binned to reduce the number of simulation runs. ATR algorithms that first quantize pixel intensity values, have been shown to be robust to these concerns, and it has been hypothesized that the quantization step reduces the sensitivity to phenomena difficult to account for when defining target type/pose representations by virtue of simply eliminating this variation from the data. These algorithms are referred to as quantization based algorithms. This quantization step also reduces the complexity of the underlying statistical models, yielding models tractable to analysis as opposed to more complicated Bayesian graphical models which generally require sampling theory based simulations.

This research is a resurgence of the study of what has been termed an “ATR Theory”. The goal of “ATR Theory” is to be able to predict algorithm performance under a variety of OCs through analysis, without resorting to empirical studies of truthed datasets where fully sampling the OC space of interest is intractable [1]. In addition to an increased understanding of algorithm performance under operational conditions, this work can also allow for sensor optimization and higher level studies regarding how to best integrate ATR features into complex sensor systems and networks, as well as contingency planning if a primary sensor for some reason becomes unavailable and a less capable system must be utilized.

To illustrate, consider the typical ATR scenario shown in Figure 1.2. The center block is the ATR algorithm which may have some configuration variables, represented by the ‘Settings’ box, which is trained using available training data. As test data is input, the algorithm calculates a test statistic, for example a likelihood score or a mean-squared error,
which is then turned into a classification decision through a decision rule, illustrated here as a simple threshold test. Typical performance analysis of ATR algorithms involves taking an algorithm, training it with an established training dataset, then testing on a separate truthed dataset yielding an empirical performance measure.

In certain cases, a benchmark training dataset may be unavailable requiring expensive data collections to be performed. Additionally, variations in the testing data not accounted for in the training dataset will yield erroneous performance figures. Here we propose modelling this variation statistically, allowing an approximation to be made on the algorithm’s relevant decision statistic. In concert with the algorithm’s decision rule, this will allow for an analytic performance measure considerate of the expected variation in the data.

As the algorithms studied here are generally unstudied, even a baseline performance analysis under AWGN is a step forward. Additionally, we begin to propose simple models for variations of concern, namely a generic target degradation, which can be applicable to many scenarios like occlusion or battle damage. Cases like these are typically outside of our control, therefore they will degrade performance and the task becomes predicting and quantifying the performance loss. We finish by providing a performance prediction approach as a sensor variable changes. In this case, the variation is in our control allowing the system to be designed to optimize performance.

Our work provides both image-to-template template-to-template approaches. In the former case, the performance results are a function of an individual image’s amplitude at each pixel location and in the latter case, the performance results will be a function of pixel amplitude for each image in the training dataset. In both cases, we rely on a second-order statistical approximation to each algorithm’s decision statistic, which we show to sufficiently approximate the true distribution of the test statistic allowing for accurate analytic performance expressions.

Despite the origin of the OC and EOC concept in the study of SAR ATR, the goal of this work was to study performance prediction approaches of quantization based algorithms
in a generally sensor agnostic framework. It is noted that sensor specific concerns can and do arise that will naturally restrict certain performance prediction approaches to a certain sensor modality, which is the case in Chapter 7. Additionally, the statistical models used to express the OC of interest may also have sensor specific considerations. Therefore, we have opted to work with mainly SAR data, however extensions to other phenomenologies are relatively direct in most cases and are also presented.

The focus will be on three algorithms in particular: multinomial pattern matching (MPM), quantized grayscale matching (QGM), and a quantized mean-squared error approach (QMSE), with the primary focus being on MPM. The first two algorithms, MPM and QGM, are known as model-based ATR algorithms and assume that in-class images are the result of realizations of a generative statistical model with class-conditional parametrizations. QGM is a template-based algorithm, which assumes that a deterministic “mean” image or template exists with which to compare candidate targets. As both the statistical model and “mean” image represent an algorithm’s representation of target classes, we may generally refer to both as a “templates” moving forward.
1.2 Contributions

Currently, quantization based algorithms are sparsely studied. Therefore, this is a fruitful research area with ample opportunity for original contributions. The motivation for this work has been established in the preceding section, and here, the proposed contributions will be highlighted.

1.2.1 General $N_q$ MPM Derivation

To begin, the seminal reference on the MPM algorithm, is unavailable and the details need to be inferred from a pre-cursor algorithm known an peaky template matching (PTM) which is restricted to only two quantization levels ($N_q = 2$). Our presented derivation differs from another available in the open literature \[3\] through use of a vector form interpretation of the underlying statistical model as opposed to scalar. This yields a more statistically rigorous model at the cost of a more complicated derivation and implementation. The differences between these two algorithm implementations have been studied and shown to be roughly equivalent and that the existing scalar form is preferred for reasons to be discussed. This analysis was presented in \[4\] and is the first proposed contribution.

Second, MPM requires a tuning procedure, as will be discussed later, in order to minimize erroneous classification results \[5\]. This tuning procedure is based on a “reward minimization” strategy designed around ignoring background pixels that tend to be constant between target classes and can offset the mismatch between target pixels of two different classes. This process is not-optional and is required for the algorithm to work as designed. The original tuning strategy of the PTM algorithm was extended to the general $N_q$ case, and a specific quantization scheme was proposed. This analysis was also presented in \[4\] and is our second proposed contribution.
1.2.2 Baseline Performance Under AWGN

The third proposed contribution is the baseline analysis of algorithm performance under AWGN for all three algorithms: MPM, QGM, and QMSE. This analysis yields the classification performance of a candidate image or target chip for a given template as a function of noise power, or alternatively SNR. The initial analysis considered only two quantization levels ($N_q = 2$) and was extended to the general $N_q$ case for MPM only. This initial analysis assumed fixed quantization thresholds, and was also extended to consider an additional quantization method referred to as “uniform”. The “uniform” quantization method uses a dynamic thresholding scheme where quantile bins are determined adaptively based on the pixel values of each image.

Investigations have not yielded any published analysis on the performance for any of these algorithms under AWGN, and our analysis allows case dependent performance comparisons to be made between the three algorithms, albeit only for the $N_q = 2$ case.

The baseline AWGN analysis for all three algorithms in the $N_q = 2$ will be published in [6]. The “uniform” quantization extension verified using IR data was presented in [7]. The extension of baseline AWGN case to the general $N_q$ case was also completed for MPM and was integrated into the performance as a function of the individual point response work submitted in [8].

1.2.3 Performance Under Target Degradation

The fourth proposed contribution is the performance of the MPM algorithm under what we term “target degradation”. This is a true sensor OC of specific interest to the ATR community and one that cannot typically be accounted for using CEM simulations and therefore will yield a performance loss. Our analysis seeks to analytically predict this performance loss.

We opt for the general “target degradation” terminology to present a general approach that can be adapted to handle more specific cases of target degradation such as battle dam-
age or occlusion. In this case, certain portions of a target will be degraded according to a Dirichlet-Multinomial model, allowing an image to be classified to be modelled as a two-component mixture model. This analysis builds on the AWGN performance analysis and adds the degradation dimension parametrized by the DM parametrization and probability of degradation. This analysis also assumes the probability of degradation for each pixel is independent and identically distributed, which is a strong assumption moderated by MPM tuning rule considerations as will be discussed.

While the goal of this research is sensor agnostic performance prediction, SAR data was again used to verify the analysis. However, it is noted that this analysis can be extended to EO/IR data as well, and was for the template-to-template performance under target degradation. This was submitted in [9].

1.2.4 Performance Under Individual Point Response Variations

The fifth proposed contribution is the performance of the MPM algorithm under ideal point response (IPR) variations using SAR data. This analysis is sensor specific, as SAR is essentially sampling the radar cross section (RCS) of a scene in the spatial-frequency domain, and unlike the previous results can in no way be extended to the EO/IR sensor domains.

The extent of this sampling is dictated by the bandwidth of the SAR waveform and the size of the synthetic aperture, and these two parameters are effectively controlling the IPR of the resulting image. Therefore we study the ATR performance trade-offs between coherently processing a large aperture to yield a single-high resolution image as opposed to multiple sub-aperture images at lower resolutions.

As the mathematics describing this process are intractable, a comprised approach to performance prediction is proposed which can also be extended to the study of more complicated variations. This approach requires training MPM templates as the parameters of an OC are varied, effectively yielding samples of the performance curve as opposed to an expression for it. This work was submitted to [8].
1.3 Outline of Dissertation

The remainder of this dissertation is outlined as follows. First, the literature review is presented in Chapter 2 before the algorithm are introduced and described in Chapter 3. The datasets used to verify the performance prediction expressions are then presented in Chapter 4. The next three chapters present the performance analyses: performance under AWGN in Chapter 5, performance under degradation in Chapter 6, and performance under IPR variations in Chapter 7. Finally, the conclusions and comments on this work as a whole are presented in Chapter 8.
Chapter 2

Literature Review

The literature review for this dissertation spans a set of different topics which are organized into two broad categories: SAR ATR and performance prediction. First, we will attempt to detail the historical development of SAR ATR, focusing on the published seminal efforts and the introduction of the OC/EOC concept. We will also discuss other ATR algorithms focusing primarily on model-based classification approaches. These model-based approaches assume candidate images or target chips are realizations of statistical models with class-conditional parametrizations. Lastly, we will focus on published efforts related to the performance prediction of ATR algorithms, the primary approaches taken to the problem, as well as available studies.

2.1 SAR ATR and Sensitivity Studies

Our work involves algorithms initially studied under two seminal SAR ATR efforts originating from Defense Advanced Research Projects Agency (DARPA) in the mid-1990’s. One of these was the “Moving and Stationary Target Acquisition and Recognition” (MSTAR) program at the Air Force Research Laboratory and the other was the “Semi-Automated IMINT Processing” (SAIP) program at MIT Lincoln Laboratory [10] [11]. These programs were broad efforts, aimed at not only the development of ATR algorithms but also
complicated implementation architectures and program management strategies, however we will focus only the aspects of the programs relevant to the research described here.

The ATR algorithm utilized in the SAIP program used a mean squared error (MSE) classifier. This approach assumes that the class-conditional references are deterministic and represented by the “average” image calculated from a training set. This is generally referred to as a template based ATR approach [12]. The class associated with the template yielding the minimum MSE score for a given candidate image is then selected by the algorithm. Novak showed this approach yielded a probability of correct identification in the high 90 percentile for both 10 and 20 target classifiers using approximately 5200 test images [13], however he also showed that the template-based approach was extremely sensitive to changes in target configuration such as additional armor-plating or fuel tanks attached to the body of the vehicle. The other notable aspect of the SAIP program was the utilization of super-resolution imaging techniques which overcome the traditional shortcomings of the more efficient Fourier based processing (mainly sidelobe artifacts) at the cost of less efficient processing [14] [15]. Lastly, under a separate program the performance of the MSE algorithm as a function of resolution and polarization was also investigated [16]. The performance evaluation methodology utilized in these publications was empirical and involved testing the algorithms on a sequestered portion of a training dataset. This approach has many shortcomings which we hope to demonstrate.

MSTAR differed from SAIP and in that it took a novel approach to SAR imaging based on a tenet known as model-based prediction [17] [2]. This model-based prediction approach utilizes computational electromagnetic (CEM) software packages to approximate the electromagnetic (EM) scattering and subsequent SAR signature given a computer-aided design (CAD) model of the target. This model-based was integrated into the MSTAR framework which utilized 3 stages: a focus of attention (FOA) stage, an index stage, followed finally by a predict, extract, match, and search (PEMS) stage. The FOA module is a detection stage where candidate chips are selected to be presented to the index stage at
a constant false alarm rate. The indexer can be considered a rough classification stage designed to select the most likely candidate hypotheses. Finally, the PEMS module iteratively searches for a final classification decision using selectively more refined target hypotheses. Because the signatures were generated using CEM simulation, parameters could be changed to match sensor OCs such as position, aspect, and squint and other extended operating conditions (EOCs) such as target configuration, articulations, and occlusions tested until the best match was found [18]. This was done in a hierarchical manner, for example first identifying the target class as a tank, then a specific model, then finally configuration and articulation. This yielded an ATR package much more robust to various OCs than the one in SAIP, however the evaluation methodology still relied on empirical simulations on a truthed dataset.

The actual ATR portion of MSTAR (the match in PEMS) is very complicated and a thorough treatment is beyond the scope of this discussion. In short, a wide variety of techniques and algorithms were used to calculate the match metric between candidate image or target chip and the class-conditional reference. The algorithms we are concerned with utilize what is known as the relative amplitude feature which was shown to yield acceptable performance across a wide variety of OCs [19]. This feature results from quantizing the pseudo-continuous pixel amplitude values into $N_q$ discrete bins using a uniform quantization strategy. For example, if $N_q = 10$ bins were to be used, the lowest $1/10^{th}$ of the pixel amplitudes would receive the lowest bin quantile, the second lowest $1/10^{th}$ would receive the next, and so on. This feature is preferred for one primary reason: it tends to be invariant to small errors in pose estimation and the underlying CAD model, which generally results in large variations in absolute amplitude. The quantization step simply eliminates these hypothetically small variations in the data.

One of the ATR algorithms that utilizes the relative amplitude feature is quantized grayscale matching (QGM) [5]. It is specifically designed around the MSTAR architecture in that it requires a predicted signature resulting from the model-based CAD simulation.
as well as a set of in-class training images. These signatures are then used to populate a reduced multinomial model parametrized by the class-conditional maximum likelihood estimates of pixel quantile realizations. We say reduced because the predicted signature from the MSTAR system is used to indicate which pixels are realizations of only $N_q$ independent underlying multinomial random variables (RVs). With the parametrized models, the likelihood of a given test image can then be calculated and the decision rule is to pick the class-conditional reference maximizing the resulting likelihood score.

Another ATR algorithm utilizing the relative amplitude feature is known as multinomial pattern matching (MPM), also referred to as the Sandia algorithm [5]. This algorithm predates QGM and is the first example of quantized data being used in a SAR ATR context [5]. It is based on a pre-cursor algorithm known as peaky template matching (PTM) which can be considered a special case of the MPM algorithm with only two quantization levels ($N_q = 2$). We note that the seminal reference on MPM is unavailable and we must therefore infer its contents based on existing works in the literature [3] [20] [5] which do contain a description of the algorithm implementation but little description of the underlying model. MPM uses the same categorical/multinomial statistical model as QGM [21], however each pixel is allowed a different distribution. A Bayesian estimate of the underlying probabilities is then used to parametrize a Dirichlet-Multinomial model in the general $N_q$ case which reduces to the Beta-Bernoulli in the special case that $N_q = 2$. Its decision rule is based on a test statistic designed to be standard normal under the hypothesis that the image originated from class-conditional reference distribution [22]. Unlike, QGM which picks the most likely match in the template set, MPM tests each reference distribution independently allowing for ‘unknown’ classification decisions inherently solving the open-set problem [23] [24].

We can draw distinctions between the MSE algorithm and the QGM and MPM algorithms in that the former is a template-based algorithm assuming the class-conditional references can be modelled with a deterministic template. The latter are model-based
ATR algorithms, not be confused with the model-based prediction approach utilized by the MSTAR framework, and assume the references are statistical models with class-conditional parametrizations.

Other model based approaches have been published, however without the quantization step. A conditionally Gaussian model was utilized by O’Sullivan resulting in above 90% correct identification for a simple 4 target problem, however performance did suffer under EOCs [25]. This approach was generalized by DeVore to a conditionally Rician able to handle complex pixel values which performed nearly the same as the conditionally Gaussian, however with much greater complexity [26]. Both of these approaches utilized a Bayesian approach that required first estimating target pose, generally considered a nuisance parameter, before a classification decision could be made. In a later work, DeVore quantitatively tested other models and suggested that a quarter-power normal is perhaps the best fit to the SAR data in his experiments [27]. The conditionally Gaussian model was shown to be the most efficient in terms of complexity by Sullivan [28].

This is far from an exhaustive study of SAR ATR approaches, but they do span the subset of model-based approaches that are amenable to analytic performance prediction as will be shown below.

2.2 Performance Prediction

Some of the work in the preceding section described the performance of the algorithms. This performance is typically evaluated in an empirical manner, using the results of empirical simulations on a truthed dataset as absolute characterizations of their performance. Unsurprisingly, it was seen that testing the algorithms at or very near their training conditions yielded good classification performance, and one of the novel aspects of the MSTAR program was the idea of also testing under OCs and EOCs to more rigorously assess algorithm performance as described by Mossing and Ross [29]. It is note worthy they also
showed that the detection problem is typically much easier to characterize than the classification problem, which makes intuitive sense.

This was definitely a step forward, however the performance evaluation methodologies still relied on empirical studies utilizing a truthed testing dataset, preferably sequestered from the algorithms developers and not contained in the training datasets. Ross et al. illustrated the shortcomings of this approach, namely that a true random sampling of the OC space of interest is intractable to impossible [1]; even with tens of thousands of test images there is no guarantee that the random space is adequately sampled to generalize the results of an empirical test to a rigorous assessment of algorithm performance. Further, this rigorous assessment of algorithm performance is of upmost importance when transitioning R&D level technology to an operational capacity.

Therefore, it becomes of importance to have analytical performance expressions capable of predicting algorithm performance as a function of OC that can then be verified and compared with simulations involving collected or simulated data. This is generally a difficult task requiring analytically tractable models that are not always able to accurately represent real world phenomenon. This is one of the benefits of the quantization based algorithms in that the quantization step has a tendency to reduce or eliminate the variation due to certain nuisance parameters allowing the use of reduced or more tractable models for various OC phenomenology [3]. Additionally, model-based ATR algorithms are useful in this context as parametrized statistical models are already postulated. This allows performance prediction through a variety of means, one of which is based in information theoretic concepts, as the work of Pasala and Malas demonstrates [30] [31]. The benefit of these approaches is that bounds on classification or detection performance are known in terms of certain information theoretic quantities [32].

Another approach was utilized by Chiang who designed a feature based Bayes’ classifier based on a parametric scattering features [33], which allowed performance prediction by parametrizing the distribution of each feature as Gaussian [34] under the EOCs includ-
ing resolution, statistical model mismatch, and correlated features.

Dudgeon has authored a comprehensive survey concerning the benefits and limitations of both Bayesian and information theoretic performance prediction approaches [35]. These approaches can be considered ‘canned’ as they generally involve stuffing an ATR problem into a mathematical vehicle with well-studied routes arriving at a final rigorous performance appraisal.

Another example of a performance prediction approach is illustrated by Irving et al. who were able to generate ROC curves of a detector using peaks extracted from the data [36]. A Poisson process was used to model both clutter and target chips allowing the parametrization of a generalized likelihood ratio test (GLRT) statistic allowing ROC curves to be generated and performance to be predicted. They key contribution here was utilizing the Poisson model which did require making some assumptions that were noted to have not been thoroughly verified. This is the approach we have chosen to follow in our research and relies more on postulating a descriptive model of the OC of interest, and expressing the associated decision statistics as a function of the OC, allowing performance metrics of interest to be analytically expressed using each algorithm’s decision rule.
Chapter 3

The Algorithms

Before moving unto the primary contributions, which involves deriving expressions describing the performance of each algorithm under specific OCs (AWGN, occlusion, and IPR), the algorithms must first be introduced and described.

3.1 Peaky Template Matching

MPM and its precursor Peaky Template Matching (PTM) were developed at Sandia National Laboratories in the mid-1990s [5]. They were not initially developed for the MSTAR program, the DARPA program briefly discussed previously. However, their performance garnered attention in the ATR community. Since the algorithm’s original development, it has also been utilized for classification of 1-D high range resolution (HRR) profiles as well as applied to other sensor modalities [3] [20].

3.1.1 PTM Statistical Model

The underlying statistical model for PTM utilizes a Beta-Bernoulli model where each quantized pixel is a Bernoulli random variable (RV) with underlying probabilities distributed as Beta. Each pixel is assumed independent, however is allowed a different probability. This
model is a result of using a Bayesian estimate of the underlying probabilities a pixel will realize a given quantization level from a set of in-class training images. This is the special case where $N_q = 2$ of the more general Dirichlet-Multinomial model utilized by MPM described in Section 3.2.

To illustrate, consider Figure 3.1 where we depict a dataset of $N$ quantized images of the same type/pose class label comprised of $K$ pixels each. As each image is quantized to $N_q = 2$ values, the entries of the dataset can take on values in the set $\{0, 1\}$. PTM assumes that each column is an independent and identically distributed (IID) realization of the $K$ independent but not necessarily identically distributed (INID) underlying Bernoulli RVs. These realizations are then used to estimate the probability $p_i$ of each pixel/RV using a Bayesian approach. This process defines the class-conditional reference distribution and is effectively training the algorithm.

![Figure 3.1: General representation of a training dataset](image)

The likelihood of a sequence of IID Bernoulli realizations (each column of Figure 3.1) can be written as

$$Pr(I_i|p_i) = p_i^{N^1_i} (1 - p_i)^{N^0_i} \tag{3.1}$$

where $N^1_i = \sum_{n=1}^{N} I_i(n)$, i.e. the count of images with a value of 1 at pixel location $i$ over the number of images training images, $N$, and similarly for $N^0_i$ except it is a count of images with value 0 at pixel $i$. Therefore, a Beta prior is chosen to yield a solution to the
posterior $\Pr(p_i|I_i)$ using Baye’s method with conjugate priors. The prior Beta distribution can be written as

$$Pr(p_i|\alpha_0, \alpha_1) = p_i^{\alpha_1-1}(1 - p_i)^{\alpha_0-1} \sim Beta\{\alpha_0, \alpha_1\}$$  \hspace{1cm} (3.2)$$

up to a constant term consisting of a ratio of gamma functions. Then, $\alpha_0$ and $\alpha_1$ can be interpreted as virtual counts of 1’s or 0’s capable of encoding any a priori information on the underlying probability. The developers of MPM chose a non-informative prior, $(\alpha_0, \alpha_1) = (1, 1)$, typically known as Laplace’s prior. It can then be shown that by conditioning on the set of the $N$ training images, the posterior distribution becomes

$$Pr(p_i|I_i) = p_i^{N_1}(1 - p_i)^{N_0}$$  \hspace{1cm} (3.3)$$

up to a constant and with $(\alpha_0, \alpha_1) = (1, 1)$, known to be Beta distributed yielding the Beta-Bernoulli model where

$$X_i \sim Bernoulli\{p_i\}$$

$$p_i \sim Beta\{N_1^i + 1, N_0^i + 1\}$$  \hspace{1cm} (3.4)$$

where $X_i$ is the Bernoulli RV representing the pixel realization at index $i \in [1 \ldots K]$ and $N_1^i$ and $N_0^i$ are the counts of realizations of 1’s and 0’s respectively at pixel $i$ summed over the $N$ training images.

Therefore, the independent Beta-Bernoulli RVs representing the class-conditional template distribution are fully parametrized by the counts of 1’s and 0’s for each pixel calculated across the $N$ training images. The derivation of the PTM test statistic in the following sections utilizes equivalent variables $\hat{p}_i$ and $N$, where $\hat{p}_i$ is the empirical probability of a
pixel realizing the high quantization level defined as

$$\hat{p}_i = \frac{N_i}{N}, \ i \in [1 \ldots K]$$

(3.5)

and we note that $\hat{p}_i$ and $N$ is an equivalent parametrization to $N^1_i$ and $N^0_i$ as $N \hat{p}_i = N^1_i$ and $N - N \hat{p}_i = N^0_i$.

### 3.1.2 PTM Test Statistic and Classification Decision

PTM bases its classification decision on a summed penalty statistic, engineered to be approximately normal under the hypothesis that the image to be classified originated from the class-conditional template distribution given in (3.4) [5] [3].

The final PTM test statistic can be calculated by first defining the quadratic penalty function

$$t_i = (X_i - \hat{p}_i)^2, \ i \in [1 \ldots K]$$

(3.6)

where $X$ is the quantized image being compared to the template distribution and the pixels are indexed by the variable $i \in [1 \ldots K]$, while $\hat{p}_i$ is the empirical probability determined by the number of training images and the counts of each quantile defined in (3.5). This penalty can be interpreted as the “un-likelihood” of a candidate image realizing a specific quantization level conditioned on the underlying empirical probability parametrizing the underlying Beta probability distribution [5].

PTM posits a null hypothesis under which $X$ is assumed to be a realization of the previously mentioned Beta-Bernoulli RVs modelling a given target class/pose template. Therefore, the test determines whether our test image $X$ originated from the previously defined distribution (the null hypothesis) or did not (the alternative hypothesis).

In order to test this hypothesis, PTM first normalizes the penalty associated with each
pixel by its first and second order moments:

\[ b_i = \frac{t_i - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} \quad i \in [1 \ldots K] \quad (3.7) \]

Realizing that under the null hypothesis, \( t_i \) is a quadratic function of the fully parametrized Beta-Bernoulli RV defined in (3.4) for a given class, the mean and variance terms required for normalization can be calculated as

\[ E\{t_i\} = \frac{1}{N+2} \left(1 - 2\hat{p}_i + N\hat{p}_i + 2\hat{p}_i^2 - N\hat{p}_i^2\right) \quad (3.8) \]

\[
\text{Var}\{t_i\} = \frac{1}{(N+2)(N+3)} \left[-4N^2\hat{p}_i^4 + 8N^2\hat{p}_i^3 + \hat{p}_i^2(-5N^2 + 4N + 4) + \hat{p}_i(N^2 - 4N - 4) + N + 1 \right] \quad (3.9)
\]

using the known moments of the fully parametrized Beta-Bernoulli RVs modelling the in-class template distribution.

Next, PTM sums these per-pixel penalties over all pixels in the image and normalizes them yielding the summed test statistic \( B \)

\[ B = \frac{1}{\sqrt{K + \hat{C}}} \sum_{i=1}^{K} b_i \quad (3.10) \]

where \( K \) is the number of pixels in the image, \( \hat{C} \) is the sum of the pixel-to-pixel covariances which will be defined in (3.12) and (3.13), and \( b_i \) was defined in (3.7). As \( b_i \) is approximately normal due to normalization through the second order, \( B \) can be considered the sum of standard normal RVs. In the case that \( X_i \) did not originate from the in-class distribution, the sum will still be normal by the central limit theorem (CLT), however we can say nothing of its mean and variance at this point. The normalization terms are designed to yield a test statistic that is standard normal if \( X_i \) originated from the in-class distribution, as the
mean of a sum of RVs is the sum of the means and the variance of a sum of correlated RVs is equal to the sum of the variance of each RV, $\sum_{i=1}^{K} \sigma_i^2 = K$, plus twice the sum of the RV-to-RV covariance terms, $\hat{C}$ [38].

Therefore, under the null hypothesis that the candidate image originated from a Beta-Bernoulli model defining the in-class distribution, the final MPM test statistic will be approximately standard normal.

$$B \sim Normal\{0, 1\} \tag{3.11}$$

and a one-sided Z-test is used to make the final classification decision [39]. The Z-test is implemented to reject large positive instantiations of the test statistic, as penalties will shift the mean of the null distribution to the right, as will be seen in Section 3.1.3. The rejection region of the Z-test is illustrated in Figure 3.2 for a critical value of $\alpha_z = 0.05$.

![Standard Normal PDF and Critical Region for $\alpha = 0.05$](image)

Figure 3.2: One sided Z-test utilized by PTM and MPM. Under the null hypothesis, the PTM test statistic in (3.16) is designed to be standard normal. Candidate target chips yielding values of the PTM test statistic in the blue region are interpreted as being unlikely under the null hypothesis at the specified level of significance, and subsequently rejected.

In implementation, the sum of the pixel-to-pixel covariances, $\hat{C}$, is estimated from the training data shown in Figure 3.1 using the sample covariance matrix. The sample
covariance matrix can be written as
\[ C = \frac{1}{N - 1} \sum_{n=1}^{N} (I_i(n) - \bar{I})^T (I_i(n) - \bar{I}) \]  
(3.12)

where \( I_i \) is the row associated with the \( n^{th} \) training image, \( n \in [1 \ldots N] \), and \( \bar{I} \) is the sample mean vector calculated across the \( N \) training images. This yields a \([K \times K]\) matrix containing the pixel-to-pixel covariance terms. Under the null-hypothesis, the unit-variance contributions on the diagonal are contained in the \( K \) term in (3.10) and the sum of the off-diagonal terms defines
\[ \hat{C} = \sum_{i=1}^{K} \sum_{j\neq i} C_{ij}. \]  
(3.13)

As \( C \) is symmetric, the sum is often calculated as twice the sum of the upper triangular components. It is noted that the calculation of \( C \) can be computationally expensive for large images, and we have seen that downsampling the training images can speed up the process without adversely effecting algorithm performance up to a factor of 4. We did not experiment with down sampling factors higher than that, and our empirical studies are discussed more comprehensively in Section 5.2.2. Additionally, it is noted that certain algorithms disregard the \( \hat{C} \) term entirely \[3\].

### 3.1.3 PTM as a Clutter Rejection Algorithm

It is important to note, that the PTM classifier was the third stage of a system that first detected potential targets in a ‘Focus of Attention (FOA) module’, then secondly segmented out the target chips, before passing them onto the PTM stage for classification \[5\]. The system was designed to pass chips at a constant false alarm rate to the classification stage that was PTM and therefore non-target/clutter chips would be processed as targets.

In order to understand how the PTM algorithm would interpret these non-target, clutter
chips, the mean of the normalized per-pixel penalty given in (3.7) can be calculated as

$$E\{b_i\} = \Pr(X_i = 1) \frac{(1 - \hat{p}_i)^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} + \Pr(X_i = 0) \frac{\hat{p}_i^2 - E\{t_i\}}{\text{Var}\{t_i\}}$$  \hspace{0.5cm} (3.14)$$

and the variance as

$$\text{Var}\{b_i\} = \Pr(X_i = 1) \left( \frac{(1 - \hat{p}_i)^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} - E\{b_i\} \right)^2 + \Pr(X_i = 0) \left( \frac{\hat{p}_i^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} - E\{b_i\} \right)^2$$  \hspace{0.5cm} (3.15)$$

where $E\{t_i\}$ was given in (3.8), $\text{Var}\{t_i\}$ in (3.9), and $\Pr(X_i = 1)$ and $\Pr(X_i = 0)$ are the probabilities that a clutter pixel will realize a high or low quantization level.

Substituting the appropriate values for $\Pr(X_i = 1)$ and $\Pr(X_i = 0)$ values, determined from the clutter statistics, into (3.14) yields negative expected per-pixel penalties (i.e. $b_i$ terms) associated with pixels having specific empirical probabilities. Recalling that the PTM test statistic is equal to a normalized sum of these penalties and a one-sided $Z$-test is to be used, it is evident that these negative penalties could potentially lead to a false alarm, where target chips consisting solely of clutter are not rejected by the $Z$-test. For this reason, these pixels must be ignored when calculating the overall MPM statistic, and this is what we refer to when mentioning the inherent clutter suppression of the PTM algorithm; i.e. the set of salient empirical probabilities is chosen to avoid rewarding (applying a negative penalty) for clutter or background pixels. Therefore, the summed test statistic calculation in (3.10) is modified to yield

$$B = \frac{1}{\sqrt{K_{\text{peak}} + \tilde{C}_{\text{peak}}}} \sum_{i \in \Gamma_{\text{peak}}} b_i$$  \hspace{0.5cm} (3.16)$$

where $\Gamma_{\text{peak}}$ is the set of pixels with empirical probabilities defined in (3.5) such that $E\{b_i|\text{Clutter}\}$ is non-negative or $\Gamma_{\text{peak}} \triangleq \{i; E\{b_i|\text{Clutter}\} > 0\}$. $K_{\text{peak}}$ is the number of
elements in the set, $|\Gamma_{\text{peak}}|$ and $\tilde{C}_{\text{peak}}$ is the sum of the covariance matrix entries in (3.12) associated with peak pixels only. It is noted that strictly greater than is important as the $\text{Var}\{t_i\} = 0$ for $\hat{p}_i = 0.5$, creating divide by zero issues when calculating the summed test statistic in algorithm implementation. It is also noted that empirical probabilities of greater than .5 is a rough rule of thumb that works in many cases, $\Gamma_{\text{peak}} \triangleq \{i; \hat{p}_i > 0.5\}$ [5], and (3.14) is a function of the number of training images and clutter statistics, therefore care should be taken that the prescribed value is valid for specific cases.

The authors of the QGM algorithm were well aware of these considerations saying, “the Sandia approach is applied for clutter rejection rather than for target classification. In particular, the published Sandia work focuses on a binary classification problem, where the objective is to discriminate between a single target type of interest (whose pose is unknown a priori) and clutter,” which we deem to be a valid criticism as the PTM process is essentially considering a binary decision test, with no mention of how to interpret a target chip that tests positive to originating from two unique template distributions [5]. However, we add that the natural choice is to select the template yielding the minimum PTM score and we have seen PTM to be a very effective algorithm for image classification. We also note that the author’s of QGM relied on references we have been unable to obtain.

To summarize, PTM assumes the class-conditional reference distributions are fully parametrized by $N$, the number of training images, and $\hat{p}_i$, the empirical probabilities each pixel will realize a high quantization value.

### 3.2 Multinomial Pattern Matching

The previous section described PTM where only two quantization levels are used ($N_q = 2$). Here we extend the model utilized by PTM to the general $N_q$ case. This yields a Dirichlet-Multinomial model where the Multinomial realizations are also known as a general categorical distribution [21]. This is a simplification of the Multinomial model which is typically
considered to be a distribution of counts. As our model is assuming a single draw, this model can be interpreted in two different ways: the 1-of-K vector form where only 1 entry of the \([N_q \times 1]\) vector can be one while the rest of the entries are zero or the scalar form where the draw is assumed an index indicating which quantile was realized. This latter form is what the published MPM implementations use [3] [20]. Both interpretations will be contrasted later in Section 5.4.2.

### 3.2.1 MPM Statistical Model

The underlying statistical model utilized by MPM is a Dirichlet-Multinomial (DM) model where each quantized pixel is assumed a realization of an independent but not necessarily identically distributed (INID) Multinomial random variable (RV) with the underlying probabilities distributed as Dirichlet. This model is a result of using a Bayesian estimate of the underlying probabilities a pixel will realize a given quantization level from a set of in-class training images [40]. In the general \(N_q = 2\) case, this model reduces to the Beta-Bernoulli model discussed previously.

To illustrate, we consider the training procedure for MPM as illustrated in Figure 3.1. Each row is a quantized and flattened training image indexed by pixel location \(i \in [1 \ldots K]\) originating from a training dataset of the same type/pose class label consisting of \(N\) training images. These images are quantized to \(N_q\) values yielding labels in the set \(\{1 \ldots N_q\}\). MPM then assumes that each column is composed of IID realizations of the \(K\) INID underlying multinomial RVs.

If the underlying probabilities, \(\vec{p}_i\), where \(\vec{p}_i\) is an \([N_q \times 1]\) vector, were known the likelihood of a pixel realizing a specific quantization level can be written as

\[
\Pr(I | \vec{p}) = \prod_{q=1}^{N_q} p_q^{N^q}
\]

where the dependence on pixel \(i\) has been suppressed and \(N^q = \sum_{n=1}^{N} \delta(I(n) - q)\), i.e.
the counts of quantile realization $q$ at pixel location $i$ across the $N$ training images. As these probabilities are unknown, they must first be estimated and MPM utilizes a Bayesian approach. A Dirichlet prior is chosen to yield a solution to the posterior $\Pr(\vec{p}|I)$ using Baye’s method with conjugate priors. The prior Dirichlet distribution can be written as

$$
\Pr(\vec{p} | \vec{\alpha}) = \prod_{q=1}^{N_q} p_{q}^{\alpha_q - 1} \sim \text{Dirichlet}\{\vec{\alpha}\}
$$

(3.18)

up to a constant term consisting of a ratio of gamma functions. Then, $\vec{\alpha}$ can be interpreted as virtual counts of quantile realizations capable of encoding any a priori information on the underlying probabilities. The available reference on MPM leaves this as a general tuning parameter that can be set to any positive value [3], however it has been found choosing $\vec{\alpha} = \vec{1}$, where this is again an $[N_q \times 1]$ vector typically known as Laplace’s prior, is an effective default selection. It can then be shown that by conditioning on the set of the $N$ training images [37], the posterior distribution on the underlying probabilities becomes

$$
\Pr(\vec{p}|I) = \prod_{q=1}^{N_q} p_{q}^{N_q + \alpha_q - 1}
$$

(3.19)

up to a constant known to be Dirichlet distributed yielding the Dirichlet-Multinomial (DM) model where

$$
\vec{X}_i \sim \text{Multinomial}\{\vec{p}_i\}
$$

$$
\vec{p}_i \sim \text{Dirichlet}\{\vec{N}_i^{q} + \vec{\alpha}\}
$$

(3.20)

where $\vec{X}_i$ is a vector valued RV representing the pixel realization at index $i \in [1 \ldots K]$ and $\vec{N}_i^{q}$ is a vector counts of each of the $N_q$ quantile realizations at pixel $i$ calculated over the $N$ training images. Therefore, the independent Multinomial RVs representing the class-conditional template distributions are fully parametrized by the counts of quantile realizations for each pixel calculated across the $N$ training images and the prior parameter

27
\( \vec{\alpha} \). Alternatively, the equivalent variable \( \vec{p}_i \) can be used and calculated as

\[
\vec{p}_i = \frac{N^q_i}{N}, \quad i \in [1 \ldots K], \quad q \in [1 \ldots N_q].
\] (3.21)

In the following section, MPM requires the calculation of two normalization terms, specifically the mean and variance of a quadratic penalty function which requires calculating the moments of the \( \vec{X}_i \) term in (3.20). While the multinomial distribution has a dependence on the number of draws, the MPM hypothesis test is designed to determine whether a single image originated from a given class conditional DM template, or did not, therefore we assume a single draw. The mean of \( X_i \) can be written as \( \mu \)

\[
\mu_{\vec{X}_i} = \frac{N^q_i + \alpha}{N + N_q \alpha} = \frac{N \vec{p}_i + \alpha}{N + N_q \alpha} = \vec{p}_i
\] (3.22)

and the variance of \( X_i \) as

\[
\text{Var}\{\vec{X}_i\} = \frac{(N^q_i + \alpha)(1 - N^q_i + \alpha)}{N + N_q \alpha}
\] (3.23)

where the denominator of these expressions results from \( \sum_{q=1}^{N_q} N^q + \alpha \). It is noted that (3.22) is the minimum mean-squared error (MMSE) estimate of the underlying probabilities conditioned on the observed counts or empirical probabilities in the training dataset, which will be utilized later in Section 3.2.3. Otherwise, this notation assumes the vector or 1-of-K form of the Multinomial distribution in the case of a single draw, sometimes referred to as a categorical distribution, and (3.22) and (3.23) are \([N_q \times 1]\) vectors \([21]\). In the more general multinomial cases, these vectors describe the moments of a distribution of counts across an arbitrary number of trials, and we note that assuming only a single trial simplifies things greatly as will be seen in Section 3.2.3 where the cross-terms can be disregarded and the computation of the higher order moments are equal to the first order moment.

The MPM template distributions are then fully parameterized by \( N, \vec{p}_i \) for \( i \in [1 \ldots K] \),
and $\hat{C}$ which was defined in (3.12). It is again noted that the certain implementations assume $\hat{C} = 0$ and do not include it in the calculation [3].

### 3.2.2 MPM Test Statistic and Classification Decision

MPM bases its classification decision on a summed penalty statistic, engineered to be approximately normal under the hypothesis that the image to be classified originated from the class-conditional template distribution given in (3.20). This discussion assumes the 1-of-K vector form and will be contrasted with the scalar form available in the literature in Section 3.2.3 [3]. Despite their implied similarities, these implementations do differ but yield approximately the same test statistics and performance as will be seen in Section 5.4.2.

Again, we assume the vector form of DM realization where $\vec{X}_i$ is a $[N_q \times 1]$ vector composed of all zeros except for 1 in the place of the $q^{th}$ row. For example, if an $N_q = 4$ scheme were used leading to quantiles in the set $\{1, 2, 3, 4\}$, and a pixel realized a quantile value of 2 then $\vec{X}_i = [0 \ 1 \ 0 \ 0]^T$. The vector $\tilde{p}_i$ consists of the empirical probabilities of each quantile observed in the training dataset as defined in (3.21). In both cases, we choose to use the convention that the top row refers to the lowest quantile, however this affects only the implementation and not the mathematical development.

The final MPM test statistic can be calculated by first defining the quadratic penalty function

$$ t_i = (\vec{X}_i - \tilde{p}_i)^2, \ i \in [1 \ldots K] $$$$ = \vec{Q}_i^T \vec{Q}_i $$

(3.24)

where by defining the term $\vec{Q}_i = \vec{X}_i - \tilde{p}_i$, we see that the error term is the squared magnitude of the difference vector between the observed realization and the empirical probabilities. This penalty has also been interpreted as the “un-likelihood” of a candidate image realizing a specific quantization level conditioned on the underlying empirical probabilities.
parametrizing the prior Dirichlet distribution, however this interpretation is more intuitive in the $N_q = 2$ case where there is only a single empirical probability value to deal with or the scalar case given in Section 3.2.3 as opposed to the vector valued form in the general $N_q$ case given here [5].

MPM posits a null hypothesis under which $X_i, i \in [1 \ldots K]$, is assumed to be a realization of the previously mentioned DM RVs modelling a given target class/pose template. Therefore, the test determines whether our test image $X$ originated from the previously defined distribution (the null hypothesis) or did not (the alternative hypothesis).

In order to test this hypothesis, MPM first normalizes the penalty associated with each pixel by its first and second order moments:

$$b_i = \frac{t_i - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}}, \ i \in [1 \ldots K]. \quad (3.25)$$

Assuming that the test image $X_i, i \in [1 \ldots K]$, originated from the class-conditional DM model defined in (3.20), $t_i$ is then a quadratic function of the fully parametrized RV and the mean term required for normalization can be calculated as

$$E\{t\} = E\{(\vec{X} - \vec{p})^T (\vec{X} - \vec{p})\}$$

$$= \sum_{q=1}^{N_q} E\{(X_q - p_q)^2\}$$

$$= \sum_{q=1}^{N_q} E\{X_q^2\} - 2E\{X_q\} \hat{p}_q + \hat{p}_q^2 \quad (3.26)$$

where the dependence on pixel index $i$ has been suppressed. Realizing that the first and all higher order moments of the DM RV $X$ are equal (for the case of a single model realization only) and specified in (3.22) this term becomes

$$E\{t\} = \sum_{q=1}^{N_q} \hat{p}_q(1 - 2\hat{p}_q) + \hat{p}_q^2. \quad (3.27)$$
Calculating the variance term is slightly more involved and starting with the identity

\[ \text{Var}\{t\} = E\{(t - E\{t\})^2\} \]

\[ = E\{t^2\} - E\{t\}^2 \]  

(3.28)

we see the \(E\{t\}\) term was just given in (3.27) and we are left with calculating \(E\{t^2\}\).

Beginning with

\[ t^2 = \left(\sum_{q=1}^{N_q} Q_q^2\right)^2 \]

\[ = \sum_{q=1}^{N_q} Q_q^2 + \sum_{q=1}^{N_q} \sum_{r=1}^{N_q} Q_q^2 Q_r \]

(3.29)

then adding the expectation and expanding the \((d_1)\) term in (3.29) yields

\[ E\{d_1\} = E\left\{\sum_{q=1}^{N_q} Q_q^2\right\} \]

\[ = \sum_{q=1}^{N_q} E\{X_q^4\} - 4E\{X_q^3\} \hat{p}_q + 6E\{X_q^2\} \hat{p}_q^2 - 4E\{X_q\} \hat{p}_q^3 + \hat{p}_q^4 \]

(3.30)

\[ = \sum_{q=1}^{N_q} \tilde{p}_q (1 - 4 \hat{p}_q + 6 \hat{p}_q^2 - 4 \hat{p}_q^3) + \hat{p}_q^4 \]

where again the first and all higher order moment terms of the DM RV \(X_q\) were given in
Similarly for the \((d_2)\) term in (3.29)

\[
E\{d_2\} = E\{\sum_{q=1}^{N_q} \sum_{r=1}^{N_q} Q_q^2 Q_r^2\} \\
= E\{\sum_{q=1}^{N_q} \sum_{r=1}^{N_q} (X^2_q - 2X_q\hat{p}_q + \hat{p}_q)(X^2_r - 2X_r\hat{p}_r + \hat{p}_r)\} \\
= \sum_{q=1}^{N_q} \sum_{r=1}^{N_q} \hat{p}_r^2 E\{X^2_q\} - 2\hat{p}_q\hat{p}_r^2 E\{X_q\} + \hat{p}_q^2 E\{X^2_r\} + 2\hat{p}_q\hat{p}_r E\{X_r\} + \hat{p}_q^2 \hat{p}_r^2 \\
= \sum_{q=1}^{N_q} \sum_{r=1}^{N_q} \hat{p}_r^2 \hat{p}_q - 2\hat{p}_q\hat{p}_r^2 \hat{p}_q + \hat{p}_q^2 \hat{p}_r + 2\hat{p}_q\hat{p}_r \hat{p}_r + \hat{p}_q^2 \hat{p}_r^2 \\
(3.31)
\]

where all the expectations of cross-terms went to zero; in the case of a single realization of the DM model only one entry of \(\vec{X}\) can be one therefore the expected value of the product of any two different entries will be zero. Again, this is not the case with the general DM model with an arbitrary number of realizations. The expectations for all the first and higher order moments were given in (3.22). This yields the final expression for the \(E\{t_i^2\}\) as

\[
E\{t^2\} = \sum_{q=1}^{N_q} \hat{p}_q (1 - 4\hat{p}_q + 6\hat{p}_q^2 - 4\hat{p}_q^3) + \hat{p}_q^4 \\
+ \sum_{q=1}^{N_q} \sum_{r=1}^{N_q} \hat{p}_r^2 \hat{p}_q - 2\hat{p}_q\hat{p}_r^2 \hat{p}_q + \hat{p}_q^2 \hat{p}_r + 2\hat{p}_q\hat{p}_r \hat{p}_r + \hat{p}_q^2 \hat{p}_r^2 \\
(3.32)
\]

and subsequently the variance of the per-pixel penalty \(\text{Var}\{t_i\}\) from (3.27) and (3.28). Again, the \(\hat{p}\) is the MMSE estimate of the underlying probability specified in (3.22).

Next, MPM sums these per-pixel penalties over all pixels in the image and normalizes them yielding the summed test statistic \(B\)

\[
B = \frac{1}{\sqrt{K + C}} \sum_{i=1}^{K} b_i \\
(3.33)
\]
where $K$ is the number of pixels in the image, $\hat{C}$ was discussed in Section 3.1.2, and $b_i$ was defined in (3.25). As $b_i$ is approximately normal due to normalization through the second order, $B$ can be considered the sum of standard normal RVs. In the case that $X_i$ did not originate from the in-class distribution, the sum will still be normal by the central limit theorem (CLT), however we can say nothing of its mean and variance without formulating a more specific alternate hypothesis. MPM does not and must test each candidate image against each template in the library. If no match is found, an “unknown” decision could be declared effectively solving the open set problem [23] [24]. If multiple matches are found, the logical result is to choose the template that yielded the smallest MPM test statistic.

Therefore, under the null hypothesis that the candidate image originated from the class-condition DM model, the final MPM test statistic will be approximately standard normal

$$B \sim \text{Normal}\{0, 1\}$$  \hspace{1cm} (3.34)

and a one-sided Z-test is used to make the final classification decision [39]. The Z-test is implemented to reject large positive instantiations of the test statistic, as penalties will shift the mean of the null distribution to the right, as will be described more thoroughly in Section 3.2.4. The rejection region of the Z-test is illustrated in Figure 3.2 for a critical value of $\alpha_z = 0.05$.

The normalization terms are designed to yield a test statistic that is standard normal if $X$ originated from the in-class distribution and the mean of a sum of RVs is the sum of the means and the variance of a sum of correlated RVs is equal to the sum of the variance of each RV, $\sum_i^K 1 = K$, plus twice the sum of the RV-to-RV covariance terms, $\hat{C}$ [38]. The additional $1/\sqrt{K + \hat{C}}$ weighting term in (3.33) is accounting for this contribution. In implementation, $\hat{C}$ is estimated from the training data shown in Figure 3.1 using the sample covariance matrix and we mention that this term is often assumed to be small and not
included in the calculation of the summed test statistic in [3]. We do note that if images are
downsampling for speed-up, care should be taken that a large enough number of salient pix-
els as discussed in Section 3.2.4 exist in the template to yield a meaningful result. Despite
the normalization terms which certainly help, the Gaussian assumption of (3.34) is still bet-
ter satisfied with more pixels due to a central limit theorem like argument and the reward
minimization process has been shown to yield “degenerate” parametrizations especially for
highly varying data as seen in Section 5.1.2.

3.2.3 Scalar Form of MPM Test Statistic

The formulation of the MPM test statistic above differs from that presented elsewhere in
the literature [3], which uses a different penalty function than the above formulation. The
author’s instead chose to define the penalty function as

\[ t_{q,i} = (1 - \hat{p}_{q,i})^2 \]  

(3.35)

where \( q \in [1 \ldots N_q] \) and \( k \in [1 \ldots K] \). This penalty can be easily understood as an \( [N_q \times K] \) matrix of penalties, and the quantile value of the test image at a certain pixel, \( X_i \), then
gives the index \( q \) into this matrix at pixel location \( i \). Therefore, \( q \) in this implementation is
the RV.

The associated normalization terms of (3.27) and (3.32) can then be calculated as [3]

\[ \hat{\mu}_i = \sum_{q=1}^{N_q} \bar{p}_{q,i} (1 - \hat{p}_{q,i})^2 \]  

(3.36)

and

\[ \hat{\sigma}_i^2 = \sum_{q=1}^{N_q} \bar{p}_{q,i} (1 - \hat{p}_{q,i})^4 - \hat{\mu}_i^2 \]  

(3.37)

where \( \bar{p}_{q,i} \) was given in (3.22). These terms are calculated from the definition of the \( n^{th} \)
moment of a discrete distribution, \( \sum x^n P(x) \), where the MMSE estimate of the underlying probabilities was used in lieu of the unknown actual probabilities and hence the \( \hat{\cdot} \) notation. As in the previous section the final test statistic is

\[
B = \frac{1}{\sqrt{K}} \sum_{i=1}^{K} t_{q,i} - \hat{\mu}_i \sum_{i=1}^{K} \hat{\sigma}_i \\
= \frac{1}{\sqrt{K}} \sum_{i=1}^{K} b_i
\]  

(3.38)

assumed to be distributed as standard normal under the hypothesis that \( X_i \) originated from the class-conditional template distribution parametrized by the empirical probabilities calculated across the training dataset. We note, this implementation does not account for any correlation present in the data and not accounted for in the model that was previously handled by the \( \hat{C} \) term in (3.33), although it can and should be included if it is found to be significant in the training data.

We note that this differs significantly from the penalty function given in the previous section which uses the 1-of-K vector formulation of the multinomial/categorical distribution [21]. This derivation assumes we draw a specific scalar quantile realization and account for the penalty associated only that value. Because of this, the previous formulation in (3.24), becomes

\[
t_{q,i} = (1 - \hat{p}_{q,i})^2 + \sum_{r=1}^{N_q} \hat{p}_{r,i}^2
\]

in notation consistent with this formulation. It is quite clear that this penalty is not equivalent to that given in (3.35) due to the additional summation term. We will refer to the MPM derivation in Section 3.2.2 as the vector form and this derivation as the scalar form of the MPM algorithm.
3.2.4 MPM as a Clutter Rejection Algorithm

This is a very important topic with respect to these algorithms and can be considered to be algorithm tuning. As a one-sided Z-test is used to make a classification decision, only positive penalties will lead to a test chip being rejected by the algorithm. What happens in practice, is that rewards (negative penalties) can be applied for background pixels, offsetting any penalties due to target mismatch and yielding false positive classification decisions. This phenomenon will be demonstrated in Section 5.4.2. Therefore when we refer to algorithm tuning or reward minimization, we are referring to selecting the set of pixels to include in the match score calculations of \( (3.33) \) or \( (3.38) \) based on their underlying empirical probabilities. These empirical probabilities are selected based on the expected value of the normalized per-pixel penalty for the background statistics as established in Section 3.1.3.

In accordance with this reward minimization strategy, the summed test penalty statistic of \( (3.33) \) or \( (3.38) \) is modified

\[
B = \frac{1}{\sqrt{K_{\text{peak}} + C_{\text{peak}}}} \sum_{i \in \Gamma_{\text{peak}}} b_i \quad (3.39)
\]

where \( \Gamma_{\text{peak}} \) is the set of pixels with empirical probabilities defined in \( (3.21) \) such that \( E\{b_i|\text{Clutter}\} \) is non-negative or \( \Gamma_{\text{peak}} \triangleq \{i; E\{b_i|\text{Clutter}\} > 0\} \). In practice, the statistics of the background pixels can be estimated from the training data.

For our studies, We choose our quantization thresholds with respect to the largest scatterer in the training dataset. The peak sidelobe level of this scatterer is then used as the lowest threshold value and any pixel values below the peak sidelobe level of the strongest scatterer will then be quantized to the lowest quantile level. For the AFRL civilian vehicle dataset, the targets are on a flat, perfectly conducting ground plane effectively guaranteeing the background RCS contribution will be minimal and we are primarily dealing with noise due to sidelobe interference \( [41] \) which will generally be below the peak sidelobe level.
determining the minimum quantization threshold. Thus, we assume background pixels will yield \( X_i = [1 \ 0 \ldots \ 0]^T \) with a probability of 1. For the development in Section 3.2.2, this yields the set of “peak” pixels

\[
\Gamma_{\text{peak}} = \{ i; (1 - \hat{p}_{1,i})^2 + \sum_{q=2}^{N_q} \hat{p}_{q,i}^2 - E\{t_i\} > 0 \} \quad \ldots \text{Vector Form} \quad (3.40)
\]

where \( E\{t_i\} \) was given in (3.27) and we have used the convention that the top row of \( X_i \) and \( \hat{p}_i \) refers to the lowest quantile. For the development in Section 3.2.3 this set of peak pixels reduces to

\[
\Gamma_{\text{peak}} = \{ i; (1 - \hat{p}_{1,i})^2 - \hat{\mu}_i > 0 \} \quad \ldots \text{Scalar Form} \quad (3.41)
\]

where \( \hat{\mu}_i \) was given in (3.36).

Lastly, while not related to algorithm tuning, we will note that the standard deviation normalization term in the denominator of (3.32) and (3.37) can evaluate to zero for certain values of \( \vec{\hat{p}}_i \) and this must be accounted for in implementation to prevent a divide by zero issue in calculating the summed penalty statistics of (3.33) and (3.38).

### 3.3 Quantized Grayscale Matching

QGM was an ATR algorithm developed under DARPA’s MSTAR program [5] utilizing the relative amplitude feature. While, MPM also utilizes this feature, its heritage is elsewhere and QGM is specifically designed around the MSTAR framework requiring modifications to be used without it. The MSTAR signature predictor yields a single predicted image based on a computational electromagnetic (CEM) simulation, which is modified to account for imaging geometry, terrain, orientation, etc. [17] [2] and QGM uses this predicted signature as an indicator vector in the training process.

Therefore, QGM relies on a similar set of truthed data for training as in Figure 3.1.
however with the additional predicted signature we will refer to as $P_i$, $i \in [1 \ldots K]$ mentioned above. The training data (of the same class as $P$) will be noted as $I_i(n)$, consistent with that used in the preceding section, for $i \in [1 \ldots K]$ and $n \in [1 \ldots N]$ where again $N$ refers to the number of training images and $K$ the number of pixels. It is also noted that $P_i$ has a hidden dependency on the type/pose class label and the notation $P[i; H]$ will be alternatively used for clarity.

We note that the following discussion follows [5] with additional discussion added where necessary.

### 3.3.1 QGM Statistical Model

While MPM utilized a more general model where each pixel is allowed a different distribution, QGM assumes that each pixel in the predicted signature $P[i; H]$ that realizes a specific quantization level originated from the same underlying RV, $\Gamma_j \triangleq \{i; P[i; H] = j\}$, or in other words $P_i$ is an indicator vector determining which pixels in the training set are IID realizations of which of the $N_q$ INID RVs. As in MPM, each RV is distributed as multinomial in the general case with $N_q > 2$ which reduces to Bernoulli in the case that $N_q = 2$. This is the case considered here and the training process involves estimating the underlying probability of success (realization of a 1) of each of these random variables, which we will refer to as $X_0$ and $X_1$

\[
X_0 \sim Bernoulli\{p_0\} \quad (3.42)
\]
\[
X_1 \sim Bernoulli\{p_1\} \quad (3.43)
\]

To these ends, QGM utilizes the maximum likelihood (ML) estimates of these probabilities using the training data $I_i(n)$. The ML estimate can be shown to be the number of successes (realizations of 1) divided by the total number of trials for each of the $N_q$ random
variables, i.e. the empirical probabilities observed in the training set, yielding \[37\]

\[
\hat{p}_0 = \frac{N^0_1}{N|\Gamma_0|} \\
\hat{p}_1 = \frac{N^1_1}{N|\Gamma_1|}
\] (3.44)

where \( N^j_i \) refers to the counts of the number of realizations of 1 associated with each RV \( X_j, j \in \{0,1\} \), in the training data, \( N \) the number of training images, and \( |\Gamma_j| \) refers to the number of occurrences of each random variable in the predicted signature \( P_i \). Again, the values are simply the empirical probability of success of each RV seen in (3.21). An example of this process is shown in Figure 3.3.

\[
\begin{array}{cc|cc|cc|cc}
 & X_1 & X_0 & X_0 & X_0 & X_1 & X_0 \\
\hline
I_i(1) & 1 & 0 & 0 & 0 & 1 & 0 \\
I_i(2) & 0 & 0 & 0 & 1 & 1 & 0
\end{array}
\]

Figure 3.3: Example QGM training scenario with \( N = 2 \) training images. The indicator vector \( P_i \) indicates which rows are realizations of which of the two Bernoulli RVs, \( X_0 \) and \( X_1 \), parameterized by the probability of success for each variable, \( p_0 \) and \( p_1 \). The ML estimates of these parameters, \( \hat{p}_0 \) and \( \hat{p}_1 \) are simply the number of successes over the number of trials: \( \hat{p}_0 = 1/8 \) and \( \hat{p}_1 = 3/4 \). Therefore, \( \Pr(X_i = m|P_i = n) = [1/8, 3/4; 7/8, 1/4] \).

These estimates are then used to populate the matrix shown in Figure 3.4 where each element contains the observed probability \( \Pr(I_i = m|P_i = j) \) calculated as described from the N training images. Each column of this matrix can be interpreted as containing the estimated probabilities of each RV, \( X_0 \) and \( X_1 \). Each row is then associated with the probability of a low or high realization. For example, given a random draw \( X_i \) from the QGM model, the first column is associated with the random variable \( X_0 \) and the first row contains \( \Pr(X_i = 0|X_0) \) and the second row contains the \( \Pr(X_i = 1|X_0) \). As the Bernoulli probabilities are complementary, \( \Pr(X_i = 0|X_j) = 1 - \Pr(X_i = 1|X_j) \), the sum of each column’s entries must equal 1, and the Bernoulli model for each RV is fully parametrized.
This matrix is important, as it is used to calculate the likelihood score of a candidate target chip, \( X_i \), \( i \in [1 \ldots K] \).

![Matrix Diagram]

Figure 3.4: Matrix encoding the likelihoods of a given QGM template. Each entry in this matrix is the ML estimate of \( \Pr(X_i = m | P_i = j) \) which is calculated as the relative frequency of each occurrence across the \( N \) training images. The overall size of the matrix will be a function of the quantization method used and \( N_q \). Our studies assume \( N_q = 2 \) with a simple quantization scheme yielding a 2x2 matrix.

It is noted that complexity is introduced in QGM through a variety of quantization methods termed: detail, region, and joint quantization [5]. These different methods utilize a segmentation algorithm to differentiate between target, background, and shadow pixels. The choice of quantization scheme determines the values of the quantized training imagery, \( I_i(n) \), and indicator variables, \( P_i \). While the details can be found in [5], detail quantization preserves the signature variability over the target region; region quantization labels each pixel as belonging to target, background, or shadow; and joint quantization combines these two schemes. For our studies, where \( N_q = 2 \), this is simplified as both \( I_i(n) \) and \( P_i \) can only take on values \( \{0, 1\} \) indicating that the underlying pixel intensity value was either above or below a threshold.

Here we note an important point, by noticing that \( |\Gamma_0| \neq |\Gamma_1| \) for an arbitrary choice of quantization threshold. This implies the estimates of the underlying probabilities for each random variable may not be based upon the same number of samples. The authors
of the QGM generally consider “uniform quantization” methods where each random variable will be equally represented in the training data guaranteeing each estimate is based upon of the same number of samples. This trade-off is a result of using the simpler ML estimates of the underlying probabilities and the QGM model does not account for the uncertainty of these estimates. Per-pixel likelihood estimates with less uncertainty (based upon more samples) will be weighted the same as those with more uncertainty (based upon less samples). Additionally, a small number of samples could lead to probability estimates of 1 or 0, commonly known as the sparse data problem. Further, estimated probabilities of 0 will lead to unbounded log-likelihood values which will become clear in the following section [5]. These issues must be considered in algorithm implementation and the pixel intensity domain selected (log, quarter-power, etc.) chosen to approximately linearize the sorted intensity values allowing for as close to a uniform quantization scheme as possible.

In summary, the indicator vector, $P[i; H]$ (also expressed using the alternative notation $P_i$ with the dependency on the type/pose label suppressed) and the matrix of estimated probabilities

$$P \triangleq \Pr(X_i = m | P_i = j)$$

(3.45)

which in the $N_q = 2$ case and realizing that $P_i = j$ indicates that pixel $i$ is a realization of the $X_j$ reduces to

$$P = \Pr(X_i = m | X_j), \ j, m \in \{0, 1\}$$

(3.46)

and therefore $P[i; H]$ and $P$ define the class conditional template distributions for the QGM algorithm.
3.3.2 QGM Classification Decision

The QGM classification decision is based on a ML decision metric where the conditional log-likelihood of a given quantized candidate image $X[i]$ given a type/pose class conditional template can be calculated as

$$
\log \Pr(X|H) = \sum_i \log \Pr(X[i]|P[i; H])
$$

(3.47)

where the probabilities are defined in the matrix given in (3.45) and (3.46). Therefore, the overall log-likelihood is just the sum of individual per-pixel log-likelihoods and the chosen class $H$ is the one that maximizes the overall log-likelihood metric

$$
\hat{H} = \arg \max_H \log \Pr(X|H)
$$

(3.48)

3.4 Quantized Mean-Squared Error

The last algorithm we will study is termed the quantized mean squared error (QMSE) algorithm. This algorithm is based on the mean squared error (MSE) approach utilized by MIT Lincoln Labs for the DARPA sponsored Semi-Automated IMINT Processing (SAIP) program [10]. As with the DARPA MSTAR program mentioned previously, the goal of this work was to develop semi-automated to fully-automated SAR ATR solutions.

Due to both MPM’s and QGM’s utilization of quantized data, we have modified the MSE classification algorithm [13] [14] to utilize quantized data. This is to account for the loss of information in the quantization step, allowing for as close to an even comparison as possible between the three algorithms.
3.4.1 QMSE Statistical Model

QMSE differs from MPM and QGM in that it is not a strictly model-based algorithm where images are assumed to be instantiations of a generative statistical model, instead QMSE is a template-based algorithm where candidate images are assumed to be noise corrupted realizations of a deterministic template \[12\]. This leads to very close analogies to the fundamental matched filtering problem discussed in signal processing textbooks \[42\].

QMSE defines the template to be the average of each pixel value calculated from the \( N_q = 2 \) quantized training data, \( I_i(n) \) shown in Figure 3.1,

\[
t_i = \frac{\sum_{n=1}^{N} I_i(n)}{N} = \frac{N_i^1}{N} = \hat{p}_i
\]  

(3.49)

where \( i \in [1 \ldots K] \) and \( N \) is the number of training images. This expression results from recognizing that \( \sum_{n} I_i(n) = N_i^1 \) i.e. the counts of the number of pixels quantized high at pixel \( i \), therefore \( t_i \) is simply the empirical probabilities previously defined in (3.5). While, MPM and QGM use the result in (3.5) to estimate the parameters of underlying Bernoulli RVs, QMSE simply assumes this is a deterministic model for each class conditional reference template, \( t_i(H) \), where the dependence on the type/pose label was previously suppressed.

3.4.2 QMSE Classification Decision

Given a quantized candidate image, \( X_i, i \in [1 \ldots K] \), the QMSE algorithm then uses the MSE criterion to determine the template with minimum MSE score. The MSE is defined as

\[
e(H) = E \{(t_i(H) - X_i)^2\}
\]  

(3.50)
where $E\{\cdot\}$ is the expectation operator. In implementation, the sample mean is used in place of the expectation operator yielding
\[
\hat{e}(H) = \frac{1}{K} \sum_{i=1}^{K} (t_i(H) - X_i)^2
\]  
and the class that minimizes this MSE metric is chosen
\[
\hat{H} = \arg \min_{H} \hat{e}(H)
\]

It is important to note, that the QMSE error in (3.50) is equivalent to the PTM quadratic penalty function in (3.6) because the templates, $t_i$ are equivalent to the empirical probabilities as shown in (3.49).

### 3.4.3 Chapter Summary

The three algorithms considered in this dissertation have been described. The first, PTM, can be considered a special case of the MPM algorithm utilizing only two quantization levels $N_q = 2$. Due to the ambiguity in interpretation of the underlying statistical model, they were presented as separate algorithms. The PTM description is presented in Section 3.1 and MPM in Section 3.2. The scalar interpretation of MPM which is considered throughout the remainder of the dissertation is in Section 3.2.3. These algorithms assume the in-class targets are representations of a DM model, which reduces to the Beta-Bernoulli in the $N_q = 2$ case, and that under the in-class hypothesis yield a test statistic that is standard Normal allowing a Z-test to be used for the classification rule. It is noted that a tuning rule must be used in practice to minimize rewarding for background regions as discussed in sections 3.1.3 and 3.2.4. A specific tuning rule is additionally discussed in section 3.2.4, which requires assuming that target and background pixels can be distinctly segmented.

QGM was described in section 3.3 and utilizes the same underlying statistical model as PTM and MPM, however assumes there are only $N_q$ INID underlying random variables.
It relies on a maximum likelihood to score in its decision logic.

QMSE on the other hand, given in section 3.4, assumes that the underlying class-
conditional target representations are not random at all, but are deterministic templates.
These templates are equivalent to the empirical probabilities that are partly used to parametrize
the PTM and MPM class-conditional distributions. The minimum mean-squared error is
then used to make the classification decision.
Chapter 4

The Datasets

In order to verify the analysis in simulation, datasets are required to train the algorithms. To these ends, we utilized two datasets approved for public-release: “The Air Force Research Laboratories Civilian Vehicle (CV) Dataset” [41] and the “Army Research Laboratories Comanche Dataset” [43] [44]. The CV dataset will be discussed in Section 4.1 and the Comanche dataset in Section 4.2.

4.1 The AFRL Civilian Vehicle Dataset for SAR

The AFRL Civilian Vehicle dataset, alternatively referred to as the Civilian Vehicle data domes, consists of full hemispherical scattering data from a set of 10 computer-aided design (CAD) models: Toyota Camry, 4 door Honda Civic, model year 1993 Jeep, model year 1999 Jeep, Nissan Maxima, Mazda MPV, Mitsubishi, Nissan Sentra, Toyota Avalon, and Toyota Tacoma. The data was generated using the XPatch® computational electromagnetic (CEM) software which utilizes the shooting and bouncing rays method to induce surface currents on the facets and then calculates the resulting far-field using the physical optics integral [45].

The actually data is distributed in Matlab ‘.mat’ format and effectively consists of SAR phase history samples. The associated frequencies are linearly spaced into 512 bins.
about a bandwidth of 5.35 GHz at a center frequency of 9.6 GHz. The azimuth vector is from $0^\circ$ to $360^\circ$ in $0.0625^\circ$ increments and elevation data is available from $30^\circ$ to $60^\circ$. We note that we typically consider only the elevation data associated with the $45^\circ$ view. An illustration of the pose sampling is shown in Figure 4.1.

Because this dataset is capable of producing high resolution 2D and 3D SAR imagery and is openly distributed, it is frequently used in SAR research. In lieu of providing additional details here, the experimental verification methodologies will be discussed in each relevant section, namely Section 5.1.1 for the AWGN performance analysis and Section 7.2.1 for the IPR analysis.

Figure 4.1: Visualization of elevation and azimuth angles used in computing simulated monostatic scattering for the AFRL CV datadomes. The viewing sphere is shown with a Tacoma CAD model. [46]
4.2 The ARL Comanche Dataset for IR

We were also provided a subset of the ARL Comanche dataset to verify our expressions [43] [44]. Unlike the CV dataset in the preceding section, this dataset is poorly documented in the literature.

The dataset consists of forward-looking infrared (FLIR) data of 10 targets viewed from aspects of $0^\circ$ to $355^\circ$ in $5^\circ$ increments. There are 22,742 total target chips and each type/view combination may have a different number of looks. Each target chip is 75 x 40 pixels, representing a field of view of approximately 9 x 4.5 meters. Associated with each chip was a range and a source ID, but no additional information was provided on the targets, sensor, or collection. It comes in “.arf” format and a file reader for MATLAB® was provided.

Due to the sparse documentation, a brief investigation was performed on the data which revealed some idiosyncrasies. 53 target chips were associated with target ID #0 and the first view. Some of the relevant statistics on this data are shown in Figure 4.2. The data is integer format with an extreme spread of approximately 8 bits ($2^8 = 512$) around an average value. The purpose of Figure 4.2 was to hopefully find a relationship between range and image intensity, which would indicate that the quantization step utilized by the algorithms is eliminating any sensitivity to range, which it did not reveal. Nonetheless, it is still safe to say that the quantization step is reducing the sensitivity to the average pixel value as the distribution around the mean value appears to be relatively consistent between all the images in the training set while the average values vary.

This dataset is used to verify the “uniform quantization” performance under AWGN in Section 5.3 and occlusion performance in Chapter 6. It is noted that this is a difficult dataset to utilize with PTM for reasons described in Section 5.1.2 mainly due to yielding highly degenerate templates. An effective re-registration method which partly solves this problem is presented in Section 6.1.3.
Figure 4.2: Average, minimum, and maximum amplitude statistics of the 53 target chips associated with ID #0 and the first view. Each dot represents a chip, and there are multiple chips available at each range. These plots do not indicate any amplitude dependence on range.
Chapter 5

Performance Under Additive White Gaussian Noise

The literature review of Chapter 2 revealed that a remarkably small amount of work has been done on the performance prediction of SAR ATR algorithms, even more so as a function of operating condition. This is certainly a fruitful research area with room for many potential contributions. Our initial goal was modest and simply to characterize the baseline performance of the algorithms in terms of image-to-template match scores under AWGN.

By first studying performance under AWGN, it can serve as a baseline to compare performance degradation caused by various OCs, as will be used in Chapter 7 to map a performance loss under IPR variations to a performance equivalent amount of SNR. It also is the first step in showing that the algorithms and required analysis is tractable for at least a simple stochastic model on test images and that the distributions of the relevant test statistics can be effectively approximated.

Additionally, it has been shown that after certain log transforms of pixel intensity values, multiplicative speckle noise becomes additive noise [47] [48]. While specific probability density functions (PDFs) have been proposed to model these effects, the AWGN results here are generalizable to more specific unimodal distributions such as Rayleigh and
Log-Normal and these results can also be interpreted as exploring performance under background/speckle noise OC in isolation.

The approach to the performance analysis of these algorithms is relatively simple. Given a target chip to be classified and an MPM, QGM, or QMSE template, we approximate the the first and second-order moments of the resulting decision statistic as a function of the noise power. This allows the performance to be studied as a fundamental scalar decision theory problem [42]. We formulate our analysis as a standard detection problem, with clear extensions to the binary classification problem due to MPM’s utility in clutter rejection. This approach yields analytic approximations under both the target plus noise and noise only hypotheses, allowing for the binary classification problem to be studied as well.

Additional to this baseline performance study in the $N_q = 2$ case, have also extended our preliminary performance analysis to the case of “uniform” quantization which requires the threshold levels to be modelled using an RV complicating the analysis. We have also extended the performance prediction approach to the general $N_q$ case for specifically the scalar version of MPM which again yielded an accurate approximation to performance. Additionally, this allows the effect of the number of quantization levels to be studied, however due to the dimensionality of the problem the “optimum” number of quantization levels remains case dependent.

## 5.1 Building an Image Database

In order to develop some test cases to demonstrate our approximations hold and yield accurate performance prediction, we utilized the datasets introduced in Chapter 4. We discuss specifics related to the creation of the AFRL CV database in Section 5.1.1 and the ARL Comanche database in Section 5.1.2.
5.1.1 AFRL Civilian Vehicle Template Database

In order to verify our analysis, a set of 4 vehicles was selected from the AFRL CV dataset described in Section 4.1: the Toyota Camry, the model year 1999 Jeep, the Mazda MPV, and the Toyota Tacoma. These vehicles were chosen to sample from the extent of available vehicle types (car, SUV, minivan, and pickup truck).

The 30° elevation data was used and image formation done by the polar format algorithm (PFA) for the targets viewed at the off-cardinal azimuths of 45°, 135°, 225°, and 315° on the center [49]. The data was projected to the ground plane and chosen to yield images with nominal 1 ft x 1 ft resolution in the range and cross-range dimensions. The data was also registered to a common image frame, eliminating nuisance parameters consisting of unknown target rotation and translation.

This process was performed as follows. First, the desired frequency domain samples were chosen to yield 1 ft range resolution in the ground plane over a 5° aspect window centered about the views described in the previous paragraph, projected to the ground plane, and interpolated to an inscribed, uniform cartesian grid for each view angle. Next, the desired extent in the cross-range was calculated to yield 1 ft cross-range resolution. Then, the resulting cross-range window was linearly slid across the available cross-range extent to yield 10 images per vehicle type and viewing angle. The 5° window was chosen to maximize signature stability, while the windowing process was intended to capture any variability that may still exist across the type/pose training set.

The result of this process yielded 160 images, 10 for each of the 4 vehicle types at 4 viewing angles. Examples of the resulting images are shown in Figure 5.1. The quantization threshold was set to -10dB from the peak value seen across all images in the training dataset.
Figure 5.1: 5 of the 10 Camry images (top) and the associated quantized representations (bottom) viewed from an aspect of $135^\circ$. These images are the result of forming sub-apertures required for nominal 1’ resolution within a $5^\circ$ window centered at $135^\circ$. This yields an approximate step-size of $0.33^\circ$ within the $5^\circ$ window. These images were also downsamplled by a factor of 4. The red pixels in the bottom images are those quantized high.

5.1.2 ARL Comanche Template Database

We also utilized portions of the ARL Comanche database described in Section 4.2 to verify the “uniform quantization” extention in Section 5.3. No pre-processing was done except for quantization, which did yield some difficulties, specifically for PTM. Certain target chips appeared nothing like the underlying trend for most target IDs. In fact, the chips associated with target ID #5 had so much variation, that no PTM salient pixels existed, $\Gamma_{\text{peak}} \triangleq \{i; \hat{p}_i > 0.5\}$ as described in Section 3.1.3, yielding a “degenerate” template. In order to combat these difficulties and yield more stable template parametrizations, we decided to focus on target ID #1 and eliminated 14 of the 53 available target chips for a total of $N = 39$ training chips. These eliminated chips were chosen in an ad-hoc manner and a subset of these chips are shown in Figure 5.2. Without an underlying table of truth conditions, we are unable to identify the underlying phenomenologies causing the variations. This yielded a PTM template with $N = 39$ training images and $|\Gamma_{\text{peak}}| = 132$ salient pixels,
Figure 5.2: An example of the chips removed from the training dataset. These chips were visibly different from the rest of the target chips for a variety of hypothesized reasons: target amplitudes inverted, target washed out, etc. Lack of truth data on the collection prevented us from determining exactly why these chips differed but their removal yielded a more useful PTM template.

which is approximately 4.4% of the total available pixels. It is worth noting that this figure is less than the 5% of peak pixels used to quantize the data, signifying that image peaks are not entirely consistent throughout the training dataset. This metric, the ratio between percentage of peak pixels and the desired threshold, can be a quick check indicator for the applicability of PTM to a candidate dataset.

The final resulting image of empirical probabilities parametrizing the template distribution is shown in Figure 5.3.

It is noted, that a pre-processing scheme was developed to combat these issues in later experiments, which will be discussed in Section 6.1.3.
Figure 5.3: Empirical probabilities for target ID #1 resulting from the reduced 39 chip training set. This set yielding $|\Gamma_{\text{peak}}| = 132$ indicating that PTM is including only 4.4% of the image pixels in the algorithm’s classification logic.

5.2 Performance Under AWGN ($N_q = 2$)

It is now time to discuss the our first primary contribution, which is an analytic performance approximation under AWGN. We formulate our analysis as a standard detection problem. This method was chosen due to criticism of PTM’s use as specifically a clutter rejector as described in Section 3.1.3.

Therefore we formulate the hypothesis test:

$$H_0 : X_i = Q(s_i + w_i)$$ (5.1)

$$H_1 : X_i = Q(w_i)$$ (5.2)

for $i \in [1 \ldots K]$ where $K$ is the number of pixels, $s_i$ is the image to be classified before quantization, and $w_i \sim N(0, \sigma^2)$ where $\sigma^2$ is assumed known. The function $Q(\cdot)$ is the binary quantization operator.

We allow for an arbitrary decision threshold, $\gamma$, noting that this can create the issues
discussed in Section 3.3.1 in the estimation of $P$.

\[ Q(t) = \begin{cases} 
1, & t \geq \gamma \\
0, & t < \gamma 
\end{cases} \quad (5.3) \]

For a given PTM template parameterized by $N$ and $\hat{p}_i$, QGM template parameterized by $P[i]$ and $P$, or QMSE template parametrized by $\hat{p}_i$ the task becomes characterizing the distribution of each scalar decision statistic under each hypothesis. For PTM this test statistic is the summed penalty statistic which is approximately standard normal given in (3.16), for QGM this statistic is the summed log-likelihood given in (3.47), and for QMSE the MSE score in (3.50). While it is direct in the case of MPM, it will be shown that the log-likelihood score of QGM and the MSE score of QMSE can also be approximated as Gaussian. These resulting distributions will be parameterized by $\sigma^2$, allowing performance to be studied using fundamental one-dimensional results from decision theory as illustrated in Figure 5.4 [50]. These prediction results will then be verified using the data of Section 5.1.1 for the fixed quantization scheme and the data of Section 5.1.2 for the uniform quantization scheme.

### 5.2.1 Performance of MPM in $N_q = 2$ case

The preliminary analysis has already been performed for MPM and the distribution of the MPM penalty for each pixel will be approximately normal with mean given in (3.14) and
Figure 5.4: Example of the Normal/Normal decision problem. The distribution of the test statistic, in this example $B$, under each hypothesis can be expressed using knowledge of the noise process, the candidate image, template distribution, and decision threshold. This allows for closed form solutions to performance measures as a function of the noise power. QGM and QMSE utilize a maximum likelihood and minimum MSE decision rules respectively and this plot is modified to account for when a single difference distribution is greater or less than zero.

Recalling the summed test statistic of (3.16) and the independence assumption in the model of the in-class distribution $X_i$ the distribution of the final MPM test statistic using the peak only version of MPM will be approximately Gaussian with mean and variance as

$$
E\{b_i\} = \Pr(X_i = 1) \frac{(1 - \hat{p})^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} + \Pr(X_i = 0) \frac{\hat{p}^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}}
$$

$$
\text{Var}\{b_i\} = \Pr(X_i = 1) \left( \frac{(1 - \hat{p})^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} - E\{b_i\} \right)^2 + \Pr(X_i = 0) \left( \frac{\hat{p}^2 - E\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} - E\{b_i\} \right)^2
$$

where $E\{t_i\}$ and $\text{Var}\{t_i\}$ were given in (3.8) and (3.9). These equations are not expanded for compactness.
follows

\[
E\{B\} = \frac{1}{\sqrt{K_{\text{peak}} + \hat{C}_{\text{peak}}}} \sum_{i \in \Gamma_{\text{peak}}} E\{b_i\} \tag{5.4}
\]

\[
\text{Var}\{B\} = \frac{1}{\sqrt{K_{\text{peak}} + \hat{C}_{\text{peak}}}} \sum_{i \in \Gamma_{\text{peak}}} \text{Var}\{b_i\} \tag{5.5}
\]

where \(K_{\text{peak}}\) is the number of peak pixels (\(\hat{p}_i > 0.5\)) and \(\hat{C}_{\text{peak}}\) is twice the sum of the pixel-to-pixel sample covariance matrices associated with peak pixels only which is calculated when training the algorithm. Therefore, all terms are fully specified given a candidate image and reference template distribution, and the remaining problem is to characterize \(\text{Pr}(X_i = 1)\) and \(\text{Pr}(X_i = 0)\) under each hypothesis in (5.1) and (5.2).

Under the alternate hypothesis \(H_1\), given in (5.2), the signal is assumed to be comprised of zero-mean IID Gaussian noise with variance \(\sigma^2\). Therefore, the probability of any pixel being quantized high, \(\text{Pr}(X_i = 1)\), is the probability that any noise realization is greater than the quantization threshold \(\gamma\), \(\text{Pr}(X_i = 1) = \text{Pr}(w_i > \gamma) = \text{Pr}(w_i - \gamma > 0)\). Therefore, under the alternative hypothesis

\[
\text{Pr}(X_i = 1|H_1) = 1 - \Phi\left(\frac{\gamma}{\sigma}\right) \tag{5.6}
\]

\[
\text{Pr}(X_i = 0|H_1) = \Phi\left(\frac{\gamma}{\sigma}\right) \tag{5.7}
\]

where \(\Phi\) is the standard normal cumulative density function. It is noted that these probabilities are independent of the pixel index \(i\) and will be constant across all pixels in the scene.

Similarly for the null hypothesis, the probability of any pixel being quantized high, \(\text{Pr}(X_i = 1)\), is the probability that any noise corrupted image pixel is greater than the quantization threshold \(\gamma\), \(\text{Pr}(X_i = 1) = \text{Pr}(s_i + w_i > \gamma) = \text{Pr}(s_i + w_i - \gamma > 0)\). Therefore,
under the null hypothesis

\[
\Pr(X_i = 1|H_0) = 1 - \Phi \left( \frac{\gamma - s_i}{\sigma} \right) \tag{5.8}
\]

\[
\Pr(X_i = 0|H_0) = \Phi \left( \frac{\gamma - s_i}{\sigma} \right) \tag{5.9}
\]

and in this case each probability is dependent on the underlying image pixel intensity, \(s_i\) which must be considered for each pixel.

This reduces the model in (5.1) and (5.2) to

\[
H_0 : B \sim N(E\{B|H_0\}, \text{Var}\{B|H_0\}) \tag{5.10}
\]

\[
H_1 : B \sim N(E\{B|H_1\}, \text{Var}\{B|H_1\}) \tag{5.11}
\]

where \(E\{B|H_j\}\) and \(\text{Var}\{B|H_j\}\) \(j \in \{0, 1\}\) are given in (5.4) and (5.5) which can be evaluated using (3.14) and (3.15) with the probabilities given in (5.8) and (5.9) for the null hypothesis or the probabilities in (5.6) and (5.7) for the alternate hypothesis.

This yields the decision scenario shown in Figure 5.4. This is the standard problem of classifying whether a test statistic originated from one of two Gaussian distributions. In this case, the Gaussian statistics are functions of the noise variance and allows for performance measures to be studied analytically, such as the probability of detection and probability of false alarm in the detection case under the baseline case of AWGN.

5.2.2 Verification of Performance of MPM in \(N_q = 2\) case

The second-order approximations of the MPM test statistic under each hypothesis derived in the previous section can now be used to predict the performance of each algorithm as a function of the noise process [39]. This is as simple as calculating the probability that the test statistic will realize a value outside of the critical region of the Z-test as described in Section 3.1.2 [42]. An example of this is shown in Figure 5.6 where we calculate the
probability of determining an in-class target to be clutter, a scenario consistent with the previous derivations. Extensions to the binary classification problem are direct.

The analysis was verified using the data generated in Section 5.1.1 for two targets. The test methodology was to first select a given PTM template, then select at random an in-class image used to generate the template. Next, a Monte Carlo (MC) simulation was used to generate a vector of IID Gaussian noise with a specific variance which was then added to the in-class image. For each MC run, the noise corrupted image was run through the PTM algorithm yielding realizations of the null hypothesis, and the noise only image was also run through the PTM algorithm yielding realizations of the alternate hypothesis. The sample means and variances calculated over 5,000 MC trials were then compared to the analytic results given in (5.4) and (5.5). This yielded Figure 5.5 which used the template associated with the Camry viewed from 45° aspect. This template had 64 peak pixels ($\hat{p}_i > .5$) with a very small pixel-to-pixel covariance sum, $\hat{C}$ defined in (3.13), value of 0.067. Test cases on templates with a greater value of $\hat{C}$ yielded similar accuracy.

It is noted that the test image and template was first downsampled by a factor of 4 to reduce the computational complexity required to mainly calculate the pixel-to-pixel covariance matrix entries in $\hat{C}$. In both cases, the in-class comparison image was chosen as the first image used to generate the template. It is clear from Figure 5.5 that our approximations for the relevant moments of the test statistics are accurate.

Next, the analysis of Section 5.2.1 was used to calculate the probability of missed detection as a function of the noise process, i.e. the probability that a target chip would be classified as clutter. This result is shown in Figure 5.6 which indicates that our assumptions are correct; the test statistics can be accurately approximated by considering only the first and second order moments.

These results demonstrate the validity of our approximations and we are able to accurately model the PTM test statistic as a function of the noise power in the $N_q = 2$ case allowing for closed form performance expressions.
Figure 5.5: The comparison between our analytic approximations to the mean and variance of the MPM test statistic in (3.16) under both hypotheses for the Camry viewed from 45° aspect. The analytic solution matches the empirical and the fluctuation in empirical variances estimates are due to simulation.

5.2.3 Performance of QGM in $N_q = 2$ case

We have described QGM as a fundamentally different algorithm than MPM, whose test statistic is by design a Gaussian RV. However, we shall see that the log-likelihood test statistic of QGM given in (3.47) can be interpreted as a weighted sum of Binomial RVs which can be manipulated to approximate the log-likelihood under each hypothesis through the second-order. As in the previous case, this allows the performance of the algorithm to be studied as the fundamental scalar Gaussian/Gaussian decision problem of Figure 5.4 where the distributions are parametrized by the power of the AWGN noise process.
Figure 5.6: The top plot shows the probability of a failing to classify an in-class target correctly using the MPM algorithm, calculated both in simulation and using our analytic approximations. The bottom plot compares the approximation to the test statistic for a specific noise variance to the empirical distribution. The approximations are able to correctly model the performance seen in empirical simulation.

Recalling (3.47), the log-likelihood of an image $X[i]$ conditioned on a given type/pose template, $H$, parametrized by $P[i; H]$ and $P$ is

$$\log \Pr(X|H) = \sum_{i=1}^{K} \log \Pr(X[i]|P[i; H])$$

Redefining $\Pr(\cdot)$ to now refer to the log-probability (i.e. log the elements of $P$) allows us to split the sum in (3.47) into a sum of sums over the pixels associated with each random
variable

\[ \Pr(X|H) = \sum_{i \in \chi_0} \Pr(X_i = n|X_0) + \sum_{i \in \chi_1} \Pr(X_i = n|X_1) \]  
\hspace{1cm} (5.12)

where \( \chi_0 \) and \( \chi_1 \) are the sets of pixels associated with random variables \( X_0 \) and \( X_1 \) respectively defined by the \( P_i \) vector. Realizing that \( \Pr(X_i = n|X_j), \ j \in \{0, 1\}, \) are simply scalar weights given by the elements of the \( [2 \times 2] \) matrix, \( P, \) this can be reduced further to

\[ \Pr(E|H) = N_0^{X_0}\Pr(X_i = 0|X_0) + N_1^{X_0}\Pr(X_i = 1|X_0) \]
\[ + N_0^{X_1}\Pr(X_i = 0|X_1) + N_1^{X_1}\Pr(X_i = 1|X_1) \]  
\hspace{1cm} (5.13)

where \( N_0^{X_i} \) and \( N_1^{X_i} \) are variables representing the number of pixels associated with each RV, \( X_0 \) and \( X_1, \) quantized low and high respectively, i.e. simple counting processes. Therefore, if we are able to accurately model the counts of quantized candidate target pixels associated with each entry of the \( P \) matrix as random variables, we can approximate the statistics of the log-likelihood score as a function of the noise power.

Starting with the simpler alternate hypothesis in (5.2), \( X_i = Q(w_i) \) where \( w_i \) is AWGN with variance \( \sigma^2, \ \forall i, \) the probability of each pixel being quantized high can be written as \( \Pr(w_i > \gamma) \) where \( \gamma \) is the quantization threshold. Therefore each pixel can be modelled as a Bernoulli RV with probability of success \( p = 1 - \Phi(\gamma/\sigma) \) where \( \Phi \) is the cumulative normal distribution function. Summing over all pixels in the sets \( i \in \chi_j, \ j \in \{0, 1\}, \) yields a binomial distribution parametrized by probability of success \( p \) and \( |\chi_i| \) indicating the cardinality of each set. The cardinality of \( |\chi_0| \) is simply the number of instantiations of \( X_0 \) in \( P_i \) and similar for \( |\chi_1| \). Also, noting that \( N_0^{X_j} \) and \( N_1^{X_j} \) are not independent but rather complementary allows the mean of the log-likelihood to be written
as

\[ E\{\Pr(E|H_1)\} = |\chi_0|(1 - p)\Pr(X_i = 0|X_0) \]
\[ + |\chi_0| p \Pr(X_i = 1|X_0) \]
\[ + |\chi_1|(1 - p)\Pr(X_i = 0|X_1) \]
\[ + |\chi_1| p \Pr(X_i = 1|X_1) \]

(5.14)

and the variance as

\[ \text{Var}\{\Pr(E|H_1)\} = |\chi_0|p(1 - p)\Pr(E[i] = 0|X_0)^2 \]
\[ + |\chi_0|p(1 - p)\Pr(E[i] = 1|X_0)^2 \]
\[ + |\chi_1|p(1 - p)\Pr(E[i] = 0|X_1)^2 \]
\[ + |\chi_1|p(1 - p)\Pr(E[i] = 1|X_1)^2 \]

(5.15)

where again \( p = 1 - \Phi(\gamma/\sigma) \).

The null hypothesis in (5.1) is slightly more complicated in that \( x_i = Q(s_i + w_i) \) and the probabilities that a pixel will be quantized high, \( \Pr(s_i + w_i > \gamma) \), are not identically distributed as each is a function of the underlying pixel value. Each pixel will still be a Bernoulli trial, however the probability of success \( p_i = 1 - \Phi(\gamma - s_i/\sigma) \) is now different for each pixel. In this case, the random variables \( N_{X_0}^\chi_{ij} \) and \( N_{X_1}^\chi_{ij} \) will be Poisson Binomial with mean given by \( \sum_{i \in \chi_j} |\chi_j|p_i \) and variance by \( \sum_{i \in \chi_j} |\chi_j|(1 - p_i)p_i \) for \( j \in \{0, 1\} \) [51].
Therefore, the mean of the log-likelihood QGM statistic in (3.47) becomes

\[
E\{\Pr(X|H_0)\} = (|\chi_0| - \sum_{i \in \chi_0} p_i) \Pr(X_i = 0|X_0) \\
+ \sum_{i \in \chi_0} p_i \Pr(X_i = 1|X_0) \\
+ (|\chi_1| - \sum_{i \in \chi_1} p_i) \Pr(X_i = 0|X_1) \\
+ \sum_{i \in \chi_1} p_i \Pr(X_i = 1|X_1) 
\]  

(5.16)

and the variance as

\[
\operatorname{Var}\{\Pr(X|H_0)\} = \sum_{i \in \chi_0} p_i(1 - p_i) \Pr(X_i = 0|X_0)^2 \\
+ \sum_{i \in \chi_0} p_i(1 - p_i) \Pr(X_i = 1|X_0)^2 \\
+ \sum_{i \in \chi_1} p_i(1 - p_i) \Pr(X_i = 0|X_1)^2 \\
+ \sum_{i \in \chi_1} p_i(1 - p_i) \Pr(X_i = 1|X_1)^2 
\]  

(5.17)

where \( p_i = 1 - \Phi((\gamma - s_i)/\sigma) \) and again the entries of \( P \) were redefined as the log-probabilities.

Therefore, as in (5.10) and (5.11), we have approximated the resulting log-likelihoods under each hypothesis as normally distributed, with mean and variance under each hypothesis parametrized by the noise variance and under the null hypothesis also the pixel intensities. Therefore, under the null hypothesis the log-likelihood of (3.47) is approximately normal with mean given by (5.16) and variance given by (5.17). Under the alternative hypothesis, the mean is given by (5.14) and variance by (5.15).

Unlike MPM which allowed direct calculation of performance metrics, QGM utilizes a maximum likelihood classification rule. Therefore, to obtain a closed form performance
metric, the difference distribution between the test statistics under each hypothesis must be calculated. The probability that the difference distribution is greater than zero is equivalent to the probability that the test statistic under one hypothesis is greater than the other, allowing performance to be determined in closed form.

If the two distributions were independent, this would be direct, however they are not. The difference distribution of two correlated Gaussian distributions can be written as

\[ X - Y \sim \text{Normal}(\mu_x - \mu_y, \sigma_x^2 + \sigma_y^2 - 2\sigma_{xy}) \]  

(5.18)

where \( \sigma_{xy} = E\{XY\} - \mu_x\mu_y \). As the \( \mu_x \) and \( \mu_y \) terms were calculated in the previous section, the remaining task is to calculate \( E\{XY\} \).

For compactness, we again redefine the probabilities in the \( P \) matrix as the log-probabilities: \( a \triangleq \log(\Pr(X_i = 0|X_0)) \), \( b \triangleq \log(\Pr(X_i = 1|X_0)) \), \( c \triangleq \log(\Pr(X_i = 0|X_1)) \), and \( d \triangleq \log(\Pr(X_i = 1|X_1)) \). This allows us to write the QGM log-likelihood scores under each hypothesis as

\[
X = \Pr(E|H_0) = N_0^{X_0}a + N_1^{X_0}b + N_0^{X_1}c + N_1^{X_1}d 
\]

(5.19)

\[
Y = \Pr(E|H_1) = M_0^{X_0}a + M_1^{X_0}b + M_0^{X_1}c + M_1^{X_1}d 
\]

(5.20)

where \( N_n^{X_j} \) and \( M_n^{X_j} \) are the Binomial and Poisson-Binomial RVs discussed in the previous section.
Calculating the second moment yields 16 terms

\[ E\{XY\} = a^2 E\{N_0^{X_0} M_0^{X_0}\} \]  
(5.21)

\[ + b^2 E\{N_1^{X_0} M_1^{X_0}\} \]  
(5.22)

\[ + c^2 E\{N_0^{X_1} M_0^{X_1}\} \]  
(5.23)

\[ + d^2 E\{N_1^{X_1} M_1^{X_1}\} \]  
(5.24)

\[ + ab E\{N_0^{X_0} M_1^{X_0}\} \]  
(5.25)

\[ + ac E\{N_0^{X_0} M_0^{X_1}\} \]  
(5.26)

\[ + ad E\{N_0^{X_0} M_1^{X_1}\} \]  
(5.27)

\[ + ba E\{N_1^{X_0} M_0^{X_0}\} \]  
(5.28)

\[ + bc E\{N_1^{X_0} M_0^{X_1}\} \]  
(5.29)

\[ + bd E\{N_1^{X_0} M_1^{X_1}\} \]  
(5.30)

\[ + ca E\{N_0^{X_1} M_0^{X_0}\} \]  
(5.31)

\[ + cb E\{N_0^{X_1} M_1^{X_0}\} \]  
(5.32)

\[ + cd E\{N_0^{X_1} M_1^{X_1}\} \]  
(5.33)

\[ + da E\{N_1^{X_1} M_0^{X_0}\} \]  
(5.34)

\[ + db E\{N_1^{X_1} M_0^{X_1}\} \]  
(5.35)

\[ + dc E\{N_1^{X_1} M_1^{X_0}\}. \]  
(5.36)

The terms in (5.26), (5.27), (5.29), (5.30), (5.31), (5.32), (5.34), and (5.35) are independent and the expectation is equal to the product of the marginal expectations. This is because the \( N_k^{X_k} \) and \( M_l^{X_l} \), \( k \neq l \), variables in these equations are associated with counts associated with the independent underlying Bernoulli RVs: \( N_n^{X_0} \) and \( M_n^{X_1} \) or \( N_n^{X_1} \) and \( M_n^{X_0} \).

The remaining terms can be split into 4 cases. Each case shares the same basic struc-
\[ E \{ N_n^{X_k} M_n^{X_k} \} = \sum_{i \in X_k} \sum_{j \in X_k \atop i \neq j} E \{ Q^n(s_i + w_i) Q^n(w_j) \} \]  
\[ + \sum_{i = j} E \{ Q^n(s_i + w_i) Q^n(w_j) \} \]  

(5.37)

(5.38)

where we note that

\[ Q^n(w) \sim \begin{cases} 
\text{Bernoulli}(\Phi(\frac{2}{\sigma})), & n = 1 \\
\text{Bernoulli}(1 - \Phi(\frac{2}{\sigma})), & n = 0
\end{cases} \]  

(5.39)

and

\[ Q^n(s_i + w_i) \sim \begin{cases} 
\text{Bernoulli}(\Phi(\frac{2-s_i}{\sigma})), & n = 1 \\
\text{Bernoulli}(1 - \Phi(\frac{2-s_i}{\sigma})), & n = 0
\end{cases} \]  

(5.40)

As the noise process is IID, the top term, (5.37), is independent and can be written as

\[ \sum_{i \in X_k} \sum_{j \in X_k \atop i \neq j} E \{ Q^n(s_i + w_i) Q^n(w_j) \} = |\chi_n - 1| E \{ Q^n(w) \} \sum_{i \in \chi_n} E \{ Q^n(s_i + w_i) \} \]  

(5.41)

and the expected value of a Bernoulli random variable is its probability, which is given in (5.39) and (5.40) for each case.

The remaining term, (5.38), is case dependent. The first case involves the counts of pixels likely to be quantized low and is associated with terms (5.21) and (5.23). The second case involves the counts of pixels likely to be quantized high and is associated with terms (5.22) and (5.24). The third case involves pixels likely to be quantized low under the null hypothesis and high under the alternate and is associated with terms (5.25) and (5.33). The last case involves pixels likely to be quantized high under the null hypothesis and zero
under the alternate, associated with the remaining terms (5.28) and (5.36). As each pixel is an independent realization, the expectation can be summed over each pixel yielding

$$\sum_{i=j} E\{Q^n(s_i + w_i)Q^n(w_j)\} = \begin{cases} \sum_{i \in \chi_n} 1 - \Phi(\frac{\gamma - s_i}{\sigma}), & (5.21), (5.23) \\ |\chi_n|\Phi(\frac{\gamma}{\sigma}), & (5.22), (5.24) \\ 0, & (5.25), (5.33) \\ |\chi_n|\Phi(\frac{\gamma}{\sigma}) - \sum_{i \in \chi_n} \Phi(\frac{\gamma - s_i}{\sigma}) & (5.28), (5.36) \end{cases}$$

(5.42)

Therefore the difference distribution is fully characterized, allowing closed form solutions to performance metrics of interest.

### 5.2.4 Verification of Performance of QGM in \(N_q = 2\) case

The results of our QGM analysis was also verified using the data generated in Section 5.1.1 for a single target. The test methodology in this case is the same. First, a given QGM template parameterized by \(P\) and \(P[i; H]\) was chosen, and an in-class image used to generate the template selected at random. Then in a MC simulation a vector of IID Gaussian noise with a specific variance was generated, added to the in-class image to instantiate a realization of the log-likehood metric under null hypothesis and left alone to instantiate a realization under the alternative hypothesis. These were both run through the QGM algorithm, and the sample means and variances calculated over 5,000 MC trials were compared to the results given in (5.16) and (5.17) for the null hypothesis and (5.14) and (5.15) for the alternate. The results are shown in Figure 5.7 and it is evident that our analysis is accurately able to predict the distribution of the test statistic under each hypothesis, through at least the second order.

Next, the final performance curve was generated using the results in Section 5.2.3 yielding Figure 5.8. The deviation between the analytic and empirical results for high noise variances cases is due to our Normal approximation to the test statistic not holding.
Figure 5.7: The comparison between our analytic approximations to the mean and variance of the QGM test statistic in (3.47) under both hypotheses for the Toyota Tacoma viewed from 225° aspect.

for large values of $\sigma^2$. Nonetheless, in the region of interest where performance goes from adequate to inadequate, our results are able to accurately predict performance.

5.2.5 Performance of QMSE in $N_q = 2$ case

While QMSE differs from MPM and QMSE by assuming an underlying deterministic model of the in-class references, as opposed to a random, the analysis again proceeds the same. The mean and variance of the relevant test statistic, the QMSE score given in (3.51), is calculated as a function of the noise power under the hypotheses given in (5.1) and (5.2).
Figure 5.8: The top plot shows the probability of a failing to classify an in-class target correctly using the QGM algorithm, calculated both in simulation and using our analytic approximations. The bottom plot compares the approximation to the test statistic for a specific noise variance to the empirical distribution.

To repeat, the MSE score given in (3.51) can be expanded to yield

\[ \hat{e} = \frac{1}{K} \sum_{i=1}^{K} (t_{i}^2 - 2t_{i}X_{i} + X_{i}^2) \]  

(5.43)

where \( t_{i} \) was the template defined in (3.49) and the model for \( X_{i} \) is given in (5.1) for the null hypothesis and in (5.2) for the alternate.

Under the null hypothesis, \( H_0 \), \( X_{i} \) is distributed as Bernoulli with probability of success \( \Pr(X_{i} = 1|H_0) = 1 - \Phi \left( \frac{2 - s_{i}}{\sigma} \right) \). All the non-central moments of the distribution are equal to this probability, therefore the expected value of the MSE test statistic under the
null hypothesis is

\[ E\{\hat{e}|H_0\} = \frac{1}{K} \sum_{i}^{K} [t_i^2 - 2t_i \Pr(X_i = 1|H_0) + \Pr(X_i = 1|H_0)] \]  \hspace{1cm} (5.44)

The expressions under the alternative hypothesis, \( H_1 \), are the same except the moments of the Bernoulli distribution have changed due to the change in the underlying signal model. In this case, \( X_i \) is distributed as Bernoulli with probability of success \( \Pr(X_i = 1|H_1) = 1 - \Phi \left( \frac{\gamma}{\sigma} \right) \) yielding

\[ E\{\hat{e}|H_1\} = \frac{1}{K} \sum_{i}^{K} [t_i^2 - 2t_i \Pr(X_i = 1|H_1) + \Pr(X_i = 1|H_1)] \]  \hspace{1cm} (5.45)

Expressing the variances under each hypothesis is more complicated. We begin by calculating \( E\{\hat{e}^2\} \) which yields the variances using \( \sigma_x^2 = E\{\hat{e}^2\} - E\{\hat{e}\}^2 \) where the dependence on hypothesis has been suppressed

\[ E\{\hat{e}^2\} = \frac{1}{K^2} E\{ \sum_{i=1}^{K} \sum_{j=1}^{K} t_i^2 t_j^2 \} \]  \hspace{1cm} (5.46)

\[ - 2t_i^2 t_j X_j \]  \hspace{1cm} (5.47)

\[ + t_i^2 X_j^2 \]  \hspace{1cm} (5.48)

\[ - 2t_i X_i t_j^2 \]  \hspace{1cm} (5.49)

\[ + 4t_i X_i t_j X_j \]  \hspace{1cm} (5.50)

\[ - 2t_i X_i X_j^2 \]  \hspace{1cm} (5.51)

\[ + X_i^2 t_j^2 \]  \hspace{1cm} (5.52)

\[ - 2X_i^2 t_j X_j \]  \hspace{1cm} (5.53)

\[ + X_i^2 X_j^2 \} \]  \hspace{1cm} (5.54)
amd evaluating the expectation for each term is direct and yields

\[
E\{e^2\} = \frac{1}{K^2} \left[ \sum_i t_i^2 \sum_j t_j^2 \right] 
\] (5.55)

\[-2 \sum_i t_i^2 \sum_j t_j E\{X_j\} \] (5.56)

\[+ \sum_i t_i^2 \sum_j E\{X_j\} \] (5.57)

\[-2 \sum_i t_i E\{X_i\} \sum_j t_j^2 \] (5.58)

\[+4 \left( \sum_{i \neq j} t_i t_j E\{X_i\} E\{X_j\} + \sum_{i=j} t_i E\{X_i\} \right) \] (5.59)

\[-2 \left( \sum_{i \neq j} t_i E\{X_i\} E\{X_j\} + \sum_{i=j} t_i E\{X_i\} \right) \] (5.60)

\[+ \sum_i E\{X_i\} \sum_j t_j^2 \] (5.61)

\[-2 \left( \sum_{i \neq j} t_j E\{X_i\} E\{X_j\} + \sum_{i=j} t_i E\{X_i\} \right) \] (5.62)

\[+ \sum_{i \neq j} E\{X_i\} E\{X_j\} + \sum_{i=j} E\{X_i\} \] (5.63)

where the \(E\{X_i\}\) is is dependent on hypothesis and the sums are from 1 to \(K\). Under the null hypothesis, \(E\{X_i\}, X_i\) is Bernoulli distributed with \(Pr(.) = 1 - \Phi((\gamma - s_i)/\sigma)\) and under the alternate hypothesis, \(E\{X_i\}\) is again Bernoulli distributed with \(Pr(.) = 1 - \Phi(\gamma/\sigma)\). All the non-central moments of a Bernoulli distribution are specified by its underlying probability and therefore the analysis is complete. It is noted that the probabilities under the null hypothesis are dependent on pixel locations, while under the alternate hypothesis the probabilities are constant. This allows the terms to be simplified which we will not document
Unlike MPM which allowed direct calculation of performance metrics and like QGM, QMSE requires calculating a difference distribution to obtain performance measures. Therefore we will follow the procedure documented in Section 5.2.3 and use (5.18) to parametrize the difference distribution. All the terms have been calculated in the previous section, except for the covariance term $\sigma_{xy}$.

For clarity we will use the variables $X$ and $Y$ to represent the second-order approximation of the distributions of the QMSE test statistic under each hypothesis

$$X = \hat{e}|H_0 = \frac{1}{K} \sum_{i=1}^{K} (t_i^2 - 2t_i X_i + X_i^2)$$

$$Y = \hat{e}|H_1 = \frac{1}{K} \sum_{i=1}^{K} (t_i^2 - 2t_i Y_i + Y_i^2)$$

and use $\sigma_{XY} = E\{XY\} - E\{X\}E\{Y\}$ to calculate the covariance which requires calculating the remaining term $E\{XY\}$ as the marginal means were given in the preceding
section yielding

\[ E\{XY\} = \frac{1}{K^2} \left[ \sum_i t_i^2 \sum_j t_j^2 \right] \] (5.66)

\[-2 \sum_i t_i^2 \sum_j t_j E\{Y_j\} \] (5.67)

\[+ \sum_i t_i^2 \sum_j E\{Y_j\} \] (5.68)

\[-2 \sum_i t_i E\{X_i\} \sum_j t_j^2 \] (5.69)

\[+ 4 \left( \sum_i \sum_{j \neq i} t_i t_j E\{X_i\} E\{Y_j\} + \sum_{i=j} t_i^2 E\{Y_i\} \right) \] (5.70)

\[-2 \left( \sum_i \sum_{j \neq i} t_j E\{X_i\} E\{Y_j\} + \sum_{i=j} t_i E\{X_i\} \right) \] (5.71)

\[+ \sum_i E\{X_i\} \sum_j t_j^2 \] (5.72)

\[-2 \left( \sum_i \sum_{j \neq i} t_j E\{X_i\} E\{X_i\} + \sum_{i=j} t_i E\{Y_i\} \right) \] (5.73)

\[+ \sum_i \sum_{j \neq i} E\{X_i\} E\{Y_i\} + \sum_{i=j} E\{Y_i\} \] (5.74)

where the \( E\{X_i\} \) is equal to to the probability of the Bernoulli random variable given by 1 – \( \Phi((\gamma - s_i)/\sigma) \) and \( E\{Y_j\} \) is equal to the probability of the underlying Bernoulli random variable given by 1 – \( \Phi(\gamma/\sigma) \) and therefore the difference distribution is fully specified.

### 5.2.6 Verification of Performance of QMSE in \( N_q = 2 \) case

Maintaining the trend, the QMSE analysis was also verified using the data generated in Section [5.1.1](#) for a single target. The test methodology remains the same; first, a given
QMSE template parameterized by \( t_i \) or equivalently \( \hat{p}_i \) was chosen, and an in-class image used to generate the template selected at random. Then in a MC simulation a vector of IID Gaussian noise with a specific variance was generated, added to the in-class image to instantiate a realization MSE metric under the null hypothesis and left alone to instantiate a realization under the alternative hypothesis. These were both run through the QMSE algorithm, and the sample means and variances calculated over 5,000 MC trials of the MSE were compared to the results given in Section 5.2.5. The results are shown in Figure 5.9.

![Figure 5.9](image)

**Figure 5.9:** The comparison between our analytic approximations to the mean and variance of the QMSE test statistic in (3.51) under both hypotheses for the Toyota Tacoma viewed from 225° aspect.

Next, the final performance curve was generated using the results in Section 5.2.5 yielding Figure 5.10. As with QGM, the deviation between the analytic and empirical
results for high noise variances cases is due to our Normal approximation to the test statistic not holding. Nonetheless, in the region of interest where performance goes from adequate to inadequate, our results are able to accurately predict performance.

Figure 5.10: The top plot shows the probability of a failing to classify an in-class target correctly using the QMSE algorithm, calculated both in simulation and using our analytic approximations. The bottom plot compares the approximation to the test statistic for a specific noise variance to the empirical distribution. The approximations are able to correctly model the performance seen in empirical simulation.

5.3 Performance of MPM Under AWGN ($N_q = 2$) with Uniform Quantization

The previous sections assumed a fixed threshold. This simplification was made based on the studied dataset of Section 5.1.1 where the peak amplitudes across the set of training
and test imagery were nominally constant. In the case of data collected from different, non-calibrated sensor systems, this will likely not be the case and therefore we must extend the analysis to consider the case of “uniform” or non-fixed quantization.

The problem setup remains the same

\[
H_0 : X_i = Q_U(s_i + w_i) \quad (5.75)
\]
\[
H_1 : X_i = Q_U(w_i) \quad (5.76)
\]

for \(i \in [1 \ldots K]\) where \(K\) is the number of pixels, \(s_i\) is the image to be classified before quantization, and \(w_i \sim N(0, \sigma^2)\) where \(\sigma^2\) is assumed known. Except in this case the quantization function is the binary uniform quantization operator written as

\[
Q_U(t) = \begin{cases} 
1, & t \geq \gamma_k \\
0, & t < \gamma_k 
\end{cases} \quad (5.77)
\]

where \(\gamma_k\) is the \(k\)th value of \(t\) when sorted from smallest-to-largest value. As our images are random under each hypothesis, \(\gamma_k\) will be a realization of a RV representing an order statistic of the data given in \(t\). Accounting for this complication is described in the following section.

### 5.3.1 Uniform Quantization and Order Statistics

The previous solution to this problem in Sections 5.2.1, 5.2.3, and 5.2.5 assumed a fixed threshold, \(\gamma\), as opposed to the random threshold given in (5.77) which simplified the problem greatly. In the latter case of (5.77), the pixel values of the test image are sorted by intensity (smallest-to-largest) and then the \(k\)th value, \(\gamma_k\) is used as a threshold \(k \in [1 \ldots K]\). Because the test images under both hypotheses, (5.75) and (5.76), are random, the resulting thresholds will also be random and their distributions can be specified using order statistics.
In general, the study of order statistics is complex, often yielding intractable expressions for both the distributions and their moments \[52\] \[53\].

In order to approximate the MPM score, the probability that a given pixel will be quantized high or low respectively needs to be calculated under each hypothesis and the remaining process is consistent with that described in Section \[5.2.1\]. Therefore, we are interested primarily in calculating the \(\Pr(X_i = 1)\) and \(\Pr(X_i = 0)\) under each hypothesis.

In the case of the alternative hypothesis \(H_1\) in \(5.76\), the image is comprised of zero-mean IID Gaussian noise with a known variance, \(w_i \sim N(0, \sigma^2)\). This is the case of determining the distribution of the \(k^{th}\) sorted value (smallest-to-largets) of \(K\) IID normal draws; \(k = 1\) would be the distribution of the minimum value and \(k = K\) would be the distribution of the maximum. In this case, we typically want the \(k^{th}\) value such that \(k = (1 - P)K\) where \(P\) is an arbitrary percentile \(\in (0, 1)\). For example, if we want to take the top 10\% of pixels, we choose \(P = .1\) yielding \(k = \text{round(.9K)}\) where the rounding operation is used to guarantee an integer threshold.

In the IID case, the expected value of the \(k^{th}\) order statistic of \(K\) draws IID draws from a normal distribution with mean, \(\mu\), and variance, \(\sigma^2\) from \(\gamma_k\) can be approximated as \[54\]
\[
E\{\gamma_k; k, K\} = \mu + \Phi^{-1}\left(\frac{r - \alpha}{n - 2\alpha + 1}\right) \sigma \tag{5.78}
\]
where \(\Phi^{-1}(\cdot)\) is the inverse normal cumulative distribution function.

While the previous equation will be useful later, it is not actually required to calculate the expected values of the \(\Pr(X_i = 1|H_1)\) and \(\Pr(X_i = 0|H_1)\), as
\[
E\{\Pr(X_i = 1|H_1)\} = P \tag{5.79}
\]
\[
E\{\Pr(X_i = 0|H_1)\} = 1 - P \tag{5.80}
\]
by construction as the thresholds \(\gamma_k\) were chosen to yield \(P\%\) of the pixels being quantized high and we will refer to \(\gamma_P\) as an equivalent threshold.
However expressing these probabilities under the null hypothesis, $H_0$, is much more difficult due to the fact that each pixel can be differently distributed due to the underlying image pixel value. Typically, published solutions to this problem are of a general theoretic variety and unsuited to a practical application such as this case [55]. We have found in work with large, high-resolution SAR images that in many instances, the intensity values of the underlying image can be assumed to be realizations of an underlying IID Gaussian process, in other words $s \sim N(\mu_s, \sigma_s^2)$ where

\[
\mu_s = \frac{1}{K} \sum_{i} s_i \quad \text{(5.81)}
\]

\[
\sigma_s^2 = \frac{1}{K} \sum_{i} (s_i - \mu_s)^2 \quad \text{(5.82)}
\]

or in other words, the sample mean and variance are used to approximate the deterministic image data as IID samples from a normal RV. This approximation allows the noise corrupted image under the null hypothesis, $s_i + w_i$, to be written as an IID gaussian RV, $s + w \sim N(\mu_s, \sigma_s^2 + \sigma^2)$, and used directly in (5.78). We do stress that the image histogram must be very close to Gaussian or the approximation will be poor and bias the approximation results in 5.2.1 and subsequently the performance prediction results will suffer. In the case that the test image cannot be modelled as IID Gaussian, we have to resort to MC simulations to calculate the expected value of the threshold, $E\{\gamma_P\}$. This is not a bad trade-off as this approximation is more likely to hold for large images where MC simulations may be computationally intractable, allowing smaller images to utilize MC simulations where the computational burden is smaller.

We then approximate the probability of any pixel being quantized high, $\Pr(X_i = 1|H_0)$, as the probability that any noise corrupted image pixel will be greater than the expected quantization threshold $E\{\gamma_P\}$, $\Pr(X_i = 1|H_0) = \Pr(s_i + w_i > E\{\gamma_P\}) = \Pr(s_i + w_i - E\{\gamma_P\} > 0)$. Therefore, under the null hypothesis the probabilities can be
written as

$$\Pr(X_i = 1|H_0) = 1 - \Phi \left( \frac{E\{\gamma P\} - s_i}{\sigma} \right)$$

(5.83)

$$\Pr(X_i = 0|H_0) = \Phi \left( \frac{E\{E\{\gamma P\} - s_i}{\sigma} \right)$$

(5.84)

and again the remainder of the analysis is given in Section 5.2.1.

## 5.3.2 Verification of PTM AWGN Prediction Under Uniform Quantization

Our performance prediction expressions were then verified in MC simulations using the ARL Comanche dataset described in Section 5.1.2. In order to verify our analysis, we randomly selected two target types, ID #0 and #5, viewed from the $0^\circ - 5^\circ$ aspect window and trained the algorithm using the process described in Sections 3.1.1 with the uniform quantization scheme in (5.77). We selected $P = .05$ such that the top 5% of pixels in each image would be quantized high. The difficulties with the dataset were noted in Section 5.1.2 which did require removing some training images to yield non-degenerate templates.

The test methodology was to first select a given PTM template, and then select an in-class image used to generate the template. Next, a Monte Carlo (MC) simulation was used to generate a vector of IID Gaussian noise with a specific variance which was then added to the in-class image. For each MC run, the noise corrupted image was run through the PTM algorithm yielding realizations of the null hypothesis, and the noise only image was also run through the PTM algorithm yielding realizations of the alternate hypothesis. The sample means and variances calculated over 5,000 MC trials were then compared to the analytic results given in (5.4) and (5.5). This yielded Figure 5.11.

Next, the performance prediction analysis was compared to that approximated using the approximated distributions of the test statistics for each hypothesis. In this case, we plot the Type II error or missed detection probability, where the algorithm incorrectly rejects
Figure 5.11: The comparison between our analytic approximations to the mean and variance of the PTM test statistic in (3.16) under both hypotheses for the target ID #1 viewed from $0^\circ - 5^\circ$ aspect. The analytic solution matches the empirical well.

It is important to note, that our approximations are not exact but the error probability is no worse than 3% throughout the entire performance curve for this test case. This typically can occur when higher order moments exist in the distribution of the test statistic, which we don’t consider. We attribute this discrepancy to the relatively small number of samples used, $|\Gamma_{\text{peak}}| = 132$, in calculating the resulting test statistics. Our previous experiments with large, high-resolution SAR images yielded approximations matching the empirical simulations almost exactly. Nonetheless, we take this approximation accuracy as a positive indicator that the approximation is generally scalable across image sizes.
Figure 5.12: The predicted PTM performance and empirical performance. The top plot shows the probability of a failing to classify an in-class target correctly using the PTM algorithm, calculated both in simulation and using our analytic approximations. The bottom plot compares the approximation to the test statistic for a specific noise variance to the empirical distribution. The approximations are able to model the performance seen in empirical simulation to within 3%.

5.4 Performance of MPM Under AWGN in the General $N_q$ Case

The previous sections discussed the performance of the algorithms in the $N_q = 2$ case only. We will now extend our analysis of PTM, MPM in the $N_q = 2$ case, to the more general case of an arbitrary number of quantization levels. A fixed threshold will be assumed, noting that the uniform quantization example of Section 5.3 can be used to find the mean thresholds values which can be substituted into (5.87) in this analysis. We will also focus on the scalar version of MPM discussed in Section 3.2.3 as opposed to the vector form of Section 3.2.2. The empirical performance of the two algorithm implementations is compared in 5.4.2 and shown to be approximately equivalent, and the scalar version is preferred due to its simplicity.
The problem begins the same,

\[ H_0 : X_i = Q(s_i + w_i; N_q, \tilde{\gamma}) \]  
\[ H_1 : X_i = Q(w_i) \]

for \( i \in [1 \ldots K] \) where \( K \) is the number of pixels, \( s_i \) is the image to be classified before quantization, and \( w_i \sim N(0, \sigma^2) \) where \( \sigma^2 \) is assumed known. The function \( Q(\cdot; N_q, \tilde{\gamma}) \) is the fixed \( N_q \) quantization operator written as

\[
Q(t; N_q, \gamma_1, \ldots, \gamma_{N_q-1}) = \begin{cases} 
1, & t \leq \gamma_1 \\
2, & \gamma_2 < t \leq \gamma_3 \\
& \vdots \\
N_q & t > \gamma_{N_q-1}
\end{cases}
\]

and the selection of quantization thresholds, \( \tilde{\gamma} \), is chosen linearly across the peak image value and the peak sidelobe level. In the case of un-windowed SAR imagery, the peak sidelobe level is approximately -13 dB, therefore the value 13 dB down from the peak is chosen as \( \gamma_1 \). The remaining values are linearly selected between this value and the peak of the image.

The approximation of the MPM test statistic in the general \( N_q \) case is direct extension of the approach given in Section 5.2.1 and we proceed the same, by calculating the mean and variance of the per-pixel property under each hypothesis. Beginning with the mean we have

\[
E\{b_i\} = \sum_{q=1}^{N_q} \Pr(X_i = q) \frac{(1 - \hat{p}_{i,q})^2 - \hat{E}\{t_{i,q}\}}{\sqrt{\text{Var}\{t_{i,q}\}}} 
\]
and for the variance

$$\text{Var}\{b_i\} = \sum_{q=1}^{N_q} \text{Pr}(X_i = q) \left( \frac{(1 - \hat{p}_{i,q})^2 - \hat{E}\{t_{i,q}\} - E\{b_i\}}{\sqrt{\text{Var}\{t_{i,q}\}}} \right)^2$$  \hspace{1cm} (5.89)

where $E\{b_i\}$ was given in (5.88). These equations are fully specified using the moments given in (3.36) and (3.37) except for the $\text{Pr}(X_i = q)$ terms. These terms are hypothesis dependent and calculated directly using the noise power, $\sigma^2$, the thresholds, $\vec{\gamma}$, and in the case of the null hypothesis of (5.85) the underlying pixel intensity, $s_i$. Finally, with expressions for these per-pixel statistics, the distribution of the summed MPM test statistic will be Normal with mean

$$E\{B\} = \frac{1}{\sqrt{K + \hat{C}}} \sum_{i=1}^{K} E\{b_i\}$$  \hspace{1cm} (5.90)

and variance

$$\text{Var}\{B\} = \frac{1}{\sqrt{K + \hat{C}}} \sum_{i=1}^{K} \text{Var}\{b_i\}$$  \hspace{1cm} (5.91)

where $E\{b_i\}$ and $\text{Var}\{b_i\}$ were given in (5.88) and (5.89) and are functions of the hypothesis dependent probabilities $\text{Pr}(X_i = q)$ which are effectively functions of the variable we are interested in studying $\sigma^2$. The calculation of these probabilities is direct from the normal distribution function and were described in (5.8), (5.9), (5.6), and (5.7) for the $N_q = 2$ case.

### 5.4.1 Verification of General $N_q$ Case Performance Prediction

Maintaining the trend, we again verified our prediction approach using the data generated in Section 5.1.1 for a single target. For a target from the database, we selected an in-class training image, corrupted it with AWGN, and calculated the summed MPM statistic using
5,000 trials at each noise variance. The results of our second-order approximations to the
distribution of the test statistic under each hypothesis are shown in Figures A.1 to A.7
which are included in Appendix A.

From examining the figures, it is clear that we have accurately modelled the first two
moments of the MPM test statistic in the general case. We must now verify that the second-
order approximation holds and that the performance can be predicted using only the first
two moments of the algorithm. This is shown in Figures A.8 through A.14 which are again
included in Appendix A. The top plot shows the predicted performance against that seen in
empirical simulation as a function of the noise power, $\sigma^2$. The black vertical line indicates
the noise power that yielded an approximate probability of missed detection (target being
declared noise) of 5%. This is an important distinction to make, as we are concerned with
the operating region where there is a relatively small probability we will miss a target, and
we see that the second order approximation holds in this region. It generally holds across
the performance curve, however we see higher order moments start to effect our prediction
approach for higher values of $N_q$.

Additionally, we see that there is an equivalent SNR gain for using more quantization
levels; the image can tolerate more noise before performance falls as the number of quan-
tization levels increase, at least at a $P_{MD} \approx .05$. However, we stress that performance as a
function of $N_q$ is a highly multivariate relationship and just because this behavior was seen
in this specific test case, does not mean it can be generalized to apply to every case. In fact,
the empirical results of Section 5.4.2 show a different trend in performance as a function
of $N_q$ because a different rejection threshold for the Z-test was utilized.

**5.4.2 Effect of Reward Minimization on MPM**

In order to compare the scalar and vector form implementations of the MPM algorithm and
demonstrate the effects of not-applying the tuning procedure described in Sections 3.1.3
and 3.2.4 additional Monte Carlo simulations were performed by selecting an in-class
image used to train each algorithm, corrupting it with AWGN according to (5.85), and calculating the MPM statistic without tuning as in (3.38) and with tuning applied as in (3.39) for both vector and scalar algorithm implementations. 5,000 trials were used for each value of \(\sigma^2\). This also yielded another test case to test the effects of \(N_q\).

The performance results for the un-tuned implementations are shown in Figure 5.13. This plots needs discussion for several reasons. The first being that the two implementations, those in Section 3.2.2 and Section 3.2.4, lead to nearly equivalent results despite their differences. This observation is consistent across all our experiments and we conclude that the scalar implementation is preferred due to its simplicity both in notation and computation. Second, this plot would indicate that there is an effective SNR gain with additional quantization levels as more noise can be tolerated without a loss in classification performance moving from 2 to 4, and finally to 8 quantization levels. However, \(N_q\) choices of 16, 32, 64, and 128 yield degenerate results as they never fail to reject the null hypothesis even when the noise completely drowns out the underlying pixel values. The explanation for this is shown in Figure 5.14. This plot shows the mean shift of the null distribution as a function of the noise power. As we have not accounted for the algorithm tuning described in Section 3.2.4, background pixels are receiving rewards. As there are significantly more background pixels, these rewards outweigh the penalties due to pixel mismatch. The behavior worsens are more quantiles are used effectively shifting the null distribution so far to the left that the one-sided Z-test shown in Figure 3.2 will never yield a rejection for the higher values of \(N_q\).

The performance results with the tuning applied as described in Section 3.2.4 are given in Figure 5.15. Again, we note that the two implementations yield nearly equivalent performance results. In terms of an optimal number of quantization levels, this plot shows significant improvement moving from 2 to more quantization levels. For \(N_q > 2\), a more interesting phenomenon tends to emerge and the performance curves change shape. We see that for the \(N_q = 8\) and \(N_q = 16\) cases that the curves are the steepest and they tend
to flatten as the number of quantization levels increase. The $N_q = 8$ and $N_q = 16$ cases can handle more noise before performance is affected but when once it is, the performance drops off relatively quickly. For the $N_q = 128$ case, we see that noise tends to yield some performance decrease sooner, however it takes almost an order of magnitude more noise power to before the transition is complete, compared to cases with a smaller number of quantization levels.

The subsequent mean shifts of the of the MPM implementations with tuning applied are shown in Figure 5.16. The plot shows that even with accounting for background statistics, negative penalties can still be applied, however the performance results obtained and shown in Figure 5.15 in this case are still coherent, as opposed to those obtained in the un-tuned test cases.

We note that the MPM label in Figures 5.13-5.16 refers to the vector form implement-
Figure 5.14: The mean shift of the null distribution as a function of AWGN without tuning applied. This negative mean shift is due to “rewarding” for background pixels resulting in erroneous performance conclusions. The higher quantization runs will never yield a rejection despite the dominance of the noise in the test images. This behavior worsens as more quantization levels are used.

In summary, we have presented a large amount of work here. The initial effort of characterizing the performance of PTM, QGM, and QMSE under AWGN was presented in section 5.2. This analysis assumed fixed quantization thresholds were available. This initial analysis was extended to the case of “uniform quantization” in section 5.3 for the PTM matching only.

Next, the analysis was extended to the general $N_q$ case in section 5.4. This required proposing a quantization rule that generally assumes target pixels can be segmented from
Figure 5.15: Empirical performance of the MPM algorithms as a function of AWGN with tuning applied. This plot indicates that a significant performance improvement can be had moving from $N_q = 2$ to more quantiles, however beyond this point the performance curve changes shape. We stress that this behavior applies to this case only and distilling a general rule of thumb remains difficult due to the highly multivariate nature of the problem.

background pixels, which allowed the general formulation of a tuning rule required to minimize rewarding for background pixels. This was presented in section 5.4.2.

All of the analysis was presented using the AFRL CV dataset for the exception of the “uniform quantization” exception which utilized the ARL Comanche dataset.
Figure 5.16: The mean shift of the null distribution as a function of AWGN without tuning applied. This negative mean shift is due to “rewarding” for background pixels resulting in erroneous performance conclusions. The higher quantization runs will never yield a rejection despite the dominance of the noise in the test images. This behavior worsens as more quantization levels are used.
Chapter 6

Performance Under Target Degradation

The previous Chapter, Chapter 5, detailed the performance of 3 algorithms under AWGN. Here, we focus on MPM specifically as described in Section 3.2 and the scalar interpretation given in Section 3.2.3 and add another dimension to the analysis yielding performance under a general target degradation model. This analysis begins with extending the image-to-template AWGN performance analysis to account for both AWGN and target degradation. This approach is then extended to a template-to-template performance model under target degradation only.

We have opted to consider a general target degradation model, the benefit of which is the generality. MPM is a model-based ATR algorithm, which assumes in-class images are realizations of a generative statistical model [5] [26] [3] [25] [27], specifically a Dirichlet-Multinomial (DM) model. Therefore, we propose modelling degraded target pixel responses using another DM model, with arbitrary parametrization, yielding a two-component mixture model. Under this model, degraded target pixels will be realized with a probability of degradation, $P_{\text{deg}}$, and the non-degraded pixels with $1 - P_{\text{deg}}$, where the probability of degradation is assumed independent of pixel location. This approach allows for a the DM parametrization $N_q$ degrees of freedom, allowing the model to be tuned to match a specific scenario.
We will see in the following sections, that in concert with the MPM tuning rule, this model tends to yield degraded ideal point responses (IPRs) in SAR imagery, that could correspond to battle damage or even occluded scenarios. In the battle damage scenario, a scattering primitive may be damaged or even removed, yielding a response that differs from expected. Occlusion, or obscuration, refers to portions of the signature being occluded from the sensor and therefore signature samples associated with occluded portions of the target again will yield unexpected responses. Similar rationale holds for EO/IR imagery. The generality of this approach is also its weakness, and there is a risk of an overly general model being unable to accurately model real-world phenomena, for example an occlusion model having some scattering return of it’s own. It will be shown that the MPM tuning rule given in Section 3.2.4 mitigates some of these concerns.

The real weakness of the approach is in the independence assumption. This is especially true for the occlusion case as most real-world occlusion processes will exhibit some type of structure. Regardless, the IID case is a logical starting point that intuitively works well with SAR data in a general target degradation scenario and potentially even for EO/IR. Future work can then extend this approach to more complicated DM models capable of modelling some correlation structure between pixels [22].

This chapter will be organized as follows. The MPM algorithm and underlying models were already introduced in Section 3.2 and 3.2.3 and the chapter will begin by formally introducing the mixture model for simulating degraded target responses under AWGN and degradation in Section 6.1.1 for the case of image-to-template match scores. Next, the performance prediction analysis is presented in Section 6.1.2 and verified in Section 6.1.3 using data from the AFRL CV dataset. This analysis is then extended to the template-to-template match scores under degradation only and verified using the ARL Comanche dataset in Section 6.2.
6.1 Image-to-Template Performance Prediction Under Degradation

We will begin with extending the previous AWGN model to include a dimension associated with degradation. We note that this analysis considers the case of image-to-template performance for a single noise corrupted and degraded test image and how classification performance is effected as the the noise and degradation parameters vary. This will be extended to account for template-to-template performance in Section 6.2.

6.1.1 A Model for Degradation

To account for degraded pixels, we extend the AWGN model given in Section 5.2 to a two-component mixture model [56]. We note that mixture models have been successfully utilized previously in the study SAR ATR [57]. This model allows the image chip under occlusion to be written as

\[ H : X_i = (1 - P_{\text{deg}})Q(s_i + w_i) + P_{\text{deg}}X_{i}^{\text{deg}} \tag{6.1} \]

where the test image, \( X_i \), is a simple two-component mixture consisting of quantized IID AWGN corrupted image, \( Q(s_i + w_i) \), occurring with probability of \( 1 - P_{\text{deg}} \) and an occluded DM component, \( X_{i}^{\text{deg}} \), occurring with \( P_{\text{deg}} \), where \( P_{\text{deg}} \) is the probability of any given pixel being degraded. Again, \( P_{\text{deg}} \) is assumed independent of pixel location, \( i \).

This model has several benefits and only requires a single assumption. First, by allowing an arbitrary model for the occluded target representation we can account for different occlusion processes. For example, we can assume that the occluded pixels will be of primarily of low intensity and thus consist of primarily low quantile realizations, or the alternate and assume they will primarily be high quantile, or assume all quantile realizations will be equally likely. This allows the model to be tuned if a priori information was avail-
Figure 6.1: Non-degraded, noise-free quantized SAR image. The pixels colored blue are not considered by the MPM algorithm and do not exist in $\Gamma_{\text{peak}}$. Note, the quantized ideal sinc nature of the IPRs.

Figure 6.2: 50% degraded, noise-free quantized SAR image. The pixels colored blue are not considered by the MPM algorithm and do not exist in $\Gamma_{\text{peak}}$. Note, the degradation of the IPR response.

able for the scenario of interest. This occluded component is parametrized as the in-class target model in (3.20). We note that, another distribution could be used and the approach described in Section 6.1.2 will be the same, with the reservation that care should be taken the modified model still yields an approximately unimodal test statistic.

The weakness in assuming that the probability of degradation, $P_{\text{deg}}$, is IID distributed across the imaged scene has been mentioned. However, the utility of this model is strengthened by the MPM tuning rule described in Section 3.2.4 and illustrated in Figures 6.1 and 6.2. The primary concern with this model is the “shotgun” like nature of the degradation process which is spatially invariant across the scene. As the tuning rule only considers quantile realizations relatively constant across all training images, large portions of the test image are ignored by the MPM algorithm. This is illustrated in the figures as the blue region, whose per-pixel penalty values are not included into the summed MPM test statistic. Therefore, the degraded pixel values at these locations are not shown.

Still referring to Figures 6.1 and 6.2, the effect of the degradation model on the salient target pixels can be inspected. The ideal sinc-like nature of the unmodified IPRs are shown in Figure 6.1 and the degraded IPRs in Figure 6.2. These figures used $N_q = 4$ and a $P_{\text{deg}}$...
value of .5. The degradation DM component was parametrized with statistics matching the background statistics, i.e. the degraded DM process was parametrized by placing counts equal to the number of training images in the lowest quantile plus the prior as described in (3.20). The effects of the degradation model are apparent in the figures, and pixels are more likely to realize lower quantization levels and the ideal mainlobe-sinc responses are modified. The degraded image could easily be considered the result of damage to the underlying scattering structure or an absorber type occlusion model.

Admittedly, more work needs to be done in the formulation of accurate degradation models, i.e. how to parametrize the DM model to correspond to specific degradation cases. However, this general approach allows us to proceed with analytic approaches and may even be an accurate model for certain cases like targets placed under thick foliage yielding a diminished RCS response or moderate battle damage effecting the ideal IPR response from certain scattering primitives like dihedrals and trihedrals.

6.1.2 Performance Analysis of MPM Under AWGN and Degradation

Examining the model in (6.1) we see it is similar to the AWGN only model of the previous chapter but now includes a dimension associated with target degradation. Therefore, we are interested in obtaining an analytic performance expression as not only the AWGN parameter, $\sigma^2$, but also the probability of degradation, $P_{\text{deg}}$, are varied. This yields a performance surface in two dimensions as opposed to the one dimensional curves of the AWGN analysis in Chapter 5. It is important to note, that the performance is also a function of the underlying pixel intensity values, quantization thresholds, and also the DM variables used to parametrize the occluded model, however these variables are considered fixed, as is the DM training set determining the MPM per-pixel penalties.

As in the previous chapter, the distribution of the summed MPM statistic will be approximated as Normal and the task becomes calculating the first and second order moments as a function of the parameters of interest. This allows the performance to be obtained from
the Z-test utilized by MPM, shown in Figure 3.2 by calculating probability that the test
statistic will exceed the critical value as a function of $\sigma^2$ and $P_{\text{deg}}$.

Although the AWGN and degraded processes are IID, the underlying pixel values are
not identically distributed and neither are the per-pixel penalty values. Therefore, we begin
by calculating the first two moments of the per-pixel penalty statistics. As the model in (6.1)
is a mixture model, this is performed for noise only case where $P_{\text{deg}} = 0$ and degradation
only case where $P_{\text{deg}} = 1$. The mean and variance of the per-pixel penalties under the
noise only hypothesis will be denoted as $E\{b_i|N\}$ and $\text{Var}\{b_i|N\}$ and the degradation only
hypothesis as $E\{b_i|D\}$ and $\text{Var}\{b_i|D\}$.

This allows standard mixture model results [56] to be used to write the per-pixel
penalty moments under the mixture distribution of (6.1) as

$$E\{b_i|H\} = (1 - P_{\text{deg}})E\{b_i|N\} + P_{\text{deg}}E\{b_i|D\}$$

(6.2)

and

$$\text{Var}\{b_i|H\} = (1 - P_{\text{deg}})\text{Var}\{b_i|N\} + P_{\text{deg}}\text{Var}\{b_i|D\}$$

$$+ (1 - P_{\text{deg}})P_{\text{deg}}\left[E\{b_i|N\}^2 + E\{b_i|D\}^2 \right] - 2E\{b_i|N\}E\{b_i|D\}$$

(6.3)

where the law of total variance was utilized to yield the variance of the mixture distribution
[38].

In accordance with (3.38) and (3.39) the moments of the summed test statistic under
the model in (6.1) can then be written as

$$E\{B|H\} = \frac{1}{\sqrt{K_{\text{peak}}}} \sum_{i \in \Gamma_{\text{peak}}} E\{b_i|H\}$$

(6.4)
and

\[
\text{Var}\{B\mid H\} = \frac{1}{K_{\text{peak}}} \sum_{i \in \Gamma_{\text{peak}}} \text{Var}\{b_i\mid H\}. \tag{6.5}
\]

Therefore, the task is to derive expressions for the moments under each case (E\{b_i\mid N\}, \text{Var}\{b_i\mid N\}, E\{b_i\mid D\}, and \text{Var}\{b_i\mid D\}). As the analysis was already utilized in the previous chapter to yield the performance under AWGN, the E\{b_i\mid N\} and \text{Var}\{b_i\mid N\} terms can be found in (5.88) and (5.89), and in notation consistent with the current analysis

\[
E\{b_i\mid N\} = \sum_{q = 1}^{N_q} \text{Pr}(X_i = q) \frac{(1 - \hat{p}_{i,q})^2 - \hat{E}\{t_{i,q}\}}{\sqrt{\hat{\text{Var}}\{t_{i,q}\}}}
\]

and for the variance

\[
\text{Var}\{b_i\mid N\} = \sum_{q = 1}^{N_q} \text{Pr}(X_i = q) \left( \frac{(1 - \hat{p}_{i,q})^2 - \hat{E}\{t_{i,q}\}}{\sqrt{\hat{\text{Var}}\{t_{i,q}\}}} - E\{b_i\mid N\} \right)^2
\]

where it is noted that the \text{Pr}(X_i = q) will be different for each pixel and calculated as

\[
\text{Pr}(X_i = q) = \Phi\left( \frac{\gamma_q - s_i}{\sigma} \right) - \Phi\left( \frac{\gamma_{q-1} - s_i}{\sigma} \right)
\]

as defined in (5.9) where \Phi is the cumulative normal distribution function and the \gamma terms are the quantization thresholds. Therefore, the per-pixel penalty statistics are the result of summing over each column of the result of a hadamard product of a probability matrix and penalty matrix at each value of \sigma^2. Each entry in the resulting vector is then a random variable approximated as standard Normal with analytic expressions for its sufficient moments.

The analysis for the degradation case again has again already been utilized in the calculation of the per-pixel normalization terms, \hat{E}\{t_{i,q}\} and \hat{\text{Var}}\{t_{i,q}\} from Chapter 3. The \hat{} notation indicates that an estimate of the probability of quantile realizations was utilized
which will again be utilized here yielding

\[
E\{b_i|D\} = \sum_{q=1}^{N_q} \tilde{p}_{q_{\text{deg}}} \frac{(1 - \hat{p}_{q,i})^2 - \hat{E}\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} \tag{6.6}
\]

where \(\tilde{p}_{q_{\text{deg}}}\) is the MMSE point estimate of the underlying random probabilities in the DM model parametrizing the degraded mixture component, \(X^\text{deg}_i\) \[22\]. The MMSE estimates can be written generally as

\[
\tilde{p}_{q_{\text{deg}}} = \frac{\tilde{n}}{\sum_{i=1}^{N_q} n_i} \tag{6.7}
\]

where \(\tilde{n}\) is an \([N_q \times 1]\) length vector consisting of the arbitrary parametrization used for the degraded model. For the verification scenario used in the following sections, \(\tilde{p}_{q_{\text{deg}}} = [N + 1, \ldots, 1]^T/(N + N_q)\) where \(N\) is the number of training images and a prior value of \(\alpha = 1\) was used for each quantile. This is consistent with what a background response would yield in the MPM training process shown in Figure 3.1.

Similarly, the variance of the per-pixel penalties under the degraded case can be expressed as

\[
\text{Var}\{b_i|D\} = \sum_{q=1}^{N_q} \tilde{p}_{q_{\text{deg}}} \left(\frac{(1 - \hat{p}_{q,i})^2 - \hat{E}\{t_i\}}{\sqrt{\text{Var}\{t_i\}}} - E\{b_i|D\}\right)^2 \tag{6.8}
\]

and it is noted that unlike the noise case in (5.9) where the expected quantile realizations differed for each pixel, \(\tilde{p}_{q_{\text{deg}}}\) is identical for each pixel index, \(i\), due to the IID model on \(X^\text{deg}_i\).

Therefore, the distribution of the test statistic under the degradation model in (6.1) is fully specified using the per-pixel moments of each mixture component given in (5.88), (5.89), (6.6), and (6.8) then combined according to (6.2) and (6.3). This yields the distri-
bution on the summed MPM test statistic according to (6.4) and (6.5) as

\[
B \sim N(\mathbb{E}\{B|H\}, \text{Var}\{B|H\})
\] (6.9)

allowing performance to be obtained from the PDF of the normal model by simply calculating the probability that the test statistic exceeds the critical value of the Z-test,

\[
P_{\text{reject}} = \text{Prob}(B > z_\alpha),
\] (6.10)

where \(z_\alpha\) is the critical value of the Z-test, 1.96 for \(\alpha = 0.05\). This performance measure would be the equivalent of falsely rejecting an in-class target.

### 6.1.3 Verification Using the AFRL Civilian Vehicle Dataset

In order to test the performance prediction approach of the MPM algorithm under both AWGN and degradation described above, the training dataset utilized in Section 5.1.1 was re-utilized. To review, the data about 4 primary viewing angles was first interpolated onto a uniformly sampled grid in K-space. The extent was chosen to yield a desired range-dimension and the cross-range extent was chosen to allow N images of a desired cross-range resolution to be chosen by linearly stepping through the available aperture.

For this experiment, the Toyota Tacoma data was used for the \(30^\circ\) elevation data viewed from \(45^\circ\) azimuth. The image formation process yielded \(N = 10\) training images which were then quantized to \(N_q = 4\) quantization levels using the quantization method discussed in Section 3.2.4. This method uses the peak sidelobe level of the peak scatterer plus an additional term to account for interference as the lowest quantization level. This effectively determines the noise floor of the image and the additional quantization levels are chosen linearly between this sidelobe value and peak amplitude value.

One of these quantized training images was shown in Figure 6.1. This figure shows 3 primary scatterers with approximately ideal IPR sinc-responses where the quantization
method was able to effectively segment and mainlobe structure of each response. The peak scatterer is on the right bordered by what appear to be sidelobes. Two more lower amplitude scatterers are there located to the left of this peak scatterer.

This image was then degraded according to the model given in (6.1). The pre-quantized amplitude values, \( s_i \), were corrupted with AWGN noise of a known noise variance quantized, then degraded with the DM model \( X^\text{deg} \) according to \( P_{\text{deg}} \). For 50 noise variance values, \( \sigma^2 \), linearly spaced on the interval \([-14,0] \) and 51 probability of degradation values, \( P_{\text{deg}} \), linearly spaced on the interval \([0,1] \), 5,000 realizations of test images were then realized. These test images were then classified by the MPM algorithm trained using the 10 training images and the values of the MPM test statistic were recorded.

The verification process is then two-fold as mentioned previously. First, we must verify the approximated distribution under the noisy degraded model given in (6.9) is able to correctly predict the the distribution of the MPM test statistic as seen in empirical simulation, i.e. the Normal approximation must be verified. This is shown in Figure 6.3. It is evident from these figures that the Normal approximation to the summed MPM test statistic is accurately modelling the empirical distribution, with some caveats.

These test cases are from the diagonal of the test case matrix, where \( \sigma^2 \) and \( P_{\text{deg}} \) are both increasing from their minimum to maximum values. The first subplot in Figure 6.3 (top-left) is the case associated with zero degradation and an inconsequential noise variance, i.e. the resulting MPM statistic is deterministic and determined by the quantized underlying pixel amplitudes only. In this case, the Normal approximation still approximately holds, where the mean of the distribution is equal to the deterministic value and the variance is just a very small number. The lack of an approximated distribution (dotted red line) is due to this small variance. Additionally, for low values of \( \sigma^2 \) and \( P_{\text{deg}} \), the Normal approximation is poor as shown in Figure 6.4. This deviation between simulated and approximated results does not affect the performance prediction result due to the distribution still being well within the failure to reject region of the Z-test.
The performance prediction results are now shown in Figure 6.5. The predicted performance matches that seen in simulation to within 6% of absolute error. The errors are due to higher order moments present in the simulated data that are not captured in the Normal approximation. These higher order moments become significant as the distribution of the MPM test statistic moves through the critical value of the Z-test, an observation consistent with the whole of this work. Nonetheless, the approximation results are predicting performance and additional moment approximations could be added to the analysis. Additionally, a relatively small number of empirical trials were utilized in these simulations which could be effecting the accuracy of the approximations as well.
Figure 6.4: For small values of $\sigma^2$ and $P_{\text{deg}}$ the normal approximation is not the best fit. This does not affect the performance prediction as the probability that the distribution extends into the rejection region of the Z-test is minimal.

### 6.2 Extension to Template-to-Template Performance Under Degradation

The analysis presented in Chapter 5 and above in Section 6.1.1 dealt with image-to-template performance, where we were able to analytically approximated the MPM test statistic as a test image was modified according to a certain model. Here, we concern ourselves with how the performance of a target class is modified when it is degraded according to a certain model.
Figure 6.5: Comparison of predicted performance and that seen in simulation. It is evident the approximations are generally capable of predicting performance within an absolute error of 6%.

### 6.2.1 The Modified Degraded Model

Extending the previous image-to-template mixture model in (6.1) to account for the template-to-template scenario

\[
H_{TT} : X_i = (1 - P_{deg})X_{i}^{targ} + P_{deg}X_{i}^{deg}
\]

where \(X_{i}^{targ}\) now refers to the in-class DM model parametrized by the training dataset. Therefore, instead of corrupting a noisy image with degradation, realizations from the in-class model are corrupted with degradation.

While the ability of the generative DM model utilized by the MPM algorithm to accurately model a set in-class images hasn’t been adequately shown, what we will refer to
as the model fit problem, it is what the algorithm assumes. This simplifies the resulting analysis greatly as by design, the resulting per-pixel penalty terms under the target mixture component will be standard Normal. The degraded mixture component approximations are the same as in Section 6.1.2.

Therefore, with some minor modifications the required analysis has already been presented. The terms in (5.88) and (5.89) previously, are now equal to 0 and 1 respectively and the remainder of the analysis is the same.

### 6.2.2 Verification Using the ARL COMANCHE Dataset

In this case, a training dataset was created using the ARL Comanche dataset introduced in Section 4.2. In order to circumvent the difficulties in Section 5.1.2, the training images were re-registered in the quantized domain. From experimentation, it was determined that the provided target chips were registered using a simple correlation procedure with integer shifts [58]. In Section 5.1.2 it was noticed that after quantization the raw data tended to yield “degenerate” templates where the resulting empirical probabilities parametrizing the MPM in-class DM distributions did not yield a suitable number of salient or peak pixels as defined in (3.41). Because of this, the data was registered after quantizing to $N_q = 2$ levels. The pixel values were first normalized to the range $[0, 1]$ and quantized such that pixels values less than 0.99 were quantized low. This process is essentially aligning the highest intensity pixel values and yielded dramatically lower mean-squared error (MSE) scores in the un-quantized domain for poorly registered raw images at the cost of slightly larger MSE scores for images already registered well as seen in Figure 6.6.

These registered images were quantized to levels: $N_q = \{2, 4, 8, 16, 32, 64, 128, 256\}$. After normalizing the data to the interval $[0, 1]$, the quantization scheme was defined by first selecting the lowest threshold to generally segment between target and background pixels consistent with the assumptions required in Section 6.1.1. The remaining thresholds were then chosen linearly from the minimum threshold to 1. It is noted that what we term
Figure 6.6: The raw data was already registered using a simple correlator, however this registration did not translate well to the quantized image domain. By first registering to quantized data in an $N_q = 2$ scheme, poor registration results were improved in the MSE sense with only a slight increase in MSE scores for images already matching well yielding more stable MPM template distributions.

A “uniform quantization” scheme is recommended in the literature when using the MPM algorithm [3]. This scheme ensures that each quantile value is realized “uniformly” and adaptively selects the thresholds independently for each image. For example in an $N_q = 4$ scheme, there would be an equal number of pixels at each quantization level. This process requires a mathematically cumbersome quantization function and we have found that by running histogram equalization on each image [59], fixed thresholds can be used in lieu of random. An example training set for an $N_q = 4$ quantization scheme is shown in Figure 6.7 with histogram equalization applied to each training image in pre-processing.

These datasets at each quantization level were then used to parametrize the class-conditional MPM distributions using the training process shown in Figure 3.1 by calculating the empirical probabilities at each pixel, $\hat{p}_i$, which along with the number of training images, $N$, parametrizes the distributions.

The verification process is again two-fold as mentioned and we will focus on results
Figure 6.7: The 33 quantized training images ($N_q = 4$) used in a verification test case. Histogram equalization was used on the pre-quantized data to ensure that quantiles were approximately uniformly distributed across each training image. Additionally, the first quantization level is generally segmenting between target and background pixels.

from a $N_q = 4$ scheme for a single target class only. First, we must verify the approximated distribution in (6.9) under the model in (6.11) is able to correctly predict the distribution of the MPM test statistic as seen in empirical simulation. This was again verified in Monte Carlo (MC) simulations by generating values of the MPM test statistic from realizations of the model in (6.11). These simulations used 10,000 trials for $P_{occ} \in [0 : 0.1 : 1]$.

Figure 6.8 shows that the analysis presented is accurately predicting the mean and the variance of the final test statistic for all values of $P_{deg}$. Figure 6.9 then shows that these moments are sufficiently modelling the true distribution of the MPM test statistic yielding an accurate performance approximation.
Figure 6.8: Moments of MPM test statistic and their approximations under degradation in the template-to-template case.

Figure 6.9: Performance and approximated performance of MPM under degradation in the template-to-template match score case. These plots show that the second order approximation holds for all values of \( P_{\text{deg}} \) and yields an accurate performance approximation.


6.3 Chapter Summary

In conclusion, we have presented an approach that extends the AWGN performance prediction approach of the MPM algorithm to include a dimension associated with target degradation. A DM model was used to parametrize the degradation for several reasons, the primary being generality. The DM is parametrized by several variables, allowing degrees of freedom for the model to be tuned to specific cases of degradation such as battle damage or occlusion. An IID assumption was utilized, which does weaken the applicability of the model, however the tuning considerations of the MPM algorithm lends credibility to the model as degraded image pixels outside of the salient set of pixels considered by the MPM algorithm are inconsequential and ignored in the analysis. The analysis was verified using AFRL CV data and shown to be valid. This analysis was extended to a template-to-template performance prediction approach which was verified using ARL Comanche data.

The hope is that given more study of target degradation models, the basic analysis here can be extended to specific cases of operating environment, where degradation parametrizations correspond to specific operating environments and cases. Additionally, this analysis could be extended to handle structured occlusion cases, at the cost of considering the variables parametrizing the correlation structure.
Chapter 7

Performance Under Individual Point Response (IPR) Variations

In the previous chapters, the parameters of these class-conditional DM distributions were able to be written as an explicit function of an operating condition of interest, AWGN in Chapter 5 and occlusion in Chapter 6. This allowed the performance to be approximated as a direct function of the parameters responsible for modelling the operating condition. Often an explicit parametrization is intractable and a more general approach is required. Therefore, by training in-class templates as an operating condition is varied, we can predict the performance at each discrete setting, effectively sampling the performance curve.

This general approach will be used to explore the role of the ideal point response (IPR) of synthetic aperture radar (SAR) imagery in MPM classification performance. SAR is essentially sampling an imaged scene in the spatial-frequency domain (K-space) which allows the image to be recovered using a Fourier inversion operation [49]. Under this imaging paradigm, ideal isotropic point targets will appear as sinc responses in the resulting image, typically referred to as the ideal point response (IPR) [60] and the mainlobe width of the sinc function for an ideal point target will be inversely proportional the size of the sampled K-space domain. Therefore, the larger K-space domain, the smaller the
mainlobe width of point target responses and the more separable closely spaced scatterers are in the image domain. It is for this reason that the IPR is commonly associated with the “resolution” of SAR imagery and is typically termed the “Rayleigh resolution” [61].

Given a sampled K-space region, it can decomposed into collections of smaller regions called sub-apertures. Images can then be formed using each of these sub-regions yielding separate looks albeit at a lower resolution (larger mainlobe width due to smaller K-space extent). Alternatively, these sub-resolution images can be coherently combined to form a single image at a higher resolution. This procedure has been utilized to speed up the image formation process [62], yield additional features to aid in the classification problem [63] [57], or estimate clutter processes useful for both segmentation and detection operations [64] [65] [66]. Here, we use this process to study trade-off in the number of looks versus resolution as relates to classification performance for the MPM algorithm. As expressing training images as an explicit function of IPR is difficult, the general approach will be utilized to predict performance.

Additionally, with the AWGN analysis presented in Section 5.4 the performance loss under IPR variations can be compared to performance loss under AWGN, allowing the IPR variation to be expressed as an equivalent loss in signal-to-noise ratio (SNR).

The chapter will be organized as follows. First, the general approach to predicting MPM performance will be described in Section 7.1 and then verified in Section 7.2 using the AFRL CV dataset. The latter section utilizes the MPM performance analysis under AWGN in Section 5.4 to map increase in IPR width to an equivalent SNR loss. Next, Section 7.3 discusses these results as well as providing guidance on how to configure MPM for increased robustness under IPR variations.
7.1 General MPM Performance Prediction Approach

We will now derive a general performance prediction approach for the case that test samples are assumed realizations of one parametrization of a DM model and will be classified as belonging to a separate DM distribution using the MPM algorithm. Recalling Section 3.2, it was stated and direct to verify, that the scalar test statistic associated with test samples originating from the in-class DM distribution will be normally distributed with zero mean and unit variance. Here, we assume the test samples did not originate from the in-class DM distribution but from a different DM parametrization. As in the other cases, we again assume the MPM test statistic will still be normally distributed and the task becomes approximating the mean and variance of the MPM statistic as a function of the alternate parametrization. Unsurprisingly, we have already shown the majority of the required analysis in previous chapters and deriving a performance expression will be relatively direct.

The major benefit to this approach lies in the ability to now express the parameters of an MPM distribution as an implicit function of an operating condition of interest. The baseline AWGN performance case in Section 5.4, where the counts or empirical probabilities at each pixel were expressed as a function of the noise variance, yielded a performance expression that is an explicit function of the OC of interest. In more difficult cases, it is intractable to express the DM parametrization as a direct function of the OC of interest as is the case with the IPR, and a compromised performance prediction approach is required.

This compromise approach requires empirically parametrizing the MPM template of interest at each OC value, but sampling from them is not required to generate a performance measure. In concert with Section 5.4, the resulting performance under the OC or variation of interest can then be mapped into a performance equivalent SNR loss. This analysis can also be used to show the intrinsic separability of MPM class-conditional templates, as opposed to performance as a function of OC. Therefore, this result will be referred to as the general MPM template-to-template performance prediction approach.

To begin, it is assumed two MPM parametrizations are available which have been
alternatively referred to as templates. These templates are parametrized by the counts and number of training images, or equivalently the empirical probabilities, as well as the prior as shown in (3.20). We then have samples drawn from the test distribution or template, \( Y_i \), that are to be classified according to the reference template, \( X_i \).

The mean of the MPM statistic under the scenario discussed can be calculated from (3.39) as

\[
E\{ B_{Y|X} \} = \frac{1}{\sqrt{K_{\text{peak}}}} \sum_{i \in \Gamma_{\text{peak}}} \sum_{q=1}^{N_q} \hat{p}_{Y,q,i}^Y (1 - \hat{p}_{q,i}^X)^2 - \hat{E}\{ t_i|X \} \sqrt{\hat{\text{Var}}\{ t_i|X \}}
\]

(7.1)

where \( \hat{p}_{Y,q,i}^Y \) is the MMSE estimate of category realizations from the test distribution given in (3.22), as mentioned previously. The remaining terms are calculated from the reference template.

Using \( \text{Var}\{ B \} = E\{ B^2 \} - E\{ B \}^2 \), the variance of the MPM statistic can be then be written as

\[
\text{Var}\{ B_{Y|X} \} = \frac{1}{K_{\text{peak}}} \sum_{i \in \Gamma_{\text{peak}}} \sum_{q=1}^{N_q} \hat{p}_{Y,q,i}^Y \left( \frac{(1 - \hat{p}_{q,i}^X)^2 - \hat{E}\{ t_i|X \}}{\sqrt{\text{Var}\{ t_i|X \}}} \right) - E\{ B_{Y|X} \}^2.
\]

(7.2)

Therefore, as in all the previous cases, the performance measure of interest can be obtained from the normal probability density function and the critical value of the Z-test.

### 7.2 Case Study Using the AFRL Civilian Vehicle Dataset

Now, we will use the general performance prediction approach given in Section 7.1 to investigate the role the IPR plays in MPM classification performance. It was mentioned that sub-aperture processing can be used to form multiple looks of a scene imaged by a SAR sensor, albeit each sub-aperture image will have a larger IPR than if the whole aperture were processed coherently to form a high resolution image. The general MPM performance
prediction will be used as the vehicle for studying these effects. By parametrizing in-class target distributions at each value of the IPR OC, which will be described in Section 7.2.1, the results in Section 7.1 can then be used directly to plot the expected performance between test images realized from a distribution at one IPR relative to a reference IPR.

The real difficulty of this problem lays in the underlying physics-based scattering principles which do not permit an analytically tractable analysis. Typically, computational electromagnetic (CEM) scattering simulations are used to generate the scattered field of a target, which is effectively the SAR phase history. Even if the approximations required to implement the forward scattering problem were perfect \([67] [68]\), most SAR imaging algorithms then utilize an idealized isotropic point scattering assumption \([69]\), if not additional approximations in the name of computational efficiency \([49]\). Therefore, we assume it is intractable to model pixel intensity values as functions of even ideal point scatterers \([70] [71]\), not to mention even more complicated canonical scattering primitives \([72] [73]\). It is important to note, that intractable does not mean impossible, and using a canonical scattering based approach is still viable but it is a complex process and still does not directly afford a compact performance expression.

By considering template parametrizations where the variation of interest is implicit in the MPM training process, efficient performance characterizations are afforded without having to resort to costly empirical simulations. If the OC permits a tractable expression, – i.e., the counts \(N_q^i\) in \([3.20]\) can be written as a compact function of the OC space of interest – an even more direct compact expression can be acquired as was discussed in Chapters 5 and 6. However, that is not the case considered here.

### 7.2.1 Development of a Training Image Database

In order to verify our analysis, a set of 4 vehicles was selected from the AFRL Civilian Vehicle dataset: the Toyota Camry, the model year 1999 Jeep, the Mazda MPV, and the Toyota Tacoma \([41]\). These vehicles were chosen to sample from the extent of available
vehicle types (car, SUV, minivan, and pickup truck).

The 30° elevation data was used and image formation done by the polar format algorithm (PFA) [49] for the targets viewed at the off-cardinal azimuths of 45°, 135°, 225°, and 315° on the center. The data was projected to the ground plane and interpolated to a uniform grid of 128 x 128 phase history samples capable of producing an image with a nominal 7-inch x 7-inch resolution in the range and cross-range dimensions. The phase history data was also rotated into a common image frame, eliminating nuisance parameters consisting of unknown target rotation and translation. This process is equivalent to that used to form the training datasets under the AWGN scenario described in Section 5.1.1.

Therefore, if the whole aperture was processed a single image at approximately 7-inch x 7-inch resolution would be obtained. In order to parametrize templates with different IPRs, the phase history was decomposed into $2^D$ mutually exclusive sub-apertures by simply dividing each sub-aperture in half at each level $D$ of decomposition $D \in \{0, 1, 2, 3\}$. This task was easy as the number of available K-space samples was a power of 2. An illustration of this decomposition process is shown in Figure 7.1. Each of the $2^{2D}$ images associated with each level of decomposition would then have $1/2^D$ the resolution of the level 0 decomposition image or alternatively $2^D$ the IPR.

The resulting images at the $D = 1$ level of decomposition are shown in Figure 7.2 along with the baseline resolution ($D = 0$) image, and the IPR broadening is apparent from examining the images. This process yields $\{1, 4, 16, 64\}$ training images associated with each level of decomposition $D \in \{0, 1, 2, 3\}$. These images were formed using the PFA algorithm and over-sampled by a factor of 2 relative to the baseline resolution image.

As this only yielded 4 testable IPR variations, a second experiment was conducted to more finely sample MPM performance as a function of IPR. This required utilizing overlapping apertures to form images at IPR factors $\{1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1\}$ and IPR factor is defined as the fraction of the available K-space extent used to form the baseline or high resolution image. We note that this does complicate the understanding as
Figure 7.1: Illustration for our decomposition scheme for 4 levels of decomposition, $D \in \{0 \ldots 3\}$. At each level, the available K-space aperture is divided in half in each dimension forming $2^{2D}$ sub-images each having $2^D$ wider IPRs than the baseline resolution image. These sub-images will then be used to form MPM DM distributions parametrized by IPR, allowing performance prediction using the general template-to-template performance prediction approach.

neighboring sub-images can contain the same scattering response due to utilizing the same portions of the aperture, however we do not consider it further.

The number of training images for each IPR factor for both experiments are shown in Table 7.1.

In accordance with the MPM training procedure discussed previously, the resulting training images were first converted to a log-scale ($20 \log_{10} |\cdot|$) and quantized to $N_q = 4$ quantization levels. The first threshold was chosen based on the peak sidelobe level of the dominant scatterer in the scene, -13dB in the case of the un-windowed imagery used here, and the remaining thresholds were then chosen linearly between the peak sidelobe level and the amplitude of the dominant scatterer. These training images were then used to populate MPM templates as shown in Figure 3.1 parametrized by the quantile counts at each pixel $N_q^i$ or alternatively the empirical probability, $\hat{p}_i$, and $N$, the number of training images, as shown in (3.20). We note that the results presented focus on the data for the Toyota Tacoma.
Table 7.1: Number of images used for each IPR factor in experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>IPR Factor</th>
<th># Training Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mutually Exclusive</td>
<td>1/1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1/4</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>1/8</td>
<td>64</td>
</tr>
<tr>
<td>Overlap</td>
<td>1/1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>64</td>
</tr>
</tbody>
</table>

Figure 7.2: The baseline resolution image and the 4 resulting images at the first level of decomposition for the mutually exclusive sub-band experiment. This corresponds to the top-right decomposition in Figure 7.1. The IPR broadening is apparent and the anisotropic nature of the imagery is starting to become evident.

viewed from 45° azimuth.

7.2.2 Verification of Performance Prediction Method under IPR Variation

Following the methodology of Section 7.1, given a series of templates parametrized as a function of operating condition, in this case the IPR, performance can be predicted by approximating the MPM test statistic through the second-order. Due to the intractability of modelling pixel quantile counts as an explicit function of the IPR, the performance will not be a direct function of the OC of interest like the AWGN or occlusion analysis, but instead
sample the performance curve at discrete values of the operating condition.

The methodology is similar to the AWGN verification of Section 5.4.1. In this case, 10,000 realizations of each class-conditional distribution were realized, and the value of the MPM test statistic for each realization was calculated using the baseline resolution template. As was the case in the previous section, we must first show that the second-order approximation of the test statistic given in (7.1) and (7.2) approximately holds. This is shown in Figure 7.3 for the experiment using mutually exclusive sub-bands. It is important to note the effect of the number of training images on the approximation accuracy, and for the second and third levels of decomposition, the approximation is a better fit. This is the case where a greater number of training images was used and will be discussed further in Section 7.3.

Therefore, this result then can show the approximated performance as a function of IPR by calculating the probability the approximated distributions exceed the critical value of the Z-test as shown in Figure 3.2. The resulting performance curves are shown in Figure 7.4 for the case of mutually exclusive sub-bands and Figure 7.5 for the case of overlapping sub-bands.

The plots in Figure 7.4 and Figure 7.5 require a bit of explanation. The magenta solid line at the top of the each plot indicates that there is only a 2.5% probability of failing to correctly identify a target if the sub-images are processed coherently (for \( \alpha_z = 0.05 \)). The rationale for this is shown in Figure 7.6. By applying a phase correction to each sub-image then coherently summing them the baseline resolution image can be reconstructed with near-perfect accuracy [62] [63]. Therefore, if the \( 2^{2D} \) sub-images at each level of decomposition were processed coherently it would yield the same template as the baseline resolution at \( D = 0 \) levels of decomposition. Alternatively, if they are processed non-coherently to yield multiple looks of a higher resolution imagery, poorer performance is obtained and can be obtained using the approximation in Section 7.1. The dash-dotted red line shows the results of this approximation and it trends well with the
Figure 7.3: The empirical distribution and the normal approximation at each level of decomposition. It is evident that although some higher order moments exist in the data, the second order approximation is adequately representing the true distribution enough to get a rule-of-thumb performance estimate. Performance is then obtained by calculating the probability the distribution exceeds the critical values of the Z-test as shown in Figure 3.2.

Finally, using the AWGN results, the performance loss due to IPR can be mapped to an equivalent loss of SNR. This can help to compare trade-offs associated with various OCs. SNR is defined to be the ratio of the average per-pixel power of the image, defined as $\sum_i^K s_i^2/K$, to the per-pixel noise power, $\sigma^2$. This was converted to dB using $10 \log_{10}(\cdot)$. 

empirical performance results shown as the dotted blue line. Further discussion on these results will be contained in Section 7.3.
Figure 7.4: Performance as a function of IPR relative to a baseline IPR. If the sub-images were processed coherently to yield a single image, it would be classified as in-class 97.5% of the time. The performance loss as a function of IPR is then shown dash-dot red line and dotted blue line. The red shows the approximated performance metric and the blue is the measured. Our performance approximation tracks well with the empirical.

These results are shown in Table 7.2 for both experiments. This table was calculated as follows: First, the maximum noise power yielding the expected 2.5% false rejection rate for $\alpha_z = .05$, was calculated. This value was then used to calculate the baseline SNR required for noise-free performance. Next, the analytic performance approximations were used to find the SNR loss relative to the baseline that yielded an equivalent performance loss.

### 7.3 Comments and Discussion

There is much to discuss regarding the results shown in the previous section. First, this approach as a whole will be commented on before moving onto the shortcomings and process improvements that could be implemented. Lastly, modifications that can be used to make the MPM algorithm more robust to IPR variations will be discussed.
Figure 7.5: Performance plot as in Figure 7.4 for the case of over-lapping sub-bands. The approximated performance still tracks well with the empirical, however as the distribution of the test statistic crosses the critical value, it is very sensitive to unconsidered higher order moments.

Table 7.2: Performance Equivalent SNR for each IPR Factor

<table>
<thead>
<tr>
<th>Experiment</th>
<th>IPR Factor</th>
<th>Equivalent SNR Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mutually Exclusive</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
<td>4.898</td>
</tr>
<tr>
<td></td>
<td>1/4</td>
<td>7.347</td>
</tr>
<tr>
<td></td>
<td>1/8</td>
<td>8.571</td>
</tr>
<tr>
<td>Overlap</td>
<td>1.0</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>-4.898</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>-1.225</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>2.449</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>3.674</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>4.898</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>6.122</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>8.572</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>9.796</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>9.796</td>
</tr>
</tbody>
</table>

Unsurprisingly, when comparing MPM samples originating from a distribution parametrized at a larger IPR than the reference template to a reference distribution parametrized at a smaller IPR, poorer performance was obtained. This is simply due to the mismatch of pixel
quantiles yielding penalties that shift the null distribution of the test statistic. Placing even a parametric isotropic point scatterer model on training images [68] and predicting quantile realizations requires solving a difficult estimation problem that is also computationally expensive. Additionally, if the isotropic point scatterer assumption does not accurately model the underlying scattering phenomenology which is expected due to the anisotropic nature of SAR imagery, among other reasons, the resulting performance prediction estimates will be inaccurate. This process also then requires simulating training imagery using the point scattering model, which again is computationally costly. The alternative presented here

Figure 7.6: The baseline resolution image at $D = 0$ levels of decomposition is shown in the top left. The top image shows the results of coherently summing the 64 sub-images at $D = 3$ levels of decomposition with the appropriate phase correction applied. The bottom left plot shows that the reconstruction is near perfect.
was shown to yield an accurate performance prediction using a relatively computationally inexpensive image formation process in the case of the IPR variations and is generalizable to other OCs of interest. It is noted that neither approach will yield a compact, closed-form performance expression as a direct function of the OC variations of interest, in this case IPR.

The approaches described here are relatively efficient given that data exists and sufficiently contains the variation of interest. It was also shown to generally track performance, nearly exactly in the case of AWGN but less accurately in the case of IPR variations. This was due to the limitations of the normal approximation not accurately modelling some higher order moments present in the empirical distributions. It is noted that a generalized or skew normal distribution could be used to potentially better model these effects under IPR variations and yield a more accurate prediction \([74]\) as observed in Figure \([7.3]\). These moments becomes crucial during the transition period, when the MPM distribution is situated near the critical value of the Z-test and even slight deviations from the normal model can yield significant errors in expected performance. This behavior is responsible for the difference in performance between the approximated and empirical results in Figures \([7.4]\) and \([7.5]\) particularly as the probability of rejecting an in-class target transitioned from 0 to 1. When the distribution is completely inside or outside of the critical region, these moments do not matter as much and the approximation is much more accurate.

It is important to mention that the performance results obtained here and shown in Table \([7.2]\) are valid only for the target studied. While a more theoretical understanding applicable to all SAR images is desirable, our results are a first step towards this goal and allow a relatively efficient performance study to be conducted given target datasets of interest. We note that the IPR factors 1 and 0.5 are common to both mutually exclusive and over-lapping sub-band experiments and yielded an equivalent SNR loss in both cases as seen in Table \([7.2]\). This is expected as an IPR factor of 0.5 yields mutually exclusive sub-bands in the over-lapping case and the templates are exactly the same. It was also observed
that the smaller IPR factors of 0.9 and 0.8 yielded an SNR gain and better performance. This is due to the tuning considerations described in Section 3.2.4. The tuning was applied to minimize rewards for matching background only, and those IPR factors yielded rewards for matching target regions that were greater than the penalties for target mismatch, shifting the distribution of the resulting test statistic to the left yielding better than expected performance. Lastly, we see that the mutually exclusive experiment associated with an IPR factor of $1/8 = 0.125$ yielded better performance than the overlapping experiment with 0.2 IPR factor. This is non-intuitive behavior as a larger IPR factor (higher resolution) is intuitively expected to yield better performance than a smaller IPR factor. Although the deviation is small, the behavior is indeed curious and could be attributed to correlation present in training images due to utilizing the same portions of the sub-aperture. Also, these IPR factors are much smaller than the baseline therefore the dynamic could be changing. We have not considered it further.

### 7.3.1 Increasing MPM’s Robustness to IPR Variations

Last, and perhaps most importantly, it was seen that the MPM algorithm is relatively robust to the IPR OC, where small deviations in IPR factor generally did not affect performance. Larger deviations led to a relatively smooth performance degradation. If robustness to IPR is desired, it can be obtained by using a smaller number of quantization levels, preferably only $N_q = 2$.

The rationale for this selection of $N_q$ can be best explained by illustrated the “tuning” process described in Section 3.2.4. This “tuning” yields a reduced set of target pixels to consider when calculating MPM penalties, $\Gamma_{\text{peak}}$. These salient pixels are shown in Figure 7.7. Although the images can vary dramatically as the IPR factor is decreased, the algorithm is generally only considering pixels within the mainlobe width of the baseline resolution image. Any penalties incurred are due to penalties arising from mismatch within this narrow region. It is also noted that, in this experiment, only 92 salient pixels were
considered which tends to weaken the normal approximation used in Section 7.1 and is responsible for the significance of the higher order moments that degrade approximation accuracy.

![Salient Pixels Considered in Experiments](image)

Figure 7.7: The “salient” pixels, $\Gamma_{\text{peak}}$, as determined by the clutter rejection process described in Section 3.2.4 are shown in red. This are the pixels that contributed to the summed MPM test statistic in (3.16) using the tuning rule in (3.41).

As the tuning process is effectively looking at only pixels corresponding to the main-lobe of the dominant scatterer in the baseline (high resolution) image, the non-matching pixels within the wider mainlobe of the wider IPR imagery are outside of this region and excluded from the calculation. Therefore, they yield no penalties and do not affect the loca-
tion of the distribution. The only mismatch is due to quantile mismatch within the narrow mainlobe region of the higher resolution baseline image. This is illustrated in Figure 7.8 and requires some explanation. The top row of plots is associated with the baseline resolution image ($D = 0$ in the mutually exclusive case) and each column shows the empirically probabilities of pixels in the training dataset realizing each quantile, $q \in \{1 \ldots N_q\}$. These images are associated with a narrower IPR and the quantization levels are distributed over a narrower region. Contrasted with the bottom row, which shows the same scenario for the $D = 3$ level decomposition case, the pixels in this region are most likely to yield a high quantile response. Therefore, where the baseline resolution template expects a distribution of quantiles over its mainlobe regions, the lower resolution templates generally only provides realizations of the highest quantile, yielding mismatch. This mismatch is solely responsible for the penalties shifting the null distribution, and can then be minimized by selecting a smaller number of quantiles. This conclusion is verified in Figure 7.9 for the mutually exclusive case. As the quantile realizations now match between the baseline and lower IPR cases, there is actually a reward and equivalent SNR gain as was seen in the IPR factors of 0.9 and 0.8 of Table 7.2.

7.4 Chapter Summary

In conclusion, we have presented a general approach to the performance prediction of the MPM algorithm. This approach requires parametrizing the MPM distributions as an implicit function of the OC of interest. By then placing a normal approximation on the summed MPM test statistic, the performance can be obtained as a function of the MPM distribution parametrizations. This approach is useful in cases where parametrizing the class-conditional distributions as a direct function of the OC of interest is intractable, as is the case with the IPR OC. Therefore, we effectively sample the performance curve at discrete values of the OC space of interest.
Figure 7.8: Empirical probabilities at each quantile, \( q \), parametrizing the distributions at \( D = 0 \) and \( D = 3 \) levels of decomposition. The dark blue pixels are excluded from the summed penalty statistic calculation.

Figure 7.9: Due to the tuning considerations of Section 3.2.4, the MPM algorithm is typically only considering pixels within the mainlobe width of the baseline resolution image. By selecting a small value of \( N_q \), the mismatch in this region can be minimized yielding an MPM configuration robust to IPR variations.
This approach was contrasted with a general approach to performance under AWGN. The simpler dynamics of this noise model permit a performance expression as an explicit function of the OC of interest, in this case the noise power, $\sigma^2$. This then allowed the performance loss due to IPR variations to be mapped to a performance equivalent SNR loss (or gain as was seen for small changes in IPR or the $N_q = 2$ scenario).

In addition to developing analytical performance expressions, the effect of IPR variations on MPM classification performance for specifically SAR imagery was studied. The trade-off explored in this case was performance improvements resulting from coherently processing the available K-space aperture to yield a single-look at a scene versus non-coherently processing the aperture to yield multiple looks at a lower resolution or equivalently larger IPR. With some reservations due to the limitations of the second-order approximation utilized in our analysis, our approximations were able to track the performance loss due to IPR variation.

It was also observed that due to the “tuning” process typically required for purposes of clutter rejection, the MPM algorithm typically only considers pixels within the mainlobe response of the peak scatterer of the baseline resolution imagery. As these pixels are generally still within the mainlobe of larger IPR imagery, any MPM penalties incurred are due to quantile mismatch within these narrow regions. It was seen that by choosing $N_q = 2$, this mismatch is effectively minimized, and the MPM algorithm is more robust to variations in IPR. Therefore, if robustness to IPR variation is desired, the minimum value of $N_q$ should be used.
Chapter 8

Conclusion

To conclude, we will now summarize the unique contributions of this dissertation, the expected impacts on the state of the art, and recommend some specific topics for future work in this area.

8.1 Summary of Contributions

To begin, the first primary contribution was the performance prediction of PTM, QGM, and QMSE under AWGN for the $N_q = 2$ case \[6\]. This work was discussed here in Section \[5\]. While still a step forward, it relied on several assumptions that simplified the problem, namely fixed quantization thresholds and only two quantization levels. The analysis presented utilized simulations developed with the AFRL CV dataset for verification. In order to reduce these assumptions, the focus was placed on specifically PTM and the analysis was extended to consider first a “uniform quantization” scheme which yielded random thresholds and several additional approximation approaches were presented \[7\]. This analysis used the ARL Comanche dataset for verification with some discussed difficulties.

Next, the existing work on PTM was extended to the general $N_q$ case yielding the MPM algorithm. This required revisiting the algorithm derivation which yielded two implementations based on the interpretation of the underlying model. The existing literature
assumed a scalar form, where the realized quantile was considered a scalar index into a penalty matrix. The implementation was described in Section 3.2.3. The more exacting vector interpretation was developed in Section 3.2 and the we have concluded the scalar form is preferred due to its simplicity [4]. This work also introduced the tuning rule utilized for the remainder of the dissertation and described in Section 5.4.2 which assumes that the targets can generally be segmented from the background. This is the third contribution.

With the MPM algorithm formally described, AWGN performance could be extended to the general $N_q$ case. This work was discussed in Section 5.4 and allowed both performance under degradation and IPR variations to be studied [9] [8]. The analysis was verified using the AFRL CV dataset.

With the basic approach developed, it was extended to more specific cases typically encountered in practice. The first of these was the performance of MPM under degradation presented in Chapter 6. Here, the underlying model utilized by MPM was extended to a two component mixture model [9]. In the first case, an image-to-template performance analysis was presented adding an additional dimension associated with target degradation. The weakness of this approach is that it utilized an IID assumption which may not be accurate in practice, however may still yield a good model fit for SAR data. The hope is that given collected target degradation data, the parameters of the degradation model can be used to accurate model specific real-world scenarios. This approach was then extended to the template-to-template approach, describing how the performance of an entire class of data will be effected under target degradation.

The second of these cases was the performance of the MPM algorithm under IPR variations. This analysis differed from the previous, in that it proposed a comprised approach that does not require postulating a mathematical model for the variations of interest. It assumes that the general variation is captured strictly in the MPM training process and allows the performance curve to be sampled by characterizing templates at each specific setting of an operating condition [8]. This approach was then utilized in a case study exploring trade-
offs in sub-aperture processing which yields multiple lower resolution images as opposed to a single high resolution image [8].

A table summarizing these contributions is shown in Table 8.1.

<table>
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<th>Task</th>
<th>Publication</th>
<th>Status</th>
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<td>IEEE AES</td>
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<td>QGM Performance Under AWGN ($N_q = 2$)</td>
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<td>QMSE Performance Under AWGN ($N_q = 2$)</td>
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<td>Uniform Quantization Scheme Considerations</td>
<td>SPIE DSS</td>
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<tr>
<td>MPM Performance Under AWGN (General $N_q$)</td>
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<td>Scalar and Vector MPM Implementations</td>
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<td>MPM Performance Under Degradation and AWGN</td>
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<td>In Review</td>
</tr>
<tr>
<td>MPM Performance Under IPR Variations</td>
<td>IEEE TAES</td>
<td>&quot;</td>
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</tbody>
</table>

Table 8.1: A summary of contributions made by this dissertation

### 8.2 Expected Impact

This work was a revival of what has been termed an “ATR Theory”. We hope the primary impact is the move towards more robust and rigorous performance appraisal of ATR algorithms. Typically, most ATR algorithms are evaluated by simply running truthed data through the ATR system yielding an empirical performance estimate. In many cases, where an extremely large amount of truthed data is available, this is sufficient. In other cases, where data collection requires large commitments of both time and money, it is inefficient and even potentially dangerous if the algorithms are expected to perform in critical environments.

While the algorithms discussed here are relatively simple and allowed for a relatively direct performance characterization, the task was previously unstudied in the open literature. This would suggest there are many other algorithms in existence that are good candidates for more rigorous performance studies. It would be a step forward to start presenting basic performance analysis as commonly as computational complexity when developing and publishing new algorithms.
In defense of the state of the art, the difficulty of these performance assessments is in postulating a model for the variations expected to be seen by the algorithm in practice. Our research yielded a distinct lack of work in characterizing something as common as occlusion which is quite common across a variety of ATR applications. Hopefully, more of these studies will be conducted in the future as the problem of designing efficient and robust algorithms first requires an adequate understanding of the environment.

8.3 Future Work

As was mentioned, the highest priority item for future work in this area is postulating and verifying statistical models for the variations of interest. Additionally, our work made a strong conditional independence assumption, which may not always be capable of accurately modelling the variations. Therefore, the task of developing accurate models for OC of concern is a relatively major effort but should be expected to yield great fruit in the development of robust and fieldable ATR systems.

Additionally, by showing where the algorithms fail a corrective action can be implemented. This procedure was demonstrated in Section 7.3.1 where the MPM algorithm was able to be configured in a specific way to yield performance independent of IPR variations. A configuration or algorithm modification for increased robustness to occlusion phenomenon is highly desirable and the type of analysis in this dissertation can help to aid insight in solving that problem.
Bibliography


Appendix A

Verification of General $N_q$ Case

Performance Prediction Plots

Figure A.1: Results of our general $N_q$ case performance prediction approach shown for $N_q = 2$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.
Figure A.2: Results of our general $N_q$ case performance prediction approach shown for $N_q = 2$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.

Figure A.3: Results of our general $N_q$ case performance prediction approach shown for $N_q = 8$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.
Figure A.4: Results of our general $N_q$ case performance prediction approach shown for $N_q = 16$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.

Figure A.5: Results of our general $N_q$ case performance prediction approach shown for $N_q = 32$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.
Figure A.6: Results of our general $N_q$ case performance prediction approach shown for $N_q = 64$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.

Figure A.7: Results of our general $N_q$ case performance prediction approach shown for $N_q = 128$ quantization levels. Our approximations of the first and second order moments of the test statistics are shown to be accurate.
Figure A.8: Results of our general $N_q$ case performance prediction approach shown for $N_q = 2$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 
Figure A.9: Results of our general $N_q$ case performance prediction approach shown for $N_q = 4$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 
Figure A.10: Results of our general $N_q$ case performance prediction approach shown for $N_q = 8$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 
Figure A.11: Results of our general $N_q$ case performance prediction approach shown for $N_q = 16$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 

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Figure A.12: Results of our general $N_q$ case performance prediction approach shown for $N_q = 32$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 

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Figure A.13: Results of our general $N_q$ case performance prediction approach shown for $N_q = 64$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 
Figure A.14: Results of our general $N_q$ case performance prediction approach shown for $N_q = 128$ quantization levels. The top plot shows the results of the analytic performance prediction approach versus the empirical results. The bottom plot shows the distribution at the noise level yielding a $P_{MD} \approx .05$. 