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Wright State University

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DEVELOPMENT OF ANALYTICAL EQUATIONS FOR OPTIMUM TILT OF TWO-AXIS AND SINGLE-AXIS ROTATING SOLAR PANELS FOR CLEAR-ATMOSPHERE CONDITIONS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Renewable and Clean Energy Engineering

By

GAURAV SUBHASH GUGALE
B.E., University of Pune, India, 2014

2016
Wright State University

________________________________
James Menart, Ph.D.
Thesis Director

________________________________
Joseph Slater, Ph.D.
Chair
Department of Mechanical and Materials Engineering

Committee on Final Examination

________________________________
James Menart, Ph.D.

________________________________
Daniel Young, Ph.D.

________________________________
Hong Huang, Ph.D.

________________________________
Robert E.W. Fyffe, Ph.D.
Vice President for Research and Dean of the Graduate School
Abstract


Solar Energy is a renewable energy source which is used widely in recent times. Photovoltaic panels collect the sun’s energy and convert it to electricity. Photovoltaic panels are being widely used in both domestic applications, commercial applications, and small-scale power generation applications. Photovoltaic panels are easy to install, they generate most of their power when electrical demands peak, prices of photovoltaic panels are dropping rapidly, photovoltaic panels require low maintenance, their operating costs are minimal, and they are highly suitable for remote applications. The amount of electricity produced by photovoltaic panels depends on the amount of sunlight the panel captures. The orientation of the panel relative to the sun’s rays is an important consideration in optimizing this energy collection.

This thesis deals with developing analytic equations that determine the optimum orientation of solar panels including the effects of a clear-atmosphere. This is done for three types of tracking: two-axis tracking, single, horizontal east-west axis tracking, and single, horizontal north-south axis tracking. While doing a literature search on the development of analytic equations that determine the optimum orientation of solar panels, it was found that Braun and Mitchell were the first to develop the equations that determine the optimum orientation of solar panels using the three types of tracking mentioned above. They developed these equations assuming there is no atmosphere on earth and that there is no reflection of the sun’s rays off the earth’s surface. Thus, the only component of solar radiation that they considered was that coming in a straight path from the sun to the earth, this is called beam radiation. The no-atmosphere optimum solar panel
orientation equations have been around for decades and they are in many textbooks on solar energy.

At this time, it appears that analytical relationships that account for the effects of the atmosphere on the optimum tilt angle of solar panels do not exist. Including the effects of the atmosphere adds two more components of solar energy to the total solar energy striking the solar panel. The beam radiation component still exists, but now diffuse solar radiation from the sun scattered by the atmosphere and all radiation from the sun reflected by the ground need to be included in any optimum tilt angle equation. Depending on the magnitude of these three types of solar radiation, the optimum panel orientation may be tilted slightly up to the sky to collect more diffuse radiation or tilted slightly down to the ground to collect more ground reflected radiation. This work derives and presents such equations for two-axis tracking, single, horizontal east-west axis tracking, and single, horizontal north-south axis tracking including the effects of a clear atmosphere. A clear-atmosphere is one in which there are no clouds. Including atmospheric effects in an analytical equation make this work unique from all the other optimum tilt angle work that has been performed for solar panels in the past.

After the analytical equations for the optimum tilt angles including the effects of a clear-atmosphere have been derived and presented for the three tracking cases mentioned above, a great deal of results are presented using these three equations. Results for a year and for a single day are presented. These results show that the differences in the optimum tilt angles determined by the no-atmosphere equations of Braun and Mitchell and the optimum tilt angles determined by the clear-atmosphere equations developed in this work are close to one another, but not the same. That is, the atmosphere changes the optimum tilt angles by up to $12.3^\circ$. While the differences in the
optimum tilt angle for no-atmosphere and clear-atmosphere may not be much, it costs nothing to readjust the solar panel to include atmospheric effects and collect a little more energy.
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A  Altitude of the location in Km.
a_{0}, a_{1}  Constants for standard atmosphere in Hottel’s clear sky model.
B  Function of the day of the year
DNI  Direct Normal Irradiance
D  Average distance between sun and earth
D_{R}  Rayleigh scatter
D_{A}  Scattering by aerosol
E  Equation of time
F_{c-s}  View factor to the sky
F_{c-g}  View factor to the ground
F_{1}  circumsolar coefficient
F_{2}  brightness coefficient
GHI  Global Horizontal Irradiance
I_{b,n}  Solar radiation incident normal to the surface of the earth.
I_{on}  Hourly solar irradiation incident normal to the surface outside the atmosphere on any day of the year.
I_{o}  Hourly solar irradiation incident on horizontal surface outside the atmosphere on any day of the year.
I_{SC}  Solar Constant
I_{cb,n}  Clear sky beam radiation incident normal to the earth’s surface
I_{cb,h}  Clear sky beam radiation incident to the horizontal surface
I_{cd,h}  Clear sky diffuse component on the horizontal surface
I_{c,h}  Total clear sky solar radiation on horizontal surface
I_{c,T}  Total clear sky solar radiation on tilted surface
I_{T,b}  Beam radiation on a tilted surface
I_{T,d}  Diffuse radiation on a tilted surface
I_{T,g}  Ground reflected radiation on a tilted surface
I_{T}  Total solar radiation on a surface tilted at slope \( \beta \) from the horizontal surface
I_{h,b} \quad \text{Beam radiation on the horizontal surface} \\
I_{h,d} \quad \text{Isotropic diffuse radiation on horizontal surface.} \\
I_h \quad \text{Total solar radiation on the horizontal surface } I_h = I_{h,b} + I_{h,d} \\
I_{T,b} \quad \text{Beam radiation on tilted surface} \\
I_{T,d} \quad \text{Isotropic diffuse radiation on tilted surface} \\
I_{T,refl} \quad \text{Ground radiation on tilted surface} \\
K \quad \text{Constants for standard atmosphere in Hottel’s clear sky model.} \\
r_o, r_1, r_K \quad \text{Correction factors for standard atmosphere in Hottel’s clear sky model.} \\
R_b \quad \text{Ratio of total radiation on the tilted surface to total radiation on the horizontal surface} \\
R \quad \text{Sun’s Radius} \\
R \quad \text{Rotation angle, angle of rotation of collector about axis when observed from the inclined end of axis, -180° to +180°. Equals zero when the normal to the surface is in the vertical plane, clockwise is positive.} \\

Greek Symbols \\
\phi \quad \text{Latitude} \\
\delta \quad \text{Declination Angle} \\
\theta_Z \quad \text{Zenith Angle} \\
\gamma_s \quad \text{Solar Azimuthal Angle} \\
\omega \quad \text{Hour Angle} \\
\alpha_s \quad \text{Solar Altitude angle} \\
\beta \quad \text{Slope of solar panel} \\
\gamma \quad \text{Surface Azimuthal Angle} \\
\theta \quad \text{Angle of Incidence} \\
\sigma \quad \text{Stefan Boltzmann constant} \\
\tau_b \quad \text{Transmittance for Beam Radiation} \\
\tau_d \quad \text{Transmission coefficient for diffuse radiation} \\
\rho \quad \text{Ground reflectivity} \\
\beta_a \quad \text{Axis tilt, angle from horizontal of the inclination of tracker axis, 0° to +90°.}
\(Y_a\) Axis azimuth, angle clockwise from north of the horizontal projection of the tracker axis, 0° to +360°. If the axis tilt is greater than zero, the vertex of the angle is at the inclined end of the axis.
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Chapter 1. Introduction

Energy is very important for the survival of humankind. We need energy for heating, cooling, communication, transportation, manufacturing, lighting, recreation, for almost every aspect of our lives. Unfortunately, we still depend on fossil fuel based energy, which is diminishing, as well as becoming expensive and unsustainable. It is an acknowledged fact that fossil fuel energy is not renewable and it is the main reason for one of the world’s most pressing problems, polluted environments. More renewable energy means fewer burned fossil fuels, which will result in reduced pollution of all types. My dream is to maximize the production of renewable, clean energy, such as solar energy, which is obtained from what can practically be considered an unlimited source. With the help of the sun’s energy, we can generate electricity or heat. For example, photovoltaic panels convert the sun’s energy into electricity and solar thermal collectors use heat absorbing panels to capture the sun’s energy to heat water for hot water applications or heat air for building space heating.

Photovoltaic (PV) panels are devices that use semiconductor materials to convert the sun’s rays directly into electrical current or an electrical voltage. Photo means light and voltaic means voltage. The photovoltaic cell produces green, renewable electricity from solar energy. PV manufacturers provide warranties for PV panels in terms of both life expectancy and efficiency. Generally, PV panels can last up to 25 years or more with an efficiency loss of 18% during 20 years of operation [1]. As compared to wind turbines, PV panels operate without any noise or moving parts. PV panels require low maintenance, operating cost are minimal, and PV panels are highly suitable for remote applications. One of the most important advantages of PV panels is they usually generate most of their power when electrical demands peak. That is, large electrical loads result when the sun is shining brightly, because high building cooling loads result; this is exactly
the time that PV panels have the most solar energy impinging upon them and thus the time when they produce the most electrical power. PV panels have been widely used in the past years in both domestic level applications (homes) and commercial applications (small-scale and large-scale power generation). PV Systems popularity has increased due to ease of installation, and a reduction in costs. PV panel prices have dropped considerably over the past 10 years. Some of the lowest costs are around $2 per watt installed. This reduction in PV prices, with the tremendous increase of PV panel applications, have placed PV panels high on the list of solar energy solutions for now and for the future.

The biggest disadvantage of PV panels is the efficiency. Compared to other renewable energy technologies like solar thermal, PV panels have low efficiency. The efficiency of PV panels ranges between 9-20%. The efficiency of a PV panel is low because only a portion of the sun’s spectrum of emitted electromagnetic radiation is used by a single material PV cell. Various research projects are being undertaken to increase the efficiency of solar panels.

Another factor that can improve the electrical energy production of a PV panel is orientating the panel towards the sun in an optimum fashion. There are two angles that describe the orientation of a solar panel. The first angle is the tilt angle, that is the slope of the panel from a horizontal plane, and the other angle is the azimuthal angle, that is the angle the projection of the panel’s normal vector onto a horizontal plane makes with due south on the earth. This thesis deals with optimum PV panel orientation for three types of tracking. The three tracking types specifically studied in this thesis are dual-axis tracking, single, horizontal east-west rotation axis tracking, and single, horizontal north-south rotation axis tracking. The factor that makes this work unique from all the other optimum orientation work that has been performed for solar panels in the past, is this work developed analytical equations that include the effects of an atmosphere without clouds and the effects of ground reflection. Up to this time optimum orientation analytical equations ignored atmospheric and ground effects completely. It is realized that there have been a number of computer studies that include all these effects. These will be discussed in the Literature Search section of this thesis.
1.1. Types of Tracking

From the vantage point of the earth, the sun travels 360 degrees from east to west around the globe. For a fixed horizontal surface, the sun is visible for 180 degrees, 90 degrees to the east of the surface and 90 degrees to the west of the surface relative to a normal vector from the center of the horizontal surface. If this flat surface is tilted upwards from the horizontal, this range of visibility can change. Also, because of the movement of the sun across the sky, a solar panel with fixed orientation will collect less solar energy. These losses can be recovered if solar panels have a tracking system which follows the motion of the sun throughout the day and throughout the year. It needs to be realized, from a fixed vantage point on the surface of the earth, the sun not only moves from east to west, but it moves higher and lower in the sky. This higher and lower movement of the sun will be called the altitude motion of the sun. The altitude motion of the sun occurs during the course of a day, but also over the course of a year. Table 1 below provides some idea of the energy that is forfeited when a collection surface is misaligned with the rays of the sun. It needs to be noted that this table does not include effects of the atmosphere.

<table>
<thead>
<tr>
<th>Misalignment Angle (degrees)</th>
<th>Percent Power Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
</tr>
<tr>
<td>1°</td>
<td>.015</td>
</tr>
<tr>
<td>3°</td>
<td>0.14</td>
</tr>
<tr>
<td>8°</td>
<td>1</td>
</tr>
<tr>
<td>23.4°</td>
<td>8.3</td>
</tr>
<tr>
<td>30°</td>
<td>13.4</td>
</tr>
<tr>
<td>45°</td>
<td>30</td>
</tr>
<tr>
<td>75°</td>
<td>75</td>
</tr>
<tr>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

In general, sun trackers can be grouped into single-axis and dual axis trackers. A tracker that rotates along one axis is called as single-axis tracker and a tracker that rotates around two axes is called a dual-axis tracker. There are two types of single-axis tracking systems that will be studied in this thesis. These are the horizontal, single-axis east-west tracking systems and the horizontal,
single-axis north-south tracking systems. With a single-axis, east-west tracking system, the solar panel rotates along a horizontal axis that lies in an east-west orientation and the rotating movement of the panel mostly tracks the altitude motion of the sun. A horizontal, single-axis north-south tracking system rotates along an axis that is orientated in the north-south direction and the rotating movement of the panel mostly tracks the east-west motion of the sun. It is possible to have single-axis trackers where the rotation axis is a slanted east-west orientation, a slanted north-south orientation, or not pointing east-west or north-south at all. These generalizations of the horizontal east-west and horizontal north-south rotation axis orientations have been addressed in the literature for the case where atmospheric and ground effects are neglected. The analytical relationships derived in this thesis that does include atmospheric and ground effects focus on horizontal east-west and horizontal north-south single-axis trackers. These will simply be called east-west trackers and north-south trackers throughout this thesis.

A dual axis tracker has two rotational axes and thus has two degrees of freedom. This type of tracking system tracks the sun’s east-west motion as well as the sun’s altitude motion. This means the solar panel can be orientated in any direction desired and the amount of solar energy impinging on the panel can be made its absolute largest value. This cannot be done with a single-axis tracker because it only has one degree of freedom and the sun is moving with two degrees of freedom. Optimum orientation of dual axis trackers including the effects of the atmosphere will also be studied in this thesis work.

1.2. Levels of Solar Radiation Calculations

For purposes of this work and as a very general way to look at optimum panel tilt models, three levels of solar radiation modeling are introduced. These three levels are:

1. No-atmosphere calculations,
2. Clear-atmosphere calculations, and
3. Cloudy-atmosphere calculations.

1.2.1. No-Atmosphere Solar Radiation

This is the first and most direct level of solar radiation modeling. For this level, radiation from the sun arrives at the earth’s surface unimpeded. Since this is what would happen if the earth had no atmosphere, this level of modeling is given the no-atmosphere label. This level of modeling
also ignores radiation reflected off the earth’s surface. Thus, the assumption is that the earth’s surface is nonreflecting. A name that is commonly used for this level of solar radiation modeling is extraterrestrial radiation. This is a very appropriate name because no-atmosphere solar radiation conditions are the exact situation that exists above the earth’s atmosphere.

It may seem that this level of calculation has no applicability to the solar radiation that reaches a solar panel located directly on the earth’s surface. This is incorrect, these models are extremely useful. When atmospheric effects are considered, solar radiation from the sun is split into a beam (direct) component and a diffuse component. The ground adds an additional component called the ground-reflected component. Of these three components, the beam radiation reaches the panel surface without interacting with the atmosphere or the earth’s surface. Thus, no-atmosphere models are the proper models for beam radiation and for beam radiation even when there is an atmosphere. Beam radiation can many times be the dominant component of solar radiation reaching a solar panel on the ground.

If absolute beam radiation magnitudes are required, the one issue that must be addressed is that the beam radiation above the atmosphere is not the same magnitude as the beam radiation at the earth’s surface. This does not affect optimum beam radiation panel orientation calculations, because the direction of the beam radiation is not changed by the atmosphere. This means that no-atmosphere models for optimum panel orientations provide excellent results for beam radiation on the surface of the earth when an atmosphere is present.

Braun and Mitchell (1983) were the first to develop mathematical equations to determine the no-atmosphere optimum orientation. They have presented these equations for the horizontal east-west single-axis arrangement, the horizontal north-south single-axis arrangement, and the dual axis tracking arrangement studied in this thesis. Braun and Mitchell have also presented optimum orientation equations for other types of tracking.

1.2.2. Clear-Atmosphere Solar Radiation

At this level, effects of the atmosphere on the solar radiation propagating through it are determined. This level of modeling considers all three components of solar radiation, beam, diffuse and ground reflected. What is not included is the effects of clouds. There are many clear-atmosphere models which estimate the clear sky solar radiation. Even on a clear day, all the extraterrestrial solar radiation does not hit the ground. This is due to the atmosphere absorbing and
scattering solar radiation. The atmosphere consists of gas molecules, dust, and particles. These concentrations vary with weather, location, and the number of pollution sources. Generally, at noon on a clear day, about 25% of the solar radiation from the sun is absorbed and scattered as it goes through the atmosphere [2]. Generally, clear-atmosphere models estimate the solar radiation incident on a horizontal surface.

At this time, it appears that no one has developed equations for calculating optimum panel orientations for clear atmosphere conditions. This is the gap this work is filling.

1.2.3. **Cloudy-Atmosphere Solar Radiation**

At this level, the calculated solar radiation includes atmosphere and cloud effects. Also included is ground reflected radiation. As is well understood by anyone who has felt cooler on a hot summer day as a cloud moves to cover the sun. The cloud does not block all energy from the sun but it certainly removes the beam component. Cloudy-atmosphere models estimate the irradiance for clear and cloudy atmospheric conditions on tilted surfaces. Because no model can predict the exact time clouds will appear and their exact location, cloudy atmosphere models tend to involve statistics. The most successful cloudy-atmosphere models are based on experimental measurements of the solar radiation reaching the earth’s surface. The most commonly available data is hourly or daily solar irradiance on a horizontal surface. However, it is necessary to calculate solar irradiance on tilted surfaces for designing flat plate collectors, photovoltaic systems, and other solar energy collecting devices. Cloudy-atmosphere models estimate the solar radiation incident on tilted surfaces.

No one has developed analytical equations for calculating optimum panel orientations under cloudy conditions; however, researchers have done numerical simulations of optimum panel orientations for these conditions. Using numerical techniques in a computer program allow for detailed hourly calculations that can be summed over a year or some other time period of interest. Computer calculations also allow for the use of statistically averaged measured data to be used to capture the effects of clouds for a typical meteorological year. Obtaining analytical relations for this case, like those obtained for the no-atmosphere case, is going to be difficult.

**1.3. Objective of the Project**
The amount of electricity produced by photovoltaic panels depends on the amount of sunlight it is exposed to. To get more sunlight, a photovoltaic panel must be positioned at the optimum angle to the sun’s rays arriving at the panel. When the photovoltaic panel is not orientated properly to the sun’s rays, it does not intercept as much light as it can; and therefore, it will not produce as much electricity as it should. From a no-atmosphere perspective, the best orientation of a solar panel is when the panel’s surface is exactly perpendicular to the sun’s rays. However, when atmospheric effects are included, this may not be the case. Atmospheric effects add a diffuse component and a ground-reflected component to the solar radiation impinging on the panel surface. Depending on the magnitude of these three components the optimum panel orientation may be tilted up to collect more of the diffuse radiation or tilted down to collect more of the ground reflected radiation. For almost every case, deviation from normal to the sun’s rays will be small, but these small differences add extra energy collection at no additional initial cost. It does not cost any more money to tilt a panel at one angle or that same angle plus 1.5 degrees. This is simply a matter of determining optimum orientations.

The specific purpose of this thesis project is to produce analytical equations that determine optimum tilt angles for solar panels that include the effects of a clear-atmosphere for horizontal east-west single-axis, horizontal north-south single-axis, and dual axis tracking arrangements. Results for optimum orientation of panels without atmosphere are well known and analytical equations have existed for some time to determine these orientations. However, relationships do not exist that account for the effects of solar radiation traveling through a scattering and absorbing medium, the air in the earth’s atmosphere. This project derives and presents such relations. As far as the author is aware, the optimum tilt angle equations shown in this thesis are the first that include atmospheric effects. This is viewed as a significant achievement. Not included in the atmospheric optimum tilt angle equations presented in this thesis are the effects of clouds. Clouds have a great effect on optimum tilt angles, but this is not the subject of this work. The work presented in thesis takes a huge step from the existing no-atmosphere, optimum tilt angle, analytical equations that have existed for quite some time, and produces analytical equations that include the effects of the earth’s atmosphere.

1.4. Outline of the Thesis
This thesis is comprised of seven chapters that discuss this project in detail. Chapter 1 has provided an introduction to solar energy and in particular different levels at which solar radiation calculations are performed. An introduction to different types of tracking arrangements and the importance of orientating the solar panels in an optimum fashion has been discussed. Chapter 2 of this thesis gives the review of different types of mathematical models that determine the solar radiation impinging on the tilted surfaces. This chapter also explains why we choose an isotropic solar radiation model for calculating solar radiation on tilted surfaces. This chapter also explains some of the important solar angles which help us in determining the positions of the sun in the sky.

Chapter 3 of this thesis is the literature survey that gives the review of the research work that is done on optimum solar panel orientations at the no-atmosphere, clear-atmosphere, and cloudy-atmosphere levels. Chapter 4 deals with equation development. In this chapter, we have derived the equations that calculate the optimum tilt angles for two-axis, single-axis east-west and single-axis north-south tracking solar panels for clear-atmosphere conditions. Chapter 5 represents a large number of results. Results which are obtained from the equations that were derived in Chapter 4 for two-axis tracking, single-axis east-west tracking, and single-axis north-south tracking are discussed. Chapter 6 is the beam transmittance study. In this chapter, the optimum tilt angle of a solar panel on a two-axis tracker is studied as a function of just the beam transmittance and the ground reflectivity. Finally, Chapter 7 presents the conclusions reached as a result of this work.

Before discussing the development of equations for the optimum orientation of a solar panel, equations for determining the amount of solar energy impinging on a surface of any orientation are discussed. This is what is done in this chapter. It is from these equations that equations for optimum tilt angles are developed. A number of radiation models are presented which determine the irradiance for either clear atmosphere or cloudy atmosphere conditions. These equations work for a panel located at any location on the surface of the earth, for any orientation.

After solar radiation passes through the earth’s atmosphere, it includes both a beam component and the diffuse component. As stated in the Chapter 1 the total solar radiation incident on a tilted surface consists of 3 components: beam radiation, diffuse radiation and reflected radiation from the ground. Beam radiation is the radiation received from the sun without being affected by the atmosphere. Diffuse radiation is the solar radiation received on the earth’s surface after being scattered by the atmosphere. The diffuse component is made up of reflections off of clouds, water vapor in the atmosphere, other particulates within the atmosphere, and the molecules that make up the atmosphere. This diffuse component consists of an isotropic diffuse component (uniform irradiance from the entire sky), a circumsolar diffuse component (forward scattering of beam radiation), and a horizon brightening component (aerosol scattering in the direction of the horizon). The models presented below for the solar radiant energy impinging on a panel use the
same method to calculate beam radiation and ground reflected radiation; the difference between these models is in the treatment of diffuse radiation [3].

The equation for the total radiant energy incident on a tilted surface in terms of the beam, each of the three diffuse components, and the ground reflected radiation is

\[ I_T = I_{T,b} + I_{T,d,iso} + I_{T,d,cs} + I_{T,d,hb} + I_{T,g}. \] (1)

All of the I quantities in this equation represent radiant energy impinging on the tilted panel per unit area. The subscript “T” means on the tilted surface. The remaining subscripts identify which radiant energy component is being considered. The subscript “b” means the beam component, the subscript “d,iso” means the diffuse isotropic component, the subscript “d,cs” means the diffuse circumsolar component, the subscript “d,hb” means the diffuse horizon brightening component, and the subscript “g” means the ground-reflected component. The quantity \( I_T \) has no additional subscripts and is the total solar energy impinging on the tilted panel per unit area. Normally the time span used for this energy collection is one hour.

### 2.1. Solar Energy Models

#### 2.1.1. Perez Model

One of the most detailed solar radiation models for determining the amount of solar energy impinging on a tilted surface is put forth by Perez et al. [4]. This model specifically deals with each of the components shown in Equation (1) above. Specifically, the Perez model represents the isotropic radiation components, isotropic diffuse, circumsolar, and horizon brightening radiation, with some complex functions. The Perez model for the solar radiation impinging on a tilted surface is

\[ I_T = I_{h,b}R_b + I_{h,d} \left[ (1 - F_1) \left( \frac{1 + \cos \beta}{2} \right) + F_1 \frac{a}{b} + F_2 \sin \beta \right] + I_h \rho \left( \frac{1 - \cos \beta}{2} \right). \] (2)

The first term on the right-hand side of this equation represents the beam component of the radiation, the second term represents all the diffuse radiation, and the third term represents the ground reflected radiation. The second term on the right-hand side is made up of three terms inside the square brackets. The first of these three terms is the isotropic diffuse radiation, the second is the circumsolar diffuse radiation and the third is the horizon brightening diffuse radiation. The quantities \( F_1 \) and \( F_2 \) are calculated from tabulated values. Also, note that a subscript “h” on the “I”
terms indicates the solar radiant energy on a horizontal surface. A drawback of the Perez model is that it is more computationally intensive than other models.

2.1.2. Klucher Model

Klucher [5] found that the isotropic sky model underestimates solar irradiance when there is increased intensity near the horizon and in the circumsolar region of the sky, i.e. under clear and partly overcast conditions. Like the Perez [4] model, the Klucher [5] model considers all the diffuse radiation components: isotropic diffuse, horizon brightening and circumsolar. During overcast sky conditions, the clearness index $F'$ becomes zero and the model reduces to one that only includes isotropic diffuse radiation. The Klucher [52] model is

$$I_T = I_{h,b} R_b + I_{h,d} \left( \frac{1+\cos \beta}{2} \right) \left[ 1 + F' \sin \left( \frac{\beta}{2} \right)^3 \right] * \left[ 1 + F' \cos \theta^2 \sin \theta \sin z^3 \right] + I_{h,\rho} \left( \frac{1-\cos \beta}{2} \right) \left( 1 + \sqrt{\frac{I_{h,b}}{I_{h,d}}} \sin \left( \frac{\beta}{2} \right)^3 \right)$$

(3)

where the first and last terms on the right-hand side represent the beam and ground reflected radiation. The middle term on the right-hand side includes all the diffuse radiation components and it cannot easily be separated into anisotropic diffuse, circumsolar diffuse, and horizon brightening diffuse like can be done with the Perez model. In this equations $F''$ is a clearness index.

2.1.3. Reindl Model

Just like the two previous models presented, the Reindl model [6] includes all the diffuse radiation components (isotropic diffuse, horizon brightening, and circumsolar). This model is a further development of the Hay-Davies [7] model given next. The Reindl model is

$$I_T = (I_{h,b} + I_{h,d} A) R_b + I_{h,d} \left( \frac{1+\cos \beta}{2} \right) \left( 1 - A \right) \left[ 1 + \sqrt{\frac{I_{h,b}}{I_{h,d}}} \sin \left( \frac{\beta}{2} \right)^3 \right] + I_{h,\rho} \left( \frac{1-\cos \beta}{2} \right).$$

(4)

The Reindl model provides slightly higher diffuse irradiance as compared with Hay-Davies model. In this equation, the circumsolar diffuse radiation is coupled with the beam radiation. This is reasonable because the circumsolar and beam radiation travel in essentially the same direction. The quantity $A$ in this equation is used to split the overall diffuse radiation into isotropic and circumsolar parts. The horizon brightening component is coupled with the isotropic scattering
term. The horizon brightening is given by \( \sqrt{\frac{I_{hb}}{I_{h}} \sin \left( \frac{\beta}{2} \right)} \) times the other quantities in the second term on the right-hand side of this equation.

2.1.4. Hay-Davies Model

In the Hay-Davies model [7], diffuse isotropic and circumsolar are used as a diffuse radiation component and horizon brightening is not considered. The Hay-Davies model is

\[
I_{T} = (I_{hb}+I_{hd}A)R_{b} + I_{hd}\left(\frac{1+\cos \beta}{2}\right)(1-A) + I_{h}\rho \left(\frac{1-\cos \beta}{2}\right).
\]

This equation can be easily seen as the Reindl model [54] given above without the horizon brightening term. As mentioned above the Reindl model added horizon brightening to the Hay-Davies model.

2.1.5. Muneer Model

In Muneer’s model [8], the shaded and sunlit surfaces are treated separately. The equation presented by Muneer for cloudy atmosphere conditions is

\[
I_{T} = I_{hb}R_{b} + I_{hd}TF + I_{h}\rho \left(\frac{1-\cos \beta}{2}\right)
\]

and that for clear atmosphere conditions is

\[
I_{T} = I_{hb}R_{b} + I_{hd}[TF(1-A) + AR_{b}] + I_{h}\rho \left(\frac{1-\cos \beta}{2}\right)
\]

where

\[
TF = \left(\frac{1+\cos \beta}{2}\right) + \frac{2B}{\pi(3+2B)} \ast \left[ \sin \beta - \beta \cos \beta - \pi \sin \left(\frac{\beta}{2}\right)^{2} \right].
\]

For cloudy atmosphere conditions, no circumsolar component is included in the model. The clear atmosphere equation of Muneer does show a circumsolar component as the second term in the square brackets on the right-hand side of Equation (7). Both the clear and cloudy condition equations of Muneer include horizon brightening which is located in \(T_{F}\).

2.1.6. Isotropic Sky Model

The last solar radiation model presented in this thesis is the isotropic sky model. The isotropic sky model [9, 10] is the simplest of all the models presented in this thesis because this model assumes all the diffuse radiation is uniformly distributed from throughout the entire sky in
all skyward directions. The circumsolar and horizon brightening radiation is assumed to be isotropic diffuse radiation in this model. While this is not true, these components tend to be small contributors to the overall solar radiation impinging on the tilted surface and thus it is not a detrimental assumption. This model is commonly used to determine solar radiative energy impingement on a panel. It may even be the most common model used. This is the model used to develop optimum orientation equations for solar panels in this thesis work. The reason for choosing this model is that it is a simple model with good accuracy for calculating the solar radiation incident on a tilted surface.

The isotropic sky model is

\[ I_T = I_{h,b}R_b + I_{h,d} \left( \frac{1 + \cos \beta}{2} \right) + I_{h,d} \rho_g \left( \frac{1 - \cos \beta}{2} \right). \]  

(9)

As can be seen there are three terms on the right-hand side of this equation. The first term is the beam component, the second is the isotropic diffuse component, and the third term is the ground reflected radiation component. The diffuse component in this equation accounts for all diffuse radiation: isotropic, circumsolar, and horizon brightening. These three terms are explained below starting with the beam component.

The beam radiation on a tilted surface is given as

\[ I_{T,b} = I_{h,b}R_b. \]  

(10)

which is the first term in Equation (9). Note that the beam radiation on a tilted surface, \( I_{T,b} \), is written in terms of the beam radiation on a horizontal surface, \( I_{h,b} \). To convert the beam radiation on a horizontal surface to the beam radiation on a tilted surface, the factor \( R_b \) is used. Of course, \( R_b \) is simply

\[ R_b = \frac{I_{T,b}}{I_{h,b}}. \]  

(11)

While this is the definition of \( R_b \), this is not the equation used to calculate \( R_b \). \( R_b \) must be calculated from the direction of the sun’s rays relative to the tilted surface and the direction of the sun’s rays relative to a horizontal surface. This is a complex process and is discussed in the sections below. The quantity \( I_{h,b} \) can be determined from experimentally measured values of \( I_h \) and \( I_{h,d} \) using the relation

\[ I_{h,b} = I_h - I_{h,d}. \]  

(12)

In this thesis work \( I_{h,b} \) is determined analytically and this will be discussed in Chapter 4.

The diffuse radiation term is
\[ I_{T,d} = I_{h,d} \left( \frac{1 + \cos \beta}{2} \right). \]  \hspace{1cm} (13)

\( I_{h,d} \) can be determined from experimental measurements, determined analytically from \( I_h \), or determined analytically from atmosphere transmittance values. The last technique will be used in this thesis work and will be described in Chapter 4. The factor \( \left( \frac{1 + \cos \beta}{2} \right) \) in Equation (13) is simply a view factor between the solar panel surface and the sky. This view factor is simply a function of the tilt of the panel from horizontal, \( \beta \) [11].

The ground-reflected radiation that impinges on the solar panel surface is given by

\[ I_{T,g} = I_h \rho_g \left( \frac{1 - \cos \beta}{2} \right). \]  \hspace{1cm} (14)

This is the third term on the right-hand side of Equation (9). This equation uses \( I_h \) which is mostly determined by experimental measurements. In this work, this quantity will be determined analytically. The quantity \( \rho_g \) is the ground reflectivity around the location of the solar panel. This is typically taken as 0.2 for non-snow covered ground and 0.7 for snow covered ground. The last factor in this equation is \( \left( \frac{1 - \cos \beta}{2} \right) \), which is the view factor between the solar panel surface and the ground. It is interesting to note that the view factor between the solar panel surface and the ground plus the view factor between the solar panel surface and the sky add to one. This means the solar panel surface only has a line-of-sight with the sky or the ground. Any surrounding buildings or trees are considered part of the ground.

### 2.2. Ratio of Beam Radiation on a Tilted Surface to the that on a Horizontal Surface

The quantity \( R_b \) in Equation (9) is determined by taking the ratio of the cosines of two incident angles as,

\[ R_b = \frac{\cos \theta}{\cos \theta_Z}. \]  \hspace{1cm} (15)

The first of these incident angles is \( \theta \), the angle between the normal of the panel surface and the sun’s rays (see Figure 1). The second incident angle is \( \theta_Z \), the angle between the normal of a horizontal surface on the earth and the sun’s rays. This angle is the zenith angle of the sun and is the complement of the altitude angle of the sun (see Figure 2).
Figure 1: Angle of incidence and slope [11].

Figure 2: Zenith angle, altitude angle, solar azimuthal angle, surface azimuthal angle, tilt angle, and angle of incidence [11].

Because problems may arise when calculating $R_b$ for time periods close to sunrise or sunset, it is best to use an integral equation for $R_b$,

$$R_b = \int_{\omega_1}^{\omega_2} \cos \theta \, dt$$

(16)
where the integrals are carried out over the desired time period. This time period is typically taken as one hour, but can be less. The problem encountered by using Equation (15) can be seen by considering the case when sunrise or sunset occurs at the midpoint of the hour. At sunrise and sunset the zenith angle is 90 degrees and so the evaluated $R_b$ is infinite. Under these circumstances, the recorded radiation is not zero so the estimated beam radiation incident on the tilted surface can go to infinity. This is not correct and this problem is eliminated when the integral form of $R_b$ is used.

The integrals in Equation (16) have been carried out and are typically written as

$$R_b = \frac{a}{b}$$  \hspace{1cm} (17)

where,

$$a = (\sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma) \left(\frac{1}{180} (\omega_2 - \omega_1) \pi + (\cos \delta \cos \phi \cos \beta + \cos \delta \sin \phi \sin \beta \cos \gamma)(\sin \omega_2 - \sin \omega_1) - (\cos \delta \sin \beta \sin \gamma)(\cos \omega_2 - \cos \omega_1).$$  \hspace{1cm} (18)

and

$$b = (\cos \phi \cos \delta)(\sin \omega_2 - \sin \omega_1) + (\sin \phi \sin \delta) \left(\frac{1}{180} (\omega_2 - \omega_1) \pi. \right.$$  \hspace{1cm} (19)

There are many angles in the integrated version of $R_b$. These angles will be discussed in the next section of this Chapter. Before discussing these angles, equations are given for $\cos \theta$ and $\cos \theta_Z$ shown in Equations (15) and (16). These are the equations that were integrated to obtain Equations (18) and (19). These are important equations in the effort to obtain the amount of radiation incident on a solar panel. The equation for the cosine of the incident angle is

$$\cos \theta = (\sin \delta \sin \phi \cos \beta - (\sin \delta \cos \phi \sin \beta \cos \gamma) + (\cos \delta \cos \phi \cos \beta \cos \omega) + (\cos \delta \sin \phi \sin \beta \cos \gamma \cos \omega) + (\cos \delta \sin \beta \sin \gamma \sin \omega).$$  \hspace{1cm} (20)

This can be written in terms of the sun azimuthal angle as

$$\cos \theta = \cos \beta \cos \Theta_z + \sin \beta \sin \Theta_z \cos(\gamma - \gamma).$$  \hspace{1cm} (21)

The equation for the cosine of the solar zenith angle is

$$\cos \theta_z = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta.$$  \hspace{1cm} (22)

The zenith angle can be seen in Figure 2 above. As can be seen the same angles that show up in Equations (18) and (19) show up here.
2.3. Important Solar Angles

As Equation (9) shows, in order to calculate solar incident radiation on a surface of any orientation, it is required to calculate the ratio of incident radiation on tilted surface to that on a horizontal surface. To calculate the beam radiation on a tilted surface we need to know the position of the sun relative to the surface normal and vertical to the earth. These angles are necessary because the terrestrial solar irradiance is a function of the sun’s motion in the sky. Terrestrial solar radiation is the solar radiation that reaches the earth’s surface. Below is an overview of the important angles which help in determining the position and motion of the sun in the sky and the orientation of the solar panel.

There are 5 angles that appear in Equations (20) and (22). Three of these angles do not require any equations and just need to be prescribed. These three angles are \( \phi \), \( \beta \), and \( \gamma \). The angle \( \phi \) can be seen in Figure 1 and this is nothing more than the latitude of the location of the panel. This latitude should be written in degrees or radians and the use of degree, minutes, and seconds should be avoided when used in any of the equations listed in this thesis. The second and third angles that do not require equations, but simply are input values, are \( \beta \) and \( \gamma \). The angle \( \beta \) is the slope of the panel from a horizontal plane and the angle \( \gamma \) is a measure of the panel’s normal projected into a horizontal plane from due south. This is the azimuthal orientation of the panel. Both \( \beta \) and \( \gamma \) are represented in Figure 2.

The remaining two angles that appear in Equations (20) and (22) are \( \delta \) and \( \omega \). The angle \( \delta \) is the declination angle of the earth’s equatorial plane relative to the rays of the sun. The declination angle depends on the tilt of the earth’s axis of rotation relative to the earth’s plane of rotation around the sun. The declination angle can be found from the approximate equation developed by Cooper [12],

\[
\delta = 23.45 \sin \left(360 \frac{284+n}{365}\right). \tag{23}
\]

This declination angle varies from 23.45° to -23.45° throughout a year. The declination angle is why we have seasons. The angle \( \omega \) is angular displacement in terms of latitudes due to rotation of the earth on its own axis at 15 degrees per hour. The earth rotates 15° per hour so at 11 am solar time the hour angle is -15°, at solar noon the hour angle is 0°, and at 1 PM solar time the hour angle is 15°. The hour angle is negative during the morning, reduces to zero at solar noon, and
becomes increasingly positive in the afternoon. To calculate the hour angle, we need to convert standard time to solar time where
\[ \omega = 15^\circ (\text{solar time} - 12). \]  
(24)
is used for the morning hours and the equation
\[ \omega = 15^\circ \times \text{solar time}. \]  
(25)
is used for the afternoon hours.

It is important to understand solar time as contrasted to local time. Local time is what we read off our clocks at a given location. Solar time is based on the position of the sun relative to the longitude of your location. To convert local time to solar time, three corrections need to be made. The first correction that is made to local time is that it is converted to standard time. Standard time and local time are exactly the same except when daylight saving time is invoked. To convert from daylight savings time to standard time one hour is subtracted from daylight savings time. The second correction that needs to be made to local time to get it converted to solar time is the deviation in the earth’s rotational speed as a function of the day of the year needs to be addressed. This variation occurs because of the elliptical orbit of the earth around the sun and the coupling of the orbital angular momentum with the rotational angular momentum of the earth. The following equation corrects for this change in the earth’s rotational speed,
\[ E = 229.2(0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos B - 0.04089 \sin B) \]  
(26)
where \( B = (n - 1) \frac{360}{365} \)  
(27)
and \( n \) is the day number starting at 1 for January 1. The third correction that needs to be applied is the difference caused between the observer’s meridian and the meridian on which the local time is referenced. This correction looks like
\[ E_L = 4(L_{\text{st}} - L_{\text{loc}}). \]  
(28)
where \( L_{\text{st}} \) is the standard meridian for the local time zone and \( L_{\text{loc}} \) is the meridian at the observers location. The last two corrections listed above can be combined into the equation
\[ \text{Solar time} - \text{Standard time} = 4(L_{\text{st}} - L_{\text{loc}}) + E. \]  
(29)
where it should be noted once again that standard time already includes the daylight saving time correction.
One last angle that should be discussed is the solar azimuthal angle, \( \gamma_s \). This angle appears in Equation (21). This angle changes throughout the day as the sun moves through the sky.

The sun azimuthal angle is just like the surface azimuthal angle in that it measures the angular displacement of the projection of the sun’s beam radiation on a horizontal plane at the location of interest from due south [11]. If the solar azimuthal angle is due south, \( \gamma_s = 0 \). When the sun is in the east the solar azimuthal angle is negative and if the sun is in the west the solar azimuthal angle is positive. The sun’s azimuthal angle can be written in terms of other angles introduced in this thesis as

\[
\sin \gamma_s = \frac{\cos \delta \sin \omega}{\sin \theta_Z} \tag{30}
\]

or

\[
\cos \gamma_s = \text{sign}(\omega) \left[ \cos^{-1} \left( \frac{\cos \theta_Z \sin \phi \sin \delta}{\sin \theta_Z \cos \phi} \right) \right]. \tag{31}
\]

<table>
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<td>( \cos \theta_Z = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta )</td>
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</table>
| 2 Solar Altitude angle (\( \alpha_s \)) | \( \alpha_s = 90 - \theta_Z \)  
\( \cos \alpha_s = \cos(90 - \theta_Z) = \sin \theta_Z \)  
\( \sin \alpha_s = \sin(90 - \theta_Z) = \cos \theta_Z \) |
| 3 Solar Azimuthal Angle (\( \gamma_s \)) | \( \sin \gamma_s = \frac{\cos \delta \sin \omega}{\sin \theta_Z} \)  
\( \cos \gamma_s = \text{sign}(\omega) \left[ \cos^{-1} \left( \frac{\cos \theta_Z \sin \phi \sin \delta}{\sin \theta_Z \cos \phi} \right) \right] \) |
| 4 Hour Angle (\( \omega \)) | Sunset hour angle:  
\( \cos \omega_s = -\tan \phi \tan \delta \)  
\( \omega_s(\text{Sunrise}) = -\omega_s(\text{Sunset}) \) |
In this chapter, the research work done on optimum solar panel orientations at the no-atmosphere, clear-atmosphere, and cloudy-atmosphere levels is discussed. In addition, work that has been done in the area of sensor based solar panel tracking is presented.

3.1. No-Atmosphere Models

For the most part, the work on developing no-atmosphere mathematical models for the optimum solar panel orientation has been done in 1983 by Braun and Mitchell. Others have contributed to this body of work, but the bulk has been done by Braun and Mitchell. The reason the no-atmosphere models have been developed many years ago is that they are the simplest to derive. All the complications that the atmosphere, clouds in the atmosphere, and reflection off the ground inject into the analysis are ignored. For no-atmosphere models, the only equation that needs to be optimized is Equation (20). While this is difficult, it becomes more difficult when atmospheric issues are injected.

3.1.1. Two-axis Tracking

This type of tracking system tracks the east-west motion of the sun across the sky and the up and down motion of the sun. Said another way this tracking system follows the sun’s azimuthal angle and the sun’s altitude angle. Because of these two degrees of freedom in the panels tracking mechanism, the panel can always be perpendicular to the sun’s rays. That is [13]
\[ \cos \theta = 1 \quad (32) \]
\[ \gamma = \gamma_s, \quad (33) \]
and
\[ \beta = \theta_Z. \quad (34) \]

Simply put the azimuthal angle of the solar panel is equal to the azimuthal angle of the sun and the tilt of the solar panel is equal to the complement of the altitude angle of the sun. The complement of the sun’s altitude angle is the zenith angle \( \theta_Z \), which is given by Equation (22). The azimuthal angle for the sun is given by Equation (31). Without atmosphere effects, this is the panel orientation that provides the most incident solar energy.

### 3.1.2. Single, East-West Horizontal Axis Tracking

In this tracking system, the axis of rotation runs east to west parallel to a horizontal plane and the panel rotates about this axis. This gives the panel good tracking capabilities for the altitude angle of the sun, but limited ability to track the sun’s east-west movements. The tilt of the solar panel for this type of tracking system is adjusted throughout the day. This means the minimum angle between the sun’s rays and the solar panel’s normal vector is [13]

\[ \cos \theta = (1 - \cos \delta^2 \sin \omega^2)^{1/2}. \quad (35) \]

The surface azimuthal angle is [13]

\[ \gamma = \begin{cases} 
0^\circ & \text{if } |\gamma_s| < 90^\circ \\
180^\circ & \text{if } |\gamma_s| \geq 90^\circ 
\end{cases} \quad (36) \]

and the tilt of the panel is [13]

\[ \tan \beta = \tan \theta_Z |\cos \gamma_s|. \quad (37) \]

As it is a single, horizontal east-west horizontal axis tracking system, the surface azimuthal angle is constrained to one of two values, \( 0^\circ \) and \( 180^\circ \). In the northern hemisphere, the \( 180^\circ \) orientation is only used on long summer days when the sun moves north of the panel.

### 3.1.3. Single, North-South Horizontal Axis Tracking

In this tracking system, the axis of rotation runs north to south in a plane parallel to a horizontal surface. This type of tracking system does a good job of tracking the sun’s east to west movement across the sky, but does not track the sun’s altitude movement well. The tilt of the solar panel for this type of tracking system changes continuously throughout the day; however, the
azimuthal angle of the solar panel only takes on two discreet values and flips from due east in the morning to due west in the afternoon [13]

\[
\gamma = \begin{cases} 
90^\circ & \text{if } \gamma_s > 0^\circ \\
-90^\circ & \text{if } \gamma_s \leq 0^\circ
\end{cases}
\] (38)

This is similar to the east-west tracking system that only allows two, discrete azimuthal orientations. The optimum tilt equation developed by Braun and Mitchell [13] for a horizontal north-south axis with continuous adjustment is

\[
\tan \beta = \tan \Theta_z |\cos(\gamma - \gamma_s)|.
\] (39)

Using this optimum tilt and azimuthal angle provides a minimum angle of incidence as given by [13]

\[
\cos \theta = (\cos \theta_z^2 + \cos^2 \delta \sin^2 \omega)^{1/2}.
\] (40)

3.1.4. Vertical Axis Tracking

Vertical axis tracking has not been discussed in this thesis, but it is an option that has been addressed by Braun and Mitchell [13]. A vertical axis tracker simply spins around the axis of a vertical pole placed on the ground. This type of tracker is known as an azimuthal sun tracker because the surface azimuthal of the solar panel follows the azimuthal angle of the sun continuously throughout the day. The tilt of the solar panel for this type of tracking system is fixed. This tilt can be set to optimize solar radiation collection throughout the year, or for a season of the year. Vertical axis tracking systems are more effective than horizontal axis tracking systems at higher latitudes [14]. The equations providing the optimum panel azimuthal angle is simply

\[
\gamma = \gamma_s,
\] (41)

where the sun’s azimuthal angle can be found from Equation (31). The optimum tilt angle of the panel is governed by the user’s energy demand and the most that can be said here is that

\[
\beta = \text{constant}.
\] (42)

For the vertical axis tracking system, the minimum angle of incidence is

\[
\cos \theta = (\cos \theta_z \cos \beta + \sin \theta_z \sin \beta).
\] (43)

3.1.5. Single, North-South Horizontal Axis Tracking

In this sun tracking mechanism, the rotation axis is inclined to the horizontal. This type of tracking system is known as a latitude-tilted axis sun tracker because the slope of the north-south
axis is at the latitude of the location where the solar panel is placed. Tilting the rotation axis of the panel at the latitude makes this axis parallel to the earth’s axis of rotation. On average over a year, this helps to minimize the incidence angle of the sun’s rays impinging on the solar panel. Because of the tilt of the north-south axis, both the azimuthal and tilt angle of the solar panel relative to due south and relative to a horizontal plane respectively vary throughout the day. For this type of tracking system, the optimum azimuthal angle is given by

\[ \gamma = \tan^{-1} \left[ \frac{\sin \theta_z \sin \gamma_s}{\cos \theta' \sin \phi} \right] + 180C_1C_2, \tag{44} \]

where

\[ \cos \theta' = \cos \theta_z \cos \phi + \sin \theta_z \sin \phi \cos \gamma_s \tag{45} \]

\[ C_1 = \begin{cases} 0 & \text{if } \left( \tan^{-1} \left[ \frac{\sin \theta_z \sin \gamma_s}{\cos \theta' \sin \phi} \right] \right) \gamma_s \geq 0 \\ -1 & \text{if } \left( \tan^{-1} \left[ \frac{\sin \theta_z \sin \gamma_s}{\cos \theta' \sin \phi} \right] \right) \gamma_s < 0 \tag{46} \end{cases} \]

and

\[ C_2 = \begin{cases} +1 & \text{if } \gamma_s \geq 0 \\ -1 & \text{if } \gamma_s < 0 \tag{47} \end{cases} \]

The optimum tilt of the panel is

\[ \tan \beta = \frac{\tan \phi}{\cos \gamma} \tag{48} \]

These optimum orientation angles provide an angle of incidence of

\[ \cos \theta = \cos \delta. \tag{49} \]

where \( \delta \) is the declination angle of the earth. Thus, this type of tracking mechanism is never more than 23.45° off the sun’s rays.

3.1.6. General Single Horizontal Axis Tracking

An equation for a general single-axis that is sloped in any direction where the panel surface is parallel to the rotation axis was developed by Marion and Dobos [15]. The axis of rotation sits at some angle with a fixed tilt of \( \beta_a \) and a fixed azimuthal angle of \( \gamma_a \). Note that \( \beta_a \) and \( \gamma_a \) are for the axis and not for the panel which are represented by \( \beta \) and \( \gamma \). Also included in Marion and Dobos relations is a rotation angle limit. This limit stops the panels from rotating past a certain angle. The reason for such an option is due to shading by adjacent panels. It does not make sense to continue to rotate an array of panels if the panels start to shade one another. If the calculated
rotation angle exceeds the tracker’s rotation limit, then the rotation angle is set to the tracker’s rotation limit.

The surface tilt angle is determined from

\[ \beta = \cos^{-1}(\cos R \cos \beta_a). \]  

(50)

The surface azimuthal angle is determined with the equations:

for \( \beta \neq 0 \) and \(-90^\circ \leq R \leq +90^\circ\)

\[ \gamma = \gamma_a - \sin^{-1} \left[ \frac{\sin R}{\sin \beta} \right], \]  

(51)

for \(-180^\circ \leq R \leq -90^\circ\)

\[ \gamma = \gamma_a - \sin^{-1} \left[ \frac{\sin R}{\sin \beta} \right] - 180, \]  

(52)

and for \(+90^\circ \leq R \leq +180^\circ\)

\[ \gamma = \gamma_a - \sin^{-1} \left[ \frac{\sin R}{\sin \beta} \right] + 180 \]  

(53)

where

\[ R = \tan^{-1}(X) + \psi, \]  

(54)

\[ X = \frac{\sin \theta_z \sin(\gamma_s - \gamma_a)}{\sin \theta_z \sin(\gamma_s - \gamma_a) \sin \beta_a + \cos \theta_z \cos \beta_a}, \]  

(55)

and

\[ \psi = \begin{cases} 
0^\circ, & \text{if } X > 0 \text{ and } (\gamma_s - \gamma_a) > 0, \text{ or if } X < 0 \text{ and } (\gamma_s - \gamma_a) < 0 \\
+180^\circ, & \text{if } X < 0 \text{ and } (\gamma_s - \gamma_a) > 0 \\
-180^\circ, & \text{if } X > 0 \text{ and } (\gamma_s - \gamma_a) < 0 
\end{cases}. \]  

(56)

3.2. Clear-Atmosphere Models

Clear-atmosphere models calculate terrestrial solar radiation based on an atmosphere without clouds. Specifically, clear-atmosphere models include direct beam radiation, diffuse radiation scattered off the atmosphere, and solar radiation reflected off the ground. Even on the clear day, all the solar radiation present above the atmosphere does not hit the ground. Generally, at noon on a clear day, about 25% of the solar radiation just above the atmosphere is absorbed and scattered as it goes through the atmosphere [2]. In the morning and evening, the solar radiation incident on a solar panel decreases more because the rays of the sun have to travel a longer path through the atmosphere.
The atmosphere consists of gas molecules, dust, and particles. This concentration varies with weather, location, and the number of pollution sources. This means the amount of solar radiation absorbed and scattered by the atmosphere varies as the composition of the atmosphere varies. Many clear-atmosphere models use a standard or typical atmosphere. Other clear-atmosphere models estimate the terrestrial solar radiation as a function of the solar elevation angle, water vapor concentration, aerosol concentration, site altitude, etc. The simple clear-atmosphere models are only a function of the solar zenith angle. Complicated models are functions of many atmospheric parameters such as aerosol concentration or perceptible water [2].

An excellent diagram of what happens to the sun’s energy as it passes through the atmosphere is shown in Figure 3. The solar flux present at the outer edges of the earth’s atmosphere is 1367 W/m² [16]. This flux magnitude is on a surface normal to the sun’s rays and is an average over the year. This number is called the solar constant. The solar constant can be looked at as the maximum radiative flux that planet earth has. As shown in Figure 3 both absorption and scattering reduce the solar constant flux as the energy traverses the atmosphere. Depending on the atmospheric conditions, absorption can account for an 11 to 30% reduction in the energy of the sun’s beam and scattering can account for a 1.6 to 11% reduction. Depending on atmospheric conditions the beam radiation can be reduced by 33 to 83%. The larger number assumes significant cloud cover and the smaller number would be more applicable to a clear-atmosphere.

The variation of the solar constant with the day of the year is shown in Figure 9. This number fluctuates ±3.3% during a year because of the earth’s varying distance from the sun. Solar radiation reaching on earth’s atmosphere varies approximately ±45 W/m² over a year. The simple equation to calculate no sky solar irradiation normal to the beam is

\[ I_{on} = I_{SC} \left( 1 + 0.033 \cos \frac{360n}{365} \right). \]  

(57)

This equation was derived by Spencer (1971) and was taken from Iqbal [16]. Spencer provides a simple and more accurate equation with an accuracy of ±0.01%.
Figure 3: Atmosphere [17].

Figure 4: Variation of no atmosphere solar radiation with time of year [16].
Radiation on a horizontal surface outside the atmosphere is given by

\[ I_o = I_{SC} \left( 1 + 0.033 \cos \frac{360n}{365} \right) \cos \theta Z. \]  

(58)

This equation can be integrated to give a hourly average as

\[ I_o = \frac{12 \times 3600}{\pi} \cdot I_{SC} \cdot \left( 1 + 0.033 \cos \frac{360n}{365} \right) \left[ \cos \phi \cos \delta (\sin \omega_2 - \sin \omega_1) + \frac{\pi(\omega_2 - \omega_1)}{180} \sin \phi \sin \delta \right]. \]  

(59)

Obtaining an integrated hourly average, as opposed to just calculating at the middle of the hour, is important at times close to sunrise and sunset.

While there are many complex clear-atmosphere solar models, there does not appear to be any clear-atmosphere models that provide the optimum orientation of solar panels. Thus, this thesis work appears to be unique. Having said this, to develop optimum orientation models for solar panels in a clear-atmosphere condition, models of the solar radiation impinging on a horizontal surface located on the ground are required. Thus, clear-atmosphere models that provide \( I_h \) and \( I_{h,d} \) are required as shown in Equations (89) and (101). Many of these types of models exists. In this literature search, a review of these models is presented.

Very simple clear-atmosphere models use only geometric calculations. The reduction of normal solar radiation outside the earth’s atmosphere to a surface on the earth during transmission through the atmosphere is a function of the zenith angle. A higher zenith angle results in higher air mass which means there will be more interaction of the solar radiation with the atmosphere.

Equations for eight of these models are:

**Daneshyar–Paltridge–Proctor (DPP) model (1978) [18,19]:**

\[ I_{cb,n} = 950.2 \left\{ 1 - e^{-0.075(90^- \theta_z)} \right\} \]  

(60)

\[ I_{cd,h} = 14.29 + 21.04 \left( \frac{\pi}{\theta_z} - \theta_z \frac{\pi}{180} \right) \]  

(61)

\[ I_{c,h} = I_{cb,n} \cos(\theta_z) + I_{cd} \]  

(62)

**Kasten–Czeplak (KC) model (1980) [20]:**

\[ I_{c,h} = 910 \cos(\theta_z) - 30 \]  

(63)

**Haurwitz model (1945) [21, 22]:**

\[ I_{c,h} = 1098 \cos(\theta_z) e^{\left( -0.057 \cos \theta_z \right)} \]  

(64)

**Berger–Duffie (BD) model (1979) [23]:**
\[ I_{c,h} = I_o 0.70 \cos \theta_z \] (65)

Adnot–Bourges–Campana–Gicquel (ABCG) model (1979) [23]:
\[ I_{c,h} = 951.39[\cos(\theta_z)]^{1.15} \] (66)

Robledo-Soler (RS) (2000) [24]:
\[ I_{c,h} = 1159.24[\cos(\theta_z)]^{1.79} e^{[0.0019*(90^\circ-\theta_z)]} \] (67)

Meinel Model (1976) [25]:
\[ I_{cb,n} = I_o 0.7AM^{0.678} \] (68)

Laue Model (1970) [26]:
\[ I_{cb,n} = I_o [(1 - 0.14h)0.7AM^{0.678} + 0.14h] \] (69)

Of the eight models presented above only the first one provides a relation for the diffuse radiation on a horizontal surface. Also, a number of the models above use a quantity called AM. This is the air mass. The air mass is calculated as a function of the zenith angle of the sun
\[ AM = \frac{1}{\cos(\theta_z)} \] (70)

and is a relative path length number. The air mass represents the path length thought he atmosphere relative to the path length through the atmosphere when the sun’s rays are normal to the earth’s surface.

There are some simple clear-atmosphere models that not only consider the zenith angle but also consider some basic parameters of the atmosphere such as relative humidity, temperature, air pressure, Rayleigh scattering, and aerosol content. Three such models are:

Kasten model [27]:
\[ I_{c,h} = 0.84I_o \cos(\theta_z) e^{(-0.027AM[f_{h1} + f_{h2}(TL-1)])} \] (71)

where
\[ f_{h1} = e\left(\frac{-h}{8000}\right) \quad \text{and} \quad f_{h2} = e\left(\frac{-h}{1250}\right) \] (72)

Ineichen and Perez [28]:
\[ I_{c,h} = C_{g1} * I_o \cos(\theta_z) e^{[C_{g2}AM[f_{h1} + f_{h2}(TL-1)]]} e^{(0.01*AM^{1.8})} \] (73)

where \( C_{g1} = 5.09e^{-5}h + 0.868 \) and \( C_{g2} = 3.92e^{-5}h + 0.0387 \) (74)

MAC model [29, 30]:
\[ I_{c,h} = I_{cb,n} \cos(\theta_z) + D_R + D_A \] (75)

and
\[ I_{cb,n} = I_o(T_0T_r - a_w)T_a \] (76)
where
\[ D_R = I_o \cos(\theta_z) T_o (1 - T_r) / 2 \] (77)
and
\[ D_A = I_o \cos(\theta_z) (T_o T_r - a_w) (1 - T_o) \omega \phi. \] (78)

In the Kasten model, atmospheric turbidity and elevation is included. The Ineichen and Perez model added some correction terms to the Kasten Model to improve the fit. The MAC model takes into consideration the absorption by the ozone layer, the absorption of water vapor, the Rayleigh scattering by molecules, and the extinction by aerosols.

There are many other complex models such as Wong and Chow [31], King and Buckius [32], Choudhary [33], Power [34], Yang [35, 36], Ineichen [37], Lingamgunta and Veziroglu [38], AHRAE [39], Hoyt [40], Lacis and Hansen [41], Josefsson [42], Carroll [43], Iqbal [16], Powell [44], EEC [45], PSI [46], HLJ [47], Kumar [48], ESRA [49], NRCC [50, 51], Salazar [52], CSR [53], MRM [54], Solis [55], EIM [56], and MLWT2.

The other clear atmosphere models are those of Hottel [47] and Liu and Jordon [10]. Hottel’s model provides a beam transmissivity for the clear atmosphere which is essentially the same as providing \( I_h \). Liu and Jordon provide a model to determine a so-called diffuse transmissivity from the beam transmissivity, which is the same as providing \( I_d \). These two models are not described in this Chapter because they are the two models used in this work and they are discussed in detail in the next chapter of this thesis.

### 3.3. Cloudy-Atmosphere Work

Like the clear-atmosphere level of modeling, the cloudy atmosphere level of modeling does not appear to have many equations available for optimum tilt angles for one axis and two-axis panels. This is understandable because this level depends on cloud cover and at this time cloud cover has to be handled in a statistical fashion. It should be stated here that the clear-atmosphere equations developed as part of this thesis work are a stepping stone to developing similar equations for cloudy atmospheres.

Recommendations for optimum panel orientations at the cloudy atmosphere level do exist for fixed solar panels for different time frames. The most common time frame is one year. The most common recommendation for a one year time frame is [11, 56]
\[ \gamma = 0 \] (79)
and
\[ \beta = \phi. \]  

(80)

Others have different recommendations than this for the tilt angle. For example, Hottel [57] recommends that

\[ \beta = \phi + 20, \]  

(81)

and Heywood [57] recommends something in the opposite direction,

\[ \beta = \phi - 10. \]  

(82)

Lewis [57] has developed an equation to calculate the optimum tilt angle for fixed solar panels to get maximum energy for a year. This equation is,

\[ \beta = \arctan \left[ \frac{\sum_{i=1}^{12} (I_{hM} \tan(\phi + \delta))}{\sum_{i=1}^{12} I_{hM}} \right] \]  

(83)

where the summation is carried out over the 12 months in a year and \( I_{hM} \) is the monthly averaged global radiation value on a horizontal surface.

Several investigators have proposed optimum slope equations for different seasons. The equations proposed by Elminir et al. [58] are

\[ \beta = \phi + 15^\circ \text{ if } \delta < 0 \]  

(84)

for the winter season (\( \delta < 0 \)), and

\[ \beta = \phi - 15^\circ \text{ if } \delta \geq 0 \]  

(85)

for the summer season (\( \delta > 0 \)).

However, other studies have used measured solar irradiation data which includes cloud and atmospheric effects for 4 sites in the state of Alabama. It was found that the optimum yearly tilt angle is [59]

\[ \beta_{opt} = \phi + 8^\circ. \]  

(86)

Furthermore, a study conducted in Europe to calculate the optimum tilt angle for a large area using measured irradiation [60] shows that the optimum yearly tilt angle including the effects of clouds is less than the latitude tilt. Thus, the optimum yearly tilt is not only a function of the latitude but also the function of the atmosphere and the cloud cover.

The most comprehensive study of optimum panel orientations including the effects of clouds was done by Matthew Lave and Jan Klessil [59]. These individuals conducted research on finding the optimum fixed orientation of solar panels and the benefits of tracking for solar panels in the continental United States. The average global solar irradiation incident on the optimum tilted solar panel was calculated for a year and was compared to the solar irradiation incident on a flat
horizontal solar panel and two-axis tracking solar panels. This study was performed using a computer program that performed detailed calculations and numerical integrations. The Page model [61] was used to determine the global irradiation at any arbitrary tilt and azimuthal angle. This study shows that there was an increase in solar radiation on optimum titled solar panels by 10% to 25% a year with increasing latitude as compared with the solar radiation falling on the horizontal, flat plate solar panel. This study also shows that there was an increase in solar radiation on two-axis tracking solar panel by 25% to 45% a year as compared to the solar radiation falling on the optimum tilted solar panel. Lave and Klessil also present solar maps of the continental United States showing the surface azimuthal angle, the optimum tilt angle, and the radiation impinging on the tracking panel, as a function of location. Figure 5 shows the optimum tilt angle for a fixed panel to obtain the maximum solar radiation for a one year time. Figure 6 shows the increase in solar radiation seen by an optimally tilted solar panel compared to the solar energy impinging on a horizontal solar panel. Lastly, Figure 7 shows the increase in solar radiation captured by a two-axis tracking panel compared to an optimally tilted fixed panel.

Figure 5: Optimum tilt angle for the solar panel to get maximum annual solar radiation falling on the solar panel [59].
Figure 6: Percentage increase in global solar radiation impinging on a solar panel which is optimally oriented versus global solar radiation impinging on the flat horizontal solar panel [59].

Figure 7: Percentage increase in global solar radiation impinging on a two-axis tracking panel versus global solar radiation impinging on the fixed solar panel which is optimally oriented [59].
Two other investigations of optimally titled panels at the cloudy-atmosphere level were done by Nakrani [62] in 2015 and Medarapu [63] in 2016. Both investigators used the Wright State developed solar code called Solar_PVHFC. This code calculates the performance of a photovoltaic, fuel cell, and hydrogen energy storage system at any location on the earth for any panel orientation for any time span desired. This program does detail hourly calculations of the solar energy impinging on a panel for any time desired using a detail anisotropic diffuse model as shown in Equation (2). These calculations use TMY3 hour horizontal plane radiation data published by the National Renewable Energy Laboratory [64]. The results are numerically integrated over the time span to obtain the total energy collected by the panels.

Nakrani [62] determined optimum tilt angles for fixed, two-axis tracking panels, and one axis, north-south trackers located in Dayton, Ohio. Different time periods were studied. For the fixed axis and the two-axis tracking panels, Nakrani [62] shows that the optimum tilts for the two-axis tracking and the fixed panels are less than the no-atmosphere conditions by about 5°. Medarapu [63] also looked at two-axis trackers and obtained the same conclusion as Nakrani [62] the panel should be tilted about 5° less than the no-atmosphere model predicts. In addition to two-axis trackers, Medarapu [63] also looked at one-axis north-south trackers, one-axis east-west trackers, and vertical axis trackers. For the east-west trackers and the north-south trackers, Medarapu [63] studied the tilt of the rotation axis. For vertical axis rotators, the tilt of the panel was studied.

3.4. Sensor Based Tracking

In the last 20 years, various methods have been proposed for sensor controlled tracking systems. Basically, all sensor based tracking systems can be classified into one of three major categories: closed-loop, open-loop, and hybrid sun-tracking systems. In the open loop system, the control system performs calculations to analyze the sun’s path using formulas such as those presented in this thesis. Feedback sensors are used in the open loop to determine the rotational angle of the tracking axis and make sure that the solar panel is positioned at the correct angles. On the other hand, in the closed loop control system, the sensor will sense the direct solar radiation falling on the solar panel and give a feedback to the control system to point the solar panel in the correct direction. Researchers have also developed a hybrid sun tracking system that contains open loop and closed loop sensors to obtain precise tracking, without difficulties when the sun is shaded.
[65, 66]. To trace the position of the sun, the above tracking methods (open loop, closed loop and hybrid system) use a microcontroller or PC-based controller.

In a closed loop tracking system, a CCD sensor or photodiode sensor is used to sense the position of the sun’s image on the receiver. If the sun’s image moves away from the receiver, then a feedback signal is sent to the control system. Due to weather conditions and different environmental factors, a closed loop control system can have difficulties tracking properly. However, this system has saved a lot of work and time by omitting high precision sun tracking alignment work. During fine weather, the accuracy of this tracking system is a few milli-radians. For this reason, closed loop tracking systems have been used for the past 20 years [65, 66, 67, 68, 69]. Kribus et al. [70] developed a closed loop controller for heliostats. This system improved the pointing error of the solar image up to 0.1 milli-radians with the help of 4 CCD cameras. The disadvantage of this system is that it is expensive, because it uses 4 CCD cameras and 4 radiometers. The solar images captured by the CCD cameras are sent to the control system for correcting the tracking error. In 2006, Luque-Heredia et al. [71] used a monolithic optoelectronic sensor as a sun tracking monitoring system. This method had an accuracy of better than 0.1°. However, for this type of tracking system to operate effectively, clear-atmosphere conditions are required. Furthermore, Chen et al. [65, 72, 73] presented digital and analog studies on the optical vernier and optical nonlinear compensation measuring principle. In 2004, Abdallah and Nijmeh [65, 74] developed a two-axis sun tracking system with the closed loop controlled system. A programmable logic controller was used to control the solar tracker in order to track the path of the sun. Closed loop sun tracking systems give better tracking accuracy; however, they lose track of the sun when the sensor is shaded or the sun is blocked by clouds.

To overcome the mentioned problem of closed-loop sun tracking systems, open loop sun tracking systems were introduced. Open loop tracking systems do not require the sun’s image as a feedback. The encoder is used as an open-loop sensor to make sure that the solar panel is accurately positioned at pre-calculated angles, which are calculated by an algorithm or a special formula. Referring to the literature [65, 75, 76, 77, 78, 79] the sun’s altitude angle and the sun’s azimuthal angle are calculated by an algorithm based on the sun’s position for a given geographical location and time. Open loop tracking methods have the ability to reach a prescribed angle within ±0.2°.

Both open loop and closed loop have their own advantages and disadvantages. That’s the reason hybrid sun tracking systems were developed. Hybrid sun tracking systems use both open
loop and closed loop sensors in order to get precise tracking accuracy and not have problems when the sun is shaded. Rubio et al. [65,80] developed and evaluated a new control strategy for a photovoltaic solar tracker. This solar tracking system operates on two tracking modes: normal tracking and safe tracking mode. The normal tracking mode uses both open loop tracking and closed-loop tracking. The open loop tracking method calculates the sun’s position in the sky and closed loop tracking method senses the sun’s position in the sky. In the safe tracking mode the orientation of the solar panel must remain in a specified boundary.
Chapter 4. Equation Development

In this section, the equation for the optimum orientation for single-axis and two-axis tracking solar panels operating in a clear atmosphere are developed. These are believed to be the only analytical equations available for optimum tilt angles for a clear-atmosphere. This is a step closer to physical reality from the no-atmosphere models presented in the previous chapter of this thesis.

Three existing models are utilized in developing the optimum tilt angle equation shown in this thesis. These are Hottel’s clear-atmosphere beam transmissivity model [47], Liu and Jordan’s diffuse radiation model [10], and the isotropic-sky model [9, 10] for determining the solar energy impinging on a solar panel at any location, at any orientation. The reason for using Hottel’s clear-atmosphere beam transmissivity model is because it provides reasonable accuracy and it is simple. The elevation above sea level of the panel’s location, the zenith angle of the sun, and the day on which the calculations are being performed are the main input parameters required. Constants in the equations can be adjusted for different climate zones. The four zones for which constants are available are tropical, midlatitude summer, midlatitude winter, and subarctic summer. This model provides the transmittance for beam radiation through a clear atmosphere. Liu and Jordan [10] have developed a relation to determine diffuse radiation for clear days. This relation is cast as a diffuse transmissivity because it determines the ratio of the diffuse radiation on a horizontal surface located at the ground level relative to the radiation on a horizontal surface at the top of the atmosphere. The only input required in Liu and Jordan’s model is the beam transmissivity from Hottel’s model. Using Hottel’s clear-atmosphere model and Liu and Jordan’s model together provides both the beam and diffuse solar radiation incident on a horizontal surface located on the surface of the earth. The last model mentioned above, the isotropic sky model, is required to
convert the horizontal surface radiation values to the solar radiation which impinges on a tilted solar panel. This model includes beam radiation, diffuse radiation, and solar radiation reflected off the ground. Thus, the optimum tilt angle equation developed as part of this thesis work includes beam radiation, diffuse radiation from the sky, and reflected radiation from the ground. Thus, the equation developed here for calculating optimum tilt angles includes a great deal more physical phenomena than the no-atmosphere optimum tilt equations currently used by a number of investigators, engineers, and students. The no-atmosphere equations were presented in the previous chapter of this thesis.

In the first section of this chapter the beam transmittance model of Hottel [47] and the so called “diffuse transmittance” as determine by Liu and Jordon’s model [10] are presented. In the second section of this chapter the Hottel beam transmittance and the Liu and Jordan diffuse transmittance are inserted into the isotropic sky model. This is the equation used to derive the optimum tilt angle equation including clear atmosphere effects. This is done in the third section of this chapter. The fourth and last section of this chapter uses the newly derived clear-atmosphere optimum tilt angle equations for some no-atmosphere cases. This is done to show the correctness of the clear-atmosphere equation derived in the third section of this chapter.

4.1. Transmittance Model

The definition of the transmittance for beam radiation though a clear atmosphere, \( \tau_b \), is [11]

\[
\tau_b = \frac{I_{\text{cb,n}}}{I_{\text{on}}}.
\] (87)

In this equation, the beam radiation on a surface normal to the sun’s rays, located on the surface of the earth considering atmospheric effects, \( I_{\text{b,n}} \), is the quantity that is desired. To get this quantity both the beam transmittance and the radiation normal to the sun’s rays at the top of the atmosphere is required. This quantity is given by Equation (57).

Since the beam transmissivity is a ratio between the normal radiation at the bottom of the atmosphere to that at the top of the atmosphere, these quantities can be replaced with the horizontal surface radiation values,

\[
\tau_b = \frac{I_{\text{cb,h}}}{I_{\text{on}} \cos \Theta_z}.
\] (88)
where $I_{cb,h}$ is the needed quantity. This means

$$I_{cb,h} = I_{on} \cos \theta_z \tau_b. \tag{89}$$

Hottel [47] provides an equation for the atmospheric transmittance as

$$\tau_b = a_o + a_1 e^{-(K \cos \theta_z)}. \tag{90}$$

The constants $a_o, a_1, K$ for a standard atmosphere with 23 km visibility are calculated from another set of constants $a'_o, a'_1$ and $K'$. The values for $a'_o, a'_1$ and $K'$ for altitudes less than 2.5 km are given by

$$a'_o = 0.4237 - 0.0082(6 - A)^2, \tag{91}$$

$$a'_1 = 0.5055 + 0.0595(6 - A)^2, \tag{92}$$

and

$$K' = 0.2711 + 0.01858(2.5 - A)^2. \tag{93}$$

where $A$ is the altitude of the location above sea level in km. For an urban haze atmosphere, these constants are given as [47]

$$a'_o = 0.2538 - 0.0063(6 - A)^2, \tag{94}$$

$$a'_1 = 0.7678 + 0.01858(6 - A)^2, \tag{95}$$

and

$$K' = 0.249 + 0.081(2.5 - A)^2. \tag{96}$$

Correction factors $r_o, r_1$ and $r_K$ are applied to $a'_o, a'_1$ and $K'$ to allow for differences in climate types. The constants $a_o, a_1, K$ are the functions of altitude of the location and haze model (visibility range). The constants $a_o, a_1, K$ can be calculated using the correction factors

$$r_o = \frac{a_o}{a'_o}, \tag{97}$$

$$r_1 = \frac{a_1}{a'_1}, \tag{98}$$

$$r_K = \frac{K}{K'}. \tag{99}$$

These correction factors for four different climate types are given in Table 3.

The transmittance for the beam radiation can only be calculated for the altitudes up to 2.5 km. As solar radiation travels through the atmosphere it is attenuated by absorbing and scattering. Not all the scattered radiation is lost, some part arrives at the surface of the earth in the form of diffuse radiation. It is important to calculate clear sky diffuse irradiance on a horizontal surface to get total irradiance falling on the surface of the earth. Liu and Jordan [10] have developed a relation between the transmission coefficient for beam and diffuse radiation for clear
days. In this report, we have used Liu and Jordan’s model to evaluate transmission coefficients for diffuse radiation, $\tau_d$ [10],

$$\tau_d = \frac{I_d}{I_o} = 0.271 - 0.294\tau_b.$$  \hfill (100)

The so called diffuse transmittance is the ratio of diffuse radiation on the horizontal surface of the earth to the extraterrrestrial radiation on the horizontal surface, $\frac{I_d}{I_o}$ or $\frac{G_d}{G_o}$.

Table 3: The correction factors $r_o, r_1, r_K$ for four different climate zones [47].

<table>
<thead>
<tr>
<th>Climate Type</th>
<th>$r_o$</th>
<th>$r_1$</th>
<th>$r_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropical</td>
<td>0.95</td>
<td>0.98</td>
<td>1.02</td>
</tr>
<tr>
<td>Midlatitude Summer</td>
<td>0.97</td>
<td>0.99</td>
<td>1.02</td>
</tr>
<tr>
<td>Subarctic Summer</td>
<td>0.99</td>
<td>0.99</td>
<td>1.01</td>
</tr>
<tr>
<td>Midlatitude Winter</td>
<td>1.03</td>
<td>1.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Clear sky diffuse radiation incident on the horizontal surface can be written as [11]

$$I_{cd,h} = I_{on}\tau_d \cos \Theta_z.$$  \hfill (101)

Total clear sky solar radiation on a horizontal surface is

$$I_{ch} = I_{cb} + I_{cd}$$  \hfill (102)

where

$$I_{ch} = I_{on}\tau_b \cos \Theta_z + I_{on}\tau_d \cos \Theta_z$$  \hfill (103)

or

$$I_{ch} = I_{on} \cos \Theta_z (\tau_b + \tau_d).$$  \hfill (104)

When the intensity of beam radiation incident normal to the surface of the earth is known, Equations (11) and (15) provide a method for estimating the intensity of diffuse radiation on a horizontal surface of the earth under a cloudless atmosphere. The diffuse and beam transmission coefficients are functions of the solar altitude, dust content, ozone content, atmospheric water vapor content, ozone content, and other radiation depleting factors [47].

In order to design the equations for calculating optimum tilt angles, we need a solar radiation model which can calculate the solar radiation on a tilted surface. The isotropic solar radiation equations is used to do this. The total solar radiation incident on a tilted surface consist of three components (beam radiation, diffuse radiation and reflected radiation from the ground).
Therefore, the equation which is developed here for calculating optimum orientations are more precise and more accurate because these equations are not only a function of the beam component, but also a function of the diffuse radiation and the radiation reflected from the ground.

4.2. Tilted Surface Radiation

To get the radiation on a tilted surface from that on a horizontal surface the isotropic sky model is used. This thesis cited several models in Chapter 2 which estimate the solar irradiation on tilted surfaces from that on a horizontal surface. The only difference between these models is the way these models treat diffuse radiation. These models use the same method to calculate beam radiation and radiation reflected from the ground. In this equation development, the isotropic sky model is used because it is simple, effective, and used extensively.

As given in Equation (9) and restated here, the isotropic sky model [9, 10] model assumes all the diffuse radiation is uniformly distributed from the entire sky and is

\[ I_T = I_b R_b + I_d \left( \frac{1 + \cos \beta}{2} \right) + I_p \left( \frac{1 - \cos \beta}{2} \right). \]

To design the equations for calculating the optimum tilt for a clear-atmosphere, \( I_b, I_d, \) and \( I \) are replaced in Equation (9) with \( I_{cb,h}, I_{cd,h}, \) and \( I_{c,h} \) for a clear-atmosphere given by Equations (89), (101), and (104),

\[ I_{c,T} = I_{on} \tau_b \cos \Theta_z R_b + I_{on} \tau_d \cos \Theta_z \left( \frac{1 + \cos \beta}{2} \right) + I_{on} \cos \Theta_z \left( \tau_b \tau_d \right) \rho \left( \frac{1 - \cos \beta}{2} \right). \]  

(105)

This is the equation that needs to be optimized as a function of the tilt angle \( \beta \). While not immediately obvious, the most difficult quantity in this optimization process is \( R_b \).

4.3. Optimum Panel Tilt Equation for Clear-Air

In order to find the optimum tilt, Equation (105) is differentiated with respect to the tilt angle of the solar panel \( \beta \) and set it equal to zero,

\[ \frac{dI_{c,T}}{d\beta} = 0 = \frac{d(I_{on} \tau_b \cos \Theta_z R_b)}{d\beta} + \frac{d(I_{on} \tau_d \cos \Theta_z \left( \frac{1 + \cos \beta}{2} \right))}{d\beta} + \frac{d(I_{on} \cos \Theta_z \left( \tau_b \tau_d \right) \rho \left( \frac{1 - \cos \beta}{2} \right))}{d\beta}. \]  

(106)

The first thing that is noticed is that \( I_{on} \) (see Equation 57) is not a function of the tilt angle and can be taken out of the derivative and cancelled,
\[ 0 = \left[ I_{on} \frac{d}{d\beta} (\tau_b \cos \theta_z R_b) \right] + \left\{ I_{on} \frac{d}{d\beta} \left[ \tau_d \cos \theta_z \left( \frac{1 + \cos \beta}{2} \right) \right] \right\} + \left\{ I_{on} \frac{d}{d\beta} \left[ \cos \theta_z (\tau_b + \tau_d) \rho \left( \frac{1 - \cos \beta}{2} \right) \right] \right\}. \]

(Cancellation 107)

(Cancelling \( I_{on} \) and substituting \( R_b \) from Equation (15) gives)

\[ 0 = \frac{d}{d\beta} \left[ \tau_b \cos \theta_z \left( \frac{\cos \theta}{\cos \theta_z} \right) \right] + \frac{d}{d\beta} \left[ \tau_d \cos \theta_z \left( \frac{1 + \cos \beta}{2} \right) \right] + \frac{d}{d\beta} \left[ \cos \theta_z (\tau_b + \tau_d) \rho \left( \frac{1 - \cos \beta}{2} \right) \right]. \]

(Simplifying 108)

(Simplifying the first term gives)

\[ 0 = \left\{ \frac{d}{d\beta} (\tau_b \cos \theta) \right\} + \left\{ \frac{d}{d\beta} \left[ \tau_d \cos \theta_z \left( \frac{1 + \cos \beta}{2} \right) \right] \right\} + \left\{ \frac{d}{d\beta} \left[ \cos \theta_z (\tau_b + \tau_d) \rho \left( \frac{1 - \cos \beta}{2} \right) \right] \right\}. \]

(109)

(From Equation (22) it is noted that \( \cos \Theta_z \) is not a function of \( \beta \) and can be taken outside of the derivative. From Equations (90) and (100) it is also seen that \( \tau_b \) and \( \tau_d \) are not a function of \( \beta \). Since \( \rho \) is a constant, this also is not a function of \( \beta \). This means Equation (109) can be written as)

\[ 0 = \tau_b \frac{d}{d\beta} (\cos \theta) + \tau_d \cos \theta_z \frac{d}{d\beta} \left( \frac{1 + \cos \beta}{2} \right) + \cos \theta_z (\tau_b + \tau_d) \rho \frac{d}{d\beta} \left( \frac{1 - \cos \beta}{2} \right). \]

(110)

(The second and third terms can be differentiated easily to get)

\[ 0 = \tau_b \frac{d}{d\beta} (\cos \theta) + \tau_d \cos \theta_z \left( \frac{-\sin \beta}{2} \right) + \cos \theta_z (\tau_b + \tau_d) \rho \frac{\sin \beta}{2}. \]

(111)

(Separating \( (\tau_b + \tau_d) \) from the third term and multiplying the above equation by 2 gives)

\[ 0 = 2\tau_b \frac{d}{d\beta} (\cos \theta) - \tau_d \cos \theta_z \sin \beta + \tau_b \rho \cos \theta_z \sin \beta + \tau_d \rho \cos \theta_z \sin \beta. \]

(112)

(Removing \( -\sin \beta \) as a common factor from the last three terms gives)

\[ 0 = \left[ 2\tau_b \frac{d}{d\beta} (\cos \theta) \right] - \sin \beta \left[ (\tau_d \cos \theta_z) - (\tau_b \cos \theta_z \rho) - (\tau_d \cos \theta_z \rho) \right]. \]

(113)

(In order to differentiate the first term of Equation (113), the formula for \( \cos \theta \) given in Equation (21)

\[ \cos \theta = \cos \beta \cos \theta_z + \sin \beta \sin \theta_z \cos (\gamma_z - \gamma). \]

and rewritten here for convenience is needed.

Taking the derivative of Equation (21) with respect to \( \beta \),

\[ \frac{d}{d\beta} (\cos \theta) = \frac{d}{d\beta} \left[ \cos \beta \cos \theta_z + \sin \beta \sin \theta_z \cos (\gamma_z - \gamma) \right] \]

(114)

gives
\[
\frac{d}{d\beta} (\cos \theta) = \{[(-\sin \beta) \cos \Theta_z] + [\cos \beta \sin \Theta_z \cos (\gamma_s - \gamma)]\}.
\] (115)

Putting this value of \(\frac{d}{d\beta} (\cos \theta)\) in Equation (113) gives,
\[
0 = 2\tau_b [(-\sin \beta) \cos \Theta_z + \cos \beta \sin \Theta_z \cos (\gamma_s - \gamma)] - \sin \beta [\tau_d \cos \Theta_z - \tau_b \rho \cos \Theta_z - \tau_d \rho \cos \Theta_z].
\] (116)

Moving the second term to the left-hand side of the equation gives
\[
\sin \beta [\tau_d \cos \Theta_z - \tau_b \rho \cos \Theta_z - \tau_d \rho \cos \Theta_z]
= 2\tau_b [(-\sin \beta) \cos \Theta_z + \cos \beta \sin \Theta_z \cos (\gamma_s - \gamma)].
\] (117)

Dividing both sides by \(\sin \beta\) gives
\[
[\tau_d \cos \Theta_z - \tau_b \rho \cos \Theta_z - \tau_d \rho \cos \Theta_z] = 2\tau_b [-\cos \Theta_z + \frac{\cos \beta}{\sin \beta} \sin \Theta_z \cos (\gamma_s - \gamma)].
\] (118)

Moving \(2\tau_b\) to the left-hand
\[
\frac{[\tau_d \cos \Theta_z - \tau_b \rho \cos \Theta_z - \tau_d \rho \cos \Theta_z]}{2\tau_b} = -\cos \Theta_z + \frac{\cos \beta}{\sin \beta} \sin \Theta_z \cos (\gamma_s - \gamma).
\] (119)

Moving \((-\cos \Theta_z)\) to the left hand side,
\[
\frac{[\tau_d \cos \Theta_z - \tau_b \rho \cos \Theta_z - \tau_d \rho \cos \Theta_z]}{2\tau_b} \cos \Theta_z = \frac{\cos \beta}{\sin \beta} \sin \Theta_z \cos (\gamma_s - \gamma).
\] (120)

Removing \(\cos \Theta_z\) as a common factor
\[
\cos \Theta_z \left\{ \frac{[\tau_d - \tau_b \rho - \tau_d \rho]}{2\tau_b} + 1 \right\} = \frac{\cos \beta}{\sin \beta} \sin \Theta_z \cos (\gamma_s - \gamma).
\] (121)

and dividing each of the appropriate terms by \(2\tau_b\) gives
\[
\cos \Theta_z \left\{ \frac{\tau_d - \rho}{2\tau_b} - \frac{\tau_d \rho}{2\tau_b} + 1 \right\} = \frac{\cos \beta}{\sin \beta} \sin \Theta_z \cos (\gamma_s - \gamma).
\] (122)

Noticing that \(\frac{\cos \beta}{\sin \beta} = \frac{1}{\tan \beta}\) and solving for this quantity gives
\[
\tan \beta = \frac{\sin \Theta_z \cos (\gamma_s - \gamma)}{\cos \Theta_z \left\{ \frac{\tau_d - \rho}{2\tau_b} - \frac{\tau_d \rho}{2\tau_b} + 1 \right\}}.
\] (123)

Noticing that \(\frac{\sin \Theta_z}{\cos \Theta_z} = \tan \Theta_z\) this equation can be written as
\[
\tan \beta = \tan \Theta_z \frac{\cos (\gamma_s - \gamma)}{\left\{ \frac{\tau_d - \rho \tau_d + \rho}{2\tau_b} \right\}}.
\] (124)

Equation (124) is the optimum tilt angle equation as a function of the zenith angle of the sun, \(\Theta_z\), the azimuthal angle of the sun, \(\gamma_s\), the azimuthal angle of the solar panel, \(\gamma\), the so-called diffuse transmissivity of the atmosphere, \(\tau_d\), the beam transmissivity of the atmosphere, \(\tau_b\), and the ground reflectivity, \(\rho\). This is the main equation for this thesis work. This is believed to be the
first time this equation has ever been presented. This equation makes it easy to see how the optimum tilt angle of the panel is a function of the sun’s zenith angle, the sun and panel azimuthal angles, the beam and diffuse transmissivities of the atmosphere, and the ground reflectivity. This equation can be used to determine the optimum tilt angles of panels that have two-axis tracking, single, east-west horizontal axis tracking, and single, north-south horizontal axis tracking including the effects of a clear atmosphere. Each of these cases are discussed individually in the next section.

4.4. Special Cases for Clear Atmosphere Optimum Panel Slope Equation

4.4.1. Two-Axis Tracking

4.4.1.1. Equation for optimum tilt for two-axis tracking

For two-axis tracking it is easy to visualize that the optimum azimuthal angle of the panel is precisely equal to the azimuthal angle of the sun,

\[ \gamma = \gamma_s. \]  

(125)

Because any value of panel azimuthal angle gives the same value for the ground reflected radiation and the diffuse sky radiation, these two radiative quantities do not affect the optimum \( \gamma \) for clear atmosphere, two-axis tracking. Thus, as it is for the no-atmosphere case, the optimum panel azimuthal angle is exactly equal to the sun’s azimuthal angle. Plugging this value into Equation (124) gives

\[ \tan \beta = \tan \theta_z \frac{1}{\left\{ \frac{\tau_d}{2\tau_b} - \frac{\rho_d}{2\tau_b} \frac{\rho}{2 + 1} \right\}}. \]  

(126)

The optimum tilt angle for a two-axis tracking solar panel under clear atmosphere conditions can be obtained using this equation.

4.4.1.2. Check on two-axis tracking equation development

As a check on the two-axis optimum tilt angle equation developed in this thesis work, it is interesting to see if Equation (126) for the optimum tilt angle of a solar panel including the effects of a clear atmosphere reduces to the no-atmosphere equation given in Chapter 3 (this is Equation 34). For no atmosphere, the values of the quantities in the denominator of Equation (124) are

\[ \tau_b = 1, \]  

(127)
\[ \tau_d = 0, \quad \rho = 0. \tag{128} \]

and

\[ \rho = 0. \tag{129} \]

This means that every term in the denominator of Equation (126) goes to zero except the 1. Thus Equation (126) becomes

\[ \tan \beta = \tan \theta_z \frac{1}{\{\bar{z} - \bar{z} - \bar{z} + 1\}}, \tag{130} \]

which simplifies to

\[ \tan \beta = \tan \theta_z \tag{131} \]

or

\[ \beta = \theta_z. \tag{132} \]

This exactly matches the equation developed by Braun and Mitchell [13] shown in Equation (34). This is evidence that the clear-atmosphere equation developed as part of this thesis work is correct.

### 4.5. Single-Axis East-West Tracking

#### 4.5.1. Equation for Optimum Tilt for Single-Axis East-West Tracking

Using the general Equation (124)

\[ \tan \beta = \tan \theta_z \frac{\cos (Y_s - \gamma)}{\{\tau_d \rho \tau_d \rho + 1\}}, \]

rewritten here for convenience, a specific equation for the optimum tilt angle of an east-west tracking system under clear atmosphere conditions can be written. As it is a single-axis east-west tracking system, the surface azimuthal angle of the solar panel will change between 0° and 180° [13]. The surface azimuthal angle is (this is Equation 36 rewritten for convenience),

\[ \gamma = \begin{cases} 0^\circ & \text{if } |Y_s| < 90^\circ \\ 180^\circ & \text{if } |Y_s| \geq 90^\circ. \end{cases} \]

The optimum orientation equation for east–west single-axis tracking for a clear atmosphere can then be declared as

\[ \tan \beta = \tan \theta_z \frac{|\cos Y_s|}{\{\tau_d \rho \tau_d \rho + 1\}}, \tag{133} \]

The optimum tilt angle for single-axis east-west tracking for the clear atmosphere was obtained by substituting the surface azimuthal angle conditions from Equation (36) into Equation (124).
4.5.1.1. Check on single-axis east-west axis equation development

As a check on the single east-west axis optimum tilt angle equation developed in this thesis work, it is interesting to see if Equation (133) for the optimum tilt angle of a solar panel including the effects of a clear atmosphere for an east-west tracking system reduces to the no-atmosphere equation given in Chapter 3 (this is Equation 37). For no-atmosphere, the values of the quantities in the denominator of Equation (133) are (these are Equations 127, 128, and 129)

\[ \tau_b = 1, \]
\[ \tau_d = 0, \]
\[ \rho = 0. \]

This means that every term in the denominator of Equation (133) goes to zero except the 1. Thus Equation (133) gives

\[ \tan \beta = \tan \theta_z \frac{|\cos \gamma_s|}{\left( \tau_d - \rho \tau_d - \rho + 1 \right)}. \] (134)

Therefore

\[ \tan \beta = \tan \theta_z |\cos \gamma_s|. \] (135)

This is the same equation developed by Braun and Mitchell [13] for no-atmosphere conditions (see Equation 37). This indicates that the clear-atmosphere optimum tilt angle equation developed in this work and shown in Equation (133) is correct.

4.5.2. Single-Axis North-South Tracking

4.5.2.1. Equation for optimum tilt for single-axis north-south tracking

Using the general Equation (124)

\[ \tan \beta = \tan \theta_z \frac{\cos(\gamma_s - \gamma)}{\left( \tau_d - \rho \tau_d - \rho + 1 \right)}. \]

for a single-axis north-south tracking system allows a specific equation for the optimum tilt angle of a north-south axis tracking system under clear-atmosphere conditions to be developed. The optimum surface azimuthal angle of a solar panel will change between +90° and -90° [13] for clear-atmosphere conditions, just like no-atmosphere conditions. The optimum surface azimuthal angle is (this is Equation 38 rewritten for convenience)

\[ \gamma = \begin{cases} 90^\circ & \text{if } \gamma_s > 0^\circ \\ -90^\circ & \text{if } \gamma_s \leq 0^\circ \end{cases}. \]
Thus the optimum tilt angle equation for a north-south single-axis tracking system for clear atmosphere conditions is

$$\tan \beta = \tan \theta_z \frac{\cos(y_s - \gamma)}{\left(\frac{\tau_d}{\tau_b} + \frac{\rho}{\tau_b} + 1\right)}$$ \hspace{1cm} (136)

4.5.2.2. Check on single-axis north-south axis equation development

As a check on the north-south optimum tilt angle equation developed in this thesis work, it is interesting to see if Equation (136) for the optimum tilt angle of a solar panel including the effects of a clear atmosphere for north-south tracking reduces to the no-atmosphere equation given in Chapter 3, Equation (39), for no-atmosphere conditions. For no atmosphere, the values of the quantities in the denominator of Equation (136) are

$$\tau_b = 1,$$

$$\tau_d = 0,$$

and

$$\rho = 0.$$ 

This means that every term in the denominator of Equation (136) goes to zero except the 1. Thus Equation (136) gives

$$\tan \beta = \tan \theta_z \frac{\cos(y_s - \gamma)}{\left(\frac{0}{2^2} + \frac{0}{2^2} + 1\right)}.$$ \hspace{1cm} (137)

Therefore

$$\tan \beta = \tan \theta_z \cos(y_s - \gamma).$$ \hspace{1cm} (138)

It can be see that Equation (39) developed by Braun and Mitchell [13] for no-atmosphere conditions is exactly the same as the limiting case of the optimum tilt equation developed for clear atmosphere conditions in this thesis work. This indicates that Equation (136) is correct.
Chapter 5. Site Results

In this chapter, the optimum tilt angle results which are calculated from the clear-atmosphere optimum tilt equation as a function of latitude and time (see Equation 124) are shown. Most results are for a latitude close to that of Dayton, Ohio, but results at other latitudes are also shown. The results presented in this chapter should provide the reader with a good understanding of the effect of a clear atmosphere on the optimum solar panel tilt angle. This is made obvious in that most of the results presented in this chapter are presented as differences from the no-atmosphere results. A few results are presented in terms of the actual optimum tilt angle to give the reader an idea of the magnitudes of the optimum tilt angle.

The program developed to solve the required equations to determine the clear-atmosphere optimum tilt angles and the no-atmosphere optimum tilt angles determines these values for every minute of the hour, for every hour of the day, for every day of the year. Thus, results are calculated for 525600 minutes which make up one year. Optimum tilt angles when there is no sun, i.e. nighttime, are set equal to zero. It should be noted that optimum tilt angles can be zero during some hours of the daytime, these zero values are easy to recognize relative to those that are set equal to zero because it is nighttime. For the nighttime hours, there is a string of zero values that occur in every 24 hour period, while for the daytime hours there may be a few that are not continuously periodic. Single graphs showing the optimum tilt angle itself over an entire year look like that shown in Figure 8. Figure 8 shows the optimum tilt angles for a two-axis tracker in a clear-atmosphere environment. Because there are 525600 data points on this limited size figure, very little information can be garnered from this graph. All it shows are peaks of 90° and minimums of 0°, but that is about the extent of the information that can be gotten from this plot. None of the
detailed hourly changes in the optimum tilt angles can be seen, even though they are there. For this reason, none of the results that are presented in this chapter are presented in this form.

The reason the maximum tilt of the solar panel goes to $90^\circ$ when the position of the sun is at sunrise and sunset is due to the dominance of the $\tan \theta_z$ term in Equation (124). At sunrise and sunset, the sun is on the horizon and therefore the zenith angle of the sun equals $90^\circ$. It is also obvious that the optimum no-atmosphere orientation of a solar panel at sunrise and sunset should be $90^\circ$ so the panel is pointing directly at the sun on the horizon. The clear-atmosphere model developed here also predicts this result because the quantity $\tan \theta_z$ in Equation (124) goes to infinity as $\theta_z$ approaches $90^\circ$. This drives the clear-atmosphere model optimum tilt angle to be $90^\circ$ at sunset and sunrise.

![Figure 8: Hourly optimum tilt angle for two-axis tracking solar panel utilizing clear-atmosphere model over one year for a latitude of 40 degrees.](image)

This chapter is divided into three sections. In each of the three sections, results for each of the three tracking orientations studied in this thesis are given. In Section 5.1 results for a two-axis tracker are given, in Section 5.2 results for a single-axis, horizontal east-west tracker are given, and in Section 5.3 results for a single-axis, horizontal north-south tracker are given. In each of these sections, four types of results are presented. Usually, the first result presented is the entire yearly results for the differences between the no-atmosphere results and the clear-atmosphere
results. Just like Figure 8 these results are for an entire year, but unlike Figure 8 results, the optimum angle differences are shown as opposed to the optimum angle itself. This allows the reader to see the envelope of the differences in the optimum tilt angles which are believed to be useful information. The second type of result presented in some of the sections is the optimum tilt angle itself plotted for four days of the year. These days are the spring equinox (March 20), the fall equinox (September 23), the summer solstice (June 21), and the winter solstice (December 21) for the year 2014. On these plots, both the no-atmosphere and the clear-atmosphere optimum tilt angles are presented. To better see the differences in the no-atmosphere and the clear-atmosphere results, the third type of results presented are the differences in the no-atmosphere and clear-atmosphere optimum tilt angles for the days listed above. The fourth type of results presented are the differences in the optimum tilt angles for the entire year plotted as a function of zenith angles. As will be seen later, this an interesting way to present the optimum tilt angle differences for an entire year.

5.1. Two-Axis Tracking

In this section, the optimum tilt angle results which are calculated from the clear-atmosphere optimum tilt equation (see Equation 126) for a two-axis tracker are shown. The envelope of hourly results for the optimum tilt angle differences is shown in Figure 9. This is a rather uninteresting plot where only the maximum and minimum differences can be seen. The maximum difference is 2.7 degrees and the minimum hourly tilt angle difference is -1.1 degrees. The maximum occurs every day of the year, while the minimum values only occur during some days in the winter. It should be noted that the yearly graphs presented in this thesis all start on January 1 at midnight. Time throughout the year is numbered by the hour from midnight, January 1. This means that spring runs between the hours 1417 to 3624, summer runs between the hours 3625 to 5832, autumn runs between the hours 5833 to 8016, and the winter runs between the hours 8017 to 1416.

The hourly results of the optimum tilt angle for the no-atmosphere and clear-atmosphere cases for a single day are plotted in Figures 10 through 13. Each figure shows a different day of the year. Figure 10 is for the spring equinox, Figure 11 is for the fall equinox, Figure 12 is for the summer solstice, and Figure 13 is for the winter solstice. The reason for plotting hourly data for a day is to allow details of the optimum tilt angles to be seen. The optimum tilt angle for these days
can also help to visualize the overall trends of the optimum tilt angles for different seasons. The spring equinox is representative of the spring season, the fall equinox is representative of the autumn season, the summer solstice is representative of the summer season, and the winter solstice is representative of the winter season.

The minimum optimum tilts for both the no-atmosphere and clear-atmosphere results is at noon because the sun is the highest in the sky at this time. The hourly optimum tilt angle over a winter day is not as shallow as compared to a summer day. This just reflects the altitude of the sun in the sky for these seasons. After sunset and before sunrise, the optimum tilt angles are set to zero because the sun is below the horizon and therefore the solar panel does not have a line-of-sight to the sun. Although, in real life, just after sunset and just before sunrise there is still light in the sky and some solar energy can be captured by the solar panel. This is due to refraction and scattering of the light from the sun which is below the horizon. The no-atmosphere and the clear-atmosphere models being used here, like most solar models presented in the literature, does not use experimental data, do not include the bending of the sun’s rays, and do not include the diffuse component of solar radiation that still exists when the sun is below the horizon. The line connecting the data point right before sunset and that data point right after sunset is just a connecting line between the data points and 0°. This can be looked at as the solar panel going to its night position pointing directly towards the sky. The line connecting the 0° data point right before sunrise and the 90° data point right after sunrise can be looked at as the solar panel returning from its night position to actively tracking the sun.

Because the differences between the no-atmosphere and clear-atmosphere optimum tilt angles shown in Figures 10 through 13 are small, separate plots of the differences between these two values are shown in Figures 14 through 17. The days of the difference plots correspond to the optimum angle plots shown in Figures 10 through 13. During the morning, evening, and for winter days we get more diffuse radiation. In morning, evening, and winter days the sun is low in the sky and the beam radiation has a longer path through the atmosphere to reach the solar panel located on the ground. As the beam radiation takes a longer path through the atmosphere, more radiation gets scattered and absorbed; therefore, the diffuse radiation goes up and the beam radiation goes down. As the beam radiation goes down, the radiation coming off the ground also tends to go down. This is why the tilt angle of the solar panel using the clear-atmosphere model is lower than the tilt angle of the solar panel using the no-atmosphere model in the morning, evening, and for
Figure 9: Angular difference of the tilt angles between the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel over one year for a latitude of 40°.

Figure 10: Hourly optimum tilt angle for a two-axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the spring equinox.
Figure 11: Hourly optimum tilt angle for a two-axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the fall equinox.

Figure 12: Hourly optimum tilt angle for a two-axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the summer solstice.
Figure 13: Hourly optimum tilt angle for a two-axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the winter solstice.

Figure 14: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 40° on the spring equinox.
Figure 15: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 40°, on the fall equinox.

Figure 16: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 40°, on the summer solstice.
Figure 17: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 40°, on the winter solstice.

winter days in Figures 10 through 13. This means the clear-atmosphere solar panel is pointing more towards the sky than the ground as compared to the no-atmosphere panel. The clear-atmosphere model is stating that diffuse radiation from the sky is larger than reflected radiation from the ground. It can be seen that the differences of the hourly tilt angles between the no-atmosphere model and clear-atmosphere model are positive in the morning, evening and throughout the day in the winter season. These deviations are small because the beam radiation is usually the dominant component of radiation for all hours and only slight variations from the direction of the beam radiation are allowed to optimize the diffuse sky radiation against the ground reflected radiation.

Figures 14, 15, and 16 at solar noon show that the differences of the tilt angles between the no-atmosphere model and clear-atmosphere model go negative. This means that the clear-atmosphere model tilts the solar panel at a larger angle, such that more ground radiation is captured by the panel. As discussed in the previous paragraph, the solar panel wants to tilt up to the sky for morning, evening, and winter hours to gather more diffuse radiation. For the midday hours in the summer, spring, and fall, ground reflected energy increases. The reason for this is that the beam
radiation is providing more reflected energy from the ground than is being scattered off the molecules in the atmosphere. Therefore, the results in Figures 10, 11, and 12 show panel tilt angles from the clear-atmosphere model being more than panel tilt angles of the solar panel using the no-atmosphere model around midday. These results are shown as negative angle differences in Figures 14, 15, and 16. In Figure 16, at solar noon the solar panel starts moving back towards the sun because of the intense beam radiation. The ground reflection increases at noon as well, but the beam radiation dominates.

To make more sense of the results shown so far in this chapter Figure 18 was prepared. Figure 18 shows the difference in the no-atmosphere optimum angle results and the clear-atmosphere optimum angle results versus the zenith angle of the sun. This plot shows results for every minute of all 8760 hours in a year for a latitude of 40°. This plot clearly shows that the effects of a clear atmosphere on the difference in the optimum angles predicted by the two models for a two-axis tracking solar panel is simply a function of the zenith angle of the sun. All the data shown in Figure 9 has been replotted in Figure 18. Figure 9 does not show much useful information, while Figure 18 does. This conclusion is verified by looking at Equations (126), (90) and (100). For a fixed location and a fixed ground reflectance, Equation (126) shows that the difference between \( \tan \beta \), where \( \beta \) is the optimum tilt angle for a clear-atmosphere environment, and \( \tan \theta_z \), where \( \theta_z \), is the optimum tilt angle for a no-atmosphere environment, is the factor

\[
\frac{1}{\left( \frac{\tau_d}{\tau_b} \rho + \frac{\rho}{\rho + 1} \right)^2}
\]

For a fixed ground reflectivity \( \rho \) this factor is only a function of the beam transmissivity \( \tau_b \) and the diffuse transmissivity \( \tau_d \). For a fixed location, Equations (90) and (100) show that \( \tau_b \) and \( \tau_d \) are only a function of the zenith angle. This means the differences in the no-atmosphere and clear-atmosphere optimum angle results are only a function of the zenith angle, as Figure 18 shows.

In Figure 18 the results are only plotted from a zenith angle of 90° to 18° because this is the range of up and down sun movement at a latitude of 40°. As will be shown in later results, other latitudes will have different ranges. The reason for stating this range of zenith angles from 90° to 18° is that a zenith angle of 90° is where the sun is on the horizon and a zenith angle of 18° is the highest point in the sky that the sun obtains when viewed from a 40° latitude.

Figure 18 shows that at sunrise and sunset (\( \theta_z = 90° \)) and at a zenith angle of 59° the differences in the optimum angles predicted by the no-atmosphere and clear-atmosphere models

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are zero. The reason for this convergence between the two models at sunrise and sunset has been explained above, but the convergence at 59° is due to other physical reasons. Since the no-atmosphere model simply points the solar panel directly at the sun, this means the clear-atmosphere model is predicting the panel should point directly at the sun as well at 59°. This has to be due to the diffuse sky and ground reflected radiation being equal in magnitude for this zenith angle of the sun. The minimum difference in the models occurs at a zenith angle of 32°. This is due to ground reflected radiation being at its largest relative to diffuse radiation.

Figure 18: Angular difference of the tilt angles between the no-atmosphere model and clear-atmosphere model versus zenith angle for a two-axis tracking solar panel over a year for a latitude of 40 degrees.

It is also interesting to note that the maximum value of the differences is 2.7 degrees which occur at a zenith angle of 83°. For larger zenith angles the difference trend goes downwards to zero at a zenith angle of 90°. It is not believed this downward trend represents reality. It is felt that the difference between the no-atmosphere result and the clear-atmosphere results should continually increase from 83° to 90°. The reason this is not happening in the model developed as part of this thesis is that Liu and Jordon’s [10] model defines their diffuse transmissivity as the ratio of the diffuse radiation on a horizontal surface at the bottom of the atmosphere relative to the solar radiation at the top of the atmosphere on a horizontal surface. This makes the diffuse radiation on
the surface of the earth go to zero as the zenith angle goes to $90^\circ$. Maybe in the future the diffuse and beam radiation model of Daneshyar, Paltridge, and Proctor [18,19] can be used. This model does not have the diffuse radiation going to zero at a zenith angle of $90^\circ$. It should also be noted that Daneshyar, Paltridge, and Proctor have the beam radiation going to zero at a zenith angle of $90^\circ$, which is not exactly realistic either.

Figures 19 through 24 show the same type of results as Figure 9 and Figures 14 through 18 show. The difference between these sets of figures is that Figures 19 through 24 are for a latitude of $20^\circ$ while Figure 9 and Figures 14 through 18 are for a latitude of $40^\circ$. It is immediately obvious that the envelopes of the yearly results are fairly similar at a $20^\circ$ latitude as compared to a $40^\circ$ latitude. The maximum and minimum differences are the same, $2.7^\circ$ and $-1.1^\circ$, but the minimum differences extend a longer period of time and the negative differences never go to zero (this can be seen in Figure 19), like at $40^\circ$, due to the location being closer to the equator. Being closer to the equator means the sun is higher in the sky.

At a latitude of $20^\circ$, the highest position of the sun in the sky is a zenith angle of $0^\circ$ (see Figure 24) as compared to a latitude of $40^\circ$ (see Figure 18) where the highest position is a zenith angle of $17^\circ$. Both zenith angle plots, that at latitudes of $20^\circ$ and that at $40^\circ$, show essentially the same trends, there is just a little lengthening that has occurred. Note that the $20^\circ$ zenith angle plot collapsed the yearly data to a single curve like the $40^\circ$ zenith angle plot.

Differences in the daily data for the spring equinox, fall equinox, summer solstice, and winter solstice can be seen by comparing Figures 14 through 17 for a $40^\circ$ latitude with Figures 20 through 23 for a $20^\circ$ latitude. The biggest difference is what happens in the middle of the day. The results at a $20^\circ$ latitude are more negative than the results for a $40^\circ$ latitude. This shows the effect of moving closer to the equator where the sun stays higher in the sky. The higher altitude sun is causing more ground reflection and thus the clear-atmosphere model is tilting the panel slightly more towards the ground than towards the sky. Another interesting difference is what occurs right at solar noon. For a $40^\circ$ latitude for the fall and spring equinoxes and the summer solstice, the differences between the no-atmosphere model and the clear-atmosphere model become less negative. The solar noon difference results are tending towards zero at smaller latitudes because the sun is getting higher in the sky and approaching being directly overhead at solar noon. This is evidenced by the fact that the summer solstice noon result is closer to zero than the fall and spring equinoxes. The winter solstice does not show this behavior at all. This behavior is caused by a
strong beam radiation component at solar noon in the summer that is causing the panel to point more directly at the sun in the clear-atmosphere model case, just as is the case in the no-atmosphere model.

Figure 19: Angular difference of the tilt angles between no-atmosphere model and clear-atmosphere model for two-axis tracking solar panel over a one year time period for a latitude of 20 degrees.

Figures 25 through 30 show the same type of results as Figure 9 and Figures 14 through 18, and Figures 19 through 24 show. The difference between these three sets of figures is the latitudes of the locations being simulated. Figures 25 through 30 are for a latitude of 60°. This is a latitude that is further away from the equator and it is a location where the sun does not get as high in the sky as the 40° latitude. Since the sun does not get as high in the sky for the 40° latitude as the 20° latitude, this means trends in the opposite direction should be seen when comparing the 60° latitude results to the 40° latitude results, as relative to when the 20° latitude results were compared to the 40° latitude results.
The envelope of the yearly results shown in Figure 25 for a 60° latitude compared to a 40° latitude shown in Figure 9 are about the same. Just like the 20° and 40° latitude results, the maximum and minimum differences are 2.7° and -1.1°; however, the minimum differences extend a shorter period of time than at 40° or 20°. The zenith angle plots show essentially the same trends for all three latitudes, except the zenith angle where the plot begins at higher values of the zenith angles for higher latitudes. It should also be noted that the 60°, 40°, and 20° latitude zenith angle plots for two-axis tracking collapsed the yearly data to a single curve. For two-axis tracking, this always happens. This means the differences in the optimum tilt angle results between the no-atmosphere model and the clear-atmosphere model are only a function of the zenith angle of the sun. This is reasonable since the beam and diffuse transmittances are just a function of the zenith angle.

Differences in the daily data for the spring equinox, fall equinox, summer solstice, and winter solstice can be seen by comparing Figures 14 through 17 for a 40° latitude with Figures 26 through 29 for a 60° latitude. Again, the biggest difference is what happens in the middle of the day, but the trends are reversed from the 20° latitude to 40° latitude comparison. The results at a 60° latitude are less negative than the results for a 40° latitude. This shows the effect of moving farther from the equator where the sun stays lower in the sky. The lower altitude sun is causing less ground reflection and thus the clear-atmosphere model is tilting the panel slightly more towards the sky than towards the ground. For the fall and spring equinoxes, the middle of the day results at 60° do not go negative at all. For summer, they go a little negative.

From all the results for two-axis tracking, it can be seen that the difference of the tilt angles predicted by the no-atmosphere model and those predicted by the clear-atmosphere model are small. These small differences in the adjustment of the solar panel will only lead to small differences in energy capture. However, since it cost nothing to readjust the orientation of a solar panel to include atmospheric effects, this extra energy capture is free.

It should also have been noticed that no plots of the optimum azimuthal angle of the panels have been given in this section. They will not be given in the sections on one axis tracking either. This is because the optimum azimuthal angles for a clear atmosphere are the same as those for a no-atmosphere. For two-axis tracking, this means the azimuthal angle of the solar panel is equal to the azimuthal angle of the sun.
5.2. Single, Horizontal East-West Axis Tracking

In this section, the optimum tilt angle results which are calculated from the developed equation for a horizontal, east-west running axis tracker that follows the altitude of the sun well, but does not track the east to west motion of the sun well are presented. These types of tracking systems are used with solar panels because they are cheaper and two-axis tracking is not a viable option if very long rows of panels are used and they are all controlled as a unit. All results presented in this section are the difference between the no-atmosphere optimum tilt angle and the clear-atmosphere optimum tilt angle. The no-atmosphere optimum tilt angles are determined from Equation (37) and the clear-atmosphere optimum tilt angles are determined from Equation (133). In this section the same results as given for the two-axis tracker are presented, except only the 40° latitude location is studied.

![Hourly optimum tilt angle differences](image)

Figure 20: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 20°, on the spring equinox.
Figure 21: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 20°, on the fall equinox.

Figure 22: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 20°, on the summer solstice.
Figure 23: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 20°, on the winter solstice.

Figure 24: Angular difference of the tilt angles from the no-atmosphere model and clear-atmosphere model versus zenith angle for a two-axis tracking solar panel over a year for a latitude of 20°.
The hourly results of the differences in the optimum tilt angles for single, horizontal east-west axis tracking over one year are plotted in Figure 25. These results are considerably different than the yearly results for two-axis tracking shown in Figure 9. It is easy to see that the shapes of the envelopes of the two plots are different, but the magnitudes are also different. The two-axis results have maximum and minimum differences of 2.7° and -1.1° respectively. The maximum difference for east-west axis tracking is 12.3°, which is considerably more than the maximum two-axis difference of 2.7°. The reason for the differences in the east-west axis tracking and two-axis tracking results is the east-west axis tracking panel is constrained to rotate around a horizontal east-west axis and cannot track the azimuthal position of the sun.

This conclusion can also be deduced by looking at Equation (126) and Equation (133). Equation (126) gives the optimum tilt for a two-axis tracking panel and Equation (133) gives the optimum tilt for an east-west axis tracking panel. The difference between these two equations is the factor $|\cos\gamma_s|$. This means the east-west axis tracker’s optimum tilt angle is a function of the azimuthal angle of the sun as well as the sun’s zenith angle, while the two-axis tracker’s optimum tilt angle is only a function of the sun’s zenith angle. These two equations, developed in this thesis work, explicitly point out the cause for the difference in two-axis and east-west axis results.

Figure 25: Angular difference of the tilt angles between no-atmosphere model and clear-atmosphere model for a two-axis tracking solar panel over a one year time period for a latitude of 60°.
Figure 26: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 60°, on the spring equinox.

Figure 27: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 60°, on the fall equinox.
Figure 28: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 60°, on the summer solstice.

Figure 29: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a two-axis tracking solar panel, for a latitude of 60°, on the winter solstice.
Figure 30: Angular difference of the tilt angles between the no-atmosphere model and clear-atmosphere model versus zenith angle for a two-axis tracking solar panel over a year for a latitude of 60°.

As with the two-axis results, daily results of the optimum tilt angles themselves are shown for east-west axis tracking for the no-atmosphere situation and the clear-atmosphere situation. These results are shown in Figures 32, 33, 34, and 35. Figure 32 is for the spring equinox, Figure 33 is for the fall equinox, Figure 34 is for the summer solstice, and Figure 35 is for the winter solstice. For the spring equinox, fall equinox, and summer solstice the results take on a somewhat different shape between the peaks at sunrise and sunset. The east-west axis results have a midday portion that changes concavity from being upwards between the sunrise and sunset peaks to being concave downwards or close to level. The two-axis results did not change the concavity between the sunrise and sunset peaks and always remained up. There is certainly a different midday behavior between the two-axis results and the east-west axis results. For the winter solstice, the results are amazingly similar.

Angular differences of the tilt angles between the no-atmosphere model and the clear-atmosphere model for an east-west axis solar panel are also plotted for single days so that the differences between these two models may be seen in more detail. Figures 36, 37, 38, and 39 show the hourly angular differences of the tilt angles between the no-atmosphere model and the clear-
atmosphere model for the spring equinox, fall equinox, summer solstice, and winter solstice, respectively. From these plots, it can be seen that the biggest tilt angle differences occur on the spring and fall equinoxes close to sunrise and sunset. As mentioned above these differences are 12.3°, which is considerably more than the maximum two-axis difference of 2.7°. These larger differences occur because a single, horizontal east-west axis tracker cannot point directly at the sun. This means the beam radiation falling on a solar panel with this type of tracking is less than it would be for a solar panel using two-axis tracking. This makes the diffuse radiation from the sky and the radiation reflected from the ground, which is also taken as being diffuse, more important. With these other components of radiation being more important, the differences from the no-atmosphere orientation become larger.

Just like as was done for the two-axis tracking panels, the difference in the no-atmosphere optimum tilt angle results and the clear-atmosphere optimum tilt angle results versus the zenith angle of the sun are plotted (see Figure 40). This plot shows results for all 8760 hours in a year for a latitude of 40°. These are the same results shown in Figure 18 but for single-axis east-west tracking arrangement. The first thing that should be noticed is that a single line is not obtained when this type of plot is produced, as it was for the two-axis tracking results. In the east-west single-axis case, limited solid regions are obtained. The reason for these solid regions is that a separate line is obtained for every sun azimuthal angle. Since Equation (133) is solved on minute intervals, a great deal of solar azimuthal angles are encountered in a day. Equation (133) clearly shows that the optimum tilt angle for single, horizontal east-west axis tracking is a function of the sun’s azimuthal angle.

While the east-west axis tracker zenith angle plot shown in Figure 40 is not as nice as the two-axis zenith angle plot shown in Figure 18, it is much better than plotting the tilt angle differences versus time for a full year as shown in Figure 31. Regions of operation can be identified. Such as the clear atmosphere model producing larger tilt angles than the no-atmosphere model for sun zenith angles less than 59° and producing smaller tilt angles for zenith angles greater than this. The region below 17° has no results because the sun does not get higher than this in the sky. Maximum differences are obtained at zenith angles close to 90° but go to zero right at 90°.
Figure 31: Angular difference of the tilt angles between the no-atmosphere model and the clear-atmosphere model for a single, horizontal east-west axis tracking solar panel over one year for a latitude of 40°.

Figure 32: Hourly optimum tilt angle for a single, horizontal east-west axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the spring equinox.
Figure 33: Hourly optimum tilt angle for a single, horizontal east-west axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the fall equinox.

Figure 34: Hourly optimum tilt angle for a single, horizontal east-west axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the summer solstice.
Figure 35: Hourly optimum tilt angle for a single, horizontal east-west axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the winter solstice.

Figure 36: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal east-west axis tracking solar panel, for a latitude of 40° on the spring equinox.
Figure 37: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal east-west axis tracking solar panel, for a latitude of 40°, on the fall equinox.

Figure 38: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal east-west axis tracking solar panel, for a latitude of 40°, on the summer solstice.
Figure 39: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal east-west axis tracking solar panel, for a latitude of 40°, on the winter solstice.

5.3. Single Horizontal, North-South Axis Tracking

In this section, the optimum tilt angle results which are calculated from the developed equation for a solar panel using single horizontal, north-south axis tracking are presented. A north-south axis tracker is better at tracking the sun from east to west across the sky than it is at tracking the altitude changes of the sun. The surface azimuthal angle for the single, north-south axis tracking solar panel will flip at solar noon from being directly east to being directly west. This type of tracking somewhat tracks both the azimuthal angle of the sun and the zenith angle of the sun. Neither is perfect, but the daily motion of the sun across the sky is tracked better than the altitude motion. Results from both the no-atmosphere and clear-atmosphere models are presented in this section. Some optimum angle results are presented, but mostly the difference between the no-atmosphere results and the clear-atmosphere results are presented. In this section, the same type of results as shown for the east-west tracker in the prior section are presented.
Figure 40: Angular difference of the tilt angles from the no-atmosphere model and clear-atmosphere model versus zenith angle for a single, horizontal east-west axis tracking solar panel over a year for a latitude of 40 degrees.

The hourly results of the differences in the optimum tilt angles for single, horizontal north-south axis tracking, over one year are plotted in Figure 41. These results are different than the yearly results for two-axis tracking shown in Figure 9 and the yearly results for the east-west axis tracking shown in Figure 31. Both the shapes of the envelopes and the magnitudes are different. The north-south axis tracking results have maximum and minimum differences of 3.5° and -1.1° respectively. This is less than the maximum difference for east-west axis tracking which is 12.3° and more than the maximum two-axis difference which is 2.7°. The minimum differences are the same for all three cases. Once again, the reason for the varying results for the different types of tracking is the constraints or lack of constraints on the particular tracking system. From an equation perspective, it can be seen that two-axis tracker optimum tilt angle is only a function of the zenith angle (see Equations 126 and 34), the east-west tracker optimum tilt angle is a function of the zenith angle and the azimuthal angle of the sun (see Equations 133 and 37), and the north-south tracker optimum tilt angle is also a function of the zenith angle and the azimuthal angle of the sun (see Equations 136 and 39). The difference between the north-south axis tracker and the east-west axis tracker is that the east-west tracker is a function of the sun azimuthal angle through $|\cos \gamma_s|$,
and the north-south tracker is a function of the sun azimuthal angle through $\cos(\gamma_s - \gamma)$ which is equivalent to $|\sin\gamma_s|$. 

To realize more detail than is shown in Figure 41, optimum tilt angles for single days are shown in Figures 42, 43, 44, and 45. Results for the spring equinox are shown in Figure 42, results for the fall equinox are shown in Figure 43, results for the summer solstice are shown in Figure 44, and results for the winter solstice are shown in Figure 45. The big difference between the north-south tracking results and those for two-axis tracking and east-west tracking is the zero-degree optimum tilt angle for north-south tracking at noon. All four days have a zero-degree optimum tilt at solar noon. The optimum tilt difference plots shown in Figures 46, 47, 48, and 49 for the same days as the optimum tilt angle plots in Figures 42, 43, 44, and 45 also go to zero at solar noon. Both the no-atmosphere model and the clear-atmosphere model provide a 0° optimum tilt angle. The optimum tilt difference plots for the spring equinox (see Figure 46), fall equinox (see Figure 47), and summer solstice (see Figure 48) all have a pronounced “W” shape between the high sunrise and sunset values. For the winter solstice (see Figure 49) a simple “V” shape is seen as opposed to a more “U” shape seen in the two-axis and east-west axis winter solstice results.

Figure 50 shows a plot of angular differences in the tilt angles between the no-atmosphere model and the clear-atmosphere model. These differences are plotted versus zenith angle over a one-year time period for a single, north-south axis tracking solar panel for the latitude of 40°. Just like the east-west tracking results shown in Figure 40, solid regions of the plot are shown. The solid region for negative tilt angles differences is a little larger than that for the east-west tracking panel and the solid region for positive tilt angle differences is more of a wide line with a big open area. The division between negative tilt angle differences and positive tilt angle differences can still be taken as 59°, but the crossing point is somewhat spread out in the north-south tracking case. Because these results are for a 40° latitude, the minimum zenith angle is still 17°.
Figure 41: Angular difference of the tilt angles between the no-atmosphere model and the clear-atmosphere model for a single, horizontal north-south axis tracking solar panel over one year for a latitude of 40°.

Figure 42: Hourly optimum tilt angle for a single, horizontal north-south axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the spring equinox.
Figure 43: Hourly optimum tilt angle for a single, horizontal north-south axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the fall equinox.

Figure 44: Hourly optimum tilt angle for a single, horizontal north-south axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the summer solstice.
Figure 45: Hourly optimum tilt angle for a single, horizontal north-south axis tracking solar panel utilizing the no-atmosphere model and the clear-atmosphere model for a latitude of 40° on the winter solstice.

Figure 46: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal north-south axis tracking solar panel, for a latitude of 40° on the spring equinox.
Figure 47: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal north-south axis tracking solar panel, for a latitude of 40°, on the fall equinox.

Figure 48: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal north-south axis tracking solar panel, for a latitude of 40°, on the summer solstice.
Figure 49: Hourly optimum tilt angle differences between the results from the no-atmosphere model and the clear-atmosphere model for a single, horizontal north-south axis tracking solar panel, for a latitude of 40°, on the winter solstice.

Figure 50: Angular difference of the tilt angles from the no-atmosphere model and clear-atmosphere model versus zenith angle for a single, horizontal north-south axis tracking solar panel over a year for a latitude of 40 degrees.
Chapter 6. Beam Transmittance Study

In this chapter, the optimum tilt angle of a solar panel on a two-axis tracker is studied as a function of the beam transmittance and the ground reflectivity. For the results in Chapter 5, the beam transmittance was calculated using Equation (90) and the ground reflectivity was held constant at 0.2. Thus, for Chapter 5 results, the beam transmittance was a function of the time of day and the day of the year. For the study done in this chapter, beam transmittance and reflectivity are made independent variables. Just like the analysis in Chapter 5, the analysis in this chapter makes the location and the parameters associated with the location input parameters that are held constant in the survey. Since the diffuse transmittance is a function of the beam transmittance according to the Liu and Jordan [10] model, the diffuse transmittance is not taken as an independent variable but calculated with Equation (100). In this chapter the effect of beam transmittance and ground reflectivity on the optimum tilt angle of a solar panel will be studied at a latitude of 40° for the four special days that were used in Chapter 5: the spring equinox, the fall equinox, the summer solstice, and the winter solstice.

In this study, two ground reflectivity’s and four beam transmissivities are surveyed. The ground reflectivity’s considered are 0.2 and 0.7. A reflectivity of 0.2 is the traditional value used to represent non-snow covered ground and 0.7 is a traditional value used to represent snow covered ground. The beam transmittances studied are 0.1, 0.3, 0.6 and 0.9. The diffuse transmittances associated with these beam transmittances are determined by Equation (100) and are shown in Figure 51. As shown in this figure, the diffuse transmittance decreases in a linear fashion as the beam transmittance increases. With the Liu and Jordan [10] model used in this work, the maximum diffuse transmittance possible is 0.271. Because the diffuse transmissivity cannot be negative, this means the highest beam transmissivity allowed by Liu and Jordon’s model is 0.92. In this survey...
the largest beam transmittance used is 0.9. A beam transmittance of 0.9 results in a diffuse transmittance of 0.0064. A beam transmittance of 0.9 is larger than the earth’s atmosphere will allow and probably a more realistic upper limit on the beam transmittance is 0.8. The lower limit on the beam transmittance is 0. While a beam transmittance this low would not be possible under the clear-atmosphere assumptions, it can be obtained if a lot of clouds are present. At a zero-beam transmittance, the diffuse transmissivity is 0.271 indicating that a great deal of the sun’s energy is being reflected back to outer space.

Figure 51: Diffuse transmittance as a function of beam transmittance.

The results of surveying the effect of beam transmittance and ground reflectivity are shown in Figures 52, 53, 54, and 55. Figure 52 is for the spring equinox, Figure 53 is for the fall equinox, Figure 54 is for the summer solstice, and Figure 55 is for the winter solstice. The solid lines in these figures are for a ground reflectivity of 0.2 and the dashed lines are for a ground reflectivity of 0.7. Each beam transmittance is assigned a different color. This coloring and line style make it easier to compare results for the same ground reflectivity or for the same beam transmissivity. The reader is reminded that all the results in these four figures are clear-atmosphere optimum tilt angles and not differences between the no-atmosphere and clear-atmosphere optimum tilt angles.
Figure 52: Optimum tilt angle for two-axis tracking utilizing the clear-atmosphere model at different beam transmittances and ground reflectivity’s for a latitude of 40° on the spring equinox.

Figure 53: Optimum tilt angle for two-axis tracking utilizing the clear-atmosphere model at different beam transmittances and ground reflectivity’s for a latitude of 40° on the fall equinox.
Figure 54: Optimum tilt angle for two-axis tracking utilizing the clear-atmosphere model at different beam transmittances and ground reflectivity’s for a latitude of 40° on the summer solstice.

Figure 55: Optimum tilt angle for two-axis tracking utilizing the clear-atmosphere model at different beam transmittances and ground reflectivity’s for a latitude of 40° on the winter solstice.
The results in all four of the figures shown in this chapter show the same trends in regards to ground reflectivity and beam transmissivity. Firstly, all the 0.7 ground reflectivity results have higher optimum tilt angles than the associated 0.2 ground reflectivity results. This occurs because a higher ground reflectivity delivers more ground reflected radiation. Thus, the clear-atmosphere model is showing that the panels are tilting at larger angles to capture more of the ground reflected radiation at the expense of the diffuse sky radiation. The tilt differences between the 0.7 and 0.2 reflectivity’s are largest around the middle of the day and they are smallest close to sunrise and sunset. Unlike the results presented in Chapter 5 where the beam transmittance varies throughout the day, the beam transmittance is constant throughout the day in these results. This means the position of the sun in the sky is causing the differences in the 0.7 reflectivity results and the 0.2 reflectivity results to change throughout the day. It is also interesting to see that the effects of ground reflectivity are largest at the smallest beam transmittances. It is believed the reason this happens is that smaller beam transmittances have a smaller beam energy component and thus the diffuse sky component and the ground reflected component are more important to the total energy captured by the solar panel.

The effect of beam transmittance can be seen by looking at either the solid lines or dashed lines in Figures 52, 53, 54, and 55. All figures show the optimum clear atmosphere tilt angles increasing with increasing beam transmittance. However, it is noticed that these increases seem to level off as the beam transmissivity increases. There is a much bigger increase in the optimum tilt angle between beam transmittances of 0.1 and 0.3, then that between 0.6 and 0.9. In fact, the changes between beam transmittances of 0.6 and 0.9 are very small. This would seem to indicate as the beam transmittance increases the optimum tilt angle becomes independent of beam transmittance. However, this is not true. What is happening is the diffuse transmittance is becoming very small (see Figure 51) and the clear-atmosphere model is pointing the solar panel directly at the sun. By looking at Equation (126) it can be determined that as the beam transmittance increases towards 0.92, the optimum tilt angles approach the no-atmosphere optimum tilt angles. They do not reach the no-atmosphere optimum tilt angle because the reflectivity term still remains. When the diffuse transmissivities approach zero, the terms with diffuse transmissivities in Equation (126) drop out of the equation. To get the reflectivity term out of the equation the ground reflectivity would have to go to zero as it is in the no-atmosphere model.
This means the 0.2 reflectivity results shown in Figures 52, 53, 54, and 55 are closer to the no-atmosphere results than the 0.7 reflectivity results.

All the clear-sky optimum tilt angle results presented in this thesis can be explained in the following manner. The clear-atmosphere model is simply trying to optimize three components of radiation that impinge on the solar panel from different directions. These three components that are being optimized are the beam radiation, the diffuse sky radiation, and the ground reflected radiation. For the most part the dominant component of these three components is the beam radiation. This is the reason the clear-atmosphere results are generally close to the no-atmosphere results. The no-atmosphere results simply consider the beam radiation and thus they point the solar panel directly at the sun. When clear atmosphere conditions are considered, a solar panel can capture more solar energy by tilting slightly towards the ground or slightly towards the sky. This skew in the optimum tilt angle from the no-atmosphere result will be small if the beam radiation is strong relative to the diffuse sky radiation and the ground reflected radiation. The skew will also be small if the diffuse sky radiation is approximately the same strength as the ground reflected radiation at the tilt angle required to maximize the beam radiation. The skew from the no-atmosphere optimum tilt will be larger if the beam radiation is weak and the diffuse sky and ground reflected radiation greatly differ from one another. Of course, constraints put on the orientation of a solar panel, such as done with one axis trackers, effect this balance of these three radiation components. If a tracking system does not allow the panel to point directly at the sun, then the beam radiation component on the panel becomes weaker opening the door for more skew in the optimum tilt angle from the no-atmosphere result. This is the reason the horizontal, east-west axis tracker results and the horizontal, north-south axis tracker results have bigger differences between the no-atmosphere and clear-atmosphere optimum tilt angles than the two-axis tracker results.
Chapter 7. Conclusions

The objective of this thesis project was to produce straightforward, simple, analytical equations that determine optimum tilt angles for solar panels that include the effects of a clear-atmosphere. These equations have been developed for three types of tracking systems: two-axis tracking, single, horizontal east-west axis tracking, and single, horizontal north-south axis tracking. These equations clearly demonstrate the relationship between optimum tilt angles and the effects of a clear atmosphere. Clear atmosphere effects are quantified through the beam and diffuse transmittances. Ground effects on the optimum tilt angle of a solar panel are included through the ground reflectivity. It is felt that the addition of these equations to the no-atmosphere optimum tilt equations that have existed for many decades is a major advancement in mathematical modeling of optimum tilt angles. These equations clearly illustrate the effects of a clear-atmosphere on these optimum tilt angles.

The fact that these optimum tilt equations properly reduce to the no-atmosphere equations in the limit of zero diffuse transmittance, zero ground reflectivity, and a beam transmittance of one provides proof that these equations are correct. A detailed proof of the development of these equations is provided in this thesis to further enhance the reader’s confidence in the correctness of these equations. A differentiation should be made between the optimum tilt equations including clear-atmosphere effects developed in this thesis and the models used for the atmospheric beam transmissivity and the atmospheric diffuse transmissivity inserted into these developed equations. Many models for clear-atmosphere beam transmittance and diffuse transmittance are available and this work used the models of Hottel [47] and Liu and Jorden [10]. These are good models but may have some inaccuracies during sunrise and sunset.
Many optimum tilt angle results and differences between the no-atmosphere and clear-atmosphere optimum tilt angles are presented in this thesis. Results for each of the three types of tracking for which optimum tilt angle equations were developed are presented. Results are presented as a function of time for a whole year or for a single day. The single day results allow the reader to see the details of the optimum tilt angles as a function of time, while the yearly results only allow the reader to see the envelope of the daily maximum and minimum optimum tilt angles throughout the year. Probably the most useful results presented in this thesis are the optimum tilt angles as a function of the zenith angle of the sun. For two-axis tracking, this collapses an entire year of results into an easy to understand single curve. For the single, horizontal east-west axis tracking and single, horizontal north-south axis tracking the results collapse to regions, as opposed to single lines. The equations developed as part of this thesis clearly show that the reason for this is the two-axis results are only a function of the zenith angle of the sun, while the east-west axis tracker and the north-south axis tracker results are a function of the zenith and azimuthal angles of the sun. For two-axis tracking, results for latitudes of 20°, 40°, and 60° are presented. For east-west axis tracking and north-south axis tracking, results are only presented for a latitude of 40°. A 40° latitude is approximately the location of Dayton, OH, the place where this thesis work was carried out.

For two-axis tracking solar panels, the presented results show that the maximum optimum tilt angle difference between the no-atmosphere model and the clear-atmosphere model over the course of a year is 2.7°. The minimum optimum tilt angle difference between the no-atmosphere model and clear-atmosphere model over a course of a year is -1.1°. For east-west axis tracking solar panels, results show that the maximum optimum tilt angle difference between the no-atmosphere model and clear-atmosphere model is 12.3°. These maximum differences in optimum tilt angles occur just after sunrise and just before sunset for the months of March, April, August and September. The minimum optimum tilt angle difference between the no-atmosphere model and clear-atmosphere model results over the course of year for the east-west axis tracker is -1.1°. For north-south axis tracking solar panels, it can be seen that the maximum optimum tilt angle difference is 3.5°. The minimum optimum tilt angle difference for this type of tracker is -1.1°. The difference of optimum tilt angles from the no-atmosphere model and the clear-atmosphere model are not large. Thus the energy capture by a solar panel using an optimum tilt angle calculated by the clear-atmosphere model and a solar panel using an optimum tilt angle calculated by the no-
atmosphere model will be small. However, since there is no additional monetary cost to using a clear-atmosphere model instead of a no-atmosphere model, why not use a clear-atmosphere model to determine the tilt angle of a solar panel with two-axis tracking, east-west axis tracking, or north-south axis tracking and collect a little more energy. This little extra energy collection has a positive effect on the economics of the solar installation. Having said this, the reader must realize the effects of clouds have not been included in any of this work. This work lays a foundation upon which analytical equations for optimum tilt angles including the effects of clouds can be developed. Laying the foundation for a cloud optimum tilt angle analytical equation may be the most important aspect of this thesis work.

A beam transmittance and ground reflectance study for a two-axis tracker was also done as part of this work. This studied helped to illuminate the competing mechanisms in the determination of optimum tilt angle for clear atmosphere conditions. The clear-atmosphere model is simply trying to optimize three components of radiation that impinge on the solar panel from different directions. These three components are the beam radiation, the diffuse sky radiation, and the ground reflected radiation. The dominant component of these three components is the beam radiation. This is the reason the clear-atmosphere results are generally close to the no-atmosphere results. The no-atmosphere results simply consider the beam radiation and thus they point the solar panel directly at the sun. When clear atmosphere conditions are considered, a solar panel can capture more solar energy by tilting slightly towards the ground or slightly towards the sky. This skew in the optimum tilt angle from the no-atmosphere result will be small if the beam radiation is strong relative to the diffuse sky radiation and the ground reflected radiation. The skew will also be small if the diffuse sky radiation is approximately the same strength as the ground reflected radiation at the tilt angle required to maximize the beam radiation. The skew from the no-atmosphere optimum tilt will be larger if the beam radiation is weak and the diffuse sky and ground reflected radiation greatly differ from one another. Of course, constraints put on the orientation of a solar panel, such as done with one axis trackers, affect the balance of these three radiation components. If a tracking system does not allow the panel to point directly at the sun, then the beam radiation component on the panel becomes weaker opening the door for more skew in the optimum tilt angle from the no-atmosphere results. This is the reason the single, horizontal east-west axis tracker results and the single, horizontal north-south axis tracker results have bigger
differences between the no-atmosphere and clear-atmosphere optimum tilt angles than the two-axis tracker results.
References


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