2017

Mathematical Modeling of a P-N Junction Solar Cell using the Transport Equations

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MATHEMATICAL MODELING OF A P-N JUNCTION SOLAR CELL USING THE TRANSPORT EQUATIONS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Renewable and Clean Energy in Engineering

By

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Abstract


While analytical models are limited in the situations that they can simulate, they are generally easier to implement than numerical models and provide a rapid view of the variables which affect a certain quantity. Analytical models are also very useful in educational situations; such as a graduate class on photovoltaics. The modeling of the interior workings of a solar cell can be complex and involved; and some of the equations can become quite lengthy. A focus of this thesis work is the derivation of the minority carrier density and minority current density equations for a p-n junction solar cell.

The equations that are derived in this thesis are presented in the book *The Physics of Solar Cells* by Jenny Nelson. This book is currently used in the graduate Photovoltaics course being offered at Wright State University. During the offering of this class suspicions arose about the correctness of the minority carrier density equations and minority current density equations presented in this book. Thus these equations had to be checked and corrected if necessary. This is done in this thesis. These equations are derived from the proper form of the transport equations and validated against numerical results and limited case results from other analytical equations. Nelson does not present a derivation for these equations and another source that provides these equations in the same form as that shown in *The Physics of Solar Cells* could not be found. *The Physics of Solar Cells* has a rather unique formulation for these equations, in that it is more general and extensive than what other sources present. The derivation work done in this thesis confirms the suspicions of the instructor of this course and shows that errors were present in these equations. The correct form of these equations is presented in this thesis.
After deriving the correct version of the minority carrier density equations and validating the corrected form against a number of published results, these equations are used to produce a large amount of survey results for GaAs solar cells. This is done by programing these equations in Microsoft Excel. Minority carrier densities are plotted for a GaAs solar cell in the dark under equilibrium conditions, in the dark with an applied voltage, under illumination where the cell is short circuited, and under illumination where the cell has an applied voltage. Surveys of the effects of the doping levels, the applied external voltage, the thickness of the p and n sides of the solar cell, the strength of the illumination on the solar cell, and the recombination speed of the minority carriers at the boundary are performed. For many of these results the minority and majority carrier densities are presented as a function of position in the cell. For the dark cases, minority and majority carriers are plotted for the p-side quasi-neutral region, the n-side quasi-neutral region, and the space charge region. For the illuminated cases carrier densities are only plotted in the quasi-neutral regions. Carrier density results are not presented for the space charge region under illuminated conditions as no analytical expressions are known to exist for this situation. Some space charge region thickness results and junction voltage results are presented for dark conditions.
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<tr>
<td>(V_{bi})</td>
<td>Bias Voltage</td>
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<tr>
<td>(\phi_n)</td>
<td>Work Function for n-type material</td>
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<tr>
<td>(\phi_p)</td>
<td>Work Function for p-type material</td>
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<td>(q)</td>
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<td>$x_n$</td>
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<td>$R$</td>
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</table>
Greek Symbols

ε  Permittivity
ϕ  Electrostatic Potential
α  Absorption Coefficient
ρ  Density
Γ  Photon Flux
τ  Carrier Lifetime
λ  Wavelength
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Acknowledgements

I would like to take this opportunity to express my sincere gratitude to my thesis advisor Dr. James A. Menart, for guiding me through this endeavor and being a source of constant inspiration. Despite battling with thoroughly preoccupied and strenuously tight schedules, you’ve spared time to render every possible effort to help me go through this thesis work, and provided infrastructure and resources as well. Dr. Menart, I want to express my sincere gratitude for guiding me through my graduate studies in Wright State University.

I would like to thank University Center for International Education for funding my thesis program and providing me financial support during the course of this study.

I would like to thank my family for being supportive and continuously motivating me through the course of thesis work.
Chapter 1 - Introduction

A solar cell or photovoltaic cell (PV) is a device that converts the photon energy of sunlight directly into electrical energy. The world’s energy demand is strongly dependent on fossil fuels, which is a major cause of increasing greenhouse gases in the atmosphere. The energy providers are inclined to look into other sources of energy to lower the greenhouse gas levels in the atmosphere. The ability to utilize the sun’s energy for electricity generation seems like an attractive prospect as it can provide pollution-free energy, which is a prime focus of energy markets around the world. Solar installations have increased in the past decade. Moreover, a solar cell has a relatively longer life-time of use with relatively lower maintenance costs. However, a solar cell is relatively expensive compared to present technologies and its efficiency is low.


Worldwide energy demand has increased since the industrial revolution and this demand is increasing at an alarming rate of 5% each year [1]. At present, the world’s energy demand is fulfilled by fossil fuels (~80-85%) [2]. However, a lot of impetus is given to use renewable sources of energy to reduce the threat to the climate. The sun provides about $1.2 \times 10^{25}$ terawatts of energy, which is many times more than human energy consumption. This surplus energy is available around the world and it can be used to replace energy from fossil fuels [3]. The
utilization of solar energy will help in reducing the greenhouse gas footprint and emission of harmful gases into the atmosphere. Solar energy has many economic and environmental benefits over the use of fossil fuels.

1.1.1. Economic Benefits of a Solar Cell

Photovoltaic installations have increased in the past decade and the most important factor that is helping this increase is the falling cost of solar panels. Researchers are finding new materials to reduce the cost of solar panels, which has considerable impact on the number of installations around the world. GTM Research group reveals a drastic increase in the PV installations from last year as shown in Figure 1 [4].

![Figure 1: Annual U.S. Solar Photovoltaic Installations 2000-2016](image)

The major holdup for the use of solar energy as a primary source of energy is its low conversion efficiency and high cost of fabrication. A majority of research is focused on improving the conversion efficiency of solar cells by using different materials. The study of carrier densities in a solar cell at different operation conditions helps researches to understand the performance of a solar cell without fabricating the actual device. A multitude of analysis tools
are available for researchers to simulate the performance characteristics of a solar cell under different operating conditions.

A variety of materials are tested with a focus to improve the efficiency and reduce the cost of photovoltaic cells. A survey conducted by the National Research Energy Laboratory illustrates the history cell efficiencies (see Figure 2).

![Best Research Cell Efficiencies](image)

**Figure 2: Best Research Cell Efficiencies [5].**

In the recent years, there has been significant growth in residential photovoltaic installations. GTM Research indicates a growth of 19% in the year 2016 as compared to 2015. The total installation of residential photovoltaic in the USA is 2583 MW DC. However, the highest growth is in the utility sector with 10,593 MW installed in 2016 in the United States. This significant growth of photovoltaic can be attributed to falling cost of photovoltaic materials as shown in Figure 3.

### 1.1.2. Environmental Benefits of a Solar Cell
The major reason for the installation of photovoltaic panels is their environmental benefits over other fuels for energy. Solar cells produce no harmful or toxic emissions and this helps in reducing greenhouse gases (CO$_2$, NO$_x$) and toxic gases (SO$_2$, particulate matter). Solar plants can be installed on degraded farmlands or lands unsuitable for farming, thus providing suitable use to barren areas. Another nice advantage of solar installations is the location of these plants can be remote and the need for expensive and intrusive transmission lines can be reduced. Solar plants help in providing electricity to rural areas, thus reducing the cost of power in remote areas [6]. The socio-economic benefit of solar energy is instrumental in increasing the installation of solar plants around the world.

![Table: US Polysilicon, Wafer, Cell and Module Prices for recent quarters [5].](image)

**1.2. Motivation of Project**

The motivation of this project was provided by my instructor of the Photovoltaic class at Wright State University. The textbook for this class, ‘The Physics of Solar Cells’ by Jenny Nelson [7] presented some general equations for calculating the electron and hole densities in a p-n junction solar cell. It was believed that there were some mistakes in the equations presented and it was decided to find a similar source for the general analytical solution of carrier number densities. A thorough literature review revealed that most sources use limiting case of the carrier density equation presented by Jenny Nelson [7]. In other words, the general solution presented by Nelson is not available in the literature or in the references presented by Nelson. The literature review done as part of this thesis work indicated that authors used different locations (p-side or n-side) for the radiation incident on the p-n junction semiconductor material, most do not use recombination velocity boundary conditions, and none have used the same conditions as used by Nelson. Therefore, it was decided to derive the transport equations for the electrons and
holes second order differential transport equation. For general solution, the boundary conditions used were those used by Nelson. The equations were solved to find a general solution for carrier densities in p and n quasi-neutral (p-QNR and n-QNR) regions of the p-n junction that constitutes the solar cell.

1.3. Objective of Project

The objective of this Master’s Thesis project was to validate the equations presented by Nelson for the minority charge carriers and the minority current densities in the p–QNR and the n-QNR of a solar cell. A systematic procedure was used to derive the general equations using the same boundary conditions as Nelson [7]. This derivation work revealed errors in the way the results were presented in the book by Nelson [7]. A thorough approach has been used to verify these equations. These equations are applicable to solar cells comprised of p-n homojunction solar cells. Equations were derived for the p-QNR minority charge carries and minority current density and the n-QNR minority carrier density and minority current density for any semiconductor material. These equations can be used to calculate carrier densities at different voltages. A comparative survey is prepared for this thesis work for different voltages and illumination conditions. No numerical calculations are performed as part of this thesis work which is focused on analytical equations. The numerical approach to calculate carrier density as done by the computer program PC1D is not used, but published results from this program are used to validate the analytical equations derived in this thesis.

The second thrust of this thesis work was to write an in-house computer program using the derived equations to calculate the carrier densities and current densities in the semi-conductor device. A Microsoft EXCEL program was prepared so that the carrier densities could be calculated for different operation conditions for p-n homo-junction solar cell. This EXCEL program was used to generate results that could be compared to published results and thus verifies the accuracy of the derivation work performed here. This program was also used to generate a good deal of survey results for a GaAs solar cell. This thesis work focuses on the modeling of GaAs photovoltaic cells; however, other types of semi-conductor materials can be used if the proper properties of that particular material can be input to the EXCEL program.
2.1. The P-N Junction

When p-type semiconductor and n-type semiconductor are brought in contact with each other, a p-n junction is formed. A p-type semiconductor is a semiconductor material with excess of holes and this is achieved by doping the semiconductor material with Boron or Gallium atoms. An n-type semiconductor has excess of electrons and we get an n-type semiconductor by doping with Phosphorus or Arsenic atoms. In a p-n junction the electrons and holes diffuse across the junction due to asymmetry of the semiconductor material. This diffusion causes a layer of fixed charges on either side of the junction as shown in Figure 4, thus forming a space charge region. The diffusion of electrons and holes across the depletion region develops an electric field that restricts the further diffusion of charge carriers.

![Crystal structure of a P-N junction](image)

*Figure 4: Crystal structure of a P-N junction [8].*
Due to electrostatic field, the minority charge carriers drift in the opposite direction of diffusion. Equilibrium is established due to balancing of drift of minority charge carriers and diffusion of majority charge carrier in the electrostatic field region. The drift and diffusion of charge carriers result in formation of built-in bias causing the electrons and holes to move out of the space charge region and the junction becoming depleted, thus appropriately named depletion region. The built-in bias in equilibrium is represented by the difference of work functions of $n$ and $p$ type materials.

$$V_{bi} = \frac{1}{q}(\phi_n - \phi_p)$$  \hspace{1cm} (1)

The built in bias can be expressed in terms of doping level.

$$V_{bi} = \frac{k_B T}{q} \ln \left( \frac{N_d N_a}{n_i^2} \right)$$  \hspace{1cm} (2)

Figure 5 illustrates the direction of drift and diffusion of charge carriers and the Energy band profile across the junction in equilibrium.

![Energy band profile](image)

**Figure 5:** Energy band profile across the p-n junction in equilibrium [9].

The external applied bias alters the potential across the junction and the junction voltage can be represented as

$$V_j = V_{bi} - V$$  \hspace{1cm} (3)

The sunlight in a solar cell causes the accumulation of holes on the p-side and electrons on the n-side, thus driving the same results as caused by positive applied bias [7].
2.1.1. Determination of Fermi Energy

The Fermi energy for a p-n junction can be calculated using the Boltzmann’s approximation and this method is widely used by the researchers.

2.1.1.1. Boltzmann Approximation

At thermal equilibrium, the Fermi level of a non-degenerate semiconductor lies within the band gap of the material [10]. The electron density in an n-type material can be represented as

\[ n = N_c e^{\frac{(E_F-E_c)}{k_B T}} \]  \hspace{1cm} (4)

where, \( N_c \) is the effective density of states in the conduction band and is given by [7]

\[ N_c = 2 \left( \frac{m_c^* k_B T}{2 \pi \hbar^2} \right)^{\frac{3}{2}} \]  \hspace{1cm} (5)

The hole density in a p-type material is

\[ p = N_v e^{\frac{(E_v-E_F)}{k_B T}} \]  \hspace{1cm} (6)

The effective density of states in the valence band is given by

\[ N_v = 2 \left( \frac{m_v^* k_B T}{2 \pi \hbar^2} \right)^{\frac{3}{2}} \]  \hspace{1cm} (7)

The intrinsic carrier density is given by

\[ n_i^2 = np = N_v N_c e^{-\frac{E_g}{k_B T}} \]  \hspace{1cm} (8)

The electron and hole density can be written in terms of intrinsic carrier density and intrinsic potential energy

\[ n = n_i e^{\frac{(E_F-E_i)}{k_B T}} \]  \hspace{1cm} (9)

\[ p = n_i e^{\frac{(E_i-E_F)}{k_B T}} \]  \hspace{1cm} (10)

where,

\[ E_i = \frac{1}{2} (E_c + E_v) - \frac{1}{2} k_B T \ln \left( \frac{N_c}{N_v} \right) \]  \hspace{1cm} (11)
and $E_i$ is the intrinsic energy near the middle of the band gap and is equal to the Fermi level of pure semiconductor in equilibrium.

The above approximation is true for a semiconductor in equilibrium for the condition

$$\frac{E_c - E_F}{k_B T} \gg 1$$ \hspace{1cm} (12)

and

$$\frac{E_F - E_v}{k_B T} \gg 1$$ \hspace{1cm} (13)

2.1.2. Workings of a Solar Cell

![Solar Panel Diagram](image)

*Figure 6: Working of a p-n junction solar cell [11].*

Illumination generates the electron-hole pairs across the p-n junction of a solar cell. When the light strikes the semiconductor material, the electron-hole pairs are generated due to the transfer of photon energy to the electrons. In an open circuit configuration, the depletion width decreases because of recombination of charge carriers generated by photons with the charge carriers in the depletion region. From Equation (3), it could be understood that the junction
Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations

Voltage decreases when a forward bias is applied to the p-n junction. Illumination causes the effect which is equivalent to that produced by the forward bias by reducing the built-in bias and removing the asymmetry.

In a short circuit configuration, the charge carriers produced by photons generate current when the device is connected to an external load as shown in Figure 6. The light generates excess charge carriers in p and n regions. The electric field at the junction helps to drive the minority charge carriers across the depletion region and the majority charge carriers through the external circuit. The forward bias generates dark current in the opposite direction and this current doesn’t allow the operation of the solar cell in the short circuit condition [2].

2.1.3. Photovoltaic Cell Materials

The Research and Development laboratories around the world are investigating solar cell chemistries to reduce the cost and increase the conversion efficiency. A variety of semiconductor materials are being studied to come up with a solar cell chemistry that can provide excellent efficiency at affordable cost. Silicon is the most widely used material for solar cells, however, many other materials like gallium arsenide, cadmium telluride and other hetero-junction cells made of Cu(In, Ga)Se$_2$ (CIGS) are being manufactured. In this research, we focused on gallium arsenide (GaAs) solar cells but the equations can be used for other types of solar cells [12].

2.1.3.1. Silicon

About 90% of solar panels used today are made of silicon. It is the most widely used photo-voltaic cell material. Multi-crystalline silicon cell has an efficiency of 21%-25% [13]. Silicon can be used in crystalline or amorphous form. The crystalline Si thin-film solar cell is cheaper than wafered cells and can be easily integrated to a module. The amorphous silicon alloy can be used to make thin sheets of Silicon, however, the amorphous structure impede the flow of electrons and holes and degrades quicker [14].

2.1.3.2. Gallium Arsenide (GaAs)

GaAs has shown an efficiency of 46%, but these are cells are expensive when compared to silicon solar cells [13]. GaAs has better electron mobility than silicon and therefore, the charge
transport is better leading to higher efficiency. The absorption length of GaAs is less than 1 micron for visible light which implies that only a few microns of GaAs material is required to absorb the light. The absorption spectra below show the sharp edges of GaAs absorption [15]. The semi-conductor materials have sharp edges for absorption coefficient because the energy below the band gap energy is not sufficient enough for the semi-conductor material to absorb any radiation. This happens because the photon energy above the band gap energy is required to excite the electrons from the valence band to the conduction band. It is observed that the absorption coefficient is not constant above the band gap energy because it depends on the wavelength of light absorbed by the semi-conductor material.

![Absorption Spectra](image)

**Figure 7:** Absorption Spectra of some common photovoltaic semiconductors [15].

### 2.1.3.3. Cadmium Telluride and Copper Indium (Gallium) Diselenide (CIGS)

The thin film CdTe solar cell is a low cost, reliable and high throughput manufacturing technology. CIGS photovoltaics are high performance solar cells and further research is carried to make it a viable option in the future [16].
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2.1.3.4. **Advanced Materials for Solar Cell**

Perovskite cells are made of perovskite material structure and can achieve an efficiency of about 22%. However, this cell technology is still under research as there are certain technical challenges before these cells can be used commercially. Other type of solar cells technologies under research are dye-sensitized solar cell, organic photovoltaics and quantum dot solar cells [13].

2.1.4. **Solar Cell Modeling Assumptions**

A solar cell can be modeled to calculate the carrier and current densities by solving the transport equations and the Poisson’s equation. The general solution of carrier and current densities can be achieved by applying necessary boundary conditions at p and n junction edges. However, the calculation becomes complicated due to the non-linear recombination term. Moreover, the complicated structure of Poisson’s equation that couples the electron and hole density provides complexity to solve the equations using mathematical tools. The complexity of the solution can be resolved by using the numerical methods.

The mathematical model is an important tool for academic purposes and the transport equations can be solved to calculate the carrier and current densities by applying some approximations. The first assumption is neglecting the electric field in the semi-conductor material. This helps in decoupling the Poisson’s equation in neutral p and n regions. The absence of electric field in the neutral regions helps us assume that the majority carriers are constant and the change in minority charge carriers influence the current in this region. Secondly, it is assumed that the recombination rate is linear in the neutral regions. The linear recombination rate simplifies the transport equation and helps in solving the equations analytically. Both these assumptions are used in this thesis work to find the general solution for carrier densities analytically [7].

2.1.4.1. **Depletion Approximation**

The depletion approximation for this thesis work is prepared in the same way as presented in Nelson. The $p$ layer has a thickness of $x_p$ and $n$ layer has a thickness on $x_n$. The depletion width on the $p$ side is $w_p$, and on the $n$ side is $w_n$ as shown in Figure 8. The junction is
free of majority carriers and for x<0, p-type material of doping $N_a$, and, for x>0, n-type material of doping $N_d$.

Figure 8: Schematic layer structure of p-n junction [7].

The electrostatic potential changes across the depletion width which changes the bias voltage $V_{bi}$ across the space charge region. The boundary conditions to calculate the electrostatic potential $\phi$ is given below

$$\phi = 0 \text{ at } x = -w_p \quad (14)$$

$$\phi = V_{bi} \text{ at } x = w_n \quad (15)$$

According to Poisson’s equation,

$$\frac{d^2 \phi}{dx^2} = \frac{q}{\varepsilon_s} N_a \text{ at } x < 0 \quad (16)$$

and

$$\frac{d^2 \phi}{dx^2} = -\frac{q}{\varepsilon_s} N_d \text{ at } x > 0 \quad (17)$$

Integrating Poisson’s equation twice to find the electrostatic potential using the boundary conditions

$$\phi = \frac{q N_a}{2 \varepsilon_s} (x + w_p)^2 \text{ for } -w_n < x < 0 \quad (18)$$

$$\phi = -\frac{q N_d}{2 \varepsilon_s} (x - w_n)^2 + V_{bi} \text{ for } 0 < x < w_p \quad (19)$$

Individual depletion widths can be obtained as
Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations

\[ w_p = \frac{1}{N_a} \sqrt{\frac{2\varepsilon_s V_{bi}}{q \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}} \]  \hspace{1cm} (20)

and

\[ w_n = \frac{1}{N_d} \sqrt{\frac{2\varepsilon_s V_{bi}}{q \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}} \]  \hspace{1cm} (21)

The total width of the depletion region is

\[ w_{scr} = w_p + w_n = \frac{1}{N_a} \sqrt{\frac{2\varepsilon_s}{q \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} V_{bi}} \]  \hspace{1cm} (22)

A good solar cell design requires wide depletion region for more charged carriers and high doping to aid high cell voltage. The depletion width increases when the \( p \) and/or \( n \) side doping is reduced. However, the built-in voltage decreases when the doping levels are reduced. Therefore, a compromise is required for doping of the \( p \) and \( n \) side of the semiconductor. A good design dopes the top layer heavily and the bottom layer lightly [7].
Chapter 3 - Models Available in the Literature

3.1. Equations for Modeling Semiconductor Devices

The Shockley diode model is used for understanding the carrier transport mechanisms in a solar cell under dark and illuminated conditions. The charge carriers behave differently under the effect of electric field and light intensity which causes deviation from thermal equilibrium behavior. However, there are some basic equations that govern the behavior of charge carriers in a solar cell. A one-dimensional representation is given below [12].

The most important equation is Poisson’s equation that presents the relationship between electric field of a p-n junction $F$ and the space charge density $\rho$

$$\frac{d^2 \phi}{dx^2} = -\frac{dF}{dx} = \frac{q}{\varepsilon_s} (-\rho_{\text{fixed}}(x) + n - p)$$

where, $\phi$ is the electrostatic potential, $\varepsilon_s$ is the static relative permittivity of the medium, $n$ is the electron density and $p$ is the hole density [7]. The density of fixed charges is given as

$$\rho_{\text{fixed}}(x) = (-N_d(x) + N_a(x))$$

The electron current density and hole current density is given by Equation (25) and Equation (26) [7]

$$J_n(x) = qD_n \nabla n + q\mu_n nF$$

and
Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations

\[ J_p(x) = -qD_p \nabla p + q \mu_p p F \]  \hspace{1cm} (26)

where, the first term on the right hand side in both the equations is the diffusion current due to carrier concentration \( \nabla p \) and the second term on the right hand side is the drift current due to electric potential \( F \). \( D_n \) and \( D_p \) are the diffusion coefficients, \( n \) and \( p \) are the electron and hole concentration, \( \mu_n \) and \( \mu_p \) are the electron and hole mobility.

The electron and hole continuity equations are given by Equation (27) and Equation (28) [7]

\[ \frac{\partial n}{\partial x} = \frac{1}{q} \frac{\partial J_n}{\partial x} + G_n - U_n \] \hspace{1cm} (27)

and

\[ \frac{\partial p}{\partial x} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + G_p - U_p \] \hspace{1cm} (28)

where, \( G_n \), \( G_p \) and \( U_n \), \( U_p \) are the photogeneration and recombination rates. Substituting Equations (25) and (26) in Equations (27) and (28) provide us with the transport equations.

\[ D_n \frac{d^2 n}{dx^2} + \mu_n F \frac{dn}{dx} + \mu_n n \frac{dF}{dx} - U_n + G_n = 0 \] \hspace{1cm} (29)

and

\[ D_p \frac{d^2 p}{dx^2} + \mu_p F \frac{dp}{dx} + \mu_p p \frac{dF}{dx} - U_p + G_p = 0 \] \hspace{1cm} (30)

Equations (29) and (30) are coupled with Poisson’s equation (Equation 23) to solve for carrier density in the semiconductor material. The effect of electric field in the transport equation makes it complex to solve it analytically. The numerical methods are used to model the transport equations for carrier densities. These equations can be solved analytically for certain situations, for instance when electric field \( F \) is assumed to be zero or constant. However, the electric field varies in the space charge region as shown in figure below and that is why the numerical modeling is more favorable approach for carrier densities for accommodating the effects of electric field and recombination.

For analytical modeling, the recombination rate is simplified to a monomolecular form and the electric field \( F \) is assumed to be constant. The transport equation simplifies to the form stated in Equation (31) and Equation (32), which can be solved analytically by solving second order differential equation. This approach is used by Nelson [7] and the results are compared in Chapter 5 of this thesis work.
Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations

\[
\frac{d^2n}{dx^2} - \frac{(n - n_0)}{L_n^2} + \frac{G}{D_n} = 0 \tag{31}
\]

and

\[
\frac{d^2p}{dx^2} - \frac{(p - p_0)}{L_p^2} + \frac{G}{D_p} = 0 \tag{32}
\]

where, the net recombination rate can be expressed as,

\[
U \approx \frac{(n - n_0)}{\tau_n} \tag{33}
\]

Figure 9: (a) Space charge density \( \rho(x) \); (b) Electric field \( F(x) \); (c) Potential \( \phi(x) \) across a p-n junction [12].

3.2. Analytical Modeling

This thesis work is based on the analytical modeling of the solar cells using the transport equations and Poisson’s equation. A thorough literature survey was conducted on the analytical approach to find carrier densities in different regions of a p-n junction solar cell. The minority
carrier densities in the n and p side of the semiconductor are surveyed. The equations presented by Nelson [7] provide the general solution for the minority charge carriers. However, a majority of literature on analytical modeling uses the limiting case for the transport equation and different forms of these equations are discussed in this chapter.

3.2.1. Minority Electron Density in p-QNR

The general solution presented by Nelson [7] for calculating minority electron density in p-quasi neutral region is shown below.

\[ n(E, x) = A_n \cosh \left( \frac{-x - w_p}{L_n} \right) + B_n \sinh \left( \frac{-x - w_p}{L_n} \right) + Y_n e^{-\alpha(x + x_p)} \]  \hspace{1cm} (34)

where,

\[ Y_n = \frac{(1 - R)ab_sL_n^2}{D_n(\alpha^2L_n^2 - 1)} \]  \hspace{1cm} (35)

The constant \( A_n \) and \( B_n \) are expressed below,

\[ A_n = n_0 \left( e^{qV/kT} \right) - Y_n e^{-\alpha(x_p - w_p)} \]  \hspace{1cm} (36)

\[ B_n = \frac{Y_n \left[ e^{-\alpha(x_p - w_p)} \left( S_nL_n/D_n \cosh \left( \frac{x_p - w_p}{L_n} \right) + \sinh \left( \frac{x_p - w_p}{L_n} \right) \right) \right] - \left( S_nL_n/D_n + \alpha L_n \right)} {\cosh \left( \frac{x_p - w_p}{L_n} \right) + S_nL_n/D_n \sinh \left( \frac{x_p - w_p}{L_n} \right)} \]  \hspace{1cm} (37)

where, \( b_s \) is the radiation impinging on the solar cell material, \( D_n \) is the electron diffusion rate, \( L_n \) is the electron diffusion length, \( \alpha \) is the absorption coefficient, \( R \) is the reflectivity of the material, \( x_p \) is the length of p-region, \( x_n \) is the length of n-region, \( S_n \) is the surface recombination velocity for electrons, \( V \) is the external bias, \( k \) is the Boltzmann constant, \( T \) is the temperature of the cell material.

In his dissertation, Schumacher had a similar analytical approach to calculate the minority electron density in p-type quasi neutral region [12]. The general solution for electron density presented in his dissertation work under illuminated condition is,

\[ n^{it} = n_0 + G\tau_e + \left[ exp \left( \frac{W_p + x}{L_e} \right) \right] \left[ n_0 \left\{ exp \left( \frac{qV_{ja}}{kT} \right) - 1 \right\} - G\tau_e \right] \]  \hspace{1cm} (38)
Several assumptions were applied by Schumacher to simplify the solution of minority electron density under illuminated conditions. The assumptions were the exhaustion region approximation, low-injection conditions, the superposition principle, no surface recombination, homogeneous photo-generation and no parasitic losses [12]. In this assumption, the effect of electric field is ignored to simplify the results. The analytical model required several assumptions to be taken into consideration to solve the transport equation for calculating electron concentration.

In the book ‘Fundamentals of Solar Cells’, the minority electron density is given by a simplified equation [10]

\[
\begin{align*}
\frac{\alpha L_n}{(\alpha^2 L_n^2 - 1)} & \left[ \begin{array}{c}
\frac{\cosh \left( \frac{y}{L_n} \right)}{L_n} - \exp(-\alpha y) \\
(SL_n \frac{D_n}{\alpha L_n^2}) \cosh \left( \frac{y_0}{L_n} \right) - \exp(-\alpha y_0) \\
+ \sinh \left( \frac{y_0}{L_n} \right) + \alpha L_n \exp(-\alpha y_0)
\end{array} \right] \\
- \left[ \begin{array}{c}
\frac{\cosh \left( \frac{y}{L_n} \right)}{L_n} \\
(SL_n \frac{D_n}{\alpha L_n^2}) \sinh \left( \frac{y_0}{L_n} \right) + \cosh \left( \frac{y_0}{L_n} \right)
\end{array} \right] \sinh \left( \frac{y}{L_n} \right)
\end{align*}
\]

For this case, the illumination is impinging on the solar cell at the end of the depletion region as shown in Figure 10. This equation holds good for zero-bias voltage case. The effect of diffusion velocity is considered by Fahrenbruch and Bube in calculating the electron density in p-quasi neutral region. However, the direction and point of radiation impinging on the solar cell material in this thesis work was different from the work presented by Nelson.

Martin A. Green presented the analytical solution for the transport equations for minority carriers in the quasi-neutral region under dark conditions. The general solution for minority electron density in the quasi-neutral region is [17]

\[
n_p(x') = n_{p0} + n_{p0} \left[ \frac{qV}{e kT} - 1 \right] e^{-\frac{x'}{L_e}}
\]  

A working example to solve for minority hole density is presented for the n quasi-neutral region but no solution is presented for minority electron density. This equation presented a limiting case of general solution obtained by solving transport equations for electrons. The effect of diffusion and recombination is ignored in this analysis. The general equation for minority hole density will be presented in the next section.
A. S. Grove presented the minority electron density his book. His solution considered the effects of illumination in the general solution of transport equation, but the effect of diffusion velocity on electron density is ignored. The general solution is presented in Equation (32) [18]

\[ n_p(x) = \left( n_{p0} + \tau_n G_L \right) \left( 1 - e^{-\frac{x}{\tau_n}} \right) \}

where \( G_L \), the generation term due to illumination and diffusion length is \( L_n = \sqrt{D_n \tau_n} \).

Rivon et al. presented a more systematic approach for analytical solution of transport equations. The effects of surface recombination were considered for the solution of charge carriers as the importance of recombination at the boundary edges cannot be neglected. Conventionally, for analytical modeling the effect of recombination in the space charge region is ignored, however, this research presented the importance of considering surface recombination as the carrier density increased at edges of SCR due to recombination [19]. However, this approximation does not include effects of surface recombination velocity. The general solution of electron density can be presented by Equation (42)

\[ n(x) = C_1 e^{-\frac{c}{\sigma m_0}x} + C_2 e^{-\frac{c}{\sigma^2 m_0}x} - Ae^{-\alpha_2 x} \]

where, \( A = \frac{C_0(\lambda)}{\alpha_2^2 - C_2 \alpha_2 - \frac{1}{L_n^2}} \).

Another analytical model was derived by Sze for the electron distribution in the p-quasi neutral region [20]. The general solution for electron density presented by Sze is
Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations

\[ n_p - n_{p0} \]

\[ = \frac{\alpha F(1 - R)\tau_n}{\alpha^2 L^2 - 1} \exp[-\alpha(x_j + W)] \left\{ \cosh\left(\frac{x - x_j - W}{L_n}\right) - e^{-[\alpha(x-x_j-W)]} \right\} \]

\[ \frac{S_n L_n}{D_n} \left[ \cosh\left(\frac{H'}{L_n}\right) - \exp(-\alpha H') \right] + \sinh\left(\frac{H'}{L_n}\right) + \alpha L_n e^{-\alpha H'} \]

\[ \frac{S_n L_n}{D_n} \sinh\left(\frac{H'}{L_n}\right) + \cosh\left(\frac{H'}{L_n}\right) \] \hspace{1cm} (43)

The assumption for this general solution is the uniformly doped semiconductor and the excess minority carrier density at the edge of the depletion layer is zero. The literature available on mathematical model did not provide a similar equation and assumptions applied by Nelson in her work. A majority of literature used different approximations to simplify the transport equations but none has presented a general solution for electron carrier density for a p-n junction homo-junction semiconductor.

3.2.2. Minority Hole Density in n-QNR

The hole density in n-Quasi Neutral Region can be solved analytically from the transport equations for holes and Poisson’s equation. The general solution of hole density in the n-region was presented by Nelson in her book in general form as shown below in Equation (44) [7]

\[ p(E, x) = A_p \cosh\left(\frac{x + w_n}{L_p}\right) + B_p \sinh\left(\frac{x + w_n}{L_p}\right) - \gamma_p e^{-\alpha(x+x_p)} \] \hspace{1cm} (44)

\[ \gamma_p = \frac{(1 - R)\alpha b_s L_p^2}{D_p \{\alpha^2 L_p^2 - 1\}} \] \hspace{1cm} (45)

where, the constant \( A_p \) and \( B_p \) can be expressed as

\[ A_p = p_0 \left( e^{qV/kT} - 1 \right) - \gamma_p e^{-\alpha(x_p+w_n)} \] \hspace{1cm} (46)

\[ \gamma_p e^{-\alpha(x_p+w_n)} \left[ \left( \frac{S_p L_p}{D_p} \cosh\left(\frac{x_n - w_n}{L_p}\right) + \sinh\left(\frac{x_n - w_n}{L_p}\right) - \left( \frac{S_p L_p}{D_p} - \alpha L_p \right) e^{-\alpha(x_n-w_n)} \right) \right] \]

\[ B_p = -p_0 \left( e^{qV/kT} - 1 \right) \left[ \frac{S_p L_p}{D_p} \cosh\left(\frac{x_n - w_n}{L_p}\right) + \sinh\left(\frac{x_n - w_n}{L_p}\right) \right] \]

\[ \left[ \cosh\left(\frac{x_n - w_n}{L_p}\right) + S_p \frac{L_p}{D_p} \sinh\left(\frac{x_n - w_n}{L_p}\right) \right] \] \hspace{1cm} (47)
where, $b_s$ is the radiation impinging on the solar cell material, $D_p$ is the hole diffusion rate, $L_p$ is the hole diffusion length, $\alpha$ is the absorption coefficient, $R$ is the reflectivity of the material, $x_p$ is the length of p-region, $x_n$ is the length of n-region, $S_p$ is the surface recombination velocity for holes, $V$ is the external bias, $k$ is the Boltzmann constant, $T$ is the temperature of the cell material.

In his dissertation, Schumacher used a similar approach to find the minority hole density in n-type quasi neutral region [12]. The general solution for hole density in n-QNR was presented in his dissertation work. It is assumed to be derived using the same process for electron density in p-QNR.

Fahrenbruch and Bube mentioned the analytical solution for hole minority carrier density in their book [10]. The solution was derived from the transport equation. The QNR is bounded by $x_n$ and $x_n'$, with $y$ measured from the front surface. This solution did not include the effects of external bias on the hole density. The minority hole density in n-QNR is expressed as

$$p_n(y) = \left[ \frac{\alpha F \tau_p}{(\alpha^2 L_p^2 - 1)} \right] \left\{ \frac{(S_p L_p D_p + \alpha L_p) \sinh \left( \frac{y_n - y}{L_p} \right)}{(S_p L_p D_p \sinh \left( \frac{y_n}{L_p} \right) + \cosh \left( \frac{y_n}{L_p} \right) - \exp(-\alpha y)} \right\} - p_{n0} \quad (48)$$

Sze presented a mathematical model of minority hole density in his book and for this derivation his assumption was to consider the excess hole density to be zero at the depletion edge. The results obtained by using the boundary is [20]

$$p_n - p_{n0} = \frac{\alpha F (1 - R) \tau_p}{\alpha^2 L_p^2 - 1} \left\{ \frac{(S_p L_p D_p + \alpha L_p) \sinh \left( \frac{x_j - x}{L_p} \right) + e^{-\alpha x} \left( S_p L_p D_p \sinh \left( \frac{x}{L_p} \right) + \cosh \left( \frac{x}{L_p} \right) \right)}{S_p L_p D_p \sinh \left( \frac{x_j}{L_p} \right) + \cosh \left( \frac{x_j}{L_p} \right)} \right\} - e^{-\alpha x} \quad (49)$$

The analytical solution presented by Sze includes the effect of radiation term ‘$F$’ in his solution; however the assumption of excess minority carriers at the depletion edge to be zero is only possible at short circuit voltage. When there is external applied voltage, there will be some excess minority charge carriers that are not included in the analysis performed by Sze. The effect
of voltage at the depletion edge is included by Nelson in her book [7]; therefore, the results of Sze could not be compared with the results of Nelson.

Martin A. Green presented a simplified analysis of transport equations. The hole density in the n-quasi neutral region for the dark conditions (no illumination) is given by

\[ p_n(x) = p_{n0} + p_{n0} \left[ \frac{qV}{e kT} - 1 \right] e^{\frac{-x}{L_h}} \quad (50) \]

He further provides an example for minority hole density in the quasi-neutral region under illuminated conditions and makes this analysis with the similar boundary conditions in dark [17].

\[ p_n(x) = p_{n0} + G \tau_h + \left[ p_{n0} \left( \frac{qV}{e kT} - 1 \right) - G \tau_h \right] e^{\frac{-x}{L_h}} \quad (51) \]

The equations for minority hole density in the n-quasi neutral region are calculated by solving the transport equations under different assumptions to simplify the results. However, the equations reviewed in this literature survey did not present a general form of the equation to calculate the minority hole densities in a p-n junction homo-junction semi-conductor material.

### 3.2.3. Carrier Densities in Space Charge Region

Nelson presented the carrier densities in the space charge region for the depletion approximation that requires \( E_{Fn} \) and \( E_{Fp} \) to be constant across the region [7]. For this approximation, the electron and hole density is given by

\[ n = n_e \left( \frac{E_{Fn} - E_i}{k_B T} \right) \quad (52) \]

\[ p = p_e \left( \frac{E_i - E_{Fp}}{k_B T} \right) \quad (53) \]

However, when the solar cell is illuminated, the Fermi levels of n and p side is not constant across the region and therefore, this assumption does not hold. The effect of electric field changes the Fermi levels (\( E_{Fn} \) and \( E_{Fp} \)) across the space charge region which makes it difficult to model the carrier density in the space charge region analytically. The complexity of solving the transport equations in the space charge region has made it difficult to solve it analytically [21]. There are some computer programs available that use the numerical approach for finding carrier densities, the most popular of them is PC1D [22].

A lengthy literature review of different books and articles was conducted to look for analytical equations to find the carrier densities in the space charge region under illuminated
conditions. However, at this point it is considered too difficult to solve the transport equations analytically for space charge, as all the references direct the reader to use numerical routines for predicting carrier density in the space charge region. Nelson presented the general equation for carrier density in space charge region under dark conditions, however, there is no solution presented for carrier density under illuminated conditions and the graph used in the book used PC1D software to plot carrier densities across the p-n junction.

3.2.4. Current Density in Quasi Neutral Regions and Space Charge Region

A few references were found where the researchers focused on modeling the current density in a solar cell analytically. The most systematic and general equation for current density is presented by Nelson [7]. The electron current density at the edge of the depletion region in the n-QNR was presented by

\[
f(E, -w_p) = -q(1-R)ab_L \frac{L_n}{a^2 L_n^2 - 1} \left\{ e^{-\alpha(x_p-w_p)} \left[ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right] - \left( \frac{S_n L_n}{D_n} + \alpha L_n \right) \right\} \\
+ \alpha L_n e^{-\alpha(x_p-w_p)} \left\{ \frac{qD_n n_0}{L_n} \left( e^{\frac{qV}{kBT}} - 1 \right) \left[ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right] \right\} \\
+ \frac{qD_n n_0}{L_n} \left( e^{\frac{qV}{kBT}} - 1 \right) \left\{ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right\}
\]

(54)

where, \(-w_p\) is the edge of depletion region.

The hole current density at the other edge of the depletion region in the p-QNR was presented by

\[
l_p = -q(1-R)ab_H L_p \frac{L_p}{a^2 L_p^2 - 1} \left\{ e^{-\alpha(x_n+w_p)} \left[ \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_p L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) \right] - \left( \frac{S_p L_p}{D_p} + \alpha L_p \right) e^{-\alpha(x_n+w_n)} \right\} \\
- \alpha L_p \left\{ \frac{qD_p p_0}{L_p} \left( e^{\frac{qV}{kBT}} - 1 \right) \left[ \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_p L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) \right] \right\}
\]

(55)

where, \(w_n\) is the edge of depletion region.

The current density for all the energies of incoming radiation can be calculated by integrating the current density over all the energies,
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\[ J_n = \int j_n(E, -w_p) \, dE \]  \hspace{1cm} (56)
and
\[ J_p = \int j_p(E, w_n) \, dE \]  \hspace{1cm} (57)

The third current is in the space charge region which is known as the recombination-generation current. Nelson included effects of generation and recombination in the space charge region as

\[ J_{scr} = \int j_{scr}(E) \, dE \]  \hspace{1cm} (58)

In her book, Nelson did not present the current density in the space charge region for a single energy. The current density in space charge region is a combination of photo-generation current and recombination current.

\[ J_{scr}(V) = \frac{q\eta_n(w_n + w_p)}{\sqrt{\tau_n\tau_p}} \left( \sinh \left( \frac{qV}{2kT} \right) \pi \frac{2}{q(V_{bi} - V)} \right) \]

\[ - q \int (1 - R)b_s e^{-\alpha(x_p-w_p)}(1 - e^{-\alpha(w_p+w_n)}) \, dE \]  \hspace{1cm} (59)

The net current density is the sum of \( J_n \) and \( J_p \) at any point and is constant throughout the p-n junction in steady state [7].

Fahrenbruch and Bube presented the current density equations differently than Nelson. The current density at point \( y \) given by Fahrenbruch and Bube is

\[ J_n(y) = \frac{q\Gamma}{(1 - \alpha^2L_n^2)} \left( \frac{S}{D_n} \right) \left[ \sinh \left( \frac{y}{L_n} \right) + \exp(-\alpha y) \right] \]

\[ - \frac{1}{\alpha L_n} \left( \frac{S}{D_n} \right) \sinh \left( \frac{y_0}{L_n} \right) + \cosh \left( \frac{y_0}{L_n} \right) \cos \left( \alpha L_n \right) \cosh \left( \frac{y_0}{L_n} \right) \]  \hspace{1cm} (60)

and

\[ J_p(y) = \frac{q\Gamma}{(1 - \alpha^2L_n^2)} \left( \frac{S}{D_p} + 1 \right) \left[ \cosh \left( \frac{y}{L_p} \right) - \exp(-\alpha y) \right] \]

\[ - \exp(-\alpha y_0) \left( \frac{S}{D_p} \cosh \left( \frac{y}{L_p} \right) + \frac{1}{\alpha L_p} \sinh \left( \frac{y}{L_p} \right) \right) - \exp(-\alpha y) \]  \hspace{1cm} (61)
The electron and hole current density calculated by Fahrenbruch and Bube [10] used a different approach than Nelson. The sun radiation for Fahrenbruch and Bube is impinging on the edge of the depletion region for p-side quasi neutral region.

Sze presented the electron and hole current density with a different approach than previously mentioned in this literature review section. He derived current density analytically with the assumption of uniform lifetime, mobility and doping [20]

\[
J_n = \frac{q\alpha F(1 - R)L_n}{\alpha^2 L_n^2 - 1} \exp\left[-\alpha(x_j + W)\right] \left\{ \alpha L_n \right. \\
\left. - \frac{S_n L_n}{D_n} \left[ \cosh \left( \frac{H'_n}{L_n} \right) - \exp(-\alpha H'_n) \right] + \sinh \left( \frac{H'_n}{L_n} \right) + \alpha L_n e^{-\alpha H'_n} \right\}
\]

(62)

and,

\[
J_p = \frac{q\alpha F(1 - R)L_p}{\alpha^2 L_p^2 - 1} \left\{ \left( \frac{S_p L_p}{D_p} + \alpha L_p \right) - e^{-\alpha x_j} \left( \frac{S_p L_p}{D_p} \cosh \left( \frac{x_j}{L_p} \right) + \sinh \left( \frac{x_j}{L_p} \right) \right) \right. \\
\left. - e^{-\alpha x_j} \right\}
\]

(63)

Martin A. Green assumed the constant generation of electron-hole pair throughout the solar cell. The hole current density is

\[
J_h = qD_{h0}n_0 \left( \frac{qV}{kT} - 1 \right) e^{\frac{-x}{L_h}} - qG L_h e^{\frac{-x}{L_h}}
\]

(64)

and a similar expression is found for \( J_e \).

In his dissertation, Schumacher mentioned that the electron flow density in the p-type quasi neutral region can be expressed as

\[
J_e(x) = qn_0 \frac{D_e}{L_e} \left[ \exp \left( \frac{W_p + x}{L_e} \right) \right] \left\{ \exp \left( \frac{qV_j a}{kT} \right) - 1 \right\} - qG L_e \exp \left( \frac{W_p + x}{L_e} \right)
\]

(65)

The equation was derived by integrating Equation (38). A diffusive hole current density in p-quasi neutral region is given by

\[
J_h(x) = qp_0 \frac{D_h}{L_h} \left[ \exp \left( \frac{W_n - x}{L_h} \right) \right] \left\{ \exp \left( \frac{qV_j a}{kT} \right) - 1 \right\} - qG L_h \exp \left( \frac{W_n - x}{L_h} \right)
\]

(66)

In this literature review, an attempt was made to compare the carrier and current densities with the equations mentioned by Nelson [7]. A lengthy review did not provide any conclusive evidence that the solutions mentioned by Nelson [7] are accurate. The research papers and books presented a systematic approach for solving the transport equations to solve for carrier density.
with different boundary conditions and assumptions. However, it was evident that other papers did not present the same results as Nelson. Therefore, to check the accuracy of the carrier and current density equations presented by Nelson [7], the general solution was derived from the transport equations by using the boundary conditions applied by Nelson. The process of derivation and results are presented in Chapter 4. A comparison is made between the equations derived in this thesis with those mentioned by Nelson and the evident differences are presented in Chapter 4. These differences have significant impact on results presented by Nelson where PC1D software was used to plot the carrier concentration throughout the semi-conductor material. A thorough comparison was made between the results obtained in this thesis work with the result of Nelson and the comparison is presented in Chapter 5.

3.3. Numerical Modeling

The direct analytical solution of the Boltzmann Transport Equation is difficult to achieve because of the effects of electric field in the space charge region. The drift-diffusion model can be used to solve the transport equations to correctly model the carrier density [2]. Aberle et al. presented a numerical optimization technology for high-efficiency silicon cells that shows low minority carrier recombination losses [23]. A sophisticated numerical method model is required to describe a real solar cell and to compare the characteristics of a solar cell [12].

There are a lot of numerical simulation techniques available in the literature. Numerical methods are important to understand solar cell operation. Mark S. Lundstrom developed one of the earlier numerical models [24]. There are analysis programs in both 1-dimension and 2-dimension: SCAP1D and SCAP2D [25]. The 2D modeling of high efficiency solar cells of Si and GaAs are modeled using this software. One of the most popular numerical programs used is PC1D. This software uses an improved numerical algorithm for predicting the carrier and current densities and is user friendly [22]. There are other numerical computer codes used worldwide like Silvaco and Crosslight. Robin et al. used the numerical program SCAPS-1D to simulate the performance of CIGS solar cells in the dark and under illuminated conditions [26]. SCAPS-1D is a 1-D simulator developed at the University of Gent [27]. Numerical methods are helpful in optimizing solar cells and save on fabrication. Gaussian defects are also included in the numerical programs to provide more real world solar cell performance characteristics.
Wu et al. studied the impacts of carrier densities in a hetero-junction solar cell. The analysis is performed using the numerical simulation model prepared by Pennsylvania State University and it is known as AMPS-1D. The high electric field in the space charge region makes it difficult to evaluate the carrier concentration in the SCR and therefore, the use of numerical modeling software is essential for simulating the performance characteristics of a solar cell and AMPS-1D has these capabilities [28].

Electron and hole density vary significantly across the depletion region when the average electron density across the quasi neutral regions and depletion region is low and therefore, it is required to use alternative methods to calculate the carrier densities in a solar cell and the most preferred way is the numerical technique [29]. McIntosh explains the effect of illumination on the excess carrier concentration in a p-n junction solar cell. When the solar cell is illuminated, the depletion region width decreases significantly which causes the excess minority charge carriers to be highest in the regions that is depleted under equilibrium conditions. This causes an uneven distribution of excess minority charge carriers and there is a sharp rise in minority charge carriers across the edge of the depletion region. The average excess charge carriers is not constant across the depletion region and that requires the use of sophisticated tools for calculation of carrier densities, such as using PC1D [29].

Version 3 of PC1D has improved the speed and convergence of one-dimensional solar cell simulations. This program uses the Newton iteration method that utilizes the linear approximation to the non-linear non-homogeneous transport equations. Iterations are necessary to calculate the carrier densities across the p-n junction [22].

Budhraja et al. performed a numerical analysis in their research paper to calculate the carrier density in a p-n junction semiconductor. A model was prepared using the lower-upper decomposition matrix method. The equations were solved numerically at every mesh point to properly estimate the carrier density, current density, electric field and voltage. The results were compared with the AMPS program under the same operating conditions [30].

Rapolu et al. developed a 2D numerical model for Si solar cells. This analysis was implemented in the program called COMSOL. COMSOL use the finite difference method. The model was developed by solving Poisson’s equation, the continuity equation, and the transport equations. The numerical method was required to calculate the carrier concentration in two dimensions [31].
Present day 3D numerical simulation software makes it possible to solve all the required partial differential equations for solar cell simulation simultaneously. A large number of results from numerical analysis of solar cells are presented by Schumacher [12]. Figure 11 illustrates the simulation results for carrier and current density under dark and illuminated conditions.

Figure 11: Carrier and current density under dark and illuminated conditions [12].
Chapter 4 - Derivation of Carrier and Current Densities

The literature survey of analytical modeling research papers and books did not present the equation similar to that presented by Nelson in her book ‘The Physics of Solar Cells’. A few researchers used a similar approach for solving the transport equations to find the carrier and current densities in quasi neutral regions and space charge regions. However, the difference was with the boundary conditions, the location where the solar energy was incident on the solar cell, and the location of the zero point on the spatial coordinate system. In this chapter, the derivation of these equations are presented and compared with the equations presented by Nelson. This derivation was required to determine whether Nelson’s equations [7] are correct or not. If Nelson’s equations turned out to be incorrect, the derivations are furthered required to determine where the inaccuracies lie. As the detailed derivation work here shows, Nelson’s equations do have errors. The differences between the equations derived in this chapter and those of Nelson are highlighted.

Also shown in this section are the equations for the electric potential distribution in the depletion region and the thickness of the depletion region under dark conditions. These equations are correct in Nelson [7] and a number of other publications. They are shown here to provide a complete listing of the equations a p-n junction.
4.1. Carrier Densities in Quasi Neutral Regions

Carrier density can be calculated using the transport equations mentioned in Chapter 3. In the quasi-neutral regions the electric field is zero and therefore, Equation (31) and Equation (32) can be reduced to

\[
\frac{d^2n}{dx^2} - \frac{(n - n_0)}{L_n^2} + \frac{G}{D_n} = 0
\]  

(67)

and

\[
\frac{d^2p}{dx^2} - \frac{(p - p_0)}{L_p^2} + \frac{G}{D_p} = 0
\]  

(68)

when the recombination is assumed to be linear. This approximation can be used to calculate the carrier concentration in the p and n neutral regions. The majority charge carriers can be assumed to be equal to the doping level. The junction is illuminated with photons having a flux density of \(b_s\) and an external applied bias \(V\).

4.1.1. Derivation of Carrier Density Equation in p-QNR

A detailed solution of second order differential equation is presented in this section using the undetermined coefficient method. The boundary conditions used for this analysis are

\[
n - n_0 = n_0 \left(e^{\frac{qV}{k_BT}} - 1\right) \text{ at } x = -w_p
\]

(69)

and

\[
D_n \frac{dn}{dx} = S_n(n - n_0) \text{ at } x = -x_p
\]

(70)

The generation term \(G\) can be expressed as

\[
g(E,x) = (1 - R(E))\alpha(E)b_s(E)e^{-\alpha(E)(x+x_p)}
\]

(71)

The transport equation for p-QNR can be written as

\[
\frac{d^2(n - n_0)}{dx^2} - \frac{(n - n_0)}{L_n^2} + \frac{g(E,x)}{D_n} = 0 \text{ for } x < -w_p
\]

(72)

\(n - n_0\) is the excess electron density in p-QNR. We can substitute excess electron density with \(n'\)
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\[
\frac{d^2 n'}{dx^2} - \frac{n'}{L_n^2} + \frac{(1 - R)ab_s e^{-\alpha(x+x_p)}}{D_n} = 0 \quad \text{for } x < -w_p \quad (73)
\]

Rearranging the equation

\[
\frac{d^2 n'}{dx^2} - \frac{n'}{L_n^2} = -\frac{(1 - R)ab_s e^{-\alpha(x+x_p)}}{D_n} \quad (74)
\]

This is a second order, non-homogeneous differential equation. The general solution of this equation can be written as

\[n' = n_c + Y \quad (75)\]

where the term \(n_c\) is called the complementary solution and the term \(Y\) is called the particular solution.

The complementary solution is the solution of

\[
\frac{d^2 n'}{dx^2} - \frac{n'}{L_n^2} = 0 \quad (76)
\]

This is a linear, second order, homogeneous differential equation with constant coefficients.

The solution of this equation is

\[n' = Ae^{m_1 x} + Be^{m_2 x} \quad (77)\]

The value of \(m_1\) and \(m_2\) can be found by solving the characteristic equation corresponding to Equation (76)

\[m^2 - \frac{1}{L_n^2} = 0 \quad (78)\]

Rearranging equation to find ‘m’

\[m = \pm \frac{1}{L_n} \quad (79)\]

Substituting this value of \(m\) in Equation (77) gives

\[n' = Ae^{x/L_n} + Be^{-x/L_n} \quad (80)\]

So, the complementary solution of the equation is

\[n_c = Ae^{x/L_n} + Be^{-x/L_n} \quad (81)\]

The particular solution for \(Y\) can be found by assuming a solution with constant \(C\), so the \(Y\) becomes

\[Y = Ce^{-\alpha(x+x_p)} \quad (82)\]

The first and second derivatives of this equation are
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\[
\frac{dY}{dx} = -\alpha Ce^{-\alpha(x+x_p)} \tag{83}
\]

\[
\frac{d^2Y}{dx^2} = \alpha^2 Ce^{-\alpha(x+x_p)} \tag{84}
\]

The particular solution for \( Y \) can be found by assuming a constant \( C \).

Substituting the values from Equations (82), (83) and (84) into Equation (74) gives

\[
\alpha^2 Ce^{-\alpha(x+x_p)} - \frac{Ce^{-\alpha(x+x_p)}}{L_n^2} = -\frac{(1 - R)ab_s e^{-\alpha(x+x_p)}}{D_n} \tag{85}
\]

Rearranging

\[
\left\{ \alpha^2 - \frac{1}{L_n^2} \right\} e^{-\alpha(x+x_p)} C = -\frac{(1 - R)ab_s e^{-\alpha(x+x_p)}}{D_n} \tag{86}
\]

and

\[
C = -\frac{(1 - R)ab_s}{D_n \left\{ \alpha^2 - \frac{1}{L_n^2} \right\}} \tag{87}
\]

Substituting the value of \( C \) from Equation (87) into Equation (82) gives

\[
Y = -\frac{(1 - R)ab_s}{D_n \left\{ \alpha^2 - \frac{1}{L_n^2} \right\}} e^{-\alpha(x+x_p)} \tag{88}
\]

Substituting the value of complementary solution, \( n_c \), and particular integral \( Y \) into Equation (75) gives

\[
n' = Ae^{x/L_n} + Be^{-x/L_n} - \frac{(1 - R)ab_s}{D_n \left\{ \alpha^2 - \frac{1}{L_n^2} \right\}} e^{-\alpha(x+x_p)} \tag{89}
\]

which can be rearranged in the following ways

\[
n' = Ae^{x/L_n} + Be^{-x/L_n} - \frac{(1 - R)ab_s l_n^2}{D_n \left\{ \alpha^2 L_n^2 - 1 \right\}} e^{-\alpha(x+x_p)} \tag{90}
\]

\[
n' = Ae^{x/L_n} + Be^{-x/L_n} + \frac{(1 - R)ab_s l_n^2}{D_n \left\{ 1 - \alpha^2 L_n^2 \right\}} e^{-\alpha(x+x_p)} \tag{91}
\]

and

\[
n' = Ae^{x/L_n} + Be^{-x/L_n} + Y_n e^{-\alpha(x+x_p)} \tag{92}
\]

where \( Y_n \) can be expressed as

\[
Y_n = \frac{(1 - R)ab_s l_n^2}{D_n \left\{ 1 - \alpha^2 L_n^2 \right\}} \tag{93}
\]

Equation (91) can also be expressed as
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\[ n - n_0 = Ae^{\frac{x}{L_n}} + Be^{-\frac{x}{L_n}} + Ye^{-\alpha(x + x_p)} \]  \hspace{1cm} (94)

The constants A and B in Equation (94) can be calculated by solving for the boundary condition on the edges of the depletion region from the Equations (95) and (96)

\[ n - n_0 = n_0 \left( e^{\frac{qV}{k_B T}} - 1 \right) \text{ at } x = -w_p \]  \hspace{1cm} (95)

and

\[ D_n \frac{dn}{dx} = S_n (n - n_0) \text{ at } x = -x_p \]  \hspace{1cm} (96)

Using the boundary condition from Equation (95), Equation (94) can be written as

\[ Ae^{-\frac{w_p}{L_n}} + Be^{\frac{w_p}{L_n}} = n_0 \left( e^{\frac{qV}{k_B T}} - 1 \right) - Y_n e^{-\alpha(-w_p + x_p)} \]  \hspace{1cm} (97)

Using the boundary condition from Equation (96), Equation (94) can be written as

\[ \left( 1 - S_n \frac{L_n}{D_n} \right) Ae^{-\frac{x_p}{L_n}} - \left( 1 + S_n \frac{L_n}{D_n} \right) Be^{\frac{x_p}{L_n}} = S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \]  \hspace{1cm} (98)

Solving Equations (97) and (98) yields the value of constants A and B,

\[ A = \frac{\left( 1 + S_n \frac{L_n}{D_n} \right) n_0 \left( e^{\frac{qV}{k_B T}} - 1 \right) e^{\frac{x_p}{L_n}} - Y_n e^{-\alpha(x_p - w_p)} \left( 1 + S_n \frac{L_n}{D_n} \right) e^{\frac{x_p}{L_n}} + S_n \frac{L_n}{D_n} Y_n e^{\frac{x_p}{L_n}} + L_n \alpha Y_n e^{\frac{x_p}{L_n}}}{2 \left( \cosh \left( \frac{x_p - w_p}{L_n} \right) + S_n \frac{L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right) \right)} \]  \hspace{1cm} (99)

\[ B = \begin{pmatrix}
\left[ n_0 \left( e^{\frac{qV}{k_B T}} - 1 \right) - Y_n e^{-\alpha(x_p - w_p)} \right] e^{\frac{x_p}{L_n}} - Y_n e^{-\alpha(x_p - w_p)} \left( 1 + S_n \frac{L_n}{D_n} \right) e^{\frac{x_p}{L_n}} + S_n \frac{L_n}{D_n} Y_n e^{\frac{x_p}{L_n}} + L_n \alpha Y_n e^{\frac{x_p}{L_n}} \\
\left( 1 - S_n \frac{L_n}{D_n} \right) n_0 \left( e^{\frac{qV}{k_B T}} - 1 \right) e^{\frac{x_p}{L_n}} - Y_n e^{-\alpha(x_p - w_p)} \left( 1 + S_n \frac{L_n}{D_n} \right) e^{\frac{x_p}{L_n}} + S_n \frac{L_n}{D_n} Y_n e^{\frac{x_p}{L_n}} + L_n \alpha Y_n e^{\frac{x_p}{L_n}} \\
\frac{\left( 1 - S_n \frac{L_n}{D_n} \right) n_0 \left( e^{\frac{qV}{k_B T}} - 1 \right) e^{\frac{x_p}{L_n}} - Y_n e^{-\alpha(x_p - w_p)} \left( 1 + S_n \frac{L_n}{D_n} \right) e^{\frac{x_p}{L_n}} + S_n \frac{L_n}{D_n} Y_n e^{\frac{x_p}{L_n}} + L_n \alpha Y_n e^{\frac{x_p}{L_n}}}{2 \left( \cosh \left( \frac{x_p - w_p}{L_n} \right) + S_n \frac{L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right) \right)}
\end{pmatrix} \]  \hspace{1cm} (100)

Substituting the value of A and B in equation 94 yields
Further rearrangement of exponential terms yield

\[
\begin{align*}
\left(1 + S_n \frac{L_n}{D_n}\right) & n_0 \left(e^{qV/k_BT} - 1\right) e^{x_n} - e^{-\alpha(x_p-w_p)} \left(1 + S_n \frac{L_n}{D_n}\right) e^{x_n} \\
& + S_n \frac{L_n}{D_n} Y_n e^{x_n} + L_n \alpha Y_n e^{x_n} \\
& \frac{x_n}{2} \left[ \cosh \frac{x_p - w_p}{L_n} + S_n \frac{L_n}{D_n} \sinh \left(\frac{x_p - w_p}{L_n}\right) \right] \\
& + n_0 \left( e^{qV/k_BT} - 1 \right) e^{(x_p-w_p)/L_n} - Y_n e^{-\alpha(x_p-w_p)} \left(1 + S_n \frac{L_n}{D_n}\right) e^{x_n} \\
& + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
& \frac{x_n}{2} \left[ \cosh \frac{x_p - w_p}{L_n} + S_n \frac{L_n}{D_n} \sinh \left(\frac{x_p - w_p}{L_n}\right) \right] \\
& + Y_n e^{-\alpha(x+x_p)} \\
\end{align*}
\]

Equation (101) can be simplified to find the excess electron density in the p-QNR. This equation is altered to obtain a final version in terms of hyperbolic functions. This approach is taken to obtain the results that can be compared with the results presented in Nelson’s book [7]. A systematic approach in taken to obtain the final results and the approach is presented in this section.

Equation (101) is rearranged to convert the exponential terms into hyperbolic functions

\[
\begin{align*}
\left(1 + S_n \frac{L_n}{D_n}\right) & n_0 \left(e^{qV/k_BT} - 1\right) e^{x_n} - e^{-\alpha(x_p-w_p)} \left(1 + S_n \frac{L_n}{D_n}\right) e^{x_n} \\
& + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
& \frac{x_n}{2} \left[ \cosh \frac{x_p - w_p}{L_n} + S_n \frac{L_n}{D_n} \sinh \left(\frac{x_p - w_p}{L_n}\right) \right] \\
& + n_0 \left( e^{qV/k_BT} - 1 \right) e^{(x_p-w_p)/L_n} - Y_n e^{-\alpha(x_p-w_p)} \left(1 + S_n \frac{L_n}{D_n}\right) e^{x_n} \\
& + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
& \frac{x_n}{2} \left[ \cosh \frac{x_p - w_p}{L_n} + S_n \frac{L_n}{D_n} \sinh \left(\frac{x_p - w_p}{L_n}\right) \right] \\
& + Y_n e^{-\alpha(x+x_p)} \\
\end{align*}
\]

\[
\begin{align*}
n - n_0 &= \left(1 + S_n \frac{L_n}{D_n}\right) n_0 \left(e^{qV/k_BT} - 1\right) e^{x_n} - e^{-\alpha(x_p-w_p)} \left(1 + S_n \frac{L_n}{D_n}\right) e^{x_n} \\
& + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
& \frac{x_n}{2} \left[ \cosh \frac{x_p - w_p}{L_n} + S_n \frac{L_n}{D_n} \sinh \left(\frac{x_p - w_p}{L_n}\right) \right] \\
& + n_0 \left( e^{qV/k_BT} - 1 \right) e^{(x_p-w_p)/L_n} - Y_n e^{-\alpha(x_p-w_p)} \left(1 + S_n \frac{L_n}{D_n}\right) e^{x_n} \\
& + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
& \frac{x_n}{2} \left[ \cosh \frac{x_p - w_p}{L_n} + S_n \frac{L_n}{D_n} \sinh \left(\frac{x_p - w_p}{L_n}\right) \right] \\
& + Y_n e^{-\alpha(x+x_p)} \\
\end{align*}
\]
Separating the cosh and sinh terms, and the simplified equation can be presented as

\[
\begin{align*}
\left(1 + S_n \frac{L_n}{D_n}\right) n_0 \left( e^{\frac{qV}{k_BT} - 1} e^{\frac{x_p - w_p}{L_n}} - Y_n e^{-\alpha (x_p - w_p)} \right) (1 + S_n \frac{L_n}{D_n}) e^{-\frac{x_p - w_p}{L_n}} \\
+ S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
+ \left[ n_0 \left( e^{\frac{qV}{k_BT} - 1} e^{\frac{x_p - w_p}{L_n}} - Y_n e^{-\alpha (x_p - w_p)} \right) (1 + S_n \frac{L_n}{D_n}) e^{-\frac{x_p - w_p}{L_n}} \\
+ S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
- 2 \left[ \cosh \left( \frac{x_p - w_p}{L_n} \right) + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \right] \right] e^{\frac{-x_p - w_p}{L_n}}
\end{align*}
\]

\[n - n_0 = \left[ \frac{1 + S_n \frac{L_n}{D_n}}{2} n_0 \left( \frac{e^{\frac{qV}{k_BT} - 1} e^{\frac{x_p - w_p}{L_n}} - Y_n e^{-\alpha (x_p - w_p)} \right) (1 + S_n \frac{L_n}{D_n}) e^{-\frac{x_p - w_p}{L_n}} \\
+ S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \\
- 2 \left[ \cosh \left( \frac{x_p - w_p}{L_n} \right) + S_n \frac{L_n}{D_n} Y_n + L_n \alpha Y_n \right] \right] e^{\frac{-x_p - w_p}{L_n}}
\]

The exponential terms are replaced with hyperbolic functions

\[
\begin{align*}
\left(1 + S_n \frac{L_n}{D_n}\right) n_0 \left( e^{\frac{qV}{k_BT} - 1} e^{\frac{x_p - w_p}{L_n}} - Y_n e^{-\alpha (x_p - w_p)} \right) (1 + S_n \frac{L_n}{D_n}) e^{-\frac{x_p - w_p}{L_n}} \\
- Y_n e^{-\alpha (x_p - w_p)} \left[ \cosh \left( \frac{x_p - w_p}{L_n} \right) \right]
\end{align*}
\]

\[n - n_0 = \left[ \frac{1 + S_n \frac{L_n}{D_n}}{2} n_0 \left( \frac{e^{\frac{qV}{k_BT} - 1} e^{\frac{x_p - w_p}{L_n}} - Y_n e^{-\alpha (x_p - w_p)} \right) (1 + S_n \frac{L_n}{D_n}) e^{-\frac{x_p - w_p}{L_n}} \\
- Y_n e^{-\alpha (x_p - w_p)} \left[ \cosh \left( \frac{x_p - w_p}{L_n} \right) \right] \right]
\]

Separating the cosh and sinh terms, and the simplified equation can be presented as
Rearranging the RHS of the equation yields

\[
\begin{align*}
n - n_0 &= \left[ n_0 \left( \frac{qV}{k_BT} - 1 \right) - \bar{\omega}_n e^{-\alpha(x_p-w_p)} \right] \cosh \left( \frac{x_p - w_p}{L_n} \right) + \bar{\omega}_n e^{-\alpha(x+w_p)} \\
&\quad + \bar{\omega}_n e^{-\alpha(x+w_p)} \left( 1 + S_n \frac{L_n}{D_n} \right) e^{-\frac{x_p - w_p}{L_n}} e^{\frac{x_p - w_p}{L_n}} + S_n \frac{L_n}{D_n} \bar{\omega}_n + L_n \alpha \bar{\omega}_n \\
&\quad + \left[ \frac{n_0 \left( e^{\frac{qV}{k_BT}} - 1 \right) - \bar{\omega}_n e^{-\alpha(x_p-w_p)} }{ \cosh \left( \frac{x_p - w_p}{L_n} \right) + S_n \frac{L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right) } \right] \sinh \left( -\frac{x - w_p}{L_n} \right)
\end{align*}
\]

Equation (105)

Some of the simple steps for rearranging the equations were not presented and the Equation (106) is rearranged to the following equation

\[
\begin{align*}
n - n_0 &= \left[ \left( n_0 e^{\frac{qV}{k_BT} - 1} - \bar{\omega}_n e^{-\alpha(x_p-w_p)} \right) \cosh \left( \frac{x_p - w_p}{L_n} \right) + Y_n e^{-\alpha(x_p-w_p)} \left( S_n \frac{L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) + \sinh \left( \frac{x_p - w_p}{L_n} \right) - \left( S_n \frac{L_n}{D_n} + \alpha L_n \right) \right) \right] \\
&\quad - \left[ n_0 \left( e^{\frac{qV}{k_BT} - 1} - \bar{\omega}_n e^{-\alpha(x_p-w_p)} \right) \cosh \left( \frac{x_p - w_p}{L_n} \right) + \sinh \left( \frac{x_p - w_p}{L_n} \right) \right] + \frac{L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right) \sinh \left( -\frac{x - w_p}{L_n} \right)
\end{align*}
\]

Equation (107)
The above equation can be written in simpler form, for excess minority electron concentration in p-quasi neutral region.

\[ n - n_0 = A_n \cosh \left( \frac{L_n}{-x - w_p} \right) + B_n \sinh \left( \frac{L_n}{-x - w_p} \right) + \gamma_n e^{-\alpha(x + x_p)} \]  

(108)

where,

\[ A_n = n_0 \left( e^{qV/k_BT} - 1 \right) - \gamma_n e^{-\alpha(x_p - w_p)} \]  

(109)

and

\[ B_n = \frac{\gamma_n \left[ e^{-\alpha(x_p - w_p)} \left( s_n \frac{L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) + \sinh \left( \frac{x_p - w_p}{L_n} \right) \right) \right] - n_0 (e^{qV/k_BT} - 1) \left( s_n \frac{L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) + \sinh \left( \frac{x_p - w_p}{L_n} \right) \right)}{\left[ \cosh \left( \frac{x_p - w_p}{L_n} \right) + s_n \frac{L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right) \right]} \]  

(110)

Finally the electron density in p-QNR can be written in the following form,

\[ n = A_n \cosh \left( \frac{L_n}{-x - w_p} \right) + B_n \sinh \left( \frac{L_n}{-x - w_p} \right) + \gamma_n e^{-\alpha(x + x_p)} + n_0 \]  

(111)

4.1.2. Comparison of this Work’s p-QNR Minority Carrier Density Equation with that of Nelson [7]

The initial objective of this thesis work is to validate the equation for minority charge carriers presented by Nelson in her book Physics of Solar Cells [7]. The equation for the electron density in p-QNR derived as part of this thesis project is presented in Equations (92), (109), (110) and (111). Equation (25), (26), (27) and (28) are presented by Nelson [7] for the electron density in p-QNR. There are some differences in the equations which are presented in this section.

The most significant difference is the value of \( \gamma \). While using the method of undetermined coefficients to solve the second order non-homogeneous differential equation, the value of \( \gamma \) obtained in this thesis differs from the \( \gamma \) value presented by Nelson [7] by a factor of a negative sign. To illustrate this further, the equations for \( \gamma \) from this work is

\[ \gamma_n = \frac{(1 - R) \alpha b_s L_n^2}{D_n \left( 1 - \alpha^2 L_n^2 \right)} \]  

(112)

and the presented in Nelson [7] is
\[ Y_n = \frac{(1 - R)ab_5L_n^2}{D_n(a^2L_n^2 - 1)} \]  

(113)

While the difference here is only a minus sign this minus sign has a significant effect on obtaining accurate results.

The second major difference in the equation for minority electron carriers is the electron density in equilibrium \( (n_0) \) term not added to the RHS of the equation presented by Nelson [7]. Nelson presented the minority charge carrier density in her book as

\[ n(E,x) = A_n \cosh\left(\frac{-x - w_p}{L_n}\right) + B_n \sinh\left(\frac{-x - w_p}{L_n}\right) + Y_n e^{-\alpha(x + x_p)} \]  

(114)

However, during the course of this thesis work, the derivation of the transport equation with the same boundary conditions provides the result for electron density in p-QNR as

\[ n = A_n \cosh\left(\frac{-x - w_p}{L_n}\right) + B_n \sinh\left(\frac{-x - w_p}{L_n}\right) + Y_n e^{-\alpha(x + x_p)} + n_0 \]  

(115)

After comparing the minority carrier density equation in p-QNR, it can be seen that electron carrier density at equilibrium \( (n_0) \) should be added to the RHS of equation presented by Nelson and this difference has some effect in calculating electron carrier density in the neutral region.

The third major difference is the value of constant \( A_n \). While comparing Equation (27), which comes from Nelson [7] and Equation (109), which comes from this thesis work, the errors were found in the value of \( A_n \). Both the equations are presented below for comparison. The equation presented by Nelson is

\[ A_n = n_0 \left( e^{qV/k_BT} \right) - Y_n e^{-\alpha(x_p - w_p)} \]  

(116)

The equation derived in this thesis work provides the value of constant \( A_n \) as

\[ A_n = n_0 \left( e^{qV/k_BT} - 1 \right) - Y_n e^{-\alpha(x_p - w_p)} \]  

(117)

The minority electron density is calculated using these three corrections and the results are compared in the next chapter.

4.1.3. **Derivation of Carrier Density Equation in n-QNR**

A detailed solution of second order differential equation is presented in this section using the undetermined coefficient method. The boundary conditions used for this analysis are
\[ p - p_0 = p_0 \left( e^{qV/k_B T} - 1 \right) \text{ at } x = w_n \] (118)

and

\[ -D_p \frac{dp}{dx} = S_p (p - p_0) \text{ at } x = x_n \] (119)

The generation term can be expressed as Equation (71). The transport equation for p-QNR can be written as

\[ \frac{d^2(p - p_0)}{dx^2} - \frac{(p - p_0)}{L_p^2} + \frac{g(E,x)}{D_p} = 0 \text{ for } x > w_n \] (120)

\( p - p_0 \) is the excess electron density in n-QNR. We can substitute excess electron density with \( p' \) as shown below

\[ \frac{d^2p'}{dx^2} - \frac{p'}{L_p^2} + (1 - R)\alpha b_s e^{-\alpha(x+x_p)} \frac{1}{D_p} = 0 \text{ for } x > w_n \] (121)

Rearranging the terms gives

\[ \frac{d^2p'}{dx^2} - \frac{p'}{L_p^2} = - (1 - R)\alpha b_s e^{-\alpha(x+x_p)} \frac{1}{D_p} \] (122)

This is a second order non-homogeneous differential equation. The general solution of this equation can be written as

\[ p' = p_c + Y \] (123)

where the term \( p_c \) is called the complementary solution and the term \( Y \) is called the particular solution.

The complementary solution is the solution of

\[ \frac{d^2p'}{dx^2} - \frac{p'}{L_p^2} = 0 \] (124)

This is a linear, second order, homogeneous differential equation with constant coefficients. The solution of the equation is

\[ p' = A e^{m_1 x} + B e^{m_2 x} \] (125)

The value of \( m_1 \) and \( m_2 \) can be found by solving the characteristic equation corresponding to Equation (124)

\[ m^2 - \frac{1}{L_p^2} = 0 \] (126)

Rearranging this equation to find ‘m’
\[ m = \pm \frac{1}{L_p} \]  

(127)

Substituting value of \( m \) in Equation (125) gives

\[ p' = Ae^{\frac{x}{L_p}} + Be^{-\frac{x}{L_p}} \]  

(128)

So, the complementary solution of the equation is

\[ p_c = Ae^{\frac{x}{L_p}} + Be^{-\frac{x}{L_p}} \]  

(129)

The particular solution for \( Y \) can be found by assuming a solution with constant \( C \), so the \( Y \) becomes

\[ Y = Ce^{-\alpha(x+x_p)} \]  

(130)

The first and second derivative of the equation is

\[ \frac{dY}{dx} = -\alpha Ce^{-\alpha(x+x_p)} \]  

(131)

\[ \frac{d^2Y}{dx^2} = \alpha^2 Ce^{-\alpha(x+x_p)} \]  

(132)

The particular solution for \( Y \) can be found by assuming a constant \( C \).

Substituting the values from Equation (130), (131) and (132) into Equation (122) gives

\[ \alpha^2 Ce^{-\alpha(x+x_p)} - \frac{Ce^{-\alpha(x+x_p)}}{L_p^2} = - \frac{(1 - R)ab_s e^{-\alpha(x+x_p)}}{D_p} \]  

(133)

Rearranging

\[ \left\{ \alpha^2 - \frac{1}{L_p^2} \right\} e^{-\alpha(x+x_p)} C = - \frac{(1 - R)ab_s e^{-\alpha(x+x_p)}}{D_p} \]  

(134)

and

\[ C = - \frac{(1 - R)ab_s}{D_p \left\{ \alpha^2 - \frac{1}{L_p^2} \right\}} \]  

(135)

Substituting the value of \( C \) from Equation (135) into Equation (130) gives

\[ Y = - \frac{(1 - R)ab_s}{D_p \left\{ \alpha^2 - \frac{1}{L_p^2} \right\}} e^{-\alpha(x+x_p)} \]  

(136)

Substituting the value of complementary solution, \( p_c \), and particular integral \( Y \) into Equation (123) gives

\[ p' = Ae^{\frac{x}{L_p}} + Be^{-\frac{x}{L_p}} - \frac{(1 - R)ab_s}{D_p \left\{ \alpha^2 - \frac{1}{L_p^2} \right\}} e^{-\alpha(x+x_p)} \]  

(137)
which can be rearranged in the following ways

\[ p' = A e^{x/L_p} + B e^{-x/L_p} - \frac{(1 - R) \alpha b_s L_m^2}{D_p \left\{ \alpha^2 L_p^2 - 1 \right\}} e^{-\alpha(x+x_p)} \]  

(138)

\[ p' = A e^{x/L_p} + B e^{-x/L_p} + \frac{(1 - R) \alpha b_s L_m^2}{D_p \left\{ 1 - \alpha^2 L_p^2 \right\}} e^{-\alpha(x+x_p)} \]  

(139)

and

\[ p' = A e^{x/L_p} + B e^{-x/L_p} + Y_p e^{-\alpha(x+x_p)} \]  

(140)

The value of \( Y_p \) can be expressed as below

\[ Y_p = \frac{(1 - R) \alpha b_s L_p^2}{D_p \left\{ 1 - \alpha^2 L_p^2 \right\}} \]  

(141)

Equation (136) can also be expressed as,

\[ p - p_0 = A e^{x/L_p} + B e^{-x/L_p} + Y_p e^{-\alpha(x+x_p)} \]  

(142)

The constants \( A \) and \( B \) can be calculated by solving for the boundary condition on the edges of the depletion region from the Equations (118) and (119). Using boundary condition from Equation (118), Equation (142) can be expressed as

\[ A e^{x_n/L_p} + B e^{-x_n/L_p} = p_0 \left( e^{qV_kU_mT - 1} \right) - Y_p e^{-\alpha(x_n+x_p)} \]  

(143)

Using boundary condition from Equation 119, Equation 142 can be expressed as,

\[ \left( 1 + S_p \frac{L_p}{D_p} \right) A e^{x_n/L_p} - \left( 1 - S_p \frac{L_p}{D_p} \right) B e^{-x_n/L_p} = L_p \alpha Y_p e^{-\alpha(x_n+x_p)} - S_p \frac{L_p}{D_p} Y_p e^{-\alpha(x_n+x_p)} \]  

(144)

Solving Equations (143) and (144) yields the value of constants \( A \) and \( B \),

\[
A = \left[ \begin{array}{c}
p_0 \left( e^{qV_kU_mT - 1} \right) - Y_p e^{-\alpha(x_n+x_p)} \frac{x_n}{L_p} \\
\left( 1 + S_p \frac{L_p}{D_p} \right) p_0 \left( e^{qV_kU_mT - 1} \right) \frac{x_n}{L_p} \\
- Y_p e^{-\alpha(x_n+x_p)} \left( 1 + S_p \frac{L_p}{D_p} \right) \frac{x_n - x_w}{L_p} + S_p \frac{L_p}{D_p} Y_p e^{-\alpha(x_n+x_p)} - L_p \alpha Y_p e^{-\alpha(x_n+x_p)} \end{array} \right] \\
\left( 1 + S_p \frac{L_p}{D_p} \right) p_0 \left( e^{qV_kU_mT - 1} \right) \frac{x_n - x_w}{L_p} + S_p \frac{L_p}{D_p} Y_p e^{-\alpha(x_n+x_p)} - L_p \alpha Y_p e^{-\alpha(x_n+x_p)} \\
2 \left[ \cosh \left( \frac{x_n - x_w}{L_p} \right) + S_p \frac{L_p}{D_p} \sinh \left( \frac{x_n - x_w}{L_p} \right) \right]
\right]^{-1}
\]  

(145)

and
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\[
B = \frac{\left(1 + S_p \frac{L_p}{D_p}\right) p_0 \left(e^{qV/k_BT} - 1\right) e^{\frac{x_n}{l_p}} - Y_p e^{-\alpha(x_n + x_p)} \left(1 + S_p \frac{L_p}{D_p}\right) e^{\frac{x_n}{l_p}}}{2 \left[ \cosh \left(\frac{x_n - w_n}{L_p}\right) + S_p \frac{L_p}{D_p} \sinh \left(\frac{x_n - w_n}{L_p}\right) \right]}
\]

Substituting the values of A and B in Equation 142 gives

\[
p - p_0 = \left[ p_0 \left( e^{\frac{qV}{k_BT}} - 1 \right) - Y_p e^{-\alpha(x_n + x_p)} \right] e^{\frac{w_n}{L_p}} e^{\frac{x}{l_p}}
\]

\[
- \frac{\left(1 + S_p \frac{L_p}{D_p}\right) p_0 \left(e^{qV/k_BT} - 1\right) e^{\frac{x_n}{l_p}} - Y_p e^{-\alpha(x_n + x_p)} \left(1 + S_p \frac{L_p}{D_p}\right) e^{\frac{x_n}{l_p}}}{2 \left[ \cosh \left(\frac{x_n - w_n}{L_p}\right) + S_p \frac{L_p}{D_p} \sinh \left(\frac{x_n - w_n}{L_p}\right) \right]}
\]

\[
- \left[ \frac{p - p_0}{p_0} \left( e^{\frac{qV}{k_BT}} - 1 \right) e^{\frac{x_n}{l_p}} - Y_p e^{-\alpha(x_n + x_p)} \left(1 + S_p \frac{L_p}{D_p}\right) e^{\frac{x_n}{l_p}} \right] e^{\frac{w_n}{L_p}} e^{\frac{x}{l_p}}
\]

\[
+ Y_p e^{-\alpha(x_n + x_p)}
\]

(147)
Rearranging further to simplify the results, the equation can be simplified to the form

\[ p - p_0 = \begin{bmatrix} p_0 \left( e^{qV/k_BT} - 1 \right) - Y_p e^{-\alpha(x_n+w_n+p)} \left[ \cosh \left( \frac{x-w_n}{L_p} \right) + \sinh \left( \frac{x-w_n}{L_p} \right) \right] \\
\left( 1 + S_p \frac{L_p}{D_p} \right) p_0 \left( e^{qV/k_BT} - 1 \right) e^{\frac{(x_n-w_n)}{L_p}} - Y_p e^{-\alpha(x_n+w_n+p)} e^{\frac{(x_n-w_n)}{L_p}} \right) + S_p \frac{L_p}{D_p} Y_p e^{-\alpha(x_n+w_n+p)} \\
\frac{2}{\cosh \left( \frac{x_n-w_n}{L_p} \right) + S_p \frac{L_p}{D_p} \sinh \left( \frac{x_n-w_n}{L_p} \right)} \cosh \left( \frac{x-w_n}{L_p} \right) \end{bmatrix} \]

Separating the cosh and sinh terms, and the equation can be simplified to the form

\[ p - p_0 = \begin{bmatrix} p_0 \left( e^{qV/k_BT} - 1 \right) - Y_p e^{-\alpha(x_n+w_n+p)} \left[ \cosh \left( \frac{x-w_n}{L_p} \right) \right] \\
\left( 1 + S_p \frac{L_p}{D_p} \right) p_0 \left( e^{qV/k_BT} - 1 \right) e^{\frac{(x_n-w_n)}{L_p}} - Y_p e^{-\alpha(x_n+w_n+p)} e^{\frac{(x_n-w_n)}{L_p}} \right) + S_p \frac{L_p}{D_p} Y_p e^{-\alpha(x_n+w_n+p)} \\
\frac{2}{\cosh \left( \frac{x_n-w_n}{L_p} \right) + S_p \frac{L_p}{D_p} \sinh \left( \frac{x_n-w_n}{L_p} \right)} \cosh \left( \frac{x-w_n}{L_p} \right) \end{bmatrix} \]

Rearranging further to simplify the results
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\[
p - p_0 = \left[ p_0 \left( e^{qV/k_BT} - 1 \right) - \gamma_p e^{-\alpha(x + x_p)} \right] \cosh \left( \frac{x - w_n}{L_p} \right) + \gamma_p e^{-\alpha(x + x_p)} \left[ \frac{L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) + \sinh \left( \frac{x_n - w_n}{L_p} \right) \right] \sinh \left( \frac{x - w_n}{L_p} \right) \]

\[
+ \gamma_p e^{-\alpha(x + x_p)} \left[ \frac{L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) + \sinh \left( \frac{x_n - w_n}{L_p} \right) \right] \sinh \left( \frac{x - w_n}{L_p} \right) \]

(150)

Some of the steps for rearranging RHS of the equation has been omitted and the simplified result can be expressed as

\[
p - p_0 = \left[ p_0 \left( e^{qV/k_BT} - 1 \right) - \gamma_p e^{-\alpha(x_n + x_p)} \right] \cosh \left( \frac{x - w_n}{L_p} \right) + \gamma_p e^{-\alpha(x_n + x_p)} \left[ \frac{L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) + \sinh \left( \frac{x_n - w_n}{L_p} \right) \right] \sinh \left( \frac{x - w_n}{L_p} \right) \]

\[
+ \gamma_p e^{-\alpha(x_n + x_p)} \left[ \frac{L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) + \sinh \left( \frac{x_n - w_n}{L_p} \right) \right] \sinh \left( \frac{x - w_n}{L_p} \right) \]

(151)

Further rearrangement is done to simplify the equation in the form presented by Nelson
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\[ p - p_0 = \begin{vmatrix} p_0 \left( e^{\frac{qV}{kTb}} - 1 \right) & -Y_pe^{-\alpha(x+w_p)} \cosh \left( \frac{x-w_n}{L_p} \right) \\ Y_p \left[ e^{-\alpha(w_n+x_p)} \left( S_p \frac{L_p}{D_p} \cosh \left( \frac{x_n-w_n}{L_p} \right) + \sinh \left( \frac{x_n-w_n}{L_p} \right) \right) \right. \\ \left. - \left( S_p \frac{L_p}{D_p} - L_p \alpha \right) e^{-\alpha(x-w_n)} \right] \\ -p_0 \left( e^{\frac{qV}{kTb}} - 1 \right) \left[ S_p \frac{L_p}{D_p} \cosh \left( \frac{x_n-w_n}{L_p} \right) + \sinh \left( \frac{x_n-w_n}{L_p} \right) \right] \\ \cosh \left( \frac{x_n-w_n}{L_p} \right) + S_p \frac{L_p}{D_p} \sinh \left( \frac{x_n-w_n}{L_p} \right) \end{vmatrix} \sinh \left( \frac{x-w_n}{L_p} \right) + Y_pe^{-\alpha(x+x_p)} \] (152)

The above equation can be written in simpler form, for excess minority hole concentration in n-quasi neutral region.

\[ p - p_0 = A_p \cosh \left( \frac{x-w_n}{L_p} \right) + B_p \sinh \left( \frac{x-w_n}{L_p} \right) + Y_p e^{-\alpha(x+x_p)} \] (153)

The equation above provides the concentration of excess minority carriers in the n-QNR. The above equation can be rearranged to find the minority hole density in the n-QNR as shown below

\[ p = A_p \cosh \left( \frac{x-w_n}{L_p} \right) + B_p \sinh \left( \frac{x-w_n}{L_p} \right) + Y_p e^{-\alpha(x+x_p)} + p_0 \] (154)

where,

\[ A_p = p_0 \left( e^{\frac{qV}{kTb}} - 1 \right) - Y_pe^{-\alpha(w_n+x_p)} \] (155)

and

\[ Y_p \left[ e^{-\alpha(w_n+x_p)} \left( \left( S_p \frac{L_p}{D_p} \cosh \left( \frac{x_n-w_n}{L_p} \right) + \sinh \left( \frac{x_n-w_n}{L_p} \right) \right) \right. \right. \]

\[ \left. \left. - \left( S_p \frac{L_p}{D_p} - L_p \alpha \right) e^{-\alpha(x-w_n)} \right] \right. - \left( S_p \frac{L_p}{D_p} - L_p \alpha \right) e^{-\alpha(x-w_n)} \] (156)

\[ B_p = \frac{-p_0 \left( e^{\frac{qV}{kTb}} - 1 \right) \left[ S_p \frac{L_p}{D_p} \cosh \left( \frac{x_n-w_n}{L_p} \right) + \sinh \left( \frac{x_n-w_n}{L_p} \right) \right]} {\cosh \left( \frac{x_n-w_n}{L_p} \right) + S_p \frac{L_p}{D_p} \sinh \left( \frac{x_n-w_n}{L_p} \right)} \]
4.1.4. **Comparison of this Work’s n-QNR Minority Carrier Density Equation with that of Nelson [7]**

Just as was done with the equation for the minority carriers in the p-QNR, comparisons between the equations derived in this thesis for the minority carrier in the n-QNR is compared to the corresponding equation presented by Nelson [7]. Similar errors were found. The hole density in n-QNR for this thesis work is presented by Equations (141), (154), (155) and (156). Equations 35, 36, 37 and 38 in this thesis are those presented by Nelson [7] for hole density in n-QNR.

Once again the most significant difference is the minus sign different in the equation for \( Y \). To illustrate this further, the equations for \( Y \) from this source is

\[
Y_p = \frac{(1 - R)ab_sL_p^2}{D_p(1 - \alpha^2L_p^2)}
\]

and that presented by Nelson [7] is

\[
Y_p = \frac{(1 - R)ab_sL_p^2}{D_p(\alpha^2L_p^2 + 1)}
\]

The second major difference in the equation for minority hole density is the sign changes that has significant effect is calculating the hole density in the n-QNR. Nelson presented the minority charge carrier density as

\[
p = A_p \cosh \left( \frac{x + w_n}{L_p} \right) + B_p \sinh \left( \frac{x + w_n}{L_p} \right) - Y_p e^{-\alpha(x + x_p)}
\]

However, during the course of this thesis work, the derivation of transport equation with the same boundary conditions provided the result as

\[
p = A_p \cosh \left( \frac{x - w_n}{L_p} \right) + B_p \sinh \left( \frac{x - w_n}{L_p} \right) + Y_p e^{-\alpha(x + x_p)} + p_0
\]

After comparing the minority carrier density equation in n-QNR, it could be seen that hole carrier density at equilibrium, \( p_0 \) should be added to the RHS of equation presented by Nelson and this difference has some effect in calculating electron carrier density in the neutral region.

There are a few sign changes in the equation for minority charge carriers in n-QNR. The \( \cosh \) and \( \sinh \) terms have sign changes that can be seen by comparing Equation (159) [7] and Equation (160). Furthermore, there is a sign change in the generation term of the minority charge
carrier density which is the final term of the minority charge density. These changes have considerable effect in calculating the minority hole density in the neutral region.

The minority hole density is calculated using these corrections and the results are compared in the next chapter.

4.2. Current Densities in Quasi Neutral Regions

Current density can be calculated using the Equations (25) and (26) mentioned in Chapter 3. In the quasi-neutral regions the electric field is zero and therefore, Equation (25) and Equation (26) can be reduced to

$$j_n = qD_n \frac{dn}{dx}$$  \hspace{1cm} (161)

and

$$j_p = -qD_p \frac{dp}{dx}$$  \hspace{1cm} (162)

Because the electric fields are essentially zero in the quasi-neutral regions, this approximation can be used to calculate the current density in the p and n neutral regions.

4.2.1. Derivation of Current Density Equation in p-QNR

The solution of the transport equations was presented in the previous sections of this chapter to calculate the carrier densities. Because the current densities are obtained from the number density equations, see Equations (161) and (162), it was reasonable to assume that there are errors in the current density equations. Therefore, some further work was performed to derive the current density equations in the neutral regions.

The electron current density can be calculated by substitution Equation (111) into equation (161) giving

$$j_n = qD_n \left[ A_n \cosh \left( \frac{-x - w_p}{L_n} \right) + B_n \sinh \left( \frac{-x - w_p}{L_n} \right) + \gamma_n e^{-\alpha(x+x_p)} + n_0 \right]$$  \hspace{1cm} (163)

Taking this derivative gives
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\[ j_n = qD_n \left[ A_n \sinh \left( \frac{-x - w_p}{L_n} \right) \left( - \frac{1}{L_n} \right) + B_n \cosh \left( \frac{-x - w_p}{L_n} \right) \left( - \frac{1}{L_n} \right) \right. \]

\[ + \left. Y_n (-\alpha) e^{-\alpha(x+w_p)} \right] \]  

Equation (164) is the general solution for electron current density through the p-QNR.

The critical location at which to calculate the electron current density is at the edge of the p-QNR that borders with the depletion region, at \( x = -w_p \). When Equation (161) is evaluated at this location, the current density is

\[ j_n(E, -w_p) = qD_n \left[ A_n \sinh \left( \frac{W_p - w_p}{L_n} \right) \left( - \frac{1}{L_n} \right) + B_n \cosh \left( \frac{W_p - w_p}{L_n} \right) \left( - \frac{1}{L_n} \right) \right. \]

\[ + \left. Y_n (-\alpha) e^{-\alpha(x_p-w_p)} \right] \]  

Rearranging the equation provides the electron current density at the edge of the depletion region in p-QNR

\[ j_n(E, -w_p) = qD_n \left[ - \frac{B_n}{L_n} - \alpha Y_n e^{-\alpha(x_p-w_p)} \right] \]  

Equation (166) contains \( B_n \) term that was calculated previously and represented by Equation (110). Substituting the value of \( B_n \) term in the above equation provides the result that contains a detailed form as explained below in Equation (164). This approach was taken to generate the equation for electron current density in the p-QNR that can be compared with the results presented by Nelson [7]

\[ j_n = \frac{-q(1 - R) \alpha b_3 L_n}{(1 - \alpha^2 L_n^2)} \left\{ e^{-\alpha(x_p-w_p)} \left[ \sinh \left( \frac{X_p - W_p}{L_n} \right) + S_n L_n \cosh \left( \frac{X_p - W_p}{L_n} \right) \right. \right. \]

\[ - \left. \left. \left( S_n L_n + \alpha L_n \right) \cosh \left( \frac{X_p - W_p}{L_n} \right) + S_n L_n \sinh \left( \frac{X_p - W_p}{L_n} \right) \right] \right\} \]

\[ + \alpha L_n e^{-\alpha(x_p-w_p)} \]  

\[ + \frac{qD_n n_0 \left( e^{qV/k_B T} - 1 \right) \left( \sinh \left( \frac{X_p - W_p}{L_n} \right) + S_n L_n \cosh \left( \frac{X_p - W_p}{L_n} \right) \right. \right. \]

\[ - \left. \left. \left( S_n L_n + \alpha L_n \right) \cosh \left( \frac{X_p - W_p}{L_n} \right) + S_n L_n \sinh \left( \frac{X_p - W_p}{L_n} \right) \right] \right\} \]
4.2.2. **Comparison of this Work’s p-QNR Current Density Equation with that of Nelson [7]**

The equation for the electron current density in p-QNR derived as part of this thesis project is presented in Equation (167). Equation (54) was presented by Nelson [7] for the electron current density in p-QNR. There is a difference of sign change in both the equations.

The equation presented by Nelson for electron current density is

\[
j(E, -w_p) = -q(1 - R)abL_n \left\{ \frac{e^{-a(x_p - w_p)} \left[ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right]}{cosh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right)} \right\} + \frac{qD_n n_0 \left( \frac{e^{qV/k_BT}}{k_BT} - 1 \right)}{L_n} \left\{ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right\}
\]

(168)

The equation obtained from this thesis work for the electron current density in p-QNR is

\[
j_n = -q(1 - R)abL_n \left\{ \frac{e^{-a(x_p - w_p)} \left[ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right]}{cosh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \sinh \left( \frac{x_p - w_p}{L_n} \right)} \right\} + \frac{qD_n n_0 \left( \frac{e^{qV/k_BT}}{k_BT} - 1 \right)}{L_n} \left\{ \sinh \left( \frac{x_p - w_p}{L_n} \right) + \frac{S_n L_n}{D_n} \cosh \left( \frac{x_p - w_p}{L_n} \right) \right\}
\]

(169)

While the difference here is only a minus sign this minus sign has a significant effect on obtaining accurate results.
4.2.3. Derivation of Current Density Equation in n-QNR

The solution of the transport equations was presented in the previous sections of this chapter to calculate the carrier densities. Because the current densities are obtained from the number density equations, see Equations (161) and (162), it was reasonable to assume that there are errors in the current density equations. Therefore, some further work was performed to derive the current density equations in the neutral regions.

The hole current density can be calculated by substitution Equation (154) into Equation (162) giving

\[ j_p = -qD_p \left[ A_p \cosh \left( \frac{x - w_n}{L_p} \right) + B_p \sinh \left( \frac{x - w_n}{L_p} \right) + Y_p e^{-\alpha(x + x_p)} + p_0 \right] \]  

Differentiating the above equation provides the result

\[ j_p = -qD_p \left[ A_p \sinh \left( \frac{x - w_n}{L_p} \right) \left( \frac{1}{L_p} \right) + B_p \cosh \left( \frac{x - w_n}{L_p} \right) \left( \frac{1}{L_p} \right) \right. \]

\[ \left. - \alpha Y_p e^{-\alpha(x + x_p)} \right] \]

Equation (171) is the general solution for hole current density. \( j_p \) was found at the edge of depletion region \( w_n \) to compare the results with the results presented by Nelson [7].

\[ j_p(E, w_n) = -qD_p \left[ A_p \sinh \left( \frac{w_n - w_n}{L_p} \right) \left( \frac{1}{L_p} \right) + B_p \cosh \left( \frac{w_n - w_n}{L_p} \right) \left( \frac{1}{L_p} \right) \right. \]

\[ \left. - \alpha Y_p e^{-\alpha(w_n + x_p)} \right] \]

Cancelling the terms in the above equation provides the hole current density at the edge of the depletion region in n-QNR

\[ j_p(E, w_n) = -qD_p \left[ \frac{B_p}{L_p} - \alpha Y_p e^{-\alpha(w_n + x_p)} \right] \]

Equation (168) contains \( B_p \) term that was calculated previously and represented by Equation (156). Substituting the value of \( B_p \) term in the above equation provides the result that contains a detailed form as explained below in Equation (174). This approach was taken to generate the equation for hole current density in the n-QNR that can be compared with the results presented by Nelson [7].
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\[ j_p = - \frac{q(1 - R) a b \ell_p}{1 - a^2 L_p^2} \ e^{-a(w_n + x_p)} \left\{ \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_{pL} L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) - \frac{S_{pL} L_p}{D_p} e^{-a(x_n - w_n)} \right\} \]

\[ \left[ \cos \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_{pL} L_p}{D_p} \sinh \left( \frac{x_n - w_n}{L_p} \right) \right] \]

\[ (174) \]

4.2.4. Comparison of this Work’s n-QNR Current Density Equation with that of Nelson [7]

The calculation of hole density in the n-QNR region calculated in the previous sections revealed some errors when compared with results presented by Nelson [7]. Based on this result validation, it was assumed that there may be some errors with electron current density equation presented by Nelson. The equation for the electron current density in p-QNR derived as part of this thesis project was presented in Equation (174). Equation (55) was presented by Nelson [7] for the hole current density in n-QNR. There is a difference of sign change between both the equations.

The equation presented by Nelson for hole current density is

\[ j_p = - \frac{q(1 - R) a b \ell_p}{a^2 L_p^2 - 1} \ e^{-a(w_n + x_p)} \left\{ \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_{pL} L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) - \frac{S_{pL} L_p}{D_p} e^{-a(x_n - w_n)} \right\} \]

\[ \cos \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_{pL} L_p}{D_p} \sinh \left( \frac{x_n - w_n}{L_p} \right) \]

\[ (175) \]

The equation obtained from this thesis work for the hole current density in n-QNR is
\[ j_p = - \frac{q(1 - R)\alpha_L L_p}{1 - \alpha^2 L_p^2} e^{-\alpha(x_n + x_p)} \left\{ \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_p L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) \right\} \]

\[
\times \left( \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_p L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) \right) - \alpha L_p \right\}
\]

\[ + \frac{qD_p p_0}{L_p} \left( e^{\frac{qV}{k_B T}} - 1 \right) \left\{ \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_p L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) \right\} \]

\[
\times \left( \sinh \left( \frac{x_n - w_n}{L_p} \right) + \frac{S_p L_p}{D_p} \cosh \left( \frac{x_n - w_n}{L_p} \right) \right) \]

(176)

While the difference here is only a minus sign this minus sign has a significant effect on obtaining accurate results.

Since the carrier density and current density equations presented by Nelson have errors and the focus of this thesis work was obtaining the correct version of these equations, in the next chapter comparisons to published results were made.
Chapter 5 - Validation of Derived Analytical Equations

In this chapter, the results from the equations derived in Chapter 4 are plotted using a Microsoft EXCEL program that was prepared during the course of this thesis work. The results are plotted for a homojunction Ga-As solar cell for both dark and illuminated conditions with and without an external voltage applied. In this chapter, the carrier density profile for short-circuit conditions and an external voltage of 0.5 V is presented with comparisons to results from the numerical program PC1D. Surprisingly, these results come from Nelson [7]. Even though Nelson presents analytical equations for number densities and currents, she does not present results from these equations. Instead, Nelson’s plotted results come from the numerical program PC1D. In addition to comparisons to PCID, comparisons of some limiting cases provide by Fahrenbruch and Bube in their book *Fundamentals of Solar Cells* are presented here.

In this thesis work, the derived equations and any additional equations shown in Chapter 3 required to provide number density and current density results are programmed in Microsoft EXCEL. Care was taken to accurately program the lengthy equations required to calculate the change carrier number densities and the current densities. A number of different operating conditions are checked.
5.1. Validation of Carrier Density Equations Under Dark Conditions

In this section, the carrier density obtained by analytical equations derived in this thesis research are compared with the carrier densities calculated using PC1D for dark conditions. The PC1D results shown here were taken from Nelson [7] and the data points are obtained by digitizing the plot presented by Nelson under equilibrium conditions [7] using software called Plot Digitizer.

5.1.1. Carrier Densities in the Dark with Equilibrium Conditions

In Figure 12, comparisons of the electron and hole number densities obtained from the analytical equations derived in this work and those obtained from the numerically based computer code PC1D are presented. These results are for a p-n junction of a GaAs solar cell with no illumination and no applied voltage. The results from the analytical equations are shown as dashed lines and the results from PC1D are solid lines. The electron number densities are shown in red and the hole number densities are shown in blue. A logarithmic scale is used for the number densities because the results cover twenty-two orders of magnitude. While a log scale does tend to mask difference between the results, a log scale is required to see detail over the entire range of number densities.

The comparisons between the analytical results and the PC1D results are excellent. This is what should be expected for this case if both the analytical equations are correct and the numerical results from PC1D are correct. It is seen that there are some differences between the analytical and numerical results, but they are small. This could be due to inaccuracies in the numerical solution from PC1D or from some of the assumptions used to derive the number density equations presented in this thesis. However, it is believed that most of these differences are due to the operator’s ability to digitize the plotted PC1D results. This is demonstrated by the ripple seen in the PC1D results shown in Figure 12. This ripple is especially noticeable in the SCR where the number densities change rapidly. The PC1D results were digitized using the software Plot Digitizer.
Figure 12: Comparison of charge particle carrier densities calculated with the analytical equations derived in this thesis to the numerical results from the computer program PCID [7]. These results are for a GaAs p-n junction in equilibrium in the dark.

5.1.2. Carrier Densities in the Dark with an 0.5 Volt External Voltage

When a p-n junction is connected to an external bias, it alters the carrier densities in the semiconductor material. This can be seen by comparing the results in Figure 13 to those in Figure 12. The minority carrier densities, the low electron and hole densities on each side of the plot, are orders of magnitude larger for the biased case than the unbiased case. For the case shown in Figure 13, the applied external bias is 0.5 V.

Once again the analytical results compare beautifully with the numerical results. Differences are seen between the dashed lines (analytical results) and the solid lines (PC1D results), but most of these differences are probably due to the operator’s ability to digitize the plotted PC1D results. The waviness in the PC1D results would indicate this. Four locations where the differences may actually be due to assumptions made in the derived analytical equations are at the bends in the curves, where the lines go from being almost horizontal to having large slopes. The analytical model assumes the SCR is devoid of free charges and is completely dominated by unbalanced fixed charge at the doping density, this is called the
depletion approximation, but PC1D does not have to make this assumption. This same thing may be showing itself in Figure 12 as well; however, this cannot be concluded conclusively from these plots because these are difficult regions to digitize.

![Graph showing carrier density comparison](image)

**Figure 13:** Comparison of charge particle carrier densities calculated with the analytical equations derived in this thesis to the numerical results from the computer program PCID [7]. These results are for a GaAs, p-n junction in the dark with an external bias of 0.5 V.

### 5.2. Validation of Carrier Density Equations Under Illuminated Conditions

In this section, the carrier density obtained by the analytical equations derived as part of this thesis research are compared with the carrier densities calculated using PC1D under illuminated conditions. Illuminated conditions are essentially having the sun’s energy impinge upon the solar cell. The photon flux \(b_s\) used is this analysis is \(3.702 \times 10^{21}\) photons/s-m\(^2\). This is essentially the spectral flux of photons with the energy to just overcome the band gap of the p-n junction multiplied by an energy band of 1 eV. This seems to be the value that is used in the PC1D results presented by Nelson, but it must be said that Nelson does not state this quantity.
5.2.1. Carrier Densities Under Illumination and Short Circuit Conditions

In Figure 14 a comparison of carrier densities produced by the analytical equations derived in this thesis to the numerical results from PC1D are shown for an illuminated GaAs p-n junction with no external applied bias. Results from the analytical equations are only shown in the QNRs and not in the SCR. At the present time there are no analytical equations available that produce good results for carrier number densities in the SCR under illuminated conditions [21]. Notice that the minority carrier densities and the carrier densities in the SCR are orders of magnitude larger than those shown for the dark condition in Figure 12. The illumination of the p-n junction makes the depletion approximation invalid.

For the most part the analytical equations derived in this thesis produce similar carrier densities as the numerical model called PC1D; however, there are some noticeable and real deviations in the minority carrier densities at the edges of the QNRs, adjacent to the SCR. At these edges the minority carrier densities predicted by the analytical equations are noticeably below those predicted by PC1D. More than likely this is due to the neglect of electric field effects in the analytical equations for the QNRs. There probably are some significant electrical field effects in the so called QNR’s right next to the SCR. These electric fields will affect the carrier number densities calculated. Another way to look at this discrepancy is that the size of the SCR is under-predicted when the minority carrier densities are neglected in its calculation. This would mean that a QNR should be bigger than was taken in this analysis. Making the QNR bigger means the region where the analytically calculated carrier densities deviate from the numerically calculated number densities would be smaller. While there are noticeable deviations in the minority carrier density results, the plot indicates the deviations between the majority carrier densities calculated analytically and those calculated with PC1D are relatively small.

In regards to the results presented in Figure 14 it needs to be said that a light absorption coefficient 7.7x10^5 m^-1 was used in the calculation of the analytical results. Just like the photo flux Nelson [7] does not specify the absorption coefficient used to obtain her PC1D results. Just like the photon flux it is reasonable to assume that a value at the band gap energy is what was used. Because there was some educated guessing undertaken in the determination of the absorption coefficient used, a figure that shows the effect of the absorption coefficient on the
results has been produced. Figure 15 shows minority carrier number densities for a GaAs p-n junction under illumination with no externally applied bias. It can be seen that the absorption coefficient has noticeable impact on the concentration of minority carriers on the p-side of the junction (left side in Figure 15), but a very small effect on the n-side of the junction (right side on Figure 15). The reason this occurs is the p-n junction is illuminated from the p-side (left side) of the junction. This means most of the photons are absorbed on the p-side of the junction causing the minority charge carriers on the p-side to increase. Very few photons make it to the n-side (right side) of the junction and thus there is little effect on the p-side minority carriers at this location. This same type of logic used to judge the effect of the absorption coefficient on the minority carrier results can be used to judge the effect of the photon flux used on the minority carrier results.

Figure 14: Comparison of charge particle carrier densities calculated with the analytical equations derived in this thesis to the numerical results from the computer program PCID [7]. These results are for a GaAs, p-n junction under illumination with an external bias of 0 V.
Figure 15: Effect of absorption coefficient on the minority carrier densities in a GaAs p-n junction under illumination with an external bias of 0 V.

5.2.2. Carrier Densities Under Illumination with 0.5 Volt External Bias

When an illuminated solar cell is connected to an external bias, the carrier densities (see Figure 16) are similar to the carrier densities for a solar cell under illumination with no external bias (see Figure 14). This indicates that a 0.5 volts external bias does not affect number densities significantly in this operation regime. The only difference between the results in Figure 14 and those in Figure 16 is the applied external bias. In addition to the curves in Figures 14 and 16 looking similar, the comparisons between the analytical results and those from PC1D are similar. Once again, the biggest differences are where the QNR meet the SCR.

5.2.3. Carrier Densities Under Illumination With Different Boundary Conditions

Additional validation of the derived analytical equations presented in this thesis can be made by comparing to results presented by Fahrenbruch and Bube [10]. Fahrenbruch and Bube present a mathematical model in their book ‘Fundamentals of Solar Cells’ that accounts for three different boundary conditions. These conditions are a given number density at the two outer
boundaries of the solar cell, the limiting case of an infinite surface recombination, and the case of zero surface recombination. In addition, Fahrenbruch and Bube complete specify their operating conditions including the photon flux illuminating the solar cell and the absorption coefficient used. This makes the comparisons more certain than the illuminated comparisons made to the PC1D results presented by Nelson [7].

![Graphical Representation](image)

**Figure 16:** Comparison of charge particle carrier densities calculated with the analytical equations derived in this thesis to the numerical results from the computer program PC1D [7]. These results are for a GaAs, p-n junction under illumination with an external bias of 0.5 V.

A schematic band diagram of a simple homojunction solar cell and the location of illumination used by Fahrenbruch and Bube [10] is shown in Figure 9. Fahrenbruch and Bube’s illumination point is a little unusual and is different than the incident point used in this work which is the point \(-x_p\) shown in Figure 11. In Fahrenbruch and Bube’s results the impinge point of the solar flux is on the interface of the SCR region and n-QNR of p-n junction. In order to make a proper comparison between the results from the analytical equations derived in this thesis and the results from Fahrenbruch and Bube changes are made to Equations (111) and (154). The major change is to the exponential terms. Nelson [7] takes the incident radiation at \(-x_p\) and Fahrenbruch and Bube take it at \(-w_p\). Thus all the \(e^{-\alpha(w_n+x_p)}\) terms need to be adjusted. The
\(\alpha L_p\) type terms do not need to be adjusted because these represent absorption at a point and are not dependent on the radiation available at that point. No changes need to be made to \(Y_n\), because it comes from the derivatives and is not a function of distance or the magnitude of the radiation. Another way to say this is that changes only need to be made to terms that contain the absorption coefficient in an exponential function, not terms that only contain the absorption coefficient. The value of the absorption coefficient \(\alpha\) is the same in any coordinate system.

The parameters used for calculating carrier densities by Fahrenbruch and Bube [10] are \(\frac{1}{\alpha} = 3\mu\text{m}, L_n = 10\mu\text{m}, D = 25\ \text{cm}^2\text{s}^{-1}\), and the photon flux \(\Gamma = 10^{17}\ \text{photons cm}^2\text{s}^{-1}\). The comparison of the carrier density results from the analytical equations derived in this thesis and those from Fahrenbruch and Bube are presented in Figure 17. Just like the comparisons made to Nelson’s published results, the comparison’s to Fahrenbruch and Bube’s published results are excellent. This is true for all three cases shown in Figure 17. To get a better look at these comparisons Figure 18 has been prepared. Figure 18 shows the same results are Figure 17, except a linear scale is used as opposed to a logarithmic scale. The linear scale does bring out a little more of the differences between this thesis results and those of Fahrenbruch and Bube, but it is still believed that this is mostly due to the inability to digitize the plotted results of Fahrenbruch and Bube with better accuracy.

![Graph showing carrier density comparisons](image)

**Figure 17:** Log plot comparisons of charge particle carrier densities calculated with the analytical equations derived in this thesis to the analytical results of Fahrenbruch and Bube [10]. These results are for a GaAs, p-n junction under illumination with an external bias of 0 V. Three different, outer boundary conditions are considered.
Figure 18: Linear plot comparisons of charge particle carrier densities calculated with the analytical equations derived in this thesis to the analytical results of Fahrenbruch and Bube [10]. These results are for a GaAs, p-n junction under illumination with an external bias of 0 V. Three different, outer boundary conditions are considered.

The comparative results shown in Figures 12, 13, 14, 16, 17, 18, and 19, are compelling evidence that the general charge carrier equations derived as part of this thesis work are correct. These figures comprise comparisons of the results from this work against two other sources [7, 10]. The first source uses a well-accepted numerical program, PC1D [7], to determine charge carrier densities and the second source was a respected book on solar cells called The Fundamentals of Solar Cells [10]. While changes to the analytical equations derived in this thesis had to be made to compare to the results in Figures 17 and 18, it still is supporting evidence for the accuracy of the derivation work done as part of this thesis. At this point it is appropriate to make that claim that the equations presented by Nelson [7] have errors, and these errors have been located and corrected by the work presented in this thesis. Equations (6.29), (6.30), (6.31) and (6.32) in Nelson [7] for the electron densities in the p-QNR should be replaced with Equations (108), (109), (110) and (111) in this thesis; and Equations (6.35), (6.36), and (6.37) in Nelson [7] for the hole densities in the n-QNR should be replaced with Equations (154), (155), and (156) in this thesis.
5.3. Validation of Current Density Equations under Illumination

The other equations derived in this thesis are current density equations for minority charge carriers in the QNRs. This is Equation (167) for the current carried by the electrons in the p-QNR and Equation (174) for the current carried by the holes in the n-QNR. These equations are building blocks for determining the total current produced by a solar cell under illumination.

To validate these equations results from the work of Fahrenbruch and Bube [10] is used. Fahrenbruch and Bube’s current density results are for the same situation as their carrier number density results shown in Figures 17 and 18. To obtain these conditions the analytical current density equations derived in this thesis have to be altered somewhat. This was done just like it was done for the carrier number density equations. This process was explained in Section 5.3.3.

As shown in Figure 19 the current density profile obtained in this research work is similar to the current density profile presented by Fahrenbruch and Bube in their book [10]. The comparisons between the two are excellent. This indicates that the current density equations derived in this work are correct and should replace Equations (6.34) and (6.39) in Nelson’s book [7].

The analytical equations derived as part of this thesis work can be used to calculate the carrier and current densities for p-n junction solar cells. These equations along with a number of other equations which have been presented in Chapter 3 of this thesis will be used to present a number of plotted results representing the performance of solar cells composed of GaAs p-n junctions. These results are shown in the next chapter.
Figure 19: Comparisons of current densities calculated with the analytical equations derived in this thesis to the analytical results of Fahrenbruch and Bube [10]. These results are for a GaAs, p-n junction under illumination with an external bias of 0V. Three different, outer boundary conditions are considered.
Chapter 6 - Solar Cell Survey Results

In this chapter five parameters that affect the performance of solar cells are presented. These five parameters are the doping number density, the external bias applied, the solar cell length, the photon flux impinging on the solar cell, and the surface recombination velocity. These five parameters can either be looked at as design parameters or operating parameters. The doping number density, the solar cell length, and the surface recombination velocity are controlled by the designer of the solar cell. The applied external bias and the photon flux impinging on the solar cell are part of the operating conditions of the solar cell. For the most part the carrier number densities are the results presented; however, some SCR widths and some built-in bias voltages are also presented.

All the results presented in this chapter are for a GaAs solar cell that is comprised of a p-n homojunction. For all but the SCR width results, the width of the p-region is kept at 1 µm and the width of the n-region is kept at 2 µm. The reason for allowing the solar cells p-side width and n-side width to increase for the SCR study is to allow a wider range of doping concentrations to be considered. The other parameters used for preparing the model are illustrated in the Appendix section.

6.1. Effect of Doping Concentrations

A properly doped p-n junction is important for improving the performance of a solar cell. The SCR width and the built-in voltage depend on the donor, \( N_d \), and acceptor, \( N_a \), doping densities used on the n-side and p-side of the solar cell respectively. A wide SCR region aids carrier collection and this can be obtained by reducing the doping levels. However, a large built-
in voltage provides a larger open circuit voltage for the solar cell which increases current flow. A larger built-in voltage is obtained by increasing the doping levels. Thus there is a trade-off in solar cell performance as a function of doping number density. While this work is not extensive enough to provide actual solar cell performance as a function of doping level, this work is in a position to show the effects of doping number density on SCR thickness, built-in voltage, and carrier number densities. These are all shown in this section for four different operating conditions: 1) equilibrium conditions in the dark, 2) a 1 volt externally applied voltage in the dark, 3) short circuit conditions with the cell under illumination, and 4) a 1 volt externally applied voltage under illumination.

6.1.1. Results for Equilibrium Conditions in the Dark

Results were generated to provide information that would increase understanding of the effect of doping on the SCR width with zero volts of externally applied voltage under dark conditions. Figure 20 presents the width of the SCR as a function of \( N_a \) while keeping \( N_d \) constant at \( 10^{16} \) donor atoms per cubic centimeter and Figure 26 presents the same information as a function of \( N_d \) keeping \( N_a \) constant at \( 10^{17} \) acceptor atoms per cubic centimeter. The calculation of the p-side SCR and n-side SCR is performed using Equations (20) and (21) respectively. The overall SCR thickness is the sum of these two values.

Both Figure 20 and Figure 21 show that the SCR region increases in width as the doping concentration becomes smaller. Both of these figures also show a leveling off of the SCR width as the doping concentrations are increased. This is interesting because harmful shrinking of SCR widths is not so important for both acceptor and donor atom concentrations above \( 10^{17} \) cm\(^{-3} \) for the conditions of this survey. Another interesting conclusion that can be drawn from the results of these figures is the SCR can encompass the whole solar cell if the p-side has a width of 1 \( \mu \)m and the n-side has a width of 1 \( \mu \)m and low enough doping concentrations are used.

In Figures 22 and 23, the effect of doping on the junction voltage is presented. It is easy to see in these plots that the junction voltage increases with increased doping, whether that is acceptor or donor atom doping. For high efficiency solar cells, higher junction voltages are desirable. The higher doping concentration leads to higher junction voltage. However, it must be remembered that higher doping concentrations decrease the SCR width as well. Therefore, a
A compromise is required to choose the doping profile for optimum results. For many of the results shown in this chapter, the acceptor concentration used is $10^{17}$ cm$^{-3}$ and the donor concentration is $10^{16}$ cm$^{-3}$. These values are around the knee in the curves shown in Figures 20 and 21.

**Figure 20:** Effect of acceptor concentration on SCR width at equilibrium conditions in the dark. The donor doping concentration for these results is $N_d = 10^{16}$ cm$^{-3}$.

**Figure 21:** Effect of donor concentration on SCR width at equilibrium conditions in the dark. The acceptor doping concentration for these results is $N_a = 10^{17}$ cm$^{-3}$. 

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Figure 22: Effect of acceptor concentration on junction voltage at equilibrium conditions in the dark. The donor doping concentration for these results is $N_d = 10^{16} \text{ cm}^{-3}$.

Figure 23: Effect of donor concentration on junction voltage at equilibrium conditions in the dark. The acceptor doping concentration for these results is $N_a = 10^{17} \text{ cm}^{-3}$.
In Figure 24, the donor doping concentration is constant at $10^{17} \text{ cm}^{-3}$ and the acceptor doping concentration is changed to analyze the effect of doping on minority carrier densities throughout the solar cell. $N_a$ is the majority hole density on the p-side of the solar cell and $N_d$ is the majority carrier density on the n-side of the solar cell. Figure 24 clearly shows the three regions of a solar cell. The region on the left side of the graph, where the carrier concentrations have a flat profile, is the p-QNR. Electric fields in this region are small and are assumed to be zero. The right side of the graph, where the carrier concentrations are again flat, is the n-QNR. Electric fields in this region are also small and are assumed to be zero. The smaller region in the middle of the graph, where the carrier concentrations change drastically, is the SCR. Electric fields in the SCR are strong and are included in this analysis.

The results in Figure 24 show that both the minority and majority carrier concentrations change with doping concentration in the p-QNR and in the SCR of the solar cell. The minority and majority carrier densities in the n-QNR do not change to any noticeable degree as $N_a$ is increased. These are not surprising results. If the doping level is changed, the carrier number densities on the side that is being doped change, while on the side that is not having its doping changed there is little no change in the QNR carrier number densities. Changes in the carrier densities throughout the entire SCR are expected and the plotted results show this to be the case.

![Figure 24: Effect of acceptor concentration on carrier densities at equilibrium conditions in the dark. The donor doping concentration for these results is $N_d = 10^{16} \text{ cm}^{-3}$.](image)

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In Figure 25, the acceptor doping concentration is constant at $10^{16}$ cm$^{-3}$ and the donor doping concentration is changed to analyze the effect of doping on carrier densities in the solar cell. Like the results shown in Figure 24 the changes in doping concentrations affect all the carrier number densities on the same side as the doping change and throughout the SCR.

![Figure 25: Effect of donor concentration on carrier densities at equilibrium conditions in the dark. The acceptor doping concentration for these results is $N_a = 10^{17}$ cm$^{-3}$.]

6.1.2. Results for Dark Conditions with a 1 Volt Externally Applied Voltage

An analysis is prepared to understand the effect of doping on the SCR width with an external voltage applied under dark conditions. The external voltage applied for these results is 1 volt. Figure 26 presents the SCR width with changes in $N_a$ while keeping $N_d$ constant at $10^{16}$ cm$^{-3}$. Figure 27 presents the SCR width with changes in $N_d$ while keeping $N_a$ constant at $10^{17}$ cm$^{-3}$. These results are useful to understand the effect of doping on SCR width. In a solar cell a wider SCR aids carrier collection. Better carrier collection produces a better performing solar cell.

The effect of doping levels on the SCR width is illustrated in Figures 26 and 27. These figures illustrate that doping should not be reduced below a certain level if the thickness of the solar cell is small. If the doping drops below a certain level the SCR becomes thicker than the
overall thickness of the solar cell. In Figure 26, the SCR width shrinks when the acceptor concentration is increased. Similarly in Figure 27, these same trends are seen as a function of the donor atom concentration. The results presented in Figures 26 and 27 are helpful in understanding the effects of doping on the SCR region thickness.

In Figures 28 and 29, the effect of doping concentration on the junction voltage of the p-n junction is presented. Both of these figures show the junction voltage increasing with increasing doping concentration. For a better performing solar cell, a higher junction voltage is desirable. While a higher doping concentration produces a higher junction voltage, it also produces a thinner SCR region. Therefore, a compromise is required to choose the doping level for optimum results. For the results presented in the following sections of this thesis, the acceptor concentration used is \(10^{17}\) cm\(^{-3}\) and the donor concentration is \(10^{16}\) cm\(^{-3}\). This was viewed as a reasonable compromise between the SRC width and the junction voltage. A compromise between the width of the SCR and the junction voltage is required when selecting doping concentrations on the p and n-sides of a solar cell.

In Figures 30 and 31 the effects of the donor doping concentration and acceptor doping concentration on the carrier densities is presented. As in the results already presented in this chapter, only one of the doping concentrations is varied while the other is held constant. Figure 30 shows the effects of varying the acceptor doping concentration while holding the donor concentration constant at \(10^{16}\) cm\(^{-3}\); while Figure 31 shows the effect of varying the acceptor doping concentration while holding the acceptor concentration constant at \(10^{16}\) cm\(^{-3}\). It is easy to see that increasing the acceptor concentration raises all the carrier concentrations in the p-QNR and in the SCR, but has little effect on carrier concentrations in the n-QNR. When the donor concentration is increased all the carrier concentrations in the n-QNR and the SCR increase, while the carrier concentration in the p-QNR stay about the same. These results are similar to those presented in Figures 24 and 25, but the carrier densities of the minority carriers are greatly increased. This is caused by the 1 volt externally applied bias to the p-n junction.
Figure 26: Effect of acceptor concentration on SCR width in the dark with an external bias of 1 V. The donor doping concentration for these results is $N_d = 10^{16} \text{ cm}^{-3}$.

Figure 27: Effect of donor concentration on SCR width in the dark with an external bias of 1 V. The acceptor doping concentration for these results is $N_a = 10^{17} \text{ cm}^{-3}$. 
Figure 28: Effect of acceptor concentration on junction voltage in the dark with an external bias of 1 V. The donor doping concentration for these results is $N_d = 10^{16}$ cm$^{-3}$.

Figure 29: Effect of donor concentration on junction voltage in the dark with an external bias of 1 V. The acceptor doping concentration for these results is $N_d = 10^{17}$ cm$^{-3}$. 
Figure 30: Effect of acceptor concentration on carrier density profiles in the dark with an external bias of 1 V. The donor doping concentration for these results is $N_d = 10^{16} \text{ cm}^{-3}$.

Figure 31: Effect of donor concentration on carrier density profiles in the dark with an external bias of 1 V. The acceptor doping concentration for these results is $N_a = 10^{17} \text{ cm}^{-3}$.

6.1.3. Carrier Densities Under Illumination with Short Circuit Conditions

An analysis was done to understand the effect of doping on the carrier concentration with the solar cell illuminated and short circuited. Shorting the solar cell allows it to deliver current,
but does not impose any retarding voltage on the cell. It should be noted that shorting a solar cell
does not cause large currents to be delivered to the external circuit, like would happen if a battery
were shorted. Batteries can be looked as a constant voltage source, while solar cells should be
looked at as constant current sources. Solar cells can only deliver a limited supply of current
based on the strength of the illumination to which the cell is exposed.

Figure 32 presents the minority and majority carrier densities with changes in \( N_a \), while
keeping \( N_d \) constant at \( 10^{16} \text{ cm}^{-3} \), and Figure 33 presents the minority and majority carrier
densities with changes in \( N_d \), while keeping \( N_a \) constant at \( 10^{17} \text{ cm}^{-3} \). These figures only show
carrier densities in the QNRs and not in the SCR. The analysis used in this thesis work is
incapable of calculating number densities in the SCR under illuminated conditions. A numerical
analysis would have to be done to find these values and this is outside the scope of this thesis.

The interesting result here is the small effect that doping has on the carrier density
concentrations. The results in Figures 32 and 33 should be compared to the results for dark
conditions in equilibrium shown in Figures 24 and 25 and to the results for dark conditions with
a 1 volt external bias applied shown in Figures 30 and 31. For the dark conditions, doping had a
large effect on all carrier densities in the QNR on the side to which doping was applied. Doping
also had a large effect on carrier densities in the SCR. Figure 32 and 33 shows that doping has a
much smaller relative effect on the minority carriers for the case with illumination where the
solar cell is short circuited. The effect on the majority carrier densities on the side which is doped
is the same for the illuminated cases as the dark cases.

While there is not a large effect on the minority carrier densities on the doped sided of the
junction, there is a noticeable effect on the size of the space charge region. Lower doping
produces a wider space charge region and thus better transport of minority carriers out of the
solar cell.
6.1.4. Carrier Densities Under Illumination with a 1 Volt Externally Applied Voltage

Results are presented to understand the effect of doping on the carrier concentrations when the solar cell is subjected to an external voltage of 1 V and illumination. Figure 34 presents
Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations

the minority and majority carrier densities with changes in \( N_a \), while keeping \( N_d \) constant at \( 10^{16} \) cm\(^{-3} \) and Figure 35 presents the minority and majority carrier densities with changes in \( N_d \) while keeping \( N_a \) constant at \( 10^{17} \) cm\(^{-3} \).

As with the results shown in the prior section, the incident radiation increases the minority electron and minority hole densities in the QNRs greatly over the dark conditions with no voltage applied. This can be seen by comparing the results in Figure 24 with Figure 34 and the results in Figure 25 with Figure 35. These density improvements are not present when comparing to the dark case with an externally applied 1 volt potential (compare Figure 30 results with Figure 34 results and Figure 31 results with Figure 35 results). The number densities are similar in magnitude and shape. This may be due to the rather large external voltage applied in these cases. For doped silicon solar cells, 1 volt is high. The open circuit voltage of a GaAS solar cell cannot exceed 1.424 volts and 1 volt is close to this limit.

For the illuminated case under short circuit conditions (see Figures 34 and 35) the change of doping concentration did not impact the minority electron concentrations on the doped side to a large degree; but there was a decline in the carrier profiles near the edges of the SCR. In addition, the depletion width decreased noticeably with increase in the doping concentration. For the illuminated case with a 1 V external bias, the minority carrier density on the doping side increase with increasing doping concentration. In addition, the carrier profiles increase near the SCR. This is different than the illuminated case that is short circuited.

Figure 34: Effect of acceptor concentration on carrier density profiles under illumination with an external bias of 1 V. The donor doping concentration for these results is \( N_d = 10^{16} \) cm\(^{-3} \).
Figure 35: Effect of donor concentration on carrier density profiles under illumination with an external bias of 1 V. The acceptor doping concentration for these results is $N_a = 10^{17}$ cm$^{-3}$.

6.2. Effect of External Bias

The forward bias reduces the junction voltage of the p-n junction solar cell. The forward bias across the SCR region reduces to $V_j = V_{bi} - V$. If the external bias is increased across the junction, then the junction voltage decreases as well. The reduction of junction voltage causes the depletion width to decrease proportional to $\sqrt{V_{bi} - V}$. The effect of external bias on the carrier densities for a GaAs solar cell in dark conditions and under illuminated conditions is presented in Figure 36 and Figure 37.

In Figure 36, the parameters are used as given in the Appendix. For this analysis, the external bias is changed from 0 V to 1.2 V. The electron and hole carrier densities of the solar cell increase with increase in the external forward bias. As can be seen in this figure the effect of external bias is large. A voltage change of 0.25 V increases the electron and hole densities by a factor of $10^4$. When the external voltages are increased to the built-in bias voltage, the minority electron density in p-QNR are the same as the majority electron densities in the n-QNR; and the minority hole densities in n-QNR are the same as majority hole densities in p-QNR.
Figure 36: Carrier densities of a GaAs solar cell in dark conditions at different external bias conditions. The electron carrier densities are shown as solid lines and the hole carrier densities are shown as dashed lines.

Figure 37: Minority carrier densities of a GaAs solar cell under illuminated conditions for different external bias voltages.

The parameters used to produce the results shown in Figure 37 are as mentioned in the Appendix. Like the other results shown in this section, the external bias is changed from 0 V to
1.2 V to calculate the carrier densities. The electron and hole density of the solar cell was increased with an increase in the external forward bias. When the voltage was changed from 0 V to 0.75 V, no increase in the electron and hole density concentrations are observed; although there is some increase near the SCR edge for the minority hole densities. When the external voltage is increased to 1 V, the minority electron density in p-QNR and the minority hole density in n-QNR increase by a factor of 10. A further increase of 0.25 V increases the carrier concentration by a factor of $10^3$. When the external voltage is the same as the built-in voltage, the majority and minority carriers are the same in the n and p quasi-neutral regions.

### 6.3. Effect of Cell Length

The length of the p-QNR and the n-QNR are two other important parameters to consider while designing a high efficiency solar cell. The thickness of the cell should be greater than the absorption length to aid efficient light absorption. The junction should be thinner compared to the thickness of the $p$ and $n$ QNR to avoid a ‘dead layer’ at the front of the cell [7]. The effect of thickness of the quasi-neutral regions is studied in this section.

#### 6.3.1. Carrier Densities for Equilibrium Conditions in the Dark

At equilibrium conditions in the dark, the electron and hole minority carrier densities are low in a p-n junction GaAs solar cell as discussed in previous sections. The optimum thickness for a GaAs solar cell in this research work is $x_p = 1 \, \mu m$ and $x_n = 2 \, \mu m$. The thickness of $p$ and $n$ region are varied to analyze the effect of cell thickness on the carrier densities.

Figures 38 and 39 present the electron and hole densities for a GaAs solar cell with parameters mentioned in the Appendix where the n-region thickness is varied between 1 $\mu$m and 200 $\mu$m. There is no change in the electron and hole densities for equilibrium condition in the dark when the n-region thickness is varied.
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Figure 38: Electron densities for a GaAs solar cell for equilibrium conditions in the dark where $X_p = 1\mu m$ and $X_n = 1\mu m, 2\mu m, 3\mu m, 20\mu m,$ and $200\mu m$.

Figure 39: Hole densities for a GaAs solar cell for equilibrium conditions in the dark where $X_p = 1\mu m$ and $X_n = 1\mu m, 2\mu m, 3\mu m, 20\mu m,$ and $200\mu m$.

Figures 40 and 41 present the electron and hole densities for a GaAs solar cell with parameters mentioned in the Appendix where the p-region thickness is varied between 1 $\mu m$ and 100 $\mu m$. There is no change in the electron and hole densities for equilibrium conditions in the dark when the p-region thickness is varied.
Figure 40: Electron density for a GaAs solar cell for equilibrium conditions in the dark where $X_n = 2\mu m$ and $X_p = 1\mu m, 2\mu m, 3\mu m, 10\mu m, 100\mu m$.

Figure 41: Hole density for a GaAs solar cell for equilibrium conditions in the dark where $X_n = 2\mu m$ and $X_p = 1\mu m, 2\mu m, 3\mu m, 10\mu m, 100\mu m$. 
6.3.2. **Carrier Densities in the Dark with a 1 Volt Externally Applied Voltage**

At external applied bias of 1 V in the dark conditions, the electron and hole minority carrier density is greater than minority carrier density in dark in a p-n junction GaAs solar. The optimum thickness for a GaAs solar cell in this research work is $X_p = 1 \mu m$ and $X_n = 2 \mu m$. The thickness of $p$ and $n$ region is varied to analyze the effect of cell thickness on the carrier densities.

Figure 42 and 43 present the electron and hole density for a GaAs solar cell with parameters mentioned in the Appendix and the $n$-region thickness is varied between 1 $\mu m$ and 100 $\mu m$. The hole density at the edge of $n$-region is greater for $X_n=1\mu m$ and the hole density decreases with increase in the width of $n$-region. However at the SCR region, the electron density is similar for all the thickness mentioned in Figure 44. A semi-infinite absorber plate has lower electron density at the edge of $p$-region. The change in $X_n$ does not affect the minority electron density in the $p$-region of the semiconductor. A thinner $n$-region aids in increasing the minority hole density in the $n$-QNR. At external bias of 1 V, the minority carrier densities increase by a factor of $10^{16}$ which can be observed from Figures 38, 39, 42 and 43. The effect of increasing the $n$-region width has minimal effect on the minority carrier densities in the solar cell under equilibrium conditions at external bias of 1 V.
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Figure 42: Electron density for a GaAs solar cell under dark conditions at external bias of 1V where $X_p = 1\mu m$ and $X_n = 1\mu m$, 2$\mu m$, 3$\mu m$, 20$\mu m$, 200$\mu m$.

Figure 43: Hole density for a GaAs solar cell under dark conditions at external bias of 1V where $X_p = 1\mu m$ and $X_n = 1\mu m$, 2$\mu m$, 3$\mu m$, 20$\mu m$, 200$\mu m$.

Figure 44 and 45 present the electron and hole density for a GaAs solar cell with parameters mentioned in the Appendix where the p-region thickness is varied between 1 $\mu m$ and 100 $\mu m$. The electron density at the edge of p-region is greater for $X_p = 1\mu m$ and the electron density decreases with increase in the width of p-region. However at the SCR region, the hole density is similar for all the thickness as mentioned in Figure 44. A semi-infinite absorber plate
has lower electron density at the edge of p-region. The change in $X_p$ does not affect the minority hole density in the n-region of the semiconductor. A thinner p-region aids in increasing the minority electron density in the p-QNR. At external bias of 1 V, the minority carrier densities increase by a factor of $10^{16}$ which can be observed from Figures 40, 41, 44 and 45. The effect of increasing the p-region width has minimal effect on the minority carrier densities in the solar cell under equilibrium conditions at external bias of 1 V.

**Figure 44:** Electron density for a GaAs solar cell under dark conditions at external bias of 1 V where $X_n = 2 \mu m$ and $X_p = 1 \mu m, 2 \mu m, 3 \mu m, 10 \mu m, 100 \mu m$.

**Figure 45:** Hole density for a GaAs solar cell under dark conditions at external bias of 1 V where $X_n = 2 \mu m$ and $X_p = 1 \mu m, 2 \mu m, 3 \mu m, 10 \mu m, 100 \mu m$. 

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6.3.3. Carrier Densities Under Illumination with Short Circuit Conditions

At short circuit voltage under the illuminated conditions, the radiation aids the electron and hole minority carrier density in a p-n junction GaAs solar as discussed in previous sections. The optimum thickness for a GaAs solar cell in this research work is $X_p = 1 \mu m$ and $X_n = 2 \mu m$. The thickness of $p$ and $n$ region is varied to analyze the effect of cell thickness on the carrier densities.

Figure 46 and Figure 47 present the electron and hole density for a GaAs solar cell with parameters mentioned in the Appendix where the n-region thickness is varied between 1 $\mu m$ and 100 $\mu m$. There is no change in the minority electron and hole density at short circuit voltage when the n-region thickness is varied. The increase in the length on the n-region has no impact on the carrier densities in the solar cell.

![Electron Density Graph]

*Figure 46: Electron density for a GaAs solar cell under illuminated conditions at short circuit voltage where $X_p = 1 \mu m$ and $X_n = 1 \mu m, 2 \mu m, 3 \mu m, 10 \mu m, 100 \mu m$.***
Figure 47: Hole density for a GaAs solar cell under illuminated conditions at short circuit voltage where $X_p = 1\mu m$ and $X_n = 1\mu m, 2\mu m, 3\mu m, 10\mu m, 100\mu m$.

Figures 48 and 49 present the electron and hole density for a GaAs solar cell with parameters mentioned in the Appendix where the p-region thickness is varied between 1 $\mu$m and 100 $\mu$m. The electron density decreases in the p-region when the thickness of the p-region is increased. For a large p-region width of 100 $\mu$m, the electron density in the p-region is extremely low. It can be seen from Figure 38 and Figure 48 that the electron density is extremely low for a GaAs solar cell in dark at equilibrium condition and for a large p-region under illumination at short circuit voltage.

In Figure 49, the minority hole density for a GaAs solar cell decreases when the thickness of p-region is increased. For a larger p-region on 100 $\mu$m, the minority hole density in the n-region is extremely low. It can be seen from Figure 39 and Figure 49 that the electron density is extremely low for a GaAs solar cell in dark at equilibrium condition and for a large p-region under illumination at short circuit voltage.

The effect of cell on the carrier densities is significant for a solar cell under illumination. Thus, it is required to model the cell length properly to increase the carrier collection in the QNR of a solar cell. The larger length on the cell decreases the carrier concentration, therefore, an optimization is required to maintain the cell length to be smaller but greater than the junction length to increase the light absorption in a solar cell.
Figure 48: Electron density for a GaAs solar cell under illuminated conditions at short circuit voltage where $X_n = 2\mu$m and $X_p = 1\mu$m, 2\mu$m, 3\mu$m, 10\mu$m, 100\mu$m.

Figure 49: Hole density for a GaAs solar cell under illuminated conditions at short circuit voltage where $X_n = 2\mu$m and $X_p = 1\mu$m, 2\mu$m, 3\mu$m, 10\mu$m, 100\mu$m.

6.3.4. Carrier Densities Under Illumination with a 1 Volt Externally Applied Voltage

At external applied bias of 1 V in illuminated conditions, the electron and hole minority carrier densities are greater than minority carrier density under illumination in a p-n junction
GaAs solar as discussed in previous sections. The optimum thickness for a GaAs solar cell in this research work is \( X_p = 1 \mu m \) and \( X_n = 2 \mu m \). The thickness of \( p \) and \( n \) region is varied to analyze the effect of cell thickness on the carrier densities.

The minority electron density in the \( p \)-region did not change when the \( n \)-region thickness is increased at external voltage of 1 V as shown in Figure 50. The minority electron density increases from edge of the quasi-neutral region to the edge of SCR. However, the effect of \( n \)-region thickness that was visible at short circuit voltage was not seen when external voltage of 1 V is applied across the \( p \)-\( n \) junction. A similar result was seen for the minority hole density in the \( n \)-region and the effect of increasing the \( n \)-region thickness did not change the minority hole density as shown in Figure 51. The minority hole density increases from edge of the quasi-neutral region to the edge of the SCR.

![Figure 50](image-url): Electron density for a GaAs solar cell under illuminated conditions at external bias of 1 V where \( X_p = 1 \mu m \) and \( X_n = 1 \mu m, 2 \mu m, 3 \mu m, 10 \mu m, 100 \mu m \).
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**Figure 51:** Hole density for a GaAs solar cell under illuminated conditions at external bias of 1 V where $X_p = 1\mu m$ and $X_n = 1\mu m, 2\mu m, 3\mu m, 10\mu m, 100\mu m$.

The minority electron density in the p-region did not change when the p-region thickness is increased at external voltage of 1 V as shown in Figure 52. The minority electron density increases from edge of the quasi-neutral region to the edge of SCR. However, the effect of p-region thickness that was visible at short circuit voltage was not seen when external voltage of 1 V is applied across the p-n junction. A similar result was seen for the minority hole density in the n-region and the effect of increasing the p-region thickness did not change the minority hole density as shown in Figure 53. The minority hole density increases from edge of the quasi-neutral region to the edge of the SCR.

**Figure 52:** Electron density for a GaAs solar cell under illuminated conditions at external bias of 1 V where $X_n = 1\mu m$ and $X_p = 1\mu m, 2\mu m, 3\mu m, 10\mu m, 100\mu m$. 
The mathematical model is helpful to understand the behavior of carrier densities across the p-n junction at different doping profile and external bias. This model can be used for a GaAs solar cell modeling to estimate the performance of the solar cell at different operating conditions.

The cell doping is an important criterion in fabrication of a high efficiency solar cell and in a crystalline solar the emitter is generally doped heavily than the base layer [7]. In this research, the p-region in the top layer when the radiation is incident and n-region is the base layer, therefore, the acceptor doping concentration is more than the donor doping concentration. This doping profile is followed so that the majority of SCR can lie in the base of the solar cell.

The cell width is another important criterion in fabrication of a high efficiency solar cell. The emitter layer or p-region in this work is made thinner than the base layer or n-region. From Figure 48 and 49, it could be concluded that increasing the width of p-region decreases the minority electron and hole density in QNR under illuminated conditions. However, increasing the width of p-region did not decrease the minority carrier density in the QNR. Therefore, the p-region should be made thinner than the n-region and the survey results provide conclusive results to model a solar cell with thinner emitter layer and thicker base layer.
6.4. Effect of Photon Flux

The amount of radiation absorbed by the solar cell has a significant impact on the minority electron and hole density. In this section, the impact of photon flux impinging on the solar cells is studied to analyze its impact on the carrier densities. The increase in photon flux density increases the electron and hole density in the quasi-neutral regions of a p-n junction GaAs solar cell used in this thesis work. Figures 54 and 55 present the variation of minority electron density in the p-QNR and minority hole density in the n-QNR.

The photon flux density of the radiation increases the carrier concentration in the QNR. The mathematical model for this research work is used to identify the effect of photon flux on the carrier concentration. A higher photon flux density is desired to have a highly efficient solar cell.

![Graph showing minority electron density in p-QNR at different photon flux densities](image)

**Figure 54:** Minority Electron Density in p-QNR at different photon flux density in a GaAs solar cell at $S_n = 0$. 
6.5. Effect of Surface Recombination Velocity

In this section, the effect of electron surface recombination velocity ($S_n$) and hole surface recombination velocity ($S_p$) on the carrier concentration in the quasi-neutral region is explored. Surface recombination velocity is studied for solar cell modeling to understand the recombination at the surface of the semiconductor material. The impurities of semiconductor material aid surface recombination. $S_n$ and $S_p$ are dependent on the movement of minority electrons and holes towards the surface. $S_n$ and $S_p$ is zero for a semiconductor when there is no recombination, however, $S_n$ and $S_p$ can attain a maximum velocity for an infinitely high surface recombination. For most of the semiconductor material, $S_n$ and $S_p$ is limited to $10^7$ cm/sec [32].

The minority carrier concentration on the surface of the semiconductor material increases when the surface recombination velocity is increased. The higher $S_n$ and $S_p$ induces more minority charge carriers towards the surface of the material. At $S = 0$, the minority charge carrier are low in a p-n junction GaAs semiconductor. However, when the $S$ is increased, the minority charge carrier concentration increases as well, however, the increase in carrier densities is very low. At $S=10^6$ cm/s, the minority charge concentration further increases but increasing the velocity further did not have any effect on the minority charge carrier density in the quasi-neutral regions. It can be assessed from Figures 56 and 57 that the maximum surface recombination
velocity achieved in a GaAs semiconductor in $10^8$ cm/sec. For this velocity, the minority carrier concentration is slightly more than the surface velocity of 0 cm/sec. The effect of surface recombination can be seen at the edges of the p and n region as high surface recombination velocity depletes the QNR for charges. The effect of surface recombination should be minimized to accommodate enough charge carriers in the QNR.

**Figure 56:** Electron density in p-quasi neutral region for a GaAs semiconductor material at Surface Recombination Velocity $S=0; 10^4; 10^6; 10^8; 10^{10}$ cm/s.

**Figure 57:** Hole Density in n-quasi neutral region for a GaAs semiconductor material at Surface Recombination Velocity $S=0; 10^4; 10^6; 10^8; 10^{10}$ cm/s.

The high surface recombination velocity is not desirable in a solar cell as it depletes the region of the minority charge carriers. The higher surface recombination velocity pulls more minority charge carriers from the region of higher concentration which can cause the quasi-
neutral regions to be depleted of charge carrier. Therefore, a surface recombination velocity needs to be low for modeling a high efficiency solar cell.
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Chapter 7 - Conclusions

The primary objective of this thesis work was to determine if there were errors in four rather lengthy equations presented in the book *The Physics of Solar Cells* authored by Jenny Nelson. These four equations are: 1) the equation for the minority carrier concentration in the p-QNR, 2) the equation for the minority carrier density in the n-QNR, 3) the equation for the minority current density in the p-QNR, and 4) the equation for the minority current density in the n-QNR. The book *The Physics of Solar Cells* is an excellent book on solar cells and has been used in the Photovoltaics class offered at Wright State University since the inception of the course; however, there was suspicion that these four equations may not be correct and it was time that someone investigated these equations. This thesis work has done this.

The first step taken in the process of checking these four equations was to do a detailed literature search to see if other authors presented these equations. The idea was to find equations for the same conditions as the four equations presented in *The Physics of Solar Cells* and compare them. If the equations compared, then the ones in *The Physics of Solar Cells* are correct and no further work needs to be done. If the equations differ, then a third source would be needed to verify which ones were correct. Of the many sources that were searched as part of this work, none presented equations that matched the conditions of those presented in *The Physics of Solar Cells*. Many sources presented versions of these four equations, but most used the equilibrium number density as the non-SCR boundary condition. This leads to much simpler equations than those presented by Nelson in her book *The Physics of Solar Cells*. The simplicity of this boundary condition is probably the reason that most textbooks use an equilibrium number density boundary condition. The equations presented in *The Physics of Solar Cells* use a recombination velocity boundary condition. This type of boundary condition leads to much more
complex equations as shown in this thesis. There was only one source that presented a version of the minority carrier density equations with recombination velocity boundary conditions. The problem with the equations presented in this source is that they applied their illumination at a much different location than that done by Nelson. Because a good comparison between these equations and those of Nelson could not be made, it was decided to derive all four of these equations from the standard transport equations.

As stated above, the second step performed in this research work was to do a detailed derivation of all four of these equations. The detailed derivation of each of these equations is presented in Chapter 4 of this thesis. These derivations revealed that each of these equations had errors in them. There were three errors found in the equation for the minority carrier concentration in the p-QNR, five errors found in the equation for the minority carrier density in the n-QNR, one error found in the equation for the minority current density in the p-QNR, and one error found in the equation for the minority current density in the n-QNR. Most of these errors were simply the wrong sign on a term, but such errors cause completely wrong results to be obtained.

A third step performed as part of this research work was to validate the four equations derived in this thesis. To do this a MS EXCEL program was written that produced quantitative results of the four equations derived in this thesis. Quantitative results from the carrier number density equations were compared to results produced from the widely used program called PC1D and to some limiting case results presented in another book on solar cells. The current density results from the equations derived in this thesis were compared to some limiting case results presented in another book on solar cells. All of these comparisons were excellent and lead to the conclusion that the four equations derived in this thesis are the correct versions of 1) the equation for the minority carrier concentration in the p-QNR, 2) the equation for the minority carrier density in the n-QNR, 3) the equation for the minority current density in the p-QNR, and 4) the equation for the minority current density in the n-QNR.

The fourth step performed in this research work was to use these derived equations to produce a number of survey results. The MS EXCEL program developed as part of this thesis work was an essential tool in carrying out these surveys. The effect of doping concentration, the effect of applied bias voltage, the effect of the cell thickness, the effect of the illumination strength, and the effect of the recombination speed were studied for four different conditions.
These conditions were a solar cell in the dark in equilibrium, a solar cell in the dark with a 1 volt external bias, an illuminated solar cell under short circuit conditions, and an illuminated solar cell with a 1 volt external bias. A large number of results were produced and are displayed in this thesis.
References


Mathematical Modeling of a P-N Junction Solar Cell Using the Transport Equations


### Appendix

Parameters used in Microsoft Excel model for GaAs semiconductor to calculate carrier and current densities.

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