Modeling of Soil Formation on The Basis of Chemical Weathering: Applications From Percolation Theory

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MODELING OF SOIL FORMATION ON THE BASIS OF CHEMICAL WEATHERING: APPLICATIONS FROM PERCOLATION THEORY

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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ABSTRACT


The concept of soil formation has been studied since the beginning of 19th century. However, until recently, there has been little concrete progress on developing an analytical result to relate soil depth or quality to measurable variables that represent the five soil-forming factors including time, parent material, topography, climate, and organisms. It has become increasingly clear that soil formation rates are closely related to chemical weathering rates. In this dissertation, we propose a theoretical approach to model soil formation process within the theoretical framework of percolation theory, which has been shown to successfully predict solute transport in heterogeneous media. From percolation theory, solute transport rate does not equal to flow rate beyond the length scale of a typical pore size, as is the case in Gaussian solute transport. Rather, it diminishes in accord with heavy-tailed solute arrival time distributions as it travels. The basis of our model relies on the hypothesis that the chemical weathering of bedrock is simultaneously the limiting factor for soil formation and most strongly limited by solute transport in porous media.
To test the hypothesis, we propose a revised method to calculate Damköhler number within the same theoretical framework to evaluate the relevant importance of solute transport in limiting chemical weathering, and results imply that chemical weathering is nearly always solute transport-limited in natural media. We then examine the proposed models by comparing predictions with field data across a wide range of climatic conditions, as well as at steep topography. Results show good agreement between predictions and field observations. We also present two applications of the proposed model to geomorphology to examine the local steady-state assumption of soil and to distinguish steady and stochastic erosion process in threshold landscapes. The applications demonstrate the potential to adopt our model into geomorphological models such as landscape evolution models, and landsliding models to predict shallow landslides.
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I. GENERAL INTRODUCTION

Soil is a vital part of the natural environment. It is essential for life on the earth, in that it provides a medium and habitat for plants, animals, and other organisms to grow. Soil also plays important roles in a variety of natural processes on the earth including the water cycle, carbon source and storage, and nitrogen cycle etc. As a central component of the critical zone, soil has been intensely studied since the beginning of the 20th century (Jenny, 1941). Understanding biogeochemical processes in soil as well as its physical evolution is essential to many other academic subjects within the scope of ecosystem studies, such as the cycling of carbon, nitrogen and other nutrients, improvement of agriculture, climate change, and landscape evolution. Despite such broad interest, there is no universal definition of "soil" across disciplines or even within soil science (Jenny, 1941). For time periods less than decades it becomes increasingly difficult to develop criteria to define what is and what is not soil (e.g., Stevens, 1968). With the debatable definition of soil, which seems not so urgent as to require universal agreement (Jenny, 1941), attempts to quantitatively define soil formation as a continuous process based on different theories have met with variable success.

I.1 Soil Formation Process

The concepts of soil formation have been examined extensively since the beginning of the 19th century. Various theories that are fundamentally different have been proposed
(Cline, 1977; Richter and Markewitz, 2001; Richter and Yallon, 2011). However, they mostly converge in their classification of the five natural soil-forming factors proposed by Dokuchaev (1883): climate, topography, parent materials, organisms and time, which are independent variables that can completely define a soil system (Jenny, 1941). When soil is defined in terms of those five soil-forming factors as "those portions of the solid crust of the earth, the properties of which vary with soil-forming factors" as Jenny describes (1941), it is possible to describe soil depth (thus, also soil production) as a function of the soil-forming factors. However, until recently, there has been little concrete progress on developing an analytical result to relate soil depth or quality to measurable variables that represent such factors. A brief description of the five soil-forming factors is given below.

In the following, the term pedology refers to the study of soil development and its associated vertical structure. With increasing maturity, different layers, in particular, the A, B, and C horizons, become distinct. The upper portion, called the A horizon, is typically dark in color due to a high organic content, while the B horizon is reddish or orange, due to metal oxides (particularly iron). A horizon is sometimes called the topsoil while B horizon is called the subsoil.

**Soil forming factors**

**Time:** Here, "time" or "age of soil" is usually not the absolute age or the "geological age of the land" (Jenny, 1941), as soil is a dynamic system. There are various concepts and
approaches to define the relative time such as "surface age", "soil residence time", and "surface exposure age" depending on the study objectives. Generally speaking, the older a soil, the more mature (the degree to which a soil possesses a well-developed stratification, or profile) and thicker it is (Jenny, 1941). Common soil production models mainly focus on how soil evolves over time.

*Parent material:* Most pedologists define the parent materials as the C horizon, or in a more accurate manner, "the initial state of the soil system" as Jenny (1941) states. Debate still exits as to whether only the weathered portions of the materials for soil-forming purpose account as parent material. If this is the case, then the weathering process of bedrock should be excluded in soil formation. In a pedological sense, only soil-forming process should be considered, so that soil formation is more affected by parent material in the sense of soil characteristics such as chemical contents; while to a geologist who includes the weathering process into soil formation, both soil formation rates and soil texture are affected by parent material.

*Topography:* Topography generally affects soil formation in two aspects: surface transport of soil, and hydraulic flux. Sediment transport is strongly relief-dependent (Montgomery and Brandon, 2002), and one should expect small erosion rates, thus thicker soils, in regions of low relief. Water is one of the prime factors transforming the parent materials to soil through weathering. The downward movement of water delivers reagents for chemical weathering, and can be affected by topography. Generally speaking,
convergent topography tends to retain more water, thus leading to a faster formation process than soil in a divergent area.

*Climate:* Climate mainly affects soil formation in the sense of moisture conditions. Factors that relate to moisture conditions include precipitation, evaporation, transpiration, and temperature (Jenny, 1941). The first 3 parameters relate to water penetration and moisture content of soil, which further affect the rate of soil formation and chemical composition of soil. Temperature generally affects soil formation rate and soil properties such as chemical contents and soil profiles by affecting chemical weathering rates and biological activities, while it also relates to water supply through the processes of evaporation and transpiration.

*Organisms:* Despite the disagreement among soil scientists as to which organisms to include within the framework of soil-forming factors, since some, like Jenny (1941) consider vegetation as a dependent variable, it is generally accepted that organisms play an important role in soil formation. Soil properties including chemical content and composition, and soil profile differentiation affected by organisms have been well-documented (Jenny, 1941) and proposed much earlier (earthworms) by Darwin (1881). In the sense of physical evolution of soil, soil depth can also be affected by biological processes, such as plants through treethrow, and animals through burrowing (Anderson and Anderson, 2010).
To summarize, the 5 soil-forming factors affect the development of soil in both its thickness and its maturity, which is characterized by soil profile, as well as the physical and chemical properties of soil. For the purpose of studying the soil production function, this research only focuses on the physical evolution of soil, i.e. the evolution of soil depth as a function of time, and two major processes are included: production of soil from the weathering of bedrock, and removal of soil from erosion. Thickness of soil depends on the coupling effect of these two processes, as weathering of bedrock increases soil depth from a slow process of disintegrating rocks to produce unconsolidated material, and erosion reduces soil depth by transporting the loosened materials away from the soil mantle. Description of soil formation function (production of soil minors erosion) requires approaches to describe these two processes quantitatively.

1.2 Soil Production Function

Function of soil production over time has been an interest of geomorphologists, geologists, and pedogenesists. It has great significance in understanding landscape evolution as well as climate change, given its key role in the global carbon cycle. As a major source and sink of carbon, soil is one of the key components in mediating the global carbon cycle. The total carbon found in terrestrial ecosystems is approximately 3170 gigatons (GT), with 80% (2500GT) of carbon stored in soil (Lal, 2008), and the soil carbon pool is about 3.1 times larger than carbon found in the atmosphere (800GT, Oelker and Cole, 2008). Soil removes CO₂ from the atmosphere and stores carbon into
the soil carbon pool through the process of soil carbon sequestration. While this process is primarily mediated by photosynthesis, it can also occur from the conversion of CO₂ into inorganic forms of carbon, through Urey reaction: \( CO_2 + CaSiO_3 \leftrightarrow CaCO_3 + SiO_2 \) (Urey, 1952), but at a comparatively low rate (Lal, 2008), i.e. through the process of chemical weathering of bedrock, which has an overall negative effect on global warming. There are relevant works demonstrating relations between carbon sequestration and soil production (Egli et al., 2012). Thus, understanding and modeling the global carbon cycle requires better understanding of soil carbon sequestration processes including converting CO₂ into inorganic carbon through soil formation, and if the predictions of carbon sequestration rates through abiotic and biotic processes can be made accurately, the reliability of future models of carbon cycling and climate change can be improved. Lal (2015) also demonstrates relations between carbon sequestration and agriculture productivity, and understanding the relevance of soil formation and carbon sequestration thus has the potential to improve soil health and agriculture management.

1.2.1 Overview of Common Soil Production Models

The origin of describing soil production as a function of soil depth first came from the idea of Gilbert (1877), who recognized the coupling of soil production, soil depth, topography and surface transport of soil (Humphreys and Wilkinson, 2007). Over the past 100 years, the disintegration of bedrock into soil mantle has been assumed to decline as soil thickness increases (Gilbert, 1877; Culling, 1965; Ahnert, 1967; Tucker and
Slingerland, 1996; Anderson and Humphery, 1989; Rosenbloom, 1994; Dietrich et al., 1995; Carson and Kirklby, 1972; Ahnert, 1987). Both Culling (1965) and Jahn (1968) found an inverse relationship between soil production and soil thickness. Cox (1980) described a hypothesis that soil production is greatest when bedrock is just exposed, and decreases exponentially with increasing depth of soil. Ahnert (1967) proposed a more complex, bell-shaped function of soil production, suggesting that rates of chemical weathering as well as soil production are greatest under a finite soil cover. In 1972, Carson and Kirkby (1972) first diagrammatically expressed the idea of Gilbert (1877) of soil production as a "humped" function with a maximum soil production rate under some non-zero soil depth, beyond which the rate of soil production declines. In 1995, Dietrich (1995) fitted the field data using an exponential decay, which was later confirmed and supported by Heimsath et al. (Heimsath et al., 1997, 2000, 2001a, 2001b). Recently, Hunt (2015a) proposed a simple power scaling for soil production based on a solute transport theory derived from percolation theory. In the main, most existing models imply that soil depths tend to increase over time, but not in a linear fashion (Huggett, 1998).

Soil production rate varies for several reasons, which all trace back to the five soil-forming factors. In the sense of physical evolution of soil, soil production rate varies depending on the parent material where soil is developing. Composition of the parent material determines how difficult it is to be altered through chemical weathering, and the property of the parent material, for example, whether it is unconsolidated material or consolidated bedrock and how fractured it is, affects the rate of water penetrating into the
media. Topography and surface curvature (Heimsath et al., 2001b) of the landform also determines the ability of soil to retain water, and climate affects the moisture content as well as the temperature of soil.

I.2.2 Well-known Soil Production Functions

While different models have been proposed, the most well-known models are the "humped" soil production function (Gilbert, 1877), and the exponential decay function (Dietrich et al. 1995, Heimsath et al., 1997) with soil depth.

The "humped" soil production function can be traced back to 1877, when Gilbert (1877) first proposed that weathering of bedrock relates to the thickness of soil mantle. It is greatest under an intermediate soil thickness, and slower under both zero and thicker soil mantles. According to Gilbert (1877), exposed bedrock or thin soil layers are less capable of holding water that is required for the chemical weathering of bedrocks. Such a "humped" soil production model was later quantitatively laid out by Culling (1960, 1965). In 1972, the "humped" soil production function was first diagrammatically expressed by Carson and Kirkby (1972), and it has been employed to fit field data and supported by several studies since then (Wilkinson and Humphreys, 2005; Heimsath et al., 2009). Later, an exponential decay of soil production rate with soil thickness is supported by field observations from Dietrich et al. (1995), and has been later confirmed, and extensively applied to quantifying soil production (Heimsath et al., 1997, 2000, 2001a, 2001b, 2010). As Dietrich et al. (1995) point out, the "humped" soil production function
will lead to a large proportion of bare rock, due to its instability at depths below that associated with the maximum production rate, and an exponential decay starting with the most rapid soil production at zero soil depth will result in fewer bedrock outcrops, generating better agreement with field observations. A better defined exponential decay function was later proposed by Heimath et al. (1997) through the approach of quantifying soil production rate from the concentration of \textit{in situ} produced cosmogenic nuclides. From Heimsath et al. (1997), an empirical soil production function (Eq. (1.1)) is proposed as

\[
\frac{\partial e}{\partial t} = P_0 e^{-kh} \tag{1.1}
\]

where \(e\) is the elevation of the bedrock-soil interface, \(t\) is time, \(P_0\) is the maximum production rate of soil (erosion of bedrock), \(h\) is the soil thickness, and \(k\) is an empirical constant (Heimsath et al., 1997). The result has then been universally applied and widely adopted in landscape evolution models. Soil production values fit to such a model are determined based on the assumption that the soil is at local steady-state, meaning that soil thickness has to be constant over time. Thus, the soil production rate (or the chemical weathering rate of bedrock) equals the erosion rate of the soil. However, the time it requires for soil to reach local equilibrium depends on the combined effect of geological, hydraulic and climatic conditions, and we will show in Chapter V that this is not always the case.
Similar techniques of quantifying soil production rate using *in situ* produced cosmogenic nuclides have been employed in several other studies since then, however, there remains support for both the "humped" soil production function (Small et al., 1999; Wilkinson et al., 2005; Heimsath et al., 2009) and exponential decay (Heimsath et al., 2000, 2001a, 2001b, 2010).

Consider the observation (Dietrich et al. 1995) that initial soil development with depths less than the depth at peak soil production are unstable. It is possible to find reasonable explanations for such instability: 1) Shallow soils are more vulnerable to disturbance from external forces like animal burrowing, 2) The ability of bedrock to retain water for chemical weathering is lower. Hydraulic flux in bedrock controls the rate of soil production, which depends on the property of the bedrock. For example, water penetrates more easily in fractured bedrock and unconsolidated media than it does into unfractured bedrock. Since the process of fracturing may be induced by vegetation growth, which is made possible only by the existence of deeper soils, modeling and predicting the behavior of shallow soils is complex.

I.2.3 Power-law Production Function Derived from Percolation Theory

Instead of an exponential decay beyond a finite soil depth, our model derived from percolation theory describes the decline using a power-law formulation. The basic equation relates soil depth to time as \( t \propto x^{D_h} \), with \( t \) as time, and \( x \) as the soil depth. \( D_h \) is a constant that only depends on the dimensionality and saturation condition of the media.
Power-law time dependence of chemical weathering rates and soil production rates has been observed in several studies (White and Brantley, 2003; Friend, 1992; Egli et al., 2014; Gunnell, 2003). White and Brantley (2003) investigate a time scale extending from weeks up to 6Myr, which is much larger, thus more conclusive than most of the studies by Heimsath and co-authors, which address only time scales from a few thousand to about 100,000 years, and are fit with the exponential decay. Similarly, soil production rates as a function of time cover time scales from months to nearly 100Myr (Friend, 1992; Egli et al., 2014; Gunnell, 2003). Moreover, the power-law decay of soil production rates on soil depth is actually observed through a great deal of examination over several sites across various regions (Hunt and Ghanbarian, 2016; Hunt, 2016). The clear correspondence of chemical weathering rates and soil production rates with time suggests some proportionality between the two. Also, a power-law formulation is consistent with the behavior of non-Gaussian transport of solute, which will be presented later in this chapter, under the hypothesis that chemical weathering and soil production are limited by solute transport. Such power-law scaling is developed within the theoretical framework of percolation theory, which is given in detail in the next section.

I.3 Theoretical Basis

The soil formation model presented in this study is derived from the previous power-law function proposed by Hunt et al. (Hunt, 2015a, Hunt and Ghanbarian, 2016) developed within the theoretical framework of percolation theory. However, the previous model of
soil production only deals with the production of soil, while neglecting the effect of erosion (Hunt and Ghanbarian, 2016). In this dissertation, a model describing the formation of soil including soil erosion is proposed. To avoid any confusion due to the existence of interchangeable concepts of "soil production" and "soil formation", in this dissertation, the term "soil production" refers to gross production of soil formed from the conversion of bedrock to loose materials, as defined by Jenny (1941), while "soil formation" refers to the net effect when removal of soil from erosion is considered.

I.3.1 Percolation Theory

Percolation theory was developed to mathematically deal with properties of random media. It was first named by Broadbent and Hammersley, who treated it as a fluid flowing into a maze (Broadbent and Hammersley, 1957). With its particular emphasis on critical phenomena developed since the 1970s (Stauffer and Aharony, 1994; Hunt et al., 2014), percolation theory has been applied in the fields of hydrology, petroleum engineering and fractal mathematics etc. (Berkowitz and Ewing, 1998). The relevance of percolation theory to this dissertation is its ability to describe the scaling of fluid flow in porous media, which will be demonstrated later in Chapter II as the controlling factor of chemical weathering of bedrock. A common example describing percolation phenomena is the fire spreading in a forest, and percolation theory addresses questions such as "whether or not the fire will penetrate through the whole forest" depending on the
percentages of trees and holes, and "how long does it take the fire to either penetrate or to be extinguished".

I.3.1.1 Main Forms of Percolation and Percolation Threshold

There are 3 main forms of percolation: bond, site, and continuum percolation. Imagine a large array of squares (Figure 1.1). Here we call the intersection of lines "sites", and the segments connecting the nearest two sites "bonds" (using the terminology from Berkowitz and Ewing, 1998). Problems dealing with the probability \( p \) of a "site" to be occupied (open) or not on the lattice are site percolation, while problems considering "bond" as the relevant entities are bond percolation. As \( p \) increases, the open sites (or bonds in bond percolation) tend to group into clusters, and at a critical probability \( p_c \) (also known as percolation threshold), when the limit of the system size approaches infinity, one giant cluster (percolation cluster) that spans (percolates) from one side to the opposite side of the lattice will form (Figure 1.1c). Unlike site and bond percolations that describe regular systems like the example above, continuum percolation deals with irregular and disordered systems such as natural porous media like soils. In a continuum percolation network, the arrangement of bond and site is irregular, and the lengths and conductivity of the bonds vary.
Figure 1.1. Illustration of percolation clusters on square lattice. (a) square lattice, (b) square lattice with 50% occupied (open) sites, i.e. $p=0.5$ (black dots), (c) square lattice with 67% occupied (open) sites. Non-isolated sites are connected by dark bonds. (Figure from Berkowitz and Ewing, 1998)

I.3.1.2 Universality of Percolation Theory

One of the most remarkable and central properties of percolation theory is known as "universality". An interesting aspect of percolation theory is that different properties of the system, such as conductivity, permeability and correlation length hold the same form of power-law scaling above but near $p_c$ (examples are listed from Eq. (1.2) to Eq. (1.5)), independent of the characteristics of the medium, and the critical exponent characterizing the power law only depends on the Euclidean dimension ($E$) of the system (Sahimi, 2011). However, the percolation threshold must be determined separately for each system. Determination of $p_c$ is difficult in complicated systems, and relates to the coordination number of the sites or bonds in the system, a value which can vary spatially in a natural medium.
\[ X^A(p) \propto (p - p_c)^\beta \quad \text{Eq. (1.2)} \]

\[ X^B(p) \propto (p - p_c)^{\beta_b} \quad \text{Eq. (1.3)} \]

\[ \xi(p) \propto |p - p_c|^{-\nu} \quad \text{Eq. (1.4)} \]

\[ g(p) \propto (p - p_c)^\mu \quad \text{Eq. (1.5)} \]

where \( X^A, X^B, \xi \) and \( g \) are the accessible fraction (the fraction of the occupied bonds or sites that are in the sample-spanning cluster), the backbone fraction (the fraction of occupied bonds or sites belong to the sample-spanning cluster that participate the fluid flow in the medium), the correlation length (the average distance between two sites that belong to the same finite cluster (Feder, 1988)), and the conductivity. Values of universal exponents \( \beta, \beta_b, \nu \) and \( \mu \) are listed in Table 1.1.

The universality of percolation theory brings the power to determine or predict the behavior of a more complicated system from results obtained in simple systems, when little details are known (Berkowitz and Ewing, 1998).

<table>
<thead>
<tr>
<th>Exponent</th>
<th>2D system ((E=2))</th>
<th>3D system ((E=3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>5/36</td>
<td>0.41</td>
</tr>
<tr>
<td>( \beta_b )</td>
<td>0.48</td>
<td>1.05</td>
</tr>
<tr>
<td>( \nu )</td>
<td>4/3</td>
<td>0.88</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.3</td>
<td>2.0</td>
</tr>
<tr>
<td>( D_f )</td>
<td>91/48</td>
<td>2.53</td>
</tr>
</tbody>
</table>
Table 1.1. Values of universal exponents and fractal dimension ($D_f$) of percolation in 2D and 3D media (Sahimi, 2011).

I.3.1.3 Correlation length, Heterogeneity, and Fractal Dimensionality

One important property of porous media is the percolation correlation length $\xi$, which characterizes the average distance between two sites that belong to the same finite cluster (Feder, 1988). Above $p_c$, the system is considered macroscopically homogenous when the length scale of observation ($L$) is larger than $\xi$, and the cluster mass $M$ scales as $M \propto L^E$, where $E$ is the Euclidean dimension of the system (Sahimi, 1993ab). In contrast, the system is heterogeneous if $L<\xi$, and $M$ scales as $M \propto L^{D_f}$, where $D_f$ is the fractal dimensionality of the cluster (universal values of $D_f$ are reported in Table 1.1). The fractal dimension can be used to characterize the structure of clusters. Instead of being self-similar, clusters in heterogeneous media are non-exact fractals that do not yield exactly the same structure over successive magnification. However, the clusters are "statistical fractals", and there is a well-defined relation between cluster size $s$ and the cluster radius $r_s$, $s \propto r_s^{D_f}$ (Berkowitz and Ewing, 1998).

I.3.1.4 Describing Flow in Porous Media Using Percolation Theory

In the case of flow in porous media, the medium can be treated as a network, with pore bodies as sites and pore throats as bonds (Chatzis and Dullien, 1977). We might consider that fluid is stored in the pore body, and resistance is given by the pore throat (Hunt et al.,
Application of percolation variants needs to be addressed cautiously due to different $p_c$ values, for example, wetting is best described as site percolation, while drying is a bond percolation problem (Sahimi, 1995). Flow problems are of interest only for $p > p_c$, and it is sufficient to consider only the backbone instead of the entire cluster (Berkowitz and Ewing, 1998), while the backbone is obtained from the dominant, optimally connected flow paths by trimming off portions that connect only at one spot (dead-ends).

I.3.2 Other Common Theories Describing Solute Transport in Porous Media

The traditional continuum mechanics approach describing the transport of solute in porous media includes Advection-Dispersion Equation (ADE) (Bear, 1972), Continuous Time Random Walk (CTRW) (Berkowitz et al., 2002; Bijeljic et al., 2004; Margolin and Berkowitz, 2000) and Fractional Advection Dispersion Equation (FADE) (Benson et al., 2000; Krepysheva et al., 2006; Meerschaert et al., 1999; Pachepsky et al., 2000). A brief summary of each approach is given here.

I.3.2.1 Advection-Dispersion Equation

ADE is a common approach studying the spatio-temporal behavior of dissolved solutes. It combines both the advection and dispersion processes (which is modeled as a diffusive process). In one dimension, ADE is (Bear, 1972),

$$\frac{\partial c}{\partial t} = -\langle v \rangle \frac{\partial c}{\partial x} + D \frac{\partial^2 c}{\partial x^2}$$

Eq. (1.6)
where \( C \) is the macroscopic mean concentration of solute, \( t \) is time, \( \langle v \rangle \) is mean flow rate, \( x \) is the space coordinate, and \( D \) is the dispersion coefficient. On the right-hand side of the equation, the first term describes advection, and the second term describes dispersion.

The chief limitations of the ADE model are that it is purely phenomenological (Sahimi, 1987), that it can only apply to specific media like homogeneous media, and it is derived based on the assumption that solute transport is Gaussian. A well-known problem of ADE model is that it cannot describe and predict the observed long tailed solute arrival time distributions (Cortis and Berkowitz, 2004) in heterogeneous media. It raises controversy in interpreting the time, space and scale dependences of transport-limited chemical reactions within porous media (Hunt et al., 2015). However, Non-Gaussian (or non-Fickian) transport is becoming recognized as the norm in natural porous media (e.g., Cushman and O’Malley, 2015), a development, which is not compatible with the ADE.

I.3.2.2 Fractional Advection Dispersion Equation (FADE) and Continuous Time Random Walk (CTRW)

Well-known models describing non-Gaussian mixing other than percolation theory include the FADE and the CTRW, both modeling power-law solute arrival time distributions that produce diminishing solute velocities as a power of the time under a wide range of conditions (Scher et al., 1991). However, these have limitations as well: instead of using the second-order derivative to describe the dispersion process, FADE replaces the diffusion term with a fractional order derivative (Benson, 1998), but the
dispersion coefficient that is determined using the FADE is still scale-dependent (Pachepskey et al., 2000); CTRW has been demonstrated to accurately model experimental data of breakthrough curves, but a non-universal parameter $\alpha$, which describes the waiting time distribution, needs to be fitted with experimental data for prediction (Sahimi, 2012). Neither the FADE nor the CTRW can predict the actual power of the power-law formulation of solute arrival time distributions.

The advantage of percolation theory is that, instead of generating an arbitrary power of solute time arrival distribution, it yields a value that relates to a known quantity from percolation theory, the fractal dimensionality of the percolation backbone $D_b$ (Lee et al., 1999; Sheppard et al., 1999; Hunt and Skinner, 2008, 2010; Hunt et al., 2011; Ghanbarian et al., 2012), which is specified mostly by the saturation conditions and the dimensionality of the flow (Sheppard et al., 1999).

I.3.3 Solute Transport Theory

Hunt and Skinner (2008) proposed a solute transport theory based on the theoretical framework of percolation theory, and demonstrate a technique to calculate spatial solute distributions as well as solute arrival time distributions of solute transport in heterogeneous media, based on the concept of critical path analysis (CPA) (Ambegaokar et al., 1971; Pollak, 1972; Friedman and Seaton, 1998; Hunt, 2001) in percolation theory. In the simplest form of CPA, hydraulic conductivity is randomly distributed, and the value of conductance connecting any two sites is independent of other conductance
values. The conceptual basis of the CPA is that flow tends to choose the path with the lowest cumulative resistance (optimal flow path), but by avoiding large resistance, the total length of the flow path might increase. The balance between minimizing cumulative resistance and shortening the flow path allows the determination of the optimal (or typical) solute arrival time, which corresponds to the peak arrival time in experiments (Hunt et al., 2015). CPA uses the percolation threshold \( p_c \) to define the smallest resistance value along the optimal flow path through the medium (Friedman and Seaton, 1998; Hunt, 2001; Hunt and Ewing, 2009), and the typical hydraulic conductivity, which is assumed to govern the flow rate can be obtained (Hunt, 2016). It has been demonstrated that CPA is capable of predicting advective flow in porous media, both in saturated and incompletely saturated conditions (Katz and Thompson, 1986; Ghanbarian and Hunt, 2012), and the predicted result of solute arrival time distribution as a function of transport distance is in agreement with simulation generated by Liu et al. (2003), and the prediction is verified by experimental observations from Jardine et al. (1993) and Cherrey et al. (2003) (Ghanbarian et al., 2012). For the development of soil formation models presented in this dissertation, we are interested in distances and times, and the typical solute arrival time (the most likely transport time) as a function of transport distance, and it is not necessary to find the typical hydraulic conductance or solute arrival time distributions using CPA. Sahimi (1994) suggested that solute along such critical path should follow the same percolation scaling law as solute transport through percolation clusters. Thus as long as fluids find the preferred flow paths, i.e. whenever
critical path concepts are relevant, scaling results for solute transport should apply. Lee et al. (1999) first proposed such a spatiotemporal scaling law of solute transport in 2D system at the percolation threshold, with the typical time \( (t) \) taken for a solute particle to travel from one side of a system to the other side for a distance of \( (x) \) being proportional to \( x^{D_b}, \quad t \propto x^{D_b} \), where \( D_b \) is the fractal dimensionality of the percolation backbone. The value of \( D_b \) varies depending on the moisture condition and the dimension of flow in the medium. Saturated conditions are usually considered as random percolation, while unsaturated conditions are considered as invasion percolation. Invasion percolation (Wilkinson and Willemesen, 1983) is used to describe the displacement of a fluid (e.g. air or water) through the invasion of a second fluid in a medium, and the value of \( D_b \) varies from the value it takes in random percolation. For application in soil formation, commonly, we assume 3D saturated conditions with \( D_b = 1.87 \) (Sheppard et al., 1999). However, there are cases that flow is 2D, for example, along a fracture plane. Glass et al. (1998) argue that unsaturated conditions are more common when flow is constrained to 2D surfaces. Values of \( D_b \) under different conditions are summarized in Table 1.2.
Table 1.2. Values of fractal dimensionality of percolation backbone ($D_b$) (from Hunt, 2015b)

<table>
<thead>
<tr>
<th>Dimension and Saturated Conditions</th>
<th>$D_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D Saturated (Random)</td>
<td>1.64</td>
</tr>
<tr>
<td>3-D Saturated (Random) or Wetting (Invasion)</td>
<td>1.87</td>
</tr>
<tr>
<td>2-D Unsaturated (Invasion)</td>
<td>1.22</td>
</tr>
<tr>
<td>3-D Unsaturated or drying (Invasion)</td>
<td>1.46</td>
</tr>
</tbody>
</table>

I.3.4 Theoretical Models

Based on the power-law formulation and the results from Sheppard et al. (1999), Hunt et al. (Hunt, 2015a; Hunt and Ghanbarian, 2016) proposed a soil production model based on the hypothesis that the total solute transport distance in a given time period corresponds to the soil depth at the end of the same time period (Hunt, 2015a). The model is developed based on the assumption that erosion of soil is negligible, and details are given in Section I.3.4.1. In this dissertation, erosion is considered, and a modified model of soil formation is presented in Section I.3.4.2.

I.3.4.1 Soil Production Model

When removal of soil from erosion is not considered, and 3D saturation is assumed, in the context of soil production, soil depth $x$ is proportional to time $t$ to the power of $D_b$ (Hunt, 2015a),
\[ t = t_0 \left( \frac{x}{x_0} \right)^{1.46} \quad \text{3D unsaturated} \]
\[ t = t_0 \left( \frac{x}{x_0} \right)^{1.87} \quad \text{3D saturated} \quad \text{Eq. (1.7a)} \]
\[ t = t_0 \left( \frac{x}{x_0} \right)^{1.64} \quad \text{2D saturated} \]
\[ t = t_0 \left( \frac{x}{x_0} \right)^{1.22} \quad \text{2D unsaturated} \quad \text{Eq. (1.7b)} \]

where \( x_0 \) is the typical particle size, \( t_0 \) is the pore crossing time, defined by the fluid flow rate, \( v_0 = x_0 / t_0 \), through a characteristic pore. In heterogeneous media a characteristic particle size is assumed to be the median diameter, or \( d_{50} \).

Thus,
\[ x = x_0 \left( \frac{t}{t_0} \right)^{\frac{1}{b}} \quad \text{Eq. (1.8)} \]

Then, by taking the time derivative of soil depth \( x \), one can obtain the equation of soil production rate (bedrock-to-soil conversion), \( R_s \),
\[ \frac{dx}{dt} = R_s = \frac{1}{1.87} \frac{x_0}{t_0} \left( \frac{t}{t_0} \right)^{-0.47} = \frac{1}{1.87} \frac{l}{\phi} \left( \frac{x}{x_0} \right)^{-0.87} \quad \text{Eq. (1.9)} \]

where \( R_s \) is the rate of soil production, the pore-scale flow rate, \( x_0 / t_0 \) is expressed as the pore-scale infiltration rate in the field (Hunt and Ghanbarian, 2016), which is calculated as infiltration rate \( I \) divided by porosity \( \phi \) of soil (Yu et al. 2017). \( I \) is the net downward flux of moisture through the soil, measured from, e.g., rainfall, and excludes the water flux returning to the atmosphere at the soil surface through direct evaporation, or from transpiration through plants.
I.3.4.2 Soil Formation Model

When studying the formation of soil, which refers to the net growth of soil in this dissertation, removal of soil by erosion should be considered. Adding the erosion term $E(t)$, on the previous model (Eq. (1.9)) to predict the formation rate of soil yields:

$$\frac{dx}{dt} = R_s - E(t) = \frac{1}{1.87} \cdot \frac{l}{\phi} \left( \frac{x}{x_0} \right)^{-0.87} - E(t) \quad \text{Eq. (1.10)}$$

In principle such an equation can be solved numerically for $x(t)$ with an arbitrary erosion rate, but to date we have only addressed the case $E(t)$ being a constant, $E$. One can obtain the soil depth $x$ after a certain time period $t$ by integrating Eq. (1.10) over $t$,

$$x = \int_0^t dt' R_s = \int_0^t \frac{1}{1.87} \cdot \frac{l}{\phi} \left( \frac{x(t)}{x_0} \right)^{-0.87} - E(t) dt' \quad \text{Eq. (1.11)}$$

Soil depth at local steady-state condition (constant soil depth) can be solved easily from Eq. (1.10) by setting $E(t) = E$, and letting $\frac{dx}{dt} = 0$ to yield,

$$x = x_0 \left( \frac{1}{1.87 E \phi} \right)^{1.15} \quad \text{Eq. (1.12)}$$

I.3.4.3 Hypotheses of the Models

Three hypotheses were made to develop the models:
1) With increasing time and length scales, solute transport becomes increasingly important in limiting chemical weathering of bedrocks. When chemical weathering is transport-limited, chemical weathering rate is proportional to solute velocity.

2) When soil production is limited by chemical weathering, which itself is limited by solute-transport, the thickness of soil produced (soil depth when erosion can be neglected) at a given time equals to the solute transport distance over the same time period.

3) The water flux in Eq. (1.9) is the pore scale net infiltration rate \( I/\phi \), where \( I = P - AET + \text{run-on} - \text{run-off} \) (\( P \) = precipitation, \( AET \) = actual evapotranspiration), and \( \phi \) is porosity. Theoretical basis and experimental facts that support each hypothesis are given in the following sections.

I.3.5 Relevance of Solute Transport to Chemical Weathering

The process of weathering typically begins at the surface of the earth's crust when bedrock is uplifted by tectonic forces. Then, alteration of bedrocks from physical breakup and chemical decay take place, a process that eventually converts parent materials into soil. As a major process of chemical weathering, mineral dissolution enables the development of soil (Maher, 2010). Although the Earth’s crust contains a very wide range of minerals, the dominant species are silicate minerals, since basalt and granite are the most important constituents of oceanic and continental crust. The slow process of silicate dissolution also plays an important role in regulating the carbon cycle by taking
up CO$_2$ through Urey (1952) reaction. One mole of silicate mineral weathered consumes one mole of CO$_2$. The kinetics of chemical weathering, in principle, mineral dissolution, has been studied extensively for decades (Maher, 2010). An inverse dependence of chemical weathering rate on time was observed in several experiments (Bain et al., 1993; Taylor and Blum, 1995; White and Brantley, 2003; Maher et al., 2004; Fantle and DePaolo, 2006). Strong correlations between chemical weathering rates and physical erosion rates are also observed (Riebe et al., 2001; Jacobson et al., 2003; West et al., 2005; Hren et al., 2007), a situation which leads, as described above, to steady-state conditions in soils. There are many factors which influence chemical weathering (Hunt and Ewing, 2009), which makes it difficult to predict chemical weathering rate as a function of time. As both the delivery of reaction reagents and the removal of weathering products are controlled by flow transport, solute transport potentially plays an important limiting factor of chemical weathering rates (Maher, 2010). Slow velocity of solute tends to slow down chemical reaction rate due to the building up of products, which drives reaction to approach thermodynamic equilibrium. Generally speaking, only with a flow rate so high that chemical reactions can never reach equilibrium, is the chemical reaction rate controlled by surface reaction (Berner, 1978). Berner (1978) first shows the distinction between transport-limited and surface reaction-limited weathering, and describes the weathering system as a well-mixed batch reactor, which is only controlled by surface reaction. However, this does not explain the inverse dependence of chemical weathering rate on time. Also, chemical weathering rates obtained in the laboratory do
not agree with those inferred from the field (White and Brantley, 2003). In particular, laboratory measured dissolution rates have been found to be as much as 5 orders of magnitude higher than field observations (Salehikhoo, et al., 2013).

There have been considerable discussions on the causes of such discrepancies, including the limiting effects from solute transport in the field (Molins et al., 2012; Dentz et al., 2011; Raoof and Hassanizadeh, 2010; Noniel et al., 2012; Maher, 2010). As mentioned in the previous section, non-Gaussian transport is normal in natural porous media. In non-Gaussian transport, solute velocity does not equal to flow rate beyond the length scale of a typical pore size, but diminishes over time (equivalently, over solute transport distance) as it travels. From the basic function of percolation theory, \( t \propto x^{D_b} \), the mean velocity can be written as \( v \approx v_0 \left( \frac{1}{t_0} \right)^{(1-D_b)/D_b} \) (Hunt et al. 2015), yielding a solute velocity that decreases over time with a power of \( 1/D_b -1 \), where \( v_0 \) is the pore scale velocity (equal to the flow rate) and \( v \) is the average velocity over the entire transport process. As the solute transport slows down, the accumulation of products in the solute produces a tendency for the reaction to reach equilibrium along the flow path, and slows down the chemical reaction rate. Once the solute velocity diminishes to a threshold that it is no longer able to drive departure from the equilibrium state of the chemical reaction, the limiting factor of chemical weathering rate will switch from surface reaction to solute transport.
The scaling of chemical reaction rate with time behaves differently under particular limiting factors as illustrated in Figure 1.2 (from Figure 4 in Maher, 2010). When chemical weathering is controlled by surface reaction, the reaction is under well-mixed conditions, and the rate is not affected from the change of flow magnitude, but can be affected by environmental conditions, such as temperature, or composition of bedrocks, which influences thermodynamics of the chemical reaction. After the limiting factor switches from surface reaction to solute transport, with hypothesis (1) that solute transport-limited chemical weathering rate is proportional to solute velocity, chemical weathering rate should follow the same scaling over time as the solute velocity does. Such hypothesis is supported by experimental facts (Figure 1.3, values digitized from Maher's Figure 4 (2010)), and has been verified by Hunt et al. (2015) in a wide range of experiments (Peng et al. 2012; Liu et al., 2008; Du et al., 2012; Zhong et al., 2005; Salehikhoo et al., 2013; White and Brantley, 2003).

As depicted in Figure 1.2, the chemical weathering rate (well-mixed reaction rate) does not change over time within the surface reaction-limited regime (the flat portion of the graph), but as the solute velocity slows down, the limiting effect from solute transport becomes increasingly important, and the chemical reaction is limited under a mixed control (curvy portion of the graph). After the limiting factor switches from surface reaction to solute transport, scaling of chemical weathering rate becomes proportional to solute velocity. From Figure 1.2, all experimental observations are within the solute transport-limited regime.
However, of course, chemical weathering of bedrock is not always limited by solute transport. In order to assess the limits of chemical reaction rates in porous media, most investigators use the Damköhler number, the ratio of a solute advection time to a reaction time (Molins et al., 2012), to evaluate the relative importance of advection rate to chemical reaction rate. When the value of the Damköhler number is larger than 1 (advection time > reaction time), reactions are considered transport limited.

Figure 1.2. Chemical reaction rate vs. flow rate. Figure from Maher (2010). $R_d$ is reaction rate, $q$ is flow rate. Circles: Clow and Drever (1996), stars: Swoboda-Colberg and Drever (1993), solid triangles from White et al. (2005), diamonds from White et al. (2008), square from White et al. (2001), open triangles pointing up from Maher et al. (2006), open triangles pointing down from Jin (2008).
Figure 1.3. Reaction rates plotted against flow rates showing agreement with a linear proportionality (values digitized from Figure 4 in Maher (2010)).

I.3.6 The Damköhler Number

The calculation of the Damköhler number is provided by an analogy to the Peclet number, $P_e$, which is defined to be the ratio of advection rate and diffusion rate. Thus, $P_e$ evaluates the relative importance of advection to diffusion (Saffman, 1959). The Damköhler number is defined as $Da_l = \tau_A / \tau_R$, where $\tau_A$ is the advection time, and $\tau_R$ is the reaction time. However, the usual calculation (Salehikhoo et al., 2013) of this ratio is flawed, as the solute transport is considered as Gaussian transport that has a constant solute velocity, which does not decay with transport distance but which decays according to a power law in non-Gaussian transport. The calculation leads to a constant advection time in the numerator. In the denominator, reaction time is calculated from the observed reaction rate,
which introduces the possible effect from advection, if the rate is slowed by transport, into the denominator (Yu and Hunt, 2017a), instead of confining it to the numerator. A revised method to calculate the Damköhler number based on non-Gaussian transport from the same percolation scaling law will be presented in Chapter II. We show that it is more common in natural media for chemical reaction to be limited by solute transport, which provides the fundamental assumption of our soil formation model, and also explains directly a portion of the several orders of magnitude discrepancy between experimental results and field observations, as flow rate conducted in laboratories are much faster than flow in natural media. The remainder of the discrepancy is then interpreted as being due to the slowing of solute velocities with increasing time or spatial scales.

I.3.7 Relevance of Solute Transport to Soil Depth

The agreement of the theoretical scaling (Hunt et al., 2015) and field observations of chemical weathering rate (Maher, 2010; White and Brantley, 2003) over large time scales suggests the approach to apply the same theoretical framework to predict soil depth (Hunt, 2016). The weathering depth of soil in a given time period should then be given in terms of the total solute transport distance over the same time period (Hunt, 2015a, Hunt, 2016), which is the second hypothesis of the theoretical model, and shows consistency with field data of 21 soils across the world in Hunt (2016). However, the actual relation between weathering depth of bedrocks and thickness of soil is still unclear. One of the issues is the
inconsistent definition of soil, i.e. which horizon is considered as soil. The agreement between field observations and predictions demonstrated in Hunt (2016) suggests a high probability of proportionality between weathering depth and soil thickness.

Hypothesis (3) relates soil production to both topography and climate by assuming that the vertical water flux in soil is the net infiltration rate, or the amount of water that percolates into the soil during a given time period. The choice of infiltration rate as pore-scale solute velocity reflects the understanding that it is the actual downward flux of the CO$_2$ carrying water that is critical to the weathering reaction (Yu and Hunt, 2017b). Such a downward flow rate may not relate simply to precipitation and evapotranspiration, if surface waters converge or diverge within the study area. Nevertheless, surface flow is not considered if run-on and run-off values are not available on site. The inference that $I$ is the controlling flux has considerable relevance in landscape and climate. Since $I = P - AET + run-on - run-off$, one can expect more rapid soil productions, thus thicker soils in convergent topography, and more humid climate. The relevance of net infiltration rate to soil depth has been shown by Hunt and Ghanbarian (2016), which demonstrate agreement of prediction by using net infiltration rate as the pore-scale velocity.

I.3.8 Defining Relevant Parameters in the Models

Given the fact that there is still no general agreement in the definitions of many of the soil properties, as well as the variations in certain soil properties, it is important to define relevant parameters involved in our model clearly before applying it for prediction.
Soil depth and soil age: There is no general agreement of the definition of soil, and it is still debatable whether (and to what extent) the weathering zone should be included when studying soil formation function. Thus, large uncertainty could be introduced from the determination of soil thickness. Another potential uncertainty comes from the determination of time. Soil is a dynamic system, and there is no absolute age i.e. the time since bedrock starts producing soil, that can be defined. Definitions of these two parameters are non-universal across the data sources that we referenced to examine our model. For example, Heimath et al., (Heimsath et al., 1997, 2000, 2001a, 2006, 2009, 2012) consider the soil depth as the soil column right above the weathered but still-in-place bedrock, Egli (2014) calculate soil depth by a different criterion using "$A/B/E/O + 1/2 (AC/CA/BC/CB/OC)$" horizons, Hunt (2015a) defines soil depth down to the $B_w$ horizon. For the determination of time, relative soil ages are usually used such as "surface age", "soil residence time", and "surface exposure age" etc. (Egli, 2014, Heimsath et al., 1997, 2000, 2001a, 2006, 2009, 2012). To address this issue and to maintain consistency, instead of applying a universal rule to determine soil depth and time, we adopt the particular authors’ convention for an individual dataset, as long as the soil depth and soil age in the dataset correspond to each other.

There are also other soil properties that affect the hydraulic property and soil formation. Here we only discuss properties that are relevant to our models: particle size and porosity.
The range of soil particle size can span several of orders of magnitude (clay, silt, sand, gravel), and there is no apparent characteristic particle size. Thus, if typical particle size is not given, we represent the characteristic particle size by the geometric mean.

According to soil classification by USGS, silt is the middle particle size (geometric mean) class in soil classification schemes, and it ranges from 2µm to 63µm, with a mean value of 32µm; here we take 30µm as the characteristic particle size for soils without information relating to soil texture or soil particle size. Most soils have porosities between 30% to 60%, with 40% to 50% being a common value (Hunt et al., 2014); thus a typical value of 40% for the porosity is assumed if no information is given.

I.4 Scope and Structure of the Dissertation

The main scope of this dissertation focuses on the physical evolution of soil by modeling its formation function on the basis of transport-limited chemical weathering rate of bedrocks. In Chapter II, a revised calculation of Damköhler number is presented, demonstrating the validity of the fundamental assumption of our models that chemical weathering of bedrocks is very likely to be transport-limited in natural media. Chapters III and IV examine our model with field data in predicting soil depth evolution as well as soil formation rates over time, and discuss how the combined effect of climatic and geological conditions contribute to the variance of soil depth across the world. Chapter V presents an application of our model in examining local steady-state conditions of soil. In Chapter VI, we apply our model to distinguish steady and stochastic erosion processes in
threshold landscapes, and present a possible application of the model to predict landslides.
II. RE-CALCULATION OF THE DAMKÖHLER NUMBER TO EVALUATE
TRANSPORT-LIMITED CHEMICAL WEATHERING

Article I.

Damköhler Number Input to Transport-Limited Chemical Weathering Calculations
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ABSTRACT: Chemical weathering of the silicate minerals in the Earth’s crust is the dominant source of variability of atmospheric carbon on time scales of about 1 Ma and up, lending significance to the ability to predict such weathering rates. Field weathering rates at such large time scales tend to be much slower than laboratory values, however. It has been proposed that the discrepancy occurs because field weathering rates are solute-transport-limited rather than reaction-kinetic-limited. To assess the relative importance of reaction kinetics and solute transport, the Damköhler number, Da, a ratio of a transport time to a reaction time can be used. Unfortunately, even in those experiments that suggest importance of transport limitations, the traditional calculation of Da yields values close to 1 and an extreme sensitivity of experimental reaction rates on Da. We develop a new method of calculating Da based on the theory of non-Gaussian solute transport, appropriate for heterogeneous porous media. Our results are that weathering rates are less sensitive to the calculated Da and that Da is now equally sensitive to input parameters, generating a much more straightforward assessment of the relevance of transport limitations to chemical weathering. Our most important single inference appears to be that, for field conditions at larger time scales (coarsely decadal and up), chemical weathering is practically always transport-limited.

KEYWORDS: Damköhler number, transport limited, chemical weathering, permeation theory, solute transport

INTRODUCTION
Chemical weathering is a process by which minerals that form in a deep subsurface environment are altered when exposed to the near surface environment and, as such, is a fundamental component of the rock cycle of geology. Although the Earth’s crust contains a very wide range of minerals, the dominant constituents of oceanic and continental crust, basalt and granite, respectively, consist mainly of aluminosilicates. The classic, Urey, reaction is a short-hand representation that indicates that 1 mol of CO₂ from the atmosphere is drawn down for every mole of silicate mineral weathered, CO₂ + CaSiO₃ ↔ CaCO₃ + SiO₂. This reaction is one of the key inputs to the modulation of the global carbon cycle and, thus, climate change, over time scales of about 1 million years to tectonic time scales of roughly 200 million years. Consequently, there is a high motivation to being able to predict the rate at which this reaction proceeds over such time scales. It has been recognized at least since 2003, however, that chemical weathering rates measured in the laboratory bear no resemblance to those inferred from the field. In particular, field-derived values are as much as 5 orders of magnitude lower than laboratory rates. While there has been considerable discussion addressing the causes of the discrepancy between field and laboratory silicate weathering rates, including the potential role of solute transport, only very recently has it been possible actually to derive a single expression that links reaction rates on the two vastly different time scales. The theoretical approach that led to this reconciliation asserts that virtually all field measurements are limited by the efficiency of the solute transport to deliver the reaction reagents (e.g., CO₂) to and remove the products (e.g., SiO₂ and CaCO₃) from the weathering front. While the derivation proves successful, its presentation lacks the needed comparison of the time scales of advection, τ_a, and reaction, τ_r. Such a comparison is conventionally given in terms of the Damköhler number, Da = τ_r/τ_a. However, the usual calculation of this ratio is flawed. There the calculation of τ_a is based on the observed reaction rate rather than its well-mixed value. Such a choice places the large advection time in the denominator of Da rather than where it belongs, if the rate is slowed by transport, in the numerator. This paper first fixes the inconsistency in the calculation of Da and then re-examines data from oft-referenced papers in this context. This correction allows for a consistent assessment of the controlling factors on chemical weathering rates at the Earth’s surface. The result is that essentially all weathering reactions at the Earth’s surface on time scales exceeding days are transport-limited as previously contended, 10,11 although many laboratory measurements are limited by the kinetics.

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As already shown, the prediction of chemical weathering rates as a function of time also generates the soil production rate as well as the sequestration rates of both carbon and nitrogen within the soil, making this a solution of considerable relevance. The link between chemical weathering and soil production is partly a matter of definition. Chemical weathering of the bedrock or unconsolidated surface materials from, e.g., fluvial,18 or glacial19 deposition, landslides,20 treethrow,21 etc., is generally understood to be the fundamental process that initiates the formation of soil. In particular, the bottom of the subsoil or Bw horizon is called the weathered depth, within indicating weathering and referring to the presence of oxides (particularly of iron), giving this soil horizon its typically red color.22 Thus, soil depth, when measured to the Bw horizon, should serve as an indicator for the total volume of weathered material, while the soil production (which is the temporal derivative of the soil depth) will be proportional to the chemical weathering rate.23 Then, the lowest order approximation for soil carbon and nitrogen sequestration rates, as pointed out in ref 19, will be that they are proportional to the soil production rate.

We discuss first the prevalence20 of non-Gaussian solute transport and heavy-tailed solute distributions25 and, within this context, the only (percolation) theoretical formulation that allows for explicit predictions.20-50 Then, we pass to the present theoretical description of heavy-tailed solute distributions from spatially variable advection in terms of percolation theory.20-52 A brief summary of evidence for the relevance of such heavy-tailed solute distributions to geochemistry is given, followed by a calculation of the Damköhler number. We emphasize that our theoretical calculations of non-Gaussian distributions give predictions of chemical weathering scaling (and related properties) when conditions are clearly in the transport-limited regime. However, as will be seen, our results are not accurate when such reactions are limited by the kinetics of well-mixed fluids nor in an intermediate regime, when part of the medium is kinetically limited and part is transport-limited. A more comprehensive theoretical approach that can treat quantitatively such a crossover as well as the transport-limited regime is yet to be developed. However, our new calculation of the Damköhler number does at least give an objective means to predict a length scale and, thus, also a time scale, at which chemical weathering becomes transport-limited.

**SOLUTE TRANSPORT THEORY**

Solute transport as developed within the percolation theory generates fat-tailed arrival time distributions, of approximately power-law form, like the continuous time random walk (CTRW),33-35 or the fractional advection dispersion equation (FADE).35-59 In contrast to both of these techniques, the percolation theory does not yield an arbitrary power but a value that is specified mostly by the flow conditions and, to a lesser extent, the properties of the medium.55 The theoretical procedure generates the entire arrival time distribution as well.52 According to Sahimi, however, this procedure may be simplified by applying the argument that, whenever the dominant flow paths are describable using the percolation theory (in the form of critical path analysis), solute transport follows basic percolation scaling laws. These two distinct perspectives lead to somewhat different predictions; however, the difference is not great, and the simplification produced using a completely analytical scaling law is worth the slight degradation in accuracy of its theoretical predictions at long time scales, particularly because it enables a simple calculation of a dimensionless number used to assess the relevance of transport vis-à-vis kinetic limitations, called the Damköhler number.55

We applied59-63 the framework of critical path analysis64 developed within the percolation theory,55-64 to a network representation of the medium55-61 to quantify all distinct solute transport paths that can be connected through the system and the fluid fluxes along those paths. Critical path analysis and concepts from the percolation theory are relevant to fluid flow in all disordered media, because the tendency for flowing fluids to exploit paths of lowest resistance is quantifiable using the percolation theory. Knowing the topology of each system of paths59 together with the associated fluid fluxes made it possible to calculate the variations in the characteristic velocity along the path and, thus, the total solute transport time.64 This probabilistic treatment of spatially variable advection, which later incorporated molecular diffusion,65 generated the solute arrival time distribution as a function of the transport distance in agreement with simulations59 and predicted59 (1) the entire observed solute arrival time distribution60-62 as a function of saturation, (2) the typical system crossing time70 for conducting particles66-68 as a function of length, (3) the known69-70 scaling of the dispersion coefficient with Peclet number, (4) and the range of observed values of the dispersivity in 2200 experiments71 as well as the Gehar "rule of thumb",72 over 10 orders of magnitude of length scale. Although previous work developed full solute arrival time distributions and, thus, the solute flux, a simple scaling theory, which gives nearly the same results for a solute velocity (anticipated by Sahimi55) is sufficient for the proposed research. Three scaling relationships together, water transport distance, solute transport along optimal paths confined near the surface, and solute transport through bulk three-dimensional media, limit crop height (h), natural vegetation height (a), and soil depth (d), respectively, as functions of time (ca. 8000 data points).

From the percolation theory, the solute transport time, t, increases more rapidly than linearly with transport distance, x, in particular, as a power of x equal to the percolation backbone fractal dimensionality, x, 1.8773 for wetting conditions or of full saturation and for which the network of pores supporting the fluid flow is fully three-dimensional; thus, the advection time is calculated as

\[ t = \int \frac{x}{L_0} \ \text{d}x = \frac{1}{1.87} \cdot \frac{L_0}{L_0} = \nu \ \text{being a pore-scale fluid velocity.}
\]

**RELEVANCE OF SOLUTE TRANSPORT TO CHEMICAL WEATHERING**

Our recent study59 assumed a proportionality of chemical weathering rates to solute transport velocities and, thus, to fluid flow rates. The fact that this is an experimental fact can be seen from Figure 1, digitized from ref 8. This fundamental result indicates transport limitations of these reactions. If, in contrast, the reaction were controlled by surface reaction rates, the rate would not be affected by the fluid magnitude, as depicted in Figure 2.

With increasing length and time scales and for certain classes of non-Gaussian transport,55 solute transport limitations
Figure 1. Reaction rates plotted against flow rates showing agreement with a simple proportionality (extracted power of 0.97 m). Note that the flow rates correspond approximately to the variability of infiltration on Earth’s surface, from about 1 mm per year to 10 meters per year. This figure was digitized with permission from ref 8. Copyright 2010 Elsevier.

Figure 2. Schematic reaction rate scaling diagram. Horizontal lines give laboratory reaction rates for well-mixed conditions at temperatures T_1 < T_2 < T_3, and solute velocities for flow rates u_1 < u_2 < u_3. Predicted reaction rates connect the horizontal line at given  T with the diagonal line at given u. In reality, one should expect a smooth crossover from one region to the other. Proportionality of a reaction rate to a flow velocity occurs only within the transport-limited portion of the diagram. Note that our calculation of the Damköhler number (eq 3) is intended only to be sufficiently accurate to diagnose the point, for a given set of experimental conditions, at which the relevant horizontal and diagonal lines cross and not the actual dependence of the total reaction time and, thus, the total reaction rate.

The basis for our calculation of the Damköhler number is provided by an analogy to the Peclet number, which determines the relative importance of advection to diffusion. The diffusion time is calculated without reference to advection, while the advection time is calculated neglecting diffusion, with the ratio evaluated. Of course, when both advection and diffusion are important, similar to when transport begins to limit weathering reactions, neither process is completely independent from the other. Existing calculations of the Damköhler number define D_k as

\[ D_k = \frac{\tau_d \cdot \Delta H}{k_{\text{eq}} / \rho} \]

where \( \tau_d \) is the advection time, \( \tau_r \) is the reaction time, \( L \) is the column length, \( \nu \) is the fluid flow velocity, \( \rho \) is the pore volume description of the fluid flow rate, \( C \) is an equilibrium concentration, \( R_{\text{cyc}} \) is the measured reaction rate normalized by surface area, and \( \rho \) is the surface area. Note that the subscript MgCO_3 makes eq 2 specific to ref 12.

This procedure underestimates the advection time by assuming that the solute velocity is constant rather than diminishing with transport distance through the column. It also has adverse consequences in conceptualization using the measured reaction rate: (1) reporting measured reaction rates in terms of the measurement-derived \( D_k \) does not test theory against experiment, although it did a fair job of collapsing data to a single relationship (our scaling procedure does so even more effectively), and (2) representing the reaction rate slowing as an increase in the factor in the denominator (\( \tau_r \)) makes \( D_k \) decrease with an increasing distance rather than increase, as would be consistent with slowing advective transport and increasing the numerator (\( \tau_d \)). In a practical sense, wide ranges of physical variables, such as column lengths and velocities, become compressed into narrow ranges of \( D_k \), accentuating the apparent dependence of reaction rates on such quantities, leading to false conclusions regarding the suitability of using \( D_k \) as a proxy for system length. Because observed reaction rates decrease with increasing length scales in accordance with the limitations provided by transport, our calculation of \( D_k \) will be performed for conditions at the column end rather than the entire column. \( D_k \) is proposed here to be best calculated as
\[
D_A = \frac{V_s}{v} = \frac{L}{\ln\left(\frac{L}{L_0}\right) - L_0 - \frac{1}{N}}
\]

(3)

where \(L\) is the column length, \(L_0\) is a particle size or pore separation, \(v\) is the fluid flow velocity, \(V_s\) is the pore volume description of the fluid flow rate, \(C\) is an equilibrium concentration, and \(R_{eq}\) is the surface-normalized reaction rate under well-mixed conditions (horizontal lines in Figure 2). In general, the well-mixed rate will be denoted by \(R_{eq}\), \(A_r\) is the surface area. We note that our calculation substitutes \((L/L_0)\) for \((L/L_0)\) in the numerator of the published calculation.2

In summary, we have calculated the advection time in the absence of a reaction and the reaction time in the absence of limitations as a result of the interaction. Individually, these correspond to the asymptotic lines in Figure 2. While a complete calculation of the reaction rate certainly will require the ability to address the interaction of kinetics and transport, the determination of the length scale at which the dominant influence crosses over from reaction kinetics to transport can be found from the intersection of the lines in Figure 2.

**MATERIALS AND METHODS**

The Damköhler number was recalculated using the proposed method (eq 3) based on the published experimental data from ref 12.

The flow-through experiment was performed at three column lengths: 5, 10, and 22 cm. Only the data set from the mixed column was used to calculate \(D_A\). Flow rates \(q\) ranging from 0.18 to 36 m/day were used to study the dissolution rates of magnesite at different flow rates. The magnesite specimen was ground and sieved to grain sizes between 354 and 500 μm. The geometric mean of the grain sizes was taken as the typical particle size. Thus, we approximated \(L_0 = 400 \mu m\) in eq 3. Porosity of the material ranged between 0.39 and 0.43. The well-mixed reaction rate \(R_{eq} = R_{eq} = 10^{-8} \text{ mol/m}^2/\text{s}\), and Brunauer–Emmett–Teller (BET) surface area = 1.87 m²/g according to ref 12.

**RESULTS**

Scaling of the Magnesite Dissolution Rate with Distance. The overall reaction rates of each column in all three column lengths increase as the flow rate increases (Figure 3a). The exponent of the scaling of the reaction rate with fluid velocity decreases from 0.28 to 0.71 to 0.68 as the column length reduces from 0.22 to 0.05 m. Our previous study3 shows that the surface reaction rate is proportional to solute velocity (itself proportional to the fluid rate) if the reaction is transport-limited. The fact that the scaling of the reaction rate with solute velocity is more likely to be linear (follow the red dashed line in Figure 3a) at a longer column length indicates that, with increasing length scales, advection time keeps increasing and solute transport limitation becomes increasingly important to the overall chemical reaction rate of the entire column. One can expect a transport-limited scaling of the overall reaction rate when the advection time is large compared to the reaction time that the limiting effect of surface reaction kinetics can be negligible. The divergence of the reaction rate from linear scaling at the same column length as solute velocity increases also indicates that slowing solute velocity has an effect similar to extending the distance on the limitation of the chemical reaction rate.

![Image of Figure 3a and 3b](image_url)

**Figure 3.** (a) Scaling of the reaction rate (mol m⁻² s⁻¹) with the flow rate ranging from 0.18 to 36 m/day at various column lengths L. (cm). The dashed line represents the theoretical scaling of the transport-limited reaction rate with flow velocity. Data were obtained with permission from ref 12. Copyright 2013 Elsevier. (b) Reaction rate versus column length ranging from 0.05 to 0.22 m. Open symbols are observed reaction rates, and lines are predicted scaling of transport-limited reaction rates. Numbers indicate flow rates q (m/day). Data were obtained with permission from ref 12. Copyright 2013 Elsevier.

The transition of the limiting effect on the chemical reaction rate as the distance and flow rate increase is more clearly shown in Figure 3b by comparing the theoretical transport-limited reaction rates predicted by the percolation theory and measured rates.

Here, the transport-limited reaction rate is calculated as

\[
R_x = R_{eq} \left( \frac{X}{X_r} \right)^{3/2}
\]

(4)

where \(R_x\) is the predicted reaction rate, \(q\) is the flow rate, \(q_r\) is a reference flow rate, \(R_{eq}\) is a reference reaction rate, \(X\) is the distance from the inlet of the column, and \(X_r\) is a reference distance. Equation 4 was derived from the basic solute transport equation (eq 1). The reference point was chosen as the reaction at the slowest flow rate (0.18 m/day) and at the longest column length (0.22 m), because at this point, the reaction is clearly limited by solute transport (Figure 3a) and the reaction rate at a higher flow rate should proportionally increase, given that the reaction rate is calculated as proportional to solute velocity.
As the flow rate increases, observed reaction rates become more divergent from prediction, indicating a lower importance of solute transport in limiting the overall reaction rate. Interestingly, the reaction rate approximately reaches steady state at the fastest flow rate and shortest distance (36 m/day and 0.05 m), meaning that the 36 m/day flow rate is just fast enough to eliminate altogether the influence of solute transport at the shortest travel distance of 0.05 m.

Damköhler Number Values. Values of the Damköhler number calculated using the proposed method (eq 3) were compared to the measured reaction rates. To simplify the figure of \(D_\text{aq}\) at various column lengths and flow rates, corresponding distances of reaction rates at various flow rates were normalized to flow rate \(q = 0.18\) m/day (Figure 4) and \(q = 36\) m/day (Figure 5), consistent with a proportionality of the reaction rate to the flow rate. Only data points \((q = 0.18\) and 0.36 m/day at three column lengths, and \(q = 1.8\) m/day at \(L = 0.22\) m) that are transport-limited (Figure 3b) could be normalized consistently because, in that case, the reactions conform to our predictions (eq 4).

In Figure 4, \(D_\text{aq}\) based on our calculation increases from 448 to 3,844 as the scaled distance extends from 0.022 to 0.22 m at flow rate \(q = 0.18\) m/day and shows a large contrast to the original calculation proposed in ref 12 (eq 2). All original \(D_\text{aq}\) values are equal to or less than 1, implying that reactions are limited by surface reaction, revealing a conflict with the fact that the distance dependence of all reaction rates shown here follows exactly that of the transport-limited reaction rates (Figure 3b). Again, such a discrepancy is a result from the underestimation of the original calculation of \(D_\text{aq}\) by considering the solute velocity as a constant and calculating reaction time using measured rates. As the distance decreases, \(D_\text{aq}\) decreases dramatically from 1.65 \(\times 10^4\) to 11 as the distance reduces from 102 to 0.05 m.

Notice that, at 0.05 m, the author stated a well-mixed reaction rate, suggesting that, at this point, the reaction is limited by the surface reaction itself and \(D_\text{aq}\) should be equal to or less than 1. However, in our calculation, \(D_\text{aq} \approx 11\). Possible reasons could be as follows: (1) the mixed control of the reaction by both solute transport and surface reaction (as the solute is transported from the inlet to the outlet of the column, the reaction passes through a surface reaction-limited regime \((D_\text{aq} \leq 1)\) and a transport-limited regime \((D_\text{aq} > 1)\); even within 0.05 m, there could be a small region near the outlet that is transport-limited, causing the overall \(D_\text{aq}\) to exceed 1) and (2) a margin of magnitude overestimation of \(D_\text{aq}\) of our calculation [given the fact that the reaction rate still decreases as the column length increases from 0.05 to 0.1 m (Figures 3 and 5)], \(D_\text{aq}\) should be less than 1 at column length \(= 0.1\) m (\(D_\text{aq} = 37\) in our calculation); consequently, any overestimation of \(D_\text{aq}\) cannot exceed a factor of 40.

In Figure 5, theoretical reaction rate scaling at flow rate \(q = 36\) m/day is shown. According to the authors, the well-mixed reaction rate is about \(10^{-3}\) mol m\(^{-2}\) s\(^{-1}\). Theoretically, the reaction is only limited by the surface reaction within a short distance and is not affected by solute velocity (flat line shown in Figure 5). As solute transport distance increases, solute velocity is slowing with a power of 1.87 and the limiting factor of the chemical reaction switches from the surface reaction to solute transport when the advection time is larger than the reaction time. Here, there is no abrupt change of measured reaction rates from the well-mixed value to the theoretical scaling. Such curvature is caused by the method in which the reaction rate was calculated. As the overall reaction rate of the entire column, the measured rate is an averaged value of reaction rates limited by both the surface reaction and solute transport.

**DISCUSSION**

As an important parameter indicating the limiting factor of the chemical reaction rate, our calculation of the Damköhler number shows a similar decreasing trend as the reaction rate increases but with a large discrepancy for individual values compared to previous results calculated in ref 12 (Figure 5). In comparison to previous results that range from 0.17 to 1, our
Figure 6. Dependence of the Damköhler number on the reaction rate \( \text{mol m}^{-2} \text{s}^{-1} \) based on previous and proposed methods. Lines are theoretical dependence of the Damköhler number on the reaction rate when the reaction is transport-limited at various flow velocities \( \text{m/day} \). Open symbols are \( D_a \) calculated using the proposed method here based on actual reaction rates. Calculations of \( D_a \) for the solid symbols are based on the previous method.\(^{12}\) Numbers indicate flow rates. Data were obtained with permission from ref 12. Copyright 2013 Elsevier.

Figure 7. Evaluation of the accuracy of the calculated Damköhler number. Single lines are theoretical scaling of reaction rates limited by solute transport. Double lines are scaling of calculated \( D_a \) with the column length when the reaction is transport-limited. The blue dashed line is the well-mixed reaction rate \( \left( 10^{-3} \text{ mol m}^{-3} \text{s}^{-1} \right) \). Open symbols are measured rates. Numbers indicate flow velocities. Data were obtained with permission from ref 12. Copyright 2013 Elsevier.

\( D_a \) shows a much wider range (from 11.5 to 33 843.85) as the reaction rate changes for 2 orders of magnitude. The wide range of \( D_a \) from our calculation results from the advection time in the numerator that increases with a power of 1.87 rather than increasing linearly with the distance and the reaction time in the denominator that is diminished using the well-mixed reaction rate instead of the measured rate. For the specific case in which the well-mixed reaction rate (and, thus, reaction time) is a constant, the Damköhler number is proportional to the advection time, thus scaling as the 1.87 power of the column length. When the reaction rate is transport-limited, one can derive the scaling of the reaction rate with the Damköhler number by

\[
D_a \propto \frac{I_{ad}}{I_{p}} = \left( \frac{L}{L_P} \right)^{1.87} \tag{5}
\]

\[
R \propto \frac{L}{t_{ad}}\propto L^{1-D_a/D_a} \tag{6}
\]

where \( v \) is the fluid velocity. Using \( D_a = 1.87^{0.05} \) for conditions of full saturation in three-dimension, the reaction rate limited by solute transport scales as the power of -0.465 with the Damköhler number (lines in Figure 6).

To examine further our calculation of the Damköhler number, theoretical scaling of transport-limited reaction rates at various flow velocities and corresponding \( D_a \) were plotted in Figure 7. At lower flow rates, \( D_a \) are close to 1 when reaction rates reach the well-mixed value, generating consistent results of
the Damköhler number and reaction rate scaling in that, as the Damköhler number increases past 1, the reaction rate slows relative to its well-mixed value. At rapid flow rate $q = 36$ m/day, the discrepancy increases. That is, because, as the flow rate increases, the scaling of the reaction rate tends to diverge earlier from the heuristic scaling as the distance decreases (as shown in Figure 7), making a shorter length for the reaction rate to meet the well-mixed value, thus resulting in a smaller corresponding $D_{m}$. It is this mixed control regime that degrades the accuracy of our Damköhler number estimation. The proposed method works reasonably well when the reaction is transport-limited.

Attempts were also made to predict the overall reaction rate of the entire column (the curve shown in Figure 5) by combining the limiting effects from surface reaction and solute transport together. If one combines effects of advection and reaction by simply adding the times of each process together, one can generate the following expression for the reaction rate:

$$\bar{R} = \frac{R_{m}}{1 + D_{m}^{-1/2}}$$  \hspace{1cm} (7)

This expression correctly reproduces the Damköhler number dependence at large $D_{m}$ while also generating the well-mixed reaction rate, $R_{m}$, in the limit of small $D_{m}$, but it does not properly reproduce the mixed condition regime and, thus, is not a candidate for the correct result.

Another approach that we developed was to separate the entire column into sections by "cut-off" length, where $D_{m} = 1$. Within that length, the reaction rate equals the well-mixed reaction rate, and beyond that length, the reaction rate was proposed to follow the scaling predicted by the percolation theory. Because the reaction rate was calculated from the concentration of the solution in the experiment as

$$\bar{R} = \frac{C_{out}Q}{A_{2}}$$  \hspace{1cm} (8)

where $\bar{R}$ is the overall reaction rate of the column, $C_{out}$ is the concentration at the outlet, $Q$ is the flow rate, and $A_{2}$ is the surface area. $C_{out}$ in eq 8 could be substituted by the sum of the two concentrations of the two sections and one can generate

$$\bar{R} = \frac{1}{1 - D_{m}} \log \frac{1}{1 - D_{m}} (1 - D_{m}^{1/2}) \log \frac{1}{1 - D_{m}}$$  \hspace{1cm} (9)

This expression reproduces the Damköhler number dependence at small $D_{m}$ but it does not properly reproduce the dependence of the reaction rate at large $D_{m}$ and, thus, is not a candidate for the correct result either. Unfortunately, both approaches, adding the advection and diffusion times on the one hand or adding the well-mixed and less mixed concentrations on the other, are unsuccessful at predicting the reaction rate under mixed control regime, although they do both generate a curved region connecting the two straight line regions in Figure 2. Thus, a more comprehensive theoretical approach that can predict the crossover from kinetic-controlled reactions to transport-limited reactions is yet to be developed.

In this discussion, we wish also to point out that, in at least one important factor, typical experimental conditions tend to accentuate the relevance of kinetic limitations, whereas field conditions tend to emphasize transport limitations. In particular, the flow velocities of ref 12 range from 0.18 m/day ($\approx$65 m/year) to 3.6 m/day ($\approx$13 410 m/year). However, under typical field conditions, flow rates will not exceed infiltration rates, which range from a fraction of 1 to 10 m/year. This discrepancy in flow rates implies that typical field values of the Damköhler number are likely to be roughly a factor 1000 larger than those in the experiments of ref 12.

Finally, we reiterate that the physical characteristic that underlies both the strong dependence upon length and time scales of such reactions as weathering rates and the non-Gaussian transport, which we use to predict such rates, is the heterogeneity of the pore space. There is, thus, no requirement to base one's approach on the particular average of the non-Gaussian transport, and other general approaches to treat heterogeneity can be used, such as the partitioning of the pore space into dual domain or multi-domain models. A general comparison of both the theoretical relationships between these diverse methods and their relative efficacy is not available. We do contend, however, that our treatment should have the advantage of greater parsimony.

**CONCLUSION**

The analysis of the experimental data shows that chemical reaction rates may be controlled by either surface reaction kinetics or solute transport. At small length scale (or rapid flow rates), the rate is limited only by reaction kinetics and not affected by solute velocity. As the distance increases (or flow rate decreases), solute transport takes place in limiting the reaction rate, forming a mixed control of the chemical reaction within the column. Only when the distance is long enough (or the flow rate is slow enough) that the limiting effect of surface reaction kinetics is so small compared to that of solute transport would the reaction rate follow the transport-limited scaling. The calculation of the Damköhler number based on the percolation theory is consistent with the scaling of the reaction rate. The apparent 1 order magnitude overestimation of the Damköhler number at the highest flow rate might be due to the fact that we are calculating overall $D_{m}$ at the end of the column. However, our calculation shows a good estimation at a slow flow rate when the entire column is more likely to be limited by solute transport. Given that the infiltration rate (thus, the flow rate for chemical weathering) in the field (between 5 × 10^{-2} and 0.01 m/day) is much slower than the slowest flow rate in the presented experiment ($q = 0.18$ m/day), the proposed method in calculating the Damköhler number and corresponding predictions of solute transport-limited weathering rates can be expected to be useful in most field experiments.

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**REFERENCES**

III. MODEL EVALUATION I: PREDICTING SOIL FORMATION FUNCTION

Article II.

Predicting soil formation on the basis of transport-limited chemical weathering

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Abstract

Soil production is closely related to chemical weathering. It has been shown that, under the assumption that chemical weathering is limited by solute transport, the process of soil production is predictable. However, solute transport in soil cannot be described by Gaussian transport. In this paper, we propose an approach based on percolation theory describing non-Gaussian transport of solute to predict soil formation (the net production of soil) by considering both soil production from chemical weathering and removal of soil from erosion. Our prediction shows agreement with observed soil depths in the field. Theoretical soil formation rates are also compared with published rates predicted using soil age-profile thickness (SAPT) method. Our formulation can be incorporated directly into landscape evolution models on a point-to-point basis as long as such models account for surface water routing associated with overland flow. Further, our treatment can be scaled-up to address complications associated with continental-scale applications, including those from climate change, such as changes in vegetation, or surface flow organization. The ability to predict soil formation rates has implications for understanding Earth’s climatic system on account of the relationship to chemical weathering of silicate minerals with the associated drawdown of atmospheric carbon, but it is also important in geomorphology for understanding landscape evolution, including for example, the shapes of hilltops, and the net transport of sediments to sedimentary basins.

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1. Introduction

As a complicated natural process (Vladychensky, 2009), soil formation is affected by five soil-forming factors: climate, topography, parent materials, organisms and time (Jenny, 1941; Dokuchaev, 1967). Defining soil formation as a continuous process helps to understand how soil and landscapes have developed and how they would be affected by climate change. Attempts to describe soil production have met with variable success. Soil depths tend to increase over time, but not in a linear fashion (Huggett, 1998), and not uniformly (data from the Heimath references, i.e., Heimath et al., 1997, 1999, 2001a, 2001b, 2005, 2009). In the main, soil depths increase more slowly the deeper the soil.

Weathering of bedrock and erosion are two major processes affecting soil depth. Weathering is the slow process of disintegrating rocks to produce materials to form soil while erosion is transporting the loosened materials away, reducing soil depth. The details of the erosion process determine whether soil depth is lowered at any given site; whenever the divergence of the soil flux is positive, there is net transport of soil away from a point. The depth is reduced if the net transport off-site is greater than the soil production on site. The focus of the present paper is not on the details of the soil transport, merely on the effects of the net soil transport on the soil depth. The existence of soil implies that, locally, conditions are more favorable to weathering than to erosion (Anderson and Anderson, 2010), a situation termed transport-limited erosion (Carson and Kirkby, 1972). Since sediment transport is strongly relief-dependent (Montgomery and Brandon, 2002), one should expect small erosion rates, thus thicker soils, in regions of low relief.

Different analytical models describing soil production have been proposed, the exponential decay (Heimath et al., 1997) of soil production with depth is almost universally applied. However, there are several reasons to prefer a power-law formulation. The most important is probably the proposed and observed correlation between soil production and chemical weathering rates as functions of time and depth (Burke et al., 2007; Dixon et al., 2009; Egli et al., 2014; Hunt and Chanbarian, 2016). The importance of this correlation lies in the demonstrated (White and Brantley, 2003) power-law dependence of chemical weathering over time scales up to 6 My, which greatly exceeds the typical range of time scales addressed in most of the Heimath references, thus clarifying the actual time-dependence. Further, Hunt and Chanbarian (2016) also showed that the results of Friend (1992) and of Egli et al. (2014) for soil production rates were consistent with the same power-law form decay as chemical weathering rates, but to even larger timescales.

Based on previous work in modeling the production of soil using a simple power law that is derived from percolation theory (Hunt and Chanbarian, 2016), here we propose a model to describe the formation...
of soil. To avoid any confusion due to the existence of interchangeable concepts of “soil production” and “soil formation,” in this paper, the term “soil production” refers to gross soil produced from the conversion of biomass, while “soil formation” refers to the net effect by considering the removal of soil by erosion.

As proposed by Hunt and Ghanbarian (2016), soil production rate diminishes with a power of \(-0.47\) with time (or a decay power of \(-0.87\) with soil depth), and it is proportional to the deep infiltration rate in the field. Such a model involves the direct relevance of two soil-forming factors: climate and time, and possible indirect relevance of topography in affecting local climate and hydraulic conditions. The model of soil formation presented in this paper will take the erosion (which itself is highly affected by topography) of soil into account. There are two other soil-forming factors left when discussing soil formation: parent materials and organisms. Effects from organisms, other than their contribution of CO2 through respiration, on soil formation are beyond the scope of this research. Besides the composition of soil resulting from parent materials, what is more of concern in modeling soil formation is probably the effects of parent rocks on production rate. The soil production function this work is based on assumed weathering of (typically fractured) bedrock or regolith is solute transport-limited, so that the production of soil can be modeled without considering how resistant the parent materials are to weathering, since the production rate only depends on solute transport. The question of when weathering rates are limited by solute transport is usually addressed using the Damköhler number, \(Da\), which is the ratio of a solute advection time to a reaction time. When \(Da > 1\), transport limitations are considered relevant. Previous work studying Damköhler number (Yu and Hunt, 2017a) shows that over timescales exceeding days chemical weathering is practically always limited by transport. The solute transport limitations arise primarily from the diminishing solute velocity over time, which decays according to a power law. An additional factor is that fluid flow rates in nature are typically orders of magnitude slower than those investigated in laboratory experiments. Each of these inputs increases time scales associated with solute advection compared to what is often inferred, and guarantee that, over time, solute transport limitations to removing weathering products exceed the limitations from the kinetics of the weathering reaction.

Our approach is based on a network picture of the medium, in which percolation concepts can deliver the statistics and topologies of the dominant flow paths, including preferential flow. In accord with this hypothesis, we use percolation exponents to describe the tortuosity of such paths. Herein lie both similarities and differences to the approach of Lin (2010), which also addresses the influences of dominant flow paths over soil networks. In contrast to Lin’s conjecture, we do not incorporate concepts of an evolution of the soil network to a state of higher organization and lower entropy, thus higher flow and solute transport rates. In the constructal law analogy invoked by Lin, one should expect an evolution towards more rapid flow and transport, with, e.g., smaller tortuosity of the flow paths. While very specific evidence for such evolution does indeed exist in river networks (which have values consistent with constructal theory at continental scales, rather than percolation theory values at all smaller scales (Hunt and Yu, 2017)), our exploration has not yet revealed such relevance at soil scales. In fact, the reverse seems to occur. Vegetation seems rather to adapt to soil networks in such a way that their roots exploit existing optimal paths of percolation theory (Hunt, 2016, 2017).

We should also mention that the “energy” formulation of Rungé (1973) bears some resemblance to ours. Rungé’s conceptual development also relates to Douchaev’s soil formation factors, but, like ours, emphasizes the importance of vertical water flow. In our formulation, based on chemical weathering, the organisms supply the CO2 for the weathering process, and the water flow rate, together with the pore separation (particle size) sets the fundamental time scale in the equation. This emphasis on water flow rates, which brings in climate (precipitation), vegetation (less evapotranspiration) plus topography (plus run-on less run-off) echoes the energy theory of Rungé (1973), as discussed by Schaezli and Schimper (2006).

Finally, in conformance with the general perspective of Banwart et al. (2011), our discussion of the relevance of erosion then compares with our modern understanding of the importance of soil loss due to human activities generally. Banwart et al. (2011) note that soil erosion has increased by nearly a factor 100 due to various human endeavors. As will be seen below, such an increase in erosion rates leads, in our predictions, to a decrease in steady-state soil thickness by a factor 100 \(1.14 = 191\), which would make soils everywhere useless for any usual human requirements.

2. Theoretical basis

Non-Gaussian (or non-Fickian) transport is becoming recognized as the norm in natural porous media (e.g., Cushman and O’Malley, 2015). Like common descriptions of non-Gaussian transport such as Continuous Time Random Walk (CTRW) (Margolin and Berkowitz, 2000; Berkowitz et al., 2002; Bijelic et al., 2004) or Fractional Advection Dispersion Equation (FADE) (Meerschaert et al., 1999; Benson et al., 2000; Parchepsky et al., 2006; Kreyenschke et al., 2006), both of which model power-law solute arrival time distributions, percolation theory generates fat-tailed arrival time distributions, of approximately power-law form. However in contrast to both of these techniques, percolation theory does not yield an arbitrary power, the power-law arrival time distributions relate to the fractal dimensionality of the percolation backbone (Lee et al., 1998; Shepard et al., 1998; Hunt and Skinner, 2008, 2010; Hunt et al., 2011; Ghanbarian-Alavijeh et al., 2012), making predictions of the actual power possible. Ability to predict this power in solute transport confers the ability to predict the power in the power-law of solute transport.

The solute transport theory was first developed in Hunt and Skinner (2008) by applying the framework of critical path analysis (Ambegaokar et al., 1971; Pollak, 1972) developed within percolation theory (Shante and Kirkpatrick, 1971; Kesten, 1982; Stauffer and Aharony, 1994; Hunt and Ewing, 2009) to a network representation of the medium (Fatt, 1956) in order to quantify all distinct solute transport paths that can be connected through the system, and the fluid fluxes along those paths. The theoretical treatment to develop an entire solute arrival time distribution has been described in a series of existing publications (Hunt and Skinner, 2008, 2010; Hunt et al., 2011; Ghanbarian-Alavijeh et al., 2012). Knowing the topology of each system of paths (Sahimi, 1993; Lee et al., 1999; Shepard et al., 1998) together with the associated fluid fluxes, made it possible to calculate the variations in the characteristics along the path, and thus the total solute transport time (Hunt and Skinner, 2008, 2010; Hunt et al., 2011; Ghanbarian-Alavijeh et al., 2012). This treatment generated the solute arrival time distribution as a function of transport distance, and the results are in agreement with simulations (Liu et al., 2003). It was also applied to relevant studies including the prediction of silicate weathering rates and laboratory experiments on reactive solute transport (Hunt, 2015), the description of soil production as a function of time (Hunt and Ghanbarian, 2016), the evaluation of relevant importance of solute transport in limiting chemical weathering (Yu and Hunt, 2017a), and the examination of the steady-state assumption in certain landscape evolution models (Yu and Hunt, 2017b).

From percolation theory, the solute transport time, \(t\), increases more rapidly than linearly with transport distance, \(x\), in particular, as a power of \(x\) equal to the percolation backbone fractal dimensionality, \(D_s\) (Lee et al., 1999). For 3D flow in saturated media and for a wide range of conditions, \(D_s = 1.87\) (Shepard et al., 1999). Sahimi and Mulchepadyay (1996) discuss limitations on this result arising from certain classes of long-range correlations in the pore space of the medium itself. Under 2D saturated conditions, such as a flow along a fracture plane, or along the walls of a cylindrical core, for example, \(D_s = 1.64\). However, field observations (Class et al., 1998) and experimental
results show that unsaturated conditions are common when flow is constrained to 2D surface, in which case that \( D_h = 1.21 \).

For unsaturated 3D flow, \( D_h = 1.46 \) for drying, but \( D_h = 1.861 \) for wetting conditions. In summary, using the exponent values given in (Shepard et al., 1999):

\[
\begin{align*}
\text{t} &= t_0 \left( \frac{x}{x_0} \right)^{1.48} \qquad (\text{3D drainage}) & \text{t} &= t_0 \left( \frac{x}{x_0} \right)^{1.87} \qquad (\text{3D saturated or imbibition}) \\
\text{t} &= t_0 \left( \frac{x}{x_0} \right)^{1.64} \qquad (\text{2D saturated}) & \text{t} &= t_0 \left( \frac{x}{x_0} \right)^{1.22} \qquad (\text{2D unsaturated})
\end{align*}
\]  

In the second expression in Eq. (1a), the distinction between 1.87 and 1.861 is neglected. Here we expect that 3D network connectivity is more common than 2D conditions, though perhaps is not universally applied. Also, since drying typically occurs due to plant transpiration, which involves either horizontal or upward moisture fluxes, we propose that any soil-formation processes occurring during drying can be neglected. Thus we can use the second expression in Eq. (1a) to describe solute transport distances.

Then we have, in the absence of soil erosion, the soil depth \( x \) as a function of time,

\[
x = x_0 \left( \frac{t}{t_0} \right)^{0.51} = x_0 \left( 1 + \frac{t}{t_0} \right)^{-0.49}
\]  

where \( x_0 \) is a typical particle size, \( x_0/x_0 \) is the net (deep) pore-scale infiltration rate. When these are all known for a given site, all adjustable parameters are eliminated in our model.

By taking the derivative of Eq. (2) one can derive the equation for soil production (bedrock-to-soil conversion, or weathered regolith creation).

\[
\frac{dx}{dt} = R_i = \frac{1}{1.87} \left( \frac{x}{x_0} \right)^{0.67} = \frac{1}{1.87} \left( \frac{x}{x_0} \right)^{0.67} - E(t)
\]  

In these expressions, \( x_0/x_0 \) is the net infiltration rate, with \( t = P - AET + R_{run-on} - R_{run-off} \). It is the average evapotranspiration, and \( \delta \) is the soil porosity (Hunt and Ghanbarian, 2010; Yu et al., 2017).

Adding the effects of an erosion rate, \( E(t) \), generates the expression of rate of soil formation (net production),

\[
\frac{dx}{dt} = R_i - E(t) = \frac{1}{1.87} \left( \frac{x}{x_0} \right)^{0.67} - E(t)
\]  

In principle such an equation can be solved numerically for \( x(t) \) with an arbitrary erosion rate, but to date we have only addressed the case \( E(t) \) being a constant, \( E \). In addition, such an equation can form the basis for a landscape evolution model if horizontal transport of soil is incorporated, but our efforts in that area are still ongoing. Soil depth at time \( t \) can be calculated from the integration of Eq. (4),

\[
x(t) = \int_0^t \left( K(E) - E \right) dt = \int_0^t \left( \frac{1}{\delta} \left( \frac{x(t)}{x_0} \right)^{0.67} - E \right) dt
\]  

Since the integral equation Eq. (5) does not have a closed-form analytical solution, soil depths are solved numerically.

3. Material and methods

Predicted soil depths using Eq. (5) were compared with published field data of alpine soils in the European Alps, Gongga Mountain, China, and Cordillera Vilcanota and Quelccaya Ice Cap region (CV-QIC), southwest of Peru, and soil developed on alluvial fan and terrace at Merced River, California (White et al., 1996; Goodman et al., 2001; He and Tang, 2008; Eglit et al., 2014). Theoretical soil formation rates for alpine soils predicted using Eq. (4) were also compared with predictions using soil age-profile thickness (SAST) method done by Eglit et al. (2014).

Soil at the Merced River terraces is dominated by sand ranging from 320 to 730 \( \mu \)m, and generates an average particle size of 530 \( \mu \)m. Particle size at Gongga Mountain, China has a much wider range from 65 to 2000 \( \mu \)m, so a geometric mean of 400 \( \mu \)m is taken as typical particle size. The papers of European Alps and CV-QIC, do not sufficiently refer particle size, thus a typical particle size of 30 \( \mu \)m is used here. Silt is the middle particle size (geometric mean) class in soil classification schemes, and a middle silt particle size would thus, in the absence of any information regarding soil texture at a given site, be the best estimate for a median particle size. Silt particle sizes range from 2 to 63 \( \mu \)m (USGS), with a mean value of 32 \( \mu \)m. The individual arithmetic means of the three principal soil particle classes, clay, silt, and sand also generates a geometric mean of 30 \( \mu \)m. This is why a typical particle size of 30 \( \mu \)m would be a reasonable choice. Net infiltration rate \( i \) is calculated by considering not only precipitation, but also the water lost to evapotranspiration and what runs off from surface flow (net run-off). Lowitich (1973) estimates that, globally, \( AET = 65\% \) of \( P \), and 24\% of \( P \) travels to streams by overland flow, leaving only 11\% of \( P \) for deep infiltration. However, it is not always the case that the difference between run-on and run-off to be negative. For local sites, there can be a net gain of water from run-on less run-off. We assume a typical porosity of 0.4 (Hill, 1968) to calculate the pore-scale infiltration rate. Relevant parameters used for each site for prediction are summarized in Table 1.

Run-off is 1.08 m/y (Juen et al., 2007) with 6\% from non-glacial melting (Kronenberg et al., 2016). Due to the lack of information of erosion rates at CV-QIC, values were estimated by considering the

<table>
<thead>
<tr>
<th>Site</th>
<th>( x_0 ) [( \mu )m]</th>
<th>( P ) [m/y]</th>
<th>( AET ) [m/y]</th>
<th>( Run-off ) [m/y]</th>
<th>Upper bound [m/y]</th>
<th>Lower bound [m/y]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E. Alps</td>
<td>30</td>
<td>1.1 to 2</td>
<td>0.27</td>
<td>1.06</td>
<td>1.975</td>
<td>-</td>
</tr>
<tr>
<td>CM*</td>
<td>400</td>
<td>0.7</td>
<td>0.32 to 0.99</td>
<td>1.825</td>
<td>2000</td>
<td>0.65</td>
</tr>
<tr>
<td>600</td>
<td>0.3</td>
<td>0.26</td>
<td>0.01</td>
<td>0.1</td>
<td>75</td>
<td>0.1</td>
</tr>
<tr>
<td>30</td>
<td>1.5</td>
<td>0.65</td>
<td>2.13</td>
<td>100</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* E. Alps = European Alps, \( P \) values are from Eglit et al. (2014), \( AET \) from the evapotranspiration map of Europe on IMPACT20 WebAtlas (IMPACT20 WebAtlas, accessed in Dec 2018), \( Run-off \) is equal to \( P \) - \( AET \) - \( Run-on \). Values of \( P, AET \), and \( Run-on \) were taken from the literature as above.

* CM = Gongga Mountain, China, \( P \) value is from He and Tang (2008), \( AET \) is between 0.6 and 0.8 m/y (Gao et al., 2007), mean value of 0.7 is taken here. Erosion rates range between 2000 and 3000 m/m/y (Juen et al., 2007). Hence the rate of 1.08 m/y is considered as the average rate.

* MC = Merced River, \( P, AET \), and \( Run-on \) are from the literature (Juen et al., 2007). \( P \) is between 0.7 to 20 \( \times 10^{-3} \) m/y (Foley et al., 2010).

* QIC = Cordillera Vilcanota and Quelccaya Ice Cap region, annual water accumulation (taken as infiltration rate here) is approximately 1.5 m/y (Goodman et al., 2001).
denudation rates across eastern Bolivian Andes (19–22°S), and the denudation rates at northwestern Peruvian Andes. For northwestern Peruvian Andes, erosion rates range from 100 to 200 m/M.yr (Abbruzzo et al., 2010). A comparable denudation rate of 150 m/M.yr was gener-
ated from the geometric mean of denudation rates across eastern Bolivian Andes (Barnes and Pelletier, 2006). Therefore, 100 to 200 m/M.yr might be reasonable for erosion rates at CV-QIC.

Given the wide range of parameters at the sites, upper and lower limits of predictions are calculated using the limits of individual parameters. Generally, upper bound is generated using highest I (obtained from highest P, and lowest AET and Run-off values), with the lowest E, and vice versa for the lower bound. But for European Alps, since the lowest precipitation of 1.1 m results a negative I, only the upper bound is considered in this case.

4. Results

Comparison of theoretical soil depths predicted at European Alps, Gongga Mountain, CV-QIC, and Merced River from Eq. (5) and the observed soil depths are shown in Figs. 1–6. The majority of the observed soil depths are within predicted limits. In Fig. 1, the discrepancy is largest at small soil ages, especially at Morteratsch. Morteratsch itself shows a large scatter of soil depths at similar soil ages. Such scatter indicates a possible unstable environmental condition or that unusual climate events occurred at Morteratsch, which could affect the agreement, since uniform parameters such as a constant erosion rate are used to generate theoretical soil depths.

Agreement of observations and predictions improves as soil age increases, suggesting that our model performs better for older soils. Depths of younger soils (shallow soil depths) tend to be more vulnerable to external influences such as trampling from animals, changes of environmental conditions, and potential stochastic nature of erosion, including effects of landslides or avalanches. As the time scale increases, parameters of environmental conditions such as infiltration rate and erosion rate are getting closer to the historical average values, and the fluctuations caused by any short-term environmental changes will be buffered, and become less important. Thus as soil age increases, it is more predictable by the proposed model dealing with ideal circumstances with relative stable environmental conditions.

Soil formation rates predicted from Eq. (4) are compared with predictions using SAST methods (Egli et al., 2014). All of our results are generated using highest infiltration rate and smallest erosion rate for particular site to predict highest soil formation rates. Fig. 5 shows results for European Alps and Fig. 6 summarizes all 3 alpine soils at European Alps, Gongga Mountain, China, and CV-QIC, southeastern Peru. Soil formation rates predicted from the two methods agree well in time scale spanning 5 orders of magnitude, with an overall slightly lower predictions from our model, especially for CV-QIC. Moreover, by taking uniform parameters in our model, soils with the same age at particular site generate identical formation rate, while various rates are obtained from SAST method. From Figs. 5 and 6, most of our predictions pass within the scatter generated from SAST method at same soil ages. By neglecting any specific micro environmental conditions for individual soil samples, our model seems to generate an average soil formation rate among soils at same soil ages.

5. Discussion

For all 4 study sites presented in this paper, majority of the field data are within the limits of maximum and minimum theoretical soil depths predicted from the proposed model (Eq. (5)). Upper and lower bounds of predictions are needed here due to the uncertainties and wide ranges of infiltration and erosion rates at each site. There are field results that lie outside the bounds of our predictions such as some soils at Morteratsch, European Alps. As a complicated process, soil formation is highly affected by environmental conditions, reflected by the high sensitivity of the soil-formation model to site parameters (particle size, infiltration rate and erosion rate). Thus specific site information is needed to generate exact predictions so that the accuracy of the model can be examined. However, the agreement between soil depths observed in the field and predictions across 4 different sites as well as the consistency of soil formation rates of alpine soils between our predictions and results from SAST methods (Egli et al., 2014) indicates possible implication of the proposed models in describing and predicting the formation of soil. By considering uniform parameters and constant erosion rates, the models presented in this paper deal with ideal circumstances with steady environmental conditions. Results show that the agreement between observation and prediction improves
as time scale spans (at older soils). Possible reasons could be 1) younger soils with shallow soil depths are more vulnerable to external influences, 2) as timescales extends, any effects from short-term changes of the environment become less important. Therefore, large timescales allow soil to get closer to ideal circumstance, and make it more predictable by our theoretical model.

Landscape is sculpted by geomorphic processes such as soil production, soil erosion, among with other things. It has become clear that quantitative modeling of soil formation is critical to understanding landscape evolution. A number of landscape evolution models are built with adaptions of existing soil production models (Heimsath et al., 1997, 2000, 2001a, 2001b; Minasny and McBratney, 1990, 2001; Cohen et al., 2010; Liu et al., 2013). Since soil property strongly influence the direction and rates of geomorphic processes (Minasny and McBratney, 2006), soil production models also contribute studies in pedogenesis models and the development of coupled geomorphological and pedogenic models (Minasny and McBratney, 2006; Welivitiya et al., 2016). Thus, the presented model in this paper will inform further study in evolution of soil and landforms. One of the applications is in examination of the steady-state assumption of certain soil development models (Yu and Hunt, 2017b), which is the simplest case in landform evolution.

The predicted soil evolution may also be relevant for Critical Zone Observatories (CZOs). In Fig. 7, we also show a comparison of our predictions using Eq. (5) with simulated results of soil evolution in Shale Hills CZO from a geomorphic-based analytical model (Liu et al., 2013). Simulation is obtained from the digitization of Fig. 9 (a) in Liu et al. (2013) (only the simulated results with initial soil depth equal to zero is digitized to match our initial condition). Value of annual infiltration rate is estimated from averaged recharge rates in 2009 and 2010 from the website of Shale Hills Critical Zone Observatory (CZO) (Susquehanna Shale Hills Critical Zone Observatory Website, accessed on July 14th, 2017). Erosion rate is about 15 m/My (Ma et al., 2010). To calculate the pore-scale infiltration rate, porosity is needed. Five series of soils are identified in the study site, and soil textures are mainly silty loam and silty-clay loam, with an averaged porosity around 0.52 down to B horizon (Lin, 2006). Kuntz et al. (2011) give a lower value of porosity of 0.45 for Eraner series. Silty loam has a d50 roughly 18 μm (Skaggs et al., 2001), which could be a reasonable estimation of the particle size. However, Lin (2010) discusses the importance of the soil structure, i.e., the soil aggregates, which act like much larger particles.

Buzawa et al. (2012) also emphasizes aggregate importance to flow paths at CZO sites including Shale Hills. Thus, particle size in Shale Hills could be larger than 18 μm. Typical particle size of soil (30 μm) might be a reasonable choice. Since our model is very sensitive to both of these two parameters, scenarios with various estimations of particle size and porosity are predicted and shown in Fig. 7. Although we don’t generate precise agreement with simulations from Liu et al. (2013), which itself has an RMSE of 0.394 m comparing with observed values, our predictions using 30 μm of particle size and 0.52 porosity are close to the predicted results. However, without the knowledge of observed soil depths and information of soil properties, it is difficult to evaluate the precision of our model in predicting soils in the Shale Hills CZO.

6. Conclusion

Despite the discrepancy in soil depth at small soil ages which itself is highly affected by external influences and shows fluctuation and
IV. MODEL EVALUATION II: PREDICTING THE VARIABILITY OF SOIL DEPTH
ACROSS DIFFERENT CLIMATIC CONDITIONS

Article III.

Article

A Simple Model of the Variability of Soil Depths

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Abstract: Soil depth tends to vary from a few centimeters to several meters, depending on many natural and environmental factors. We hypothesize that the cumulative effect of these factors on soil depth, which is chiefly dependent on the process of biogeochemical weathering, is particularly affected by soil porosity (i.e., solute) transport and infiltration from the land surface. Taking into account evidence for a non-Gaussian distribution of rock weathering rates, we propose a simple mathematical model to describe the relationship between soil depth and infiltration flux. The model was tested using several areas in mostly semi-arid climate zones. The application of this model demonstrates the use of fundamental principles of physics to quantify the coupled effects of the five principal soil-forming factors of Dokuchaev.

Keywords: soil formation; percolation; infiltration; erosion

1. Introduction

The concepts of soil formation have been extensively examined, starting from the beginning of the 19th century (Justus von Liebig; see http://www.madehow.com/knowledge/Justus_von_Liebig.html), and thereafter modified and refined by many world-renowned soil scientists, e.g., Charles Darwin [1] in England, Vasily Dokuchaev [2] in Russia, and Grove Karl Gilbert [3]. George Nelson Coffey [4], and Eugene W. Hilgard [5] in the United States. The conceptual approaches to a pedogenic theory proposed by many scientists are fundamentally different, and have been revisited many times [6-8]. Although these theories are conceptually different, they all generally converge over the idea of Dokuchaev’s five natural soil-forming factors: biota impact, climate impact, initial material, terrain, and time, as quoted in both Glinka [9] and Jenny [10]. Dokuchaev, as quoted in Glinka [9], emphasized the necessity to determine the solution of the soil-forming factor equation, stating that:

In the first place we have to deal here with a great complexity of conditions affecting soil; secondly, these conditions have no absolute value, and, therefore, it is very difficult to express them by means of figures; finally, we possess very few data with regard to some factors, and none whatever with regard to others. Nevertheless, we may hope that all these difficulties will be overcome with time, and the soil science will truly become a pure science.

The consideration of these theories and factors provide a more comprehensive view of soil formation than either can do alone [11] for different types of landscapes, including those dominated by exposed bedrock, or fertile soils.
For later reference, soils are most commonly divided into three horizons or layers: O, A, and B, although the E, P, and C horizons are also fairly commonly discussed. In short, the O layer, present in forests, but not grasslands, is dominated by organic material, e.g., decaying plant or animal matter. The A and B layers were defined originally by Dokuchaev; the A horizon being the topsoil or humus, which is typically brown or black due to its high organic content, while the B layer is called the subsoil, and is typically more brightly colored due to the presence of clay minerals and iron oxides. However, even the full traditional classification scheme does not capture the modern understanding of soil evolution completely.

The evaluation of the lower boundaries of soil is important for many scientific and practical applications, such as agricultural and hydrological studies [12]. The depth of the soil mantles the Earth’s surface tends to vary from a few centimeters to several meters or even tens of meters [13], depending on many natural and environmental factors. For example, Hillel [14] described soil as the “top meter or so of the Earth’s surface, acting as a complex biophysical organism”. It is well recognized that an understanding and a prediction of how and where water infiltrates from the land surface and moves through the vadose zone within a landscape controls the process of biogeochemical weathering, and soil hydrology can be used to explain soil morphology and an ecosystem’s dynamical functions [12]. It has been proven that soil is being transformed globally from natural to human-affected material, the lower boundary of soil is much deeper than the sol num historically confined to the O to B horizons, and most soils are a kind of paleosol, being products of many soil-forming processes that have ranged widely over the lifespans of most soils. In other words, a soil’s polygenesis is dependent on fluxes of matter and energy, which are thermodynamically transforming soil systems [15,16]. Nevertheless, when one compares predictions with data for soil depth, it is necessary at least to hypothesize the relevance of theory to a particular boundary, which we have consistently chosen to be the bottom of the soil Bw horizon, an oxidation depth.

We pose the question: are these soil fluxes dependent on infiltration from the land surface, and does the spatial variability of infiltration underlie soil depth variability?

Water flow, soil erosion (and deposition), and soil formation all affect soil depth. Soil erosion is chiefly accomplished through advective processes such as overland flow and rainsplash [17], though soil creep and a number of other processes contribute as well. Biogeochemical weathering, a basis of soil formation, requires water to carry reaction reagents to the weathering front, and reaction products away [18], and thus relates to deep infiltration. Erosion rates vary over about 4 orders of magnitude, from a fraction of a meter per million years in the interior of Australia, to a maximum of over 1000 m per million years in the Himalayas [19]. Precipitation rates vary from 2 mm/year in the Atacama Desert to 10 m/year in the New Zealand Alps (and in many other regions). The soil production rate is linearly proportional to precipitation [18,20], and has been reported as an exponentially diminishing function of soil depth [21], though the “humped” function was reported in several recent studies; see Heimsath et al. [22]. Soil erosion and soil production must be correlated, which is a stipulation guaranteed by the conditions of an apparent steady-state landscape evolution, but is also possible if the fundamental physical processes controlling the soil formation processes are related, even when steady-state conditions do not apply.

Historically, soil formation $s$ has been represented in terms of a single formula, in which the principal factors of formation $s = f(cl, o, r, p, t)$, are represented independently from each other, e.g., $cl$ for climate, $o$ for organism, $r$ for topography (relief), $p$ for parent material, and $t$ for time [9,10]. A guiding convention has been that soil is predominantly formed due to biogeochemical weathering, as a combination of physical, chemical, thermal, and biological processes together causing the disintegration of rocks, an evolutionary process that does not stop with the initial formation of soil. These processes, themselves, are limited by the atmosphere–rhizosphere, subsurface interaction, and in particular, infiltration from the land surface and solute transport in the unsaturated (vadose) zone. For example, organisms and precipitation supply the CO$_2$ necessary to drive silicate-weathering
processes [23]. Soil processes are also affected by erosion, deposition, (e.g., aeolian) plant root uptake, microbial processes, infiltration, and evapotranspiration.

In the following, we present the general model. Then, we test it: first to see whether the values of the typical input variables generate values in accord with typical soil depths around the globe, then to see whether it reproduces variability in soil depth in accord with that observed over variable climatic input and what is known about parent material particle size variability with respect to topography (slope). Finally, we consider the implications of our model treatment and its wide range of applicability for landscape evolution concepts and discussions of agricultural sustainability.

2. General Model

The model for soil formation derives from percolation theory for solute transport in porous media. Percolation theory can be applied to enumerate the dominant flow paths when the medium is highly disordered. Chemical weathering in situ is shown to be transport-limited [24,25], but the lower boundary of chemical weathering is the bottom of the B horizon. The soil depth, neglecting erosion, is taken to be the distance of solute transport [18]. Percolation concepts that relate solute transport time and distance thus relate soil age and depth through the process of transport-limited chemical weathering [24].

It has been shown [18] that, when erosion (and deposition) can be neglected,

$$x = x_0 \left( \frac{t}{t_0} \right)^{1/D_b}.$$  \hspace{1cm} (1)

Equation (1) describes the evolution of soil depth $x$ as a function of time $t$. This expression has been derived based on the results of field and laboratory investigations of solute transport [25,26] associated with the chemical weathering of soil. It has also been assumed that this equation can be used to describe the bottom depth of the Bw horizon. Here, $D_b = 1.87$ is the fractal dimensionality of the percolation backbone for vertical flow, with 1.87 valid for full saturation and three-dimensional connectivity. In percolation theory, the backbone is obtained from the dominant, optimally connected flow paths by trimming off portions that connect only at one spot, called dead-ends. The mass fractal dimension of the backbone provides the scaling exponent relating time and distance. Predominantly downward flow occurs also under wetting conditions, but in this case the correct exponent is only slightly different, with $D_b = 1.861$, an insignificant difference from 1.87. In this expression, $x_0$ is a characteristic particle size of the soil parent material, and $(x_0/t_0) \equiv 1/\phi$, with

$$I = P - \text{run-on} - \text{run-off},$$

where $I$ is the net infiltration rate; $\phi$ is the soil porosity (used to change the Darcy velocity to the pore-scale velocity); $P$ is precipitation; and AET is the average evapotranspiration. The soil production function is then given by

$$dx/dt = (1/D_b) \left( \frac{x_0}{t_0} \right) \left( \frac{x}{x_0} \right)^{D_b} - 1.$$  \hspace{1cm} (2)

Erosion is assumed to be taken into account by subtracting a constant, $E$, from the right-hand side of the differential equation. When soil erosion and soil production processes are equal in magnitude, $dx/dt = 0$, and the soil depth, $x$, is given by the equation

$$x = x_0 \left( \frac{I}{1.87\phi E} \right)^{1.15}. $$  \hspace{1cm} (2)

The power 1.15 of Equation (2) is $1/(D_b - 1)$.

Equations (1) and (2) implicitly represent a combination of the effects of climate, topography, and evapotranspiration. Equation (2) does not contain a time variable, since this equation represents the solution of Equation (1), consistent with an asymptotical convergence of the soil formation depth to a steady state value.
3. Predicting a Typical Soil Depth

If a typical soil depth is a depth you would expect to measure, this equates the term, in a narrow sense at least, to a mean soil depth. However, the actual values of soil depths vary from zero to tens of meters. What is a typical soil depth? Batjes [26], who considered the 4353 soils in the World Inventory of Soil Emission (WISE), used it to build up UNESCO’s (United Nations Educational, Scientific, and Cultural Organization) database of 106 soil types presented on its soil map, assuming a characteristic soil depth of 1 m. Montgomery [27] gives a mean soil depth of 1.39 m, with a mean soil depth of 2.74 m for native vegetation, and 2.01 m for soil production areas. Hillel suggested that 1 m is a typical soil depth. What value would be suggested by Equation (2)?

Take \( \Delta = 30 \mu m \), the size of a typical silt particle. Silt is the middle particle size (geometric mean) class in all soil classification schemes, and \( 30 \mu m \) is the middle (arithmetic mean) of the silt range. The same value, \( 30 \mu m \), is also the geometric mean of the individual arithmetic means of the three principal soil particle classes, clay, silt, and sand. This particular length scale relates most closely to parent material, whether the soil is weathering from a bedrock with a specific mineral size, or whether it is forming on, e.g., an alluvial deposition. To calculate a mean infiltration rate, we must consider not only the precipitation, but also the water lost to evaporation and transpiration as well as what runs off. These variables relate to climate, the hydraulic conductivity of the substrate, and to the role of plants in the water cycle. Schlesinger and Jasechko [28] estimate that, globally, transpiration constitutes 61% of AET, and returns approximately 39% of P to the atmosphere. Thus, AET represents a mean fraction \( 0.61 \) of \( P \). Lvovich’s [27] estimation that \( AET = 65\% \) of \( P \) is almost identical, and he also gives a global mean precipitation of 834 mm. Lvovich [29] estimates that a global mean of 24% of \( P \) travels to streams by overland flow, leaving only 11% of \( P \) for deep infiltration. The mean terrestrial \( P \) is reported as between 850 mm and 1100 mm [30], with a mean of 975 mm. Sixty-four percent of 975 mm is 624 mm, leaving 351 mm for \( P - AET \). However, 11% of 975 mm is only 102 mm. On any local site, however, the difference between the run-on and the run-off can be either positive or negative. Thus, these estimates suggest that the amount of water reaching the base of the soil should be a column of water somewhere between 102 mm and 351 mm. Alternatively, we can consider the mean global AET over cold, temperate, and tropical, forested and non-forested, regions. Using the six different values given by Peel et al. [31] for these biomes generates an AET value of 654 mm, which is fairly close to the value inferred from Schlesinger and Jasechko [28], and implying \( AET = 321 \) mm. The actual infiltration rate is obtained from I through division by the porosity. We assume a typical porosity of 0.4, leading to values of \( I/P \) between 255 mm/year and 878 mm/year (using a combination of Schlesinger and Willmott's numbers), or between 225 mm/year and 735 mm/year (using the numbers of Lvovich). These values average to 566 mm/year, or 480 mm/year, depending on the particular estimates applied, and are reasonable. A typical erosion rate of about \( E = 30 \) m/Myear = \((1 \text{ m/Myear}) \times (1000 \text{ m}/\text{Myear})^{1/3}\), is obtained from the geometric mean of the range of erosion rates discussed in Bierman and Nichols [19]. Using \( \Delta = 0.0003 \text{ m}, I/P = 806 \text{ mm/year}, \) and \( E = 30 \text{ m/Myear}, \) and the first range of \( I \) values given, the result for \( x \) is \( 0.48 \text{ m} < x < 1.53 \text{ m} \), while for the second range of \( I \) values given, \( 0.42 \text{ m} < x < 1.81 \text{ m} \). Both the arithmetic and geometric means of both ranges cluster around 1 m.

4. What Can We Say about the Variability of Soil Depths?

Let us consider first the ratio \( I/E \), which, raised to the power 1.15, has the potential to produce the greatest variation (range) in soil depths. In fact, this ratio should be quite insensitive to \( P \), since both \( I \) and \( E \) tend to increase with increasing precipitation. For example, Dunne et al. [32] found a linear relationship between \( I \) and \( P \), in general accord with the previously cited tendency for AET to be roughly half of \( P \). Reiners et al. [33] reported a linear relationship between \( P \) and erosion, \( E \). Their study utilized a rainfall gradient at similar temperatures across the Cascade Mountains in Washington State, United States. Along this transect one should thus expect roughly constant soil depths, and the ratio \( I/E \) should, in the absence of steep topography, remain relatively invariant.
What about trends with temperature? Data from Sanford and Selnick [34] revealed a tendency for the fraction of precipitation lost to AET to increase with increasing temperature, particularly in conjunction with aridity. Thus, the conclusions of Heimsath et al. [35] regarding the results of their Australian measurements, “[the suite of results from different field sites indicates that erosion rates generally increase with increasing precipitation and decreasing temperature,” indicate that the processes of soil formation may be dependent on evapotranspiration. Consequently, the water potentially available for either infiltration or overland flow, \((P - AET)\), serves as a predictor of \(E\) and soil formation, rather than simply \(P\). We, therefore, hypothesize that both the numerator and denominator in Equation (1) would contain a proportionality to the quantity \((P - AET)\), meaning that weather conditions (within a specific climatic zone) would have far less influence on soil depth than commonly assumed. However, \(I\) and \(E\) can be expected to have a complementary dependence on the partitioning of water to overland flow, which brings in the effect of topography. The relationship between the potential evaporation, evapotranspiration, precipitation, and runoff was considered in great detail by Budýko [36], and many soil scientists and hydrologists followed Budýko’s approach, e.g., Gentine et al. [37].

Concerning topography, regions with steeper topography will tend to have higher overland flow, and thus higher erosion rates, resulting in lower infiltration and soil formation rates. As an example, Burbank et al. [38] found that erosion rates and precipitation in the Himalayan mountains were not correlated (in contrast to Reiners et al. [33]), and attributed their anomaly to the strong tendency for the precipitation to decline where the slope was increasing. Notably, however, the declining precipitation with increasing slope should lead to a diminution in soil production compared with erosion, and a higher probability of bedrock exposure, as is indeed the case in this region. Divergent topography, with concomitant divergence in surface water flux and therefore soil transport, will produce thinner soils than convergent topography, as noted in Heimsath et al. [39], a tendency intensified by steeper topography generally. Our reasoning, though it may be accentuated in reality by lateral soil transport [39], does not depend on such transport, and is merely a consequence of the greater infiltration values in topography that is convergent and not so steep that soil covering is missing entirely.

How does soil depth depend on the particle size of original sediments? This question is more nuanced than the previous question. Soil depths should nominally be proportional to particle sizes. However, erodibility has a strong dependence on particle size, first increasing with increasing size from clay to silt, and then decreasing with increasing size at larger sizes. The seeming anomaly at small particle sizes is due to the cohesive forces between clay grains, which are typically charged. Thus, as long as \(I\) is principally precipitation-limited, a decrease in particle size below silt size tends to reduce soil erosion, on account of the increasing cohesive forces between the grains. Therefore, \(P_0\) for finer soils at least, should be positively correlated with the erosion rate, \(E\), and the two stated influences will tend to cancel. However, at larger particle sizes, increasing particle sizes should tend to decrease \(E\), accentuating the tendency for soils to be deeper; although, if the argument is turned around, a greater importance of erosion will tend to remove finer components, leading to a coarser soil. Sandy soils should thus have the deepest weathering horizons, although for larger particle sizes, the term soil is not characteristically employed. If \(I\) is principally hydraulic conductivity-limited, however, then greater precipitation rates, \(P\), will not tend to increase \(I\), but will tend to increase water run-off and erosion, \(E\), leading to much thinner soil depth, regardless of particle size. Thus, unweathered crystalline rock with very low hydraulic conductivity values will tend to be exposed, unless it is buried through, e.g., fluvial deposition. Similar conclusions hold for increased slope angles, which will increase \(E\) and, more likely, decrease \(I\), both of which should lead to thinner soils.

Finally, it is worth noting that, especially for very low soil formation rates and erosion rates in, e.g., continental interiors such as Australia [40], soil formation rates do tend to be larger than soil erosion rates, consistent with the predicted power-law decay of the soil production function (rather than the oft-assumed exponential form of Heimsath et al. [41]). The slow decay toward a steady-state soil production value leads to an increased tendency of soils in arid regions not to attain steady-state
conditions [40], and for their depths to be smaller than that predicted from steady-state landscape evolution assumptions.

5. Comparison With Data: Mainly Climate

Below, we use data of White et al. [42], He et al. [43], Egli et al. [44] the Heimsath group [35,36,45–47], to confirm the relative consistency of soil depths across climatic gradients, but not across variation in topography. The San Gabriel Mountain data of Southern California [45] demonstrate the variation of soil depths along a gradient in topographic relief, and thus erosion rates, but not of climate. We have found particle size data for only five of the data sets below, and even in some of these cases we had to generate a median particle size from graphic representations of what are considered to be typical distributions of particle diameters for a given texture [48].

In southeastern Australia, where many of the Heimsath group’s field sites are located, precipitation tends to increase inland up to the escarpment, and then decrease with increasing altitude. The decrease in P with increasing altitude is mirrored by a diminution in AET from between 600 mm/year and 700 mm/year, to between 500 mm/year and 600 mm/year [49]. The Frog’s Hollow and Brown Mountain sites at about 1000 m elevation have more limited vegetation cover and cooler temperatures compared with Nunnock River and Snug, both factors that tend to reduce evapotranspiration.

In the San Gabriel Mountains, “the landscape varies from gentle, soil mantled and creep dominated in the west to steep, rocky and landslide dominated in the east”, accompanied by an increase in erosion rates from about 35 m/Myear to over 200 m/Myear, with the boundary to landside dominated at about 200 m/Myear. The actual soil depth extremes were taken from Figure 3 of Heimsath et al. [45], and restricted to non-landslide-dominated slopes. On landslide-dominated slopes, the soil depth was “patchy”, a scenario not addressed here. Thus, the variation in soil depth from west to east along the San Gabriel Mountains is a result of a variation in the erosion rate due to changes in mountain slope, rather than a variation in, e.g., climatic variables. Net infiltration rates are calculated as $I = P - AET - Run-off$, given the fact that run-off tends to be higher than run-on, and can play a role in water loss on site (according to Looivich [29]). 24% of precipitation flows into the ocean through surface run-off globally, while only 11% goes into deep infiltration. A summary of predicted and observed soil depths over two orders of magnitude of erosion rates is given in Table 1. Our predicted mean soil depth across 12 sites on four continents is 1.14 m, while the observed mean soil depth across those sites is 0.81 m.

In Figure 1, we compare predicted and observed soil depths, forcing the linear fit to go through the origin (for the San Gabriel Mountains, mean values are used here). We have an overall 43% overestimation for the mean soil depths across 12 sites, with less than 1% discrepancy at 3 sites, and 5 out of 12 underestimations (−22% on average) along with 7 overestimations (−40% on average). A large fraction of the overestimation comes from Merced River (84%), which has a slow average erosion rate and might not have reached a steady-state condition, and from east of the San Gabriel Mountains (200% to 500%) with a very shallow observed soil depth of 3 cm, which contributes a large discrepancy to the percentage, if not the actual discrepancy. There are a number of other potential reasons for overestimation. Our choice of an arithmetic mean for the observed soil depths tends to minimize the influence of shallower soil depths in the reporting of regional values for soil depth, but your shallow soils still tend to reduce a mean depth compared with a predicted steady-state depth value. Other sources of errors could come from reducing an entire particle size distribution to a median particle size, the accuracy of P, AET, and Run-off rates, and the porosity of the soils, particularly since we do not have a means to address local variability for most sites. Note that removing Merced River (maximum predicted value) will result in reducing $R^2$ from 0.934 to 0.679, while decreasing the numerical pre-factor from 1.66 to 1.05, making the relationship nearly one-to-one.
Table 1. Predicted soil depths from reasonable infiltration and given erosion rates.

<table>
<thead>
<tr>
<th>Station</th>
<th>Region</th>
<th>$E$ (m/Myr)</th>
<th>$I_p$ (m/yr)</th>
<th>$P$ (m/yr)</th>
<th>Predicted <em>x</em> (m)</th>
<th>Observed Mean $x$ (m)</th>
<th>Reference Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown Mountain</td>
<td>AU</td>
<td>14</td>
<td>0.22</td>
<td>0.69</td>
<td>0.95</td>
<td>0.62</td>
<td>[35,49]</td>
</tr>
<tr>
<td>Frog’s Hollow1</td>
<td>AU</td>
<td>10</td>
<td>0.2</td>
<td>0.72</td>
<td>1.29</td>
<td>1.5</td>
<td>[36,46]</td>
</tr>
<tr>
<td>Frog’s Hollow2</td>
<td>AU</td>
<td>27</td>
<td>0.2</td>
<td>0.72</td>
<td>0.41</td>
<td>0.43</td>
<td>[36,46]</td>
</tr>
<tr>
<td>Snug</td>
<td>AU</td>
<td>30</td>
<td>0.26</td>
<td>0.87</td>
<td>0.42</td>
<td>0.41</td>
<td>[35,49]</td>
</tr>
<tr>
<td>Nunnock River</td>
<td>AU</td>
<td>35</td>
<td>0.45</td>
<td>0.91</td>
<td>0.78</td>
<td>0.62</td>
<td>[35,49]</td>
</tr>
<tr>
<td>Coos Bay</td>
<td>OR</td>
<td>35</td>
<td>0.7</td>
<td>0.92</td>
<td>0.43</td>
<td>0.4</td>
<td>[36,46,55,56]</td>
</tr>
<tr>
<td>Gougga Mountain</td>
<td>CH</td>
<td>127</td>
<td>0.55</td>
<td>1.55</td>
<td>0.22</td>
<td>0.33</td>
<td>[44,56,57]</td>
</tr>
<tr>
<td>European Alps</td>
<td>EU</td>
<td>35</td>
<td>0.7</td>
<td>0.92</td>
<td>0.43</td>
<td>0.4</td>
<td>[36,46,55,56]</td>
</tr>
<tr>
<td>Tennessee Valley</td>
<td>N.CA</td>
<td>35</td>
<td>0.7</td>
<td>0.92</td>
<td>0.43</td>
<td>0.4</td>
<td>[36,46,55,56]</td>
</tr>
<tr>
<td>Merced River</td>
<td>C.CA</td>
<td>17.5</td>
<td>0.1</td>
<td>0.31</td>
<td>2.12</td>
<td>3.87</td>
<td>[42,50]</td>
</tr>
<tr>
<td>San Gabriel Mountain-west</td>
<td>S.CA</td>
<td>35</td>
<td>0.55</td>
<td>0.81</td>
<td>0.63 to 1.3</td>
<td>0.95</td>
<td>[43,54,56]</td>
</tr>
<tr>
<td>San Gabriel Mountain-east</td>
<td>S.CA</td>
<td>200</td>
<td>0.35</td>
<td>0.61</td>
<td>0.09 to 0.18</td>
<td>0.03</td>
<td>[43,54,56]</td>
</tr>
</tbody>
</table>

Notes: * Region: AU = Australia, OR = Oregon, N.CA = North California, C.CA = Central California, S.CA = South California, CH = China, EU = Europe. 
| For the European Alps, $P$ is from Egli et al. [44], $I_p$ and $P$ values from the Impact Database [58]. 
| For the Tennessee Valley, $P$ is from Neelam et al. [58], $I_p$ and $P$ values from Webster et al. [57]. Here we take $p = 0.4$ as typical porosity. 
| For all data sets except Gougga Mountain and the 4 Californian sites, a typical particle size of 30 μm was chosen to calculate predicted depth, since most of the soils in the Tennessee Valley fall into the clay loam category [98] with median particle size 10 μm [48]. 
| He et al. [43] report a typical particle size of 400 μm for Gougga Mountain, while et al. [42] give 500 μm for Merced River. 

Figure 1. Comparison of the predicted soil depth via Equation (2) versus the observed depths for 12 sites (open circles) from all around the world. The dashed red line represents the 1:1 line. See Table 1 for further details.

6. Comparison with Data: Slope Angle

Let us consider specifically the slope angle dependence of soil depth as exhibited by the data from the San Gabriel Mountains [45] and a result from Norton and Smith from 1930 as reported in Jenny [10]. We apply Equation (2) with the known value of $P - AET - run-off$ from Table 1, and a 30 μm median particle diameter. In order to address the slope dependence of soil depth, Equation (2) requires a slope-dependent erosion rate. Although it is not within our capability to predict such a function, Montgomery and Brandon [60] reported in their Figure 1 an empirical function for the
slope angle dependence of erosion rates in the Olympic Mountains in Washington. Incorporating this empirical input makes it possible to use Equation (2) to predict the slope angle dependence of soil depth. The comparison is shown in Figure 2. In order to make this equation predictive, the input of the erosion rate function (Montgomery and Brandon [60]) is critical. This function tends to produce a rapid reduction in soil depths to nearly zero as slopes of about 30 degrees are exceeded (since zero values cannot be plotted on a logarithmic graph, such values were converted to 0.001 for both axes). In spite of the considerable scatter in field values, it appears that our prediction captures the essential trends accurately. Note that the use of either 20 μm or 30 μm for the fundamental particle size will result in an overestimation of the soil depth at zero slope, but an underestimation at larger slopes, since the latter values are deeper than the zero slope depths, a result attributed by the authors [45] to the effects of soil deposition.

![Figure 2](image)

**Figure 2.** Predicted and observed soil depth as a function of slope. The Norton and Smith data were digitized from Jenny [10], but extend only to a slope of 8 degrees. The Heimath et al. [45] data were reported in a Table and extend over the range of 6 degrees to 32 degrees. The erosion function input was digitized from Figure 1 of Montgomery and Brandon [60]. It was given in Table 1 here, and a typical particle size of 30 μm, close to the 20 μm value considered to be most likely to characterize the San Gabriella mountain slopes, was applied. In order to reduce the scatter in the reported data, we give the mean soil depth at any particular slope value, although this will attach additional weight to the locations with deeper soils.

7. Implications for Geomorphological Studies of Natural and Agricultural Landscapes

The existing landscape evolution models require a large number of inputs, such as soil production and soil transport as functions of depth, parent material, topography, climate, and organisms, while delivering several quantities of interest. The outputs include: (1) regional denudation rates, which are important for understanding neo-tectonics; (2) spatial variability of soil erosion; (3) spatial variability of soil depth; and (4) spatial variability of landforms. When adapted to landscapes with human interference, such as agriculture, the second and third of these products may take on additional significance regarding sustainability. The first and fourth results typically involve larger time scales than the human time scales resolved by agricultural practices. Many processes are relevant to the evolution of landscapes, and these can have different impacts at different time scales and in different locations. We point out that the choice of soil production model has an impact on the results of such landscape evolution models, and that our soil production function may help to resolve some difficult problems in landscape evolution modeling.
When an exponential model for soil production is assumed, which delivers a maximum soil production rate modulated by an exponential decay, one finds for a steady-state soil thickness the negative of the logarithm of the ratio of the erosion rate to the maximum soil production rate, \(-\ln(E/R_m)\) [61]. For a wide range of typical values for \(E\) and maximum soil production rate, \(R_m\) (as reported by Helmsath and co-workers), it turns out that this formula yields soil depths ca. 1 m. But the key reason for an essentially consistent soil depth that lies in the correlation between \(E\) and \(P\); through the factor \(P - AET\) is missing in Roering [61] (both run-off and net infiltration, \(I\), increase with \(P - AET\)). In Roering’s treatment, the relative consistency in the output arises from the logarithmic phenomenology, which is very slowly varying in comparison with our power law. Consider, however, what happens if \(I\), which is the upper limit of soil production rates in our treatment, is substituted for \(R_m\) in the Roering soil thickness relationship. Since maximum soil production rates [62] are ca. 3000 m/Myear, but infiltration rates can be approximately three orders of magnitude larger, even with Roering’s logarithmic dependence, substituting \(I\) for \(R_m\) would increase the predicted soil depth considerably. Nevertheless, replacement of \(R_m\) by \(I\) in Roering’s result generates the same argument in the logarithm as appears in our power law. This is a significant correspondence. Our result expressed in Equation (2) generates, in principle, a much more sensitive function of parameters such as infiltration and erosion rates to the soil depth. However, the tendency of each to increase with increasing \(P - AET\) makes this ratio rather insensitive to changes in climate. However, see what happens if the erosion rate changes by over an order of magnitude due to topography, such as in the San Gabriel Mountains. Our result predicts a better than order of magnitude change in soil depth, as observed, as well as the approximate functional form of the soil depth as a function of slope angle (Figure 2), whereas that of Roering predicts a variation less than a factor 2 (compare the actual soil depth distinction between the western and eastern provinces of a factor approximately 20).

Although the most important topic may thus relate to absolute values of soil production and erosion, issues in the local variability of soil production and erosion relate to the shapes of the topography as well, and here understanding is also lacking. More generally, the use of common landscape models [63-65] does not allow for the prediction of the wide range of observed shapes of landscapes. From Roering [61], “linear transport models [use of the diffusion equation for soil downslope transport] produce constant curvature, not planar slopes, necessitating integration of various downslope transport mechanisms. Put simply, the [introduced] flux-slope nonlinearity enables nearly steep and planar (low convexity) sideslopes to erode at rates commensurate with highly convex hilltops.” As Roering points out, such problems have been addressed by incorporating soil depth-dependent transport as well as soil production into landscape evolution models [66,67], which allows an increase in soil transport rates downslope even in the case of planar slopes. However, with our result of a soil formation function which is highly dependent on infiltration, the low infiltration rate on hilltops and slopes (as compared to hollows) will tend to produce a smaller soil production rate, which could, without this strong dependence on infiltration, otherwise be interpreted as resulting from a larger erosion rate. Perhaps a portion of the difficulties encountered by landscape evolution models is that they do not incorporate sufficient local variability in soil production rates due to the convergence/divergence of surface flow.

Our results also have implications for agricultural systems. In particular, we can predict what a new steady-state soil depth will be if the soil erosion rate is increased by an order of magnitude or more, as results from traditional agricultural practices. While it is possible to write an accurate result for the soil production in terms of the instantaneous depth and the erosion rate, which allows a more rigorous prediction of the time frame over which the soil adapts to the new erosion conditions, several factors suggest that it may be better to calculate this time scale simply by taking the quotient of a typical original soil depth (say 1 m) and dividing by the erosion rate. One complicating factor is that the soil production function may change when the soil is very shallow, at least if the bedrock does not have a hydraulic conductivity comparable to that of the soil. In such a case, the relevant infiltration rate may be much smaller in a very shallow soil, and the soil production function comparably reduced.
Such a situation could lead to a “humped” soil production function [22], an instability resulting in total soil loss, and essentially, a two state system, where it may be difficult for natural systems to evolve between the two, i.e., once the soil is lost, it does not return. Results for our predicted steady-state soil depths are given in Table 2 below. In the case of calculations for the time required to strip a landscape of soil, we used an arbitrary starting depth of 1 m, in approximate accord with our general predictions, Roering’s [61] equation, and, as it turns out, with steady-state depths calculated from Equation (2) in accord with the input erosion rates given by Montgomery [27].

### Table 2. Predicted steady-state soil depths for conditions reported by Montgomery [27].

<table>
<thead>
<tr>
<th>Status or Condition</th>
<th>$\epsilon E^a$ (mm/yr)</th>
<th>Predicted Depth $^a$ (m)</th>
<th>Time $^a$ (yr)</th>
<th>$E^b$ (mm/yr)</th>
<th>Predicted Depth $^b$ (m)</th>
<th>Time $^b$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional agriculture</td>
<td>3.954</td>
<td>0.0038</td>
<td>254</td>
<td>1.537</td>
<td>0.011</td>
<td>651</td>
</tr>
<tr>
<td>Conservation agriculture</td>
<td>0.124</td>
<td>0.21</td>
<td>5064</td>
<td>0.082</td>
<td>0.33</td>
<td>12,200</td>
</tr>
<tr>
<td>Natural vegetation</td>
<td>0.053</td>
<td>0.54</td>
<td>0.013</td>
<td>2.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soil production $^c$</td>
<td>0.036</td>
<td>0.85</td>
<td>0.017</td>
<td>2.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geologic erosion</td>
<td>0.173</td>
<td>0.14</td>
<td>0.029</td>
<td>1.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean soil depth $^d$</td>
<td>0.51</td>
<td>1.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $^a \epsilon E$ denotes mean erosion rates. Predicted depth is calculated using $\epsilon E$, $I = 0.5$ m/year, and $x_0 = 0.00003$ m. $T$ is the corresponding time (approximate) to reach steady state starting from a soil depth of $x = 1$ m, $T = x/\epsilon E$; $^b E$ denotes median erosion rates. Predicted depth is calculated using $E, I = 0.5$ m/year, and $x_0 = 0.00003$ m. $T$ is the corresponding time (approximate) to reach steady state starting from a soil depth of $x = 1$ m, $T = x/E$; $^c$ The rate is not an erosion rate, but the soil production rate, and the depth is the soil depth that would generate such a production rate; $^d$ Mean soil depth is calculated from averaging individual values in the column above it.

Using a mean erosion (or soil production) rate, one finds a mean soil depth of 0.51 m, while using a median erosion rate, the result is 1.95 m, consistent with our understanding that 1 m is a typical soil depth. An important result is that, for traditional agriculture, using a typical soil depth of 1 m generates a time until virtually complete soil loss of between 250 and 650 years, a result that is consistent with Montgomery’s [68] assertion that limits on agriculture placed by soil loss were critical in setting the period of domination of a number of classical civilizations at about 500 years.

Consider some of the individual values of soil depths in Table 2. With native vegetation, the range of depths extends from 54 cm to 2.74 m. The soil production values suggest a range of soil depths from 85 cm to 2.01 m. However, the geologic erosion rates indicate steady-state soil depths from 14 cm to 1.09 m. (Note that soil degradation, i.e., soil depletion, is taking place due to a combination of factors, such as deforestation (30%), agricultural activities (28%), overgrazing (35%), overexploitation (7%), and industrialization (1%) [69]. Among factors of physical degradation are water erosion (35%), wind erosion (29%), chemical degradation (12%), and compaction/crusting (4%) [69]. Some of the more interesting results, however, may be the implied steady-state soil depths for normal agriculture, which lie between 4 mm and 11 mm, whereas the corresponding depths for conservation agriculture range from 21 cm to 33 cm. Thus, it is clear that, while normal agriculture is not sustainable, conservation agriculture is also quite marginal, as most crops need more than 33 cm of soil to thrive.

### 8. Summary

Based on the concept that the soil forming processes combined are dependent on the infiltration rate, we developed a simple model for the prediction of the soil depth. The model is verified by comparison of predicted and actual soil depths for a range of climates with emphasis on semi-arid zones, as well as along a slope gradient. Discrepancies between the data and model calculations remain, but they are comparatively small, taking into account the variety of soil forming factors: topography, climate, organisms, and parent material. When expressed as a function of time, it is shown to describe the temporal development of soil production. However, according to Dokuchaev and later soil scientists, it is equally important to be able to express a range of other soil forming factors, such as carbon and nitrogen content and other chemicals.
In our discussion and comparison with data, some details are still missing, such as (in most cases) site-specific porosity, or run-off values. The relevant porosity may be the effective value, which excludes pores that do not connect, or connect to flow paths at only one point, and likely excludes also internal water adsorption into clay minerals. We also have not addressed issues of climate change, which could require significant alterations of parameter values over time. It is to be hoped that a more detailed investigation of such parameter values and their potential temporal variation will improve the accuracy of our predictions. Additionally, we have not addressed specific flow paths, which may depend on a wide variety of factors, such as the slope angle, or whether the soil is graded or layered. All of these factors may introduce as yet unaccounted for variability. In spite of these omissions, the theoretical approach appears to account properly for soil formation factors over a wide range of climates and slope angles.

Author Contributions: Yu (writing, soil data location and digitization, climate and run-off data search, analysis), Hunt (conception, Faybeshenko (writing, relationships to fundamental soil formation models and history of soil science), Ghanbarian (writing and analysis).

Conflicts of Interest: The authors declare no conflict of interest.

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V. MODEL APPLICATION I: DISTINGUISHING LOCAL STEADY-STATE AND
NON-STEADY-STATE SOILS

Article IV.

An examination of the steady-state assumption in soil development models with application to landscape evolution

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ABSTRACT: Soil depth and soil production are highly complicated phenomena, generated from a complex interaction of physical, biological and chemical processes. It has, nevertheless, become increasingly clear that soil formation rates are closely related to chemical weathering rates. Somewhat paradoxically, it is likewise becoming apparent that such biogeochemical reactions as slowly transform rock to soil are limited by physical processes, such as flowing water and the formation of fractures. We have formulated a theoretical approach that relates soil formation rates to chemical weathering rates, and, likewise, to solute transport rates. For such a theoretical framework to be relevant, the solute transport rates cannot equal those of the flowing water, as is the case in Gaussian solute transport. Rather, solute transport must be slowed in accordance with heavy-tailed solute arrival time distributions.

The inference is that the traditional advection-dispersion equation formulation for solute transport is inadequate in the typically heterogeneous geological media that weather to form soils. Here we examine the implications of this soil production model on the assumption of the approach to steady state. Particularly at slow erosion rates we find that many soil columns are not in equilibrium. This tendency may be accentuated in dry climates. Copyright © 2017 John Wiley & Sons, Ltd.

KEYWORDS: soil production; steady state; chemical weathering

Introduction

Soil production is a complicated process (Huggett, 1998), about which little is directly known or knowable; on account of the vast timescales involved. Attempts to define soil production as a continuous process starting with initial exposure at the Earth’s surface (Jenny, 1941) have met with variable success, as for shorter periods less than decades it becomes increasingly difficult to develop criteria for what is and what is not soil (e.g., Stevens, 1968). Soil depths tend to increase over time, but not in a linear fashion (Huggett, 1998), and not uniformly (e.g., Heimath et al., 1997, 1999, 2001a, 2001b, 2005), in the main, soil depths increase more rapidly the shallower is the soil. However, it was suggested as early as the 19th century that soil production, associated with shallow soils covering bedrock, may be so slow that it is insufficient to sustain such soils (Gilbert, 1877), an idea apparently confirmed in one of Heimath’s studies (Heimath et al., 2009). Such a discussion leads to the concept of a ‘humped’ soil production function, which first increases with increasing depth but then, beyond a certain depth, decreases. Even the fundamental processes involved in soil production vary depending on whether the soil is developing within recently deposited or exposed uncemented media, such as alluvium, glacial moraines or landslide deposits, or, alternatively, from bedrock, whether fractured or not. In the latter case an entire array of physical weathering processes that work in tandem with chemical processes (Dietrich et al., 2003; Anderson and Anderson, 2010; Roering, 2008; Braun et al., 2016) is essential to help generate soil-sized particles and, indeed, to generate the flow paths through, for example, fracturing of the bedrock, that facilitate the entry of surface waters to the medium. Soil production rates vary for other reasons as well: variations with surface curvature (Heimath et al., 2001b) and slope (Montgomery and Brandon, 2002— in steady state, at least), water content, upflow area (Liu et al., 2013) or precipitation (Amundson et al., 2013) have been documented. The results of the theoretical approach considered here have been shown to generate useful predictions for either bedrock or uncemented media, though it is important to consider potential differences in interpretation of the theoretical results.

Although the focus here is on physical evolution of soil, it should be kept in mind that a wide range of biological processes affects soils. Dokuchaev (1883) first formulated the five soil formation factors: climate, topography, time, organisms and parent material. Of the organisms, Darwin’s earthworms (1881) are perhaps the most famous. In any case, however, using the argument that biogeochemical weathering is critical to the formation of soil provides the basis for the functional form of our soil production function, and involves the respiration of plants and other organisms in providing the CO₂ for the Urey reactions (Urey, 1952; Berner, 1992). Other factors
enter explicitly, such as time, or implicitly, such as topography, through erosion rates, or finally, climate and parent material through flow rates.

In addition to the complications arising from its development, soil is also a complex system, the surface by incremental (such as soil creep, e.g. Heimsath et al., 1999, 2001b) and catastrophic (such as landslides or solifluction) surface processes, as well as by overland flow, by plants through trough-cutting and wind erosion, and by the erosive effect of the fast-moving river systems (references to all these processes are found in Anderson and Anderson, 2010). Divergence in the surface soil flux leads to soil erosion or deposition. Spatially dependent surface processes (dependent on distance to a divide, slope or substrate heterogeneity) (Roering, 2008) can lead to spatially variable erosion rates which also vary with surface convergence and flow routing. Depending on the functional relationship of such processes to slope angle, one can generate convexo-convex, or planar, slopes (Roering, 2008). Locally variable soil production will have an impact on the resulting landscape form as well.

Since it has become clear that soil production is critical to understanding landforms, chemical weathering, as a component of soil production, is a relevant question. Chemical weathering is an important component of soil production, the development of coupled geomorphological and pedogenetic models has been given some priority. Some of these models include Liu et al. (2013), Dimuth et al. (2016) and Arnaud et al. (2016). In principle, such models can take into account all the processes relevant to hillslope evolution and address any timescales, without invoking surface processes which are then realized through increased clarity introduced by the explicit role of non-Gaussian transport, solute and water transport speeds in soil formation, it becomes possible to understand the relevance to soil production of climatic and geological variables, which constrain the water flow that drives the solute transport and thus the weathering (Hunt and Ghanbarian-Alavijeh, 2016; Braun et al., 2016).

The purpose of this work was to check whether, in contrast to the exponential formulation of soil production, the proposed power law decay of soil production rate allows an inference to be made regarding the relevance of the steady state assumption for real soils. Following Phillips (2010) we focus on sites with low depth decay. A power law decay allows us to address soil removed by erosion processes, or added by soil production, and to neglect the specifics of the wide range of processes mentioned briefly above. For simplicity, we will consider the erosion rate to be a constant in time, though future studies can easily relax this assumption.

In order to come to quantitative conclusions regarding the relevance of steady-state conditions, a reliable model for soil production as a function of soil depth is required. While different models have been proposed, including a power law (Braun et al., 2016; Hunt, 2015a, 2015b; Hunt et al., 2015), the exponential decay (Heimsath et al., 1997) of soil production depth is almost universally applied. However, there are reasons to prefer a power law formulation. The most important is probably the many observed cases where soil production and chemical weathering rates as functions of time and depth (Burke et al.; 2007; Dixon et al., 2009; Egl et al., 2010; Hunt and Ghanbarian-Alavijeh, 2016). Chemical weathering rates are characterized in terms of power laws (White and Brantley, 2003; Maheu, 2010).

Any conceptual model (as in the soil sciences communities) is the inference from one variable to another variable (in this case, the soil depth), which must otherwise be assumed to evolve independently, might evolve distinctly. It will become clear that calculations of soil age based on the residence times of carbon may be complicated and, in some cases, impossible. These and other complications suggest that this particular model is not only an opening in a discussion of the concept of the occurrence of steady-state soils and, beyond this, for landscapes.

Other aspects of our modeling, illuminated in this study may also be of importance in geomorphology. Our soil production function is proportional to a vertical (depth) integral. The dependence on vertical flow should produce local variations in soil depth from variable partitioning between runoff and infiltration, as well as larger-scale variability. The soil depth due to climate associated with gradients in precipitation
and evapotranspiration. Such a perspective unifies cause and effect over a range of length scales, but may also have an impact on the interpretation of local landscape forms, such as of slope curvature.

The advantage of our model is that all its parameters relate directly to quantities that can be specified locally. The two parameters are a length scale, suggested to be the median particle diameter, \( d_{50} \), and the local infiltration rate, whose most important input is from climatic variables, but also incorporates some effects of local topography in the routing of surface water. These concrete relationships may enhance accretion landscape modeling at spatial scales from local to continental.

Theoretical Basis

Because we address effects of solute transport on chemical weathering, and thus the rate of formation of soil, the theoretical basis for this work is more concretely grounded in the theory of ‘anomalous’ solute transport than in numerical geomorphological modeling. Nevertheless, since our model is closely related to a parallel line of research (Braun et al., 2016) that treats the geomorphological modeling more explicitly, we then refer to that closely related paper to address the geomorphological ramifications more thoroughly.

Braun et al. (2016) state in their abstract:

We present here a new model for the formation of regolith on geologic timescales by chemical weathering based on the assumption that the rate of chemical weathering is primarily controlled by the ability of groundwater to transport solute away from the reacting solid-fluid interface and keep the system from reaching equilibrium (saturation). This allows us to specify the rate of propagation of the weathering front as linearly proportional to the pore fluid velocity which we obtain by approximating the water movement in the regolith layer. The surface of the regolith layer is affected by mass transport and erosion. The main prediction of the model is that the geometry of the regolith, i.e., whether it is thickest beneath topographic highs or topographic lows, is controlled by the value of a dimensionless number, which depends on the shape of the soil profile, the hydraulic conductivity, and local precipitation rate, but is independent of the chemical weathering rate. The model also predicts that in anorganic areas, regolith thickness increases as the square root of time, whereas in organic environments, a steady state regolith thickness can be achieved, when the propagation of the weathering front is equal to erosion rate.

From Hunt (2015b), in describing the equation \( x = x_0 \left( \frac{t}{t_0} \right)^{0.53} \) for the soil depth, and its derivative, \( \frac{dx}{t} = \left( \frac{t}{t_0} \right)^{0.53} \),

\( \frac{\text{The ratio of length to time scales required } \frac{x_0}{t_0} \text{ gives a steady-state pore-scale fluid velocity, } \omega, \text{ and the statement, } \frac{\text{Accounting for a constant soil erosion rate, } \omega, \text{ would lead to a stable-state solution } x = x_0 \left( \frac{t}{t_0} \right)^{1.53} \text{, one sees that our model yields the same conclusions as that of Braun et al. (2016), since } E \text{ is very small in continental interiors and large in orogenic environments, while the power 0.53, predicted for small erosion rates, is essentially indistinguishable from 0.5, proposed by Braun et al. (2016). Moreover, the basis for the derivation is conceptually equivalent: consider the statements of Hunt et al., (2015) in addressing the scaling of chemical reactions in porous media (in the abstract) "Our calculated functional dependence of the solute velocity acts as a proxy for chemical reaction rates" and then "we have the result that the solute velocity scaling looks very much like diffusion (rate 0.53 power) and the pore-scale solute velocity is identical to the pore-scale fluid velocity..." From Hunt (2015b) on the formation of weathering, "Greater tortuosity of the solute transport paths leads to a reduced thickness that increases more slowly with time, since it takes correspondingly longer for the solute to reach the weathering front, and reaction products to be removed." Thus, in Hunt's paper, as in Braun et al. (2016), the removal of the products of the reaction is taken as a limiting factor in the rate at which the reaction proceeds. In Braun et al. (2016), specific flow path configurations relative to the medium stratigraphy are detailed. In our case, we point out that the basis for the relevance of percolation scaling is the heterogeneous distribution of minerals in the subsurface, whether bedrock, (near) surface clasts, unconsolidated media or soil. Importantly, the integral of the solute velocity over time is the total solute transport distance, which relates a soil depth directly to the weathering rate. Where does this peculiar exponent, 0.53, come from in the context of this work?"

Non-Gaussian (or non-Fickian, "anomalous") transport is becoming recognized as the norm in natural porous media (e.g., Cushman and O’Malley, 2015). Common descriptions of non-Gaussian transport in terms of the continuous time random walk (CTRW) (Berkowitz et al., 2002; Bijelic et al., 2004; Margolin and Berkowitz, 2000) or fractional advection dispersion equation (FADE) (Benson et al., 2000; Krapivsky et al., 2006; Meerschaert et al., 1999; Pachepsky et al., 2000), both of which model power law solute arrival time distributions. Under a wide range of conditions, such power laws are special in that they describe the scaling of solute velocities that diminish according to a power of the time (Scher et al., 1991).

But neither the CTRW nor the FADE can predict the actual power. In percolation theoretical treatments, power-law arrival time distributions relate to the fractal dimensionality of the percolation backbone (Lee et al., 1999; Sheppard et al., 1999; Hunt and Skinner, 2008, 2010; Hunt et al., 2011; Gharianian-Alavijeh et al., 2013), in contrast to the predictions of the actual power possible. Hunt and Gharianian-Alavijeh (2016) show that the remaining parameters in the scaling relationship between soil age and soil depth, namely the fundamental time and space scales, can also be determined from site-specific values of a median particle size and the deep (net) infiltration rate, respectively.

Percolation theoretical treatments of solute arrival time distributions have been considered for about 30 years (Gist et al., 1990; Koplik et al., 1988; Sahimi and Iradian, 1988; Sahimi, 1994, 2013; Bunde and Havlin, 1996; Lee et al., 1999; Malae et al., 2000). All of the cited treatments were based on specific percolation disorder and proximity to the percolation threshold, which appears, on the surface at least, to exclude their applicability from the typical well-connected media encountered near the Earth’s surface. More recently Hunt and Skinner (2008) developed a technique to calculate spatial solute distributions as well as solute arrival time distributions for well-connected media with wide ranges of local flow rates due to heterogeneity in, for example, a pore size distribution. The theoretical construction is based on concepts first developed in the context of critical path analysis (Ardbegasgaard et al., 1971; Oplaski, 1972; Friedman and Seaton, 1994; Hunt, 2001), a means to calculate characteristic flow rates in a heterogeneous medium, which determines the largest possible value of a flow-limiting pore size using percolation theory. While such a treatment is thus specifically adapted to typical heterogeneous porous media near the Earth’s surface, its level of universality is such that Sahimi’s argument (Sahimi, 1994), that percolation scaling arguments for solute transport should apply whenever critical path concepts are relevant, is valid.
The theoretical treatment to develop an entire solute arrival time distribution has been described in a series of existing publications (Hunt and Skinner, 2008, 2010; Hunt et al., 2011; Chambanbar-Alavijeh et al., 2012). In short, a pore-scale network of intercommunication connecting neighboring pores is assumed. Using this distribution, the probability that an optimal flow path between planes separated by a distance \( x \) can be connected, which passes through no conductance smaller than some arbitrary value, is calculated. This procedure is based primarily on the cluster statistics of percolation and transformations of variables from cluster volume to cluster length, and from length to conductance probability to conductance value. Then the topology of the path so defined in terms of percolation variables, together with the distribution of conductance values on the path, is used to calculate the expected arrival time, \( t_{\text{g}} \), for solutes traveling over such a cluster characterized by minimum conductance \( g \). Finally, \( t_{\text{g}} \) is used to transform the result for the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length). It is then possible to calculate the probability that a finite system is characterized by a particular control (conductance per unit length).

Past applications of solute transport to reaction processes in porous media have been to long-term predictions of silicate weathering rates and laboratory experiments on reactive solute transport (Hunt et al., 2015), principally from the Hanford site. In these comparisons it has been shown that a single-medium model generates simultaneous agreement with all the experiments considered. Furthermore, the same percolation conditions, 3D flow connectivity and saturated (or unsaturated) conditions were appropriate for all experiments except one (Liu et al., 2008), which required 2D flow connectivity and unsaturated conditions (Hunt et al., 2015). That experiment was interpreted to have been performed under conditions of prevalent wall flow, a known complication for coarser Hanford site soils, in which large particles near the walls generate highly permeable flow paths with such large pores that it is difficult to maintain local conditions of full saturation. Thus, with a single exception, in principle all the experiments would fit on the same universal curve given by dield from the second half of Equation 1.4. A difficulty with the percolation theory for saturated (or wetting) conditions, and 3D network connectivity. The same conditions are assumed here, but the theoretical solution is obtained from the fundamental fractal scaling theory of percolation (Sahimi, 1994; Lee et al., 1999; Sheppard et al., 1999).

Lee et al. (1999) show that for solute entering the left side of a region from a point on a line (or plane, in 3D), the most likely time to traverse the system and exit on the right side a distance \( x \) downstream is proportional to \( x^2 \). Thus \( x \) is a transport distance. Here, \( x_{\text{D}} \) is the fractal dimension of the percolation backbone. Such a relationship is known from the CTIRW and the FADE as well, but the power corresponding to \( x_{\text{D}} \) is in those systems an adjustable parameter. The percolation backbone fractal dimensional for 3D flow in saturated media is, for a wide range of conditions, \( x_{\text{D}} = 1.87 \) (Sheppard et al., 1999). Sahimi and Mahdavipour (1996) discuss limitations on this result arising from certain classes of long-range correlations in the medium itself. If the flow is 2D, such as along a fracture plane, or along the walls of a cylindrical core, for example, under saturated conditions, \( x_{\text{D}} = 1.64 \). However, when flow is constrained to 2D surfaces, both in the field (Class et al., 1998) and in experiments, unsaturated conditions are common. In that case, \( x_{\text{D}} = 1.21 \). For unsaturated 3D flow, \( x_{\text{D}} = 1.46 \) for drying, but \( x_{\text{D}} = 1.87 \) for wetting conditions. In summary, using the exponent values given in Sheppard et al. (1999):

\[
t = t_0 \left( \frac{x}{x_{\text{D}}} \right)^{1.46} \quad (1a)
\]

\[
t = t_0 \left( \frac{x}{x_{\text{D}}} \right)^{1.87} \quad (1b)
\]

\[
t = t_0 \left( \frac{x}{x_{\text{D}}} \right)^{1.64} \quad (2a)
\]

\[
t = t_0 \left( \frac{x}{x_{\text{D}}} \right)^{1.22} \quad (2b)
\]

In these equations, \( t \) is the time required for solutes to be transported a distance \( x \), \( x_{\text{D}} \) is a fundamental length scale, which is assumed to be a pore separation or particle size, and \( t_0 \) is a fundamental timescale, defined by the fluid flow rate, \( v_0 = x_0/t_0 \), through a characteristic pore. In heterogeneous media a characteristic particle size is assumed to be the median diameter, or \( d_{\text{med}} \). Note that if the fundamental length scale, \( x_0 \), is as large as meters, in accord with human

Applications to Silicate Weathering Rates

In Braun et al. (2016), the authors distinguished between three weathering regimes: one controlled by reaction kinetics at large flow rates, one controlled by advective transport at intermediate flow rates, and one controlled by diffusion at very slow flow rates. While we tend to agree with this assessment, we point out that the diminishing solute velocity with time tends to raise the importance of advection relative to reaction kinetics with increasing timescales even when flow rates are large (Yu and Hunt, 2017). The question of whether reaction rates are limited by solute transport is usually addressed using the Damkohler number, \( D_a \), which is the ratio of a solute advection time to a reaction time (Salehishkoo et al., 2013). When \( D_a > 1 \), transport limitations are considered relevant. In Yu and Hunt (2017) \( D_a \) is calculated explicitly for non-Cauchian transport and it is shown that over timescales exceeding days chemical weathering is practically always limited by transport, rather than reaction kinetics. Yu and Hunt (2017) showed that the chemical weathering rates measured by Salehishkoo et al. (2013) could be predicted accurately when the Damkohler number was much larger than 1. These authors did not, however, address the possible complications at very low Peclet number characterized by slow diffusion and high dispersion. Diffusion of solutes through soil is an important process that controls the availability of reactants and the rates of chemical reactions. Diffusion is a transport process that occurs through the movement of solute particles from regions of higher concentration to regions of lower concentration. In the context of soil science, diffusion plays a crucial role in the processes of nutrient uptake by plants, the transport of contaminants in the subsurface, and the movement of water and gases through soil pores. Diffusion is driven by concentration gradients and is often described by Fick’s laws of diffusion. The rate of diffusion is influenced by various factors, including the diffusivity of the solute, the temperature, and the porosity and tortuosity of the soil matrix. Diffusion is a slow process compared to advection and can be a limiting factor in the transport of solutes in soils.
introduced solutes (Hunt and Ewing, 2016), the velocity of solute transport is the same as that of water at scales up to a few meters, meaning that Gaussian transport models are less severely compromised.

In the case of certain long-range correlations of local conductances, other values of $D_o$ are possible (Sahimi and Mulkapadhyay, 1996). However, the relevance of other such values of $D_o$ in soil production (Hunt and Ghanbarian-Alavieh, 2016) and chemical weathering (Hunt et al., 2015) has not yet been demonstrated. Only in weathering rind development (Hunt, 2015b) was the relevance of other exponents demonstrated, mostly when the flow and transport network was connected two-dimensionally (e.g., in fractures). In two dimensions, both inhibition and drainage produce the same exponent, in contrast to the case of 3D network connectivity, which we expect here to be mostly, though perhaps not entirely, universal. Thus, since drying typically occurs due to plant transpiration, which involves either horizontal or upward moisture fluxes, we propose that any soil formation processes occurring during drying can be neglected. Thus we can use the second expression in Equation (1a) to represent solute transport distances. Use of $x_o$ as a median particle size, $b_o$ a typical pore crossing time, and $x_o/b_o$ as the net (deep) infiltration $I$ eliminates all adjustable parameters in calculating a transport distance, when these are known for a given site. If $x_o/b_o$ represents a total solute transport distance, $dx/dt$ represents a solute velocity. The rate of chemical weathering, as well as the soil production function, as functions of increasing time have been argued to be described (Hunt et al., 2015; Hunt and Ghanbarian-Alavieh, 2016). In the absence of erosion, then, the soil depth $x$ as a function of time $t$ is just the integral of the soil production function over the time that the medium has been in place, which returns the solute transport distance as a function of time:

$$x = x_0 \left( \frac{1}{b_0} \right) t = x_0 \left( \frac{t}{b_0} \right)^{0.53}$$  \hspace{1cm} (2)

Note that Equation (2) cannot be considered the fundamental equation for soil modeling, as it contains implicitly a specific history of the medium, but does not address the soil production as a function of the current state of the medium. The latter will be the focus below. In Equation (2) the characteristic particle size, $x_0$, is best treated as a median particle size, $d_{50}$. Defining $b_o$ under field conditions is more difficult, since one would need, in principle, to be able to determine precipitation, evapotranspiration, porosity and the difference of run-on and run-off at the particular site where the soil is produced. This ultimately brings in the necessity of routing surface water, beyond our present scope. Also, the derivation of Equation (2) neglects the role of soil erosion or deposition. In order to treat soil erosion, it is necessary first to generate an equation for the rate of soil production, $R_o$:

$$\frac{dx}{dt} - R_o = \frac{1}{1.87} \left( \frac{x}{x_0} \right)^{-0.47} - I$$  \hspace{1cm} (3)

where $x_o$ is the particle size, $x$ is soil depth, $I$ is the soil age, and $x_o/b_o$, the mean flow rate, equates to the deep infiltration rate, $I$ (Hunt and Ghanbarian-Alavieh, 2016). In the consideration of steady-state conditions below, however, the variable $x$ as age of the soil does not make any sense, since in that context the history of how long the soil has been at steady state is lost. In that context, we may continue to use $x$ as a particle residence time, but this does not necessarily correspond to the soil age, even when steady state has not been reached. The choice of $I$ as deep infiltration rate reflects the understanding that it is the actual downward flux of the CO$_2$-carrying water that is critical to the weathering reaction. This flow rate can be affected by climate (precipitation), plants (transpiration), surface water routing (run-off), or substrate hydraulic conductivity.

We prefer the final form of Equation (3), since it immediately allows adaptation to include the process of erosion. Thus the relationship of soil production to $x/b_o$ (or residence time) is no longer unique when erosion occurs, but the relationship between soil depth and soil production is unaffected. Adding the effects of an erosion rate, $E(t)$, generates

$$\frac{dx}{dt} - R_o - E(t) = \frac{1}{1.87} \left( \frac{x}{x_0} \right)^{-0.47} - E(t)$$  \hspace{1cm} (4)

In principle, such an equation can be solved numerically for $x(t)$ with an arbitrary erosion rate, but so far we have only addressed the case of $E(t)$ being a constant. $E$ in addition, such an equation can form the basis for a landscape evolution model if horizontal transport of soil is incorporated, but our efforts in that area are still ongoing.

**Steady state**

Equation (4) can be solved easily for the case of steady-state conditions ($dx/dt=0$), to yield

$$x = x_0 \left( \frac{1}{1.87} \right)^{1.13} \left( \frac{t}{b_0} \right)^{1.13}$$  \hspace{1cm} (5)

Note that the important roles of vegetation and temperature are subsumed here in $I$, which in the absence of overland flow is proportional to $(P-AET)$, where $P$ is the precipitation rate and AET is the actual evapotranspiration. In the present analysis, we have used $P-AET$ to estimate $I$, although two important effects, which tend to cancel each other, are omitted. We will show why, at least to first approximation, this is a reasonable approach. The mean terrestrial $P$ is reported as between 850 mm and 1100 mm, with a mean of 975 mm (Willmott et al., 1994), although Lovelock (1973) reports 834 mm. A mean of these two values is approximately 903 mm. Lovelock (1973) estimates global mean AET as 65% of $P$ (Schlesinger and Joseyeho (2014) give 64%). Thus $P-AET$ is approximately 35.5% $P$, 321 mm yr$^{-1}$.

Consider now that transformation of an incident atmospheric water flux to a pore-scale flow velocity requires division by the porosity, which typically varies between 0.3 and 0.6 (Hillel, 2004), but may be guessed at about 0.4. Run-off is estimated (Lovelock, 1973) to average about 23% of precipitation worldwide. Thus $P-AET - \text{run-off}$ is only about 13% of $P$. Consequently, the expected net (deep) infiltration value $I$ is approximately 13% of 321, or 424 mm yr$^{-1}$, which compares reasonably well with 325 mm yr$^{-1}$. However, areas with a higher fraction of $P$ lost to AET, such as Australia, could have a lower run-off fraction, meaning that our approximation will cause a theoretical underestimation. Such an approximation will also introduce scatter into the predictions due to local run-off uncertainty.

Higher temperatures tend to increase AET and reduce soil development. Also, the soil depth is proportional to the particle size and nearly to the ratio of two rates: infiltration to erosion. For reasonable values of $x_0=30$ mm and a (deep) infiltration rate of 1 m yr$^{-1}$, and erosion rate of 100 m Myr$^{-1}$ generates a steady-state soil depth of slightly over 2 m. But a decrease of

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the net infiltration rate to 0.1 m yields (with other parameters the same) a steady-state soil depth of about 15 cm. Thus the typical 10-100 cm depths found by the Heimath group (Heimath et al., 1997, 1999, 2001a, 2001b, 2005, 2009) for climates with precipitations ranging from 200 mm to 2 m (and evapotranspiration accounting for something more than half of the precipitation) are in general accord with our predictions of the values of steady-state soil depths. However, as we will see, this overall accordance is not verified in most specific cases.

The papers of Heimath et al. (1997, 1999, 2001a, 2001b, 2005, 2009) do not reference particle sizes. However, soil ternary diagrams distinguish particles (in order of increasing size) clay, silt and sand. A middle silt particle size would thus, in the absence of any information regarding soil texture at a given site, be the best estimate for a median particle size. Silt particle sizes range from 2 to 63 µm (US Geological Survey), with a mean value of 32 µm, or a geometric mean of 11 µm. Where we did not have information on soil particle size, we used a value of 30 µm for the median particle size, particularly since semi-arid regions tend to have somewhat larger particle sizes. According to Sanderman and Amirudin (2009), however, most of the soils at Tennessee Valley – the most humid region of those considered – fit into the clay loam category, with an average of 36% sand, 24% silt and 40% clay; in other words, slightly more clay than sand. Thus the median particle size should be slightly below the median silt particle of 11 µm. Accordingly, for this site only, we assumed 10 µm median particle size.

Materials and Method

Predicted soil depths assuming both steady-state and non-steady-state conditions were compared with field data at five study sites (Snug, Brown Mountain, Tin Camp Creek, Frogs Hollow and Nunrock River) in southeastern Australia, and two Californian sites (San Gabriel Mountain and Tennessee Valley) published by Heimath et al. (1997, 2000, 2001a, 2006, 2009, 2012). As noted above, a typical value of ρ = 30 µm was taken as the fundamental length scale for all datasets, except for a 10 µm particle size for Tennessee Valley. Individual infiltration rates i at each study site are shown in Table 1 (Hunt and Churbanker-Alavieh, 2016). Erosion rates along with the soil depths (Heimath et al., 1997, 2000, 2001a, 2006, 2009, 2012) for individual soil sample were used as values for E in Equation (5).

Soil depths assuming steady-state condition were predicted using Equation (3). For non-steady-state assumption, integration of Equation (4) is needed to generate soil depth over a certain period of time since soil started to be produced (which was reported as ‘age’ in Heimath et al., 2000, 2001a, for Nunrock River and Frogs Hollow sites):

\[ x = \frac{1}{f_0} e^{-k_0} = \frac{1}{f_0} \left( 1 - \frac{1}{x_0} \right)^{0.67} - E \]  

Since the integral equation (Equation 6) does not have an analytical solution, soil depths were solved numerically. Unlike the prediction assuming steady-state condition, ‘soil ages’ are needed for prediction under non-steady-state conditions. Although, in the context of our derivation, it is convenient to think about the age of a soil, it is more productive to begin by replacing this term with a typical soil residence time. Given the fact that the information of ‘soil ages’ are only available for two of the seven study sites (Nunrock River and Frogs Hollow), a published method (Ivy-Ochs and Kaber, 2008) in cosmogenic nuclides dating was used to determine the ‘ages’ that are missing based on the 10 Be and 26 Al concentrations published along with the soil depths at the rest of the five sites. The ‘age’ of soil is calculated as

\[ C_i(t) = \frac{P_{i0}}{\lambda} \left( 1 - e^{-\lambda t} \right) \]  

where, C_i (atoms g⁻¹) is the concentration of 10 Be or 26 Al; P_i0 is the nuclide production rate at the sampling site; t = 6 for 10 Be and t = 36.8 for 26 Al at sea level and high latitude (≥60°) according to Heimath’s group (Heimath et al., 2000); λ = ln(2)λ_i0 is the radioactive decay constant, and λ_i0 = 1.500 (000) yr for 10 Be and t_i0 = 7.01 (000) yr for 26 Al. t is the exposure time of the bedrock where the concentration of nuclides was determined. Here, we consider the exposure time t as the time period over which the soil is forming. We make the same assumption that the authors stated in the papers (Heimath et al., 1997, 2000, 2001a, 2006, 2009, 2012) that the initial concentration of the nuclide within the bedrock at depth H where the samples were taken is 0 at t = 0. At t = 0, soil starts to form at the exposed rock surface as a consequence of weathering, and at the same moment cosmogenic nuclides (10 Be or 26 Al in this case) within the rock at depth H from the rock surface (depth = 0) starts to accumulate following the function described in Equation (7). Over a certain time period of t as long as at the material at depth H from the surface of the soil is still in rock form and has not been weathered, accumulation of the nuclides should follow the same function with time no matter how much rock has been altered into soil on top of it. Therefore, the exposure age calculated from Equation (7) based on the concentration of nuclide C_i determined within the bedrock at depth H corresponds to the time period over which the soil is forming. However, according to Ivy-Ochs and Kaber (2008), the exposure age Equation (7) is used for calculating surface exposure age of exposed rock surfaces (H = 0), and production rate of nuclide within a target rock declines exponentially with a depth (H, Heimath et al., 1991), so that the value of P_i0 in Equation (7) might be different and need adjusting with depth. This might lead to a possible underestimation of the exposure age.

Calculation of t from Equation (7) was verified by comparing with published values of ‘age’ at Nunrock River and Frogs Hollow (Heimath et al., 2000, 2001a). The discrepancies for 16 out of 19 data points of the two datasets were less than 1%. The highest discrepancy was 2.85% (Table II).

Table 1. Precipitation (P) and actual evapotranspiration (AET) rates at the study sites (Hunt and Churbanker-Alavieh, 2016)

<table>
<thead>
<tr>
<th>Site</th>
<th>P-AET (mm yr⁻¹)</th>
<th>AET (mm yr⁻¹)</th>
<th>Precipitation (mm yr⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR</td>
<td>0.11</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>FH</td>
<td>0.1</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>TG</td>
<td>0.3</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>SN</td>
<td>0.26</td>
<td>0.65</td>
<td>0.91</td>
</tr>
<tr>
<td>BMT</td>
<td>0.06</td>
<td>0.65</td>
<td>0.71</td>
</tr>
<tr>
<td>TV</td>
<td>0.6</td>
<td>0.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: NR, Nunrock River (southeastern Australia); FH, Frogs Hollow (southeastern Australia); TC, Tin Camp Creek (southeastern Australia); SN, Snug (southeastern Australia); BMT, Brown Mountain (southeastern Australia); TV, Tennessee Valley (California Coast); SG, San Gabriel Mountains (California).
Table II. Verification of calculated soil residence time based on concentrations of cosmogenic nuclides with published data

<table>
<thead>
<tr>
<th>Be (atoms g⁻¹)</th>
<th>N (atoms g⁻¹)</th>
<th>Averaged age (kyr)</th>
<th>Published age (kyr)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frog Hollow</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>103 217</td>
<td>592 709</td>
<td>16.75</td>
<td>16.6</td>
<td>-0.926</td>
</tr>
<tr>
<td>99 159</td>
<td>608 001</td>
<td>16.62</td>
<td>16.5</td>
<td>-0.752</td>
</tr>
<tr>
<td>202 519</td>
<td>1 059 721</td>
<td>31.62</td>
<td>31.2</td>
<td>-1.336</td>
</tr>
<tr>
<td>392 381</td>
<td>2 505 639</td>
<td>68.45</td>
<td>66.6</td>
<td>-2.773</td>
</tr>
<tr>
<td>422 549</td>
<td>2 637 396</td>
<td>72.92</td>
<td>70.9</td>
<td>-2.852</td>
</tr>
<tr>
<td><strong>Nunnoock River</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>513 300</td>
<td>3 059 000</td>
<td>87.01</td>
<td>86.81</td>
<td>-0.258</td>
</tr>
<tr>
<td>177 400</td>
<td>624 100</td>
<td>18.07</td>
<td>18.02</td>
<td>-0.268</td>
</tr>
<tr>
<td>107 100</td>
<td>601 500</td>
<td>17.20</td>
<td>17.16</td>
<td>-0.241</td>
</tr>
<tr>
<td>249 100</td>
<td>1 353 000</td>
<td>39.69</td>
<td>39.58</td>
<td>-0.267</td>
</tr>
<tr>
<td>168 300</td>
<td>1 080 000</td>
<td>29.01</td>
<td>28.94</td>
<td>-0.344</td>
</tr>
<tr>
<td>148 000</td>
<td>829 100</td>
<td>23.80</td>
<td>23.74</td>
<td>-0.250</td>
</tr>
<tr>
<td>195 700</td>
<td>1 087 000</td>
<td>31.42</td>
<td>31.34</td>
<td>-0.260</td>
</tr>
<tr>
<td>234 600</td>
<td>1 402 000</td>
<td>39.15</td>
<td>39.22</td>
<td>0.189</td>
</tr>
<tr>
<td>262 900</td>
<td>1 503 000</td>
<td>42.98</td>
<td>42.86</td>
<td>-0.276</td>
</tr>
<tr>
<td>87 590</td>
<td>522 300</td>
<td>14.47</td>
<td>14.53</td>
<td>0.409</td>
</tr>
<tr>
<td>319 000</td>
<td>1 870 000</td>
<td>52.98</td>
<td>53.23</td>
<td>0.463</td>
</tr>
<tr>
<td>166 500</td>
<td>1 012 000</td>
<td>27.91</td>
<td>27.8</td>
<td>-0.378</td>
</tr>
<tr>
<td>229 900</td>
<td>1 309 000</td>
<td>37.42</td>
<td>37.32</td>
<td>-0.272</td>
</tr>
<tr>
<td>152 800</td>
<td>931 600</td>
<td>25.63</td>
<td>25.55</td>
<td>-0.304</td>
</tr>
</tbody>
</table>

Note: Data are from Heimath et al. (2000, 2001a). Averaged age is calculated soil residence time. Published age is published soil residence time by Heimath et al. (2000, 2001a).

Results

Comparison of predicted soil depths assuming steady-state condition (Equation (5)) and observed field data at five Australia sites is shown in Figure 1. Here, the bounds on prediction are taken from relevant variability in infiltration rates among the five study sites. The highest infiltration rate (i = 0.31 m yr⁻¹, Snug) and lowest infiltration rate (i = 0.1 m yr⁻¹, Frog Hollow) were used to generate the upper and lower bounds of theoretical soil depths. The majority of the observed soil depths are within the predicted limits and follow the scaling predicted in Equation (5). However, there are a few data points (some soils at Snug, BMT, and TCC) that are outside the prediction (not only the predicted bounds, but the scaling), especially at small erosion rates, indicating a possible non-steady-state condition at lower erosion rates.

To further examine any possible non-steady-state condition, relevant parameters including soil depths and erosion rates published, corresponding soil residence time calculated from Equation (7) based on cosmogenic nuclides concentrations and limits of infiltration rates among sites are plugged into Equation (6) to predict soil depths assuming non-steady-state condition (Figure 2). A better agreement at slower erosion conditions is generated without the assumption of steady-state condition. Here, we want to emphasize that although non-steady-state was assumed here, steady-state condition can be reached at high erosion rate when ‘soil age’ t is larger than the time (t) it takes for production rate R_e to diminish to equal the erosion rate E.

Therefore, soil depths predicted in Figure 2 do not exclude possible steady-state conditions, which is also indicated by the predictions lining up on a straight line (overlap with predictions assuming steady-state condition, not shown in the figure) at large erosion rates.

The better agreement between theoretical and field data for soils at small erosion rates without assuming steady-state condition indicates that these soils are probably not at equilib-
rium, while the consistency between predictions assuming both steady-state and non-steady-state conditions, and observed soil depths at large erosion rates, suggest that most of these soils have reached equilibrium condition.

Results for San Gabriel Mountain (SGM) and Tennessee Valley (TV) are shown in Figures 3 and 4. For SGM, the infiltration rate was determined from the evapotranspiration and precipitation map across the USA (Sanford and Selnick, 2013). Due to the wide ranges of evapotranspiration (0.31–0.4 m yr$^{-1}$) and precipitation (0.51–0.75 m yr$^{-1}$) showing on the map, and the various micrometeorites across Los Angeles County, where SGM is located, the infiltration rate at SGM was determined as a range with upper bound = 0.75–0.31 m yr$^{-1}$ and lower bound = 0.51–0.4 m yr$^{-1}$. For the TV site, typical particle size was 10 μm; infiltration rate was referenced from Table I.

Soils at both SGM and TV sites are more likely to have reached steady-state condition. There is good agreement between predictions assuming both conditions. Since our calculation for soil depth at non-steady-state include the scenario of steady-state condition, the discrepancy between the two would get smaller as soil approaches steady state. In Figure 3, the soil depth determined from steady-state conditions are reached. For the SGM site, the majority of the data are confined by theoretical boundaries, and at TV data are distributed around the theoretical soil depth with a power of $-1.038$ (very close to $-1.149$ from Equation (3)) indicating that it is likely that soils at TV have reached steady-state condition.

**Discussion**

The comparison of our predictions and field data of soil depths at the seven study sites examined in this paper reveals that most soils have reached equilibrium except some of those that have slow erosion rates in southeast Australia. When steady-state condition is reached, consistency will be generated between observations and predictions assuming both steady-state and non-steady-state conditions (as shown in Figures 3 and 4 and in Figures 1 and 2 at high erosion rates). Discrepancies will appear if steady state has not been reached (Figure 1 at slow erosion rates).

Soils that are off the predictions when steady-state condition is assumed generate better agreement without such assumption. However, the production rates (equal to erosion rates assuming steady-state condition) to zero when calculated from non-steady-state conditions are reached. For the SGM site, the majority of the data are confined by theoretical boundaries, and at TV data are distributed around the theoretical soil depth with a power of $-1.038$ (very close to $-1.149$ from Equation (3)) indicating that it is likely that soils at TV have reached steady-state condition.

**Figure 3.** Predicted soil depth assuming steady-state conditions (SS) and non-steady-state conditions (NSS) at San Gabriel Mountain. Upper bound f = 0.44 m yr$^{-1}$, lower bound f = 0.11 m yr$^{-1}$. Particle size = 30 μm. Data from Heimath et al. (2012). [(Colour figure can be viewed at wileyonlinelibrary.com)]

**Figure 4.** Predicted soil depth assuming steady-state conditions (SS) and non-steady-state conditions (NSS) at Tennessee Valley. Infiltration rate = 0.6 m yr$^{-1}$ (Table I). Particle size = 10 μm. Data from Heimath et al. (1997). [(Colour figure can be viewed at wileyonlinelibrary.com)]

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processes that cause fluctuation of erosion rate. There are factors that could cause variations in soil transport along the surface, and by taking a constant erosion rate, our model only deals with ideal circumstances: (3) the uncertainty in erosion rate and the time period over which the soil is forming. Since Equation (7) deals with exposed rock surface, nuclide production rate at any given depth is probably lower than that at the surface, leading to a possible underestimation of $r$ that is used in our model, and the published erosion rates we are taking into the model were calculated based on steady-state conditions, which might not be the actual values; (4) other physical and biological processes that affect soil formation that are not taken into account in our model. These reasons are major difficulties in predicting soil depth accurately (as shown from the $r = 1$ line shown in Figure 5; RMSD = 22.6%). In Figure 5, a power of 0.76 was generated from prediction against observation at all seven study sites. Since shallow soil depth is more vulnerable to outside forcing, such as trampling from animals, the removal of the three shallowest soil samples at depth 0.03 m resulted in closer conformance with linearity (a power of 0.84, though not shown in Figure 5). Thus the power of 0.76 is, as is, our model as correct. Besides the difficulties mentioned above, one main reason that we are not able to address the accuracy of the model is the lack of data. Therefore, future study is needed, with more information of relevant parameters (e.g., particle size) to examine the accuracy of the presented model.

We note that the plateau in soil depths at low erosion rates is not due to our assumption, but can be explained by the existence of a critical scale factor of about 0.5 (Hieblen et al., 1997) that will lead to the inability of the model to predict depth. However, such an exponential cutoff in soil depths is not consistent with the existence of deep weathering in (low-relief) tropical soil profiles, where weathering depths (lowertemperate) may reach 30 m at 5 Myr (Hill et al., 2000), 50 m at 16 Myr (Mignon et al., 2002) or up to 200 m (Gardner, 1957) for Mesozoic age laterites. Consider first the particular case of a 50 m depth at 16 Myr. The infiltration rates applied above result from about 0.1 to 0.6 mm yr$^{-1}$. Taking the particle size we assumed for nearly all sites as 30 μm, an infiltration rate of 0.5 mm yr$^{-1}$ towards the upper end of values used – and using 16 Myr for $t$, the result from $x = 0.33$ (Equation 2) is 63 m for $x$. Thus the same formulation and same parameters used in this paper, which is within 25% of the known depth of chemical weathering in laterites with age 16 Myr – a timescale almost 1000 times as long as those considered in the literature discussed here. The time of formation of the Australian laterites, roughly Mesozoic in age (Gardner, 1957), is not known, but could be from 2 to 10 times as long, since their formation ceased at some point in the late Mesozoic or early Cenozoic. The longest formation time estimate (10 times as long) would generate a depth of 212 m. Consequently, the present formulation has the potential to be applicable to a much wider range of times, circumstances, and results.

For further information, consult Table III regarding a range of thickness predictions over great ages (exceeding 1 Myr) using Equation (3) with uncharged parameter values for $x$ and $t$, namely 30 μm and 0.5 m yr$^{-1}$, respectively. Half the predictions are valid within 40%, and the range of 10% 25%, even though erosion is neglected in each case, and no variation in particle size or infiltration rate was included. To assess the relevance of our chosen infiltration rate for global deep weathering datasets that include a range of climates, more than half of the land surface has an AET between 30% and 80% of the precipitation (Sanford and Sehlnick, 2013); although AET can exceed 90% at high temperatures in, for example, the desert southwest. Thus for tropical soils at least, a reasonable first estimate for $I = AET$ (neglecting the moment surface water routing) is a value of 50% of 1 m: about 0.5 m or perhaps a little less.

A regression of the predicted versus observed depths yields $z = 0.23x + 18$ – a rather large intercept – but an $R^2$ value of 0.93. If the discrepancies are considered to be due to erosion, one finds, to lowest order, a geometric mean erosion rate of 2.4 m Myr$^{-1}$.

**Conclusion**

A model of soil production tied to solute transport-limited chemical weathering has been applied to address the question of whether common assumptions regarding steady-state conditions of soil columns are justified. In the present study, we find that steady-state conditions are more nearly attained at higher erosion rates, consistent with an earlier suggestion by Phillips (2010), but we do not conclude that this result will necessarily translate to other studies or other regions. In less arid regions, for example, soil production function values will tend to be larger, and steady-state conditions can be attained.

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Table III. Predicted and measured depths of bauxites and latelites using Equation (2)

<table>
<thead>
<tr>
<th>Age (Myr)</th>
<th>Predicted depth (m)</th>
<th>Measured depth (m)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>87</td>
<td>100</td>
<td>−13</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>18</td>
<td>+122</td>
</tr>
<tr>
<td>5</td>
<td>34</td>
<td>30</td>
<td>+13</td>
</tr>
<tr>
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<td>18</td>
<td>2</td>
<td>+800</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>15</td>
<td>+100</td>
</tr>
<tr>
<td>3.5</td>
<td>28</td>
<td>25</td>
<td>+12</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>12</td>
<td>+16</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>13</td>
<td>+62</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>22</td>
<td>+36</td>
</tr>
<tr>
<td>16</td>
<td>63</td>
<td>50</td>
<td>+26</td>
</tr>
</tbody>
</table>

Equation (2) is 63 m for $x$. Thus the same formulation and same parameters used in this paper, which is within 25% of the known depth of chemical weathering in laterites with age 16 Myr – a timescale almost 1000 times as long as those considered in the literature discussed here. The time of formation of the Australian laterites, roughly Mesozoic in age (Gardner, 1957), is not known, but could be from 2 to 10 times as long, since their formation ceased at some point in the late Mesozoic or early Cenozoic. The longest formation time estimate (10 times as long) would generate a depth of 212 m. Consequently, the present formulation has the potential to be applicable to a much wider range of times, circumstances, and results.

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A model of soil production tied to solute transport-limited chemical weathering has been applied to address the question of whether common assumptions regarding steady-state conditions of soil columns are justified. In the present study, we find that steady-state conditions are more nearly attained at higher erosion rates, consistent with an earlier suggestion by Phillips (2010), but we do not conclude that this result will necessarily translate to other studies or other regions. In less arid regions, for example, soil production function values will tend to be larger, and steady-state conditions can be attained.

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Figure 5. Observed soil depth versus predicted soil depth at all seven study sites. Infiltration rate varies between sites (Table II). SCM has infiltration rate ranging from 0.11 to 0.44 m yr$^{-1}$; average infiltration rate 0.27 m yr$^{-1}$ was used to predict soil depth. Dashed line represents $r = 1$. RMSD (root mean square deviation) = 22.6%. Data from Hill et al. (2000, 2001a, 2006, 2009, 2012). (Colour figure can be viewed at wileyonlinelibrary.com)
at shorter time intervals. Our prediction is also limited by the incomplete information of variations in relevant parameters (most particularly particle size, but also net infiltration rates). Also, there are uncertainties from the time period for soil production and erosion rates due to wind-erosion by air, or published by the authors based on the concentration of cosmogenic nuclides. The neglect of other physical and biological processes may also affect the accuracy of our prediction; however, we consider the overall discrepancy (Figure 5) between observation and prediction with all limitations mentioned above to be acceptable. Given the fact that salinity levels in the soil are slow (and it keeps slowing down with time), it may still be one of the major limiting factors of soil formation when all other relevant processes are considered.

Improvements in our modeling treatments are needed, even if they would not affect the conclusions of the present paper. First, it will be important somehow to access data regarding particle size at individual sites. It will also be important to generate more specific climatic data, with storm frequency, intensity, air temperature and other measurements important to providing local rates of deep infiltration, and their fluctuations and variability. Even if such data are not easily available, long-term climatic data should be accessed, in case the areas investigated are subject to significant climate change over the period of development of the soil. This eventually appears likely, in view of the common age ranges given as 10 000–100 000 years, or the duration of the most recent glacial advance.

Nevertheless, if our power law soil production function is correct, there is no strict limit on the development of soil as implicit in an exponential soil production function. While soil production rates do continue to decline with increasing depth, they do not cease below an implied skin depth. In line with this, even deep tropical weathering results and associated basalt deposit formation appear to be treatable within the same framework as developed here. Further, all of our soil production function parameters can be obtained from site-specific investigations. The dependence on local infiltration rates may have relevance to modeling hillslope form, while the input from climatic variables should allow, in principle, application of the same results to continental scales. Such an application could help model geomorphic effects of past, or future, climate change or tectonics.

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Yu J, Hunt AG. 2017, Damkohler number input to transport-limited chemical weathering and soil production calculations. ACS Earth and Space Chemistry 1: 30-38.
VI. MODEL APPLICATION II: POSSIBLE IMPLICATION FOR SHALLOW LANDSLIDE PREDICTION THROUGH DISTINGUISHING STEADY-STATE AND STOCHASTIC EROSION PROCESSES

VI.1 Introduction

Studies of steep topography have been of continuing interest to geoscientists and, particularly, geomorphologists. The morphologic characteristics of landscapes reflect the complex feedback between tectonics and climate-driven processes in sculpting the topography. The tectonically active landscapes are often exposed to natural hazards, such as landslide, debris flows, floods and earthquakes. Therefore, understanding the development of soil on steep hillslope is essential, as such regions are typically interpreted as threshold landscapes (DiBiase et al., 2012). Many studies have been reported regarding quantitatively understanding the relationship between erosion rate and topographic elements including hillslope gradient, topographic relief, hilltop curvature, and drainage density (e.g., Gilbert, 1877; Ahnert, 1970; Montgomery and Brandon, 2002; Binnie et al., 2007; Roering, 2008; DiBiase et al., 2010; Hurst et al., 2012; DiBiase et al., 2012). General conclusions have been slow to develop.

Ahnert (1970) reported a linear relation between erosion rate and mean local relief at mid-latitude drainage basins. However, several studies have demonstrated that the linear relationship breaks down as the mean slope increases and approaches a threshold angle of
stability $S_c$ (e.g., Carson and Petley, 1970; Schmidt and Montgomery, 1995; Ouimet et al., 2009; Montgomery and Brandon, 2002; Binnie et al., 2007; DiBiase et al., 2010), at which downslope sediment fluxes become infinite (Roering et al., 2007). In this case, sediment flux switches from creep-related process to mass wasting (DiBiase et al., 2012), and landslides can occur, such that hillslope lowering prevents hillslope from becoming steeper than $S_c$, and erosion rate and topographic relief become decoupled (Schmidt and Montgomery, 1995; Burbank et al., 1996; Montgomery, 2001; Montgomery and Brandon, 2002).

These published descriptions of the relationship between erosion rate and mean slope provide us an opportunity to study soil development on hillslopes by combining the slope-dependent erosion rate with our soil formation model (Yu and Hunt, 2017b). More importantly, soil depth is one of the essential factors controlling shallow landsliding, along with slope angle and slope shape (Iida, 1999), and soil depth is more important for predicting occurrence of shallow landsliding than other factors Okimura (1987). Thus, understanding soil depth development on steep topography is fundamental for understanding and predicting occurrence of landsliding at threshold landscapes.

In this chapter, we adopt the non-linear relationship between erosion rate and mean slope proposed by Montgomery and Brandon (2002) into our soil formation model to predict the slope-dependence of steady-state soil depth on hillslopes. We then compare our predictions with mean observed soil depths at various slope angles to examine the
validity of our model. By assuming a steady-state condition of the soil mantle, the predicted soil depths are effectively assumed to be at stable conditions without the disturbance from landslides, such that soils within such regions are assumed to have constant soil erosion rate dominated by soil creep. However, it is clear that there is evidence of shallow landslides on steep topography at some study sites, potentially causing an underestimation of the prediction comparing with observation. We discuss the relevance of our prediction to landslides, and demonstrate an implication of our model in distinguishing regions that have potential occurrence of shallow landsliding.

VI.2 Materials and Methods

The basic relationship between long-term erosion rate and the mean slope, that we adopted to predict the dependence of soil depth on slope, is obtained from an empirical equation published in Montgomery and Brandon (2002, equation 1), who studied the relationship between erosion rate and slope at Olympic Mountains. Relevant site-specific parameters to set the relation include a background erosion rate due to chemical weathering ($E_0$), a rate constant related to sediment transport coefficient ($K$), and a threshold slope gradient ($S_c$). In order to make predictions, we keep all the default values for Olympic Mountains, and generate the identical curve (Yu et al., 2017) of erosion rate vs. mean slope as shown in their Figure 1. We also digitized the observed data in the field with error bars included (0.0001 m/yr of uncertainty of erosion rate from the original
Here we apply Eq. (1.12) to describe the steady-state soil depth as a function of erosion rate.

\[ x = x_0 \left( \frac{1}{1.87 \phi E} \right)^{1.15} \]

Then erosion rates corresponding to each slope angle are plugged into Eq. (1.12) to predict the dependence of soil depth on mean slope (Figure 6.1). Here, we set \( I = 0.22 \text{m/yr} \), a typical particle size \( x_0 = 30\mu\text{m} \), and the porosity \( \phi = 0.4 \) in Eq. (1.12) as best values for the Heimsath et al. (2012) studies of the San Gabriel Mountains (SGM) (Yu et al., 2017). This enables us to evaluate our predication by comparing observed soil depths on hillslopes at SGM and other 5 sites.

Eq. (1.12) is a power-law prediction of soil depth on erosion rate, \( E \). It contains a numerical prefactor that depends on particle size and the deep infiltration rate. On a bilogarithmic plot of soil depth against slope angle, the only quantity which varies is, to lowest approximation, \( E \), and the net effect of the remaining constants is only to raise or lower the curve by a constant value. Rather than generating a new prediction for each combination of numerical factors, we move each data set vertically, using the ratio of its combined numerical prefactor to that of the SGM. The details are given below.

Multiple soil depths observed at the same mean slope in SGM (Heimsath et al., 2012) are averaged to reduce scatter (Yu et al., 2017). Soil depth from the model (Eq. (1.12)) is proportional to particle size, and is nearly proportional to infiltration rate (to the power of
1.15). Since soil particle size can range orders of magnitudes, our model is even more sensitive to soil type (or substrate particle size) than to the infiltration rate. Yu et al. (2017), based on published values (Schlesinger and Jasechko, 2014; Lvovich, 1973; Peel et al., 2010), summarizes a range of 11% to 35% of annual precipitation that goes to deep infiltration. Here, we take the mean percentage (23%) of precipitation to estimate $I$ for each site. Then, the values are compared with infiltration rate (0.22m/yr) in SGM. We also compare the particle size at different sites. We then normalize the soil depth based on the combined effect caused by the variability in both the particle size and infiltration rate, and the values are listed in Table 6.1. No adjustment is made if the ratio is less than a factor of 2.

<table>
<thead>
<tr>
<th>Site</th>
<th>SGM</th>
<th>PL</th>
<th>AM</th>
<th>Sterling</th>
<th>Lesvos</th>
<th>RdJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ (m/yr)$^b$</td>
<td>0.220</td>
<td>0.253</td>
<td>0.207</td>
<td>0.14</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$x_0$ (µm)</td>
<td>30</td>
<td>30</td>
<td>1095</td>
<td>30</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Factor$^c$</td>
<td>1.00</td>
<td>1.17</td>
<td>33.88</td>
<td>0.60</td>
<td>3.00</td>
<td>22.00</td>
</tr>
</tbody>
</table>

Table 6.1. Particle size and infiltration rate information across sites

a. SGM = San Gabriel Mountain, PL = Plastic Lake Basin, AM = Apennine Mountains, RdJ = Rio de Janeiro

b. Infiltration rate is estimated as 23% of annual precipitation for each site if run-on and run-off values are not available. $I$ value in SGM is referenced from Yu et al. (2017), precipitation in Plastic Lake is 1.1m/yr (Buttle et al., 2004), 0.9m/yr for Apennine
Mountains (Salciarini et al., 2006) and 0.4 m/yr for Sterling, northeastern Colorado (Moore et al., 1993).

c. Factor used to adjust soil depths is calculated as: factor = (particle size on site/30) * 
(infiltration rate on site/0.22)^1.15

Soils in SGM are mainly loams on the hillslopes (Rulli and Rosso, 2005), which has median particle size ranging from 20 to 40 μm. Here we take 30 μm as typical particle size (Yu et al., 2017). For Apennine Mountains, central Italy (Salciarini et al., 2006), soils are mainly talus and are much coarser than SGM. We estimate the particle size based on the hydraulic conductivity (10^{-5} to 10^{-2} m/s) on the site. From information provided by Aqtesolv, USGS and USDA, (accessed in Sep. 2017), comparable hydraulic conductivities are found within the category of coarse sand, which has particle size range from 600 to 2000 μm. Geometric mean of 1090 μm is taken as the typical particle size.

Soils at Sterling are mainly fine loamy and fine silty with percentage of sand ranges from 42% to 54% (Moore et al., 1993), which is likely to be loams, and soil texture is close to that in SGM. For Lesvos, Greece (Bakker et al., 2005) and Rio de Janeiro (RdJ), Brazil (Fernandes et al., 2004) we take a factor of 3 and 22 to normalize the soil depths just to make the values comparable, without the knowledge of infiltration rate and soil particle size in the field. For Lesvos, it is difficult to estimate infiltration rate and soil particle size, because the climate there is characterized by spatial variance in precipitation (Kosmas et al., 2000), with a rainfall gradient of 45% from east to west (Bakker et al., 2005), and the
soil texture (ranges from clay to coarse-grain soils) is greatly affected by parent materials and climatic conditions (Kosmas et al., 2000). For RdJ, there is no direct data for soil depths. According to the authors, the upper portion has soil depth less than 2m, while in the middle portion, soil depth is between 4 to 5m, and at the lower portion, soil is as deep as 12m (Fernandes et al., 2004). We estimate the corresponding mean slope (13.44 degrees) at the middle portion based on the statement that "elevations decrease from 975 to 20m in about 4km”, and estimating the mean slope (25 degrees) at the upper portion by taking the average of mean slope at middle portion and the mean threshold slope (37 degrees) within the region. The greatest soil depth is assumed to be obtained at near zero slope. Very limited information of climate or soil particle size is available, but climate is more humid at RdJ than it is in SGM. Rainfall is very intense during summer at RdJ, which can be as high as 480mm of rainfall accumulation in just 3 days at low elevation, and 700mm at higher elevation (500m) within in the same short period (Fernandes et al., 2004), and more precipitation, thus more infiltration would result in deeper soils.

Observed soil depths after normalization to the same conditions are plotted in Figure 6.1 to compare with our predictions. For sites without values available at slope close to zero, the deepest soil depths obtained (mostly in the valleys) in the field for each site are used as estimations of soil depths at zero slope.
VI.3 Results and Discussion

From Figure 6.1, in the main, our prediction (blue line) based on the empirical equation proposed by Montgomery and Brandon (2002) agrees well with observed data with an overall underestimation, especially in SGM. As slope angle increases, the discrepancy increases as well. Prediction (open diamond with error bars) generated from observed erosion rate along the hillslope (Montgomery and Brandon, 2002) including 0.0001 m/yr uncertainty of erosion rate shows a better agreement with field data compared with that predicted from the empirical equation. Almost all observed soil depths are within the predicted range, except some underestimations in SGM at large slope angle.
Figure 6.1. Comparison of predicted and observed soil depths on hillslope. Solid line is the prediction adopting the empirical relationship between erosion rate and slope angle proposed by Montgomery and Brandon (2002). Open diamonds are the predictions using observed erosion rate with 0.0001m/yr of uncertainty (Montgomery and Brandon 2002). No upper error bar is present if the observed value is already less than 0.0001m/yr.

There are several reasons which could cause the discrepancy: 1) the sensitivity of our model to particle size and the infiltration rate. The prediction generated from our model is proportional to particle size and the infiltration rate to the power of 1.15. There are certainly uncertainties in how we determine the normalizing factor to adjust the soil depths at different sites. Estimation of infiltration rates for each study site can be inaccurate without the actual information of AET, run-on and run-off values, and can vary with slope angle. For individual sites, typical particle size is taken as the $d_{50}$ of the soil class, which eliminates the variation in particle size, 2) the uncertainty originating from the empirical equation that we adopted for prediction. As shown in Figure 6.1, the soil depths predicted based on observed erosion rates (Montgomery and Brandon, 2002) demonstrates a better agreement. As discussed by Montgomery and Brandon (2002), multiple relationships between erosion rate and mean slope are observed as the slope steepens. Therefore, a single empirical equation is close but might not be perfectly accurate to model the entire relationship, 3) the validity of transferring a same relationship across sites having different geologic conditions. The empirical equation (Montgomery and Brandon, 2002) involves 3 parameters: background erosion rate from
chemical weathering \((E_0)\), a rate constant related to sediment transport coefficient \((K)\), and a threshold slope gradient \((S_c)\), all of which might be site-specific. To evaluate the effect from each parameter on the relationship, we change the values of \(E_0\), \(S_c\) and \(K\) to demonstrate how much difference it would cause (Figure 6.2). Results show slight effect from the threshold angle \(S_c\) (double dash line). Changing \(E_0\) mostly affects soil depth at small slope angle, and the difference diminishes as slope increases (double solid line). As for in our case, effect from the rate constant \(K\) (dot line) might be more important than the other two parameters, as changing \(K\) reduces the discrepancy between predications and observations at steep slope angles. DiBiase et al. (2010) proposed a different non-linear relationship between erosion rate and mean slope at SGM, but the two relationships can be matched up if the parameters in the empirical equation are adjusted, suggesting a potential limitation of applying the same relationship generated from a single site to other sites. 4) the occurrence of shallow landsliding at steep slope, which might be the most important factor causing the underestimation. Due to reasons such as the variability in regolith properties and hydrologic conditions, shallow landslide can sometimes occur even below the threshold slope, which can be a range of slopes other than a single slope angle. In any case, the stochasticity in the landslide recurrence intervals will introduce significant variability in soil depths. Moreover, there are evidences of shallow landsliding in some of the study sites on steep topography.
Figure 6.2. Effects from changing the relevant parameters (threshold angle: $S_c$, sediment flux coefficient: $K$, background erosion rate: $E_0$) in the empirical equation (Montgomery and Brandon, 2002) on our prediction. Solid line represents predictions using original parameters.

In the Olympic Mountains, soil is only sustained from shallow landsliding at slope below 25 degrees (Montgomery and Brandon, 2002), even though the threshold angle is at 40 degrees. Heimsath et al. (2012) reported shallow landslides at slopes higher than 30 degrees, with a steepest observed slope of 45 degrees. Landslide scars are observed in both Apennine Mountains (Salciarini et al., 2006) and RdJ (Fernandes et al., 2004). Thus, potential occurrence of shallow landslides should be considered at steep slopes. In this case, our predictions of soil depths assuming steady-state condition can be shallower than
observations between the separation of landsliding, meaning that the landscapes are at unstable condition, and the regions are susceptible to shallow landsliding.

Figure 6.3 shows a schematic diagram generated from our model demonstrating the divergence of soil depth with and without the occurrence of landslides. Landslides are set aperiodically within time scale of 100,000 years. Soil depth in the scenario without landsliding is predicted from Eq. (1.11) to generate soil depth at arbitrary age,

\[
x = \int_0^t dt' R_s = \int_0^t \left( \frac{1}{1.87} \left( \frac{x(t)}{x_0} \right)^{-0.87} - E \right) dt' \\
\text{Eq. (1.11)}
\]

Long-term erosion rate \((E)\) is calculated from the total soil removed in the landsliding scenario dividing by the time scale. When landsliding is considered, soil is mainly removed by mass wasting from landslides, and zero erosion is assumed between the separation of landsliding, thus soil depth is proportional to time (landslide scar age) to the power of \(1/D_b = 0.53\). Such power scaling is actually observed (Figure 6.4) in studies of landsliding recovery (Shimokawa, 1984; Trustrum and De Rose, 1987). In Figure 6.3, the slope becomes unstable once the soil turns thicker than steady-state soil depth.
Figure 6.3. Schematic illustration of soil depths with and without disturbance of shallow landsliding over time. Occurrence of landslidings are generated aperiodically at age of 20,000, 40,000, 70,000, 80,000, 90,000 and 100,000.
Figure 6.4. Scaling of soil development on the age of landslide scars. Data from Shimokawa (1987) is digitized from Iida (1999, Figure 11). First data point at 4 yrs is neglect. For Data from Trustrum and De Rose (1987), first data point at 13 yrs is neglect. Multiple soil depths at 15.341 yrs are averaged to reduce scatter.

Thus, our predictions assuming steady-state condition can set a boundary to separate stable and unstable zones along hillslopes. Soils thinner than the steady-state soil depth and below the boundary are at stable zones, as it is still gaining depth and the production rate is higher than erosion rate. Soils thicker than the steady-state soil depth are at unstable zones, where shallow landsliding can occur. Iida (1999) shows a theoretical boundary of stable and unstable zones on hillslopes by calculating a critical soil depth, which relates to slope angle, soil cohesion, and the hydraulic condition. We digitized the boundary curve as well as the observed soil depths at Hamada City in Shimane Prefecture,
Japan (Iida, 1999, Figure 6), and generated our steady-state soil depth with the empirical equation proposed by Montgomery and Brandon (2002) by setting the threshold angle to be 60 degrees and using a typical particle size of 30µm and infiltration rate of 0.22m/yr. Our predictions are then adjusted by multiplying by a factor of 17 to normalize the climatic and geologic conditions to those at Hamada City, with its higher precipitation of 1.7m/yr (Iida, 1999) and assuming a much coarser particle size. From the soil map of Shimane Prefecture (The National Land Information Division of Japan, accessed Oct. 2017), soils are mainly brown forest soils, which has a nutty structure and is mainly loamy to silty loamy in soil texture (Kamoshita, 1955). A 0.001m/s hydraulic conductivity (Iida, 1999) suggests a coarse soil texture, and Shiau et al. (2017) shows a 65% sand, 18% silt and 17% clay composition of soil texture in forest soils. Thus, in the main, soil in Hamada City is more likely to be sandy loam with more silt than clay. According to a vadose zone modeling report (RAC DSR Report, Table 1-2-2), $d_{50}$ for sandy loam is about 260µm. The result is shown in Figure 6.5. Soil depths higher than the boundary are susceptible to shallow landsliding. Overall, the boundary generated from steady-state soil depth is lower than Iida's (1999) result, suggesting that from our prediction, there are more soils at unstable condition than the original result. In Iida's Figure 10 (Iida, 1999), there are landslide scares observed at the stable zone determined in his paper, however, it is still difficult to verify our result without the information of how landslide scars are distributed in the site or to evaluate the discrepancy comparing with Iida's result by just estimating relevant parameters in the field. In the main, both of
the two models indicate unstable soils at steep slope angles. Soil formation rate decreases as soil depth becomes thicker, while the erosion rate is larger at steep slopes. Thus, at higher slope angles with shallower soil depths, soil production is more rapid. Therefore, if landsliding in the study site is truly set by extreme events such as rainstorms, there will be soils that are thicker than predictions, and susceptible to landsliding. A less severe event needed to trigger landsliding will reduce the number of unstable soils.

Figure 6.5. Predicted boundary of stable and unstable zones on hillslopes in Hamada City in Shimane Prefecture, Japan. Circles are observed soil depth (Iida and Tanaka, 1997), solid line represents theoretical boundary generated by Iida (1999). Both Iida's data and prediction are digitized from Figure 6 in Iida (1999).
VI.4 Conclusion

The soil formation model derived from percolation theory can be used to predict soil depth dependence on slope by adopting an appropriate relationship between erosion rate and slope angle. At steep slope where erosion of soil is mass wasting-dominated or susceptible to shallow landsliding, the predicted soil depth provides a potential method to distinguish soils that are unstable, and to generate a stochastic understanding of landsliding. Moreover, soil depth development on hillslope is one of the controlling factors for recurrence interval of shallow landsliding (Iida, 1999), therefore, the presented model in this paper has a potential to be combined with landsliding studies focusing on other determining factors, such as hydrologic conditions, or drainage area, to develop models for landsliding predictions.
VII. GENERAL CONCLUSIONS

In this dissertation, we present a power-law soil formation model derived from the solute transport theory developed within the framework of percolation theory. The basis of our model relies on the hypothesis that the chemical weathering of bedrock is simultaneously the limiting factor for soil formation and most strongly limited by solute transport in porous media. The hypothesis is supported on several bases: 1) by the values of the Damköhler number that we calculated using experimental data in Chapter II, the results of which imply that chemical weathering is nearly always solute transport-limited, and 2) by the predictive capabilities conferred by the use of solute transport results for soil depth models, as discussed in Chapters III – VI.

The revised calculation of the Damköhler number, $Da_T$, based on percolation theory is consistent with the scaling of weathering reaction rates, and demonstrates that, particularly at low experimental flow rates, solute transport becomes the limiting factor for chemical weathering. The increased relevance of transport limitations at low experimental flow rates suggests that solute transport limitations on chemical reaction in soils are even more important than in experiment, given that the slowest flow rate of 0.18m/d in experiments is still much larger than what is encountered in the field. In Chapter III and Chapter IV, we examine the proposed soil formation model using field data across a wide range of climatic conditions, as well as at steep topography, and explain the causes of variance in soil thickness based on the factors involved in the model.
Results show good agreement between predictions and field observations. While some discrepancies remain, these are comparatively small, and may be due partly to the difficulty of determining the appropriate field parameters. Consider that, in lab experiments, the flow rates, times, reaction rates, and column lengths are usually all given explicitly, but in the field, the soil formation has taken place under uncontrolled conditions over periods from decades to millions of years.

Specific difficulties include: 1) Missing details of the study sites. As shown in Eq. (1.10) to Eq. (1.12), the predictions generated from the models depend on information of particle size ($x_0$), infiltration rate ($I$), porosity ($\phi$), and long-term erosion rate ($E$). Even though reasonable estimations are made, if details of the study sites are missing, these estimations have introduced discrepancies, 2) Simplifications of relevant parameters. The model only takes a single value (usually, the average value) to represent values of certain parameters. For example, median particle size, $d_{50}$, is considered as the particle size, and the erosion rate is a constant in time. However, given that soil formation is a dynamic process, particle sizes can have a wide range, while $d_{50}$ might also be changing over time. Prediction of soil depth is very sensitive to particle size, as there is nearly a linear proportionality between the two at small erosion rate. The erosion rate can also change abruptly if the study sites have experienced severe climate change. Considering average values is the easiest, and might be the most effective way for predictions, however, simplification will reduce the accuracy of the models, especially at short time scales. 3) Instability of shallow soils. We usually find larger discrepancy at shallower or younger
soils. A possible explanation is that shallow soils are more vulnerable to external
disturbance like animal trampling. The development of soils at early stages also depends
on the property of bedrock. 4) The determination of soil depth. As mentioned in Chapter I,
definition of soil is not universal across disciplines. Large uncertainty can be introduced
from different criteria in determining soil depth and its corresponding soil age. The
unclear concept of "what is soil" also makes it challenging to define the relation between
total solute transport distance and soil depth. Extensive data examined in Hunt (2015a) as
well as the results generated from the proposed model show very similar temporal scaling
of transport distance and soil thickness, which suggests the proportionality between the
two. However, the exact relationship remains unclear. 5) Other omissions not considered.
From the five soil-forming factors, we address the effect from time, climate, and
topography, while chemical properties of parent material and effect from biological
activity represent variability that is unaccounted for. In spite of the discrepancies and the
discussed omissions, the presented model appears to account properly for soil formation
factors, through basic soil information, and soil water flux.

In Chapter V and Chapter VI, we demonstrate two applications of the model to
geomorphology. Chapter V suggests the capability of the model in distinguishing local
steady-state and non-steady-state conditions of soils. Despite limitations and uncertainties
discussed above, results show that the local steady-state assumption, which is also the
fundamental hypothesis of the exponential decay model of soil production, is not valid
for some of the soils, especially at small erosion rates. It also suggests the potential of
adopting the soil production model into landscape evolution models. In Chapter VI, we show that with a known dependence of erosion rate on slope angle, the proposed model can successfully predict soil depth dependence on slope. Such application is especially meaningful at steep topography, where the erosion process can be either mass wasting-dominated i.e., shallow landslides, or soil-creep dominated. By comparing the assumed steady-state soil depth and the actual soil depth, one can distinguish the unstable zones that are susceptible to shallow landslides. Moreover, as soil depth is one of the essential controlling factors of shallow landslides (Iida, 1999), our model assuming steady-state condition has the potential to combine with landsliding models focusing on other controlling factors to develop landsliding prediction models.

In summary, this dissertation presents the theoretical framework of a soil formation model on the basis of chemical weathering, and demonstrates theoretical as well as experimental results that support the fundamental hypothesis relating soil formation to chemical weathering. The model is verified through comparing with field data across various climatic conditions, and has been applied to distinguish local steady-state and non-steady-state conditions, and predict soil depth dependence on hillslope such that unstable zones susceptible to shallow landslides along steep topography can be distinguished. There are limitations of the model mainly due to the missing details of the study site, the simplification of relevant parameters, and the omission of some other possible factors affecting soil-forming process, which should be addressed in future studies. However, there is general good agreement between predictions and observations,
and the discrepancies observed are still comparatively small, considering the many factors that can affect soil depth. Improvement can be made by addressing those limitations, but it appears as though the conclusion obtained from the presented model would not be affected. One of the potential advantages of the model is that all parameters involved in the model are quantities that can, in principle, be directly determined locally, even though they are not all customarily measured. If further testing confirms the predictions, the model can be adapted to any landscape evolution models, as well as to climate change models that account for CO₂ sequestration through the Urey reaction (1952) with the promise of helping improve the accuracy of estimating carbon cycling in the ecosystem.
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