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Joint Determination of Rack Configuration and Shelf Space Allocation to Maximize Retail Impulse Profit

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JOINT DETERMINATION OF RACK CONFIGURATION AND SHELF SPACE ALLOCATION TO MAXIMIZE RETAIL IMPULSE PROFIT

A thesis submitted in partial fulfillment of the
requirements for the degree of
Master of Science in Industrial and Human Factors Engineering

by

UTTAM KARKI
B. E., Tribhuvan University, 2017

2019

Wright State University

WRIGHT STATE UNIVERSITY

GRADUATE SCHOOL

12/11/2019

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Uttam Karki ENTITLED Joint Determination of Rack Configuration and Shelf Space Allocation to Maximize Retail Impulse Profit BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Industrial and Human Factors Engineering.

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ABSTRACT

Karki, Uttam. M.S. IHE, Department of Biomedical Industrial and Human Factors Engineering, Wright State University, 2019. Joint Determination of Rack Configuration and Shelf Space Allocation to Maximize Retail Impulse Profit

For brick-and-mortar retailers to be successful, it is critical for them to optimally design their rack layout and place products in order to draw attention of shoppers. Literature suggests that racks placed at acute (or obtuse) angles to the main aisle frequented by shoppers can enhance visibility of products compared to racks placed orthogonally (i.e., 90˚). Placing products with high impulse purchase potential in the resulting highly visible locations on the rack can increase shopper impulse purchases. However, placing racks at angles other than 90˚ can increase the required floor space. Additionally, while reducing the height of the racks just below eye-height enhances visibility, it, however, reduces the number of available locations per product and increases restocking costs.

To effectively trade off the benefits of visibility (in turn, impulse profit) and limitations of space and restocking costs, we propose the Joint Rack Configuration and Shelf Space Allocation (JRC-SSA) problem. The JRC-SSA jointly determines rack decisions (orientation and height) and product decisions (placement and number of locations) in order to maximize a retailer's impulse profit (after discounting for space and restocking costs). As JRC-SSA is an extension of the classical SSA that has been shown to be NP-hard, and that the visibility estimation is not in a closed analytical form, standard

mathematical programming solvers are not suitable. Consequently, we employed the population-based Particle Swarm Optimization (PSO) framework and designed five subroutines to efficiently find a (near) optimal solution to the JRC-SSA.

Using realistic data collected from a major US retailer and that available in the existing literature, we conducted a comprehensive experimental study to derive managerial insights. Results indicate that product decisions were impacted by the angle of the rack; if a high impulse product was placed on the front face near to the endcap in a 90˚ rack, the same product was now placed on the back face in an acute-angled rack. We also noticed that acute-angled racks increased impulse profit over 90˚ racks at low space costs; shorter racks were prominent for low restocking costs. Overall, configurations exist where a retailer can realize up to 8.2% increase in profit through the JRC-SSA compared to a 7 ft height rack placed at 90˚ orientation.

We expect that these, and several other insights discussed in our study, will help retailers in quantitatively evaluating their current rack designs and product placements, and optimize them, to increase shopper experience and, in turn, impulse profit.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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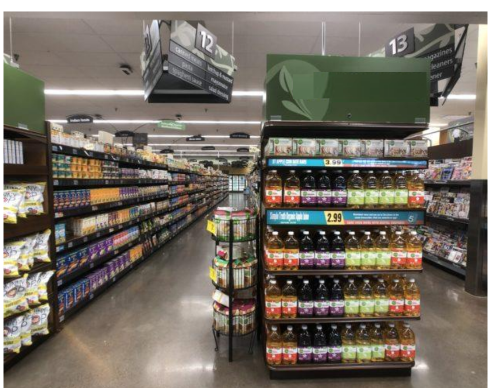
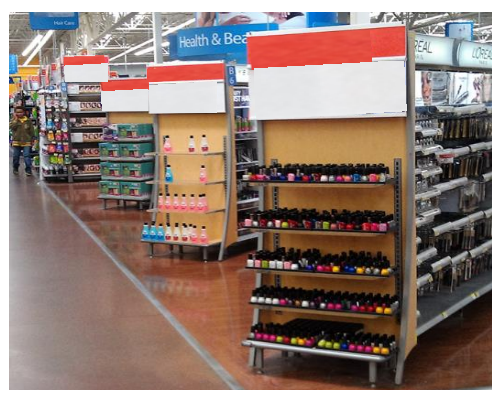
1. INTRODUCTION

Although online retailing has increased in popularity in the recent years, nearly 90% of total retail purchases still come from the traditional brick-and-mortar stores (Levy, 2017). Based on a survey of more than 1,000 shoppers, 70% of shoppers responded that they prefer to shop in physical store of one of the retail chains than its e-commerce (Timetrade, 2017.). Physical stores play a key role over e-commerce in meeting shopper needs for instant gratification, trying out and seeing the products, easy return policy, and spending time with friends and families (Jakovljevic, 2019).

In a physical store, shopper's experience is usually influenced by how they navigate and associate with products in a store (Bitner, 1992; Lu & Seo, 2015). This experience is usually determined by the extent to which products are exposed to them. Product exposure on a rack aids in shopper's interaction with products and plays a prominent role encouraging stores' revenue (Cairns, 1962; Cairns, 1963; Anderson, 1979), as shopper's will only buy what they see (Ebster & Garaus, 2015).



(a) Traditional rack layout



(b) Racks placed at an angle at a leading retailer

(c) Product placement on a rack height < 4 ft

Figure 1: Rack layout and product assignment in retail stores

One way to increase product exposure (and in turn retailer revenue) is to focus on the placement of products on the rack. This problem of placing products in the most visible locations on a rack and allocating appropriate number of locations increasing product visibility is often referred to as the shelf space allocation (SSA) problem (Cox, 1970; Borin et al., 1994; Amrouche & Zaccour, 2007; Flamand et al., 2016; Frontoni et al., 2017). Nearly all approaches to solve the SSA problem, however, assume the rack to be 7 ft high (above shopper eye-height) and placed at 90˚ to the shopper's travel path. This means that the high visibility areas on the rack are prespecified and assumed to be constant.

Recent literature in retail layout suggests that rack design can be a key determinant of what shoppers see and experience during a store visit, and in turn, maximizes retailer revenue. Specifically, racks placed non-orthogonally to the shopper path can increase product visibility on the rack (Mowrey et al., 2018; Guthrie & Parikh, 2019). Additionally, reducing the rack height can reduce occlusion and further enhance visibility (Guthrie & Parikh, 2019). Such innovative rack designs can be seen at stores of several leading retailers; e.g., Walmart places racks at an angle in the Cosmetic section, Kroger uses curved racks, and DSW uses low-height racks (<4 ft).

Clearly, while both the retail layout and SSA literature focuses on retailer's revenue by better exposure of products, they take alternate paths. On one hand, the retail layout literature assumes that product decisions (placement and faces) are known a priori and solve for only the rack decisions (orientation and height). In contrast, nearly all SSA approaches assume that rack decisions are known a priori, and subsequently just solve the product assignment problem. This begs the following questions:

- How do rack decisions (orientation and height) interact with product decisions (location and allocation)?
- How much benefit would a joint determination of rack and product decisions garner to the retailer compared to the assumption of a standard rack (7 ft high at 90˚ to the shopper travel)?

Through this study, we attempt to bridge the gap between these two streams of research in retail store planning (i.e., rack configuration and shelf space allocation) and, subsequently, explore the synergies that can further benefit a retailer. In so doing, we account for a small section of the entire store in which we consider a single rack located between two racks in an aisle. As the middle rack will have occlusion in exposure due to the rack placed in front and back side of it (see Figure 2) and are less exposed to the shoppers. Hence, we picked one of those middle rack as a representative rack in our study and make the following contributions. First, we propose an optimization model that determines the optimal rack orientation and height, along with product placement and faces. The objective of this model is to maximize the total impulse profit after discounting for the rack area and restocking costs. We model impulse profit as a function of the visibility probability of the rack locations, along with product impulsivity and profit. Altering the orientation of rack alters the space required by the rack, which is captured by estimating the required area. Similarly, reducing the height of the rack would reduce the number of available product locations, in turn, impacting the total inventory of each product on the rack. This would impact the frequency of restocking, which is captured through the restocking costs. Second, because the SSA problem has been shown to be NP-hard, and that the visibility estimation for impulse profit is not in a closed analytical form, we propose a heuristic based on Particle Swarm Optimization (PSO) to find the (near) optimal solutions. Third, using realistic data from existing literature and that available from a retailer, we conduct a comprehensive experimental study and identify key insights of practical relevance to a retailer. Finally, we demonstrate the benefits of our integrated approach with a traditional SSA approach that focuses only on the product decisions (assuming given rack orientation and height).

Our experiments suggest that the location of products on the rack depends on the angle of the rack; if a high impulse product was placed on the front side near to the endcap in a 90˚rack, then the same product would be placed on the backside in an acute-angled rack due to substantially different visibility profiles. Further, the number of facings allocated to the products changed substantially with changes in the rack angle and height. We also noticed that acute-angled racks were more prominent than 90˚ racks when area cost was low; racks just below eye-height were more prominent for low restocking costs. We noticed up to 8.2% increase in profit through the JRC-SSA compared to solving the SSA assuming 7 ft height rack placed at 90˚ orientation.

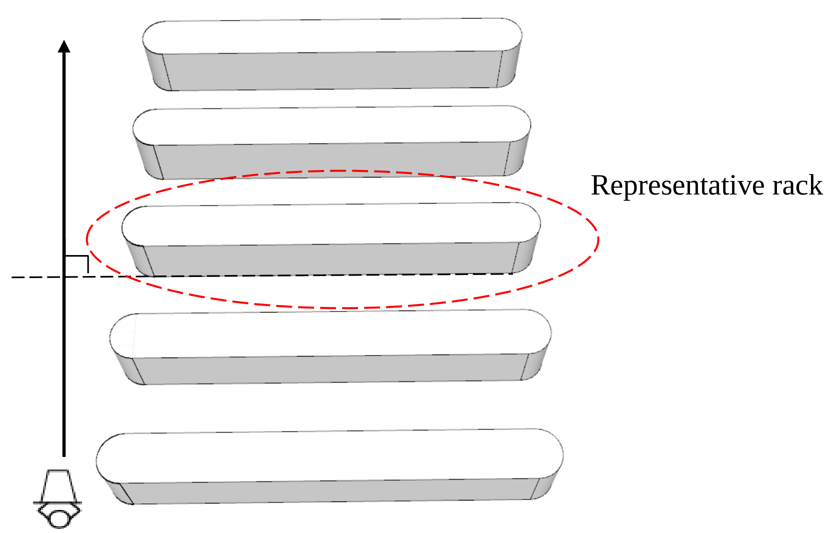


Figure 2: Representative rack on a given store layout

With these foundations, we now present details of our study organized as following outline. Review of the relevant literature is presented in section 2. Our proposed optimization model for JRC-SSA problem and particle swarm optimization approach to solve JRC-SSA model are discussed in section 3 and 4 respectively. We present our experimental design in section 5 and section 6 summarizes our key findings and discuss potential future research.

2. LITERATURE REVIEW

Retail rack design and shelf space allocation are two isolated streams of research in retail planning. While the former focuses on optimizing the rack-level decisions, the latter focuses on product-level decisions. Considering that our study spans across both these streams of literature, we now summarize key research in each of these streams and the corresponding gaps that form the basis of our study.

Literature on retail facility layout has traditionally focused on optimizing the retailer's revenue by optimizing the department placements and rack configuration. Peters, Klutke, and Botsali (2004) were the first to propose a department location assignment model considering three different types of retail layouts; aisle, hub-and-spoke, and serpentine. Although they maximized the impulse revenue generated from the layout, they assumed that a product will only be considered as visible if a shopper is standing next to the product along their path. To address the department sizing and placement problem, Yapicioglu and Smith (2012) proposed a bi-objective model where they maximized the store revenue. They determined the exposure based on fixed customer traffic zones. They assumed that high traffic zones will have high number of shoppers in those areas, thus making department highly visible; i.e., they considered visibility as a function of those traffic zone and department sizes. Recently, Hirpara and Parikh (2019) proposed a model to optimally place the departments in a store by explicitly accounting for changes in the shopper path with changes in the department layout. They derived up to *k*-shortest paths to pick products in a shopper's planned purchase list and considered department as visible if it was along the shopper path.

While the above approaches used a high-level measure of visibility, more recent approaches have taken a more fundamental approach by using a shopper's field of vision to develop refined estimates of visibility and use it towards optimizing rack configurations. Mowrey and Parikh (2018) proposed the Retail Rack Layout Problem that optimized the rack orientation and number of columns across multiple racks. In their proposed non-linear optimization model, they used a visibility measure by considering shopper's horizontal field of view. They observed that, for a given space constraint, acute-angled racks can substantially increase visibility with only a marginal decrease in rack locations. Depending on the duration of exposure, acute or obtuse-angled rack can increase product exposure from 213-226% in small head turn and 17-18% in large head turn over 90° rack orientation. Guthrie and Parikh (2019) extended this visibility measure by considering both horizontal and vertical field of vision, and considering curved racks and racks of varying heights. The resulting 3D estimation problem was solved using an analytical-computational approach. They later use these estimates in solving the Rack Orientation and Curvature Problem of identifying the optimal rack angle and curvature to maximize impulse profit, after discounting for space cost (Guthrie and Parikh, 2019). Depending on the system parameters, an angled rack orientation that increase floor space by 18% can increase exposure by 530% while moderate increment in floor space (<5%) can still increase exposure by 48% (Guthrie, 2018). They found that rack height, orientation, and curvature, in that order, affected the visibility and, in turn, impulse profit. However, this work was limited in that it assumed a prespecified set of product decisions (placement and number of locations).

Another related area in retail planning is Shelf Space Allocation (SSA), which employs the fact that high impulse potential products are sensitive to changes in the shelf space (Curhan, 1972; Desmet and Renaudin, 1998). Accordingly, the objective of the SSA problem is to determine the best placement and location assignment across multiple products, along limited shelf space, in order to maximize expected revenue (Murray, 2010). Hwang, (2009) proposed a model to design shelf space and product allocation problem to maximize the retailer's profit and solved it using genetic algorithm. Ghoniem et al. (2014a) proposed a mixed-integer nonlinear model optimizing product assortment and pricing decision in order to maximize retailer's profit. They found that jointly planning retail categories can save 5%-65% of profit and prevent suboptimal assortments. Ghoniem et al. (2014b) proposed a mixed-integer programming model to maximize the average impulse buying profit per customer by determining the shelf space allocation for individual products categories. Zhao et al. (2016) proposed a joint optimization model to solve shelf space allocation and product display location problem, where they also accounted for multi-item replenishment; items replenished individually, and items replenished jointly. A simulated annealing based hyper-heuristic algorithms was proposed and found that joint replenishment policy leads to a higher profit than that of the model for the individual replenishment policy.

Flamand et al. (2016) solved an optimization problem with product location and shelf-space allocation as decision variables in order to maximize the impulse profit per basket. They found that assigning products with high impulse purchase along high customer traffic densities increases the average impulse profit per basket. In an extension to this work, Flamand et al. (2018) considered the product affinity and disaffinity constraint to maximize the overall store's profit proposing a store-wide shelf space allocation model. Similarly, Frontoni, (2017) proposed a model to minimize the out of stock cases by optimally re-allocating the shelf space. They proposed an integer linear programming model with a space elastic demand function.

Although the SSA literature is fairly matured, almost all of the proposed approaches assumed a 7 ft, 90° rack. But the retail rack configuration literature suggests that the visibility profile on a rack can alter significantly based on the rack orientation and height. No known models or analysis exist that suggest what may happen to these product decisions if the rack configuration was altered.

Our study fills this exact gap by proposing a novel, joint approach towards identifying the optimal rack and product decisions. We do this by accounting for changes in the area requirement for non-90˚ racks and changes in the restocking costs for shorter racks. We now present our proposed model for JRC-SSA.

3. AN OPTIMIZATION MODEL FOR THE JRC-SSA PROBLEM

Our proposed optimization model determines the (i) rack height, (ii) rack orientation, (iii) product sequence, and (iv) number of product locations on a given rack. The objective is to maximize the marginal impulse profit after offsetting the cost of area and restocking of the products on the shelves. We make the following assumptions in building our model:

- We solve the problem for a single rack which is a representative rack that is an intermediate and the visibility of the locations on it are known.
- All product categories have already been allocated to the rack and must be assigned.
- The shopper is walking along the main aisle heading towards a planned purchase list when encountering the rack under consideration; the visibility of a product on this rack is considered from the main aisle.

Table 1 and 2 shows the parameters and decision variables used in the optimization model, which are also illustrated in Figure 3.

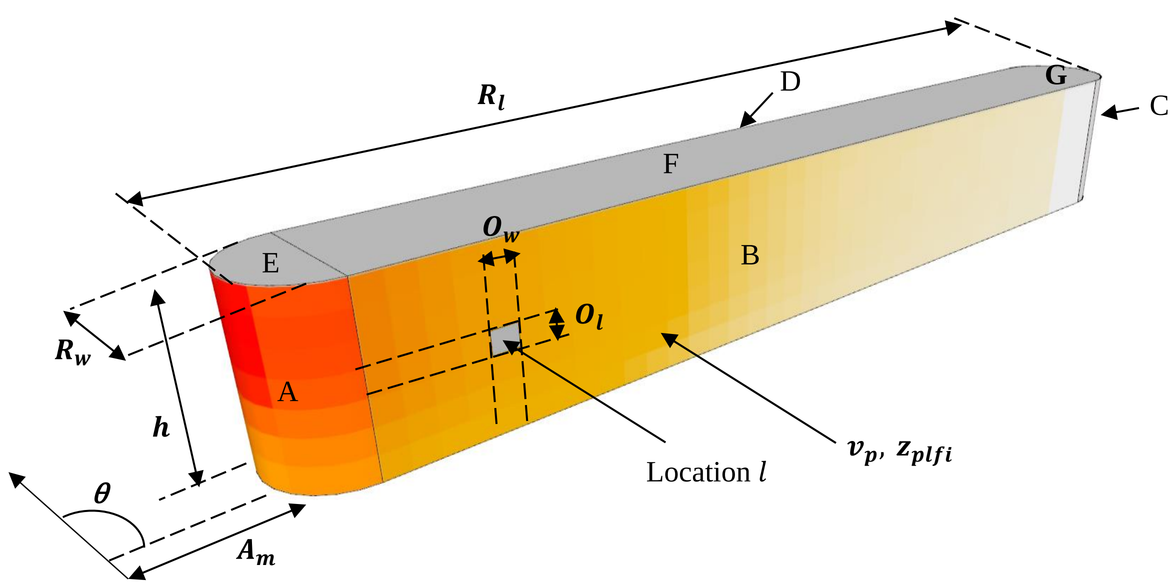


Figure 3: Representative rack with parameters and decision variable

Table 1: Parameters used in the model

Notation	Definition
I	Set of allowable rack heights; $i \in I$
L	Set of locations on the rack; $l \in L$
F	Set of rack faces; $f \in F$
P	Set of product categories; $p \in P$
R_l, R_w	Length and width of the rack (ft)
L_p^{max}, L_p^{min}	Maximum and minimum shelf locations that can be assigned to product category p
A_c	Distance between two successive racks (ft)
A_w	Width of the main aisle (ft)
O_l, O_w	Length and width of a location (ft)
I_p	Probability of products to be purchased from category p if seen
P_p	Profit generated from the product in category p
N_p	Number of products in category p that can be stacked in a location l
D	Number of days the store is open annually
S	Number of shoppers per day visiting the store
S_e	Shopper's eye height (ft)
Ω_h, Ω_v	Horizontal and vertical field of regard of a shopper
DOV	Shopper's depth of vision (ft)
C	Cost of floor space ($/ft^2)
R	Restocking cost ($/restock)

Table 2: Decision variables used in the model

Notation	Definition
h	Height of the rack (ft)
θ	Rack orientation (°)
y_h	1, if height of the rack is h; 0, otherwise
z_{plfi}	1, if product category p is assigned to location l on face f at height h; 0, otherwise
l_p	Locations allocated to product category p
r_p	Number of visits made for restocking product category p
l_{tot}	Total number of locations on the rack
v_p	Probability of visibility for product p during the shopping path
a	Required floor space (ft^2)

We now propose the following optimization model to jointly solve the retail rack layout and shelf space allocation problem (JRC-SSA).

$$\text{maximize} \qquad SD \sum_{plfi} \left(I_p P_p z_{plfi} v_p - Rr_p \right) - Ca$$

subject to

$$v_p = f_1(z_{plfi}, l_p, g(\theta, h, R_l, R_w, A_c, A_w, S_e, O_l, O_w, DOV, \Omega_h, \Omega_v)) \tag{1}$$

$$a = f_2(\theta, R_l, R_w, A_c, A_w, O_l, O_w) \tag{2}$$

$$\sum_{pfi} z_{plfi} = 1 \quad \forall\, l \tag{3}$$

$$L_p^{min} \leq \sum_{lfi} z_{plfi} \leq L_p^{max} \quad \forall\, p \tag{4}$$

$$\sum_{plf} z_{plfi} \leq y_h \quad \forall\, i \tag{5}$$

$$\sum_{h} y_h = 1 \tag{6}$$

$$\sum_{i} i y_i = h \tag{7}$$

$$l_{tot} = 2h\left(\frac{R_l}{O_l}\right) + 2(4h) \qquad \forall\, i | h > 4 \tag{8}$$

$$l_{tot} = 2h\left(\frac{R_l}{O_l}\right) + 2(h+2) + R_w\left(\frac{R_l}{O_l}\right) \qquad \forall\, i | h \leq 4 \tag{9}$$

$$r_p \geq \left(\frac{v_p I_p}{N_p l_p}\right) \ \forall\, p \tag{10}$$

$$\theta \in [30^0, 150^0] \tag{11}$$

$$0 \leq v_p \leq 1 \ \forall\, p \tag{12}$$

$$a \geq 0 \tag{13}$$

$$y_h, z_{plfi} \in \{0,1\} \qquad \forall\, i, p, l, f \tag{14}$$

The objective of JRC-SSA is to maximize the marginal impulse profit generated by the model. Notice the nonlinearity in the first term ($z_{plfi} \cdot v_p$). To estimate marginal impulse profit, we first calculate the impulse profit and subtract total space cost and restocking cost from the impulse profit generated. Constraint (1) calculates the product's visibility based on number of locations allocated to a product and their location's visibility. Constraint (2) calculate the required floor space and constraint (3) ensures every location on a rack needs to be assigned with a certain product category p. Constraint (4) bounds the number of locations allocated to a product category p. Constraint (5) ensure the rack should have a certain height in order to assign the product category and Constraint (6) and (7) ensures only one rack height can be selected from the allowable set of rack heights. Constraint (8) and (9) calculates the total number of locations on a rack for a given rack height, which the non-linear. Constraint (10) calculates total number of minimum annual restocks to be made for each product category p. Constraint (11) bounds the rack orientation between 30°and 150°. Constraint (12) bounds v_p values between 0 and 1. Constraint (13) describe that the required floor space is non-negative and constraint (14) explains the binary decision variables in the optimization model.

Recall that JRC-SSA integrates key decisions related to rack configuration (height and orientation) and shelf space allocation. Existing literature to address the SSA problem suggests that it is a NP-hard problem, for which no known exact procedures are available (Flamand et al., 2016; Murray et al., 2010). Similarly, recent literature in optimizing rack configuration points to the lack of a closed-form expression to estimate product visibility, v_p (Guthrie & Parikh, 2019). Both these observations, along with the non-linearity in the objective function and a constraint, compound the complexity of the JRC-SSA and render it difficult to be solved using state-of-the-art mathematical programming approaches. In light of this, we propose a metaheuristic approach based on the particle swarm optimization (PSO) framework to solve the JRC-SSA problem.

4. A PARTICLE SWARM OPTIMIZATION BASED HEURISTIC

4.1 PSO Description

The Particle Swarm Optimization framework mimics the social behavior of flocks of birds, swarm of bees, and fish schools (Sun et al., 2004; Prasannavenkatesan and Kumanan, 2011). A number of successful applications of PSO have been reported; e.g., facility layout (Ohmori, 2010; Kundu, 2012; Mowrey and Parikh, 2018) and supply chain (Prasannavenkatesan and Kumanan, 2011; Park and Kyung, 2013). A finite number of particles are initialized in PSO to find the best possible solution in the search space. After each iteration, a particle's position and velocity are updated based on the particle's previous velocity, previous position, and global best position (discussed later in this section).

In our proposed PSO procedure, a solution is represented as a vector of the decision variables ordered as rack height (h), rack orientation (θ), sequence in which product categories will be placed on the rack, and their corresponding facings. An example representation with 5 product categories would be as follows: $\{h, \theta, 4, 2, 1, 5, 3, 11, 16, 22, 14, 15\}$, where positions #3-#7 indicate the sequence of product categories and positions #8-#12 indicate their corresponding number of locations on the rack.

We enhanced the standard PSO framework by incorporating five subroutines to effectively search the solution space and evaluate the candidate solutions: *Rack Design* subroutine, *Product Assortment* subroutine, *Product Assignment* subroutine, *Product Visibility* subroutine, and *Impulse* subroutine. At each iteration, the candidate solution (represented by a particle) goes through all these subroutines yielding a potential global best solution. The below pseudo-code summarizes the overall algorithm. We now explain each subroutine.

```
Initialize population of particles with random positions and velocities
Do
  For each particle:
     Evaluate feasibility of the encoded solution
     If Feasible:
        Convert encoding to rack layout (Rack Design subroutine)
        Convert encoding to product category assortment (Product Assortment subroutine)
        Place products on a rack based on assignment rule (Product Assignment subroutine)
        Estimate v_p (Product Visibility subroutine)
        Evaluate fitness function (Impulse subroutine)
           If fitness value is greater than global best:
              Set current solution as global best
           If fitness value is greater than neighborhood best:
              Set current solution as neighborhood best
           If fitness value is greater than particle best:
              Set current solution as particle best
         Else:
           Reject solution
     Else:
         Reject solution
  End
```

4.1.1 Rack design subroutine: This subroutine determines both the rack height and rack orientation. For these, we use the smallest position value (SPV) rule to convert a real-valued number into a feasible integer value between prespecified lower and upper bounds on the height and orientation (Kaur and Tiwari, 2012).

For instance, let the value of position #2 (representing θ in the particle) be -25. We first generate a sequence from X_{min} to X_{max} of length equal to the number of possible solutions as $\frac{X_{max}-X_{min}}{(N-1)}$, where X_{max} and X_{min} are the upper and lower bounds of the search space. Here, N is the number of possible parameter values; N for rack orientation = 181 (i.e., {0˚, 1˚, …, 180˚}). An example calculation of a sequence of length 181 to encode rack orientation for X_{max} = 50 and X_{min} = -50 can written as {-50, -49.44, -48.44, …, 49.44, 50}.

We then subtract the position value (i.e., -25) from this sequence and take the absolute value of each position in this sequence yielding a new non-negative sequence. The position of the smallest value index in this non-negative sequence represents the solution of that parameter in that iteration. Continuing with the previous example, after subtracting the position value of θ = -25 from sequence and considering the absolute value, we get {25, 24.44, 23.89, …, 0.55, 0.00, 0.55, ……, 74.44, 75}. In this new non-negative sequence, 0.00 is the smallest value and its position index is #45. Hence, we set θ = 45˚ as our rack orientation in current solution. It is easy to place bounds on this sequence by assigning a big number M for values outside the bound to ensure that the chosen position is within the bounds.

4.1.2 Product assortment subroutine: This subroutine determines the sequence of product categories to be assigned on a rack and the corresponding number of locations. We again use the SPV rule to convert real-valued numbers into integers.

To understand this better, suppose the values from position #3 - #7 (representing product assignment sequence for 5 product categories) are {10.02, -15.9, 35.61, -45.11, 21.35}. In the sequence, position index that has the smallest value is chosen as the first product to be assigned, second smallest as the second product and so on. For example, the 4th position index (-45.11) is the smallest value in the sequence, and hence product category 4 is assigned first. Similarly, 2nd position index (-15.9) is the second smallest value in the sequence and product category 2 is assigned second. Hence the final product assignment sequence will look like {4, 2, 1, 5, 3}. Table 3 shows the position vector and product assignment sequence for 5 product categories.

Table 3: Generating product category sequence based on position vector

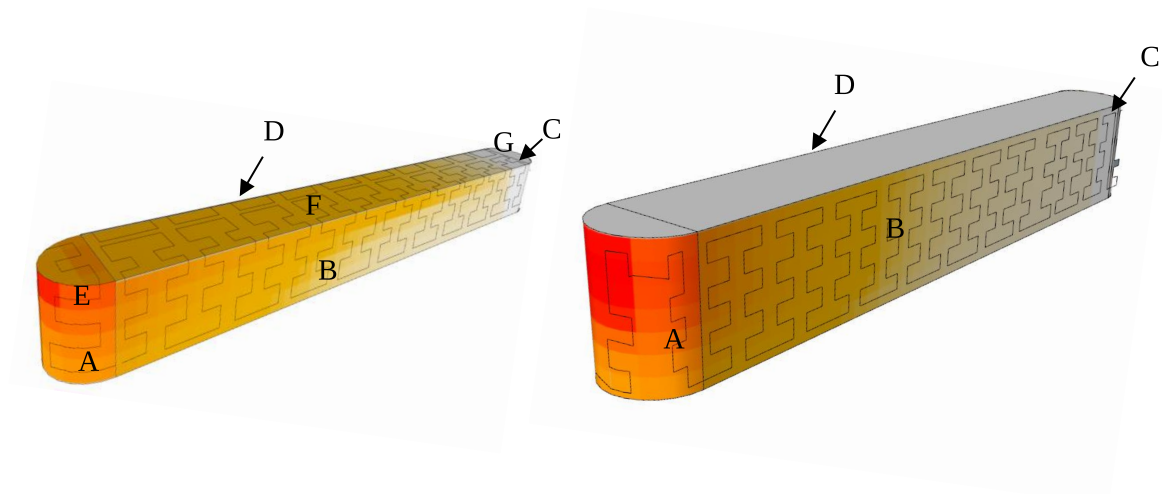
Position vector	10.02	-15.9	35.61	-45.11	21.35
Product category sequence	3rd	2nd	5th	1st	4th

To encode the number of locations, suppose the total number of locations (l_{tot}) is 616 and the value from position #8 - #12 is represented by {-37, 44, -10, 18, -35}. First, we determine the multiplication factor by taking the ratio of l_{tot} to the sum of the absolute value of position #8-#12. Then we multiply the absolute value of each position by the calculated multiplication factor giving us a real number that represents the number of locations to be assigned to the product sequence generated above. Since the number of locations cannot be fractional, we round all the positions and get the integer value of the number of locations for the product sequence. In above case, multiplication factor was found to be 4.278 and the final number of locations would be {158, 188, 43, 77, 150}. If rounding exceeds the total number of locations on the rack, then we reduce the locations allocated to product category with the lowest $I_p P_p$ value. Similarly, if rounding leads to not utilizing all the rack locations, then we first assign locations to product categories with locations less than minimum locations (if such is the case due to rounding) and then to a product category with the highest $I_p P_p$ value.

4.1.3 Product assignment subroutine: This subroutine assigns product categories to the rack based on the sequence and locations determined in the above subroutine. To do this, we use space-filling curve to facilitate product adjacency and reduce the chance of irregular shapes. Consider an example for rack height of 7 ft as shown in Table 4 and a prespecified space-filling curve (based on the preference of the retailer) shown in Figure 4. In Figure 4, the filling pattern for space-filling curve is determined by the integer number from 1 to l_{tot} ; shown in Figure 4 itself. In Figure 4, "1" represent the starting position of space-filling curve and moves towards l_{tot} in increasing order. This subroutine assigns product categories to this rack in the form shown in Figure 4.

Table 4: Product categories and locations allocation

Product categories	1	2	5	12	11	8	7	4	6	10	9	3
Number of locations	22	32	31	59	80	23	77	80	71	80	29	32



(a) Space filling curve on 4 ft rack height *(b) Space filling curve on 7 ft rack height*

Figure 4: Space filling curve on 4 ft and 7 ft rack height

4.1.4 Product category visibility subroutine: For a given particle (which represents the assignment of product categories on a rack placed at a specified angle and height), this subroutine calculates v_p (the probability a product category p is seen at least once by the shopper). From expression (1) in our proposed optimization model, the probability a product category p to seen at least once by the shopper, $v_p = f_1\left(z_{plfi}, l_p, g(\theta, h, R_l, R_w, A_c, A_w, S_e, O_l, O_w, S_h, DOV, \Omega_h, \Omega_v)\right)$. We employ the approach presented in Guthrie and Parikh (2019) to derive the function f_1, which uses information about the location of product category p (z_{plfi} and l_p) and the probability of a location seen at least once by the shopper (which depends on the rack decisions, aisle widths, and shopper attributes) as given by $g(\theta, h, R_l, R_w, A_c, A_w, S_e, O_l, O_w, S_h, DOV, \Omega_h, \Omega_v)$.

4.1.5 Impulse subroutine: This subroutine calculates the expected marginal impulse profit for each particle as $SD \sum_{plfi}\left(I_p P_p z_{plfi} v_p - Rr_p\right) - Ca$. Area cost is determined based on the rack height and orientation obtained using the procedure described by Guthrie and Parikh (2019). Essentially, that approach creates a bounding box around the rack and incorporates cross-aisle and main aisle area. Restocking cost for each product category (in each particle) is estimated based on the annual demand of that product category, number of locations assigned to it, and the quantity per location, $r_p \geq \left(\frac{v_p I_p}{N_p l_p}\right) \ \forall\, p$.

4.2 Solution Updating

At each iteration i, the position of the particle is represented by X_{ij}^t and velocity by V_{ij}^t. The position and velocity of particles are updated as of equation (1) and (2).

$$V_{ij}^t = K(V_{ij}^{t-1} + C_1 r_1\left(Pbest_{ij}^{t-1} - X_{ij}^{t-1}\right) + C_2 r_2\left(Gbest_j^{t-1} - X_{ij}^{t-1}\right)) \tag{1}$$

$$X_{ij}^t = X_{ij}^{t-1} + V_{ij}^t \tag{2}$$

In equation (1), r_1 and r_2 are the uniform random number between [0, 1] and determine the rate of movement towards local best or personal best solution. C_1 and C_2 are the acceleration constant and K is the constriction coefficient. Preliminary experiments suggested that dynamically raising the value of C_2 in comparison to C_1 improved solution quality and convergence. While we set C_1=2.05 per suggestion by Clerc and Kennedy (2002), we initiate C_2= 0.4 and increase it by 0.2 after first 1000 iterations and then after every 500 iterations. We set K set to 0.7282 (Clerc and Kennedy, 2002). The randomness in velocity might cause the particle's velocity to move towards infinity, hence we incorporated limits on velocity as -50≤ V_{max} ≤50. When particle's velocity crosses these bounds, its velocity value is set to its nearest bounds. Similarly, for a particle's position, limits were added as -50≤ X_{max} ≤50. We used no further improvement (> 0.05%) in global solution for 1000 iterations and maximum iterations of 10,000 as the stopping criteria.

5. EXPERIMENTAL STUDY

In order to understand the sensitivity of solutions generated by the PSO for various system configuration parameters and generate managerial insights, we conducted a comprehensive experimental study using realistic data, as discussed below.

5.1 Data collection

Data for our experiments were collected from two nearby retail stores. Details about type of products assigned, assignment locations, number of locations, rack dimensions and orientation were recorded from both the retailers. Since the data collected were at the product level, we screened the product information and grouped them into a product category level. For instance, sugar from Domino and Great Value were combined under product category Sugar.

Table 5: Data from Retailer 1

Product category	Impulse purchase rate	Profit per unit ($)
Baking/chocolate	0.2600	2.91
Kraft spreads	0.2793	2.05
Chili	0.4450	0.70
Pasta sauce	0.2625	1.05
Biscuits/rolls	0.2601	1.03
Jell-O	0.2468	1.06
Canned fruit	0.4490	0.57
Cat food	0.0794	3.08
Japanese food	0.2633	0.76
Macaroni	0.2554	0.39
Sugar	0.0705	1.09
Beans	0.2541	0.27

Table 6: Data from Retailer 2

Product category	Impulse purchase rate	Profit per unit ($)
Taco seasoning	0.0759	2.35
Rice	0.0782	2.25
Precooked beans	0.2271	0.26
Beans	0.2541	0.27
Spaghetti	0.0678	4.65
Tuna	0.2570	0.92
Ranch dressing	0.2604	0.74
Mayonnaise	0.2554	0.39

A total of 20 products from Retailer 1 were grouped into 12 product categories and 18 products from Retailer 2 into 8 product categories. Impulse purchase rates of the product categories were obtained from (Flamand et al., 2016). Similarly, per unit profit for each product categories are based on estimates used in (Guthrie and Parikh, 2019). Tables 5 and 6 summarize this data from the two retailers.

Table 7 summarizes the layout and shopper parameters we used in our analysis. The shopper field of regard (horizontal and vertical) and eye-height were per Guthrie and Parikh (2019).

Table 7: Layout parameters

Parameter	*Value*
Shopper's vertical field of regard, up and down from center, $(\phi_v + \Omega_v)$	45°
Shopper's horizontal field of regard, left and right from center, $(\phi_h + \Omega_h)$	45°
Shopper's depth of view (DOV)	50 ft
Shopper eye-height (S_e)	5 ft
Cross aisle and main aisle width (A_c and A_m)	8 ft and 10 ft
Rack length and width (R_l and R_w)	40 ft and 5 ft

5.2 PSO Performance

Preliminary experiments with the above data suggested that 40 particles – each particle is a candidate solution – was sufficient to get high-quality solutions in a reasonable time. We coded the PSO based meta-heuristic in R programming language with parallel implementation. All the experiments were implemented on Intel(R) Core™ i7-8750H CPU@2.20 GHz, 12 cores 16 GB RAM personal computer.

We used two metrics to evaluate the PSO performance; variation in the objective function 'within particles of a run' and 'between runs.' To do so, we ran 5 instances of the model with parameter values; \$20/ft^2 annual space cost, \$4/restock as restocking cost, 100% profits per product and 1000 as shopper's volume. Additionally, store opening days was assumed to be 365 days. All 5 instances were run for stopping criteria of maximum 10,000 iterations or no improvement in objective function (greater than 0.05%) by 1,000 iterations. Table 8 summarizes the results for all 5 instances with variation in objective function and computational time.

Table 8: Comparison of PSO solutions and computational time

Instance	Best layout		Objective	Iterations	Computational time (hours)	Within particle variation in this run
	h	θ				
1	4 ft	30°	$1,144,765	1100	1.51	0%
2	4 ft	30°	$1,147,138	1036	1.37	0%
3	4 ft	30°	$1,152,321	1037	1.52	0%
4	4 ft	30°	$1,150,394	1117	1.62	0%
5	4 ft	30°	$1,150,389	1057	1.54	0%

Notice in Table 8 that all the particles converged to a global best solution in each of the 5 runs. The variation 'within particles of a run' was 0% and 'between runs' ranged from 0.167-0.656%. The mean objective function across the 5 runs was $1,149,001 with a standard deviation of $3,013. The average computation time was 1.512 hours. These findings provided sufficient evidence that our PSO was robust. We, therefore, used this PSO implementation to conduct our experiments and generate managerial insights.

5.3 Experimental Factors

We considered three levels of annualized space cost/ft^2. Based on our literature, the annual floor space cost ranges from $16/ft^2 in Cleveland, OH to $293.02/ft^2 in Los Angeles, CA. Hence, we used $20/ft^2, $50/ft^2, $100/ft^2 as representative values. In addition, three different values of restocking cost; $4, $10, $80 per restock (including labor and equipment cost) were considered for experimental study.

We also considered two levels of profit per products. While 100% represented the data we had collected, 50% tried to emulate situations when the products had a lower profit margin. For instance, beauty products, phone accessories, activewear, and similar are the high-profit products (Widmer, 2019), whereas milk and bread are examples of low-profit margin products. The facings of each product category are bounded between 20 and 80. To evaluate stores with low and high customer traffic, we used 250 and 1000 shoppers per day, with the assumption that the store open 365 days in a year. Table 9 summarizes these parameters and their values.

Table 9: Parameters values used in experimental study

Parameters	*Levels*	*Values*
Space Cost	3	\$20, \$50, \$100
Restocking Cost	3	\$4, \$10, \$80
Profit per product	2	100%, 50%
Number of Shoppers	2	250, 1000

Impulse profit per product category was calculated by taking a product of impulse purchase rate and unit profit; i.e., $I_P P_P$. We grouped the product categories in Table 5 into three different levels; high, medium and low, based on impulse profit. Table 10, 11, 12 and 13 summarizes the solutions from the 36 experiments we conducted. In these table, “Assignment” column represents the product categories assigned to different faces on a rack. The top row in this column represents product categories and bottom row (italic font and highlighted in light grey) denotes number of locations assigned to those product categories. The last column “7ft, 90°” indicates the objective function of such a layout with optimized product assignment; we do this by fixing θ = 90 and h = 7ft in the PSO; the “%-diff” indicates the %-increase in the objective function realized through a rack that is either short, angled, or both. Notice that increase of up to 8.2% can be realized using our proposed JRC-SSA approach. Key observations that explain such increases are discussed below.

Table 10: Summary of results from 250 shoppers and 100% profit level

S	**Profit**	***C* $/ft²**	***R* $**	***h***	***θ***	**JRC-SSA Objective**	**Assignment**				**7ft, 90°**	
							A/E	**B/F**	**C/G**	**D/F**	**Objective**	**% Diff from JRC-SSA**
250	100%	$20	$4	4 ft	30°	$256,475.5	5 *32*	5, 3, 10, 11, 12 *32, 80, 80, 80, 80*	12 *80*	12, 7, 6, 9, 4, 1, 8, 2 *80, 66, 42, 22, 22, 21, 21, 22*	$249,744.3	2.6%
			$10	4 ft	30°	$245,152.2	4 *31*	4, 8, 12, 10, 11 *31, 61, 80, 80, 60*	11, 7 *60, 80*	7, 6, 5, 3, 2, 9, 1 *80, 41, 35, 25, 25, 25, 25*	$243,264.9	0.8%
			$80	7 ft	90°	$176,076.6	1 *33*	1, 9, 6, 7, 12, 8, 11 *33, 31, 51, 80, 31, 80, 31*	11 *31*	11, 10, 5, 3, 4, 2 *31, 59, 80, 77, 31, 22*	-	-
		$50	$4	7 ft	90°	$229,004.6	2, 3 *20, 20*	3, 6, 5, 8, 11,12 *20, 20, 54, 80, 80,80*	12 *80*	12, 10, 9, 7, 4, 1 *80, 80, 80, 62, 20, 20*	-	-
			$10	7 ft	90°	$222,761.9	7, 1 *20, 20*	1, 8, 6, 5, 9, 12 *20, 20, 39, 80, 80, 80*	12 *80*	12, 11, 10, 3, 4, 2 *80, 79, 80, 74, 24, 20*	-	-
			$80	7 ft	90°	$159,362.1	1 *30*	1, 8, 4, 5, 6, 12 *30, 29, 33, 80, 79, 80*	12, 7 *80, 80*	7, 10, 11, 3, 9, 2 *80, 29, 33, 79, 34, 30*	-	-
		$100	$4	7 ft	90°	$195,073.1	1, 2 *20, 20*	2, 5, 3, 8, 9, 12 *20, 20, 31, 80, 80, 80*	12, 11 *80, 80*	11, 10, 7, 6, 4 *80, 80, 77, 28, 20*	-	-
			$10	7 ft	90°	$182,955.7	3, 2 *21, 21*	2, 6, 8, 10, 12 *21, 31, 80, 80, 80*	12, 11 *80, 80*	11, 9, 7, 5, 4, 1 *80, 80, 80, 21, 21, 21*	-	-
			$80	7 ft	90°	$125,080	1 *30*	1, 8, 4, 6, 7, 10 *30, 29, 30, 74, 79, 80*	10, 12 *80, 29*	12, 5, 11, 9, 3, 2 *29, 80, 29, 66, 60, 30*	-	-

Table 11: Summary of results from 250 shoppers and 50% profit level

S	**Profit**	***C*** **$/ft²**	***R*** **$**	***h***	***θ***	**JRC-SSA Objective**	**Assignment**				**Result for 7ft 90°**	
							A/E	**B/F**	**C/G**	**D/F**	**Objective**	**% Diff**
250	50%	$20	$4	4 ft	90°	$117,626.4	1,2 *20, 21*	2, 8, 9, 5, 10, 12 *21, 24, 34, 74, 78, 77*	12 *77*	11, 7, 6, 4, 3 *80, 80, 38, 21, 21*	$116,966.2	0.6%
			$10	7 ft	90°	$110,723.6	4, 1 *20, 20*	1, 8, 3, 5, 11, 12 *20, 20, 35, 80, 80, 80*	12, 10 *80, 80*	10, 9, 7, 6, 2 *80, 80, 75, 26, 20*	-	-
			$80	7 ft	90°	$64,633.47	1 *48*	1, 2, 3, 9, 7, 12 *48, 57, 80, 29, 80, 28*	12, 11 *28, 28*	11, 5, 10, 6, 4, 8 *28, 70, 28, 80, 55, 33*	-	-
		$50	$4	4 ft	90°	$95,724.9	2, 6 *20, 20*	6, 8, 4, 7, 9, 11 *20, 21, 37, 67, 80, 80*	11, 10 *80, 77*	10, 12, 5, 3, 1 *77, 37, 64, 38, 27*	$94,952.6	0.8%
			$10	7 ft	90°	$88,510.3	4, 1 *20, 20*	1, 3, 8, 6, 9, 11 *20, 21, 23, 73, 80, 80*	11, 12 *80, 80*	12, 10, 5, 7, 2 *80, 80, 80, 38, 21*	-	-
			$80	7 ft	90°	$43,859.5	1 *55*	1, 8, 3, 9, 5, 10 *55, 39, 80, 28, 80, 27*	10, 12 *27, 27*	11, 7, 6, 4, 2 *27, 79, 80, 53, 41*	-	-
		$100	$4	7 ft	90°	$59,597.3	1, 2 *20, 20*	2, 8, 3, 7, 10, 11 *20, 20, 42, 80, 80, 80*	11 *80*	11, 12, 9, 5, 6, 4 *80, 75, 80, 75, 24, 20*	-	-
			$10	7 ft	90°	$52,426.6	9, 1 *20, 20*	1, 8, 4, 5, 7, 11 *20, 20, 31, 77, 80, 80*	11, 12 *80, 80*	12, 10, 3, 6, 2 *80, 80, 80, 28, 20*	-	-
			$80	7 ft	90°	$8,310.3	2 *45*	2, 4, 3, 6, 10, 12 *45, 44, 80, 80, 51, 24*	12, 5 *24, 25*	5, 11, 7, 9, 1, 8 *25, 24, 80, 68, 64, 31*	-	-

Table 12: Summary of results from 1000 shoppers and 100% profit level

S	**Profit**	***C*** **$/ft²**	***R*** **$**	***h***	***θ***	**JRC-SSA Objective**	**Assignment**				**Result for 7ft 90°**	
							A/E	**B/F**	**C/G**	**D/F**	**Objective**	**% Diff**
1000	100%	$20	$4	4 ft	30°	$1,144,765	7 *32*	7, 3, 4, 11, 10 *32, 80, 80, 80, 80*	10 *80*	10, 5, 12, 8, 6, 1, 2, 9 *80, 53, 33, 36, 26, 25, 25, 25*	$1,051,370.3	8.2%
			$10	4 ft	30°	$1,105,375	1 *25*	1, 5, 8, 12, 11 *25, 80, 80, 80, 80*	11 *80*	11, 6, 10, 7, 3, 4, 2, 9 *80, 53, 33, 36, 26, 25, 25, 25*	$1,020,944.5	7.6%
			$80	7 ft	30°	$780,033.3	12 *80*	12, 6, 8, 7, 9 *80, 80, 23, 68, 78*	9, 4 *78, 80*	4, 11, 1, 2, 3, 5, 10 *80, 51, 31, 23, 43, 36, 23*	$779,189.5	0.1%
		$50	$4	4 ft	30°	$1,086,466	3 *25*	3, 9, 10, 11, 12 *25, 67, 70, 77, 80*	12, 5 *80, 80*	5, 6, 7, 4, 2, 8, 1 *80, 44, 29, 24, 24, 24, 24*	$1,033,494.2	4.9%
			$10	4 ft	30°	$1,042,947	8 *26*	8, 4, 11, 12, 10 *26, 80, 57, 80, 80*	10, 7 *80, 65*	7, 3, 9, 2, 6, 1, 5 *65, 44, 27, 27, 27, 28, 27*	$996,799	4.4%
			$80	7 ft	90°	$760,499.6	1 *32*	1, 8, 3, 5, 9, 11 *32, 31, 53, 80, 79, 44*	11, 12 *44, 31*	12, 10, 7, 6, 4, 2 *31, 41, 80, 80, 32, 33*	-	-
		$100	$4	4 ft	90°	$1,004,133	1, 3 *20, 21*	3, 4, 7, 9, 12 *21, 22, 37, 71, 80*	12 *80*	12, 10, 11, 6, 8, 2 *80, 80, 20, 80, 36, 21*	$984,377.5	2.0%
			$10	7 ft	90°	$952,346.1	2, 3 *21, 21*	3, 9, 5, 4, 11, 12 *21, 21, 41, 80, 80, 80*	12 *80*	12, 10, 7, 6, 8, 1 *80, 80, 80, 69, 22, 21*	-	-
			$80	7 ft	90°	$714,437.5	4 *31*	4, 1, 6, 7, 9, 12 *31, 36, 80, 80, 73, 31*	12, 10 *31, 31*	10, 5, 11, 3, 8, 2 *31, 80, 31, 80, 31, 32*	-	-

Table 13: Summary of results from 1000 shoppers and 50% profit level

S	**Profit**	***C*** **$/ft²**	***R*** **$**	***h***	***θ***	**JRC-SSA Objective**	**Assignment**				**Result for 7ft 90°**	
							A/E	**B/F**	**C/G**	**D/F**	**Objective**	**% Diff**
1000	50%	$20	$4	4 ft	30°	$541,476.2	2 *32*	2, 7, 9, 12, 10 *32, 80, 80, 80, 80*	10 *80*	10, 5, 6, 11, 8, 3, 4, 1 *80, 68, 33, 23, 23, 23, 23, 23*	$503,712.4	6.4%
			$10	4 ft	30°	$506,355	1 *32*	1, 5, 6, 10, 12 *32, 80, 76, 80, 54*	12, 7 *54, 74*	7, 11, 3, 8, 4, 9, 2 *74, 28, 29, 28, 29, 29, 29*	$481,931.6	4.8%
			$80	7 ft	90°	$303,767	8 *29*	8, 2, 3, 6, 5, 12 *29, 46, 74, 73, 80, 29*	12, 10 *29, 29*	10, 11, 9, 7, 1, 4 *29, 30, 30, 80, 80, 36*	-	-
		$50	$4	4 ft	90°	$492,941.2	1, 8 *20, 20*	8, 4, 6, 9, 5, 11, 12 *20, 20, 32, 48, 74, 66, 79*	12 *79*	12, 10, 7, 3, 2 *79, 71, 80, 37, 21*	$484,357.7	1.7%
			$10	7 ft	90°	$459,162.1	1, 2 *20, 20*	2, 8, 5, 7, 11 *20, 26, 68, 80, 68*	11, 12 *68, 79*	12, 10, 9, 3, 6, 4 *79, 80, 80, 52, 23, 20*	-	-
			$80	7 ft	90°	$284,621.1	8 *39*	8, 1, 3, 9, 10, 12 *39, 80, 80, 80, 21, 21*	12, 5 *21, 21*	5, 11, 7, 6, 4, 2 *21, 21, 80, 80, 49, 44*	-	-
		$100	$4	7 ft	90°	$448,238.9	1, 2 *21, 21*	2, 3, 5, 9, 10, 12 *21, 25, 61, 80, 80, 80*	12 *80*	12, 11, 7, 8, 6, 4 *80, 80, 80, 46, 21, 21*	-	-
			$10	7 ft	90°	$411,120.2	1, 8 *22, 22*	8, 5, 4, 11, 9, 12 *22, 42, 80, 22, 80, 80*	12 *80*	12, 10, 7, 3, 6, 2 *80, 80, 80, 61, 25, 22*	-	-
			$80	7 ft	90°	$245,285.1	2 *40*	2, 8, 4, 6, 10, 5, 12 *40, 27, 48, 79, 27, 80, 27*	12, 11 *27, 27*	11, 7, 9, 3, 1 *27, 80, 43, 80, 58*	-	-

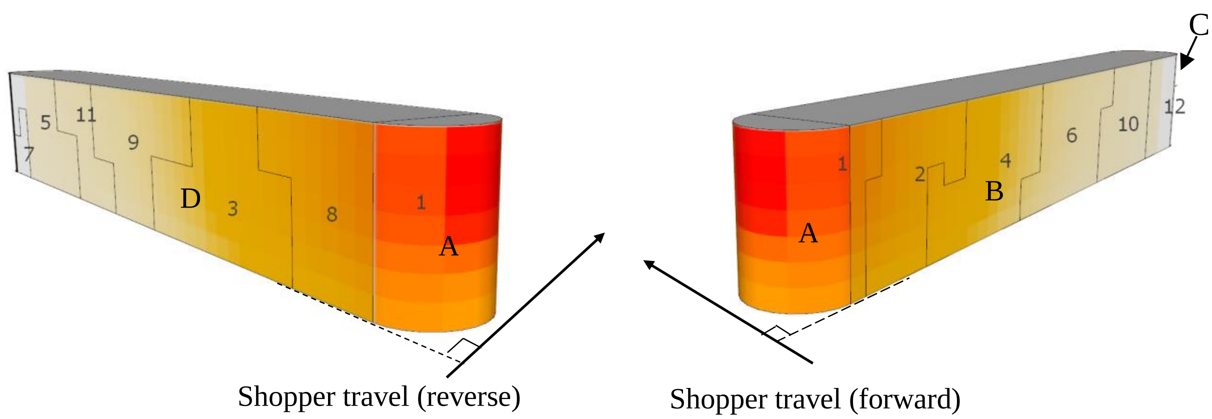
Observation 1: The orientation of rack impacts the location of high impulse potential products. We observed that, in all 36 instances, rack orientation affected the location of the high impulse potential products. Product category with the highest $I_P P_P$ values were located on highly visible faces; i.e., face A and B when θ = 90°, and face A and D when θ = 30°.

To understand this further, consider Table 11 that summarizes the product categories with their $I_P P_P$ in a non-ascending order.

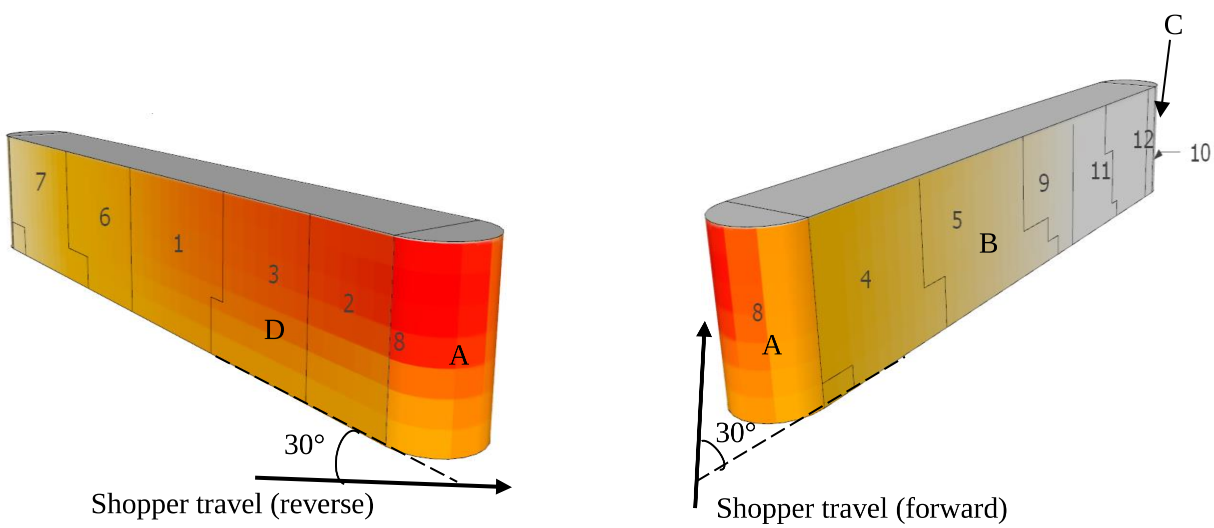
Table 11: Product categories and their $I_p P_p$ values

Order	Product category	$I_P P_P$
1	Baking/chocolate	0.7567
2	Kraft spreads	0.5740
3	Chili	0.3131
4	Pasta sauce	0.2748
5	Biscuits/rolls	0.2673
6	Jell-O	0.2621
7	Canned fruit	0.2549
8	Cat food	0.2442
9	Japanese food	0.1995
10	Macaroni	0.1008
11	Sugar	0.0770
12	Beans	0.0682

Figure 5 shows the example allocation of product categories on a rack placed at two different orientations, θ = 90˚ and θ = 30˚, both at height 7 ft. The visibility index of rack locations is represented by darker and lighter shades; darker region being the most visible and lighter being the least. We can observe that when rack orientation is 90°, face A and part of faces B and D (closer to face A) tend to be the most visible areas on a rack. Clearly, the assignment of product categories #1-#4 (high $I_P P_P$ values) on highly visible faces will produce high impulse profit. However, these assignments change when θ=30˚; notice that product category #1. Further, product categories #2 and #3 are now on face D (which were previously on faces A and B). A number of other product categories also moved across the faces. This is because when θ=30˚, face D is a lot more visible compared to when θ=90˚. Similarly, face C has increased visibility, while faces A and B have decreased visibility, which resulted in product category #10 now placed on face C (as $I_P P_P$ for #10 > $I_P P_P$ for #12).



(a) Product assignment on a 7 ft rack oriented at 90°



(b) Product assignment on a 7 ft rack oriented at 30°

Figure 5: Product assignment on a rack at different orientation (arrows indicate the direction of shopper travel)

Observation 2: High expected impulse profit products are often assigned number of locations closer to their lower bound.

We observed that high expected impulse profit products (product categories #1, #2 and #3) are often assigned number of locations closer to their lower bound. Recall that, impulse profit generated is highly dependent on visibility of the products across shopper's path. But the number of such highly visible location on a rack face are limited. Consequently, products with high I_pP_p values compete across such limited space, whereby each gets facings closer to their lower bound to allow for the other products to access space in order to maximize the objective value. This can be noticed in Table 10 - 13 where product categories #1-#3 frequently have facings in the range of 20-40 (in the second row of assignment column); recall, we use 20 as the lower bound on the number of facings per product category.

So, why not assign maximum number of locations to high I_pP_p values? To better understand this, recall constraint (1) in the proposed optimization model, where v_p is probability of visibility for product p during the shopping path ($0 \leq v_p \leq 1$). Clearly, assigning visible locations to a product increases v_p, which in turn increases expected impulse profit generated by that product category. However, increasing the number of highly visible locations to product category p has diminishing returns in terms of increases in v_p (see Figure 6). That is, while assigning more visible locations will increase v_p, the rate of such an increase in v_p is much lower. However, if this product category is still assigned higher number of highly visible locations, then this would decrease the available number of highly visible locations for other product categories with reasonably high $I_P P_P$ values. This will result in lower v_p for those products and a reduced overall objective function value. Our proposed PSO is able to effectively trade-off the number of locations across product categories with high $I_P P_P$ in an attempt to maximize the objective function value.

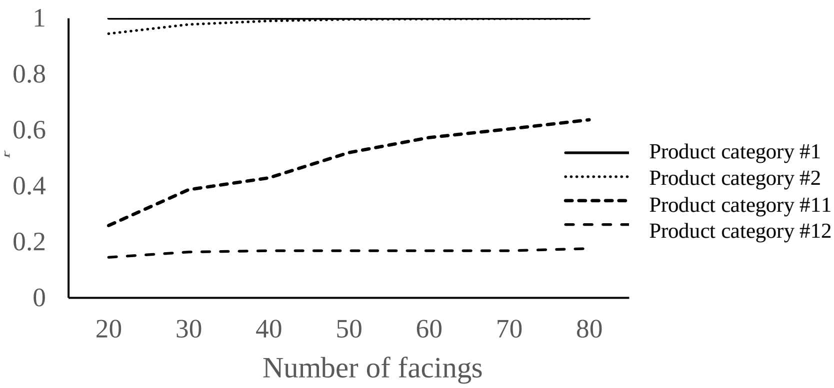


Figure 6: Products v_p values at different number of facings allocation

To verify the above proposition empirically, we compared our solution to a greedy approach where we set the locations for three high $I_P P_P$ to their upper bound and the three lowers $I_P P_P$ products to their lower bound. The resulting solution was 14% lower than the objective value obtained via the PSO (see Table 12).

In contrast, product categories with low $I_P P_P$ values were assigned locations closer to their upper bound in order to increase their v_P and, in turn, increase the objective function value; e.g., product categories #11 and #12 were each assigned 80 locations (see Figure 7).

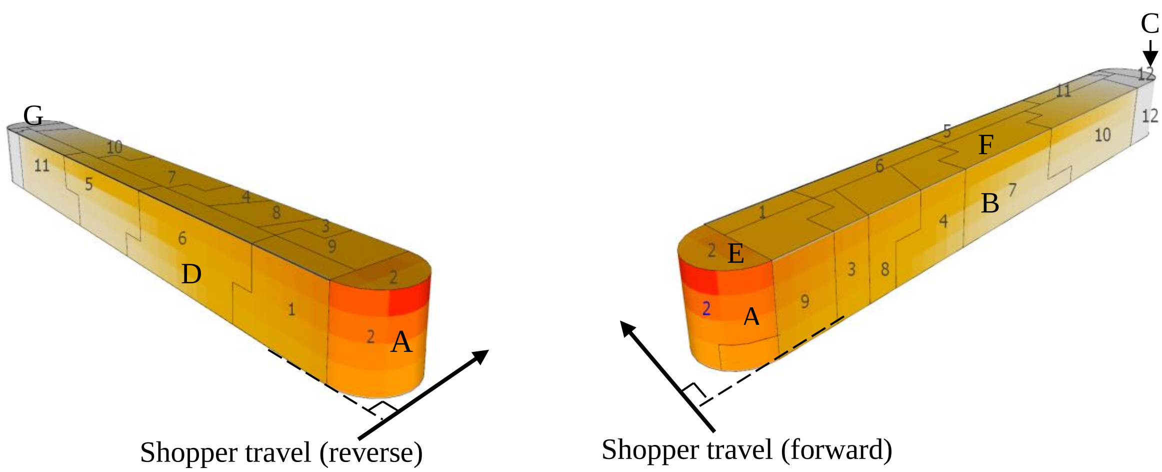


Figure 7: Number of facings assigned to high and low impulse potential products

Table 12: Comparison between number of locations assigned to different product categories

	Rack layout		*Objective*	*Assignment*			
	h	θ		*A/E*	*B/F*	*C/G*	*D/F*
PSO	4 ft	30°	$245,152	4 *31*	8, 12, 10, 11, 7, *61, 80, 80, 60, 80*	6 *41*	5, 3, 2, 9, 1 *35, 25, 25, 25, 25*
Greedy	4 ft	30°	$211,529 (-14%)	4 *45*	4, 8, 12, 10, 11, 7, 6, 5, 3 *45, 45, 20, 20, 20, 45, 44, 44, 80*	3 *80*	3, 2, 9, 1 *80, 80, 45, 80*

Observation 3: Rack orientation is sensitive to area cost; acute angles favored for lower area cost.

Table 13 summarizes the rack layout at varying space and restocking cost for 1,000 shoppers and 100% product profit.

Table 13: Rack layout at varying area and restocking cost

		Restocking cost		
Area cost		*\$4/restock*	*\$10/restock*	*\$80/restock*
	\$20/ft²	4 ft, 30°	4 ft, 30°	7 ft, 30°
	\$50/ft²	4 ft, 30°	4 ft, 30°	7 ft, 90°
	\$100/ft²	4 ft, 90°	7 ft, 90°	7 ft, 90°

For a fixed restocking cost, we observed that as the space cost increases, θ switches from 30˚ to 90°; see Table 13. To understand this, consider Figure 8 that illustrates floor space at different rack orientations, θ. When θ changes from acute (30°) to orthogonal (90°), the required floor space decreases with the minimum occurring at θ=90°. Similarly, changing rack orientation from 90° towards obtuse (150°) again increases the floor space. However, the opposite effect is realized with respect to visibility, where it increases as θ moves from 90° to 30°. Clearly, there exists a trade-off between total space cost and total visibility; see Figure 8.

Also notice that the profile of the objective function with changes in θ and prespecified area (\$20/ft^2 and \$50/ft^2) and restocking (\$4/restock) costs is shown in Figure 9 rack height *h* and product decisions (placement and number of locations) were still decision variables. Observe the

W-shaped tri-modal nature of the objective function, with two primary peaks at θ=30˚ and 150˚ and a secondary peak at θ=90˚. We also observed that as the area cost changes, the θ=90˚ becomes the primary peak, and thus the optimal rack orientation (figures not shown).

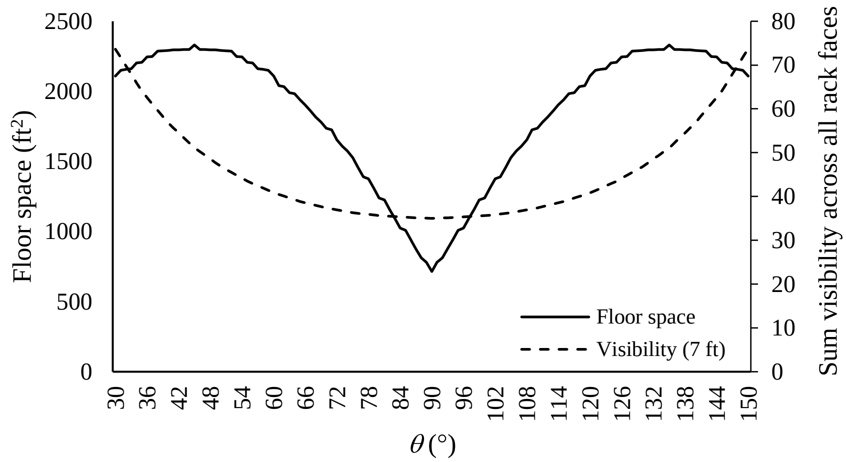


Figure 8: Area and total number of visible locations for varying θ

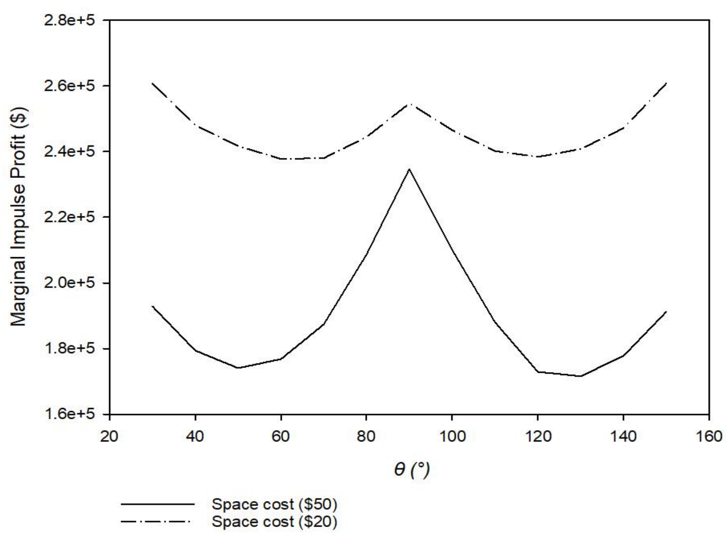


Figure 9: Objective function at different rack orientation for 250 shoppers, 100% profit level and $4/restock

Observation 4: Rack height is sensitive to restocking cost; shorter racks favored for lower restocking costs.

A similar trend was observed with changing restocking costs on the optimal rack height (see Table 13). At low restocking costs, rack height of 4 ft was observed to be the optimal height. To understand this, recall that as the rack height decreases, the number of available locations for the product categories on the rack reduces. This means that for the same expected demand of a product category, the number of restocks increases, which increases the restocking cost. However, in case of racks lower than eye-height, the top faces (E, F, and G) are now exposed. Further, accordingly to Guthrie and Parikh (2019), in a layout with shorter racks, the occlusion created by racks prior to a given rack is much less (resulting in higher visibility of locations) compared to that created by racks above eye-height (see Figure 10). Both these effects, availability of top faces and lower occlusion, increase the number of visible locations on the rack and, in turn, the potential for higher impulse profit. So in the case when restocking cost is low, the increase in the number of restocks is offset by the increase in the available number of visible locations (with some locations having higher visibility due to the reduced occlusion effect mentioned earlier).

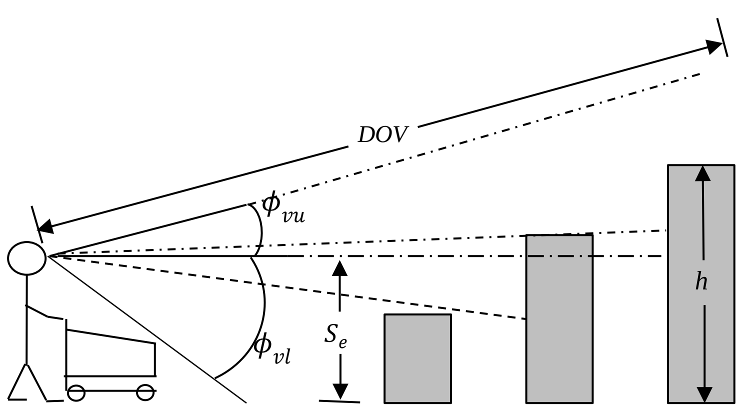
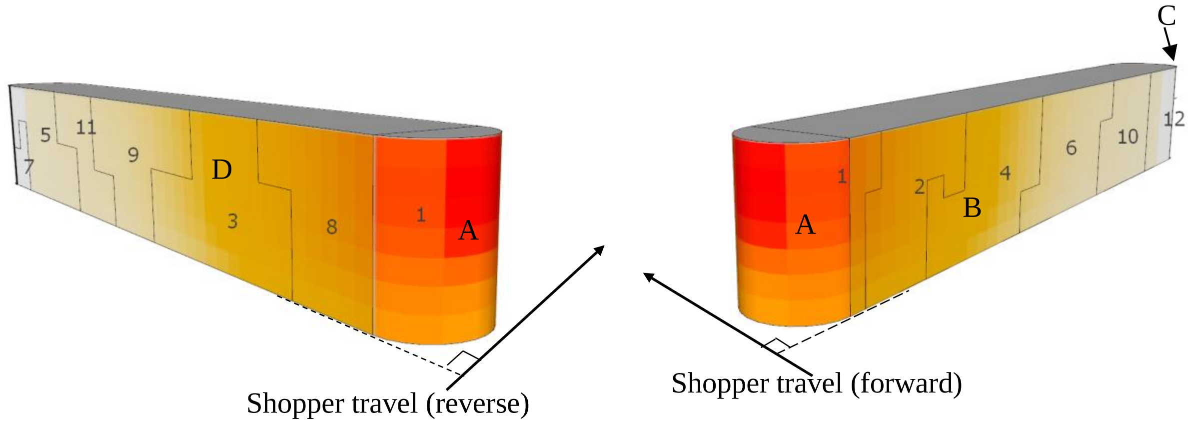
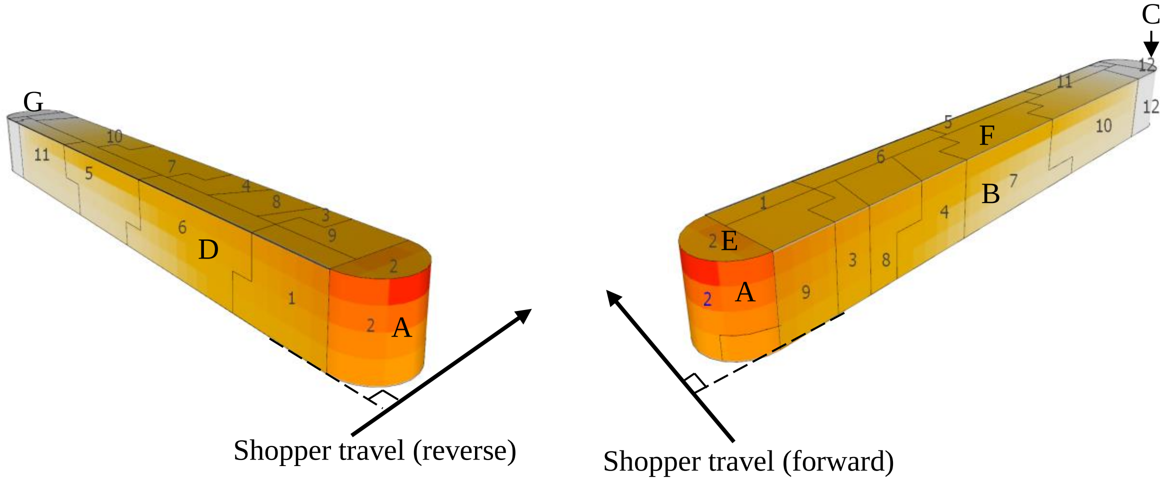


Figure 10: Shopper's field of vision at different rack height



(a) 7 ft rack height at 90° orientation



(b) 4 ft rack height at 90° orientation

Figure 11: Rack orientation at changing area and restocking cost

We also compared the amount of loss in the benefits if a 7 ft high rack was used instead of the optimal 4 ft rack on a specific configuration. Table 14 summarizes the results of the PSO-generated solution and that of a 7 ft high rack (in which the height was fixed, and all other decisions were derived). A loss of over 5% was observed when not using the optimal height.

However, as the restocking cost increased, additional visibility gained through a 4 ft rack could not offset the increase in the restocking cost, leading to a 7 ft high rack as being optimal.

Table 14: Comparison between 7 ft and 4 ft rack height at \$20/ft² area cost and \$4/restock

	Rack layout		Objective	Assignment			
	h	θ		A/E	B/F	C/G	D/F
Result from PSO	4 ft	30°	\$255,559.5	9 *25*	6, 12, 10, 11 *72, 65, 79, 79*	11, 7 *79, 80*	7, 3, 8, 2, 1, 4, 5 *80, 46, 24, 25, 25, 24, 24*
Height fixed to 7 ft	7 ft	30°	\$242,027.2 (-5.29%)	8, 3 *21, 80*	3, 10, 12, 11 *80, 80, 80, 80*	11 *80*	11, 7, 6, 4, 1, 5, 2, 9 *80, 80, 60, 25, 30, 36, 22, 22*

6. CONCLUSIONS AND FUTURE RESEARCH

Deciding the rack configuration and allocating products on a rack are two key decisions frequently encountered by retailers. These decisions have a direct effect on what shopper see (and experience) in the store and, in turn, impulse profit. Realizing that the two streams of literature, rack layout and shelf space allocation, have evolved separately and that there is a gap in our understanding of the interaction between these two decisions, we proposed the Joint Rack Configuration and Shelf Space Allocation (JRC-SSA) problem. The objective of JRC-SSA is to determine the optimal retail rack layout (height and orientation) and product decisions (placement and number of locations) in order to maximize the potential marginal impulse profit after accounting for space and restocking costs. To this extent, we proposed an optimization model and adopted the particle swarm optimization framework to solve JRC-SSA efficiently.

Our experiments suggested up to 8.2% increase in the marginal impulse profit increase with the JRC-SSA compared to only solving SSA problem assuming a 7 ft, 90° oriented rack. Further, the placement of products on the rack altered considerably with changes in the rack orientation. For instance, at 90° orientation, high impulse potential products were placed on Faces A and B on the rack, whereas the same products were now placed on Faces A and D when the rack was orientated at 30°. We also observed that while rack orientation gravitated to acute angles for low area costs, rack height gravitated towards shorter heights for low restocking cost.

This research can be extended in many ways. First, to keep the problem complexity manageable and effectively derive insights, we assumed a single representative rack in our study. It would be worthwhile to extend our model to a layout with multiple racks, with each identical to the other or each allowed to have its own optimal orientation and height. Doing this, however, will increase the problem complexity and the proposed PSO must be enhanced or another algorithm may need to be designed. Second, while we considered a shopper passing by our representative rack on her way to a planned purchased elsewhere, incorporating the shopper’s travel into the aisle of this rack and accounting for the exposure of products in the overall visibility probability would be worthwhile.

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