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One-Dimensional Kinetic Particle-In-Cell Simulations of Various Plasma Distributions

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ONE-DIMENSIONAL KINETIC PARTICLE-IN-CELL SIMULATIONS OF VARIOUS PLASMA DISTRIBUTIONS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science

By

RICHARD N. VANDERBURGH

B.S., Wright State University, 2019

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GRADUATE SCHOOL

December 2nd, 2020

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Richard N. Vanderburgh ENTITLED One-Dimensional Kinetic Particle-In-Cell Simulations of Various Plasma Distributions BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science.

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ABSTRACT

Vanderburgh, Richard N. M.S. Department of Physics, Wright State University, 2020. One-Dimensional Kinetic Particle-In-Cell Simulations of Various Plasma Distributions.

A one-dimensional kinetic particle-in-cell (PIC) MATLAB simulation was created to demonstrate the time-evolution of various plasma distributions. Building on previous plasma PIC programs written in FORTRAN and Python, this work recreates the computational and diagnostic tools of these packages in a more user- and educational-friendly development environment.

Plasma quantities such as plasma frequency and species charge-mass ratios are arbitrarily defined. A one-dimensional spatial environment is defined by total length and number and size of spatial grid points. In the first time-step, charged particles are given initial positions and velocities on a spatial grid. After initialization, the program solves for the electrostatic Poisson equation at each time step to compute the force acting on each particle. Using the calculated force on each particle and the “leap-frog” method, the particle positions and velocities are updated and the motion is tracked in phase-space. Modifying parameters such as spatial perturbation, number of particles, and charge-mass ratio of each species, the time-evolution for various distributions are examined.
The simulated distributions examined are categorized as the following: Cold Electron Stream, Electron Plasma Waves, Two-Stream Electron Instability, Landau Damping, and Beam-Plasma. The time evolution of the plasma distributions was studied by several methods. Tracking the electric field, charge density and particle velocities through each time step yields insight into the oscillations and wave propagation associated with each distribution. One key diagnostic missing from the original FORTRAN code was the electric field dispersion relation. The numerical dispersion relation allows for further insight into modelling plasma oscillations/waves in addition to the kinetic/field energies and electric field tracking present in the original code. Simulated results show agreement with other kinetic simulations as well as plasma theory.
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Chapter 1: Introduction

Plasma is often considered the fourth state of matter. As opposed to classical electrically neutral gas, plasma is composed of ionized gas molecules and electrons. Most of the observable universe consists of matter in the form of plasma: stars, intergalactic dust, lightning, and even some parts of the Earth’s atmosphere. In modern times, engineers and scientists have harnessed plasma for the development of technologies such as neon signs, fluorescent lights, semiconductor etching, and tokamaks (a device which uses magnetic confinement to produce controlled thermonuclear fusion power).

The ionosphere is the ionized region of Earth’s atmosphere, consisting of ions and other charged particles. Therefore, this region possesses plasma-like characteristics. Between altitudes of 60-1,000 km, most of the ionization in this layer is provided by the sun. When radio waves are sent through the atmosphere, the ionosphere interacts with the signal due to the electromagnetic interaction. One effect of this interaction is the absorption/reflection of radio signals due to ions. [1]

Figure 1.1 shows the various regions of the Earth’s ionosphere. Each region is characterized by the temperature and density of electrons and ionic species present. The interaction of space/terrestrial satellite communications (SATCOM) with the ionosphere depends on the layer of propagation, as each layer possesses unique electromagnetic properties.
Figure 1.1: Layers of the ionosphere differ by the altitude above the Earth. Depending on factors like the altitude and time of day, the various layers will contain different temperatures and densities of electrons and ionic species. During the day, the sun’s rays excite gaseous molecules, which then radiate. The formation of these ionic species produces complicated behavior for computational modeling. [14]

In a plasma simulation, SATCOM signals may be represented by a perturbation in a charge distribution. Through defining an initial setup of plasma parameters and initial distribution, the time evolution is approximated through computation of equations of motion and electromagnetic interaction.
For a fully physical representation of any arbitrary plasma distribution, any number of particles must be tracked through time according to position, velocity, and acceleration. Representing the plasma particles in phase space, particle positions and velocities are tracked in space and time. A 1-dimensional representation is often sufficient to demonstrate real physics. Figure 1.2 shows a one-dimensional plasma consisting of sheet-charges, which are non-uniform exclusively along the x-direction.

![Diagram](image)

Figure 0.2: Representation of a 1D electrostatic plasma model. Self and applied fields are along the x-axis with no variation in either y or z. [1]

Although this work does not model plasma phenomenon unique to the ionosphere, the simple examples demonstrated are necessary steps to building a fully physical code.
In addition to theoretical and experimental research, computational simulations have yielded powerful results in understanding plasma physics. Within the realm of plasma simulation, the methods used for various physical situations can be broken into two groups—kinetic and fluid. Kinetic models are suitable for calculating the motion of discrete particles interacting with electric and magnetic fields. The drawback to kinetic models has historically been the heavy computational requirements, as opposed to a fluid simulation. The benefit of a kinetic simulation is the ability to resolve microscopic particle effects, which are not represented in a fluid model. In the case of a single charged particle, (either an electron, ion, or other charged particle), the motion is fully described by the Lorentz force \([3]\),

\[
\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}
\]  

(1.1)

Where \(q\) is the charge of particle, \(\vec{v}\) is the particle velocity, and \(\vec{E}\) and \(\vec{B}\) are the electric and magnetic fields at the location of the particle. In the case where multiple particles are present, the motion is still affected by the Lorentz force, however \(\vec{E}\) and \(\vec{B}\) depend on charge and current densities \(\rho\) and \(\vec{J}\). After defining an initial distribution of charges, the time-evolution of a plasma can be approximated through computational methods. For this work, a kinetic particle-in-cell approach was used to model the time evolution of various plasmas.
Chapter 2: Particle-in-Cell (PIC)

Calculation of the motions and fields of the plasma particles requires a spatial grid to track the positions, as well as the charge and current densities of the distribution. Figure 2.1 shows an arbitrary 2-dimensional plasma distribution with a spatial grid defined by Cartesian x and y coordinates. [2]

![Figure 2.1: PIC Mathematical spatial grid. Particles reside between grid points, but fields and densities are calculated exclusively at the grid points. [2]](image)

Particles with charge and mass can travel between the gridlines, however, electric and magnetic fields are calculated along the gridlines exclusively.
2.1: Computational Cycle

The “leap-frog” method allows for a computationally efficient method of integration. Figure 2.1.1 shows the sequence in the cycle of computation. First, the simulation parameters (particle positions, velocities, etc.) are defined. Next, a weighting method is used to calculate charge and current densities $\rho$ and $J$ along the gridlines. Integration of these densities yields the electric and magnetic fields $E$ and $B$. These fields are then weighted along the gridlines to find the force $F$ for each particle.

![Diagram of computational cycle](image)

Figure: 2.1.1: Computational cycle for the PIC simulation. The particles are numbered $i = 1, 2, … \text{NP}$; with grid indices $j$, which are scalers in 1D [2]
2.2: Integration of Equations of Motion

The first-order differential equations of motion for each particle are shown in Equation’s 2.2.1 and 2.2.2. The force $\vec{F}$ becomes a scaler in 1-dimension and only depends on the charge $q$ of each particle and electric field $E$. [2]

$$m\frac{d\vec{v}}{dt} = \vec{F} = qE$$  \hspace{1cm} (2.2.1)

$$\frac{d\vec{x}}{dt} = \vec{v}$$  \hspace{1cm} (2.2.2)

These equations are replaced by the finite-difference equations,

$$m\frac{\vec{v}_{new}-\vec{v}_{old}}{\Delta t} = \vec{F}_{old}$$  \hspace{1cm} (2.2.3)

and

$$\frac{\vec{x}_{new}-\vec{x}_{old}}{\Delta t} = \vec{v}_{new}$$  \hspace{1cm} (2.2.4)

Figure 2.2.1 shows the “leap-frog” integration method and time-centering.

The computer advances $\vec{v}_t$ and $\vec{x}_t$ to $\vec{v}_{t+\Delta t}$ and $\vec{x}_{t+\Delta t}$ even though the initial positions and velocities are not known at the same time. The initial conditions for particle velocities and positions given at $t = 0$ must be changed to fit in the flow of time. First, $\vec{v}_0$ is pushed back to $\vec{v}_{-\Delta t/2}$ using the force $\vec{F}$ calculated at $t=0$.

Second, the energies calculated from $\vec{v}_t$ (kinetic) and $\vec{x}_t$ (electric field potential) must be adjusted to appear at the same time. The leap-frog method has error, with the error vanishing as $\Delta t \to 0$. This work uses the “leap-frog” method in all the
examples, because it is both simple (easy to understand, and with minimum storage) and surprisingly accurate [2].

Figure 2.2.1: Leap-Frog Integration Scheme. There are more accurate force integration algorithms (higher-order Runge-Kutta methods, for example) which can be implemented, at the cost of higher computation demands, however. [2]
2.3: Integration of the Field Equations

For the electrostatic case, the differential equations to solve are

\[ \mathbf{E} = -\nabla \phi \text{ or } E_x = -\frac{\partial \phi}{\partial x} \text{ and } \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \text{ or } \frac{\partial E_x}{\partial x} = \frac{\rho(x)}{\varepsilon_0} \]

This set of differential Maxwell’s equations are solved for the Electrostatic Case [2]. When combined, these equations provide Poisson’s equation to provide the formula for electrostatic potential. [2]

\[ \nabla^2 \phi = -\frac{\rho(x)}{\varepsilon_0} \text{ or } \frac{\partial^2 \phi_x}{\partial x^2} = -\frac{\rho(x)}{\varepsilon_0} \]  

(2.3.1)

There are numerous approaches to solving Poisson’s equation, one being a solution using discrete Fourier transforms. For this approach to be feasible, we enforce periodic boundary conditions. If a particle moves left of 0, then it is placed on the opposite side of the numerical grid, at L. If the particle moves right of L, the reverse happens, and is placed back at 0. The Fourier transform of charge density provides the following identity between potential and charge density in k-space.

\[ \phi(k) = \frac{\rho(k)}{\varepsilon_0 k^2} \]  

(2.3.2)

The formula above is used to obtain potential from charge density using Fourier Transform, where the \( \frac{\partial^2}{\partial x^2} \) operator has been replaced by \(-k^2\). Next, an inverse Fourier transform is performed to solve for \( \phi(x) \). The last step is to find the grid’s electric field from the negative gradient of the electric potential. The
method used for solving Poisson’s equation utilizes the fast Fourier transform (FFT) and its inverse (IFFT). [2]

Figure 2.3.1: Progression of charge density $\rho(x)$ to $\rho(k)$ to $\phi(k)$ finally to electrical potential $\phi(x)$. [2]

Figure 2.3.2: Example Plots taken from a Cold Plasma Using MATLAB. The plots show the progression from charge density $\rho(x)$ to $\rho(k)$ to $\phi(k)$ finally to electrical potential in real space, $\phi(x)$. [2]
2.4: Charge Density and Electric Field Weighting

Calculation of the force acting on each plasma particle requires defining the charge density grid point. Depending on the desired level of accuracy, different weighting schemes are used. The most simple weighting schemes are known as nearest-grid-point (NGP) and cloud-in-cell (CIC). Figure 2.4.1 shows the particle shape “as seen by the spatial grid”.

![Figure 2.4.1: Effective particle shape as seen by the spatial grid [8]](image)

Although the NGP scheme is computationally simple, the calculation yields a square particle shape as “seen by the grid” as a square. This all-or-nothing
method deposits charge density in the cell closest to the particle. Using the first-order CIC weighting yields a triangular particle shape as “seen by the grid”. This method deposits charge density at not just a single cell, but the next closest cell as well.

\[
\text{NGP: } \rho_i = \frac{q}{\Delta x} \quad (2.4.1)
\]

\[
\text{CIC: } \left\{ \rho_i = \frac{q}{\Delta x} \left[ \frac{x_{i+1} - x}{\Delta x} \right] = \frac{q}{\Delta x} \left[ \frac{x_i + \Delta x - x}{\Delta x} \right] \right\} \text{ and } \left\{ \rho_{i+1} = \frac{q}{\Delta x} \right\} (2.4.2)
\]

Equations 2.4.1 and 2.4.2 show the Nearest-Grid-Point and Cloud-In-Cell Weighting Schemes [2]

It should be noted that all plasma simulation examples in this work are implemented using CIC. Similar to the charge density weighting, the electric field acting on each particle is calculated using a linear weighting on the \( j \)th cell from each for each \( i \)th particle.

\[
E_i = [1 - \left( \frac{x}{\Delta x} - j \right)] E_j + \left[ \frac{x}{\Delta x} - j \right] E_{j+1} \quad (2.4.3)
\]

Equation 2.4.3 shows the electric field weighting used for all examples shown [2]
Chapter 3: PIC Simulation Choice of Parameters and Diagnostics

3.1: Plasma Parameters

To guarantee the PIC simulation obeys real physics, fundamental plasma parameters must be considered. One of the most fundamental plasma parameters is known as the plasma frequency, \( \omega_p \). [3]

\[
\omega_p = \sqrt{\frac{nq^2}{\varepsilon_0 m}} \tag{3.1.1}
\]

Equation 3.1.1 shows the formula for a plasma frequency of electrons against neutralizing background species. In all the examples in this work, the plasma frequency is defined to calculate the mass \( m \) and charge \( q \) from the arbitrarily defined charge-to-mass ratio \( q/m \).

\[
q = \frac{L \ast \omega_p}{\varepsilon_0 i \ast N \ast q/m}
\]

\[
m = \frac{q}{q/m}
\]

Another fundamental plasma parameter is known as the Debye Length, \( \lambda_D \). This is defined as the distance traveled by a particle at the thermal velocity in \( 1/2\pi \) of a plasma cycle. It can be interpreted as the shielding distance around a test charge and the scale length inside which particle-particle effects occur most strongly and outside of which collective effects dominate. This constrains the
spatial grid size to be dependent on the arbitrary thermal velocity and plasma frequency.

\[ \lambda_D = \frac{v_{\text{thermal}}}{\omega_p} \rightarrow \Delta x \approx \lambda_d \]  

(3.1.2)

Equation 3.1.2 shows the formal definition of the Debye Length, leading to a constrained spatial grid size. This work focusses on the collective behavior of collisionless plasmas at wavelengths longer than the Debye length, \( \lambda \geq \lambda_D \). [2]

The useful information in a one-dimensional simulation is significantly less computationally intensive than a three-dimensional case. Instead of modeling the three-dimensional number of particles, \( N_D \approx 10^6 \), the one-dimensional \( N_D \approx 10^2 \) is used. A collisionless plasma is characterized by \( N_D \gg 1 \) and \( L \gg \lambda_D \). [2]

In order to obtain time-dependent oscillations in the plasma simulation, an initial spatial perturbation is defined:

\[ x_1 \cos(2\pi x_i \frac{\text{mode}}{L} + \theta_x) \]  

(3.1.3)

Where \( x_1 \) is the amplitude and \( x_i \) is the position of the \( i \)th particle. This creates bunching in some areas of the plasma distribution, and greater separation between charges in other areas. In areas where the bunching is tighter, the electric field will push the particles away from each other.
3.2: Waves in Plasma

To demonstrate the wave phenomenon of a plasma, a fluid formulation is used to derive the equations. Fluid models describe many of the fundamental results of plasma physics, including the phenomena demonstrated by the examples shown in this paper. In this work, a fluid approach is used to derive the dispersion of two examples: cold plasma oscillations and electron plasma waves.

The first equations needed for the derivations are the continuity and momentum equations. For every species \( s \) present in the plasma, there is a plasma fluid continuity equation, [3]

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v}_s) = 0
\]  
(3.2.1)

and the plasma fluid momentum equation,

\[
n_s = \left( \frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s \right) = \frac{q_s n_s}{m_s} (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - \frac{\gamma k_B T_s}{m_s} \nabla n_s
\]  
(3.2.2)

Where \( n_s \) and \( \mathbf{v}_s \) are the particles’ density and velocity of species \( s \), \( q \) and \( m \) are its individual particles’ charge and mass, and \( T \) is the temperature of the particle species. Since this work is only looks at the electrostatic case, solving for the wave phenomenon only requires Poisson’s equation combined with equations 3.2.1 and 3.2.2, whereas a fully electromagnetic model would incorporate Faraday’s and Ampere’s Laws as well.
The periodic motion of the plasma examples discussed in this work is expressed through Fourier analysis as a superposition of sinusoidal oscillations with frequencies \( \omega \) and wavelengths \( \lambda \). The simplest wave is a single component of this decomposition. When the oscillation amplitude is small, a waveform is mostly sinusoidal; and there is only one component.

A sinusoidal oscillation, for example, in the electric field—is represented by Equation 3.2.3. [3]

\[
E(r, t) = E_0 \exp[i(k \cdot r - \omega t)]
\]  
(3.2.3)

Where

\[
k \cdot r = k_x x + k_y y + k_z z
\]  
(3.2.4)

Looking at only the x-component of the wave propagation, \( E(x, t) \), a two-dimensional FFT of the electric field data in space in time is used to obtain electric field as a function of \( k \) and \( \omega \). The calculation of the \( k \) and \( \omega \) Fourier amplitudes is then compared to the theoretical dispersion to assess the accuracy of the physics shown by the PIC simulation.

Figure 3.2.1 shows an example of a plasma dispersion relation. The diagram depicts the dependency of angular frequency \( \omega \) with wave number \( k \) of an electron plasma wave. This system will be discussed more thoroughly in Chapter 4.2.

Depending on the classification of wave, different mathematical dependencies are derived for the dispersion relation. Equations 3.2.5 and 3.2.6 show the phase
velocity and group velocities $v_\phi$ and $v_g$ as functions of $\omega$ and $k$. If a wave possesses a dispersion relation with $v_g = 0$, the curve will be characterized as having a horizontal slope, as $v_g = \frac{d\omega}{dk} = 0$. In this case, there is no wave propagation. If the dispersion relation has a non-zero slope, then the wave propagates. As a check to make sure a wave follows real physics, $v_g$ must be less than the speed of light, and therefore the largest allowed $\omega$ is $ck$.

$$v_\phi = \frac{\omega}{k} \quad (3.2.5)$$

$$v_g = \frac{d\omega}{dk} \quad (3.2.6)$$

![Figure 3.2.1: Example of a Theoretical Plasma Dispersion Relation](image)
3.3 Plasma Energies

Calculation of various kinetic and potential energies yield further insight into the plasma characteristics. The total kinetic energy for each species is defined by the following equation at each time step.

\[ KE_{\text{species}} = \frac{1}{2} \sum_{i} m_i v_i^2 \]  

(3.3.1)

Where \( m_i \) and \( v_i \) are the mass and velocity of the \( i \)th “superparticle” for each species summed over the total number \( N \). The kinetic energies are further divided into drift and thermal energies. [2]

\[ KE_{\text{drift}} = \frac{1}{2} \sum_{i} m_i \langle v_i \rangle^2 \]  

(3.3.2)

and

\[ KE_{\text{thermal}} = \frac{1}{2} \sum_{i} m_i (\langle v_i^2 \rangle - \langle v_i \rangle^2) \]  

(3.3.3)

In addition to kinetic energy, the total electrostatic field energy is approximated by the following equation at each point in time.

\[ ESE \propto \frac{1}{2} \sum_{k} \rho_k \phi_k^* \]  

(3.3.4)

Where \( \rho_k \) and \( \phi_k^* \) are the discrete Fourier transforms of the charge density and potential, respectively. The electrostatic field modal energies are found the following equation.

\[ ESE_{\text{modal}} \propto \frac{1}{2} \rho_k \phi_k^* \]  

(3.3.5)
Chapter 4: Plasma PIC Simulation Examples

4.1 Cold Plasma Oscillations

A “cold” plasma is the simplest plasma distribution to model. This example consists of a distribution containing massive, and therefore immobile ions with spatially perturbed electrons. The ions are considered fixed in space for this example, and are therefore not represented in the simulation. First, an initial distribution of stationary electrons is defined. Standing waves are formed in the charge density when electrons are given a spatial perturbation of the form defined by Equation 3.1.3. In regions of higher charge density, electric fields accelerate the electrons to travel from their equilibrium positions.

After the electrons are displaced, consecutive electric fields grow in the opposite direction to restore the neutrality of the plasma by pulling the electrons back to their original positions. Due to their inertia, the electrons will overshoot and oscillate around their equilibrium positions at the plasma frequency, \( \omega_p \). This oscillation is fast enough that the massive ions do not have time to respond to the oscillating field and are approximately fixed. [3]

Figure 4.1.1 shows the cold plasma phase space time evolution. A sinusoidal spatial perturbation causes standing waves in charge density and therefore electric field. Spatial input variables are as follows: \( L=2\pi, nt=150, dt=0.2, ng=32 \). \( L \) is the
length of the physical space represented by the simulation, $nt$ is the number of
time steps the particle motions and fields are updated, $dt$ is the temporal length of
each time step, and $ng$ is the number of gridpoints. Species input variables for the
electrons are defined as: $N = 64$, $wp = 1$, $qm = -1$. $N$ is the number of particles,$wp$ is the species plasma frequency, and $qm$ is the charge-mass ratio. Each of these
parameters are usually set to be different for each species represented in the
simulation. All other possible simulation parameters are set to zero for the cold
plasma example. Figures 4.1.1 to 4.1.6 were all generated with a spatial
perturbation using Equation 3.1.3 with $mode=1$ and $x1=0.001$. 
Figure 4.1.1: The cold plasma phase space shows the initial uniformity of the distribution function, which then oscillates with a sinusoidal standing wave. Each electron possesses simple harmonic motion around its respective initial position.

Figure 4.1.2: The Cold Plasma initially possesses a sinusoidal charge density due to the spatial perturbation, then oscillates harmonically for the duration of the simulation.

Figures 4.1.2 and 4.1.3 show the charge densities and electric fields (respectively) of a cold plasma through time. The non-uniform charge distribution seeks net neutrality, but once the electrons are evenly spread, their new non-zero velocities push the particles to oscillate at the plasma frequency.
Figure 4.1.3: The electric field oscillation behaves similar to the charge density oscillation, as the electrostatic model means that the electric field is merely the negative of the charge density’s derivative in space.

Figure 4.1.4 shows the time evolution of the kinetic, electric potential, and total energies. Each energy is found using Equations 3.3.1, 3.3.2, and the sum of the two equations, respectively. At $t = 0$, the energy is stored entirely in the form of electric potential, since all particles possess velocities of zero. After the electrons are displaced from equilibrium, kinetic energy grows and exchanges with electric potential at a period dependent on $\omega_p$. 
Figure 4.1.4: Cold Plasma Energies show a periodic exchange between the kinetic and potential energies, tracking the total energy demonstrates the non-conserving property of this simulation.

Figure 4.1.5 is taken from another paper using the kinetic PIC method to model a cold plasma. [6] Comparing with Figure 4.1.4, the two plots possess the same shape, although the magnitudes differ due to different initial plasma parameters.
Figure 4.1.5: This plot shows the cold plasma energies of another simulation. [6]

Figures 4.1.6, 4.1.7, and 4.1.8 show the excitation in electrostatic field energy due to the first 3 modes of spatial perturbation, respectively. The electrostatic modal energies are found using Equation 3.3.5.
Figure 4.1.6: With an initial spatial perturbation mode set to 1, the modal electrostatic energies of the cold oscillation are found using Equation 3.3.5 [2].

The initially defined mode of spatial perturbation causes an excitation in the electrostatic energy corresponding with the same wavelength. This modal correspondence is confirmed with the electrostatic modal energy being highest when compared to the other modes. This correspondence between the spatial perturbation and electrostatic modal energies can be seen in Figures 4.1.7 and 4.1.8, where two more simulations were run with identical parameters, only differing by the initial spatial perturbation.
Figure 4.1.7: Cold plasma electrostatic modal energies with a spatial perturbation mode set to 2. The electrostatic mode with the highest energy corresponds to the spatial perturbation mode in this example
Figure 4.1.8: Electrostatic modal energies with excitation of mode 3.

In addition to modeling the time evolution of the electrostatic modal energies, the dispersion relation of the electric field oscillation is found using analytical and numeric methods. The following conditions are used in conjunction with the fluid continuity and momentum conservation Equations 3.2.1 and 3.2.2 to generate the theoretical dispersion relation for a cold plasma. [5] Figure 4.1.9 shows the theoretical dispersion for a cold plasma with plasma frequency, $\omega_p$.

- $\nabla \times E = 0$ (Longitudinal waves);
- $B = 0$ (Unmagnetized plasma);
- $v_e = v_{e1} = 0$ (Initially Immobile)
Figure 4.1.9: Theoretical dispersion relation for a cold plasma, found analytically using the fluid continuity and momentum conservation equations combined with the conditions of a cold plasma. [2]

Figures 4.1.10 to 4.1.14 show the dispersion relation for a cold plasma with plasma frequencies $\omega_p$ and modes of excitation. These plots agree well with the theoretical relation $\omega = \omega_p$. Comparing to Figure 4.1.9, the group velocities are all $v_g = \frac{d\omega}{dk} = 0$, since these are standing waves and do not propagate. Figure 4.1.11 is taken from another paper for comparison. [6] The disagreement between the theoretical and numeric dispersion in this other paper’s simulation may be due to the chosen parameters. The parameters chosen were 2048 particles, 256 grid, a $4\pi$ grid length points, 150 time steps, a $dt = 0.1$, with a sinusoidal spatial perturbation of 0.001 of amplitude.
Figure 4.1.10: Cold plasma numerical dispersion relation with $\omega_p=1$ and spatial perturbation mode=1.
Figure 4.1.11: This numerical dispersion plot is taken from another PIC simulation. The disagreement between the theoretical black line and Fourier amplitudes may be due to the chosen parameters. [6]
Figure 4.1.12: Cold plasma numerical dispersion relation with $\omega_p=2$ and spatial perturbation mode=1.

Figure 4.1.13: Cold plasma numerical dispersion relation with $\omega_p=2$ and spatial perturbation mode=2
Figures 4.1.10, 4.1.12, and 4.1.14 show the same cold plasma simulation only differing in $\omega_p$ and the mode of spatial perturbation. Altering $\omega_p$ changes the temporal rate of harmonic oscillation, whereas altering the mode of spatial perturbation changes the wavelength of oscillation. $\omega_p$ affects the computer defined parameters of charge $q$ and mass $m$, which in turn effect the force acting on the particles. The force defines the motion of the particles, which then in turn dictate the electric fields, as the position of each particle determines the spatial charge densities.
4.2 Electron Plasma Waves

Figure 4.2.1: Theoretical dispersion relation for electron plasma waves [3]

The plot above shows the theoretical dispersion relation for electron plasma waves. Electron plasma waves are among the most fundamental phenomenon in plasma physics. These waves are electrostatic in nature and propagate in unmagnetized plasmas. They are high frequency waves and the ions are treated as approximately unperturbed. The characteristics associated with Electron Plasma Waves are as follows:

- \( \nabla \times \mathbf{E} = 0 \) (Longitudinal waves);
- \( \mathbf{B} = 0 \) (Unmagnetized plasma);
- \( v_e = v_{e1} \parallel \mathbf{E} \)

Given the assumption of fixed ions, generating the dispersion relation only requires solving electron continuity and momentum equations and Poisson’s
equation. The resulting electric field dispersion relation is represented by the following equation. [5]

\[ \omega^2 = \omega_p^2 + 3v_{th}^2k^2 \] (4.2.1)

Where \( v_{th}^2 \) is the Maxwellian thermal velocity of the electrons. Spatial input variable are defined as follows: \( L=50, nt=800, dt=0.0999/2, ng=500 \). The species input variables are defined as: \( N=32,000, qm = -0.01, vt1 = 1, v0 =0 \). [5] Figures 4.2.2, 4.2.4, and 4.2.5 were all created with \( vt1 = 1 \), producing an initial distribution function with a “hot” Maxwellian velocity distribution.

The following plots show a correspondence between the theoretical dispersion curve and the Fourier amplitudes of the electric field data. Although the plots do not show as much agreement as Figure 4.2.3 (taken from another paper), the parabolic curve of the theoretical dispersion relation is suggested in the numerical Fourier amplitudes.
Figures 4.2.2: Numerical dispersion relation of electron plasma waves of $\omega_p=1$ with theoretical curve for comparison.

Figure 4.2.3: Numerical dispersion relation of electron plasma waves. [5]
Figure 4.2.4: Dispersion relation of electron plasma waves of $\omega_p = 2$.

Figure 4.2.5: Dispersion relation of electron plasma waves of $\omega_p = 3$
4.3 Two-Stream Instability

An instability consisting of counter-streaming electrons was modeled by defining each stream with opposing initial drift velocities, $v_0$. Since the initial velocities are single valued, the initial distribution may be considered as “cold”.

Spatial input variables were defined: $L = 2\pi$, $n_{sp}=2$, $n_t=300$, $dt=0.2$, $n_g=32$. Species input variables (both streams): $N = 128$, $w_p = 1$, $q_m = -1$, and $v_0 = \pm 1$. Both streams were given perturbation settings of $mode=1$, $x_1=0.001$. All other parameters were set to zero.

Figure 4.3.1: Phase-space time evolution of two-stream instability.
Figure 4.3.1 shows the phase-space evolution of the two streams. From the initial time $t = 0$ to approximately $t = 15$, there is linear behavior in the growth of the perturbation. After the maximum electric field is reached, nonlinear behavior develops as the electron streams become thermalized.

![Electron-Electron Stream Charge Density](image)

Figure 4.3.2: Two-stream time evolution of charge density.
Figure 4.3.3: Two-stream electric field time evolution.

Figures 4.3.2 and 4.3.3 show the charge density and electric field evolution of the two streams. Figure 11-4 shows the kinetic, electric potential, and total energy. The electric field growth coincides with a drop in kinetic energy in both streams. It should be noted that each stream possesses identical kinetic energies in time; since the distributions are symmetric in phase space.
Figure 4.3.4: Two-stream kinetic, electric potential, and total energies.

The figure above shows the kinetic, electric potential, and total energies. Exponential growth of electric potential energy in first phase of time evolution can be seen. The electric field growth coincides with a drop in kinetic energy in both streams. It should be noted that each stream possesses identical kinetic energies in time; since the distributions are symmetric in phase space.
Figure 4.3.5: Time evolution of the electrostatic modal energies, with most of the energy in the excited mode 1 until the instability reaches maximum electric field. After this point in time the streams become thermalized and the other modes developed comparable energies.
Figure 4.3.6: The two-stream velocity distribution time evolution shows divergence in the velocities after approximately time=10 (arbitrary units). Thermalization is reached at about time=30, as the velocity distribution starts to possess a Maxwellian shape.

The plots above show the velocity distribution time-evolution of the two-stream instability. Initially, the streams each have drift velocities of $-v_0$ and $+v_0$, but after the instability reaches maximum the distributions begin to approach a near-Maxwellian shape.
Figure 4.3.7: Drift energies for each electron stream. Both streams possess identical energies for each case. The drift energies initially hold at a steady value, until maximum instability, when the thermal energy takes over.
Figure 4.3.8: Time evolution of the thermal energies of each steam show identical behavior between the two streams. During the exponential increase in electric field, the velocities start to become thermalized simultaneously.
4.4 Landau Damping

As opposed to a cold plasma, a thermal plasma possesses a Maxwellian velocity distribution. Even though the electrons are assumed to be collisionless, the electrons gain kinetic energy and the electric field amplitude decays in time. When individual electrons move in the electric field, they can diminish their energy (electron velocity larger than phase velocity of wave) or receive additional energy from the wave (electron velocity less than phase velocity of wave). The energy balance for a swarm of electrons depends on the quantity of “cold” and “hot” electrons. For a Maxwellian distribution, the quantity of “cold” electrons with a velocity of zero is more than quantity of “hot” electrons. This leads to the damping of the electric field perturbation. [2]

Spatial input variable are defined for this example: \( L=2\pi, nt=300, dt=0.1, ng=256 \). The species input variables of \( N =8192, wp = 1, wc = 0, qm = -1, vt1 = 0.5, v0 =0 \). A spatial perturbation was given with \( mode=1, x1=0.02 \). Figure 12-1 shows the initial phase space, with a Maxwellian distribution with a mean velocity of zero.

Figures 13-2 and 13-3 show the time evolution of the charge density and electric field. Initially the electric field has a smooth sinusoidal shape, but overtime flattens due to Landau damping.
Figure 4.4.1: Initial distribution function represented in phase space. As opposed to the cold plasma and two-stream examples, and similar to the electron plasma waves, the electrons are given an initial Maxwellian velocity distribution.
Figure 4.4.2: Looking at the charge density evolution, the initially rough sinusoidal shape quickly decays and loses coherence due to Landau damping.
Figure 4.4.3: Analogous to the charge density shown in Figure 4.4.2, the electric field also loses coherence through time due to Landau damping.
Figure 4.4.4: The time evolution of the velocity distribution shows how the Maxwellian shape is virtually unchanged while the electric field decays.
Figure 4.4.5: The thermal energy of the Landau damping example shows an initial increase in thermal energy, coinciding with the loss of electrostatic field energy in Figure 4.4.6.

Figures 4.4.5 to 4.4.7 show the time-evolution of the thermal, field, and mode energies, respectively. Initially the field energy oscillates primarily in the excited mode due to the initial spatial perturbation, but dissipates due to Landau damping. The electrons not travelling at the phase velocity defined by the spatial perturbation take away energy from the electros carrying the sinusoidal “signal”.
Figure 4.4.6: The electrostatic field energy shows an exponentially decaying oscillation, characteristic of Landau Damping of the electric field.
Figure 4.4.7: Landau damping presented in multiple electrostatic field energy modes. The initial spatial perturbation in the particle positions gives mode 1 the maximum energy. After approximately time=10, however, the second and third modes possess comparable energies. This spreading of electrostatic energy between the modes is an indication of thermalization in the electron velocities.
4.5 Beam-Plasma

Like a two-stream instability, a beam-plasma consists of two “cold” species. The beam consists of stationary ions, while the plasma is made of mobile electrons with a drift velocity. Separate parameters are defined for each species: charge-to-mass $qm$ and number of particles, $N$. Spatial input variables were provided, $L=2\pi$, $nsp=2$, $nt=1200$, $dt=0.2$, $ng=64$. Species input variables of $N = 512$, 64, $wp = 0.03$, 1, $qm = -1$, 0.001, $v0 =1$, 0, respectfully for electrons and ions. A spatial electron perturbation was defined: $mode=1$, $x1=0.01$. All other simulation parameters are set to zero.

Figure 4.5.1 shows the beam-plasma phase space evolution. Giving an initial spatial perturbation to the plasma causes an instability to grow, similar to the two-stream example. During the linear phase of growth, the spatial and electric field perturbation (and therefore electric field) in the electron plasma grows exponentially in time.
Figure 4.5.1: The beam-plasma phase space shows a similar behavior as the two-stream example. The blue dots represent mobile electrons with an initial drift velocity of 1. The red dots represent ions with a much lower charge-mass ratio.
Figure 4.5.2: The beam-plasma electric field time evolution shows a mostly sinusoidal shape, as the electrons are the primary factor in electric field. Electrons are much higher in number and have a much higher plasma frequency, the computer parameter which determines charge and mass when combined with the charge-mass ratio.

Figure 4.5.2: The Beam Plasma simulation shows the exponential growth and subsequent oscillation after saturation in electric field energy. Figure 13-4 shows the modes of electric field energy.
Figure 4.5.3: Similar to the two-stream, the electrostatic field energy increases exponentially until about time=110. After this time, the field energy starts to oscillate, exchanging primarily with the electrons’ kinetic energy.
Figure 4.5.4: The modal electrostatic field energies show that the first mode is dominant, as the electrons are spatially perturbed to excite that wavelength.
Figure 4.5.5: The time evolution of the beam-plasma velocity distribution shows that the heavy ions maintain their velocities much more than the electron plasma, which starts to develop a Maxwellian distribution near the end of the run.

Figures 4.5.6, 4.5.7, and 4.5.8 show the thermal, drift, and kinetic energies of each species, respectively. The exponential growth in electric field coincides with growth in both species’ thermal energies, although the increase is more significant in the stationary ions.
Figure 4.5.6: The thermal energies of both the electrons and ions increase exponentially, coinciding with the increase in electric field.
Figure 4.5.7: These plots show the evolution of the drift energies in each species. The electrons lose some drift energy due to thermalization, although almost all the kinetic energy still consists of drift energy. The ion beam gains some drift energy, but is 12 orders of magnitude less than the gain in thermal energy in the species.
Figure 4.5.8: Most of the electrons’ kinetic energy consists of drift energy, as the species is given an initial drift velocity. The rest of the electron kinetic energy is thermal velocity, which can be seen in Figure 4.5.7. Most of the ions’ kinetic energy consists of thermal energy, which can be seen in Figure 4.5.6.
Chapter 5: Summary

This work was initially intended to demonstrate a fully physical ionospheric plasma system, but the difficulty of creating a complete model proved too immense. Instead, the work became focused on how to translate FORTRAN and Python code to MATLAB, and how to guarantee the accuracy of the generated output by comparison with the original code. In addition to the calculation of the electric field acting on each particle, a more physically complete simulation requires other physical considerations. The magnetic fields as well as particle collisions of each particle must be added to the computational toolkit to generate a more complete set of plasma interactions. [2].

Relative to a computational fluid dynamic approach to plasma, a particle-in-cell approach is more computationally taxing. Despite the computational cost, however, useful plasma phenomenon is representable with PIC. This work ultimately consists of demonstrating the relatively elementary wave properties of a cold stream, electron plasma waves, two-stream, Landau Damping, and electron-beam systems. Comparing with expected theoretical results derived from the fluid continuity and momentum equations as well as other physics, agreement was obtained using the PIC method.
Bibliography


   porl2.tripod.com/sitebuildercontent/sitebuilderfiles/dmreport1.pdf.


Appendix

%ES1 - Main Program Code By Richard Vanderburgh at Wright State University 2020

clear variables
close all
clc

FIRST_EE

INIT
  t=0;
SETRHO
FIELDS
dxdt = dx/dt;
  \%{
    \text{axis tight manual} \% this ensures that getframe() returns a
    \text{consistent size}
  v = VideoWriter(sprintf(example));
  open(v);
  \%
  }

  for t=1:nt
    \%
    if mod(t,ixvx)==0|| t==1 && ixvx~=0
    \% if t==1
    \% phasecounter=1;
    \% PhaseSpacePlots=figure;
    \% hold on
    \%
    \% for species=1:nsp
    set(0,'CurrentFigure',PhaseSpacePlots)
    h1=subplot(3,3,phasecounter) ;
    \%subplot(2,2,1)
    scatter(x(:,species)/dxi,vx(:,species)*dxdt, '.');
    \%set(gca,'FontSize',12)
    grid on
    ylim([-3 3]);
    xlim([0 L]);
    \%
    \% hold on
    \%
    \% if t==1
    title(['Time = ',num2str((t-1)*dt-dt/2),']);
xlabel('Position (arb. units)');
ylabel('Velocity (arb. units)');
elseif \( t \approx 1 \)
title(['Time = ',num2str((t)*dt-dt/2),]);
xlabel('Position (arb. units)');
ylabel('Velocity (arb. units)');
end
phasecounter=phasecounter+1;
hold off
drawnow
%frame = getframe(gcf);
%writeVideo(v,frame);
end
if \( t==nt \) && \( ixvx~0 \)
set(0,'CurrentFigure',PhaseSpacePlots)
suptitle([sprintf(example), 'Phase Space'])
saveas(PhaseSpacePlots,[sprintf(example), 'PhaseSpace.png'])
end
if \( t==1 \)
SETV
end
ACCEL
MOVE
FIELDS

for species=1:nsp
te(t,1)=te(t,1)+ke(t,species);
end
te(t,1)=te(t,1)+EnergiaP(t,1);
if mod(t,irho)==0|| t==1 && irho~0;

    if \( t==1 \)
rhocounter=1;
ChargeDensityPlots=figure;
hold on
    end
set(0,'CurrentFigure',ChargeDensityPlots)
h2=subplot(3,3,rhocounter);
%subplot(2,2,2)
plot(gridx,real(rho),'LineWidth',2)
xlim([0 L]);
ylim([-4 -0]);
grid on
hold on
xlabel('Position');
ylabel('Charge Density');
if t == 1
    title(['Time = ',num2str((t-1)*dt)]);
elseif t ~= 1
    title(['Time = ',num2str(t*dt)]);
xlabel('Position (arb. units)');
ylabel('Charge Density (arb. units)');
grid on
end
rhocounter=rhocounter+1;
drawnow
%hold off
end
if t==nt && irho~=0
    set(0,'CurrentFigure',ChargeDensityPlots)
suptitle([sprintf(exam),
        'Charge Density'])
saveas(ChargeDensityPlots,[sprintf(example),
        ' ChargeDensity.png'])
end
%
for species=1:nsp
    % Clear out old charge density.
    for j = 2:ng+1
        rho(j) = rho0;
    end
    rho(1) = 0;
end
if mod(t,iphi)==0 || t==1 && iphi~=0
    if t==1
        phicounter=1;
        PotentialPlots=figure;
        hold on
    end
    set(0,'CurrentFigure',PotentialPlots)
h3=subplot(3,3,phicounter) ;
    %subplot(2,2,3)
    plot(gridx,real(phi),'LineWidth',2)
xlim([0 L]);
    %ylim([-3 -1]);
    grid on
    %hold on
    xlabel('Position (arb. units)');
ylabel('Potential (1arb. units)');
if t == 1
    title(['Time = ',num2str((t-1)*dt)]);
elseif t ~= 1
    title(['Time = ',num2str(t*dt)]);
xlabel('Position (arb. units)');
ylabel('Potential (arb. units)');
grid on
end
drawnow
phicounter=phicounter+1;

hold off
end
if t==nt && iphi==0
    set(0,'CurrentFigure',PotentialPlots)
suptitle([sprintf(example), 'Electric Potential'])
saveas(PotentialPlots,[sprintf(example), 'ElectricPotential.png'])
end
if mod(t,iE)==0 || t==1 && iE==0
    if t==1
        Ecounter=1;
        ElectricFieldPlots=figure;
        hold on
    end
    set(0,'CurrentFigure',ElectricFieldPlots)
    h4=subplot(3,3,Ecounter); %subplot(2,2,4)
    plot(gridx,real(E(t,:)), 'LineWidth',2)
xlim([0 L]);
    ylim([-1 1]);
    grid on
    hold on
    xlabel('Position (arb. units)');
ylabel('Electric Field (1arb. units)');
    if t == 1
        title(['Time = ',num2str((t-1)*dt)]);
    elseif t ~= 1
        title(['Time = ',num2str(t*dt)]);
xlabel('Position (arb. units)');
ylabel('Electric Field (arb. units)');
    grid on
end
Ecounter=Ecounter+1;
drawnow
hold off
end

if t==nt && iE~=0
set(0,'CurrentFigure',ElectricFieldPlots)
suptitle([sprintf(example), 'Electric Field'])
saveas(ElectricFieldPlots,[sprintf(example), 'ElectricField.png'])
end

if mod(t,ifvx)==0 || t==1 && ifvx~=0
if t==1
fvxcounter=1;
fvxPlots=figure;
hold on
end
set(0,'CurrentFigure',fvxPlots)
h5=subplot(3,3,fvxcounter);

hist(vx);
grid on
if t==1

    title(['fvx at time ',num2str((t-1)*dt-dt/2),']);
xlabel('velocity');
elseif t ~= 1
    title(['fvx at time ',num2str((t)*dt-dt/2),]);
xlabel('velocity');
    %xlim([-3 3]);
    %ylim([0 50]);
end
fvxcounter=fvxcounter+1;
drawnow
hold off
end

if t==nt && ifvx~=0
set(0,'CurrentFigure',fvxPlots)
suptitle([sprintf(example), 'Velocity Distribution'])
saveas(fvxPlots, [sprintf(example), 'Velocity Distribution.png'])
end
			%t*dt-dt/2
end

%suptitle([sprintf(example), 'Electric Field with wp= ',
num2str(wp(1)) ' and Mode= ' num2str(mode(1))])
%saveas(ElectricFieldPlots, [sprintf(example),
'ElectricField.png'])
%close(v);
%plotspectrnew
wk

PLOTTING

%End of Simulation :)