Interferometry Analysis Method for Colliding Plasma Generated with Exploded Wires

Michael D. Gruesbeck

Wright State University

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Interferometry Analysis Method for Colliding Plasma Generated with Exploded Wires

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

by

MICHAEL D. GRUESBECK
B.S., Purdue University, 2015

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Wright State University

Michael A. Saville, Ph.D., P.E.
Thesis Director

Fred D. Garber, Ph.D.
Interim Chair, Department of Electrical Engineering

Barry Milligan, Ph.D.
Vice Provost for Academic Affairs, Dean of the Graduate School

Committee on
Final Examination

Michael A. Saville, Ph.D., P.E.

Josh Ash, Ph.D.

Yan Zhuang, Ph.D.

Building upon recent work to estimate electron and ion densities from paired interferograms, the current work develops a model for the 2D interference phase function. Unlike the previous work that only estimated a radial plasma profile from images of a singly exploded straight Cu wire, we estimate 2D properties from a colliding plasma generated from two simultaneously exploded wires. First, a 2D phase model is proposed for the interference patterns of the images taken at 1064 nm and 532 nm. Then, the model parameters are estimated using Fourier analysis. Secondly, the plasma region in each image is partitioned into subregions and analyzed using the Abel transform under the assumption of locally cylindrical conditions. The approach allows for analysis of the 2D plasma profile using semi-automated analysis of a time series of interferograms.
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Introduction

1.1 Motivation

Communication is critical for any mission, especially when a manned spacecraft is returning home after a trip to space. During these missions though when the spacecraft travels through the ionosphere, communication with the spacecraft ceases. This is called reentry blackout and is caused by an envelope of plasma around the aircraft, created by the heat from the compression of the atmosphere by the spacecraft [11]. It was not until the creation of the Tracking and Data Relay Satellite System (TDRSS) that the reentry blackout would be solved. This allowed NASA to communicate by satellite relay with the spacecraft from above reentry where the plasma envelope did not exist [11]. However, the reentry blackout still is a problem as other worlds are explored. Both the Mars Pathfinder and the Huygens probe endured communications blackout as they entered Mars’ and Titan’s atmospheres respectively. The recently landed Mars rover Perseverance also experienced brief communication blackout windows during its descent. With the goals and aspirations of sending humans to Mars, it would be critical to minimize or eliminate these windows of time with no communications. Understanding the properties such as electron and atomic densities of these colliding plasmas could prove key in solving these communications blackouts. To study these properties, we will be using interferometry.

Interferometry is a technique in which electromagnetic waves are superimposed causing interference [3]. Interferometry became a popular diagnostic tool after the efforts of
Dr. Thomas Young. Dr. Young, guided by the knowledge that when two sound waves overlapped beats were produced, began attempting to prove this wave nature of light. The dilemma is that when two independent light sources combine, they are naturally incoherent. Dr. Young analyzed the intensity pattern using a single light source to highlight an opaque screen with two pin holes that allowed light through. The experiment has since been studied and explained with Maxwell’s Equations that showed how light is a form of electromagnetic radiation and the intensity pattern of Young’s double slit experiment results from the interference of two waves. This process was successful in producing alternating bright and dark bands, or interference fringes [8]. Below in Fig. 1.1 is an diagram of Young’s Experiment.

![Interference Pattern](image)

Figure 1.1: Young’s double slit experiment [9].

Young’s breakthrough has lead to the development of multiple types of interferometers as diagnostic tools for a wide array of research including the properties of plasma.

Our goal is to develop a phase function model for these interferograms of a colliding plasma. Once the model is created, it can be evaluated to estimate the plasma properties. However, unlike previous iterations of the experiment, the two-wire experiment does not feature cylindrical symmetry. If the data is partitioned in such a way that cylindrical sym-
metry can be reasonably assumed in the region, then each partition can be evaluated to estimate the plasma properties with the Abel Transform.

1.2 Challenges

One of the most difficult challenges when working with optical interferometry is the ability to repeat the process and get the same results. The lens equipment needs precise measurements to ensure that the lenses are at proper focal length, and that they are aligned precisely. If not secured properly, even a small bump can cause misalignment. Also, the equipment to fire the lasers, ablate the wires and record the images need to be in sync to ensure proper time frame data is acquired. Furthermore, the electrical load, the wires for our experiment, must be precisely inserted. If there is extra load, or if it is tainted with other particles not of the load it can drastically change the outcome of the experiment. To ensure repeatability, a checklist of procedures is kept and used prior to each experiment.

Another challenge with the experiment is the plasma itself. A plasma is the fourth state of matter. A plasma is made up of ionized particles. These particles are turbulent, with collisions occurring regularly and the plasma volume grows.

Environmental factors also can affect the outcome of the experiment. Humidity, air flow, temperature, and pressure all can affect the output of the interferometer and the expansion rate and direction of the plasma. These atmospheric conditions need to be monitored and kept the same from one experiment to another. The Plasma Physics Lab of the Sensors Directorate, Air Force Research Lab has developed a measurement system and procedure that allows repeatable and reproduceable results [7]. To ensure that any minute change does not affect the outcome of the experiment, prior to the start of the experiment a reference interferogram is captured for comparison against the data results of the experiment. The current effort investigates a colliding plasma that is generated from two simultaneously exploded wires. The optical path and camera set up is similar to the prior work reported
in [7].

Figure 1.2: A reference image (a) is captured prior to the experiment. The experiment data is show in (b).

A reference interferogram is captured prior to beginning of the experiment Fig. 1.2(a) to remove the changes that may occur with changing atmospheric conditions or alignment adjustments. During the experiment an interferogram Fig. 1.2(b) is captured at the time desired per the experiment parameters.

For analysis, a primary challenge will be dealing with the asymmetry of the dual exploding wires. For the single wire case, there was cylindrical symmetry about the wire axis as the plasma volume expanded outwards. This allowed for use of the Abel Transform to estimate the plasma properties. For the dual wire case though, that symmetry does not exist globally across the entire interferogram. However, by partitioning the plasma region, we can make a safe assumption that locally in that partition there is cylindrical symmetry such that we can apply the Abel Transform to the region.

1.3 Research Hypothesis

Our goal is to advance from the single wire ablation case work of Hamilton [7] onto a more complex dual wire ablation case. We will create a model for the interferogram based
on the spatial frequency and initial phase and then generate a two-dimension phase map for the plasma region. Finally, the plasma region will be partitioned properly to fit our necessary assumption of cylindrical symmetry to estimate the electron density and atomic density plasma properties.

1.4 Outline of Thesis

The rest of the document is organized as follows. Chapter 2 provides relevant information to the current state of the analysis plasmas. In Chapter 3, the experimental process for obtaining the interferometric data is described. The algorithm for processing this data is explained in detail. Chapter 4 presents the results of the model and application of the Abel Transform. In Chapter 5 summarizes the work completed and suggests further opportunities for this research.
Background

In this chapter, previous experiments detailing processing and analysis methods will be surveyed. This survey will include interferometry image processing algorithms used in previous wire ablation experiments and methods used in interferometry in general to guide our image processing algorithm defined in chapter 3.

2.1 Previous Work

In our experiments we are expanding on the previous works by Sarkisov [13] and Hamilton [7] from a single wire ablation case to a dual wire ablation case. Therefore, we will review their work and similar experiments involving dual wire ablation interferometry.

The experiment by Sarkisov et al [13] used dual wavelength interferometry to analyze plasma generated by single wire ablation. The load was made of aluminum wires with a length of \(20 \text{ mm}\). The diameter of the load varied from 10 to 38 \(\mu\text{m}\). Sarkisov derived the equation to equate fringe order shift to atomic and electron densities below [13].

\[
\delta = \frac{2\pi \alpha}{\lambda} \int N_a \, dx - 4.49 \times 10^{-14} \lambda \int N_e \, dx ,
\]  

(2.1)

Sarkisov measured a fringe order shift for wavelengths 532 nm and 1064 nm. Since this equation cannot be solved directly, therefore, the Abel transform was applied to get a profile output for the atomic density, \(N_a\). While Sarksiov did use two wavelengths, only the 532-
nm wavelength was used to determine atomic density. He found the atomic density to be approximately $4.3 \times 10^{19} \text{ cm}^{-3}$. Sarkisov stated that the dual wavelength results would be reported at a later time.

Sarkisov also used a technique called shadowgraphy to determine expansion velocity of the plasma. This process uses a shadowgram to measure the distance the plasma traveled over time to determine its velocity. Sarkisov estimated the wire ablation generated plasma to be expanding at a speed of $3.5 \text{ km s}^{-1}$ [13].

For the experiment conducted by Hamilton [7], they used dual wavelengths of 532 nm and 1064 nm to ablate a single wire. Experiments were conducted on copper wire of diameter 25 $\mu$m and aluminum wire loads with a length of 10 mm. To process these interferometric images, Hamilton applied a window function in the frequency domain to filter noise. Then, a cubic spline fit was used to locate the fringe lines since there was both curvature and tilt present in the image [7]. From these fit functions the fringe order shift was measured. Hamilton measured the peak fringe order shift to report maximum atomic and electron densities. Hamilton reported a maximum electron density of $2.07 \times 10^{18} \text{ cm}^{-3}$ and maximum atomic density of $4.56 \times 10^{19} \text{ cm}^{-3}$ for copper load [7]. For aluminum load a maximum electron density of $3.22 \times 10^{18} \text{ cm}^{-3}$ and maximum atomic density of $6.15 \times 10^{18} \text{ cm}^{-3}$ [7].

The experiments conducted by Disawal et al [4] were not done on wire ablation, but their processing techniques are useful in the modeling of the interferograms. The fringe shift order is equivalent to phase by a simple scaling of $2\pi$. For each fringe order, the phase gains $2\pi$ radians. Disawal worked on automating the detection of collimation position of an incoherent beam in Lua interferometry using a Fourier fringe analysis technique [4]. They determined that the intensity of the interferometer image could be written as

$$ I(x, y) = A + B \cos [\psi(x, y) + \alpha(x, y)] ,$$

(2.2)
where $\psi$ is the phase and $\alpha$ is the deflection angle [4]. Disawal defines the phase angle as follows

$$\psi(x, y) = \tan^{-1} \left( \frac{\text{Im}\{C(x, y)\}}{\text{Re}\{C(x, y)\}} \right), \quad (2.3)$$

though it is unclear what $C$ is defined as to the reader [4]. The phase $\psi$ is wrapped with $\psi \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$. Disawal adopted a phase unwrapped process to unwrap the phase, though does not explicitly state the unwrapping algorithm.

While also not an experiment on wire ablation, Takeda describes a method of analysis for precision interferometry called Spatial-carrier fringe pattern analysis [14]. This spatial carrier technique allows for the use of a single interferogram to find carrier frequency and phase. The intensity of the interferogram is in the form of

$$g(x, y) = a(x, y) + B(x, y) \cos [2\pi f_o x + \phi(x, y)], \quad (2.4)$$

where $f_o$ is the spatial carrier frequency and $\phi$ is the phase [14]. After filtering in the frequency domain with a Fourier Transform, the data is returned to the spatial domain with the following form

$$c(x, y) = \frac{1}{2} b(x, y) \exp \{i\phi(x, y)\}, \quad (2.5)$$

where the phase can be obtained using a simple inverse tangent function

$$\phi(x, y) = \tan^{-1} \left( \frac{\text{Im}\{c(x, y)\}}{\text{Re}\{c(x, y)\}} \right), \quad (2.6)$$

with $\phi \in [-\pi, \pi]$ [14]. This wrapped phased is then corrected using an unwrapping algorithm, though one in particular is not mentioned.
2.2 Technical Challenges

While the previous works on wire ablation plasma generation reported maximum values for atomic and electron densities, we are working on generating a pixel by pixel density map for each image over time. That has lead us to explore other interferometry analysis methods such as spatial frequency and phase analysis. By finding a phase, we believe that we can generate a density map for the entire plasma region rather than just where the fringe lines appear.

Furthermore, when calculated the initial phase, this function is a wrapped phase within the limits of $[-\pi, \pi]$. This phase has to be unwrapped using an algorithm. Prior to unwrapping the initial phase is passed through a low pass filter to reduce noise to assure that no mistakes are made in the unwrapping due to noise.

Another challenge is resolving the electron and atomic densities. With single wire case it is easier as the plasma generates a cylindrical profile. With the two wire case that is not necessarily true, so the plasma region will have to be broken down into subregions where its safe to assume cylindrical symmetry such that we can apply the Abel transform.

Finally, we make an assumption in our experiment that we are fully vaporizing the load. This assumption is neccessary according to Sarkisov [13] in order to separate the polarizability constant from the atomic density. Without this assumption the estimation of atomic density would not be possible. However, in practice this may not be happening.

2.3 Methods of Analysis

Looking at the methods of analysis for the interferogram, Sarkisov evaluated the plasma properties of a single exploding wire case. This case allowed for cylindrical symmetry. Sarkisov chose to map the fringe lines by hand by gathering points on the fringe line [13]. Then, he fit the data with a curve. The Abel transform was applied to the data to estimate
the atomic and electron density of the plasma. Furthermore, Sarkisov used shadowgrams to estimate the expansion velocity of the plasma.

The method of analysis used by Hamilton was quite different than that of Sarkisov. Hamilton chose to fit the Fringe lines with a cubic spline [7]. From here, he manually estimated the maximum fringe shift by measurement and estimated the maximum atomic and electron density. While this method did not produce a radial profile, the maximum densities were on the same order of Sarkisov’s results.

Both Disawal and Takeda chose to evaluate the interferogram different from the above methods. Knowing the behavior of the intensity function operated as a sinusoid, both chose to use a Fourier Transform to evaluate the spatial frequency and phase of the interferogram [4] [14]. Each chose to use an unwrapping function on the phase function to create a phase model of the interferogram.

We will be using a combination of these methods to evaluate our interferograms. First, we will model the interferogram with a spatial frequency and initial phase obtained from the Fourier Transform. Then, we will map the phase of the interferogram and use this phase function to determine fringe order. The data will be partitioned to safely assume cylindrical symmetry so that it can be evaluated by the Abel Transform.

2.4 Summary

In this chapter several journal articles were surveyed to gain an understanding of methods for processing interferograms of plasma generated by exploding wires. We have studied the work of Sarkisov [13] and Hamilton [7] who use fringe order to estimate the atomic and electron densities. We also explored other methods of evaluation of interferograms. Disawal used Fourier analysis to find phase [4]. In Takeda’s work [14], he derived a method using Fourier to find the spatial frequency and initial phase of an interferogram.
Methodology

3.1 Research Hypothesis

For this experiment, we will estimate the electron density, atomic density, and the plasma expansion velocity from colliding plasma generated by dual wire ablation. To do this we will acquire data in the form of interferometric images from the lab experiment as outlined below. We will acquire spatial frequency and initial phase to model the data and generate a two-dimensional phase map. This phase map will allow for a two-dimensional estimation of atomic and electron densities for the plasma generated by two exploding wires.

3.2 Approach

The approach used is outlined below. First, the data is collected from the lab experiment. Then, a preprocessing algorithm is applied to the data. A calculation is completed for the line integrated electron density and atomic density. Finally, an estimation of the atomic density, electron density, and plasma expansion velocity can be made.

3.2.1 Preprocess of Interference Images

The preprocessing of the interferometric data images involves a four step algorithm. The data images will first be aligned. Once aligned, the images will be filtered and passed
through an amplitude leveling algorithm. The data will then be quantized into an eight bit output.

3.2.2 Solving for Line Integrated Density

In this part of the approach, we will start with the fringe order model derived by Sarkisov [13] that equates fringe order to a linear combination of line integrated electron density and line integrated atomic density. With a dual wavelength, we can generate a system of equations and solve separately for line integrated electron density and atomic density based on the fringe order from each wavelength.

3.2.3 Electron Density, Atomic Density, and Plasma Expansion Velocity

From the calculated line integrated density functions we can then estimate the electron density and atomic density using the Abel Transform. The plasma expansion velocity will then be estimated over time.

3.3 Lab Experiment

The lab experiment is setup nearly identical to the lab experiments completed in Hamilton’s work [7] with two changes. These changes which were noted through email (A. Hamilton, personal communications, January 29, 2021) are a change to the interferometer and the addition of a second exploding wire.

The same laser is used [5], that produces the first and second harmonic wavelengths 532 nm and 1064 nm respectively. We will use the same size plasma chamber for this experiment that has a volume of 10 cm$^3$. A diagram of the plasma chamber is below in
Fig. 3.1. Prior to execution of the experiment the plasma chamber is vacuumed to a sub 10 mTorr pressure.

The load for this experiment is a pair of aluminum wires that are used for the load with a density of $2.69 \text{ g cm}^{-3}$ [1]. The aluminum wire has a diameter of 15 $\mu$m and a length of 1.0 cm. Each wire is secured to the side of the electrodes, one of the left and other on the right, with a rubber band. The wires are 3.2 mm apart. The current flows in the same direction in both wires at the same time.

Figure 3.2: Diagram of optical bench [7].
The optical bench where the experiment is setup is identical to the experiment in Hamilton’s work [7]. Fig. 3.2 shows the layout for the experiment. One exception was made in changing the interferometer. In Hamilton’s work, an air wedge interferometer [10] was used. In this experiment, the lab will be using a Mach-Zehnder interferometer. The Mach-Zehnder interferometer is useful because the fringe lines are generally straight and parallel, also it allows for the adjustment of the fringe spacing in the interference pattern [2].

### 3.4 Data Structure

The data output are interferometric images captured by a pair of ccd cameras. Prior to the experiment a reference image is captured for each of the two wavelengths 532 nm and 1064 nm respectively. Then, each camera has a set delay timer to capture the image at a specific timestamp in the experiment. There is a total of four images for each experiment, one reference for each wavelength and then a corresponding image at the specified timestamp of capture. These images are of dimensions 2504 by 3326 pixels.

We are making a pair of critical assumptions about the data that we are collecting. First, because the plasma is generated in a vacuum chamber that is pumped to less than 10mTorr we assume that any disturbance in the phase output is created by the plasma flow. Secondly, we assume that during ablation, the entire wire is vaporized leaving behind no solid state. This assumes full ionization of the aluminum atoms.

### 3.5 Preprocessing Algorithm

#### 3.5.1 Image Alignment

With our goal to model the data with spatial frequency as phase similar to Eq. 2.4 we first must align the data images. Due to the nature of the data collection, the images are about
the z-axis, with the 532 nm being a reflection of the 1064 nm image. To remedy this, the 532 nm image is reflected by

\[ I(m, n) = I(a + 1 - m, n) \] (3.1)

where \( a \) is the number of pixels in the y-direction and \( m, n \) are pixel numbers in their respective axis. An example of this process is below in Fig. 3.3. All data images being used were captured in the lab at time stamp 150 ns unless explicitly stated otherwise.

![Figure 3.3: Image reflection for alignment.](image)

To finish alignment, control points need to be established to center the images. Four control points on each reference image are used for alignment. Two points on each electrode where it meets the wire. Fig. 3.4 shows the cpselect tool in Matlab that allows for control point selection. The image on the left is the 1064-nm reference image and on the right is the 532-nm reference image.
With the control points, we can align the images. To do so we center the image with the following

\[ C = \frac{A + B}{2} \]  

(3.2)

where \( A \) is control point 1, the left wire control point and \( B \) is control point 2, the right wire control point in Fig. 3.4. We do this for the anode and cathode. \( C \) is the centerpoint for the cathode and anode, and operates as a 0 mm anchor in the z-direction on the cathode, or top electrode. Next, the pixel’s z-dimension is scaled using the control points and the known separation of the cathode and anode. Similarly, the y-dimension is scaled using the control points and the known width of the cathode. The pixel dimensions in units of mm are 0.05 mm x 0.05 mm. This allows us to have same size image chips for the Abel Transform computation that comes later. A comparison of pre-alignment and post alignment images is shown below in Fig. 3.5.
Figure 3.5: Interferograms pre-alignment (top) and post-alignment (bottom).
With the images aligned and scaled, we make an observation that majority of the image is in a plasma free state. The plasma region is the area of interest to us, therefore we crop the image to the plasma region for analysis. Furthermore, to reduce any anomalies that could occur at the electrodes, they also will be cropped out. The cropped dimensions of the image are $[-3, 3]$ by $[2.5, 6.5]$. An example of the crop is below in Fig. 3.6.

Figure 3.6: Cropped region for modeling of 1064 nm 150 ns interferogram.

The cropped images in Fig. 3.6 are filtered and leveled as described in [6] so the phase
function can be estimated from

\[ \tilde{I}(y, z) = I(y, z) - A(y) = B(y) \cos\left[2\pi f_o(y)z + \phi_y,0\right], \]  

(3.3)

where \( A(y) \) is the mean of the pixel amplitudes at \( y \), and \( B(y) = 1 \) as a result of the leveling operation [6]. Equation 3.3 is a modified version of Takeda’s spatial frequency equation in [14]. The smoothing and leveling is helpful to automatically extract the fringe order from Fig. 3.6 where the intensity is non-uniform.

### 3.5.2 Image Modeling and Phase Mapping

To model the interferogram we are going to follow the modified version of Takeda’s [14] method for determining the intensity of an interferogram in Eq. 3.3. The intensity has a spatial frequency, \( f_o(y) \) as seen in the bright to dim to bright fringe pattern in the \( z \)-direction and an initial phase \( \phi_y,0 \) in the \( y \)-direction. An example of the cropped region is shown in Fig. 3.6. To model this interferogram we need to find the spatial frequency and initial phase. This was done using a Fourier Transform along the \( z \)-axis

\[ \tilde{I}(y, f) = \mathcal{F}\{I(y, z)\}. \]  

(3.4)

In this instance \( \mathcal{F} \) is the discrete Fourier Transform. Since we understand the frequency response of a sinusoidal function, the spatial frequency will be a maxima in the frequency domain,

\[ f_o(y) = \arg \max_f \tilde{I}(y, f), \]  

(3.5)

and the initial phase can be obtained from the value of \( \tilde{I} \) at the maxima by

\[ \phi_y,0 = \tan^{-1}\left( \frac{\text{Im}\{\tilde{I}(y, f_o(y))\}}{\text{Re}\{\tilde{I}(y, f_o(y))\}} \right). \]  

(3.6)
Since this initial phase is bound $[-\pi, \pi]$ we use an unwrapping algorithm to give the unwrapped initial phase. The phase and frequency are passed through a smoothing convolution filter to reduce noise and ensure continuity along the y-axis. A model of the interferogram is then generated using the frequency and phase

$$\bar{I}(y, z) = \cos(2\pi f_0(y)z + \phi_{y,0}) .$$ \hspace{1cm} (3.7)

An example model is displayed in Fig. 3.7

![Interferogram Model](image)

**Figure 3.7: Model of 1064-nm 150-ns interferogram.**

The phase map is generated by

$$\Phi(y, z) = 2\pi f_0(y)z + \phi_{y,0} .$$ \hspace{1cm} (3.8)
With the two-dimensional phase map we can calculate the fringe order used to solve for electron and atomic density by making a comparison between the reference image and the phase map

\[ \delta(y, z) = \frac{\Phi(y, z)}{2\pi}, \]  

(3.9)

where \( \delta(y, z) \) is the two-dimensional fringe order. An example of this fringe order map is seen in Fig. 3.8 below.

Figure 3.8: Fringe order map of 1064-nm 150-ns interferogram.
3.6 Algorithm for Electron Density, Atomic Density, and Plasma Expansion Velocity

To estimate the electron density and atomic density we begin with the model derived by Sarkisov [13] that relates fringe order to line integrated atomic and electron densities. To solve for the densities separately, two equations are needed, one for each wavelength.

\[
\delta_1 = \frac{2\pi\alpha(\lambda)}{\lambda_1} \int N_a \, dx - 4.49 \times 10^{-14} \lambda_1 \int N_e \, dx \tag{3.10}
\]

\[
\delta_2 = \frac{2\pi\alpha(\lambda)}{\lambda_2} \int N_a \, dx - 4.49 \times 10^{-14} \lambda_2 \int N_e \, dx \tag{3.11}
\]

Here we have denoted subscript 1 to be the 532-nm wavelength and subscript 2 to correspond to the 1064-nm wavelength and \( \delta \) referring to their respective fringe order. \( N_a \) is the atomic density, \( N_e \) is the electron density, and \( \alpha(\lambda) \) is the polarizability constant for aluminum.

3.6.1 Solving for Line Integrated Density

The cylindrical plasma volume has a circular cross-section as shown below in Fig. 3.9 where the \( y \)-direction is transverse to both the wave propagation and cylinder’s axis of symmetry. For each location \( y_0 \), the path of integration through the column is a chord (shown as the orange line segment). The relative fringe shift is a positive number where each integer modulus represents a full line shift.
The line integral for the electron density is simplified as the line-integrated electron density profile
\[ \chi_e(y_0) = \int_{-\infty}^{\infty} N_e(r) \, dx, \] 
and then use a change of variables \( x = \sqrt{r^2 - y_0^2} \) which results in
\[ \chi_e(y_0) = 2 \int_{y_0}^{\infty} \frac{N_e(r) r}{\sqrt{r^2 - y_0^2}} \, dr, \] 
and is recognized as the forward Abel Transform. In similar fashion, the line-integrated ion density is
\[ \chi_a(y_0) = 2 \int_{y_0}^{\infty} \frac{N_a(r) r}{\sqrt{r^2 - y_0^2}} \, dr, \] 
where we assumed earlier that the wire was completely vaporized allowing the polarizability constant \( \alpha(\lambda) \) to be factorable from \( N_a \) [13].

By substituting Eq. 3.13, Eq. 3.14 into Eq. 3.10 and Eq. 3.11 we the equations can be simplified to
\[ \delta_1 = \frac{\alpha}{\lambda_1} \chi_a - b \lambda_1 \chi_e, \]
\[ \delta_2 = \frac{\alpha}{\lambda_2} \chi_a - b\lambda_2 \chi_e, \]  
\[ (3.16) \]

where \( b = 4.49 \times 10^{-14} \text{ cm}^{-1} \) and \( \alpha = 8.1 \times 10^{-24} \text{ cm}^3 \) for 532 nm and \( 7.3 \times 10^{-24} \text{ cm}^3 \) for 1064 nm [12].

The solution to this system of equations for \( \chi_e \) and \( \chi_a \) are

\[ \chi_a = \lambda_1 \frac{\lambda_1 \delta_2 - \lambda_2 \delta_1}{\alpha(\lambda_1^2 - \lambda_2^2)} = \frac{b\lambda_1}{\Lambda} \delta_2 - \frac{b\lambda_2}{\Lambda} \delta_1, \]  
\[ (3.17) \]

\[ \chi_e = \frac{\lambda_2 \delta_2 - \lambda_1 \delta_1}{b(\lambda_1^2 - \lambda_2^2)} = \frac{\alpha}{\lambda_1 \Lambda} \delta_2 - \frac{\alpha}{\lambda_2 \Lambda} \delta_1, \]  
\[ (3.18) \]

where the determinate \( \Lambda = \frac{\alpha b}{\lambda_1 \lambda_2} (\lambda_1^2 - \lambda_2^2) \) must be non-zero.

### 3.6.2 Inverse Abel Transform

The inverse Abel transform requires that the data have cylindrical symmetry. Since the dual wire colliding plasma does not innately have cylindrical symmetry, the data must be partitioned in such a manner that we can assume so. Since the Abel Transform requires cylindrical symmetry, we found it best to partition each image into four sections to ensure the assumption of cylindrical symmetry is appropriate. The wire axes were chosen because about the wire is natural cylindrical symmetry, also the center of the electrodes suited best to maintain cylindrical symmetry, assuming equal contribution of plasma from each wire. Fig. 3.10 is an example of the partition.
Figure 3.10: Example partitioning of 532 nm 150 ns interferogram.

Once partitioned, the data can be passed through the inverse Abel Transform to estimate the atomic density and electron density

\[
N_a(r) = -\frac{1}{\pi} \int_r^{\infty} \frac{\delta \chi_a}{\delta y} \frac{r}{\sqrt{y^2 - r^2}} \, dy
\]  

\[
N_e(r) = -\frac{1}{\pi} \int_r^{\infty} \frac{\delta \chi_e}{\delta y} \frac{r}{\sqrt{y^2 - r^2}} \, dy.
\]  

The results are posted in Chapter 4.
3.7 Summary

In this chapter the methodology of the experiment was discussed. The experimental lab setup was, highlighting changes from the previous iteration. Then a preprocessing algorithm was given, detailing steps for alignment, smoothing, leveling, and quantizing the data. Next, calculations of phase, fringe order and line integrated densities were completed. Finally, the data was partitioned for symmetry and processing with the inverse Abel transform to get an estimate of electron and electron density as well as a calculation of plasma expansion velocity. These results were recorded in the next chapter.
Analysis

4.1 Modeling Results

First results we will observe is how well the model generated fit the original data. To do this analysis the original chip and the model were loaded, and data points were selected. The datapoint was selected by locating corresponding fringes, then selecting a point with the highest intensity of that fringe at the same y-value. An example of datapoint selection is below in Fig. 4.1.

These datapoints were gathered for all timeframes for the 532-nm and 1064-nm wavelengths. Since we knew the pixel size to be 0.05 mm the number of pixels difference and
the percent difference were calculated knowing there are 81 pixels in the z-axis for the image. The results are below in Table 4.1 for 532 nm timeframes and Table 4.2 for 1064 nm timeframes.

Table 4.1: 532-nm Model Comparison. O is the original data chip, while M is the model.

<table>
<thead>
<tr>
<th>Time Stamp</th>
<th>O (mm)</th>
<th>M (mm)</th>
<th>—O-M— (mm)</th>
<th>—O-M— (Pix)</th>
<th>Error(%)</th>
</tr>
</thead>
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<tr>
<td>50</td>
<td>4.15</td>
<td>3.9</td>
<td>0.25</td>
<td>5</td>
<td>6.17</td>
</tr>
<tr>
<td>100</td>
<td>3.5</td>
<td>3.3</td>
<td>0.2</td>
<td>4</td>
<td>4.94</td>
</tr>
<tr>
<td>150</td>
<td>3.55</td>
<td>3.7</td>
<td>0.15</td>
<td>3</td>
<td>3.70</td>
</tr>
<tr>
<td>200</td>
<td>3.45</td>
<td>3.55</td>
<td>0.1</td>
<td>2</td>
<td>2.47</td>
</tr>
<tr>
<td>300</td>
<td>4.2</td>
<td>4.35</td>
<td>0.15</td>
<td>3</td>
<td>3.70</td>
</tr>
<tr>
<td>350</td>
<td>4.6</td>
<td>4.9</td>
<td>0.3</td>
<td>6</td>
<td>7.41</td>
</tr>
<tr>
<td>400</td>
<td>3.75</td>
<td>4</td>
<td>0.25</td>
<td>5</td>
<td>6.17</td>
</tr>
<tr>
<td>450</td>
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<td>3.65</td>
<td>0.05</td>
<td>1</td>
<td>1.23</td>
</tr>
<tr>
<td>500</td>
<td>3.5</td>
<td>3.35</td>
<td>0.15</td>
<td>3</td>
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</tr>
<tr>
<td>550</td>
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<td>2.47</td>
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<tr>
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<td>3.7</td>
<td>0.05</td>
<td>1</td>
<td>1.23</td>
</tr>
<tr>
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<td>0.2</td>
<td>4</td>
<td>4.94</td>
</tr>
<tr>
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<td>3.9</td>
<td>0.15</td>
<td>3</td>
<td>3.70</td>
</tr>
<tr>
<td>800</td>
<td>3.7</td>
<td>4.05</td>
<td>0.35</td>
<td>7</td>
<td>8.64</td>
</tr>
</tbody>
</table>
The model fit the 532 nm original data chips well, with the greatest error being 0.35 mm, or 7 pixels. With the 1064 nm the model did not fit as well, with the largest error being 0.65 mm, or 13 pixels.

### 4.2 Abel Transform Results

The Abel Transform produces estimates for two plasma properties, the electron density, and the atomic density. An example of the atomic density and electron density for the 150 ns time frame are below in Fig. 4.2. The rest of the time frames can be seen in Appendix A.
For the 150 ns time frame, the electron density is on the order of $10^{17} \text{ electrons cm}^{-3}$ and the atomic density is on the order of $10^{19} \text{ ions cm}^{-3}$. This shows approximately a 1% ionization rate. The highest atomic density is reported in the location of the ablated wires, while the highest electrons density is reported in the center between the two ablated wires for this time frame. The electrons appear to be far more diffuse about the plasma region than the atoms. Below is the maximum densities found for each time frame in Fig. 4.3.
The maximum atomic density and maximum electron density both appear to be random. While expected to naturally decay over time due to recombination, that does not appear to be happening in the time frame studied for these colliding plasmas.
Conclusion

5.1 Summary

Our goal was to build upon previous work of estimated electron and atomic densities using dual wavelength interferometry. We have developed a 2D model for the interference data. When compared to the original images, the model of the 532-nm data had a maximum fringe alignment error of 9% error for the 532-nm and 16% maximum for 1064-nm. There could be multiple reasons for this error. First, when we set the pixel size, we use interpolation to estimate values at the specific pixel, which could introduce some error. Also, there may be some error in the initial phase unwrapping algorithm. Next, we produced atomic density and electron density estimates in two dimensions. The colliding plasmas produced maximum density values as expected according to previous work on single wire cases. The maximum atomic density was on the order of $10^{19}$ cm$^{-3}$, while the electron density was on the order of $10^{17}$ cm$^{-3}$. These density values produced a ionization rate of approximately 1%, which is also similar to the single wire cases of previous work. One phenomenon noticed was that in previous single wire work, the densities decayed over time. However, that appears not to be the case in the two-wire experiments according to the data produced.

When comparing the electron and atomic density data over time in Appendix A, there is a noticeable trend with the data. Both the electron density and atomic density maximums appear to be concentrated near the wire axes in the very early time frame. Though the electron density is much more diffuse while the atomic density is more concentrated about the
wire axes. As time progresses, the electron density is the first to migrate towards the center of the plasma region while the atomic density begins to become diffuse. This is primarily caused by the differing velocities between electrons and ions. When the electrons begin to accumulate near the center of the plasma region collisions are occurring and changing the velocities of those electrons. In the later time frames, the atomic density also accumulates near the center with the electron density.

If we evaluate the fringe order with the above observations about the electron and atom movements, a trend develops in the fringe order. Early in the sequence when the atoms and electrons are both gathered about the wire axes there is a positive numerical fringe order. During this time the atomic density is dominant across the plasma region, because the atoms are two orders of magnitude denser than that of the electrons. Starting at approximately the 200-ns timeframe, the electrons start to accumulate near the center of the plasma region. Here it is observed a positive fringe order near the wire axes, and a negative fringe order in the center of the plasma. Where there is a positive fringe order, the atomic density is dominant. However, by observing the central region of the atomic and electron density profiles, in the center there is negligible amounts of atoms and a concentration of electrons. The negative fringe order is associated with the dominance of the electron density in the region. This continues until the 700-ns time frame when both the atomic density and electron density is maximum in the center of the plasma region. Once again, a positive fringe order is observed as the atomic density is again dominant in the region.

5.2 Recommended Future Work

For the future, finding a mathematical method of analysis that does not require a specific symmetry should be paramount. While the Abel Transform worked for this specific instance, it required partitioning to safely assume the cylindrical symmetry requirement. This occasionally had some disconnects when fitting the density estimates output from the
Abel transform together and required a smoothing filter pass. If there was a method that had no symmetry restrictions and could be passed the entire data set in a single calculation would produce better results.

When observing the colliding plasma through the different time frames, the plasmas first are independent without interaction, then expand into the central region where the collisions occur. Over time, the atoms and electrons accumulate in the center of the plasma region. The fringe order that occurs at this point, post 700 ns appears similar to the later stages of a single wire case. Since the maximum densities did not appear to exponentially decay during the observed time frames like the single wire case, it may be of interest to observe later time frames from 800 ns to approximately 1200 ns to observe the behavior of the colliding plasma. It may be a case where the colliding plasma begins to behavior similarly to that of a single wire case at this point.

We produced results for observations in the y-z plane for ablated wires. With a working 2D model, the wires can be observed from different angles and meshed to produce a 3-dimensional analysis of colliding plasma over time. This would give a more complete picture of electron and ion movements in 3-dimensional space which could prove pivotal in understanding the flow of electrons. This insight could be useful in finding or creating ways to communicate through a plasma, such as entering and leaving an atmosphere.
Bibliography


Appendix A

Data for All Timeframes

In the following pages, a group of sub-figures are presented for each time frame. These images are arranged as follows: (a) chip of the unprocessed 1064-nm image, (b) model of the filtered and leveled 1064-nm chip, (c) chip of the unprocessed 532-nm image, (d) model of the filtered and leveled 532-nm chip, (e) atomic density estimate, (f) electron density estimate.
Figure A.1: 50 ns chips, models, and densities.
(a) 1064-nm original,
(b) 1064-nm model,

(c) 532-nm original,
(d) 532-nm model,

(e) atomic density,
(f) electron density,

Figure A.2: 100 ns chips, models, and densities.
Figure A.3: 150 ns chips, models, and densities.
Figure A.4: 200 ns chips, models, and densities.
Figure A.5: 300 ns chips, models, and densities.
Figure A.6: 350 ns chips, models, and densities.
Figure A.7: 400 ns chips, models, and densities.
Figure A.8: 450 ns chips, models, and densities.
Figure A.9: 500 ns chips, models, and densities.
Figure A.10: 550 ns chips, models, and densities.
(a) 1064-nm original,

(b) 1064-nm model,

(c) 532-nm original,

(d) 532-nm model,

(e) atomic density,

(f) electron density,

Figure A.11: 600 ns chips, models, and densities.
Figure A.12: 700 ns chips, models, and densities.
Figure A.13: 750 ns chips, models, and densities.
Figure A.14: 800 ns chips, models, and densities.