2023

Effects of Elastic Anisotropy on Residual Stress Measurements Performed Using the Hole-Drilling Technique

Joshua T. Ward
Wright State University

Follow this and additional works at: https://corescholar.libraries.wright.edu/etd_all

Part of the Mechanical Engineering Commons

Repository Citation
https://corescholar.libraries.wright.edu/etd_all/2798

This Thesis is brought to you for free and open access by the Theses and Dissertations at CORE Scholar. It has been accepted for inclusion in Browse all Theses and Dissertations by an authorized administrator of CORE Scholar. For more information, please contact library-corescholar@wright.edu.
EFFECTS OF ELASTIC ANISOTROPY ON RESIDUAL STRESS MEASUREMENTS PERFORMED USING THE HOLE-DRILLING TECHNIQUE

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

By

JOSHUA T. WARD
B.S.M.E., Wright State University, 2022

2023
Wright State University
WRIGHT STATE UNIVERSITY
COLLEGE OF GRADUATE PROGRAMS AND HONORS STUDIES
April 26, 2023

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION
BY Joshua T. Ward ENTITLED Effects of Elastic Anisotropy on Residual Stress
Measurements Performed Using the Hole-Drilling Technique BE ACCEPTED IN PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in
Mechanical Engineering.

________________________
Harok Bae, Ph.D.
Thesis Director

_____________________________
Raghavan Srinivasan, Ph.D., P.E.
Chair, Mechanical and Materials
Engineering Department

Committee on Final Examination:

____________________________
Harok Bae, Ph.D.

____________________________
Mark Obstalecki, Ph.D.

____________________________
Mitch Wolff, Ph.D.

____________________________
Shu Schiller, Ph.D.
Interim Dean
College of Graduate Programs & Honor Studies

DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited
ABSTRACT

Ward, Joshua T. M.S.M.E., Department of Mechanical and Materials Engineering, Wright State University, 2023. Effects of Elastic Anisotropy on Residual Stress Measurements Performed Using the Hole-Drilling Technique

In the present work, the variation in through-thickness residual stress profiles driven by elastic anisotropy is investigated using the incremental hole-drilling method. The standardized hole-drilling technique allows for the calculation of in-plane stresses based on measured surface strains, however, these calculations assume elastic isotropy. The assumption of elastic isotropy allows for the material constants to be reduced down to two values, however, this assumption is invalid for many materials used in aerospace design. These materials are often times elastically anisotropic, which leads to inaccuracy and uncertainty in measured stress profiles. An interference fit ring and plug sample was designed, using a finite element model, to impart a predictable stress profile for C260 brass with varying levels of crystallographic texture. Utilizing the finite element model, simulated and experimental hole-drilling measurements can be compared. The correlation study between the virtual and experimental stress profiles aids in quantifying the magnitude of errors caused by assumption of elastic isotropy. Understanding the potential sources of the errors will allow us to develop new modeling and experimental approaches that can incorporate elastic anisotropy into residual stress hole-drilling measurements.
# Table of Contents

## Introduction

1.1 Research Objective ......................................................................................... 1  
1.2 Thesis Overview ............................................................................................... 1  

## Background

2.1 Residual Stress Hole-Drilling Method ................................................................. 3  
2.2 Previous Work ................................................................................................... 5  

## Methodology

3.1 Ring and Plug Samples ....................................................................................... 10  
3.1.1 Ring and Plug Manufacturing and Assembly Process .................................... 11  
3.2 Computational Analysis ..................................................................................... 13  
3.2.1 Simulation Framework .................................................................................. 14  
3.3 Experimental Analysis ....................................................................................... 19  
3.3.1 Material Property Measurements .................................................................. 20  
3.3.2 Residual Stress Measurements ..................................................................... 23  
3.3.3 Heat Treatment Trials ................................................................................... 26  

## Simulation Framework

4.1 Isotropic vs. Anisotropic Samples ................................................................. 29  
4.1.1 Orientation Optimization ........................................................................... 30  
4.1.2 Orientation Optimization Results ............................................................... 32  
4.2 Sample Design ................................................................................................. 33  

## Anisotropic Material Constant Analysis

5.1 3-Point Bending Test ....................................................................................... 36  
5.1.1 Beam Design ................................................................................................ 38  
5.1.2 3-Point Bending Results ............................................................................. 41  
5.2 Through Transmission Ultrasound .................................................................. 42  
5.2.1 Through Transmission Ultrasound Theory ............................................... 43  
5.2.2 Through Transmission Ultrasound Results ................................................. 45  
5.3 Resonant Ultrasound Spectroscopy ................................................................. 46  
5.3.1 Dimensional Sensitivity Study ................................................................. 49  
5.3.2 Modal Matching Optimization ..................................................................... 57  
5.3.3 RUS Results ............................................................................................... 62
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Measurements</td>
<td>64</td>
</tr>
<tr>
<td>6.1 Simulated Experimental Samples</td>
<td>64</td>
</tr>
<tr>
<td>6.2 Results</td>
<td>66</td>
</tr>
<tr>
<td>Conclusions</td>
<td>76</td>
</tr>
<tr>
<td>7.1 Summary</td>
<td>76</td>
</tr>
<tr>
<td>7.2 Future Work</td>
<td>77</td>
</tr>
<tr>
<td>Bibliography</td>
<td>79</td>
</tr>
</tbody>
</table>
## List of Figures

2.1 Strain Gage Rosette for Residual Stress Hole-Drilling Measurements ............... 4  
2.2 Residual Stress Redistribution Caused by Hole-Drilling .......................... 4  
2.3 Calibration Coefficients Representation .................................................. 6  
2.4 Uniform vs. Non-Uniform Residual Stress Profiles ................................... 7  
2.5 Different Types of Strain Gage Rosettes ................................................ 9  

3.1 Ring and Plug Notations ............................................................................. 10  
3.2 Ring and Plug Sample with Digital Strain Gage ......................................... 14  
3.3 Modeled Ring and Plug Sample with Mesh Plot ........................................ 15  
3.4 Radial and Hoop Stress Distribution for Ring and Plug Sample ..................... 16  
3.5 Anisotropic Radial and Hoop Stress Distribution for Ring and Plug ................. 19  
3.6 Inverse Pole Figure of Plane Density after Cold-Rolling Reductions ............... 22  
3.7 DART machine from Hill Engineering ...................................................... 25  
3.8 Phase Diagram Showing Temperature Ranges of Brass Treatments ............... 26  
3.9 Stress vs. Depth for non-Heat Treated C260 Brass Samples ........................... 28  
3.10 Stress vs. Depth for Heat Treated C260 Brass Samples ............................... 28  

4.1 Definition of Θ Used in Orientation Simulation Study ..................................... 30  
4.2 Hoop Stress Profiles for Plugs with Varying Angles Θ and Isotropic Ring .................. 31  
4.3 Hoop and Radial Stress Profiles for Various Orientation Angles .................. 32  
4.4 Hoop and Radial Stress Profiles of Varying Geometric Designs ..................... 35  

5.1 Small Scale 3-Point Bending Test Apparatus ............................................. 37  
5.2 Strain Gage for Bending Test and its Application on a Sample ....................... 37  
5.3 Beam Design and Cutting Layout for 3-Point Bending Test .......................... 39  
5.4 Strain vs. Force for a Repeatability Experiment of Bending Apparatus ............ 40  
5.5 Stress vs. Strain for Varying Reduction Rates of C20 Brass .......................... 41  
5.6 Wave Propagation and Particle Direction for Longitudinal and Shear Waves ...... 43  
5.7 Experimental Setup of Through Transmission Ultrasound Measurements .......... 44  
5.8 Traditional RUS Experimental Setup ...................................................... 47  
5.9 LASER-RUS Experimental Setup ............................................................. 49  
5.10 RUS Sample with a 4 x 8 x 1 Geometry .................................................. 50  
5.11 Sensitivity of Resonances to the Variation of $E_x$ and $E_z$ .......................... 51  
5.12 Parameter Sensitivity for Brass Constants with Geometric Variation ............. 54  
5.13 Weighted Sensitivity for Brass and Steel Property Geometric Variation ......... 56  
5.14 Bending and Torsional Modes of Parallelepiped ....................................... 58  
5.15 Modal Changes Based on Property Variation of Simulated Resonances ........... 58  
5.16 Normalization of Simulated Modes for Image Comparison of Optimization ...... 60  
5.17 Workflow of Developed RUS Optimization Inversion Algorithm ................... 62  

DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Radial and Hoop Stress Distributions for 0% Reduced Brass Ring and Plug</td>
<td>65</td>
</tr>
<tr>
<td>6.2</td>
<td>Radial and Hoop Stress Distributions for 75% Reduced Brass Ring and Plug</td>
<td>65</td>
</tr>
<tr>
<td>6.3</td>
<td>Plot of Stress Values from Simulations for 0% and 75% Reduced Samples</td>
<td>66</td>
</tr>
<tr>
<td>6.4</td>
<td>Through-Thickness Stress Profiles from 0% Reduced Brass of Standard Hole</td>
<td>67</td>
</tr>
<tr>
<td>6.5</td>
<td>Through-Thickness Stress Profiles from 0% Reduced Brass of Large Hole</td>
<td>67</td>
</tr>
<tr>
<td>6.6</td>
<td>Biaxial Gage Layout for Relived Strain Measurements from Ring Removal</td>
<td>68</td>
</tr>
<tr>
<td>6.7</td>
<td>Superposition of Assembly Induced Stress for 0% Reduced for Standard Hole</td>
<td>70</td>
</tr>
<tr>
<td>6.8</td>
<td>Superposition of Assembly Induced Stress for 0% Reduced for Large Hole</td>
<td>70</td>
</tr>
<tr>
<td>6.9</td>
<td>Strain Profiles of Superimposed Experimental vs. Simulated Samples</td>
<td>72</td>
</tr>
<tr>
<td>6.10</td>
<td>Stress as a Function of Depth for Two Spatial Locations of Plug</td>
<td>75</td>
</tr>
</tbody>
</table>
# List of Tables

3.1 Orthotropic Material Constants for Ring and Plug Simulations .................. 18
4.1 Amplitude of Radial and Hoop Stress for Various Ring/Plug Designs ............... 32
4.2 Material Constants used in Dimensional Ring and Plug Study .................... 34
5.1 In-Plane Elastic Moduli Computed from 3-Point Bending Tests .................... 42
5.2 Material Constants Measured Using Through Transmission Ultrasound ............ 46
5.3 Geometric Ratios Used in Sensitivity Study ........................................ 52
5.4 Material Constants of Steel and Brass used in Sensitivity Study .................. 53
5.5 Formatting of Residuals used in Sensitivity Study .................................. 53
5.6 Weighting of Constants for Overall Sensitivity Metric ............................ 55
5.7 Orthotropic Elastic Constants Calculated from RUS Optimization ................. 63
6.1 Relieved Strains and Corresponding Stresses from Ring Removal .................. 68
6.2 Strain and Percent Difference for Experimental vs Simulated Hole-Drilling ...... 72
6.3 Material Constants Used in Hole-Drilling Analysis ................................... 73
6.4 Percent Error between Experimental and Simulated Stress Profiles ............... 74
Nomenclature

A  Calibration Constant
B  Calibration Constant
b  Width of Sample
C  Calibration Constant
c  Wave Velocity
D  Grid Center Line Diameter
D_o  Hole Diameter
E  Elastic Modulus
\(\varepsilon\)  Frequency Difference
\(\bar{\varepsilon}\)  Average Frequency Difference
F  Force
f  Frequency
G  Shear Moduli
h  Height of Sample
I  Isotropic Simulation Frequency
L  Length between Supports
P  Pressure
\(\rho\)  Density
R  Nominal Radius
R_i  Assembly Inner Radius
R_o  Assembly Inner Radius
r  Member Radius
\(\sigma\)  Stress
S  Sensitivity
\(\Theta\)  Angle between X-axis
v  Poisson’s Ratio
V  Property Variation Frequency

Subscripts

e  Experimental
h  Hoop Direction
i  Inner Value
l  Longitudinal
m  Modeled
o  Outer Value
RD  Rolling Direction
r  Radial Direction
s  Shear
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TD</td>
<td>Transverse Direction</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X-Direction</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Y-Direction</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Z-Direction</td>
<td></td>
</tr>
</tbody>
</table>
Acknowledgement

I would like to thank the many people who helped to bring this work to life, especially Dr. Mark Obstalecki, my AFRL sponsor. His support has been immeasurable and his guidance, invaluable. I would also like to thank Mr. Bryce Ullman, who has aided in this work. I would like to thank Dr. Harok Bae for his constant reassurance and guidance throughout this work. I would also like to thank Dr. Mitch Wolff for his help and support. Finally I would like to thank the many people at AFRL that provided their expertise on the manufacturing side of this project.
Dedicated to
My fiancée, my mother, my father, my brother, and my sister
Introduction

1.1 Research Objective

Residual stresses are self-equilibrating stresses that remain in a piece of material after the removal of external boundary conditions. Accounting for residual stress during the design process is critical to understanding a component’s performance. This is because residual stresses can be beneficial, such as residual stresses induced by shot peening, which can improve fatigue life through surface compressive stresses, and thus highlighting the importance of accurate measurement techniques [1]. The research captured in this thesis focuses on characterizing the effects of elastic anisotropy on residual stress profiles measured using the hole-drilling technique.

1.2 Thesis Overview

This study is an effort to understand the impact of elastic anisotropy on residual stress measurements. The current standard for residual stress calculations using the hole-drilling method is based on the assumptions of elastic isotropy. Emphasis is placed on developing new elastic material constant measurement tools using non-destructive evaluation techniques and implementing the identified characteristics into a material performance prediction model. Furthermore, the presented work focuses on employing the new material characterization tools in
order to generate finite element models for experimental and simulated strain profile comparisons.

Chapter 2 introduces the hole-drilling method and gives background on its development. Chapter 3 describes the finite element modeling and simulations using SolidWorks for computational analysis of hole-drilling residual stresses, the experimental analysis techniques used for measuring residual stresses, material characterization, and the heat treatment trials for stress relief proposed in this work. The simulation framework and its use in sample design will be discussed in Chapter 4. The methods for material characterization such as 3-point bending, through transmission ultrasound, and resonant ultrasound spectroscopy will be discussed in Chapter 5. Chapter 6 describes the experimental measurements process. This thesis concludes with a summary and a discussion highlighting opportunities to extend the work.
Background

2.1 Residual Stress Hole-Drilling Method

The residual stress hole-drilling method is a semi-destructive technique to evaluate the residual stress profile near the surface of a sample, which has been standardized for linear-elastic isotropic materials by the American Society for Testing and Materials [2]. The method was first investigated by Rendler and Vigness in 1966 where they developed a repeatable process for measuring residual stress by drilling holes into a sample [3]. This method begins by applying a strain gage rosette to the surface of a sample as seen in Figure 2.1. A hole, between 0.04 and 0.16 inches in diameter, is incrementally drilled into the geometric center of the gage. As the material is removed the relieved surface strains surrounding the hole are measured through the stain gages as seen in Figure 2.2. These incremental strains can be used to calculate a through thickness in-plane stress profile using a series of equations that have been standardized in ASTM E837 [2].
Figure 2.1: EA-05-125RE-120 strain gage. The first term (EA) refers to the gage layout (0° - 90° - 225°). The second term is the Self Temperature Compensation (STC) value and is material specific. The first part of the third term (125) is the grid length of the gage in mm. The second part of the third term refers to the shape of the gage tab layout. The last term is the nominal resistance in Ohms.

Figure 2.2: Material removal causes a redistribution of internal stresses to equilibrate for the removed sample. This redistribution of stress can be measured in strain on the surface, through the use of a strain gage rosette. [2]
The residual stresses measured are assumed to be uniform across the diameter of the hole. Two methods were defined in the ASTM standard, the blind-hole analysis and the through-hole analysis. In the blind-hole analysis, the incremental material removal was typically done at steps that are 1/40 of the diameter of the hole, as these were the depths that the calibration coefficients were tabulated in ASTM E837-13a [2]. However, in recent updates with ASTM E837-20, holes can now be drilled at any step, as the calibration coefficients are now calculated using a 15-coefficient polynomial acquired through finite element analysis [4]. The relieved strains are then measured through the strain gage applied onto the surface, through the three channels as seen in Figure 2.1. Relieved strains refer to the set of three values obtained from the three independent strain gages that make up the rosette. The sensitivity to relieved strains at the surface decreases as the hole depth increases, which increases measurement uncertainty at larger depths. Using the through-hole analysis the strains are measured once before and once after the hole is drilled, giving only an average of the through thickness stress profile. For this research the blind-hole approach was used.

2.2 Previous Work

The original hole-drilling method as used by Rendler and Vigness was an empirical relationship between relieved strains and principal stresses. Their model was limited however to through thickness or full depth holes. Intermediate steps could not be accounted for in the model, where only through-thickness stress profiles could be calculated. They were able to calculate in-plane principal stresses and the principal angle, with a set of equations derived using elastic theory. These equations are reduced into matrix form resembling Hooke’s law, assuming elastic isotropy, and can be seen in Equation 1.
Here $A$, $B$, and $C$ are calibration constants that are dependent on the geometry of the strain gage used, the elastic constants of the material, the radius of the hole, and the depth of the hole.

As the capabilities of Finite Element Methods (FEM) increased, more progress was made in advancing the technique through the computation of through thickness calibration coefficients [6]. The computation of calibration coefficients allowed for residual stresses to be evaluated at incremental steps as a function of depth of the hole, whereas prior it was only an average across the whole thickness of the sample [2]. The calibration coefficients are formatted as a lower triangular matrix that shows relative impact of the measured relieved strains at various incremental steps to the overall stress state and can be visualized as shown in Figure 2.3. The calibration coefficient method was developed as the integral method by Schajer and has been adapted in the ASTM today [5]. A set of non-dimensional calibration coefficients was defined to be independent of hole size, and elastic constants, and could be used for uniform and non-
uniform stress states, shown in Figure 2.4. This method states that the measured strains are impacted by each prior material removal step, which can be modeled as an integral of the strain over the depth of the hole.

![Figure 2.4: Geometry of the hole and residual stresses for (a) Uniform Stresses and (b) Non-uniform Stresses [2]](image)

Schajer went on to develop a method for measuring residual stresses in orthotropic materials through the use of non-dimensional compliance matrices [7]. Here the stress is related to the strains through a combination of a calibration matrix and a compliance matrix. Hooke’s law for an orthotropic material is used to calculate stress from measured strains shown in Equation 2.

\[
\begin{align*}
\varepsilon_x &= \frac{\sigma_x}{E_x} - \frac{\sigma_y v_{yx}}{E_y} \\
\varepsilon_y &= \frac{\sigma_y}{E_y} - \frac{\sigma_x v_{xy}}{E_x} \\
\gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}}
\end{align*}
\]
. An adaptation of the standardized approach for calculating residual stresses from relieved strains was made accounting for orthotropic symmetry, rewritten in a similar matrix form as Equation 1, and shown in Equation 3.

\[
\frac{1}{\sqrt{E_x E_y}} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \tau_{xy} \\ \sigma_y \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}
\] (3)

This combination matrix, c, composed of \( c_{nm} \) is dependent on orthotropic material constants, hole diameter, and the geometry of the strain gage rosette, as the displacement field is solved for numerically, and integrated over the gage area. This method works under the assumption of a uniform through-thickness stress distribution. It can be shown that the combination matrices for the blind-hole analysis converge at large hole depths to the results of a through-hole plane stress solution which is defined as the limiting state of the measurement. This shows that you must drill to a depth large enough to reach the limiting state, which is dependent on the ratio of the out-of-plane shear moduli to the in-plane directional moduli. The computed stresses become less accurate at depths lower than that of the limiting state, highlighting the need for further investigation into the calculations of residual stresses in non-isotropic materials.

In the early 2000’s the residual stress hole-drilling method was standardized in ASTM E837 [2]. This standard defined the hole-drilling method into sets of equations based on stress profile assumptions (uniform vs. non-uniform), and included 20 increment calibration matrices for the three different types of strain gage rosettes shown in Figure 2.5. Since the standard’s initial publication it has been updated in 2013 and most recently in 2020, with changes to the calibration coefficients [2, 4]. Originally a lower triangular matrix of values that were calculated through finite element simulations, which was updated to a 15 coefficient polynomial. These polynomials were derived for different ratios of the diameter of the hole to the diameter of the
gage centers, and account for variation in experimental conditions while allowing for more control over depth profiles as opposed to the defined incremental steps required [4]. Other work has been conducted in regards to the calibration coefficients and how they are influenced by various parameters such as the shape of the end-mill, and drilling parameters [8, 9].

In 2020 Olson, Dewald, and Hill developed a method for quantifying uncertainty in residual stress measurements using the hole-drilling technique [10]. The uncertainty metric captures two sources, the first being the strain uncertainty, regarding the precision of the gages, and the second being the choice of regularization parameters α. The number of hole depth steps causes the calibration matrices to become numerically ill-conditioned, and because of this small errors in the measured strains can cause proportionally larger errors in the stress calculations. To mitigate these errors a smoothing process is implemented using Tikhonov regularization [11-13]. The regularization parameter, α, controls the degree of regularization, and has a direct effect on smoothing the stress results. Repeatability experiments were conducted and used to validate the uncertainty analysis, which has been implemented into the workflow on the DART system used for hole-drilling measurements which will be discussed in Section 3.3.2.

Figure 2.5: Types of hole-drilling strain gage rosettes

DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited
Methodology

3.1 Ring and Plug Samples

Ring and plug samples are a type of interference fit assembly that are comprised of the inner member (plug) and outer member (ring) shown in Figure 3.1. A shrink fit assembly refers to a member being assembled into another member with a slightly smaller cavity, resulting in a stress distribution. This means that the plug and ring have a radial interference, notated δ in Figure 3.1. Analytical solutions to calculate stress distributions are available for ring and plug samples assuming the material is elastically isotropic and therefor the only material constants needed are E and v. Using these analytical solutions we can validate a set of finite element simulations that we can then extend to include elastically anisotropic material constants. The solutions of isotropic materials also assume that the lengths, the thickness out of plane in Figure 3.1b, of the inner and outer member are the same.

![Figure 3.1: Notation for Ring and Plug samples. (a) Unassembled parts; (b) after assembly](image)

The resulting residual stress distribution in a ring and plug sample is the result of both the stress induced due to assembly, as well as any existing residual stresses due to material processing history. Ring and plug samples were chosen because the developed stress
distributions, assuming elastically isotropic materials and equal lengths, make it easy to observe any variations caused by elastic anisotropy. For ring and plug samples the hoop and radial stress, \( \sigma_h \) and \( \sigma_r \) respectively, at any radii can be calculated using Equations 4-5 respectively, where the pressure, \( P \), can be calculated using Equation 6 [14].

\[
\sigma_h = \frac{P_l r_i^2 - P_o r_o^2 - r_i^2 r_o^2 P_o - P_l}{r_o^2 - r_i^2} \tag{4}
\]

\[
\sigma_r = \frac{P_l r_i^2 - P_o r_o^2 + r_i^2 r_o^2 P_o - P_l}{r_o^2 - r_i^2} \tag{5}
\]

\[
P = \frac{\delta}{R \left[ \frac{1}{E_o} \left( \frac{R_o^2 + R^2}{R_o^2 - R^2} + \nu_o \right) + \frac{1}{E_i} \left( \frac{R_i^2 + R^2}{R_i^2 - R^2} + \nu_i \right) \right]} \tag{6}
\]

The equations above can be reduced to the form below assuming a plug with no hole which will cause \( R_i \) and \( P_i \) to be equal, resulting in Equation 7.

\[
\sigma_h = \sigma_r = -P \tag{7}
\]

Equation 7 predicts a spatially uniform, equal biaxial stress state with a magnitude inversely proportional to the pressure within the plug. The equations assume elastic isotropy; any deviation in elastic constants from isotropy will result in spatial variation in stress distributions in the plug. The deviation from isotropy and its effect on stress distributions will be further discussed in section 3.2.1.

3.1.1 Ring and Plug Manufacturing and Assembly Process

To maximize the variation of stress distributions in the plug, caused by elastic anisotropy, materials were selected such that the plug would have the lower yield strength. This was to
ensure that the optimized stress distribution would not yield the outer member, thus compromising the assembly. Another factor that can influence the variation in the stress distribution is the orientation of the principal material axes, such as rolling direction (RD) or transverse direction (TD), in relation to the ring and plug. For example, if a ring and plug were both made of elastically anisotropic materials, the samples could be oriented such that the stiffest directions are aligned with one another, or potentially the stiffest direction of the plug is aligned with the most compliant direction of the ring. This relative orientation of material directions between the ring and plug is defined as the axis orientation of the samples, and will be further discussed in Section 4.1.1. The radial interference, $\delta$, was selected to induce 80% to the yield strength of the limiting material, which was to be used for manufacturing the plug, to ensure purely elastic deformation. More will be discussed about the ring and plug design in Section 4.2.

Samples are assembled by heating the ring and cooling the plug to cause thermal expansion and contraction respectively. The ring was brought to a temperature of 500°F in air. This temperature was chosen as the highest temperature the ring can be handled using heat resistant gloves while maintaining dexterity. The plug was submerged in liquid nitrogen, which will cool down to about -320°F. The expansions and contractions due to heating and cooling respectively will allow the two pieces to fit into one another with clearance. The plug was removed from the liquid nitrogen using tongs and placed on a ceramic plate, used to minimize thermal conductivity and extend assembly time. As the plug is placed on the ceramic plate, the ring is pulled from the oven and placed around the plug. Due to the clearance between the two pieces at the extreme temperature no force is required to assemble the samples. After the ring is placed around the plug, the ring can be rotated for a few seconds to align the desired axes of the
two components. The sample equilibrates back to room temperature within about two hours, inducing the residual stress distribution.

3.2 Computational Analysis

Finite Element Analysis (FEA) allows rapid iterations in sample design by varying material constants virtually, emulating physical assemblies. FEA of ring and plug samples was conducted to calculate expected stress distributions based on material constants, interference fits (δ), and ring and plug axis orientation. The iterative process optimized sample design under the imposed constraints, which maximized the spatial stress distributions in the plug.

SolidWorks simulations were used to perform digital hole-drilling tests. The simulations were conducted by creating a ring and plug model and placing a digital strain gage rosette using a split line feature on the surface of the sample. Material was then removed using the cut function and the changes in strains were recorded for each incremental removal. The simulations compared the elastically anisotropic ring and plug samples and to capture potential errors in the stress calculations by highlighting the difference between calculated and simulated through thickness stress profiles. A sample with a digital gage and a hole drilled into the center of the gage is shown in Figure 3.2.
Figure 2.2: Ring and plug sample with a digital gage (shown in the red circle) applied to the surface with hole drilled into sample

### 3.2.1 Simulation Framework

FEA allows for rapid iteration through different design parameters of interest for ring and plug samples. Material constants, interference, and sample dimensions all affect the stress distributions of ring and plug samples. These parameters were input into the sample design workflow for rapid variation to maximize stress variation in the anisotropic samples, and maximize stress magnitude in the isotropic samples, without yielding. An example ring and plug simulated model shown in Figure 3.3.
With the samples modeled, stresses and strains were to be predicted using digital strain gages. Each piece was assigned isotropic material constants equivalent to a 304 stainless steel (E = 28000 ksi, $v = 0.29$) [15]. A shrink fit contact set was then applied to the outer surface of the plug and the inner surface of the ring. Running an FEA requires the definition of boundary conditions. For this model, two different boundary conditions were applied. The first boundary condition was a fixed rotation condition applied to both the ring and the plug. This boundary condition fixed the samples from rotating in relation to one another, and forced the desired sample axes to be aligned accordingly. The second boundary condition was a fixed base boundary condition. In typical FEA models there is an external force requiring the model to be fixed in all three dimensions at some point on the model. The model was fixed from movement in the thickness direction, which is normal to the diametric faces of the ring and plug.
The radial and hoop stress distributions for an isotropic ring and plug are shown in Figure 3.4. As discussed in Section 3.1, the stress field of the plug for an elastically isotropic material is uniform spatially and the hoop stress and radial stress are equal in magnitude. When a material starts to deviate from isotropy the stress fields are no longer spatially uniform nor are the magnitudes of the radial and hoop stresses equal. Through comparison of the finite element simulation results and the analytical solution obtained using Equations 1-3, we can gain confidence in the model. This was done by running multiple different ring and plug samples, and comparing the stress values from the analytical solutions, to that of the simulations. With the verification of the isotropic case, the simulation framework can be validated for the anisotropic case.

The FEA is predicting strain and stress fields throughout the discretized model made up of the elements shown in the mesh of Figure 3.3. Hooke’s Law states that strain is proportional to the applied stress within the elastic limit of a solid shown in Equation 8.

\[ \sigma_i = C_{ij} \varepsilon_j ; \quad (i, j = 1, 2, \ldots, 6) \]  

Figure 3.4: Radial (left) and hoop (right) stress distribution of ring and plug sample, where E = 28000 ksi, ν = 0.29, and the radial interference δ = 0.0025"
In this equation the stiffness matrix, \( C_{ij} \), has 81 different constants. However, due to symmetry of the stress and strain tensors this allows for a reduction to 36, and this is shown in Equation 9.

This can be further reduced by assuming the material is orthotropic. This means that the material has three orthogonal planes of symmetry, which allows the number of constants to be reduced from 36 to 9, and this is shown in Equation 10. The furthest reduction based on symmetry that can be made is to isotropy, which reduces the number of elastic constants in the \( C \) matrix down to two, and is shown Equation 11.

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{23} \\
\varepsilon_{13} \\
\varepsilon_{12}
\end{bmatrix}
\] (9)

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{23} \\
2\varepsilon_{13} \\
2\varepsilon_{12}
\end{bmatrix}
\] (10)

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{13} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{23} \\
2\varepsilon_{13} \\
2\varepsilon_{12}
\end{bmatrix}
\] (11)

where \( C_{44} = 0.5(C_{11} - C_{12}) \)

SolidWorks allows the user to input orthotropic material constants. Orthotropic materials have nine independent constants, which include Young's modulus or directional moduli for the three orthogonal axes \((E_x, E_y, E_z)\), a shear moduli for the three axis pairs \((G_{xy}, G_{yz}, G_{zx})\), and a
Poisson’s ratio for the three axis pairs ($v_{xy}$, $v_{yz}$, $v_{xz}$). A simulation was ran with the nine parameters matching an isotropic assumption except with $E_y = 2E_x$ shown in Table 3.1, for both the ring and plug to highlight the variation, and the results are shown in Figure 3.5. The hoop and radial stress now follow a sinusoidal pattern around the azimuth where the peaks and valleys align with the x-axis and y-axis. The magnitude of this sinusoid is determined by the ratio of directional moduli between the ring and the plug. This model was designed with the stiffer axes of the ring and the plug being aligned, meaning $E_{x,\text{Ring}} = E_{x,\text{Plug}}$, $E_{y,\text{Ring}} = E_{y,\text{Plug}}$, … etc.

Table 3.1: Orthotropic material constants for ring and plug simulation

<table>
<thead>
<tr>
<th>Property</th>
<th>Ring</th>
<th>Plug</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$ (ksi)</td>
<td>28000</td>
<td>28000</td>
</tr>
<tr>
<td>$E_y$ (ksi)</td>
<td>56000</td>
<td>56000</td>
</tr>
<tr>
<td>$E_z$ (ksi)</td>
<td>28000</td>
<td>28000</td>
</tr>
<tr>
<td>$G_{xy}$ (ksi)</td>
<td>11200</td>
<td>11200</td>
</tr>
<tr>
<td>$G_{yz}$ (ksi)</td>
<td>11200</td>
<td>11200</td>
</tr>
<tr>
<td>$G_{xz}$ (ksi)</td>
<td>11200</td>
<td>11200</td>
</tr>
<tr>
<td>$v_{xy}$</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$v_{yz}$</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>$v_{xz}$</td>
<td>0.29</td>
<td>0.29</td>
</tr>
</tbody>
</table>

With the development of this workflow further studies can be conducted for sample design. The rapid simulation design will allow for interferences to be tested, axis orientations (will be discussed further in Section 4.1.1) and material changes. This means different material pairings between ring and plug, dimensions, and other factors can be modeled to calculate predicted stress distributions that will maximize the spatial variation of stress based on material constants. This will be used to optimize the sample to maximize stress variation and allow for expected stress and strain values to be compared to experimental measurements.
Experimental Analysis

Ring and plug sample design optimization is driven by material constants, the residual stress profiles prior to assembly, and the interference. In order to accurately predict the stress distributions in anisotropic ring and plug assemblies using the finite element framework, accurate material constants need to be determined. Textbook values for material constants are generally limited to elastically isotropic materials, thus elastic moduli measurement techniques were explored. Three different techniques were used to quantify material constants, 3-point bending tests, through transmission ultrasound measurements, and Resonant Ultrasound Spectroscopy (RUS). 3-point bending was beneficial for measuring in-plane directional moduli ($E_x, E_y, E_z$) but is unable to be used to measure the other constants. Through transmission ultrasound is able to be used to measure the shear moduli ($G_{xy}, G_{yz}, G_{xz}$), but assumptions about isotropy must be made to calculate the remainder of the constants. RUS is able to calculate the full anisotropic stiffness matrix, but requires an initial guess at the material constants for accurate measurement. Due to these constraints and limitations, RUS was selected as the primary moduli measurement technique.
measurement technique, while 3-point bending and through transmission ultrasound were used as generate initial guesses for RUS. Another advantage of RUS is the fact that all elastic constants can be measure simultaneously from one sample from a small, rectangular parallelepiped, whereas mechanical test methods typically require multiple samples that are relatively large. These methods will be further discussed in Chapter 5.

Measuring existing residual stresses in the stock material is the next part of the design process. Samples have residual stresses induced during most manufacturing processes that contribute to the post-assembly stress profiles. The induced stresses on the ring and plug prior to assembly had to be characterized, as assuming a zero stress state pre-assembly would not be valid. The resulting stress distribution of the assembled piece would deviate from the simulated model designed without residual stresses. The stresses can be measured before and after assembly in order to quantify the independent assembly induced residual stresses. Quantifying the pre-existing residual stresses induced by the manufacturing history, as well as the post-assembly residual stresses can allow for the principle of superposition to be used to find the assembly induced stress alone. In order to minimize the preassembly stress samples can be stress relieved through heat treatment trials.

3.3.1 Material Property Measurements

Many materials exhibit some level of elastic anisotropy resulting from their processing history. Some metal production processes such as extruding or rolling can induce a crystallographic texture driven by plastic flow. Texture can drive anisotropy at the macroscopic scale if the material is anisotropic at the crystal scale. The anisotropic ratio, $A$, of a single crystal can be calculated for cubic crystals, using Equation 12.
\[ A = \frac{2C_{44}}{C_{11} - C_{12}} \]  

(12)

The anisotropic ratio for isotropic materials is unity. Many materials exhibit some level of anisotropy, but materials such as aluminum or tungsten have values very close to unity and are sometimes assumed to be isotropic. Other materials such as copper or nickel have values of \( A \) that are larger than 1, and therefor exhibit anisotropy at single crystal scale. When texture is induced a preferred crystal orientation is created, and when the material has single crystal anisotropy, the result is a material that is anisotropic at the continuum scale.

Manufacturing processes can be used to tune the texture; one example is rolling. Rolling a material can cause its crystals to reorient into a preferred crystal orientation. For example, a single crystal of C260 brass has an anisotropic ratio of 3.78. Due to this level of anisotropy at the single crystal scale, a textured sample should exhibit anisotropy at the continuum scale [16]. Through Electron Back Scatter Diffraction (EBSD) it can be seen that C260 brass with minimal prior processing appears isotropic because of the minimal texture. According to Duggan, C260 brass texture evolution during the rolling process begins at a 40% reduction, with the maximum texture developed at an 80% reduction, shown in Figure 3.6, as observed through optical, X-ray and electron metallography [17]. The reduction indicating the change in the dimension normal to the rollers with respect to the original dimension. For example, if a 1-inch beam was rolled down to a final thickness of 0.75-inches that would be a 25% reduction.
With C260 brass’s ease of machining and ability to tune its texture, this material was
selected for the design of plugs for ring and plug samples. The brass was rolled to three different
reductions, 25%, 50% and 75% thickness reduction, and characterized to influence sample
manufacturing choices. At each of the rolled states, the material constants were measured using
3-point bending, Through Transmission Ultrasound (TTU), and Resonant Ultrasound
Spectroscopy (RUS). The next step is calculating the residual stress profiles for all of the various
sets of material constants defined by the varying amounts of texture induced by the different
reductions.
3.3.2 Residual Stress Measurements

The processing history of a sample plays a large role in the development of residual stresses. Rolling causes plastic deformation to occur, and as a result residual stresses are induced. Also the machining of the material into the rings and plugs can induce residual stresses. For example, shot peening causes a compressive surface stress, rolling causes a tensile surface stress distribution in the direction of the rolling with a compressive stress towards the center of the sample in the reduced dimension. In order to accurately and efficiently measure residual stress, a system called the Device for Automated Residual stress Tests (DART) was used.

The DART is a system designed and procured from Hill Engineering (US Patent 10,900,768). The DART, shown in Figure 3.7 allows for complete hole drilling measurements to be made in under an hour. This system requires human interaction for the sample set-up and specimen surface zeroing procedure. To set the sample up the geometric center of the gage is located using an integrated optical camera. The zero depth for a given measurement is calibrated by incrementally drilling at the center of the gage until the surface has been 70% scratched by the end-mill. Once the sample is set up and the machining parameters (such as the end-mill speed and feed rate) are defined the system performs the hole drilling measurement autonomously. The system collects data from the three strain gages, and using the methodology outlined in the ASTM Standard (2), the in-plane residual stresses are calculated at the various incremental depths. To ensure accuracy in the residual stress calculations the ASTM 837-20 was coded in Matlab. By using the strain outputs of the measurements, the stresses are independently calculated for comparison. The data analysis code was reengineering using the ASTM standard...
to build confidence in the DART system, as well as allow for easier reprocessing, should the data require it.

A combination of standard hole size (0.08” diameter) and large hole (0.16” diameter) measurements were made. In order to only account for the residual stresses due to the assembly process, measurements were made in two stages. Hole drilling measurements were conducted on the assembled ring and plug sample. Biaxial gages were then applied to the sample in multiple locations, and the ring was then removed using a wire EDM. The radial and hoop strains are measured before and after the ring is removed, and using Hooke’s law, the radial and hoop stresses that were relieved can be predicted. Once the ring has been removed, another set of hole drilling measurements is completed on the plug to measure any residual stress profile that is present due to the processing history of the sample. Using the principle of superposition the stress due to the assembly can be calculated using the two sets of hole-drilling data. These data can then be compared to the stress calculated with the relieved strains after ring removal.
Figure 3.7: DART machine used for making residual stress measurements, courtesy of Hill Engineering
3.3.3 Heat Treatment Trials

Residual stresses are present in the samples from the processing history, and the received stress state is unknown and undesirable for sample design. The ring and plug assemblies are being designed to 80% of the yield stress of the limiting material, which is C260 brass. Any unknown residual stresses present could potentially cause the sample to yield during assembly. To relieve the unwanted residual stress the samples can be heat treated. A partial phase diagram highlighting the hot working, recrystallization, and stress-relieving temperature ranges for C260 brass is shown in Figure 3.8 [18]. C260 brass or Cartridge brass is 70% copper and 30% zinc. For stress relief the temperature ranges between 260-300°C, and the treatment duration is one hour per one inch of thickness.

![Copper-Zinc partial phase diagram showing the different temperature ranges of α, α+β, and β brasses](image)

Figure 3.8: Copper-Zinc partial phase diagram showing the different temperature ranges of α, α+β, and β brasses [18]
While utilizing this procedure can relieve some of the residual stresses, it will not relieve all residual stress. For that reason samples were made to test the stress relief heat treatment procedure to quantify the remaining residual stress after heat treatment. Samples were manufactured from the as received state of C260 brass, and were cut into one by one by half inch cubes. Six samples in total, three of which were heat treated at 280°C for 30, 60, and 90 minutes, and the other three being the control group. The as received sample stress profiles as a function of depth are shown in Figure 3.9 with the stress profiles after heat treatment shown in Figure 3.10.

In these samples the x-direction was aligned with the rolling direction of the sample, and the y-direction with the transverse direction. The surface stress in the x-direction was reduced by about 50% of the stress on average of the entire depth. The y-direction stress was reduced by roughly 40% on average. The shear stresses in all measurements were relatively constant where the magnitude of the shear stress remained below 1 ksi throughout the heat treatment study. The magnitude of relieved stress is comparable for the three different times. With the stress relief being comparable for all times, the 30 minute procedure was utilized for future heat treatments. This procedure will be utilized for all samples prior to assembly to ensure minimal residual stresses in the samples prior to assembly. With all of the experimental aspects of the present work discussed, the next step is to identify the computational tools that aided in these studies.
Figure 3.9: Cartesian stress as a function of depth for non-heat treated brass samples

Figure 3.10: Cartesian stress as a function of depth for heat treated brass samples
Simulation Framework

4.1 Isotropic vs. Anisotropic Samples

Ring and plug samples are being designed to investigate the difference between isotropic and anisotropic materials, and the stress distributions induced from the interference fit assembly process. The design variables of the ring and plug samples include the material constants of both the ring and the plug, the interference between the ring and plug, the geometry of the ring and plug, and the axis orientation between the ring and the plug. Samples are to be manufactured with plugs being made of the textured and un-textured C260 brass. These samples will have identical geometries, including the ring, to highlight the variation in stress and strain profiles due solely to elastic anisotropy. This means that two possible ring and plug sample pairings are possible. The sample could have either an isotropic ring, or an anisotropic ring, where the plug is manufactured from the various rolling conditions of brass. Sample design was optimized by comparing simulated stress magnitudes of sample designs with varying axis orientations, as well as the two different sample pairings.
4.1.1 Orientation Optimization

A series of simulations were conducted for the two different ring and plug sample pairings. The first set of simulations involved an isotropic ring, and an elastically anisotropic plug, while the other set is modeled with an elastically anisotropic ring, and an elastically anisotropic plug. It is important to note that the isotropic ring and anisotropic plug pair does not require multiple simulations as the stress distribution is not affected by plug orientation because the isotropic ring exhibits the same stiffness in every direction. The anisotropic ring and anisotropic plug had eight different simulations run where the angle, $\Theta$, between the stiffest axes was between $0^\circ$ and $150^\circ$ in $XX$ degree increments. Any further rotations would simply be a repeated data set, as the orthogonal axes would have the same stiffness at $180^\circ$ intervals apart. The angle, $\Theta$, is defined between the x-axis of the ring and the x-axis of the plug as shown in Figure 4.1. The directional moduli were varied between the axes so that $E_x = 16000$ ksi and $E_y = 11200$ ksi, and the in-plane Poisson’s ratio was kept constant at $v = 0.32$. When the ring was isotropic it was assigned a value of $E = 16000$ ksi and $v = 0.32$. These constants are selected to produce a $30\%$ difference in the directional moduli and do not represent a specific material.

Figure 4.1: Definition of $\Theta$ for orientation simulation study
Results show that the stress distributions for the hoop stress are no longer uniform spatially, as in Figure 3.5, when the plug was assumed to be elastically isotropic. Figure 4.2 shows the stress distributions for the varying angles of $\Theta$ for the elastically anisotropic ring and plug (a-h), as well as the elastically anisotropic plug and elastically isotropic ring (i). These distributions now follow a sinusoidal pattern as a function of the angle around the azimuth. The goal of the sample design process is to maximize the spatial variation of stress inside the plug, and to quantify the amplitude of the sinusoidal pattern of the stress distribution using the simulations. The spatial stress distributions for the radial stress, and the hoop stress were plotted as functions of the angle around the azimuth as shown in Figure 4.3. Upon inspection it can be noted that the amplitudes of the radial and hoop stress sinusoidal distributions exhibit less variation with respect to the axes orientation or the material selection between isotropic and anisotropic rings. The calculated amplitudes of these sinusoids are in Table 4.1.

![Figure 4.2: Hoop stress plug profile for varying angles of $\Theta$ and an isotropic ring](image)
The largest amplitude of the radial stress sinusoidal patterns occurred when the principal axes of the ring and plug were oriented 45° apart while the minimum amplitude was found when

Table 4.1 Amplitude of radial and hoop stress for various ring/plug designs

<table>
<thead>
<tr>
<th>Θ</th>
<th>σ_r (ksi)</th>
<th>σ_h (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.73</td>
<td>2.50</td>
</tr>
<tr>
<td>30</td>
<td>1.06</td>
<td>2.53</td>
</tr>
<tr>
<td>45</td>
<td>1.19</td>
<td>2.49</td>
</tr>
<tr>
<td>60</td>
<td>1.16</td>
<td>2.43</td>
</tr>
<tr>
<td>90</td>
<td>1.05</td>
<td>2.22</td>
</tr>
<tr>
<td>120</td>
<td>1.17</td>
<td>2.42</td>
</tr>
<tr>
<td>135</td>
<td>1.20</td>
<td>2.49</td>
</tr>
<tr>
<td>150</td>
<td>1.07</td>
<td>2.52</td>
</tr>
<tr>
<td>180</td>
<td>0.73</td>
<td>2.50</td>
</tr>
<tr>
<td>Isotropic</td>
<td>1.10</td>
<td>2.60</td>
</tr>
</tbody>
</table>

4.1.2 Orientation Optimization Results

The largest amplitude of the radial stress sinusoidal patterns occurred when the principal axes of the ring and plug were oriented 45° apart while the minimum amplitude was found when
the axes were aligned. However, looking at the hoop stress, the highest amplitude occurred when the ring was elastically isotropic. The magnitudes of all sinusoidal stress distributions of hoop stress were higher than that of the radial stress sinusoidal distributions. Therefore, the hoop stress will be used as a driving factor in designing the samples. This analysis led to the selection of an isotropic ring with an anisotropic plug. It is important to highlight that this design will require that the material constants of only one material need to be fully characterized, whereas the ring’s material constants can be assumed from textbook values. In conclusion an isotropic material will be used to make the rings, while the textured brass will be used as the anisotropic plug.

As discussed earlier, the plug is to be the limiting material in the assembly. The plug is to be manufactured out of C260 brass, which has an estimated yield strength of 52 ksi. Therefore a material that is commercially available with a yield higher than 52 ksi and can withstand temperatures up to 500°F without altering texture or yield strength is needed. After an exhaustive material search 1045 steel was chosen, which has an estimated yield strength of 65.3 ksi [19].

4.2 Sample Design

Two of the three design variables, i.e., the material property combination and ring-plug orientation have been investigated and described in Section 4.1. The analysis resulted in a design that will utilize an elastically isotropic ring and an elastically anisotropic plug. The other two design variables include the geometries of the ring and plug, as well as the magnitude of the interference between the ring and the plug. As shown in Equation 6 the pressure is calculated using the radii of the ring and plug, the material stiffness, and the interference. The next step in the design process is to identify the geometries and the interference to maximize spatial variation of stress.
Equation 6 reveals that increasing the outer diameter of the ring can increase the pressure on the mating surface between the ring and the plug which in turn increases the magnitude of stress within the assembly. The commercially available C260 brass stock is four inches wide in the transverse direction, limiting the maximum diameter of the plug. Manufacturing a three inch diameter plug provides enough remaining material within a 4 inch by 4 inch area surrounding the plug to manufacture samples for 3-point bending tests, through transmission ultrasound and RUS. Four different ring outer diameters with two different interferences were simulated to observe the differences in stress states. The first interference was calculated for a case in which the Von Mises maximum stress was equal to 50% of the yield of the assembly, which is based on the limiting material of the plug being C260 brass, and the second interference is 80% of the yield. Note, heating the ring to 500°F the yield strength will decrease. The yield strength of the heated 1045 steel is roughly reduced from 77 ksi to roughly 53 ksi [20]. The simulations were run with the material constants for the ring and plug as found in table 4.2. The radial and hoop stresses as functions of the angle around the azimuth for the eight different design conditions are shown in Figure 4.4. The stresses were normalized for comparison.

Table 4.2: Material constants of the ring and plug used in dimensional study

<table>
<thead>
<tr>
<th>Property</th>
<th>Plug</th>
<th>Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$ (ksi)</td>
<td>16000</td>
<td>29000</td>
</tr>
<tr>
<td>$E_y$ (ksi)</td>
<td>11200</td>
<td>29000</td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.32</td>
<td>0.29</td>
</tr>
</tbody>
</table>
The sinusoidal patterns have comparable amplitudes for all but the two highest ring diameters with the larger interference. These two distributions are roughly 20% higher on average. However, after surveying commercially available stock materials, the largest ring that can be manufactured from available bar stock is the 5.5” sample. In comparing only the 4” and 5.5” ring diameters for the two different interference fits, it has been shown that the 5.5” ring with the 0.00375” interference has the largest stress amplitude.

Figure 4.4: Radial (left) and hoop (right) stress as a function of the angle for various ring sizes and interferences
Anisotropic Material Constant Analysis

5.1 3-Point Bending Test

A 3-point bending test is performed to measure elastic material constants. In the 3-point bending experiment, a rectangular cross-sectional beam is placed between two supports, and is loaded in the center between the supports. As the load is applied, the displacement is measured at the directly opposite face and location as the applied load, and a stress strain plot can be generated. In this study, a strain gage was used to measure the axial and transverse strains, as opposed to measuring the displacement. The elastic modulus can then be obtained from the slope of the stress strain curve. The stress at the center of the beam can be calculated using Equation 12, where $F$ is the force, $L$ is the length of the sample between supports, $b$ is the base dimension of the sample, and $d$ is the height of the sample [21].

$$
\sigma = \frac{3 F L}{2 b d^2}
$$

(12)

The force was applied through the use of calibrated weights loaded onto a hanger supported on the beam through a rounded clevis. The rounded clevis had a cylindrical pin that was in contact with the top surface of the sample. When the load was applied this cylindrical contact best models a line contact load applied at the geometric center along the length of the beam. Weights were added in five pound increments with a maximum load of forty five pounds as to prevent yielding of the sample. A small scale 3-point bending apparatus was designed and 3D printed as shown in Figure 5.1. The hanger was loaded onto the clevis and the initial strain values were recorded. The sample was then incrementally loaded with strain values recorded at every
increment. The strains on the bottom surface of the beam were measured through the use of a CEA-09-062UT-350 strain gage, and are shown applied to a sample in Figure 5.2. The load is applied to the opposite face, directly opposite to the gage. The gage has two resistors allowing for two strains to be measured simultaneously. The transverse strain measured through gage 1, and the axial strain measured through gage 2. This allows for both the directional moduli of the axial direction (E_a) and the Poisson’s ratio of the axial-transverse plane (v_{at}) to be measured.

Figure 5.1: Assembled (left) and exploded view (right) of the small scale bending test apparatus

Figure 5.2: CEA-09-062UT-350 strain gage (left) and the same gage applied onto a sample beam (right) for the 3-point bending test
5.1.1 Beam Design

Accurate measurement of the elastic moduli requires the implementation of geometric constrains on the beam. The beam dimensions being used follow the guidelines for 3-point bending found in ASTM D790-17 [22]. The standard states that beam dimensions should have a height-base-length ratio between a 1:1:12 to a 1:4:16. For our experiments, we used a 1:2:14 where H ≈ 0.25”, B ≈ 0.5”, and L ≈ 3.5”. Finite element analysis of a 3-point bending test showed that beams with this geometry can experience a 5.1% difference between the measured and actual elastic moduli, and a 10.5% difference between the Poisson’s ratios. The errors were calculated using FEA and comparing the elastic constants supplied to the model to that measured by simulating a 3-point bending test with digital biaxial strain gages. The error in the Poisson’s values is caused by the 3-point bending apparatus design. The two supports prevent anticlastic bending, and therefore a loss in sensitivity to the transverse strain. However, since these values serve as initial guesses for the RUS workflow this error is acceptable.
During the rolling process, the surfaces in contact with the rollers experience the highest degree of plastic deformation, potentially producing a gradient of texture, and varying elastic constants, through the material’s thickness. The beams were cut from the center of each rolling condition to mitigate the varying elastic constants through depths, keeping the same neutral axis, shown in Figure 5.3. The plate on the bottom left of Figure 5.3 represents the plate size for all the rolling conditions. The beams represent the various rolling conditions and the cutting of the beams to maintain the neutral axis. The two different beam orientations put the length along the rolling direction, and the transverse direction. The two orientations allow for both in-plane directional moduli, and an in-plane Poisson’s ratio to be measured. Four beams were made for each rolling condition and each orientation for a total of 8 beams per rolling condition.

Figure 5.3: Beam design for the 3-point bending test for the various reductions and orientations of C260 brass
A repeatability experiment was performed to ensure the printed 3-point bending apparatus did not introduce any error into the measurements. This potential error could be caused by non-uniform loading conditions, incorrectly applied load through clevis positioning, and non-equal reaction forces at edge contacts caused by geometry of prints. Sample measurement hysteresis was investigated by taking measurements as the sample was loaded from 0 to 45 pounds, and similarly as it was incrementally unloaded back to 0 pounds. The 3-point bending apparatus was then disassembled and reassembled, and the prior testing procedure was followed again, which generates a total of five sets of strain data for the same sample as shown in Figure 5.4. The strains do not deviate much with the highest difference in strain being 35.2 µε at the maximum applied load, with an average difference per step of less than 2 µε. This variation is acceptable, and therefore, it can be assumed that the apparatus does not introduce any error into the system.

Figure 5.4: Strain vs force for a beam bending repeatability test
5.1.2 3-Point Bending Results

Beams of all reductions and orientations were tested and the resulting elastic moduli and Poisson’s ratios were calculated using Equations 13 and 14. In equation 13 \( \sigma \) is calculated using Equation 12.

\[
E = \frac{\sigma}{\varepsilon_l} \quad (13)
\]

\[
v = -\frac{\varepsilon_t}{\varepsilon_l} \quad (14)
\]

The measured moduli were averaged over the four different beams. The plots of the stress-strain data were normalized for consistent comparison between the slopes of the different reduction samples. The normalized stress strain curves are shown in Figure 5.5. The different directional moduli and the in-plane Poisson’s terms are summarized in Table 5.1.

![Stress vs Strain for Varying Reduction Rates](image)

Figure 5.5: Stress vs strain for varying reductions of C260 brass measured using 3-point bending
The directional moduli at the 25% reduction show a difference of roughly 5%. These data confirm that rolled texture development of C260 doesn’t begin until around a 40% reduction. The 50% reduced sample exhibited an 8.4% difference between the directional moduli which is the largest observed difference. The 75% reduction sample has the lowest difference between the directions with a 3.4% difference. It should be noted that these values serve as initial guesses for the RUS optimization which will be discussed in section 5.3.2.

### 5.2 Through Transmission Ultrasound

The Through Transmission Ultrasound (TTU) method is a nondestructive evaluation technique that can be used to measure the elastic constants of solids by using ultrasonic wave transducers. The velocity of these waves is measured to estimate Young’s modulus, Poisson’s ratio, and shear modulus. For an elastically isotropic material, the application of the TTU method is straightforward and will be discussed more in the next section. However, since TTU is used primarily as an initial guess for the following RUS optimization, which will be discussed in section 5.3.2, the errors from the measurements are acceptable. The TTU technique will be used to check the elastic moduli obtained from the 3-point bending test and measure the other elastic constants such as the three shear moduli and three Poisson ratios.

<table>
<thead>
<tr>
<th></th>
<th>0% Reduction</th>
<th>25% Reduction</th>
<th>50% Reduction</th>
<th>75% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{RD}$ (ksi)</td>
<td>16,500</td>
<td>16,500</td>
<td>15,000</td>
<td>15,800</td>
</tr>
<tr>
<td>$E_{TD}$ (ksi)</td>
<td>16,100</td>
<td>17,400</td>
<td>16,500</td>
<td>16,300</td>
</tr>
<tr>
<td>$V_{RD,TD}$</td>
<td>0.370</td>
<td>0.343</td>
<td>0.357</td>
<td>0.360</td>
</tr>
<tr>
<td>$V_{TD,RD}$</td>
<td>0.323</td>
<td>0.297</td>
<td>0.394</td>
<td>0.287</td>
</tr>
</tbody>
</table>
5.2.1 Through Transmission Ultrasound Theory

TTU measurement of the Young’s modulus, Poisson’s ratio, and shear modulus requires two different types of excitation. The Young’s moduli can be measured using longitudinal waves, whereas the Poisson’s ratio, and shear moduli can be measured with shear waves. Figure 5.6 shows the physical differences between the directions of the wave propagations with respect to the direction of the particle motion. These waves are pulsed through the sample at a constant frequency. One transmits the signal, and the other receives the signal with the aid of a pulser-receiver.

The two transducers are placed in contact with one another to get a baseline time for the wave measurement. The sample is then placed between the two transducers which causes the time that

DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited
the signal is received to increase. Using these two measurements the velocity can be calculated as the change in length divided by the change in time. The change in time is simply the difference between the time signal from the receiver \( dt = t_1 - t_0 \), and the change in length is the size of the sample \( dl = L \). This experimental setup can be visualized in Figure 5.7.

The velocities can then be related to the elastic moduli, shear moduli, and Poisson’s ratio through Equations 15, 16, and 17 respectively [23, 24].
\begin{align}
E &= 4\rho c_s^2 \left( \frac{3}{4} - \left( \frac{c_s}{c_l} \right)^2 \right) \\
G &= \rho c_s^2 \\
v &= \left( \frac{1}{2} - \left( \frac{c_s}{c_l} \right)^2 \right) \left( \frac{1}{1 - \left( \frac{c_s}{c_l} \right)^2} \right)
\end{align}

The axes of the shear moduli and the Poisson’s term are dependent on the orientation of the sample, where the first subscript is the axes normal to the transducer face, and the second subscript is the direction of the wave particle motion. These axes only affect the measurements of the shear wave velocity \(c_s\). Equation 14 shows that the shear moduli is only dependent on the shear wave velocity. This relationship does not assume that the material is elastically isotropic. Equations 13 and 15 are dependent on both the longitudinal and shear wave velocities, and are both derived under the assumption of elastic isotropy.

5.2.2 Through Transmission Ultrasound Results

The following results were calculated for the various measurements using TTU. The data has been reduced and is shown in Table 5.2. It is important to note that for all samples, the \(x\)-direction is aligned with the rolling direction of the brass plate, the \(y\)-direction with the transverse direction, and subsequently the \(z\)-direction with the normal direction.
The 0% and 25% reductions appear to be elastically isotopic as there is less than 1% variation in the directional constants. At 50% and 75% reduction, there are roughly 10% variations in the directional moduli. The results follow the same trends found using 3-point bending. The magnitude of the constants are off slightly from those calculated using 3-point bending, but the relative difference is comparable. The resulting elastic constants obtained from the tests were fed into the RUS technique as initial guesses.

### 5.3 Resonant Ultrasound Spectroscopy

Resonant Ultrasound Spectroscopy (RUS) is a nondestructive evaluation technique that can be used to calculate the entire 6 x 6 stiffness matrix of materials [25, 26, 27, 28]. RUS uses solid body resonances of the material to calculate the elastic constants through an inversion algorithm. The traditional RUS experimental setup is shown in Figure 5.8. There are two different modal frequency and shape prediction methods used in the inversion algorithm currently. The first method is the Rayleigh-Ritz numerical approximation technique, and the second is an FEA
vibrational technique. Both of the techniques are able to calculate the resonances, as well as the mode shapes of a sample. However, each method has pros and cons which will be discussed as follows.

The Rayleigh-Ritz method is a numerical method that can be used to approximate eigenvalues and eigenvectors (mode shapes) [29]. Resonances of a material are dependent on multiple variables (or factors), i.e., material stiffness (C matrix), density, sample geometry, and the boundary conditions of the sample. The Rayleigh-Ritz method solves an inversion algorithm to calculate the resonances of a sample. The technique is utilized by holding all variables constant except the stiffness. The stiffness is then iteratively changed until the resonances calculated match the experimentally measured resonances using an optimization method. The issue lies, however, in the boundary conditions in which are coupled to the geometry of the sample. This is because the Rayleigh-Ritz method is executed by solving a stationarity integral over the volume of a solid body. The integrals can be computationally expensive for irregular geometries, but manageable for simple shapes such as cylinders, rectangular parallelepipeds, and a few other shapes. Any slight variations from these geometries can cause large discrepancies in

Figure 5.8: Components for traditional RUS experimental setup. A sample is placed between two ultrasonic transducers, one of which is used to excite the sample at a signal supplied from the function generator, and the other is used to measure the output signal through the sample.
the volumetric integrals. The discrepancies will propagate errors throughout the resonance calculations, and therefore skew the data. If the resonances are incorrect, the inversion algorithm will likely converge to a stiffness matrix that is incorrect. Although this technique requires more care in sample preparation/manufacturing it is computationally inexpensive overall. Thanks to the reduced simulation time, this method can typically solve the inversion problem in less than an hour on a workstation.

The second method is using an FEA vibrational technique. Using FEA allows us to compute the resonance responses of the samples with complex geometries. The inversion approach follows the same procedure of iteratively solving for the resonant frequencies while changing the stiffness matrix until the simulated resonant frequencies match the experimental data. The FEA variation technique trades off sample preparation time with computational time. Even for a simple sample, it takes roughly 2-3 minutes per simulation, depending on mesh size, when there are small perturbations from a characteristic geometry, such as rectangular parallelepiped. This prolongs solving the inversion problem to roughly 16-18 hours.

RUS can be used to measure resonances, and with the development of Laser RUS systems, displacement fields or mode shapes can also be measured [30, 31]. The experimental setup for Laser RUS is shown in Figure 5.9. The existing technique only utilizes the resonance frequencies, so it is possible that a simulated data set could match with the experimental resonances, but not the resonance mode shapes. To avoid these issues, a new optimization framework was developed that takes both the resonances and mode shapes into account. Also, because sample geometry plays a large factor in both techniques (Rayleigh-Ritz and FEA) a
A sensitivity study was performed to identify an optimal sample geometry that maximizes the sensitivity to material constant variations.

![Experimental setup of LASER-RUS system](image)

Figure 5.9: Experimental setup of LASER-RUS system. A sample is placed on two ultrasonic transducers, mostly to minimize boundary effects. One transducer excites the sample with a given sample from the function generator, and the output signal is measured from the Laser Vibrometer.

### 5.3.1 Dimensional Sensitivity Study

A rectangular parallelepiped was selected as the RUS sample geometry to balance sample manufacturing and computational time. A rectangular parallelepiped is a hexahedron (having six faces) with three sets of parallel faces and right angles between faces. This geometry is ideal because of the ease of derivation of the stationarity integral which is used in the resonance calculations using the Rayleigh-Ritz approach [32]. It also requires minimal finite element modeling because of the simplicity of the geometry and decreases the computational cost of the simulation studies. However, the dimensions of the sample have a large effect on the resonances, and because of this an optimal sample geometry is desired.
Initial RUS measurements were made on a sample of 75% reduced brass with a size of \( \frac{1}{2}'' \times 1'' \times \frac{1}{8}'' \) shown in Figure 5.10, producing and aspect ratio of 4 x 8 x 1 in the X, Y, and Z directions respectively. Upon attempting the inversion algorithm it was found that the sample was not sensitive to certain material constant changes as highlighted in Figure 5.11. As simulations were performed with the model geometry, changes in certain material constants were not effecting the resonances. It was observed that changes in the directional moduli \( E_x \) cause the differences between experimental and simulated resonances to change. However, similar changes to the directional moduli \( E_z \) showed no effect on the differences between the resonances. The reason for this was found to be the differences in the dimensional aspect ratio, as the z dimension was four times smaller than the next closest dimension. It was hypothesized that as the ratio between the smallest dimension and the next closest increases, sensitivity can be lost, and an

![Figure 5.10: RUS sample with a 4 x 8 x 1 geometry](image-url)
ideal geometry must exist that produces maximum sensitivity to most of the material constants. For this reason a sensitivity study was performed to identify an ideal geometry.

During RUS optimization of material constants the initial guess of material constants is typically leverages isotropic values as they are readily available for most materials. For this reason isotropic material constants were used as the baseline for this study. Samples were created with a varying aspect ratio highlighted in Table 5.3. The sample space reflects an upper triangular matrix because swapping the X- and Y-directions would reflect a physical 90° rotation of the sample, but the elastic material constants would remain the same. The green represents dimensions used, the gray represents the reflected geometries not used and the black represents sample sizes not tried because of degeneracy. Degeneracy is caused by samples with dimensions that are close to one another, and can be physically described as the overlapping of resonances, where the two modes are a simple 90° rotation of one another and cannot be differentiated.

![Frequency Difference vs. Simulated Frequency](image)

**Figure 5.11**: Sensitivity of resonances to the variation of Ex (left) and Ez (right)
For each sample geometry an isotropic assumption was made, with the constants mimicking brass, and steel to explore two different ranges of possible material constants. This was done to confirm if the ideal geometry was material dependent or not. Through the varying geometries each of the nine parameters, including three dimensional moduli, three Poisson’s terms, and three shear moduli, were varied between roughly ±5-10%. The two different material constant variation sets are shown in Table 5.4. The resonances were then compared to the isotropic value, and the difference between the two was calculated as shown in Table 5.5. Similar percentage changes in the constants yielded similar changes in the resonances. An average difference was then calculated for each resonance, with which the overall error was then calculated using an L2 norm as shown in Equations 18 and 19. This value was called the sensitivity, which represented the average change in frequency in relation to the change of a specific property. This was completed for all nine parameters, and all 66 dimension sets for both materials. The results are shown in Figure 5.12.

Table 5.3: Aspect ratios used in sensitivity study, $Z = 1$. Green representing aspect ratios used, black representing ratios not used due to degeneracy, and grey representing ratios not used due to symmetry.
Table 5.4: Material constant variations of assumed brass and steel, for dimensional sensitivity analysis for RUS measurements

| C260 Brass | E_x, E_y, E_z (ksi) | 14000 | 15000 | 16000 | 17000 | 18000 |
| G_xy, G_zz, G_yz (ksi) | 5000 | 5500 | 6000 | 6500 | 7000 |
| ν_xy, ν_xz, ν_yz | 0.3 | 0.31 | 0.32 | 0.33 | 0.34 |
| 304 Stainless Steel | E_x, E_y, E_z (ksi) | 26000 | 27000 | 28000 | 29000 | 30000 |
| G_xy, G_zz, G_yz (ksi) | 12000 | 10700 | 11200 | 11700 | 12200 |
| ν_xy, ν_xz, ν_yz | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 |

Table 5.5: Formatting the residuals using in the sensitivity calculation. I is the frequency for the isotropic case, V is the frequency of one of the varied parameter cases, ε stands for the residual that is represented by I - V

| Isotropic Simulation Frequency (I) | Varied Simulation Frequency (V) | Residual (ε) |
| I_1 | V_1 | ε_1 |
| I_2 | V_2 | ε_2 |
| . | . | . |
| I_n | V_n | ε_n |

\[
\bar{\varepsilon} = \frac{(\varepsilon_5 - \varepsilon_4) + (\varepsilon_4 - \varepsilon_3) + (\varepsilon_3 - \varepsilon_2) + (\varepsilon_2 - \varepsilon_1)}{4} \tag{18}
\]

\[
S = \sqrt{\sum_{k=1}^{n} |\bar{\varepsilon}_k|^2} \tag{19}
\]
In these plots values less than one represent a decrease in sensitivity, whereas a value greater than one represents an increase in sensitivity. The increases in sensitivity are not due completely to the geometry but also are dependent on the material properties investigated. The ideal geometry to maximize sensitivity to $E_x$ was a 5 x 14 x 4, but for $v_{yz}$ was a 5 x 7 x 4. To compute an overall sensitivity the constants were summed after being weighted according to Table 5.6. These weights were based on the ranges of physical constants. Directional moduli have the most potential variation with respect to an assumed isotropic value for differing alloys and textures and the highest magnitude. The shear moduli are usually around half or less than half of the directional moduli. For elastically isotropic C260 brass, the Young’s modulus is around 16,000 ksi, where the shear moduli is about 5,800 ksi. The Poisson’s ratios for most metals typically range between 0.2 and 0.45. With such small variation the Poisson’s terms were

Figure 5.12: Parameter sensitivity for brass properties with geometric variation
given the lowest weight. The total weights for the directional moduli are 60%, the shear moduli are 30% and the Poisson’s are 10%.

Using the weights described above the parameters were summed for both the brass and the steel material constant sets. From the results shown in Figure 5.13, both plots have a relatively similar gradient to them, however the magnitudes are smaller for the steel set. Both sets have a maximum sensitivity at the aspect ratio of 1.25 x 1.75 x 1, which corresponds to the sample size of 5 mm x 7 mm x 4 mm. For the brass property set this sample geometry had an increase in sensitivity by over 120% whereas the steel property set had an increase of roughly 60%. With the optimal geometry defined, the development of the new inversion tool was the next step for the RUS technique.

Table 5.6: Weighting of material constants for total sensitivity analysis

<table>
<thead>
<tr>
<th>Property</th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$E_z$</th>
<th>$G_{xy}$</th>
<th>$G_{xz}$</th>
<th>$G_{yz}$</th>
<th>$\nu_{xy}$</th>
<th>$\nu_{xz}$</th>
<th>$\nu_{yz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
</tr>
</tbody>
</table>
Figure 5.13: Total weighted sensitivity for brass (top) and steel (bottom) property variations
5.3.2 Modal Matching Optimization

The typical inverse matching method considers only resonance frequency values to calculate the error between an experimental and simulated dataset. If the error is not acceptable, the method iteratively changes the constants to minimize the error. However, both the resonant frequency and displacement fields (and mode shapes) are obtained from RUS for each resonance. Modal matching problems are generally considered difficult optimization problems, with a large number of local minima. Another complicating factor is that it is possible that some of the resonances switch positions with one another in frequency space as material constants are varied. For example, a bending mode and a torsional mode are shown in Figure 5.14. The bending mode (left) is more sensitive to changes in the directional moduli, whereas the torsional mode (right) is more sensitive to the shear moduli. In Figure 5.11, some of the resonances change frequency values quite drastically, whereas others are not affected at all. The modes that were most affected were bending and extensional modes. The inversion algorithm however, was designed to pair frequencies for the error calculation based on the eigenvalue number, $\lambda$. These eigenvalues can
switch depending on which material constants were being varied. Examples of mode switching are shown in Figure 5.15.

Figure 5.14: Bending (left) and torsional (right) modes found using finite element analysis

Figure 5.15: Eigenvalue changes of resonances based on property changes
In these two separate simulations the material constant $v_{yz}$ was changed from 0.3 in run 1, to 0.34 in run 2. This caused the two sets as well as a few others (when only looking at the first 75 resonances) to have modes that moved in relation to one another. It is interesting to note that in run 1, the mode that moved from $\lambda = 10$ to $\lambda = 11$, had a very minimal change in the frequency itself, but the frequency of the other mode changed enough to move the order. The same behavior was noted in the second pair. If the existing inversion algorithm were to be implemented the resonances would still be matched regardless of incorrect mode shapes, because only the $\lambda$ value is used.

In a study conducted by Longo et al., an inclusion of modal shapes into the optimization problem has been developed by using an image correlation method [33]. Using wood samples, an orthotropic material, RUS measurements were conducted, using a laser-RUS system, and experimental mode shapes were measured. They then used an FEA software to calculate the resonances and mode shapes. Using the image correlation the experimental and simulated modes were matched together, and the subsequent matching frequencies used in the error calculation. However, a portion of the error was based on the correlation coefficients of the matching modes in addition to the difference in simulated and experimental frequencies.

In the present work, the new tool developed uses the mode shapes measured using laser RUS and compares them to simulated displacement fields calculated using the Rayleigh-Ritz approach. First, an initial simulation was run with the properties guessed to get a baseline mode comparison. The modal responses of the initial run are screened to identify the “good modes” from the experiment. During the measurement, some of the windowed frequency sweeps are not true resonances, and so the data is unnecessary to be matched, or other times if the data is too
noisy it can be removed from the matching pool. Using a self-developed app created in MATLAB, which optimized the initial manual matching process the experimental and simulated modes could be swept through and manually matched to identify the modes wanted for the matching process. These matches are then the defined experimental modes to be searched through during the optimization. The simulated modes are normalized through histogram equalization, pixel resizing, and the experimental modes are normalized through intensity redistribution, and then histogram equalization. This is due to the fact that the experimental data is only scanned in a grid pattern over the surface whereas the simulated data is calculated over the entire surface. To allow the displacements and overall mode shapes to be compared the data format has to match. This was done by reducing the simulated data mesh grid size to match the grid pattern experimentally used. The initial simulated mode, and the normalized simulated mode are shown in Figure 5.16.

Once the images were normalized the following process was completed. The first experimental mode that was defined as a mode for the search is selected and a window of ± 30

Figure 5.16: Simulated mode shape initially (left) and after normalization (right)
kHz is created around the experimental resonance value to generate a searching area. All simulated modes in that window are then compared against the experimental mode through an L1 norm using the displacement values of the grids. The match percentage was used to identify the closest match, where the highest match percentage was deemed the closest mode, and was calculated using Equation 20, where m and n are the numbers of rows and columns in the grid and u is the displacement.

\[
Percent \ Match = 100 \times \left(1 - \frac{\sum_{0 \leq i \leq m} \sum_{0 \leq j < n} |u_e(i,j) - u_s(i,j)|}{\sum_{0 \leq i \leq m} \sum_{0 \leq j < n} (u_e(i,j) + u_s(i,j))}\right)
\]

If the highest percentage match is less than 80% then no match is assigned, and the pair is ignored. This process is repeated for all the “good modes” selected previously. Once all the experimental modes are swept through, the resonances of these matching modes are then assigned as a pair. The objective function is then calculated using Equation 21 and the optimizer is seeking to minimize this value.

\[
error = \frac{1}{n} \sum |f_e - f_s|
\]

The objective function is corresponding to the average frequency error of each match. The function accounts for the number of modal matches. However, the number of matches can also be used to identify error. At the beginning of this workflow, the clear modes are picked from the experimental data. The optimizers that automated modal matching then only search these modes. It is possible that a simulation could solve for a very low error, only because the number of
modes matched was very low, and the frequencies matched very well for these pairs. To counteract a low number of modal matches that may result in an artificially low error an error penalization was introduced. If the ratio of number of matched modes to the number of “good modes” was less than 70% than the error was divided by said ratio to artificially increase the error. This optimization tool was put together utilizing the optimization toolbox and the optimization function, \textit{fmincon}, in MATLAB. The workflow is shown in Figure 5.17. The red indicates the previous method, where a defined eigenvalue pairing was used for frequency comparison, whereas the blue indicates the new method that includes a modal matching to identify the frequency pairs for error calculation.

Figure 5.17: Workflow developed for RUS optimization used in material constant calculation

5.3.3 RUS Results

The following results were calculated for the various reductions using the automated mode matching resonance optimization. The data has been reduced and can be found in Table 5.6. For these samples the x-direction is equivalent to the transverse direction, the y-direction is equal to the rolling direction, and the z-direction is equivalent to the normal direction in relation to the rolling of the samples.
Texture development does not begin to occur until after 50% reduction, as shown in Table 5.6. The 0% reduction, or as received, brass has minimal differences in the in-plane directional moduli $E_x$ and $E_y$. In addition, the shear moduli are all within ±0.5% of each other, highlighting that the material can be assumed to be elastically isotropic. At 25% reduction, the moduli increase but the relative differences between the directional moduli stay the same at around ±300 ksi, and again the shear moduli are all within about ±0.5% of each other, allowing for the assumption of elastic isotropy. At 50% reduction, the moduli decrease to the original values, but the relative changes between moduli is still minimal. At the 75% reduction, a variation of about 9% is found between the moduli. This difference can cause notable variation in the spatial distribution of residual stresses induced by the assembly of the ring and plug specimens. Due to the minimal differences in the directional moduli for the 25% and 50% reduced brass, ring and plug samples were manufactured from the as received brass, and the 75% reduction only.

Table 5.7 Elastic moduli measured using Resonant Ultrasound Spectroscopy

<table>
<thead>
<tr>
<th></th>
<th>0% Reduction</th>
<th>25% Reduction</th>
<th>50% Reduction</th>
<th>75% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$ (ksi)</td>
<td>16,500</td>
<td>17,100</td>
<td>16,500</td>
<td>16,900</td>
</tr>
<tr>
<td>$E_y$ (ksi)</td>
<td>16,200</td>
<td>17,400</td>
<td>16,200</td>
<td>15,600</td>
</tr>
<tr>
<td>$E_z$ (ksi)</td>
<td>16,300</td>
<td>16,900</td>
<td>16,200</td>
<td>17,100</td>
</tr>
<tr>
<td>$G_{xy}$ (ksi)</td>
<td>6,060</td>
<td>5,690</td>
<td>5,710</td>
<td>5,960</td>
</tr>
<tr>
<td>$G_{yz}$ (ksi)</td>
<td>6,070</td>
<td>5,790</td>
<td>5,440</td>
<td>5,520</td>
</tr>
<tr>
<td>$G_{xz}$ (ksi)</td>
<td>6,040</td>
<td>5,760</td>
<td>6,320</td>
<td>5,950</td>
</tr>
<tr>
<td>$V_{xy}$</td>
<td>0.320</td>
<td>0.321</td>
<td>0.352</td>
<td>0.4074</td>
</tr>
<tr>
<td>$V_{yz}$</td>
<td>0.340</td>
<td>0.348</td>
<td>0.328</td>
<td>0.315</td>
</tr>
<tr>
<td>$V_{xz}$</td>
<td>0.323</td>
<td>0.355</td>
<td>0.361</td>
<td>0.322</td>
</tr>
</tbody>
</table>
Experimental Measurements

6.1 Simulated Experimental Samples

Two ring and plug samples were designed, each composed of a 1045 steel ring that was assumed to be elastically isotropic and a C260 brass plug, which was assumed to be elastically isotropic in the as received condition, and elastically anisotropic at the 75% reduction. The expected uniform distributions of the radial and hoop stresses, on the left and right, respectively, for the as received brass ring and plug assembly are shown in Figure 6.1. The hoop and radial stress are equal in the plug, with a value of -22.6 ksi. The expected stress distributions for the 75% reduced brass ring and plug assembly are shown in Figure 6.2. The stress varies as a function of the angle around the azimuth. The stress profiles as functions around the azimuth for both the 0% reduction, and 75% reduction samples are shown in Figure 6.3.
Figure 6.1: Radial (left) and hoop (right) stress distributions for 0% reduced brass assembly

Figure 6.2: Radial (left) and hoop (right) stress distributions for 75% reduced brass assembly
A stress variation of 2 ksi is expected in the hoop stress as a function of the angle, with a 1 ksi variation in the radial stress for the 75% reduced brass. The 0% reduction brass is expected to have a uniform stress distribution spatially.

![Radial and Hoop Stress as a Function of Angle Around Azimuth](image)

Figure 6.3: Stress Profiles for both 0% reduction and 75% reduction brass ring and plug samples

### 6.2 Results

After the assembled ring-plug samples were measured, the ring was removed using a wire EDM to measure the relieved strains. Post-removal measurements were made on the plug, allowing for assembly induced stresses to be calculated using the principle of superposition. The stress profiles for the 0% reduction are shown in Figures 6.4 and 6.5 for the cases of the standard and large hole measurements, respectively.
Figure 6.4: Through thickness stress profiles for standard hole measurements of 0% reduced brass

Figure 6.5: Through thickness stress profiles for large hole measurements of 0% reduced brass
The stress profiles shown in both Figure 6.4 and 6.5, show the stresses are comparable to each other for varying measurements. This confirms the expectation of uniform through thickness and spatially. However, the magnitude of the stress is lower than expected by about 25%. Once the ring was removed the sample was gaged as shown in Figure 6.6 with biaxial gages that allow for the measurement of radial and hoop strains to be measured. The relieved radial hoop strains and subsequent stresses are shown in Table 6.1.

![Figure 6.6: Gaging layout for ring removal strain measurements](image)

<table>
<thead>
<tr>
<th>Gage #</th>
<th>$\Delta \varepsilon_R$ ($\mu \varepsilon$)</th>
<th>$\Delta \varepsilon_H$ ($\mu \varepsilon$)</th>
<th>$\sigma_r$ (ksi)</th>
<th>$\sigma_h$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>659.4</td>
<td>676.5</td>
<td>16.1</td>
<td>16.3</td>
</tr>
<tr>
<td>2</td>
<td>686.0</td>
<td>666.0</td>
<td>16.5</td>
<td>16.3</td>
</tr>
<tr>
<td>3</td>
<td>677.6</td>
<td>662.4</td>
<td>16.3</td>
<td>16.2</td>
</tr>
<tr>
<td>4</td>
<td>676.6</td>
<td>676.7</td>
<td>16.4</td>
<td>16.4</td>
</tr>
</tbody>
</table>

Table 6.1: Relaxed strains and stresses of ring removal process of 0% reduced brass plug
As the stress profiles align with a spatially uniform equal-biaxial stress state, the datasets were averaged. The stress profiles of the plug alone were measured and then using superposition the assembly induced stresses were calculated. The obtained stress profiles are compared to the stresses measured from the ring removal process for standard and large hole measurements in Figures 6.7 and 6.8 respectively.

Comparing the measured stress profiles, both through superposition, and the strain relief of the ring removal show comparable values. The measured values are roughly 25% lower than the expected stress state as predicted through the finite elements models. The deviated stress measurements had been observed in prior ring and plug samples. The potential causes of the deviation may include instrumentations, measurements, sample preparation, lab environmental conditions, etc., which can be a topic of future and continuing study.
Figure 6.7: Superposition of assembly induced stress from standard hole-drilling measurements, compared to stress measured from ring removal process.

Figure 6.8: Superposition of assembly induced stress from large hole-drilling measurements, compared to stress measured from ring removal process.
To compare the elastically anisotropic sample to the simulated FEM models it is necessary to ensure that the experimental and simulated samples are comparable to one another. With the hoop and radial stresses being the desired metrics for comparison between assumed isotropy and elastic anisotropy, it is first required to compare the strains of the two data sets. If the strains of the two data sets are comparable, then the stresses can be compared from the simulated dataset, which takes into account the elastic anisotropy of the materials, and the experimental dataset, which assumes elastic isotropy.

For the experimental data, two different hole-drilling measurements were completed. The first measurement was made while the sample was assembled, whereas the second measurement was made at the same angle, once the ring had been removed. Using these two datasets the strain as a function of depth, for assembly induced stress only can be calculated using the principle of superposition. The same method can be followed for the resulting residual stresses. With the stresses as a function of the azimuth angle being periodic, and repeat after 180°, only the strains and resulting stresses for Θ = 0°, and 90° will be investigated. The strains as a function of depth, for Θ = 0°, 90° is shown in Figure 6.9. The simulated and experimental strains are fairly comparable to one another. The average and percentage differences in με are given in Table 6.2. Knowing that the strains are comparable to one another, the stresses are then compared.
Figure 6.9: Strain profiles for the superposition of experimental sample and simulated sample for $\Theta = 0^\circ$ and $\Theta = 90^\circ$

Table 6.2: Difference in strain between experimental and simulated datasets

<table>
<thead>
<tr>
<th>Gage #</th>
<th>Difference (µε)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Theta = 0^\circ$</td>
<td>$\Theta = 90^\circ$</td>
</tr>
<tr>
<td>1</td>
<td>-6.7</td>
<td>-7.4</td>
</tr>
<tr>
<td>2</td>
<td>6.9</td>
<td>11.5</td>
</tr>
<tr>
<td>3</td>
<td>-10.6</td>
<td>16.3</td>
</tr>
</tbody>
</table>
The current standard to calculate stresses from the strains utilizes only two material constants, the directional moduli, $E$, and the Poisson’s ratio, $v$. Utilizing the material constant measurement techniques two different sets of constants were measured in-plane of the hole-drilling measurements. The two directional moduli, $E_x$, and $E_y$, as well as the two respective Poisson’s ratios, $v_{xy}$, and $v_{yx}$. The analysis was run for both of these sets of constants using the current standard, and compared to the simulated stress values, knowing the strains from the experimental and simulated datasets matched within 8%. The constant sets properties are outlined in Table 6.3. Figure 6.10 shows the stress as a function of depth for the locations of $\Theta = 0^\circ$ and $\Theta = 90^\circ$ for the two different material constant sets.

<table>
<thead>
<tr>
<th></th>
<th>Constant Set 1</th>
<th>Constant Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (ksi)</td>
<td>15,556</td>
<td>16,938</td>
</tr>
<tr>
<td>$v$</td>
<td>0.374</td>
<td>0.407</td>
</tr>
</tbody>
</table>

The expected stress profile of a ring and plug sample is a constant through thickness stress, however, the calculated stress profiles, shown in Figure 6.4, are very noisy. This highlights that the current standard, which assumes elastic isotropy, is not well equipped to handle samples that are elastically anisotropic. I believe this is caused by the violation of the assumption of elastic isotropy that is used when deriving the equations utilized in the ASTM standard, as well as the calculations of the calibration matrices, that also assume elastic isotropy. The stress state, although expected to be constant, was averaged to calculate an average through
thickness stress. This average stress was compared to the average through thickness stress of the simulated data sets, and the error percent reported in Table 6.4.

Table 6.4: Average error for the two different material constant sets and two different angles

<table>
<thead>
<tr>
<th>Gage #</th>
<th>Constant Set 1</th>
<th>Constant Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Theta = 0^\circ)</td>
<td>(\Theta = 90^\circ)</td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>-15.97</td>
<td>-17.65</td>
</tr>
<tr>
<td>(\sigma_h)</td>
<td>-13.55</td>
<td>-8.76</td>
</tr>
</tbody>
</table>

The error is shown to be between 4-18% for the two difference constant sets, for the two different locations. The error in radial stress was always larger than that of the hoop stress. The errors between radial and hoop stress for the specific locations and constant sets were closer for the 0° location, with a difference between 2-4%, whereas the differences in percent error of the 90° location for both constant sets was between 9-10%. Seeing this potential error in the case of the stress magnitude of roughly 22 ksi, shows that the average values could be off by anywhere between 0.8 – 4 ksi. The typical uncertainty for hole-drilling is between 0-1 ksi. These errors are well outside the uncertainty and range, and highlight the need for further development.
Figure 6.10: Stress as a function of depth for $\Theta = 0^\circ$ (top) and $\Theta = 90^\circ$ (bottom) for different material constant sets
Conclusions

7.1 Summary

The baseline of comparison used to highlight the effects of elastic anisotropy on residual stress measurements was the ring-plug sample manufactured from as received brass. The brass material was assumed to be elastically isotropic. The elastic constants were measured using 3-point bending, through transmission ultrasound, and resonant ultrasound spectroscopy. The results of the three different measurement techniques confirm the assumption of elastic isotropy with a variation in directional properties of 3% or less. The residual stress analysis was then performed using the average of the directional moduli, and compared to the stress calculated through the relieved strains from the ring removal process. The variation in the stresses measured from the hole-drilling and the ring removal was less than 2%. The difference in expected stress magnitudes from simulations, and analytical solutions is roughly 25%. Although additional investigation is required to understand the sources of the deviation the stress difference is uniform through the stress depth profile, as well as a uniform spatial stress distribution for the elastically isotropic ring and plug sample.

To show the effects of elastic anisotropy, a ring-plug sample was manufactured out of textured C260 brass. The elastic material constants measured from 3-point bending, through transmission ultrasound, and resonant ultrasound spectroscopy show a roughly 10% difference between the directional moduli. The difference in simulated and experimental stress values for radial stress is at least -11%, with a maximum difference of -17.65%. The hoop stress has a lower average percent difference with a minimum of -3.80%, however, a maximum of -13.55%.

DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited
In comparison to the variation in strains the error was no more than 8%, which shows that there is error introduced simply by the assumption of elastic isotropy. When assuming elastic isotropy it is shown that there can be error anywhere between 4-18% in the measured residual stresses.

With the variation in stress calculated from the various methods, it has been shown that there are effects of elastic anisotropy on residual stress hole-drilling measurements. Although the effects due to material elastic anisotropy are smaller than expected, the need for an updated methodology that can address elastic anisotropy is still present.

### 7.2 Future Work

It was shown that there are effects in residual stress measurements caused by elastic anisotropy. One of the issues found was that the material may not have had enough variation in the directional properties to show the significant differences in stress values. For this reason, more ring-plug samples will be designed with other elastically anisotropic materials, i.e., additively manufactured samples. The samples will be made of other materials that will have higher variations in directional properties, and accentuate the spatial variation of residual stresses.

Another potential topic for future research is the deviation of the stresses measured from the simulated stress. Future study will be conducted to identify the source of this error, as to minimize sources of variation between simulated and experimental samples. This is needed to derive the relationships that will be used in an elastically anisotropic residual stress hole-drilling calculation technique.
Finally, the RUS optimization technique will continue to be developed. As a quick and nondestructive evaluation technique, it is of interest to continue to develop the capabilities of this method. Efforts will be placed on the development of high temperature capabilities, as well as the potential to involve residual stress measurements.
Bibliography


DISTRIBUTION STATEMENT A. Approved for public release. Distribution is unlimited


