The effective slip length and vortex formation in laminar flow over a rough surface

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A. Niavarani and N.V. Priezjev, "The effective slip length and vortex formation in laminar flow over a rough surface," *Phys. Fluids* **21**, 052105 (2009).

A. Niavarani and N.V. Priezjev, " Rheological study of polymer flow past rough surfaces with slip boundary conditions," *J. Chem. Phys.* **127**, 144902 (2008).

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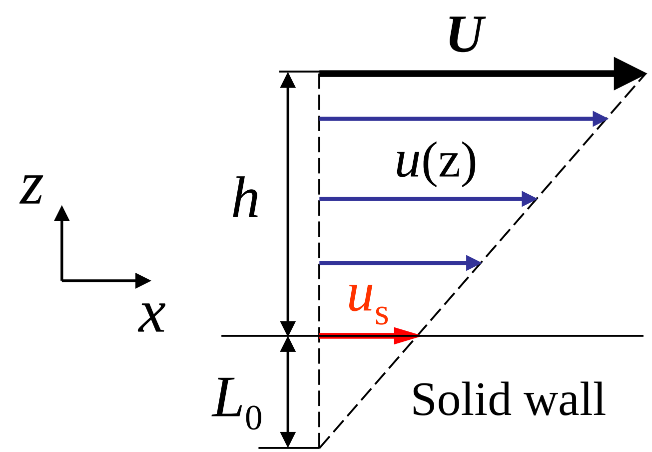
Introduction

- The validity of the no-slip boundary condition is well accepted in macroflows, however, in micro and nanoflows molecular dynamics (MD) simulation and experimental studies report the existence of a boundary slip.

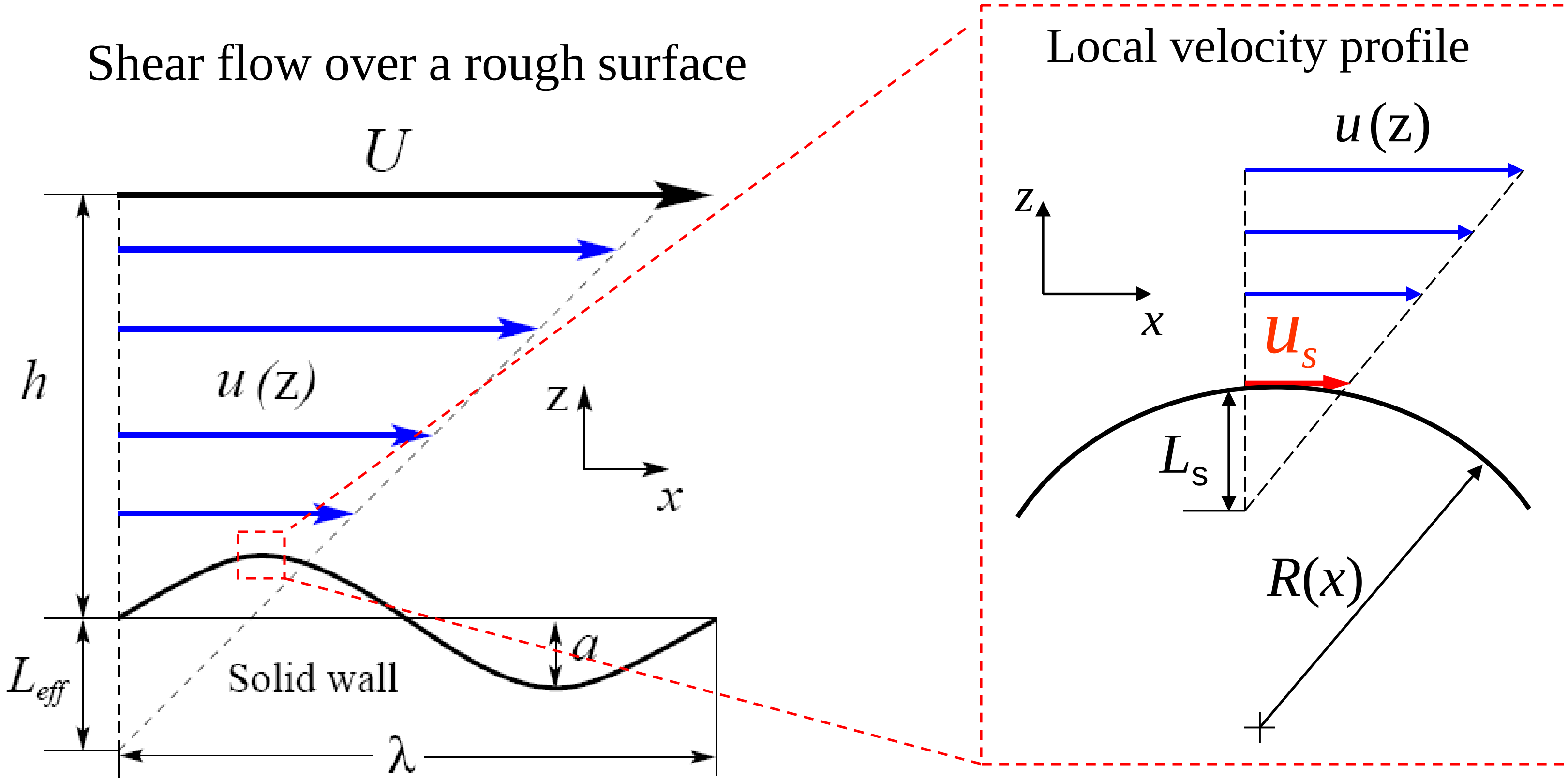


$$u_s = L_0 \dot{\gamma}$$

$$\dot{\gamma} = \frac{du(z)}{dz}$$

- The boundary condition and surface topology are major factors affecting the flow pattern near the solid surface and the formation of recirculation zones.
- In microfluidic channels the flow separation can modify wall shear stress distribution, enhance mixing efficiency and promote convective heat transfer.
- In this study the effects of surface corrugation, local slip boundary conditions, and the *Re* number on flow pattern and effective slip length are studied.

The effective slip length L_{eff} and intrinsic slip length L_0



$$z(x) = a \sin(2\pi x / \lambda)$$

$$u_s(x) = L_s \frac{\partial u(z)}{\partial n} \qquad \frac{1}{L_s} = \frac{1}{L_0} - \frac{1}{R(x)}$$

L_{eff} is the effective slip length, which characterizes the flow over macroscopically rough surface

L_s : local slip length

L_0 : intrinsic slip length can be estimated from MD simulations

$R(x)$: local radius of curvature

Details of continuum simulations (Finite Element Method)

Equations of motion (penalty formulation):

$$\nabla \cdot \mathbf{u} = -\frac{p}{\Lambda}$$

$$\rho(\mathbf{u} \cdot \nabla u) = \Lambda \nabla(\nabla \cdot \mathbf{u}) + \mu \nabla^2 u$$

$$\rho(\mathbf{u} \cdot \nabla v) = \Lambda \nabla(\nabla \cdot \mathbf{u}) + \mu \nabla^2 v$$

Boundary condition:

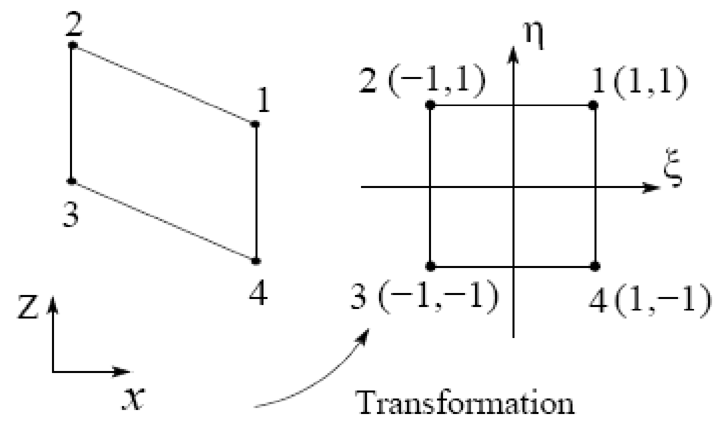
$$u_t = L_0[(\vec{n} \cdot \nabla) u_t + u_t / R(x)]$$

$R(x)$: local radius of curvature

(+) concave, (–) convex

L_0: intrinsic slip length

Bilinear quadrilateral elements



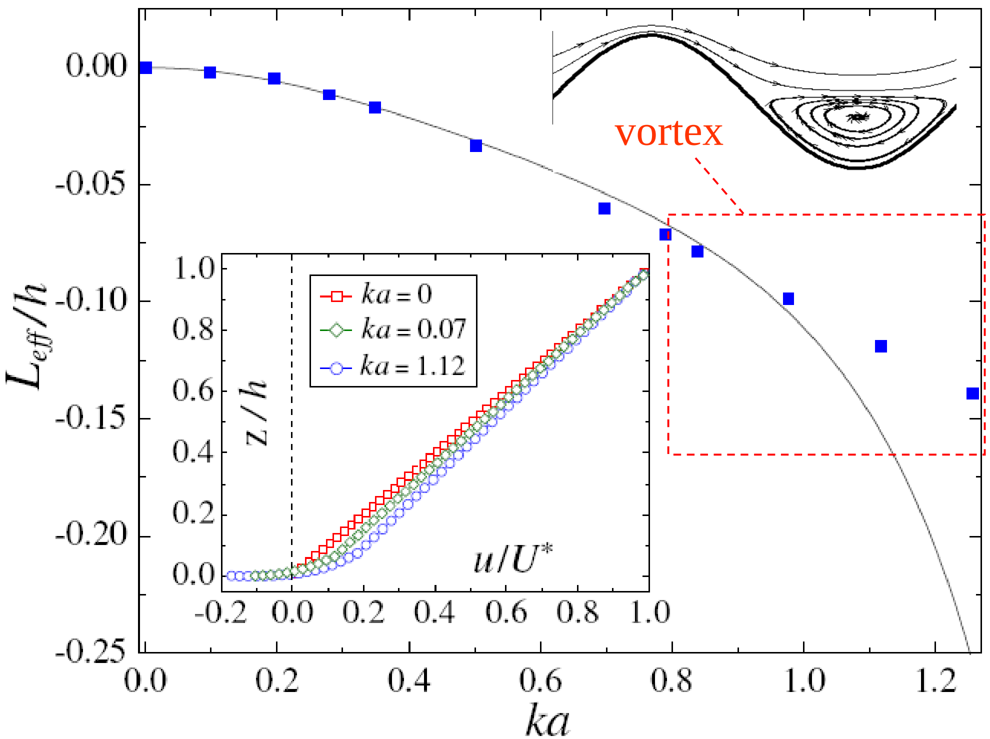
shape function $N_i = \dfrac{(1+\xi_i\xi)(1+\eta_i\eta)}{4}$

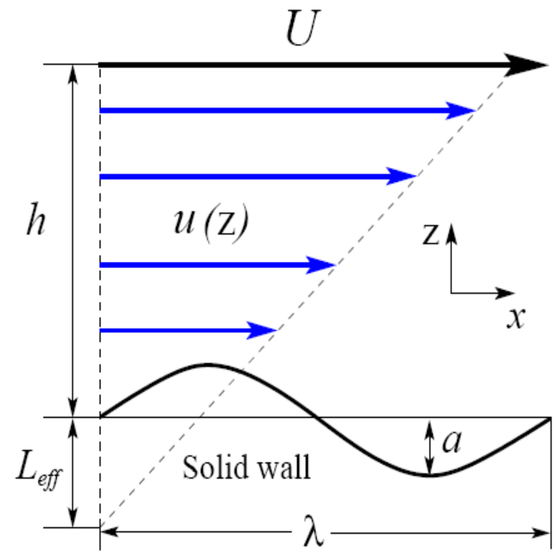
Galerkin formulation:

$$\Big[\int_\Omega \rho N_i \Big(\bar{u}_i v_j \frac{\partial N_j}{\partial x} + \bar{v}_i v_j \frac{\partial N_j}{\partial z}\Big)\Big] + \Big[\int_\Omega \Lambda \frac{\partial N_i}{\partial z}\Big(\frac{\partial N_j}{\partial x} u_j + \frac{\partial N_j}{\partial z} v_j\Big) d\Omega\Big] + \Big[\int_\Omega \mu \Big(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\Big) v_j d\Omega\Big] = RHS_z$$

Effective slip length as a function of wavenumber *ka* with no-slip boundary condition

Stokes solution, $L_0 = 0$





$$ka = \frac{2\pi a}{\lambda}$$

wavelength λ is fixed
while amplitude *a* is varied

(■): numerical simulation
(—): analytical solution, fourier expansion of streaming function

$$L_{eff} = \frac{L_0\omega_\infty(ka) - ka^2\omega_0(ka)/(1+2kL_0)}{1+k^3a^2L_0}$$

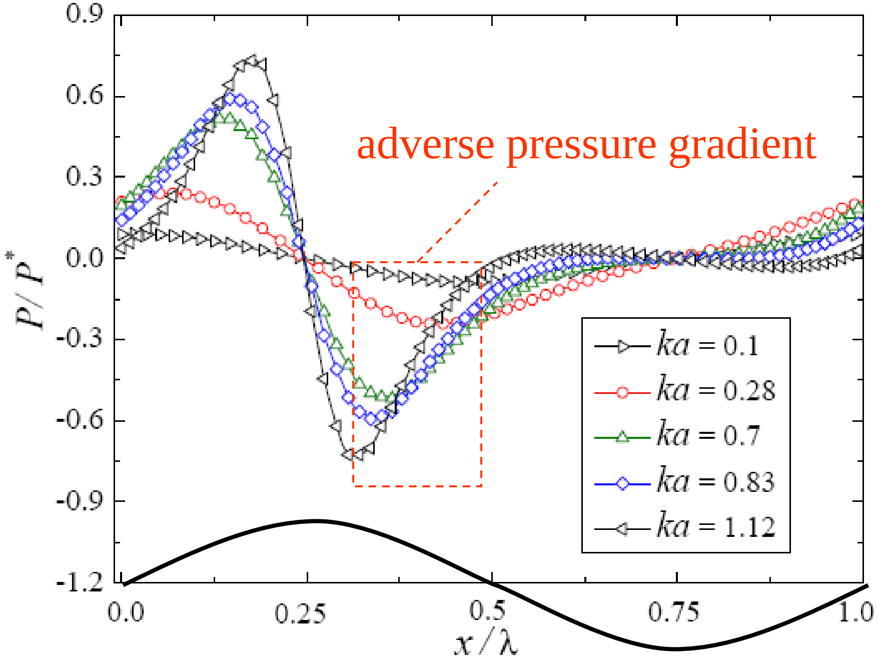
Range of applicability $ka \le 0.5$

Einzel et al., Phys. Rev. Lett. **64**, 2269 (1990)

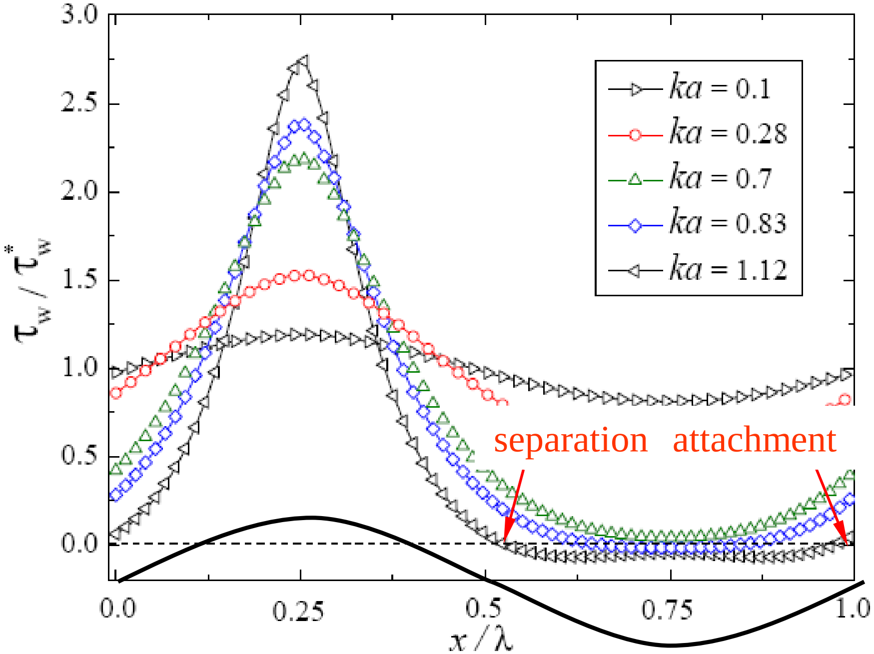
- Effective slip length decays as a function of *ka*
- Velocity profiles are linear in the bulk region
- A flow circulation appears at *ka* = 0.79

Pressure and shear stress profiles from Stokes solution for $L_0 = 0$ as a function of ka

Pressure profiles along the curved surface



Wall shear stress profiles

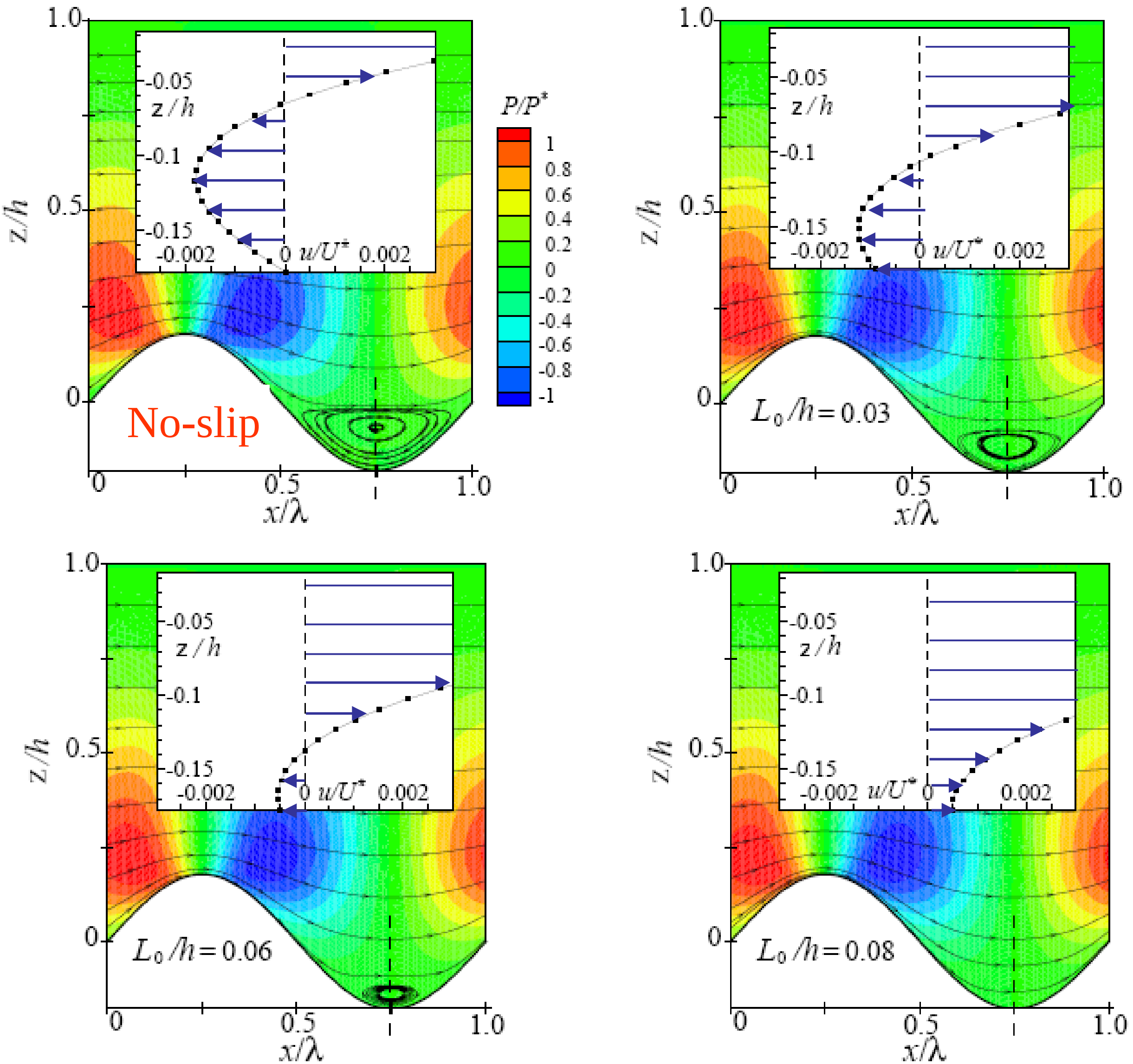


- Adverse pressure gradient increases at larger wave amplitudes a
- The adverse pressure gradient combined with the wall shear stress slows down the flow after the peak, which leads to flow separation at large amplitudes

- The wall shear stress τ_w becomes zero in the valley at the separation and attachment points
- The wall shear stress at the peak of corrugation increases at larger a

$$\tau_w = \mu\left(\frac{\partial u_t}{\partial n} + u_t / R(x)\right)\Big|_w$$

Pressure contours and streamlines with the local slip boundary condition



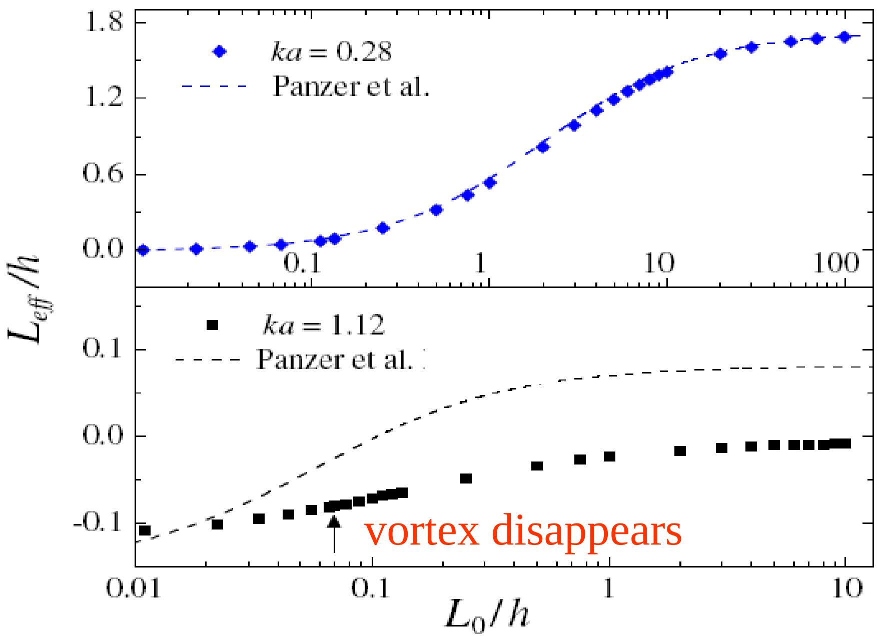
$$ka = \frac{2\pi a}{\lambda} = 1.12$$

With increasing the slip length L_0, the vortex gradually vanishes

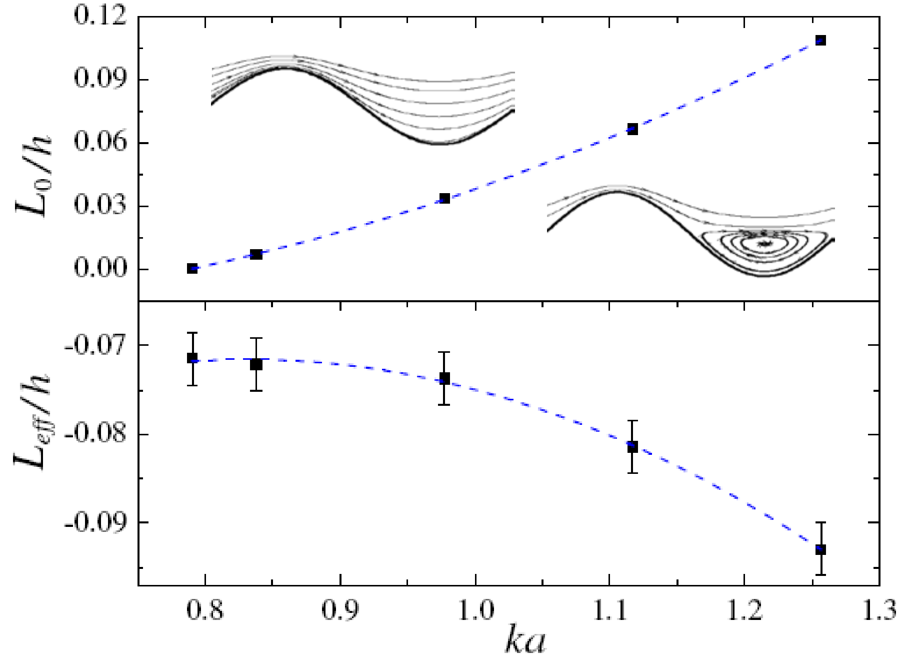
As the vortex becomes smaller the flow streamlines penetrate deeper into the valley and the effective slip length increases

Effective slip length from Stokes solution as a function of *ka* and local slip length L_0

Effective slip length as a function of L_0



Onset of vortex formation as a function of *ka*

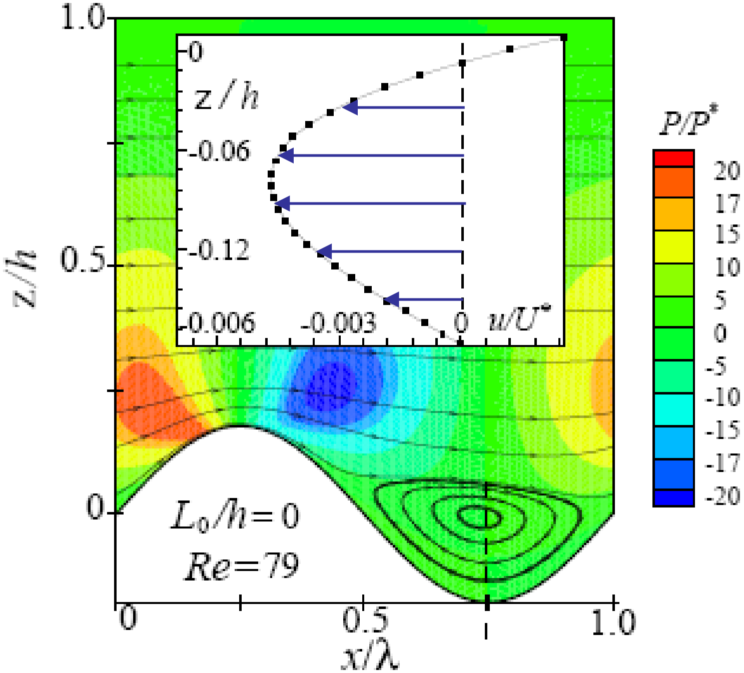


- At large amplitude the analytical results overestimate our numerical results
- The effective slip length saturates to constant value as L_0 increases
- While effective slip length L_{eff} increases, the the recirculation zone becomes smaller

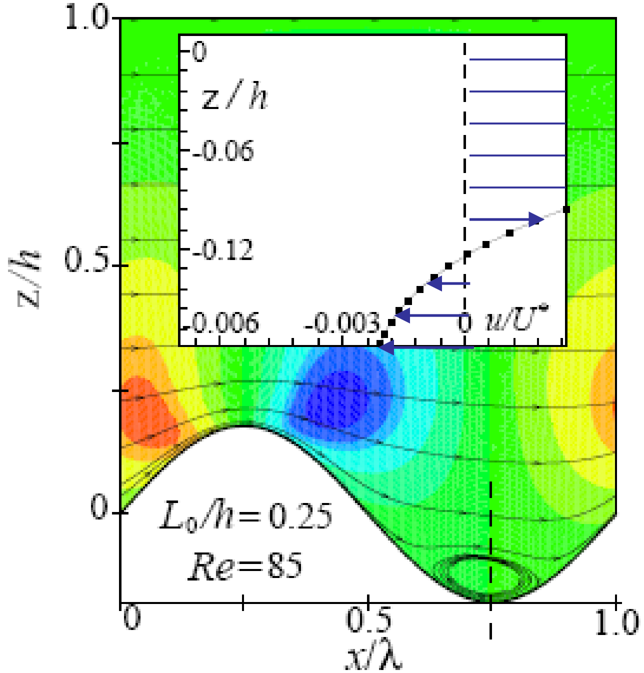
- As corrugation amplitude increases, the amount of local slip required to remove the vortex from the valley increases
- If the flow circulation is present in the valley, L_{eff} is negative

Inertial effects on the pressure contours and velocity profiles

No-slip boundary condition



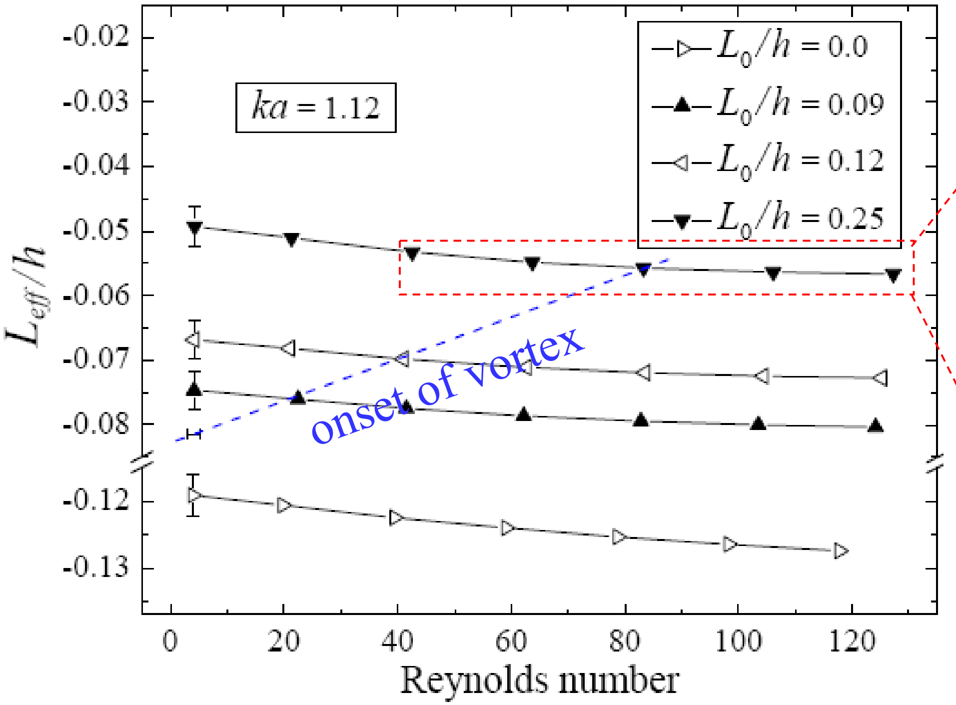
Local slip boundary condition



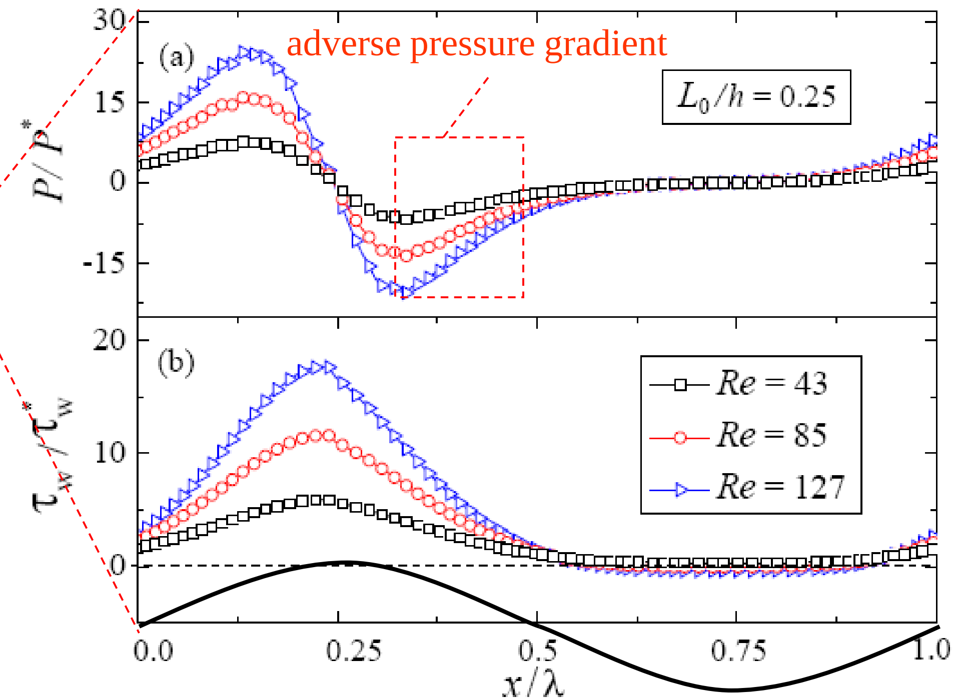
$$\mathrm{Re} = \frac{\rho U^* h(1 + L_{eff}/h)}{\mu}$$

- Due to the inertial term in the Navier-Stokes equation, the vortex in the bottom of the valley becomes asymmetric
- In the presence of local slip condition the vortex size decreases and streamlines are deformed to follow the boundary curvature (similar to the Stokes flow case).

Effective slip length as a function of local slip length and Reynolds number



Pressure and wall shear stress profiles



- With increasing *Re* the streamlines move away from the lower boundary and the no-slip plane is shifted into the bulk region and the effective slip length becomes smaller
- Below the blue dashed line, the circulation is always present in the valley and recirculation zone grows as *Re* increases

- The adverse pressure gradient and the wall shear stress on the right side of the peak increase as the *Re* number becomes larger and a vortex appears in the valley when *Re*>85

Important conclusions

- In the case of Stokes flow with the local no-slip boundary condition the effective slip length decreases with increasing corrugation amplitude and a vortex is formed in the valley for $ka \geq 0.79$.
- In the presence of the local slip boundary condition along the wavy wall, the effective slip length increases and the size of recirculation zone is reduced.
- The vortex vanishes at sufficiently large values of the intrinsic slip length L_0.
- Inertial effects promote vortex formation in the valley and reduce effective slip length.
- The growth or decay of the vortex as a function of either Reynolds number or intrinsic slip length is accompanied by the decrease or increase of the effective slip length [a control mechanism for vortex formation in microfluidic channels].

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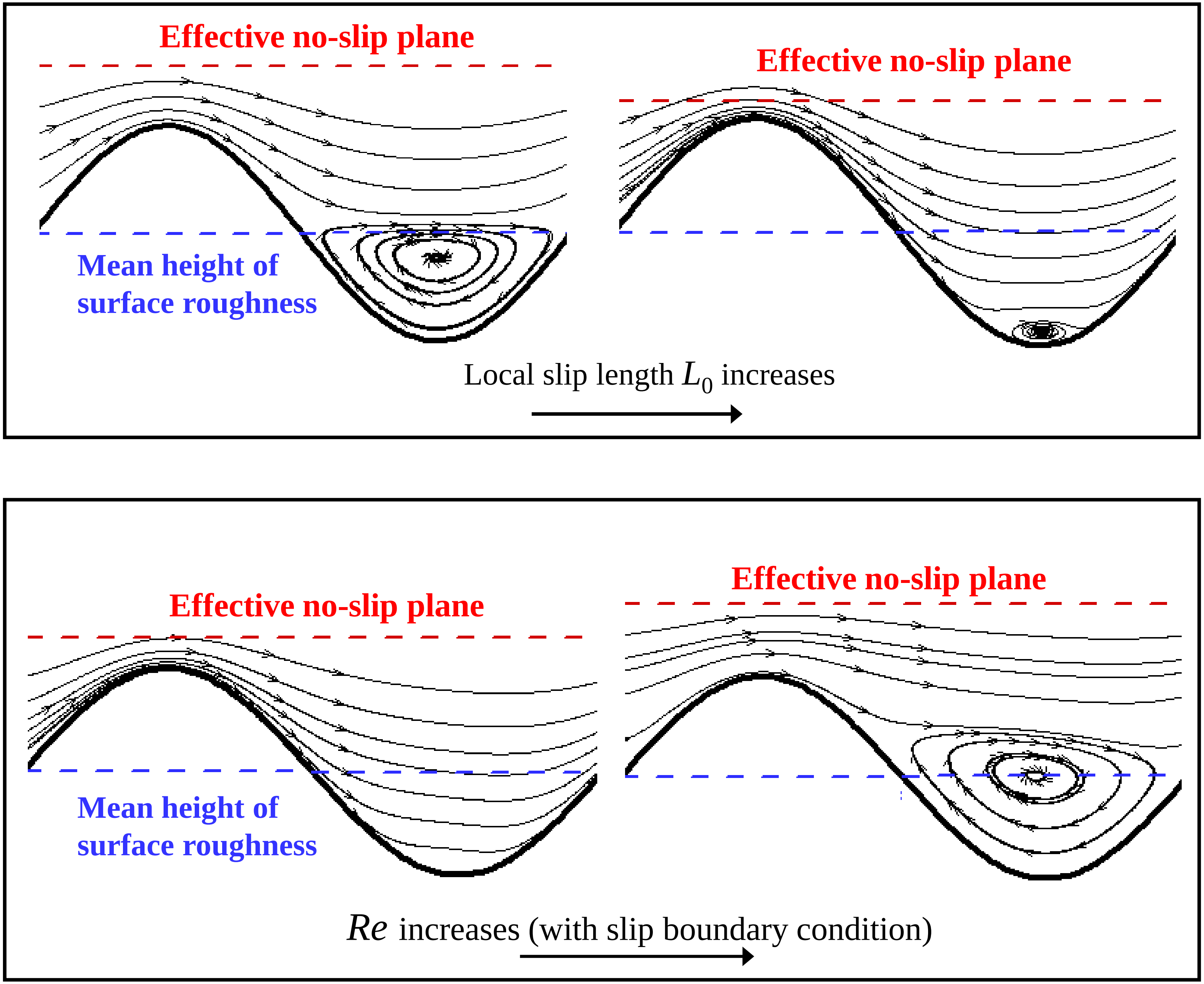
Effective no-slip plane
Effective no-slip plane
Mean height of
surface roughness
Local slip length L_0 increases
Effective no-slip plane
Effective no-slip plane
Mean height of
surface roughness
Re increases (with slip boundary condition)