2011

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A Proof that $P \neq NP$

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September 2010

Abstract
We demonstrate the separation of the complexity class $NP$ from its subclass $P$.

Preliminaries
Preliminary definitions and background can be found in [Sudkamp, 2006], and the following are taken from [Sudkamp, 2006].

[Sudkamp, 2006, Section 8.7]: Every nondeterministic Turing Machine can be simulated by a deterministic Turing Machine. Hence, they give rise to the same notion of computability.

[Sudkamp, 2006, Definition 8.8.1]: A deterministic (k-tape) Turing Machine enumerates a language $L$ if all of the following hold.

- The computation begins with all tapes blank.
- With each transition, the tape head on tape 1 (the output tape) remains stationary or moves to the right.
- At any point in the computation, the nonblank portion of tape 1 has the form $B\#u1\#u2\ldots\#uk\#\quad$ or $\quad B\#u1\#u2\ldots\#uk\#v$ where $u_1, u_2, \ldots$ are in $L$ and $v$ is a string over the tape alphabet.
- A string $u$ will be written on tape 1 preceded and followed by $\#$ if, and only if, $u$ is in $L$.

[Sudkamp, 2006, Theorem 8.8.6]: A language is recursively enumerable if, and only if, it can be enumerated by a deterministic Turing Machine.

The following is easily shown from the above. We include a proof for completeness.

**Theorem 1**
A language is recursively enumerable if, and only if, it can be enumerated by a nondeterministic Turing Machine.

**Proof.**
By the results cited above, a language is recursively enumerable if, and only if, it can be enumerated by a deterministic Turing Machine, while deterministic Turing Machines can simulate nondeterministic ones (and vice versa). qed.

Results
We now proceed to the new results.
Theorem 2
Every set of non-negative integers is recursively enumerable.

Proof.
Let S be an arbitrary set of non-negative integers. Let L be the language containing exactly those strings over \( \{0,1\} \) which are binary representations of a number in S.

Now consider the following (1-tape) nondeterministic Turing Machine M, where q0 is the start state, and B stands for a blank read from the tape.

![Turing Machine Diagram]

Obviously, there is a computation of M which produces L (and therefore S). By Theorem 1 we have that L, and therefore S, is recursively enumerable. Since S was chosen arbitrarily, any set of non-negative numbers is recursively enumerable. qed.

Corollary 1
The set of all subsets of the non-negative integers is countable.

Proof.
Since every Turing Machine can be described by a finite string (or, use Gödel numbering), the set of all Turing Machines is countable. Since every subset of the non-negative integers can be enumerated by a Turing Machine (Theorem 2), the set of all these subsets must be countable. qed.

Corollary 2
The theoretical foundations of Computer Science are contradictory.

Proof.
Georg Cantor has shown (using a diagonalization argument) that the set of all subsets of the non-negative integers is uncountable, which contradicts Corollary 1. qed.

Corollary 3
\( P \neq NP \).

Proof.
Since the theoretical foundations of Computer Science are contradictory, the statement follows immediately. qed.

References