Low Cost Lightweight Mode Forming System for Angle of Arrival Estimation

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Low Cost Lightweight Mode Forming System
for Angle of Arrival Estimation

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Engineering

By

MARK ANTHONY STEWART
B.S.E.E., Wright State University, 2005

2009
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ABSTRACT

Stewart, Mark Anthony, M.S. Egr., Department of Electrical Engineering, Wright State University, 2009. Low Cost Lightweight Mode Forming System for Angle of Arrival Estimations

Current mode forming systems have high angular resolution and operate over large bandwidths. System size, complexity, and cost restrict the use of these systems to ground-based or large-airframe platforms. A spiral antenna in conjunction with mission-specific modularized planar microstrip mode formers would allow for angle of arrival estimations on smaller platforms such as a medium-sized UAV. This thesis investigates the mode forming characteristics of a planar four arm spiral antenna driven with a planar microstrip mode former designed to operate over a single personal communications band. The theoretical foundation for a closed-form angle of arrival estimation given by Penno and Pasala[3] in “Angle Estimation with a Multi-Arm Spiral Antenna” is analyzed and is compared with models simulated in Numerical Electromagnetics Code. The results of both the theoretical and simulated models are compared to measured data from each realized subsystem and the system as a whole. A determination is made that the simple model presented by Penno and Pasala[3] is not sufficient to determine angle of arrival information, and a more sophisticated model is needed.
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1. Introduction

1.1. Thesis Motivation

The ability to quickly locate enemy communications is one of the most important tasks in modern warfare. A lightweight and low cost direction finding system for use on a small unmanned aerial vehicle platform would increase the military’s ability to perform this function, and is the motivation for this thesis. A compact planar mode former that is modularized to be mission specific in conjunction with a planar four arm spiral antenna would demonstrate the utility of such a system to the military, and is the focus of this effort. The feasibility of such a system was studied and a proof-of-concept prototype was built and tested. Special care was taken towards designing a system that will be relatively small, lightweight, and has a low enough production cost to be considered expendable.

1.2. Thesis Objectives

The following were the objectives of this thesis:

- Outline the system parameters for the proposed system
- Discuss and compare the most common methods of direction finding
- Provide a justification for the method of direction finding used in this effort
- Discuss the theory and characteristics of a four arm spiral antenna
- Define a theoretical basis for Angle of Arrival (AOA) measurements
- Describe the function and characteristics of a mode former
- Discuss the design and development of an appropriate mode former
- Define a theoretical model of the chosen mode former
• Measure the physical prototype mode former using a Vector Network Analyzer
• Use the prototype mode former in conjunction with a flat four arm spiral antenna produced by Douglas Glass[1] from his master’s thesis entitled “INVESTIGATION OF CYLINDRICALLY-CONFORMED FOUR-ARM SPIRAL ANTENNAS” to produce a Angle of Arrival system
• Provide theoretical patterns and AOA estimations from the system
• Measure the patterns produced by the system
• Provide a comparison of the theoretical and measured results

1.3. System Specifications and Requirements

For this effort the proposed system was designed to work with only one communication device. This proof of concept can be extended to other devices in the future. Different modules could be placed on the vehicle depending on which type of communication device is being located. The module in this effort was designed for use with the cellular Personal Communications Service (PCS) which operates in two bands, 1850-1910 MHz for the Mobile Station, and 1930-1990 for the Base Station. Since the focus of this system is to spatially locate the mobile device, the system was designed to work between 1850 and 1910 MHz and has a center frequency of 1880 MHz.

The proposed system, including a spiral antenna and a processing device, is intended to operate on a small unmanned aerial vehicle. Thus weight and size are a critical issue. The entire system is contained within a 15cm cube and weighs less than 10 lbs.
1.4. Discussion of Scattering Parameters

The scattering parameters, or S-Parameters, was the basis on which most of the results given by simulations and measurements were analyzed. Therefore a basic understanding of this type of network analysis is given in this section to be sure that the reader can interpret the results. The following is based on the discussion of the scattering matrix given by Pozar [1].

The scattering parameters give a full description of a network as seen at its N ports. This form of network analysis relates the voltage waves that are incident on the ports to those reflected back from the ports. The scattering matrix is defined as:

\[
\begin{bmatrix}
V_1^- \\
V_2^- \\
\vdots \\
V_N^-
\end{bmatrix}
= 
\begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1N} \\
S_{21} & S_{22} & \cdots & S_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
S_{N1} & S_{N2} & \cdots & S_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1^+ \\
V_2^+ \\
\vdots \\
V_N^+
\end{bmatrix}
\]

or

\[ [V^-] = [S][V^+] \] (1.1)

where \( V^+ \) is the amplitude of the voltage wave incident on port n, and \( V^- \) is the amplitude of the voltage wave reflected from port n. A specific element of the scattering matrix can be determined by

\[ S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+=0 \text{ for } k \neq j} \] (1.2)
$S_{ij}$ is found by driving port $j$ with an incident wave of $V_j^+$, and measuring the reflected wave amplitude, $V_i^-$, coming from port $i$. The incident waves on all other ports except the $j$th port is terminated in matched loads to suppress the reflections, which means they are set to zero. $S_{ii}$ is the reflection coefficient into port $i$, and $S_{ij}$ is the transmission coefficient from port $j$ to $i$.

1.5. Simulation Method

1.5.1. Microwave Office

Microwave Office is a software package from Applied Wave Research that provides for the analysis of electromagnetic circuits and systems. Certain components of the system are of a planar microstrip design and were simulated using this software. These designs were modeled as certain combinations of transmission lines and in some cases additional lumped element devices. The S-Parameters are given for each design as an output of a simulation over a certain frequency band, and are compared to the theoretical results that would be expected of such a device.

The version of Microwave Office that was used for the simulations in this effort was a rather old one and did not have a provision for a full wave analysis of microstrip bends. An issue arose during this investigation concerning microstrip bends in one of the planar components. This issue is discussed in detail later in this document.
1.5.2. Numerical Electromagnetics Code

The simulation software that was used in this effort to predict the resulting patterns radiated from the spiral antenna that is fed with the mode former in this investigation was the Numerical Electromagnetics Code 4 XD (NEC4x) from the Livermore Laboratory. This code allows for the modeling of the spiral antenna as a segmented wire structure. It uses the Method of Moments solution to calculate the current on each wire and then produces the field pattern and polarization of the antenna.

The model that was used in this effort was one produced by Glass[1]. In his thesis he presents a detailed discussion on the theoretical basis for which NEC4x works along with a description on the production of this model. To produce the patterns that were simulated in this effort, the frequency and phase progression across the terminals of the antenna were changed during the simulations. The model was otherwise unchanged. The outputs of these simulations were given in tabulated data form contained in a text document. The fields were then plotted using MATLab™.

The majority of PCS based mobile stations have a vertically polarized antenna. As such, only the data from the simulation that relates to the vertical polarization of the spiral antenna was considered in this effort.

1.6. Measurement Method

1.6.1. Vector Network Analyzer

A vector network Analyzer (VNA) is a device that measures the magnitude and phase characteristics of network. It accomplishes this by comparing the incident signal that is transmitted from the analyzer with either the signal that is transmitted through the
network or the signal that is reflected back from the input of the network. The measurement results in a set of S-Parameters that can be stored in tabulated form in a text document.

The VNA used to measure the characteristics of the components fabricated in this effort was the Anritsu 37347C with an operating range of 40MHz to 20GHz and is shown in Figure 1.1. Prior to each test the VNA was calibrated using the standards produced by Anritsu and the procedures outlined in their manual.

![Figure 1.1 The Anritsu 37347C Vector Network Analyzer](image)

**1.6.2. Spectrum Analyzer**

A spectrum analyzer is device that shows the spectrum of an incoming signal. Each frequency component contained in the input signal is displayed as a power level...
corresponding to that frequency. The spectrum analyzer that was used in this investigation was the Anritsu MS2721A, and is shown in Figure 1.2.

Figure 1.2 The Anritsu MS2721A spectrum analyzer

1.6.3. Desktop Antenna Measurement System

The device used to measure the radiated patterns in this effort is the Desktop Antenna Measurement System (DAMS) produced by Diamond Engineering. It consists of a static tripod, a tripod with an elevation and azimuthal rotation, a controller, and PC based software. This system is used in conjunction with a VNA which acts as a transmitter and receiver. The software controls the movement of the rotation table and the frequency sweep of the VNA recording the $S_{21}$ for each point in space verses frequency. There is no outside interference due to the fact that the signal transmitted by the VNA is modulated and the receiver only processes that signal. The effects from multipath can be mitigated by properly range gating the VNA. The DAMS is shown in Figure 1.3.
Figure 1.3 Diamond Engineering’s Desktop Antenna Measurement Systems (DAMS)
2. Direction Finding

Direction finding is a crucial component in the modern battlefield. Locating the source of enemy signals allows for the gathering of intelligence, tracking, and coordination of attack. There are multiple methods to perform direction finding. The most common will be discussed in this chapter, including the positive and negative aspects of each method. The viability for this project will also be determined.

2.1. Two Element Comparison

The simplest form of direction finding can be done with a two-element array of antennas, and comparing either the phase or amplitude of the output of those antennas. A plane wave signal that impinges on the two-element array perpendicular to the plane of the array or at boresight there will have zero phase and amplitude difference between the two outputs antenna. However, if the signal impinges on the array at some angle off of bore sight it will be detected by one antenna before the other, as illustrated in Figure 2.1. There will then be a phase and amplitude difference between the outputs of the two antennas. The phase difference between elements will be the electrical distance that the signal must travel to reach the second antenna. The physical distance that the signal will travel will be \( d \cos \theta \). To calculate the electrical distance \( \psi \), we use the following equation:

\[
\psi = \text{Radians per Wavelength} \times \text{Physical Distance} \quad (2.1)
\]

\[
\psi = kd \cos \theta = \frac{2\pi}{\lambda} d \cos \theta = \frac{2\pi f}{c} d \cos \theta
\]
where

\[ c = \text{speed of light} \]

\[ f = \text{frequency of signal} \]

This electrical distance will be the phase difference between the outputs of antenna 1 and antenna 2. The angle of the incoming signal is given by:

\[ \theta = \cos^{-1}\left(\frac{c\psi}{2\pi f d}\right) \]  

(2.2)

![Figure 2.1 The Two Element Antenna Array (from Balanis[2])](image)

The advantages of this method are its simplicity, low cost, and small size. There are a two disadvantages to using this method. The first is that this method only works
azimuthally. No elevation information can be found. The second is that the equation for finding the incoming angle of the signal is obviously frequency dependent. The frequency of the signal must be known in order to accurately estimate the angle of arrival.

2.2. Beam Steering

Another method of direction finding involves a phased array of antennas. The radiation pattern for this array is determined by the amplitude and phase of the current at each antenna element. The phased array can have its beam steered electronically by changing the phase of each element. Multiple beams can be formed simultaneously using a phase control device that allows for all phase progressions to be present on the elements of the array at once. There are three types of arrays that are presented here; linear, planar, and circular. The equations of the following sections are adapted from Balanis[2].

2.2.1. Linear Arrays

A linear array typically consists of N identical antenna elements each fed with a signal of uniform amplitude, and with a phase difference of $\beta$ between each element. The geometry of linear array is shown in Figure 2.2.
Assume an N element array is positioned symmetrically along the positive and negative Z-axis. The Pattern Multiplication Theorem states that the total radiated field from the array is equal to the field of a single element of that array positioned at the origin multiplied by the array factor. The array factor is given as:

\[ AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad (2.3) \]

where

\[ \psi = kd \cos \theta + \beta \]

It can be shown that the array factor in a normalized form can also be written as:

\[ AF = \frac{1}{N} \left[ \sin \left( \frac{N}{2} \psi \right) \right] / \left[ \sin \left( \frac{1}{2} \psi \right) \right] \quad (2.4) \]
Since the phase progression between elements is a controlled variable the maximum radiation can be scanned to any direction. If each element had a variable phase shifter that a continuous scan could be made. With this configuration the amplitude of the combined array output could be compared at each point in progression of the phase shift. A maximum at one point would constitute an incoming signal at that angle of arrival.

In order to find the relationship between the phase progression and the angle of the beam it will be assumed that the maximum is at 90 degrees. At this beam angle the phase difference between the outputs should be 0. Therefore the phase excitation between the elements would be:

\[
\psi = kd \cos \theta + \beta |_{\theta=\theta_0}
\]

\[
0 = kd \cos \theta_0 + \beta
\]

\[
\beta = -kd \cos 90^\circ
\]

\[
\beta = 0
\]

This method improves on the first method by the fact that it has a greater directivity, and the ability to scan the beam. Depending on the step size of the phase progression, very accurate results can be obtained. The downside to this method is the use of more antennas and the need for phase shifters adds to the size and the weight of the overall system. Fine resolution phase shifters are very expensive and therefore drive up the cost of the system. The total system power requirements are also increased due to the power needed for the phase shifters, and the power for the control system. One other negative aspect is the fact that this method still only produces azimuthal direction finding.
2.2.2. Planar Arrays

By adding antenna elements along a second axis the geometry shown in Figure 2.3 is obtained. Like the linear array the phase can be varied element to element in a well-defined progression along each axis to scan the main beam in any direction in the plane.

![Figure 2.3 Geometry of a planar antenna array](image)

The antenna factor for the planar array is:
The normalized antenna factor for the planar array can be shown to be:

\[
AF = \sum_{n=1}^{N} \sum_{m=1}^{M} e^{i(m-1)\psi_x} e^{i(n-1)\psi_y}
\]

\[
where:
\psi_x = kd_x \sin \theta \cos \phi + \beta_x
\]

and

\[
\psi_y = kd_y \sin \theta \sin \phi + \beta_y
\]

The normalized antenna factor for the planar array can be shown to be:

\[
AF = \left[ \frac{1}{M} \frac{\sin \left( \frac{M}{2} \psi_x \right)}{\sin \left( \frac{\psi_x}{2} \right)} \right] \left[ \frac{1}{N} \frac{\sin \left( \frac{N}{2} \psi_y \right)}{\sin \left( \frac{\psi_y}{2} \right)} \right]
\]

This configuration of this antenna array would give both the azimuthal and elevation information needed. However to have a high resolution there would need to be a large number of antennas. There would also need to be MxN phase shifters, and the control mechanism for the shifting would be very complex. For the intended UAV application here the weight, size, and power consumption of this system would be too large.

2.3. Circular Arrays

The circular array has antenna elements which are evenly spaced in a circular ring and is shown in Figure 2.4. Assuming that N isotropic elements are in the x-y plane at a radius of \(a\) the normalized field of the array is given as
\[ E_n(r, \theta, \phi) = \sum_{n=1}^{N} a_n \frac{e^{-jkR_n}}{R_n} \]  

(2.7)

where \( R_n \) is the distance from the \( n^{th} \) element to a point in space. The distance when \( r \gg a \) can be shown to be

\[ R_n = r - a \sin \theta \cos(\phi - \phi_n) \]

Figure 2.4 Geometry of a circular antenna array

Assuming that \( r \gg a \) we can also assume that \( R_n \approx r \) and therefore
where

\[ \phi_n = 2\pi \frac{n}{N} \]

\[ l_n = \text{amplitude of the } n^{th} \text{ element} \]
\[ \alpha_n = \text{phase excitation of the } n^{th} \text{ element} \]

To direct the peak of the main beam in the \((\theta_0, \phi_0)\) direction the phase excitation of the \(n^{th}\) element becomes

\[ \alpha_n = -ka \sin \theta_0 \cos (\phi_0 - \phi_n) \]

and the equation then becomes

\[ E_n(r, \theta, \phi) = \frac{e^{-jkr}}{r} \sum_{n=1}^{N} l_n e^{j[\alpha_n + ka \sin \theta \cos (\phi - \phi_n) - \sin \theta_0 \cos (\phi_0 - \phi_n)]} \]  \hspace{1cm} (2.10)

Once again this is a great system for direction finding. All needed information can be gathered at a high resolution. However the physical structure would be too large, the control system would be complex, and would need high power to operate. Size, complexity of control, and power consumption have eliminated all the viable designs so far and will cause this one to be eliminated also.
2.4. Spiral Antennas

The previous designs need multiple antenna elements and in most cases the angular resolution increases with the total number of elements in the array. This of course causes the size and complexity of control to increase also. This leads to higher power consumption and the need for a large amount of power to be available to the system. What would be optimal is a system that utilizes only one antenna and can be controlled with a passive phase system. This would limit the size and power consumption and ultimately reduce the weight of the system.

Such a system does exist and would include a planar spiral antenna feed by a planar microstrip mode former. Such a system requires zero power consumption and is small enough to packaged in the size limitations required of this effort. The spiral antenna and mode former will be discussed in the following chapters.
3. Spiral Antennas

As stated in the previous chapter the spiral antenna appears to be an ideal candidate for this effort. The fact that only one antenna needs to be used which produces both elevation and azimuthal information on the impinging signal keeps the size and weight of the system down to a manageable level. The spiral antenna also operates over a broad frequency range as high as a 10:1 bandwidth. This chapter will discuss the common geometries used for spiral antennas, the modal operation of a spiral antenna, the antenna model, the fabricated spiral antenna, and derive the basis for calculating angle of arrival estimations.

3.1. Common Geometries

There are two common geometries used for planar spiral antennas. They are the logarithmic or equiangular and the Archimedean spiral. Both types of spirals have modal operation. The basis for the angle of arrival estimations is derived for the equiangular spiral, but the antenna used for this effort is an Archimedean. Section 3.3 discusses angle of arrival estimations and how the results presented for an equiangular spiral antenna under certain circumstances, can also be applied to the Archimedean spiral geometry. Since the characteristics of both types of spirals are needed for that discussion, both are discussed in the following paragraphs.

In an equiangular spiral the distance between each revolution of an arm increases as the angle traversed increases. Equiangular four-arm spiral antennas with varying flare rates are shown in Figure 3.1 to Figure 3.3. The radius of the n\textsuperscript{th} arm of an N-arm spiral is given by,
\[ r_n(\phi) = be^{a(\phi-\alpha_n)} \]  

where \( b \) is the initial radius, ‘\( a \)’ is the flare rate, and \( \alpha_n \) is the rotation angle relative to the x-axis. The rotation angle is defined as,

\[ \alpha_n = \frac{2\pi}{n} \]

Figure 3.1 Equiangular Spiral of 3 and a half turns with a flare rate of 0.2
In an Archimedean spiral the distance between each revolution of an individual arm is constant as the angle increases. Archimedean four-arm spiral antennas with
varying flare rates are shown in Figures 3.4 to 3.6. The radius of the $n^{th}$ arm of an N-arm spiral is given by,

$$r_n(\theta) = a(\phi - \alpha_n) + b.$$  \hfill (3.3)

where $b$ is the initial radius, $a$ is the flare rate, and $\alpha_n$ is the rotation angle relative to the $x$-axis. The rotation angle for the Archimedean is the same as given in (3.2).

Figure 3.4 Archimedean Spiral of 3 and a half turns with a flare rate of 0.0075
Figure 3.5 Archimedean Spiral of 3 and a half turns with a flare rate of 0.00125

Figure 3.6 Archimedean Spiral of 3 and a half turns with a flare rate of 0.0004
Upon visual inspection of Figure 3.3 and Figure 3.6 it can be seen that as the flare rate of the equiangular spiral decreases it tends to look like an Archimedean spiral. This is an interesting and important characteristic that will be exploited in the derivation of the angle of arrival estimates in a later section.

3.2. Modal Operation

The property of the spiral antenna that makes it an ideal candidate for the system proposed in this effort is its ability to produce angle of arrival information. This capability is a result of exciting the antenna in different modes. The number of modes that a spiral antenna is capable of producing is equal to the number of n arms of the antenna minus one. The phase excitation for the $k^{th}$ arm of a spiral antenna with n arms for Mode $N$ is

$$\psi_k = 360^\circ \left( \frac{k - 1}{n} \right) \cdot N, \ k = 1, 2, ..., N. \quad (3.4)$$

The different phase progression across the terminals will cause the antenna to produce different field patterns as shown in Figure 3.7. In each of these modes the total phase progression is an integer multiple of 360°. By comparing the amplitude of two modes, the elevation of an incoming signal can be obtained, and by comparing the phase relationship between those same two modes, the azimuthal information can be obtained. For this system modes one and two will be used. Figure 3.8 illustrates how the azimuth angle is determined by subtracting the mode one and mode two output phases. In this figure the antenna surface is in the x-y plane where mode one has a circumference of one
wavelength, and mode two has a circumference of two wavelengths. Figure 3.9 shows the
difference of the output phases in relation to the physical azimuthal angle of the antenna.

Figure 3.7 Field Patterns from different modes of a four arm spiral antenna (from Penno and
Pasala[3])
Figure 3.8 Azimuthal angle determination from the difference in modal outputs (from Glass[1])

Figure 3.9 Relation of the difference of phase between modes 1 and 2 and the physical location on the antenna (from Corzine and Mosko[5])
All antenna patterns in this thesis will be presented in the format outlined in Figure 3.10.

Figure 3.10 Presentation format of antenna patterns (adapted from Glass[1])

### 3.3. Angle of Arrival Estimations

To use a spiral antenna to obtain angle of arrival information the antenna must support both modes one and two. In order to support these modes a well defined phase progression must be present across the terminals of the arms. A mode former accomplishes this task and will be discussed in a later chapter. Closed form expressions for the output signals of the mode former can obtained with the understanding of how a spiral antenna radiates.
Penno and Pasala\cite{3} model the modal output and derive closed form expressions for calculating angle of arrival estimations with an equiangular spiral antenna. Their model is based on two assumptions. The first is that there is no mutual coupling between the arms of the spiral. Secondly the amplitude of the current is constant throughout. The results obtained from Penno and Pasala\cite{3} will then be used to derive expressions for the Archimedean spiral antenna which is used in this effort.

The open circuit voltage at the terminals of the antenna is given by,

\begin{equation}
V_{oc} = \frac{1}{I_e} \int_{v_{ol}} \vec{E}_{inc} \cdot \vec{J}_t \, dv
\end{equation}

\( \vec{E}_{inc} \) is the incident electric field, \( \vec{J}_t \) is the current density distribution on the antenna, and \( I_e \) is the terminal current. The length traveled along the arm of a spiral will be used to calculate the current density. The length along an arc can be expressed as,

\begin{equation}
ds = \sqrt{dx^2 + dy^2}
\end{equation}

The transformation from polar to Cartesian coordinates for \( x \) and \( y \) are defined as,

\begin{align*}
x &= r \cos \phi \\
\text{and} \\
y &= r \sin \phi
\end{align*}

Taking the differential of \( x \) and \( y \) gives,

\begin{align*}
dx &= \cos \phi \, dr - r \sin \phi \, d\phi \\
\frac{dy}{dx} &= \sin \phi \, dr + r \cos \phi \, d\phi
\end{align*}
and squaring both results in,

\[ dx^2 = \cos^2 \phi \, dr^2 + r^2 \sin^2 \phi \, d\phi^2 - 2r \cos \phi \sin \phi \, dr \, d\phi \]

and

\[ dy^2 = \sin^2 \phi \, dr^2 + r^2 \cos^2 \phi \, d\phi^2 + 2r \cos \phi \sin \phi \, dr \, d\phi \]

Putting these results into equation (3.2) we have,

\[ ds = \sqrt{\cos^2 \phi \, dr^2 + r^2 \sin^2 \phi \, d\phi^2 + \sin^2 \phi \, dr^2 + r^2 \cos^2 \phi \, d\phi^2} \]

\[ ds = \sqrt{dr^2 + r^2 d\phi^2} \]

\[ ds = \frac{dr^2}{\sqrt{d\phi^2 + r^2 \, d\phi}} \]  

(3.3)

Taking the derivative of \( r_n \) with respect to \( \phi \) and squaring the results gives,

\[ dr_n^2 = a^2 b^2 e^{2a(\phi - \alpha_n)} \, d\phi^2 \]

Putting this result into (3.3) gives,

\[ ds = \sqrt{a^2 b^2 e^{2a(\phi - \alpha_n)} + b^2 e^{2a(\phi - \alpha_n)}} \, d\phi \]

\[ ds = b e^{a(\phi - \alpha_n)} \sqrt{a^2 + 1} \, d\phi \]  

(3.4)

Integrating from \( \alpha_n \) to \( \phi \).

\[ L_n = S = b \sqrt{a^2 + 1} \int_{\alpha_n}^{\phi} e^{a(\phi - \alpha_n)} \, d\phi \]
The distance along the spiral arm to a point that has a radius of $r_n$ is,

$$L_n = \frac{b}{a} \sqrt{a^2 + 1 \left(e^{a(\phi-a_n)} - 1\right)}$$

$$L_n = \sqrt{1 + \frac{1}{a^2} \left(b e^{a(\phi-a_n)} - b\right)}$$

The distance along the spiral arm to a point that has a radius of $r_n$ is,

$$L_n = (r_n - b) \sqrt{1 + \frac{1}{a^2}} \quad (3.5)$$

Assuming no mutual induction, the current on each arm of the spiral can be expressed as,

$$I_k(l) = I_{T,k} e^{-j\beta l} \quad (3.6)$$

$I_{T,k}$ is the terminal current on the $k^{th}$ arm, $\beta = \frac{2\pi}{\lambda}$, and $l$ is the length along that arm from the terminal. An N-arm spiral antenna has N-1 modes of operation. Each mode of operation requires a certain phase progression across the terminals. This phase progression for mode 1 is defined as,

$$\psi = -k \frac{2\pi}{N}, \quad k = 1, 2, ..., N$$

The currents at each terminal are then,

$$I_{T,k} = I_0 e^{-j\frac{2\pi k}{N}}, \quad k = 1, 2, ..., N \quad (3.7)$$

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The currents on each arm of the spiral are initially out of phase. However, as the current travels outward on each arm of the antenna there will be points at which the current on adjacent arms will be in phase and will sum together and will allow the antenna radiate from these regions. Figure 3.11 shows three points, P₁, P₂, and P₃, which are on adjacent arms of the spiral antenna. If the spiral is tightly wound, with the ‘a’ term in the exponent having a small value, the difference in radius between these three points can be considered negligible. The difference in length along the spiral arms between points P₁ and Pₖ is given by,

$$\Delta l = k \frac{2\pi r}{N},$$

and the phase difference due to the difference in length is,

$$\Delta \theta_1 = \beta \Delta l.$$
The total phase difference due to the terminal phase progression and the difference in length is then,

\[ \Delta \theta = \Delta \theta_1 - k \frac{2\pi}{N} \]

\[ \Delta \theta = -k \frac{2\pi}{N} (1 - \beta r) \]

When

\[ r = \frac{\lambda}{2\pi} \]

then

\[ \Delta \theta = 0, \]

and the currents at P\(_1\) and P\(_k\) are in phase. A circular region whose radius is \( \frac{\lambda}{2\pi} \) and has a circumference of \( 1\lambda \) is defined as the active region, and constitutes a mode 1 radiation.

By changing the phase progression across the terminals to \( -k \frac{4\pi}{N} \), the active region shifts out to a circular region of \( \frac{\lambda}{\pi} \) having a radius of \( 2\lambda \). This would constitute a mode 2 radiation.

The active circular region of the radiating antenna can be broken into \( N \) segments, with each segment corresponding to one of the \( N \) arms. The length traveled along one of the arms to a “virtual” terminal at the beginning of each segment is given by,

\[ L_n = (r_n - b) \sqrt{1 + \frac{1}{a^2}} \]  \hspace{1cm} (3.8)
The azimuthal coordinates of these virtual terminals for each $k^{th}$ arm are,

$$\phi_{n,k} = \frac{2\pi}{N} (k - 1) + \frac{1}{a} \ln \left( \frac{n\lambda}{b} \right), \quad k = 1, 2, ..., N \quad (3.9)$$

The currents in the segments are given by,

$$\tilde{I}_k = I_{T,k} e^{-j \beta L_n} e^{-jn(\phi - \phi_{n,k})}, \quad \phi_{n,k} \leq \phi \leq \phi_{n,k} + \frac{2\pi}{N} \quad (3.10)$$

The current in the active region previously defined due to mode-n is,

$$\tilde{I}_n = I_0 e^{j \psi_n} e^{-jn\phi} \tilde{a}_\phi, \quad 0 \leq \phi \leq 2\pi \quad (3.11)$$

To determine the phase, this expression evaluated at $\phi = \phi_{n,1}$,

$$I(\phi_{n,1}) = I_0 e^{j \psi_n} e^{-jn\phi_{n,1}} = I_0 e^{j \psi_n} e^{-jn \frac{1}{a} \ln(n\lambda/b)} \quad (3.12)$$

is made to be equal to the current at the virtual terminal at $\phi = \phi_{n,1}$,

$$I_0 e^{-j \beta L_n} = I_0 e^{j \psi_n} e^{-jn \frac{1}{a} \ln(n\lambda/b)} \quad (3.13)$$

which gives

$$\psi_n = -\beta L_n + n \frac{1}{a} \ln \left( \frac{n\lambda}{b} \right) \quad (3.14)$$

The current is then given by the expression,

$$\tilde{I}_n = I_0 e^{j \left( -\beta L_n + n \frac{1}{a} \ln(n\lambda/b) \right)} e^{-jn\phi} \tilde{a}_\phi, \quad 0 \leq \phi \leq 2\pi \quad (3.15)$$
With the resultant expressions for the current found, the induced current density distribution can be determined. With the antenna located in the x-y plane the unit vector in the direction of the propagation of the incident wave is given by,

\[ u_i = -[\sin \theta_0 \cos \phi_0 \hat{x} + \sin \theta_0 \sin \phi_0 \hat{y} + \cos \theta_0 \hat{z}] \]

The plane wave is given by

\[ \vec{E} = E_0 \hat{\rho} e^{-j\beta u_i \cdot r} \quad (3.16) \]

where the unit vector \( \hat{\rho} \) is the polarization of the incident field. In this discussion there are two polarizations of interest. The first is the \( \theta \) polarization and is shown in Figure 3.12.
It can be seen by the geometry that the polarization vector will have components in the x, y, and z-direction. To begin alpha is defined as,

$$\alpha = 180 - 90 - \theta_0 = 90 - \theta_0$$

Consequently the component in the z-direction is given by,

$$\cos \alpha \hat{z} = \cos(90 - \theta_0) \hat{z} = \sin \theta_0 \hat{z}$$

The projection onto the x-y plane is given by,

$$\sin \alpha = \sin(90 - \theta_0) = \cos \theta_0$$

with the component of the projection in the x-direction expressed as,

$$\cos \theta_0 \cos(\phi_0 + 180) \hat{x} = -\cos \theta_0 \cos \phi_0 \hat{x}$$

and the component of the projection in the y-direction expressed as,

$$\cos \theta_0 \sin(\phi_0 + 180) \hat{y} = -\cos \theta_0 \sin \phi_0 \hat{y}$$

The polarization vector for a $\theta$ polarization is then,

$$\hat{p}_\theta = -\cos \theta_0 \cos \phi_0 \hat{x} - \cos \theta_0 \sin \phi_0 \hat{y} + \sin \theta_0 \hat{z}$$

The second polarization of interest is one directed in the $\phi$ direction. The geometry of this situation is shown in Figure 3.13. There is no component in the z-direction. The component in the x direction is,

$$-\sin \phi \hat{x},$$
and the component in the y-direction is given by,

\[ \cos \phi \hat{y} \]

So the polarization vector for a \( \phi \) polarization is then,

\[ \hat{p}_\phi = -\sin \phi_0 \hat{x} + \cos \phi_0 \hat{y} \]

The unit vector tangent to the current is similar to the \( \phi \) polarization but has an angle of \( \phi' \) instead of \( \phi_0 \), and is given as,

\[ \hat{a}_{\phi'} = -\sin \phi' \hat{x} + \cos \phi' \hat{y} \]

Therefore, the term involving the dot product of polarization vector and tangent to the current vector can be expressed for each of the polarizations by the following.
\[
\hat{p}_\theta \cdot \hat{a}_\phi' = (- \cos \theta_0 \cos \phi_0)(- \sin \phi') - (\cos \theta_0 \sin \phi_0)(\cos \phi') \\
= \cos \theta_0 (\cos \phi_0 \sin \phi' - \sin \phi_0 \cos \phi') \\
\hat{p}_\theta \cdot \hat{a}_\phi = \cos \theta_0 \sin (\phi' - \phi_0)
\]

and

\[
\hat{p}_\theta \cdot \hat{a}_\phi' = \sin \phi_0 \sin \phi' + \cos \phi_0 \cos \phi = \cos (\phi' - \phi_0)
\]

In the expression for the incident field the ‘r’ term that resides in the exponent will only have terms in the x and y-direction. This is due to the fact that the arms of the antenna lie in the x-y plane. The interaction between the incident field and the antenna will occur at the circular active region located at \( r = \frac{n\lambda}{2\pi} \) from the center of the antenna. The ‘r’ term can then be expressed as,

\[
r = \left(\frac{n\lambda}{2\pi}\right)(\cos \phi' \hat{x} + \sin \phi' \hat{y})
\]

So the \( \hat{u}_i \cdot r \) term in the expression for the incident field then becomes,

\[
e^{-j\beta \hat{u}_i \cdot r} = e^{-j\left(\frac{2\pi}{\lambda}\right)\left(\frac{n\lambda}{2\pi}\right)(- \cos \phi' \sin \theta_0 \cos \phi_0 - \sin \phi' \sin \theta_0 \sin \phi_0)} \\
= e^{-jn \sin \theta_0 (- \cos \phi' \cos \phi_0 - \sin \phi' \sin \phi_0)} \\
e^{-j\beta \hat{u}_i \cdot r} = e^{jn \sin \theta_0 \cos (\phi' - \phi_0)}
\]

and the incident field can then be expressed for each polarization as

\[
\vec{E}_\theta = E_0 \cos \theta_0 \sin (\phi' - \phi_0) \ e^{jn \sin \theta_0 \cos (\phi' - \phi_0)} \quad (3.17)
\]
and

\[ \vec{E}_\phi = E_0 \cos(\phi' - \phi_0) e^{j m \sin \theta_0 \cos(\phi' - \phi_0)} \]  \hspace{1cm} (3.18)

The induced open circuit voltage is

\[ V_{oc,k} = \oint \vec{E}_{inc} \cdot \hat{t}_t^k \, dl_m' \]  \hspace{1cm} (3.19)

\( V_{oc,m} \) is the induced voltage at the \( m^{th} \) terminal, \( \hat{t}_t^k \) is the normalized current at that terminal and is given by

\[ \hat{t}_t^k = \frac{\vec{I}_k |_{\phi=\phi_{n,k}}}{I_{T,k}} = e^{-j \beta L_n \hat{e}_k} \]  \hspace{1cm} (3.20)

where \( \hat{e}_m \) is the unit vector in the direction of the current at point \( L_m \). The mode former’s outputs are a weighted sum of the terminal voltages. The output of mode \( n \) of an \( N \)-arm spiral can be expressed as

\[ M^n = \sum_{k=1}^{N} W_{n,k} V_{oc,k} \]

with

\[ W_{n,k} = e^{-j \left( \frac{2 \pi}{N} \right) (k-1)n} \]
This is the N-point discrete Fourier transform of the terminal voltage of the spiral. The voltage for mode \(n\) is then

\[
M^n = \sum_{k=1}^{N} \oint_{C_k} E_0(\hat{p} \cdot \hat{k}) e^{-j\beta \hat{u}_r \cdot r} \left[ W_{n,k} e^{-j\beta t_m} \right] dl_m
\]

(3.21)

\(C_k\) is the contour corresponding to the \(k^{th}\) arm of the spiral. The active region that contributes to the \(n^{th}\) mode consists of the circular region with radius of \(\frac{n\lambda}{2\pi}\). In the active region the normalized current can be given as

\[
\tilde{I}_n = \frac{I_n}{I_0} = e^{i\left(-\beta L_n + \frac{n\lambda}{2}\ln\left(\frac{b}{\beta}ight)\right)} e^{-jn\phi} \hat{a}_\phi.
\]

(3.22)

The mode \(n\) output can then be expressed as

\[
M^n = \sum_{k=1}^{N} \oint_{S_k} E_0(\hat{p} \cdot \hat{a}_\phi) e^{-j\beta \hat{u}_r \cdot r} \left[ e^{i\left(-\beta L_n + \frac{n\lambda}{2}\ln\left(\frac{b}{\beta}\right)\right)} e^{-jn\phi} \right] dl_m
\]

(3.23)

\(S_k\) is the segment of the active region of the \(k^{th}\) spiral arm. The contributions from each of the active regions can be added together and the mode \(n\) output is then

\[
M^n = \int_{0}^{2\pi} E_0(\hat{p} \cdot \hat{a}_\phi) e^{-j\beta \hat{u}_r \cdot r} \left[ e^{i\left(-\beta L_n + \frac{n\lambda}{2}\ln\left(\frac{b}{\beta}\right)\right)} e^{-jn\phi} \right] \left(\frac{n\lambda}{2\pi}\right) d\phi.
\]

(3.24)

This can be evaluated in closed form to produce expressions for the modal outputs for each polarization. For the \(\theta\) polarization the modal output becomes,

\[
M_\theta^n = \int_{0}^{2\pi} E_0 \cos \theta_0 \sin(\phi' - \phi_0) e^{in\sin\theta_0 \cos(\phi' - \phi_0)} \left[ e^{i\left(-\beta \theta_L + \frac{n\lambda}{2}\ln\left(\frac{b}{\beta}\right)\right)} e^{-jn\phi} \right] \left(\frac{n\lambda}{2\pi}\right) d\phi'.
\]
Upon making the substitution

\[ u = \phi' - \phi_0 \]

the modal output becomes,

\[
M_{\theta}^{\text{n}} = E_0 e^{i \left( -\beta L_n - \frac{n}{a} \ln \left( \frac{n \lambda}{b} \right) \right) + \frac{n \lambda}{2 \pi} \cos \theta_0} \int_0^{2\pi} \sin(u) e^{j n \sin \theta_0 \cos \phi' e^{-j n u} d\phi'}
\]

Substituting the sin term with its exponential form and factoring out the constants gives,

\[
M_{\theta}^{\text{n}} = E_0 e^{i \left( -\beta L_n - \frac{n}{a} \ln \left( \frac{n \lambda}{b} \right) \right) + \frac{n \lambda}{2 \pi} \cos \theta_0} e^{-j n \phi_0} \int_0^{2\pi} (e^{j u} - e^{-j u}) e^{j n \sin \theta_0 \cos u} e^{-j n u} du
\]

rearranging the exponentials in the integral leads to,

\[
M_{\theta}^{\text{n}} = E_0 e^{i \left( -\beta L_n - \frac{n}{a} \ln \left( \frac{n \lambda}{b} \right) \right) + \frac{n \lambda}{2 \pi} \cos \theta_0} \int_0^{2\pi} (e^{-j (n-1) u} - e^{-j (n+1) u}) e^{j n \sin \theta_0 \cos u} du
\]

and distributing the sine and cosine exponentials the equation then becomes

\[
M_{\theta}^{\text{n}} = E_0 e^{i \left( -\beta L_n - \frac{n}{a} \ln \left( \frac{n \lambda}{b} \right) \right) + \frac{n \lambda}{2 \pi} \cos \theta_0} \int_0^{2\pi} e^{-j (n-1) u} e^{j n \sin \theta_0 \cos u} du - \int_0^{2\pi} e^{-j (n+1) u} e^{j n \sin \theta_0 \cos u} du
\]
An integral representation of the Bessel function of the first kind of order \( n \) is

\[
J_n(x) = \frac{j^{-n}}{2\pi} \int_0^{2\pi} e^{jx\cos\phi} e^{j(n\phi)} d\phi
\]

and keeping in mind that cosine is an even function the integration terms in the modal output can then be expressed as,

\[
\int_0^{2\pi} -e^{jn\sin\theta_0 \cos(-u)} du - \int_0^{2\pi} -e^{jn\sin\theta_0 \cos(-u)} du
\]

\[
= -2\pi j^{n-1} J_{n-1}(n \sin \theta_0) + 2\pi j^{n+1} J_{n+1}(n \sin \theta_0)
\]

\[
= j2\pi j^n [J_{n-1}(n \sin \theta_0) - J_{n+1}(n \sin \theta_0)]
\]

Making the modal output with a \( \theta \) directed polarization then becomes

\[
M_\theta^n = j^n \frac{n\lambda}{2} E_0 e^{j(-\beta L_n + n_1^2 \ln(n_2/b))} e^{-jn\phi_0} \cos \theta_0 [J_{n-1}(n \sin \theta_0) + J_{n+1}(n \sin \theta_0)] (3.27)
\]

For the \( \phi \) polarization the modal output is given by

\[
M_\phi^n = \int_0^{2\pi} E_0 \cos(\phi' - \phi_0) e^{jn \sin \theta_0 \cos(\phi' - \phi_0)} \left[e^{j(-\beta L_n + n_1^2 \ln(n_2/b))} e^{-jn\phi'}\right] \left(\frac{n\lambda}{2\pi}\right) d\phi'
\]

\[
(3.28)
\]

Making the same substitution as the first polarization
the modal output can be then becomes,

\[ M_{\theta}^n = E_0 \left( \frac{n \lambda}{2 \pi} \right) \left[ e^{i \left( -\beta l_n + \frac{n \lambda}{l} \ln \left( \frac{n \lambda}{b} \right) \right)} \right] \int_0^{2\pi} \cos u e^{jn \sin \theta_0 \cos u} e^{-j n \phi_0} du \]

\[ M_{\phi}^n = E_0 \left( \frac{n \lambda}{2 \pi} \right) \left[ e^{i \left( -\beta l_n + \frac{n \lambda}{l} \ln \left( \frac{n \lambda}{b} \right) \right)} \right] e^{-j n \theta_0} \int_0^{2\pi} \cos(-u) e^{jn \sin \theta_0 \cos(-u)} e^{-j n u} du \]

Then \[ M_{\phi}^n = E_0 \left( \frac{n \lambda}{2 \pi} \right) \left[ e^{i \left( -\beta l_n + \frac{n \lambda}{l} \ln \left( \frac{n \lambda}{b} \right) \right)} \right] \frac{1}{2} e^{-j n \theta_0} \int_0^{2\pi} e^{(n+1)(-u)} e^{jn \sin \theta_0 \cos(-u)} du + \int_0^{2\pi} e^{(n-1)(-u)} e^{jn \sin \theta_0 \cos(-u)} du \]

Again using the evenness of the cosine the integration terms in the expression become

\[ \int_0^{2\pi} e^{(n+1)(-u)} e^{jn \sin \theta_0 \cos(-u)} du + \int_0^{2\pi} e^{(n-1)(-u)} e^{jn \sin \theta_0 \cos(-u)} du \]

\[ = 2\pi j^{n+1} J_{n+1}(n \sin \theta_0) + 2\pi j^{n-1} J_{n-1}(n \sin \theta_0) \]

\[ = 2\pi j^{n+1} [J_{n+1}(n \sin \theta_0) - J_{n-1}(n \sin \theta_0)] \]

Making the modal output with a $\phi$ directed polarization then becomes

\[ M_{\phi}^n = j^{n+1} \frac{n \lambda}{2} E_0 \left[ e^{i \left( -\beta l_n + \frac{n \lambda}{l} \ln \left( \frac{n \lambda}{b} \right) \right)} \right] e^{-j n \theta_0} [J_{n+1}(n \sin \theta_0) - J_{n-1}(n \sin \theta_0)] \] (3.29)

These expressions for the modal outputs apply to an equiangular spiral antenna. However, the antenna used for this effort will be an Archimedean spiral. The radius of the $n^{th}$ arm of an N-arm Archimedean spiral is given by,
\[ r_n(\phi) = a(\phi - \alpha_n) + b. \]

where \( b \) is the initial radius, \( a \) is the flare rate, and \( \alpha_n \) is the rotation angle relative to the \( x \)-axis. The rotation angle is defined as,

\[ \alpha_n = \frac{2\pi}{n} \]

The Taylor Series Expansion for the equiangular spiral equation can then be expressed as,

\[ b e^{a(\phi - \alpha_n)} = b \sum_{k=0}^{\infty} \frac{[a(\phi - \alpha_n)]^k}{k!} = 1 + a(\phi - \alpha_n) + O([a(\phi - \alpha_n)]^2) \quad (3.30) \]

The argument for the difference in path length for adjacent arms requires that the difference in radius be negligible. As the difference in radius tends to zero, so does the flare rate. This means that ‘\( a \)’ would a very small number and the terms in the Taylor series expansion after the 1\textsuperscript{st} power term can therefore be ignored. If this is the case then the expression for the radius of the \( n \textsuperscript{th} \) arm of an equiangular becomes,

\[ r_n = b + ab(\phi - \alpha_n) \]

This expression is very similar to the expression for the Archimedean spiral. If the following substitutions are made then the \( L_n \) expression derived for the equiangular spiral can be used for the Archimedean case. The radius of the \( n \textsuperscript{th} \) arm of an \( N \)-arm Archimedean spiral is now given by,

\[ r_n = cb(\phi - \alpha_n) + b \]

where \( c \) is the flare rate ‘\( a \)’ divided by the initial radius ‘\( b \)’. The distance along the spiral to a point with a radius \( r_n \) for the Archimedean is then,
Using the expressions derived here the modal outputs for an equiangular spiral with some modification can be applied to the Archimedean spiral. Those expressions become,

\[ L_n = (r_n - b) \sqrt{1 + \frac{1}{c^2}}. \]

As previously stated only the vertical (θ) polarization is of interest in this effort. Plotting the previous expressions we can see the vertically polarized pattern formed by modes one and two for the Archimedean spiral. Figure 3.14 and Figure 3.15 show the pattern for mode one and Figure 3.16 and Figure 3.17 show the pattern for mode two.

\[
M_\theta^n = j^n \frac{n\lambda}{2} E_0 e^{j\left(-\beta l_n + \ln\left(\frac{n\lambda}{b}\right)\right)} e^{-j n \theta_0} \cos \theta_0 \left[ J_{n-1}(n \sin \theta_0) + J_{n+1}(n \sin \theta_0) \right]
\]

(3.31)

and

\[
M_\phi^n = j^{n+1} \frac{n\lambda}{2} E_0 \left[ e^{j\left(-\beta l_n + \ln\left(\frac{n\lambda}{b}\right)\right)} \right] e^{-j n \theta_0} \left[ J_{n+1}(n \sin \theta_0) - J_{n-1}(n \sin \theta_0) \right]
\]

(3.32)
Figure 3.14 Vertical slices of the theoretical pattern formed by mode one

Figure 3.15 Horizontal slices of the theoretical pattern formed by mode 1
Figure 3.16 Vertical slices of the theoretical pattern formed by mode two

Figure 3.17 Horizontal slices of the theoretical pattern formed by mode two
To calculate the angle of arrival estimate the ratio of the expressions for modes one and two are evaluated

\[
\frac{M_{\theta}^2}{M_{\theta}^1} = -\frac{\lambda E_0 e^{j\left(-\beta L_2 + 2\frac{1}{c} \ln\left(\frac{a}{b}\right)\right)}}{j \frac{\lambda}{2} E_0 e^{j\left(-\beta L_1 + 2\frac{1}{c} \ln\left(\frac{a}{b}\right)\right)}} e^{-j2\theta_0 \cos \theta_0 [J_1(2 \sin \theta_0) + J_3(2 \sin \theta_0)]} e^{-j\phi_0 \cos \theta_0 [J_0(\sin \theta_0) + J_2(\sin \theta_0)]}
\]  

(3.33)

which when reduced is

\[
\frac{M_{\theta}^2}{M_{\theta}^1} = 2 \frac{J_1(2 \sin \theta_0) + J_3(2 \sin \theta_0)}{J_0(\sin \theta_0) + J_2(\sin \theta_0)} e^{-j\left[\beta(L_2 - L_1) + \frac{1}{c} \ln\left(\frac{a}{b}\right)\right] + \phi_0}
\]  

(3.34)

Overlaying the vertical slices of the vertical plots as in Figure 3.18 it can be seen that for \(0^\circ \leq \theta \leq 90^\circ\) the magnitude difference between mode two and mode one is unique. This amplitude of this expression describes the elevation angle as shown in Figure 3.19. Secondly, the phase of this expression describes the azimuthal angle as shown in Figure 3.20.
Figure 3.18 Overlay of vertical cuts of mode one and mode two patterns

Figure 3.19 Difference in magnitude of mode two and mode one in relation to the elevation angle
3.4. Planar Four Arm Spiral Model

The Archimedean spiral produced by Glass[1] has the following properties:

- Outer Diameter (in) 5.684
- Lower Frequency (GHz): 0.661
- Inner Diameter (in): 0.282
- Upper Frequency (GHz): 13.323
- Strip Width (in): 0.141
- Arm Spacing (in): 0.203
- Number of Rotations: 3.25

Running Nec4x on the numerical model produced by Glass[1] with perfect excitation of the terminals of each arm produces the predicted patterns for the spiral antenna. Figure 3.21 shows the geometry of the model, Figure 3.22 and Figure 3.23 show the pattern...
formed by mode one, and Figure 3.24 and Figure 3.25 show the pattern formed by mode two. The standard vertical cuts in Figure 3.22 do not occur at the peak radiation. Figure 3.26 shows a vertical cut at the peak radiation.

Figure 3.21 Geometry of the spiral antenna NEC model
Figure 3.22 Vertical slices of the NEC4x simulated pattern formed by mode one.

Figure 3.23 Horizontal slices of the NEC4x simulated pattern formed by mode one.
Figure 3.24 Vertical slices of the NEC4x simulated pattern formed by mode two

Figure 3.25 Horizontal slices of the NEC4x simulated pattern formed by mode two
Comparing the mode 1 pattern of the NEC model with the theoretical mode 1 pattern reveals that the two patterns are very similar. The only deviation is below sixty degrees where the NEC pattern starts to “square” off. In contrast mode two of the theoretical and NEC models diverge greatly. The NEC model produces a “flower” pattern with four nulls located at 55, 145, 235, and 325 degrees.

Looking at the step right before Penno and Pasala’s[3] final result there were 4 quarter circles that were combined to give the complete circular region. These individual active regions for each of the arms were modeled in NEC. The geometry is shown in Figure 3.27 and the resultant patterns are shown in Figures 3.28 thru 3.32. The patterns for this quartered loop with the exception of rotation in the azimuth plane are identical to the patterns of the NEC modeled spiral antenna.
Figure 3.27 Geometry of the quartered loop NEC model

Figure 3.28 Vertical slices of the NEC4x simulated quartered loop pattern formed by mode one
Figure 3.29 Horizontal slices of the NEC4x simulated quartered loop pattern formed by mode one

Figure 3.30 Vertical slices of the NEC4x simulated quartered loop pattern formed by mode two
Figure 3.31 Horizontal slices of the NEC4x simulated quartered loop pattern formed by mode two

Figure 3.32 Vertical slices of the NEC4x simulated quartered loop pattern formed by mode two rotated to the maxima
There is an obvious difference between the two NEC models and the closed form expression presented by Penno and Pasala[3]. Mode one in all cases appears to be similar while mode two diverges for the NEC models dramatically from the Penno and Pasala[3] model. The most likely conclusion is that there are assumptions that are not valid in a realized form and also that there are physical phenomenon that are not being modeled.

The theoretical model looks to predict the modal behavior of the spiral antenna and not the model antenna itself. There are assumptions in the initial steps of the derivation that, in a realized antenna, cannot be discounted. The final equations for the theoretical field have constant amplitude current. This presents a problem in the realization of a spiral antenna. The amplitude of the current as it travels along the arm will not be constant for three reasons. First, the arms of the antenna are not a perfect conductor and there will be loss as the signal travels along the line. Secondly, the arms of the antenna are curved which will cause outward radiation. This will cause there to be even more loss as the signal travels along the arm. Finally assuming that there is no mutual coupling between adjacent arms is not valid. The derivation requires that the arms of the antenna are wrapped tightly. These two criteria are mutually exclusive. The more tightly wrapped spiral arms become the more mutual coupling increases. All three of these factors are taken into account with the NEC model of the spiral antenna, and are the likely cause of the diverging results.

To examine this concept let us consider an antenna model that physically resembles the active circular regions presented in the Penno and Pasala[3] paper, where each element has constant current amplitude, and the mutual coupling is limited. A sixteen element circularly disposed horizontal dipole array fits all of these criteria. The
radius of the array will be of the same length as the circular regions presented in the Penno and Pasala[3] discussion. Figure 3.33 shows the geometry of this configuration. Using equation 3.4 to determine the element to element phase progression and modeling this configuration in NEC4x results in the patterns in Figures 3.34 thru 3.37.

Figure 3.33 Geometry of the sixteen element array NEC model
Figure 3.34 Vertical slices of the NEC4x simulated sixteen element array pattern formed by mode one.

Figure 3.35 Horizontal slices of the NEC4x simulated sixteen element array pattern formed by mode one.
Figure 3.36 Vertical slices of the NEC4x simulated sixteen element array pattern formed by mode two

Figure 3.37 Horizontal slices of the NEC4x simulated sixteen element array pattern formed by mode two
As we can see from the previous figures the patterns from the sixteen element circularly disposed array are very similar to the results of the Penno and Pasala[3] paper. However, following the same requirements in that derivation, two separate arrays of sixteen elements would be needed, one array for mode one and another array for mode two, each with its own corresponding radius. A thirty two element antenna array is not feasible for this system if it is to maintain a low cost with minimal size and weight. We can consider an eight element array and see if the results still match the theoretical results from Penno and Pasala[3]. Figure 3.38 shows the geometry of this configuration while Figures 3.39 thru 3.42 show the resultant patterns.

Figure 3.38 Geometry of the eight element array NEC model
Figure 3.39 Vertical slices of the NEC4x simulated eight element array pattern formed by mode one

Figure 3.40 Horizontal slices of the NEC4x simulated eight element array pattern formed by mode one
Figure 3.41 Vertical slices of the NEC4x simulated eight element array pattern formed by mode two

Figure 3.42 Horizontal slices of the NEC4x simulated eight element array pattern formed by mode two
The resultant patterns from the eight element model are still very similar to the Penno and Pasala[3] model. The only divergence is some slight “squaring” of the pattern in mode two below sixty degrees. Again two separate arrays would be needed to model the spiral and sixteen elements, while certainly much more desirable than thirty two, still commands a complex phase system and more than the available real estate. Let us take this one step further and model a four element array. The geometry is shown in Figure 3.43 and the resultant patterns in Figures 3.44 thru 3.47.

Figure 3.43 Geometry of the four element array NEC model
Figure 3.44 Vertical slices of the NEC4x simulated horizontal dipole array pattern formed by mode one.

Figure 3.45 Horizontal slices of the NEC4x simulated horizontal dipole array pattern formed by mode one.
Figure 3.46 Vertical slices of the NEC4x simulated horizontal dipole array pattern formed by mode two

Figure 3.47 Horizontal slices of the NEC4x simulated horizontal dipole array pattern formed by mode two
Looking at the patterns produced by the four element array we can see that like the spiral antenna it greatly diverges from the patterns produced by the Penno and Pasala[3] model in mode two. The four element array in fact looks very similar to the patterns of the spiral antenna with mode one matching almost perfectly and mode two matching just as well although rotated in the azimuthal plane.

According to Mosko[5] the spiral antenna radiates when the adjacent arms of the antenna are in phase and add together and the phase progression between the arms is preserved. Modeling the current on the arms of the spiral antenna shows that this occurs at only one location for each of the arms in each of the modes, and that it occurs at a slightly different angular location for each of the modes. Figure 3.49 illustrates these locations for mode one and Figure 3.50 illustrates the locations for mode two.
Figure 3.49 Locations were adjacent arms are in phase and current progression is preserved for mode one

Figure 3.50 Locations were adjacent arms are in phase and current progression is preserved for mode two

Modifying the four element circular array demonstrated previously by locating the dipoles at the positions indicated in Figure 3.49 and Figure 3.50 and simulating this in
NEC it can be shown that the mode two pattern for the circularly disposed array of four dipoles is identical to the pattern of the spiral with mode two excitation. Figure 3.51 and Figure 3.54 show the geometry of the array for modes one and two respectively, while Figure 3.52 and Figure 3.53 show the simulated mode one pattern, and Figure 3.55 and Figure 3.56 show the simulated pattern for me two.

Figure 3.51 Geometry of the mode one four element array NEC model based on the spiral
Figure 3.52 Vertical slices of the NEC4x simulated four element array NEC model based on the spiral formed by mode one

Figure 3.53 Horizontal slices of the NEC4x simulated four element array NEC model based on the spiral formed by mode one
Figure 3.54 Geometry of the mode two four element array NEC model based on the spiral

Figure 3.55 Vertical slices of the NEC4x simulated four element array NEC model based on the spiral formed by mode two
Figure 3.56 Horizontal slices of the NEC4x simulated four element array NEC model based on the spiral formed by mode two

It appears that for this antenna operating at the given frequency that a circularly disposed four element horizontal dipole array better models the antenna’s performance than the Penno and Pasala[3] model. One of the reasons that the Penno and Pasala[3] model works well for mode one is that the mode one dipoles are located a quarter wavelength apart and the phase progression between each element is a quarter wavelength. Therefore the pattern from each dipole sums together such that the area in between elements is filled. However, in the case of the spiral antenna the radius of the mode two dipoles moves outward from the origin doubling the length between and the phase progression between elements doubles. This allows for the development of nulls between the elements to produce the “flower” pattern. To demonstrate this phenomenon let us look back at the eight element circular array. With the eight element array excited
in mode two the number of elements allow for the dipoles to be relatively close together even though the radius has increased. Both modes one and two had the circular pattern with no nulls as shown in the horizontal slices. If we were to excite this in mode three the radius of the elements would move outward and the phase progression would double again. It the proposed hypothesis holds true then we would expect to see a flower pattern in the horizontal slices of mode three. This is in fact what happens as illustrated in Figure 3.57.

![Figure 3.57 Horizontal slices of the NEC simulated 8 element array with mode three excitation](image)

Although accurate models have been obtained for the spiral antenna for both mode one and mode two a complication has arisen regarding the angle of arrival estimation calculations. The signal will not be strong enough in the nulls of the pattern to
determine any angular information. This will cause azimuthal ambiguity that cannot be resolved, and the spiral antenna’s usefulness in regards to angle of arrival information will be limited to certain angular regions and have large “blind spots”. The calculated difference in magnitude between mode two and mode one of the NEC simulated spiral to give elevation information is shown in Figure 3.58. The calculated phase difference to produce azimuthal information is shown in Figure 3.59 and we can see that the linear fit of that curve looks very similar to the theoretical phase difference show in Figure 3.20. However, it can be seen that there will be ambiguity in a region of approximately thirty three degrees to the left and right of each null as shown in Figure 3.60.

![Figure 3.58 Difference between the magnitude of mode one and two of the NEC simulated spiral in relation to the elevation angle](image-url)
Figure 3.59 Difference between the phase of mode one and two of the NEC simulated spiral in relation to the azimuth angle

Figure 3.60 Areas of ambiguity of the azimuthal phase relationship
4. The Butler Matrix

4.1. Overview

One way to dispense with the need for phase shifters for beam steering or mode forming is to use a Butler matrix. The theories and equations presented here in this section are adapted from Pattan [6].

A Butler matrix is composed of passive four-port quadrature couplers and fixed phase shifters. It has N inputs, and N outputs, and can be used to drive an array of N antennas. A Butler matrix produces N orthogonally spaced beams overlapping at the -3.9dB level, and each beam has the full gain of the array. A rudimentary Butler matrix is shown in Figure 4.1 with the output phase for the corresponding inputs.
The phase progressions of the outputs when feeding M2 are the opposite of the outputs when feeding M1. Similarly, feeding M4 produces the opposite outputs that feeding M3 does. The symmetry of the output phase of the structure seems to indicate that this Butler matrix will produce equivalent beams on both sides of the bore sight. Using the equation derived in section 3.2 for the relationship between phase progression and beam angle, the location of the beams can be found. The phase relationship for each input to each output is:

\[ M1 = -45^\circ \]
\[ M2 = +45^\circ \]
M3 = +135°
M4 = -135°

Putting these values into equation the following beam angles shown in Figure 4.2 were obtained:

θ1 = 75.5225°
θ2 = 104.4775°
θ3 = 41.4096°
θ4 = 138.5904°

Figure 4.2 Resultant Beams for Given Butler Matrix

One valuable property of the Butler matrix is that if all inputs are fed simultaneously, all of the beams are simultaneously produced. Consequently, all beams are available simultaneously to detect an incoming signal, and therefore no switches are needed to make the array scan. Whichever output has the highest voltage would be the direction the signal is coming from. There are two disadvantages in the use of the butler
matrix. One is that it cannot detect a signal that arrives broadside to the array, and the second is that it is narrow-banded. The mode former is narrow banded due the band limitations in the components that comprise it, and this fact will be detailed later in this chapter.

The Butler matrix would be useful for the UAV application. The fact that it is passive and needs no switching saves the weight of batteries to power it. However, a large number of elements would be needed for a high resolution, and the elevation of the incoming signal can still not be obtained unless a planar array is used.

We could dispense of the need for a large number of elements in a planar array by employing a spiral antenna. All modes of the antenna would be available simultaneously and the AOA method for the spiral antenna could be used.

4.2. Quadrature Coupler

The main component in the mode former is a quadrature hybrid. This device is a 3 dB directional coupler with a 90° phase difference in the outputs. The geometry of a hybrid is shown in Figure 4.3, and operates in the following manner. With all ports matched, a signal applied to port 1 is evenly divided between ports 2 and 3 with a 90° phase shift between them. No signal appears at port 4, which is the isolated port. The theory discussed in the following section is adapted from Pozar[4].

4.2.1. Theory

The quadrature hybrid has a high degree of symmetry. Any port can be used as the input port with the output ports being on the opposite side of the hybrid. Thus each
row of the scattering matrix for the hybrid can be obtained as a transposition of the first row.

To find the scattering matrix for the hybrid this four-port network will first be drawn in normalized form as shown in Figure 4.4. Each line represents a transmission line with impedance normalized to $Z_0$, and the excitation at port one is a wave of unit amplitude. The four-port network will then be decomposed into two decoupled two-port networks. The first two-port network, shown in Figure 4.5, is a result of the symmetry of the excitation and is the even-mode excitation. Figure 4.6 shows the odd-mode network that is a result of the anti-symmetric excitation. The incident waves for each network will
have amplitudes of $\pm \frac{1}{2}$, and therefore the amplitudes of the waves emerging from each port can be found by the following equations:

\begin{align*}
B_1 &= \frac{1}{2} \Gamma_e + \frac{1}{2} \Gamma_o \quad (4.1a) \\
B_2 &= \frac{1}{2} T_e + \frac{1}{2} T_o \quad (4.1b) \\
B_3 &= \frac{1}{2} T_e - \frac{1}{2} T_o \quad (4.1c) \\
B_4 &= \frac{1}{2} \Gamma_e - \frac{1}{2} \Gamma_o \quad (4.1d)
\end{align*}

Where $\Gamma_e$ and $\Gamma_o$ are the even- and odd-mode reflection coefficients, and $T_e$ and $T_o$ are the even- and odd-mode transmission coefficients.

Figure 4.5 Even Mode Decomposition of the Quadrature Coupler (from Pozar[4])
Figure 4.6 Odd Mode Decomposition of the Quadrature Coupler (from Pozar[4])

The even-mode excitation network is composed of three individual components in cascade: an open-ended shunt, a transmission line, and another open-ended shunt. The ABCD parameters for a shunt are given as:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
Y & 1
\end{bmatrix},
\]

The admittance of an open ended shunt is \( Y = \frac{1}{Z} \tan \psi_1 \), where \( \psi_1 = \beta_1 L_1 \). \( \beta_1 \) is the propagation constant and \( L_1 \) is the length of the shunt. With the normalization of the characteristic impedance the ABCD parameters become:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
\tan \beta_1 & 1
\end{bmatrix}. \quad (4.2)
\]

The ABCD parameters for a transmission line with the normalized impedance are given as:
The ABCD parameters for the even-mode network can be obtained by multiplying the parameters for each of the cascaded components giving:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_e = \begin{bmatrix}
j \tan \varphi_1 & 0 \\
1 & 1
\end{bmatrix} \begin{bmatrix}
\cos \varphi_2 & j \frac{1}{\sqrt{2}} \sin \varphi_2 \\
\sqrt{2} \sin \varphi_2 & \cos \varphi_2
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
j \tan \varphi_1 & 1
\end{bmatrix}
\]

with the final result being:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_e = \begin{bmatrix}
\cos \varphi_2 - j \frac{1}{\sqrt{2}} \sin \varphi_2 \tan \varphi_1 & j \frac{1}{\sqrt{2}} \sin \varphi_2 \\
j \sqrt{2} \cos \varphi_2 \tan \varphi_1 + j \sin \varphi_2 (\sqrt{2} - \frac{1}{\sqrt{2}} \tan^2 \varphi_1) & \cos \varphi_2 - j \frac{1}{\sqrt{2}} \sin \varphi_2 \tan \varphi_1
\end{bmatrix}
\]

The conversion from ABCD parameters to S-parameters for $S_{11}$, which is equivalent to $\Gamma$ is:

\[
S_{11_e} = \Gamma_e = \frac{A_e + B_e/Z - ZC_e - D_e}{A_e + B_e/Z + ZC_e + D_e}
\]
Since A and D are identical they will cancel in the numerator and have twice the value in the denominator. Pulling out the common factor of $j \frac{1}{\sqrt{2}} \sin \varphi_2$ in both the numerator and denominator results in the following:

$$\Gamma_e = \frac{\tan^2 \varphi_1 - 2\sqrt{2} \cot \varphi_2 \tan \varphi_1 - 1}{2\sqrt{2} \cot \varphi_2 (\tan \varphi_1 - j) + 3 + j2\tan \varphi_1 - \tan^2 \varphi_1}. \quad (4.6)$$

The conversion from ABCD parameters to S-parameters for S21, which is equivalent to $T$ is:

$$S21_e = T_e = \frac{2}{A_e + \frac{B_e}{Z_e} + Z_e C_e + D_e}. \quad (4.7)$$

Substituting the values into the equation and pulling like terms together gives:

$$S21_e = T_e = \frac{2}{j \frac{1}{\sqrt{2}} \sin \varphi_2 \left[2 \cot \varphi_2 (\tan \varphi_1 - j) + 3 + j2\tan \varphi_1 - \tan^2 \varphi_1\right]} . \quad (4.8)$$

Similarly the odd-mode network is composed of three individual components in cascade, but with the open-ended shunt replaced with a shorted shunt. The admittance for a shorted shunt is $Y = -j \frac{1}{Z} \cot \varphi_1$. The ABCD parameters for the odd-mode excitation become:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j \cot \varphi_1 & 1 \end{bmatrix} \begin{bmatrix} \cos \varphi_2 & j \frac{1}{\sqrt{2}} \sin \varphi_2 \\ \frac{1}{\sqrt{2}} \sin \varphi_2 & \cos \varphi_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j \cot \varphi_1 & 1 \end{bmatrix}$$
With the final result being:

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_o = \begin{bmatrix}
\cos \varphi_2 + \frac{1}{\sqrt{2}} \sin \varphi_2 \cot \varphi_1 & j \frac{1}{\sqrt{2}} \sin \varphi_2 \\
-j 2 \cos \varphi_2 \cot \varphi_1 + j \sin \varphi_2 (\sqrt{2} - \frac{1}{\sqrt{2}} \cot^2 \varphi_2) & \cos \varphi_2 + \frac{1}{\sqrt{2}} \sin \varphi_2 \cot \varphi_1
\end{bmatrix} \tag{4.9}
$$

The results in equation 4.9 gives the odd-mode reflection and transmission coefficients as:

$$
\Gamma_o = \frac{2 \sqrt{2} \cot \varphi_2 \cot \varphi_1 + \cot^2 \varphi_1 - 1}{2 \sqrt{2} \cot \varphi_2 (- \cot \varphi_1 - j) + 3 - j 2 \cot \varphi_1 - \cot^2 \varphi_1} \tag{4.10}
$$

and

$$
T_o = \frac{2}{j \frac{1}{\sqrt{2}} \sin \varphi_2 [2 \sqrt{2} \cot \varphi_2 (- \cot \varphi_1 - j) + 3 - j 2 \cot \varphi_1 - \cot^2 \varphi_1]} \tag{4.11}
$$

The hybrid has the following values:

$$
\varphi_1 = \beta_1 L_1 = \frac{2 \pi \lambda}{\lambda \frac{1}{8}} = \frac{\pi}{4}
$$

And

$$
\varphi_2 = \beta_2 L_2 = \frac{2 \pi \lambda}{\lambda \frac{1}{4}} = \frac{\pi}{2}
$$
Plugging these values into 4.8-4.11 will result in the following coefficients:

\[ \Gamma_e = 0 \]
\[ T_e = -\frac{1}{\sqrt{2}}(1 + j) \]
\[ \Gamma_o = 0 \]
\[ T_o = \frac{1}{\sqrt{2}}(1 - j) \]

Putting these results into equation 4.1 gives:

\[ B_1 = 0 \]
\[ B_2 = -\frac{j}{\sqrt{2}} \]
\[ B_3 = -\frac{1}{\sqrt{2}} \]
\[ B_4 = 0 \]

There is no reflection at port 1. Ports 2 and 3 are at half-power with a 90° phase shift between them. No power is output to port 4. The scattering matrix will then be:
The difference in phase between ports is critical to the operation of a mode former. Since a mode former is comprised of hybrids, this makes the phase relationship between ports on the hybrid critical as well. Therefore, $B_2$ and $B_3$ are the critical s-parameters, and the relationship between these two will be closely monitored.

### 4.2.2. Design

The results obtained in the previous section are only valid for a single frequency. The hybrid needed for the system proposed needs to cover a band from 1850 MHz to 1910 MHz. Therefore, the hybrid will be designed at the center frequency of 1880 MHz and will be realized in a planar microstrip form. The following values are obtained for the hybrid when using this frequency:

\[
S_{\text{hybrid}} = \begin{bmatrix}
B_1 & B_2 & B_3 & B_4 \\
B_2 & B_1 & B_4 & B_3 \\
B_3 & B_4 & B_1 & B_2 \\
B_4 & B_2 & B_3 & B_1
\end{bmatrix} = \begin{bmatrix}
0 & j & 1 & 0 \\
j & 0 & 0 & 1 \\
1 & 0 & 0 & j \\
0 & 1 & j & 0
\end{bmatrix}
\]

- $Z_1 = 50 \, \Omega$
- $L_1 = 1.5245 \, \text{cm}$
- $Z_2 = \frac{50}{\sqrt{2}} \, \Omega$
- $L_2 = 1.4805 \, \text{cm}$
- $\varepsilon_{\text{eff}1} = 6.8386$
- $\varepsilon_{\text{eff}2} = 7.2512$
4.2.3. Simulation Results

The values above were used for a simulation in Microwave Office of the proposed hybrid from a frequency range of 0.5 times of the center frequency to 1.5 times the value of the center frequency. Figure 4.7 shows the results of $S_{11}$, $S_{21}$, $S_{31}$, and $S_{41}$ for this simulation. The results of this simulation match the standard results of any hybrid which are discussed in many microwave engineering textbooks. An example from Pozar’s text is shown in Figure 4.8.
Figure 4.7 S Parameters of the Simulated Quadrature Coupler

Figure 4.8 Theoretical S Parameters for a Quadrature Coupler (from Pozar[4])
4.2.4. Measurement Results

The quadrature coupler was realized in a planar microstrip form on Rogers 3010 substrate material. The AutoCAD drawing is shown in Figure 4.9, and the fabricated coupler is shown in Figure 4.10. The S-Parameters of the hybrid were measured using the VNA and the results of those parameters are plotted in Figure 4.11. Comparing these results to the plots from the simulation shown in Figure 4.7 and from the theoretical plots from Pozar in Figure 4.8 we can see that they are very similar in shape and magnitude.

Figure 4.9 AutoCAD Drawing of Fabricated Quadrature Coupler
Figure 4.10 Fabricated Quadrature Coupler

Figure 4.11 Measured S Parameters of the Fabricated Quadrature Coupler
From the discussion on spiral antennas in Chapter 3 we know that the terminal to terminal phase progression is well defined and that the magnitude of the excitation at each terminal must be equal. Therefore a comparison of adjacent ports was conducted on the fabricated coupler to assure that the magnitude was consistent and the phase progression was well preserved. Figure 4.12 is a plot of the difference in magnitude between adjacent ports from a frequency range of half of the center frequency to one and a half times the center frequency, and Figure 4.14 shows the difference in phase over the same frequency range. From these plots it seems that if only port to port phase was a determining factor the hybrid would be useful in a band of about 200 MHz. However upon inspection of the port to port magnitude comparison it appears that it will only operate well very near the center frequency. Figure 4.13 and Figure 4.15 show a comparison of the magnitude and phase over the bandwidth of interest. The phase appears pretty constant, but the magnitude varies about one half dB. As we see the further away from the design frequency we get the more the amplitude varies, and the required phase relationship falls apart. This is why the mode former is narrow banded.
Figure 4.12 Difference in Magnitude between Ports 2 and 3

Figure 4.13 Difference in Magnitude between Ports 2 and 3 in the Operating Band
Figure 4.14 Difference in Phase between Ports 2 and 3

Figure 4.15 Difference in Phase between Ports 2 and 3 in the Operating Band
4.3. Layout

The configuration of the mode former that will give the desired phase progression across the terminals of the spiral antenna is shown in Figure 4.16. The mode former will require four hybrid couplers and five phase shifters. As discussed in Chapter Two variable phase shifters are very expensive and need a control system. To avoid this, my design will employ fixed length transmission line phase shifters. The advantage of using the fixed line phase shifters is that they can be realized in planar microstrip form on the same substrate as the hybrids. This allows for a complete mode former to be placed on one piece of substrate. They are also passive and occupy, at least in this frequency range, a small area. The disadvantage to using these types of phase shifter is that there is a ninety degree phase shift at one single frequency. As the operating frequency moves away from the center the phase shift will move away from ninety degrees.

The layout shown in Figure 4.16 was rearranged so there were no crossing transmission lines and that configuration is illustrated in the AutoCad drawing in Figure 4.17.
Figure 4.16 Layout of the proposed mode former

Figure 4.17 AutoCad layout drawing of the mode former
4.4. Simulation Results

The layout detailed in Figure 4.17 was simulated in Microwave Office using Rogers 3010 substrate properties for mode one and mode two. The results of the simulation are nearly identical to that of the theoretical results, and indicate that the mode former could operate well over a band of about 150 MHz. The difference in magnitude between the simulated mode one and mode two outputs is plotted in Figure 4.18 and Figure 4.20, and the difference in phase between the same outputs is shown in Figure 4.19 and Figure 4.21. Comparing the simulated values for modes one and two with the theoretical values, shown in Table 4-1 and Table 4-2, we can see that the deviation from the theoretical values is negligible.

Figure 4.18 Difference in magnitude between the mode one outputs of the simulated mode former
Figure 4.19 Difference in phase between the mode one outputs of the simulated mode former

Table 4-1 Magnitude and phase difference between mode one outputs of the simulated mode former

| Frequency (GHz) | $|S31/S31|$ | $\angle S31/S31$ | $|S31/S41|$ | $\angle S31/S41$ | $|S31/S51|$ | $\angle S31/S51$ | $|S31/S61|$ | $\angle S31/S61$ |
|----------------|------------|----------------|------------|----------------|------------|----------------|------------|----------------|
| 1.850          | 1.000      | 0.000          | 0.997      | -90.114        | 1.000      | -178.545       | 1.002      | -268.395       |
| 1.860          | 1.000      | 0.000          | 0.999      | -89.975        | 1.002      | -179.042       | 1.003      | -268.832       |
| 1.870          | 1.000      | 0.000          | 1.000      | -89.872        | 1.003      | -179.549       | 1.004      | -269.296       |
| 1.880          | 1.000      | 0.000          | 1.000      | -89.823        | 1.003      | 179.944        | 1.004      | -269.795       |
| 1.890          | 1.000      | 0.000          | 1.000      | -89.836        | 1.004      | 179.404        | 1.003      | -270.330       |
| 1.900          | 1.000      | 0.000          | 0.999      | -89.908        | 1.003      | 178.842        | 1.002      | -270.897       |
| 1.910          | 1.000      | 0.000          | 0.999      | -90.042        | 1.003      | 178.248        | 1.000      | -271.484       |
Figure 4.20 Difference in magnitude between the mode two outputs of the simulated mode former

Figure 4.21 Difference in phase between the mode two outputs of the simulated mode former
Table 4-2 Magnitude and phase difference between mode two outputs of the simulated mode former

| Frequency (GHz) | | Simulated Mode 2 Outputs (Linear/Degrees) |
|----------------|--------------------------|
|                | | [S32/S32] | [S32/S42] | [S32/S52] | [S32/S62] |
| 1.850          | 1.000 | 0.000 | 0.998 | -181.507 | 1.003 | 1.540 | 1.002 | -179.932 |
| 1.860          | 1.000 | 0.000 | 0.997 | -180.996 | 1.003 | 0.970 | 1.001 | -179.970 |
| 1.870          | 1.000 | 0.000 | 0.997 | -180.504 | 1.003 | 0.440 | 1.000 | -179.995 |
| 1.880          | 1.000 | 0.000 | 0.997 | -180.014 | 1.003 | -0.070 | 1.000 | -179.999 |
| 1.890          | 1.000 | 0.000 | 0.998 | -179.531 | 1.004 | -0.550 | 1.001 | -180.001 |
| 1.900          | 1.000 | 0.000 | 0.999 | -179.057 | 1.004 | -1.040 | 1.002 | -180.018 |
| 1.910          | 1.000 | 0.000 | 1.001 | -178.584 | 1.004 | -1.540 | 1.003 | -180.057 |

4.5. Measurement Results

The mode former was realized on Roger’s 3010 substrate and is shown in Figure 4.22. The mode former’s S-Parameters were then determined with the VNA for both mode one and mode two. The difference in magnitude between the simulated mode one and mode two outputs is plotted in Figure 4.23 and Figure 4.25, and the difference in phase between the same outputs is shown in Figure 4.24 and Figure 4.26. Comparing the simulated values for modes and two with the theoretical values, shown in Table 4-3 and Table 4-4, we can see that the deviation from the theoretical values is fairly substantial when compared with the simulated model.
Figure 4.22 Fabricated mode former

Figure 4.23 Difference in magnitude between the mode one outputs of the fabricated mode former
Figure 4.24 Difference in Phase between the mode one outputs of the fabricated mode former

Table 4-3 Magnitude and phase difference between mode one outputs of the fabricated mode former

| Frequency (GHz) | \(|S31/S31|\) | \(\angle S31/S31\) | \(|S31/S41|\) | \(\angle S31/S41\) | \(|S31/S51|\) | \(\angle S31/S51\) | \(|S31/S61|\) | \(\angle S31/S61\) |
|----------------|----------------|-----------------|----------------|-----------------|----------------|----------------|----------------|----------------|
| 1.850          | 1.000          | 0.000           | 0.970          | -92.266         | 1.126          | -177.916       | 1.076          | -267.046       |
| 1.860          | 1.000          | 0.000           | 0.973          | -92.187         | 1.113          | -178.841       | 1.059          | -267.079       |
| 1.870          | 1.000          | 0.000           | 0.980          | -92.019         | 1.098          | -179.617       | 1.051          | -266.988       |
| 1.880          | 1.000          | 0.000           | 0.988          | -92.146         | 1.086          | 179.553        | 1.047          | -266.991       |
| 1.890          | 1.000          | 0.000           | 0.997          | -92.243         | 1.071          | 179.025        | 1.047          | -266.956       |
| 1.900          | 1.000          | 0.000           | 1.007          | -92.372         | 1.055          | 178.590        | 1.048          | -266.913       |
| 1.910          | 1.000          | 0.000           | 1.014          | -92.675         | 1.040          | 178.460        | 1.055          | -266.924       |
Figure 4.25 Difference in magnitude between the mode two outputs of the fabricated mode former

Figure 4.26 Difference in phase between the mode two outputs of the fabricated mode former
Table 4-4 Magnitude and phase difference between mode two outputs of the fabricated mode former

| Frequency (GHz) | $|S32/S32|$ | $\angle S32/S32$ | $|S32/S42|$ | $\angle S32/S42$ | $|S32/S52|$ | $\angle S32/S52$ | $|S32/S62|$ | $\angle S32/S62$ |
|----------------|-----------|-----------------|-----------|-----------------|-----------|-----------------|-----------|-----------------|
| 1.850          | 1.000     | 0.000           | 1.043     | -187.628        | 0.925     | -1.366          | 0.935     | -182.970        |
| 1.860          | 1.000     | 0.000           | 1.052     | -188.430        | 0.932     | -2.109          | 0.948     | -183.373        |
| 1.870          | 1.000     | 0.000           | 1.057     | -189.314        | 0.935     | -2.702          | 0.957     | -184.186        |
| 1.880          | 1.000     | 0.000           | 1.054     | -189.995        | 0.940     | -3.210          | 0.966     | -184.949        |
| 1.890          | 1.000     | 0.000           | 1.049     | -190.423        | 0.943     | -3.828          | 0.969     | -185.493        |
| 1.900          | 1.000     | 0.000           | 1.036     | -190.646        | 0.945     | -4.370          | 0.967     | -185.793        |
| 1.910          | 1.000     | 0.000           | 1.019     | -190.573        | 0.943     | -4.862          | 0.963     | -186.056        |

4.6. Error Analysis

The difference of magnitude and phase between the simulated and fabricated mode formers outputs for both mode one and mode two and show in Table 4-5 and Table 4-6. For the center frequency of 1.88 GHz the maximum deviation of magnitude from the required parameters for mode one and mode two is 0.082 and 0.083 respectively. This difference doesn’t seem that significant but could lead to antenna pattern being skewed. Regarding the phase progression, which in previous discussions was required to be precise, we see that for mode one and mode two the deviation from the required values of phase are 2.308 and 9.981 degrees respectively. The mode one deviation is relatively small but the mode two deviation is not. This may cause the shape of the pattern to be different from the simulation and also cause more problems with azimuthal ambiguity.
Table 4-5 Difference of magnitude and phase angle between the simulated and actual outputs of the mode former for mode one

| Frequency (GHz) | Δ|A1| | Δ∠A1 | | Δ|A2| | Δ∠A2 | | Δ|A3| | Δ∠A3 | | Δ|A4| | Δ∠A4 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.850          | 0.000| 0.000| -0.027| -2.152| 0.127| 0.629| 0.074| 1.349 |
| 1.860          | 0.000| 0.000| -0.026| -2.212| 0.111| 0.201| 0.056| 1.753 |
| 1.870          | 0.000| 0.000| -0.021| -2.147| 0.095| -0.068| 0.047| 2.308 |
| 1.880          | 0.000| 0.000| -0.012| -2.232| 0.082| -0.391| 0.043| 2.804 |
| 1.890          | 0.000| 0.000| -0.002| -2.407| 0.067| -0.379| 0.045| 3.374 |
| 1.900          | 0.000| 0.000| 0.008| -2.464| 0.052| -0.252| 0.046| 3.984 |
| 1.910          | 0.000| 0.000| 0.015| -2.633| 0.038| 0.212| 0.055| 4.560 |

Table 4-6 Difference of magnitude and phase angle between the simulated and actual outputs of the mode former for mode two.

| Frequency (GHz) | Δ|A1| | Δ∠A1 | | Δ|A2| | Δ∠A2 | | Δ|A3| | Δ∠A3 | | Δ|A4| | Δ∠A4 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.850          | 1.000| 0.000| 0.045| -6.121| -0.078| -2.906| -0.067| -3.038 |
| 1.860          | 1.000| 0.000| 0.055| -7.434| -0.071| -3.079| -0.052| -3.403 |
| 1.870          | 1.000| 0.000| 0.060| -8.810| -0.068| -3.142| -0.043| -4.192 |
| 1.880          | 1.000| 0.000| 0.057| -9.981| -0.063| -3.140| -0.034| -4.950 |
| 1.890          | 1.000| 0.000| 0.051| -10.892| -0.061| -3.278| -0.032| -5.492 |
| 1.900          | 1.000| 0.000| 0.038| -11.589| -0.059| -3.330| -0.035| -5.775 |
| 1.910          | 1.000| 0.000| 0.018| -11.989| -0.061| -3.322| -0.041| -5.999 |
One possible factor contributing to the output errors of the fabricated more formers is the design of the phase shifters. The Microwave Office simulation did not do a full wave solution, but only model the mode former as combinations of microstrip transmission lines. It was thought that the radius of bends in the phase shifter might be two small and thus causing radiation from each bend. The lines of the phase shifter might also be located to closely together cause some mutual coupling.

To test this theory three microstrip transmission lines were fabricated. The baseline case was a 50Ω microstrip line with a length of 2 cm. The second added a ninety degree phase shifter of the current design to that 2 cm line, while the third employed a phase shifter in which the radius of the bends were increased and the line were located further apart. The designs for these three cases are shown in Figures 4.27 to 4.29.

![Figure 4.27 AutoCad drawing of the 2cm microstrip line](image)
The fabricated lines were then tested with the VNA to determine their transmission properties. The difference between the two phase shifted lines and the base line of 2 cm was taken and shown in the plot of Figures 4.30 and 4.31. As we can see...
from these plots the current delay line actually only has a phase delay of approximately 86.8 degrees at the center frequency while the expanded line has a delay of exactly 90 degrees. This shows that the current delay line is a contributing factor to the error in the mode former’s outputs. The biggest deviation in phase comes from output two of the mode former. This line has no phase shifters. While it would seem that this one should have the least error we find this not the case. All of the phase relationships are defined relative to output one, and for mode two output one has two phase shifters in line. Thus the biggest difference in the number of phase shifters also has the biggest deviation is phase. This proves that the current phase shifter design is in fact a major contributor to the error.

Figure 4.30 Difference in phase between the shifted microstrip lines and the base line from .5 to 1.5 time the center frequency.
The magnitude and phase relationships for both mode one and mode two of the fabricated mode former was used to replace the excitation parameters of the spiral antenna in the NEC model. The simulation was performed with the new excitations and the resultant patterns are shown in Figures 4.32 to 4.35. As we can see these results have changed slightly from the original simulation. Instead of the “squaring” of the mode one pattern above 60 degrees it start to produce a flower pattern much like the pattern of the four element array. We can also see a slight skewing of the beams, but overall the difference between the outputs of the simulated and fabricated mode formers has not affected the shape of the pattern too dramatically. This leads to the conclusion that
although the mode former has some deviations the patterns will still form. However this will cause the error angle of arrival estimations.

Figure 4.32 Phi cuts from the results of the NEC model with the fabricated mode former's mode 1 excitation.
Figure 4.33 Theta cuts from the results of the NEC model with the fabricated mode former's mode 1 excitation.

Figure 4.34 Phi cuts from the results of the NEC model with the fabricated mode former's mode 2 excitation.
The calculated difference in magnitude between mode two and mode one of the re-simulated NEC spiral to give elevation information is shown in Figure 4.36, and the calculated phase difference to produce azimuthal information is shown in Figure 4.37. As we can see the elevation determination now has some ambiguity and would not be resolvable above approximately seventy-four degrees. The difference in phase between modes one and two now spans $6\pi$ instead of $2\pi$. This allows for every measurement of phase difference the possibility of occupying one of three locations.
Figure 4.36 Difference between the magnitude of mode one and two of the NEC resimulated spiral in relation to the elevation angle

Figure 4.37 Difference between the phase of mode one and two of the NEC simulated spiral in relation to the azimuth angle
4.8. Spiral Antenna with Fabricated Mode Former

The spiral antenna substructure was modified from the one produced by Glass[1] to be smaller and lighter. In performing his measurements it was found that large deal of strain was put on the DAMS system’s rotation devices. The mode former was then fed in both a mode one and mode two excitation and the patterns were measured. During the pattern measurement process for the horizontal slices there was a mechanical breakdown of the rotation device for the DAMS system. Therefore, rotation had to be performed by hand and at higher increments of azimuthal degrees. This unfortunately has meant a lower angular resolution for the horizontal plots.

Figures 4.38 and 4.40 show the vertical slices of the measured patterns while Figures 4.39 and 4.41 show the horizontal slices of the measured pattern. We can see from these figures that the patterns look similar the re-simulated patterns shown before. The beams are slight skewed and the flowering does occur in mode one above sixty degrees. However, we can also see that the patterns have some anomalies in their shape and they are not symmetric. This causes some ambiguities to the elevation pattern as shown in Figure 4.42. It appears that nothing above approximately 45 degrees could be resolved in the elevation calculation. Figure 4.43 shows that difference in phase is similar to that shown in the original spiral simulation. There are large ambiguous regions surrounding the locations of the nulls.
Figure 4.38 Vertical slices of the spiral antenna pattern formed by mode one

Figure 4.39 Horizontal slices of the spiral antenna pattern formed by mode one
Figure 4.40 Vertical slices of the spiral antenna pattern formed by mode two

Figure 4.41 Horizontal slices of the spiral antenna pattern formed by mode two
Figure 4.42 Measured amplitude difference between mode one and mode two in relation to elevation angle.

Figure 4.43 Measured phase difference between mode one and mode two in relation to the azimuthal angle.
5. Conclusions

5.1. Summary of Work

The motivation for this thesis was a low cost and lightweight mode former for angle of arrival estimations. Different antenna configurations were explored and it was determined that the best choice of antenna would be the four arm spiral. The spiral antenna was chosen because of its compactness, its ability to operate over a broad range of frequencies, and the fact that it can be excited in different modes. It is in the comparison between the magnitudes and phase of the signal impinging on these modes that the angle of arrival information can be obtained. Previous theoretical work had been done by others to find a closed-from solution to calculate the angle of arrival with the four arm spiral antenna. This work was investigated by simulating the spiral antenna in Numerical Electromagnetics Code and experimental data was taken for validation.

A mode former capable of supporting the four arm spiral antenna in a mode one and mode two excitation was simulated, designed, and fabricated. This mode former was tested using a VNA to determine its S-Parameters. The results were compared to the simulation to verify that that mode former was capable of supporting the antenna.

During the simulation of the spiral antenna it was found that the closed-form solution would not be able to be used as a method of calculating the angle of arrival for this spiral antenna. Other models were considered and presented as viable models for the spiral. However, the spiral in both simulation and realization exhibited multiple nulls in mode two that cause large areas of ambiguity in the estimation of angle for elevation and
azimuth. The spiral could be used in this capacity but at a cost. Although it is ideal for its size and broad band frequency range, it is not optimal for angle of arrival. The determination would have to be made on what parameters can be sacrificed. There are other options that may provide full angular resolution, but at the cost of size and frequency range.

5.2. Recommendations for Further Study

The first recommendation would be to fabricate another mode former with the alternative design for the phase shifters. It could then be determined if the mode former could at least allow the spiral antenna to produce symmetrical patterns. This would improve the elevation estimation at all angles of Phi, and allow for closed-form solution to be used for an elevation determination. Another area to be considered for future research may be to investigate the alternative models presented in more in depth. It may be possible for one of these antenna systems to be made compact and low cost enough to be modularized as the mode former was intended to be. In particular an eight arm spiral antenna should operate in a similar manner to the eight element circular array. If this holds true there will be no nulls in the pattern for modes one and two and therefore no azimuthal ambiguity.
6. References


