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Distributed Fault Detection for a Class of Large-Scale Nonlinear Uncertain Systems

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Distributed Fault Detection for a Class of Large-Scale Nonlinear Uncertain Systems

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

by

Qi Zhang

Wright State University

2011
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Qi Zhang ENTITLED Distributed Fault Detection for a Class of Large-Scale Nonlinear Uncertain Systems BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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ABSTRACT

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In the distributed large-scale system, the behavior of any subsystem is not only influenced by variables belonging to it (local variables), but also by the variables in other subsystems during its interaction with neighboring subsystems. The effect of the fault in one subsystem will be quickly propagated to other subsystems due to their interconnections. Currently, most of the fault detection and diagnosis schemes are focused on centralized system which do not consider the interaction terms and can not efficiently detect the faults.

In this thesis, a distributed fault detection scheme is developed for a class of large-scale nonlinear uncertain systems with unstructured modeling uncertainty. For each subsystem in the large-scale system, a fault detection estimator (FDE) is designed by utilizing local measurements and certain communicated information from neighboring FDEs associated with subsystems that are directly interconnected to the particular subsystem under consideration. Under certain assumptions, adaptive threshold for fault detection in each local subsystem is derived, and its robustness property with respect to modeling uncertainty and interactions among interconnected subsystems is also investigated. Also, the fault detectability conditions characterizing the class of faults in each subsystem that can be detected by this approach is analyzed. A simulation example of automated highway systems is used to illustrate the effectiveness of the proposed method.
List of Symbols

Chapter 1

$U$ Input signal of the system
$N$ Nonmeasurable disturbance signal
$\Theta$ Unknown process parameters
$Y_{\text{min}}$ lower threshold in limit value checking
$Y_{\text{max}}$ upper threshold in limit value checking
$G_l$ observer gain in the sliding mode observer
$\rho, P_2$ scales in the sliding mode observer

Chapter 2

$\zeta_i$ known nonlinearity of the nominal dynamics of the $i$th subsystem
$\varphi_i$ modeling uncertainty in $i$th subsystem
$h_{ij}$ direct interconnection between the $i$th subsystem and the $j$th subsystem
$\beta_i$ time profile of a fault in $i$th subsystem
$E_i$ fault distribution matrix
$\phi_i$ nonlinear fault function
$T_i$ transformation matrix
$\rho_{i1}, \rho_{i2}$ known nonlinearity of the nominal dynamics of the $i$th subsystem in the canonical model
$\eta_{i1}, \eta_{i2}$ modeling uncertainty of $i$th subsystem in the canonical model
$H_{ij}^1, H_{ij}^2$ direct interconnection between the $i$th subsystem and the $j$th subsystem in the canonical model
$\sigma_{i1}$ a Lipschitz constant
$\sigma_{i2}(\cdot)$ a known bounding function
$\gamma_{ij}^1, \gamma_{ij}^2$ known Lipschitz constants

Chapter 3

$L_i$ gain matrix for the $i$th FDE
$\nu_{ip}$ threshold for the $p$th output of the $i$th FDE
$T_0$ time when the fault occurs
$T_d$ time when the fault is detected

Chapter 4
\( \psi_i \) distance between the \( i \)th and the \((i - 1) \)th vehicle

\( v_i \) the \( i \)th vehicle's velocity

\( \xi_i \) driving/braking force applied to the \( i \)th vehicle

\( \varpi \) positive constant

\( m_i \) \( i \)th vehicle vehicle mass

\( D_i \) \( i \)th vehicle aerodynamic drag

\( d_i \) constant frictional force

\( \tau_i \) engine/brake time constant

\( \vartheta_i \) magnitude of disturbance signal

\( \theta_1 \) magnitude of an actuator fault

\( \theta_2 \) magnitude of a process fault

\( g_1 \) structure of an actuator fault

\( g_2 \) structure of a process fault
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Dedicated to my family
Introduction

With a fast development of modern technologies in industry, the complexity of the control system keeps increasing, correspondingly, there emerge significant applications of distributed large-scale systems in many aspects such as intelligent vehicle highways systems, formation control of unmanned aerial vehicles, power generation and distribution systems, telecommunication networks and water distributions, etc. To achieve reliable and safe operation of these distributed large-scale industry systems, the effective fault diagnosis approaches are indispensable.

The fault diagnosis procedure is monitoring the system and generating all information about the abnormal behavior of its components. In general, Fault detection, Fault isolation and Fault identification are mainly three parts of the fault diagnosis [2]. We can know wether there is a fault or not in a system during the process of **Fault detection** ; We can know some information about the fault such as location, in the process of **Fault isolation** and we can know the the size and type or nature of the fault in the **fault identification** process. There has been a strong interest and intensive research activities in the area of fault detection and fault diagnosis in centralized and distributed nonlinear systems in recent years.

Recently, there are significant applications of the large-scale distributed nonlinear systems in industry. A system is defined as 'distributed' if it can be considered as being constituted by a number of subsystems communicating with each other, so that the behavior of any single subsystem is influenced by variables belonging to it, and also by the influ-
ence of a proper subset of all the other subsystems. Examples of such distributed systems include advanced automotive control systems which consists of over 50 interconnected microcontrollers controlling different vehicle functionalities, intelligent vehicle highway systems [1], formation control of unmanned aerial vehicles, and large-scale critical infrastructure systems including power generation and distribution systems, telecommunication networks, water distribution networks, etc. In contrast to the centralized system, an occurrence of a failure in a subsystem will not only affect the local subsystem, but also propagate the affects to the neighboring subsystem via the interactions among the subsystems which may seriously deteriorate the whole system performance or cause the system unstable. Since such distributed control systems need to operate reliably at all times, despite the possible occurrence of faulty behavior some subsystems, the design and application of fault diagnosis schemes based on the distributed system is a crucial step in achieving reliable and safe operations.

Based on the fault diagnosis requirements for the large-scale distributed system mentioned above, recently, there are significant research activities involved in the area of derivation and application of fault diagnosis approaches for distributed systems. Currently, most of the fault diagnosis methods are based on a centralized architecture. The performance of centralized fault diagnosis methods are inevitable affected by the limited amount of computation power available at the single computation node and the constrains of communication bandwidth in a distributed system. In recent years, there is a trend of applying fault diagnosis methods based on distributed architectures on distributed systems to overcome the deficiency of the centralized methods. In this thesis, we focus on solving the fault detection (FD) problem for a class of large-scale nonlinear distributed systems.

In the following sections, first, some typical centralized fault detection methods are reviewed. Then, for the distributed systems, currently existing fault detection methods are introduced. Finally, the research objective is introduced,
1.1 Model-Based Centralized Fault Detection Methods

Model-based fault detection methods are based on a mathematical model describing the relations between measurable variables (input signals and output signals) to detect faults in the process, actuators and sensors. Figure 1.1 shows a general scheme for process model based fault detection. The residual can be generated by the detection methods based on the information of the inputs signals and the outputs signals. Then, the generated residual is compared to the thresholds, and the fault detection decision is made. The process model based methods include observers, parity equation, and parameters estimation methods, etc.

![Scheme for the model based fault detection.](image)

Figure 1.1: Scheme for the model based fault detection.

1.1.1 Observer Based Fault Detection Method

Observer-based fault detection (FD) method is a one of the major methods in model based FD area. According to the original work by Luenberger[3], the observer based approach is a useful tool of fault diagnosis in dynamical system([4], [5], [6], [7] and [8]). The observer based FD approach can estimate the unknown internal state or the unknown outputs of the system from the measurements by unliving different type of observer, and
then construct the residual by a properly weighted output estimated error.

For the linear systems, if the process can be represented as

\[ \dot{x}(t) = Ax(t) + Bu(t) \]  
\[ y(t) = Cx(t) \]

(1.1)

where the input \( u(t) \) and the output \( y(t) \) are assumed to be known. The following state observer can be used to obtain the unmeasurable state variable if the observer gain is properly designed.

\[ \hat{x}(t) = A\hat{x}(t) + Bu(t) + He(t) \]  
\[ e(t) = y(t) - C\hat{x}(t) \]

(1.3)

(1.4)

For the nonlinear systems, the main obstacle in getting the solution of the observer-based faults detection problem is the lack of a universal approach for nonlinear observer design. Recently, some inspired approaches such as sliding mode techniques [9], modern differential geometric approaches [10], and adaptive scheme[11] have been incorporated with observer based fault diagnosis methods.

1.1.2 Parity Equation Methods

The key problem in the model-based fault detection method design is how to develop the robust residual generation for fault diagnosis of dynamic uncertain system. The residuals can be obtained easily by parity equation methods. In the parity equation methods, we compare the actual process behavior with the behavior of a nominal process model. We take the difference between the actual process outputs and those from the nominal model as the residuals. We can obtain the desired fault diagnosis properties by using the linear trans-
formations of these residuals. In the design of parity equation, it is necessary to find the residual that satisfies required response properties. The residuals have directional or structural properties corresponding to particular faults and are generated in order to enhance fault isolation.

There exist different approaches for the design of parity equations according to different models, such as time continuous state-space models [8], transfer function models [12], [13] and some special nonlinear models in which the artificial neural network can be directly be applied to generate output residuals [14].

1.1.3 Parameters Estimation Methods

The parameters estimation methods estimate the unknown parameters by measuring the inputs and outputs signals if the basic structure of the model is known, and fault diagnosis can be conducted based on the derivation of estimated system model parameters from their nominal values.

According to [15], for the linear processes, the methods of least square methods, extended least squares unbiased least square and recursive least squares are typical parameters estimation methods. The task is to determine the unknown parameters in the system model from a number of measured input and output signals. For the nonlinear processes, some specifical model such as Hammerstein model, Weiner model, and Volterra series can be applied with the parameters estimation methods.

Unfortunately, for the centralized fault diagnosis approaches introduced above, they are unfeasible for fault diagnosis of distributed systems. First, the task of computing in realtime may be limited by the amount of computation power available at the single computation node. Moreover, if the measurements from the actual system are transmitted through a communication network, the available bandwidth also affects the FDI performance. Therefore, due to the limitations of centralized approaches in the distributed systems, the development of distributed fault diagnosis approach is necessary to improve the fault diagnosis
performance.

1.2 Fault Detection Methods of Distributed Systems

In recent years, the area of distributed fault diagnosis has attracted significantly increasing attention (see, e.g., [16, 17, 18, 19] and the references cited therein). In [16], the overlapping decompositions strategy is applied on the large-scale system. The distributed system can be mapped into sets of interconnected subsystems. Based on the measurable local state and the transmitted variables from the neighboring subsystem, a local fault detector is designed for each subsystem, moreover, a specially designed consensus-based estimator is used to make the diagnoser reach a common decision about the variables which are affected by faults.

[19] makes use of the sliding mode observer to address the problem of decentralized actuator fault detection and estimation for a class of non-linear large-scale systems. First, a sliding mode observer is developed together with an appropriate coordinate transformation to find the sliding mode dynamics. Then, based on the features of the observer, an equivalent output error injection is used to estimate the decentralized fault. The modeling certainty is assumed to have certain structure and a non-linear bounds. By exploiting the structured modeling uncertainty, the decentralized reconstruction scheme is given.

By using an on-line neural network approximation technique, [20] adopted distributed estimators methods. The interconnection between neighboring subsystems is approximated by the neural network and the approximation information is sent to the local fault detector in each subsystem to help make a fault detection decision.

The distributed fault detection methods introduced above are novel and interesting. However, there is a lack of suitable methodology to address the FDI problem for a general large-scale nonlinear uncertain systems, and the FDI for distributed and large-scale systems is still an open area. The problem of fault diagnosis for distributed systems is challenging
because of the complex structure, time delay of signals transmission, interactions among subsystems and the structureless uncertainties in the distributed system.

1.3 Research Motivation

Based on the above discussions, most of the existing fault diagnosis approaches are with a centralized architecture, they are not suitable for fault diagnosis of distributed systems due to computation constrains and transmission constrains. Fault diagnosis of distributed systems is a new area and limited research work has been done for fault detection and isolation in a general distributed nonlinear system with unstructured uncertainty. The fault detection methods of distributed systems introduced in the previous section have not considered the unstructured uncertainty in their models and are not developed for a general distributed nonlinear system model. In addition, most of them are still based on a centralized architecture.

In this thesis, our objective is to develop a distributed fault detection method for a class of large-scale nonlinear systems. It includes the following three objectives:

First, we develop a distributed fault detection scheme for a class of large-scale nonlinear systems in the presence of unstructured modeling uncertainty.

The unstructured modeling uncertainty is a class of modeling uncertainty that appears possibly in all state equations without being pre-multiplied by a known distribution matrix that satisfies certain conditions.

In some research work of fault diagnosis, the distribution matrices of the structured modeling uncertainty are assumed to satisfy some certain rank conditions, so that it may allow the designed FDI schemes to completely decouple the fault from modeling uncertainty. In contrast, we want to detect any faults as soon as possible in the distributed system with unstructured modeling uncertainty by utilizing the proposed fault detection method.

Second, some important properties of the fault detection scheme is investigated in
this thesis. Specifically, the analysis focuses on: (i) investigation of adaptive thresholds
for distributed fault detection and its robustness property with respect to modeling uncer-
tainty and interactions among interconnected subsystems; (ii) derivation of fault detectabil-
ity conditions characterizing the class of faults in each subsystem that can be detected by
the proposed method.

Finally, we apply the proposed distributed fault detection method on fault detection of
vehicles in an automated highway system.

This thesis is organized as follows. In Chapter II, the problem of distributed fault
detection is formulated. Chapter III describes the details of the distributed fault detection
scheme and investigates several important technical issues, including the design of adap-
tive thresholds, the robustness and fault detectability (sensitivity) properties. In Chapter
IV, an application example of automated highway systems is presented to illustrative the
effectiveness of the method. Finally, Chapter V gives some concluding remarks.
Problem Formulation

The architecture of a large-scale nonlinear dynamic system composed of $M$ subsystems is shown in Figure 2.1. Each subsystem interacts with other subsystems, the information of local state variables, local controller signal, and output of the local system will be transferred to the neighboring subsystems through interaction which affects dynamics of reboring systems. Also, the dynamic of the local system is affected by the information from neighboring systems.

In a large-scale nonlinear dynamic system, there are $M$ subsystems interconnected with each other. We can use the following differential equations to represent each subsystem:

\[
\begin{align*}
\dot{x}_i &= A_i x_i + \zeta_i (y_i, u_i) + \varphi_i (x_i, u_i, t) + \sum_{j=1}^{M} h_{ij} (x_j) + \beta_i (t - T_0) E_i \phi_i (y_i, u_i) \\
y_i &= C_i x_i
\end{align*}
\] (2.1)

where the state vector, input vector and output vector of the $i$th subsystem are represented by $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{l_i}$, respectively, ($n_i \geq l_i$), for $i = 1, \cdots, M$. In addition, $\phi_i : \mathbb{R}^{l_i} \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{q_i}$ and are smooth vector fields. The constant matrices $E_i \in \mathbb{R}^{n_i \times q_i}$ and $C_i \in \mathbb{R}^{l_i \times n_i}$ with $q_i \leq l_i$ are of full rank, and the pair $(A_i, C_i)$ is observable.

In addition, the known nonlinearity of the nominal dynamics of the $i$th subsystem is represented by the smooth vector field $\zeta_i : \mathbb{R}^{l_i} \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{n_i}$ in (2.1), the modeling uncer-
Figure 2.1: The architecture of a large-scale nonlinear dynamic system.
tainty is described by the smooth vector \( \varphi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \times \mathbb{R}^+ \mapsto \mathbb{R}^{m_i} \), and the dynamics change of the \( i \) th subsystem caused by the occurrence of a fault can be represented by 

\[
\beta_i(t - T_0) E_i \phi_i(y_i, u_i).
\]

Where, \( \phi_i(y_i, u_i) : \mathbb{R}^{l_i} \times \mathbb{R}^{m_i} \mapsto \mathbb{R}^{q_i} \) is a smooth vector representing the nonlinear fault function, \( \beta_i(t - T_0) \) is the time profile function of a fault which occurs at \( T_0 \) (some unknown time) and \( E_i \) is distribution matrix for a fault in the \( i \)th subsystem. Furthermore, the direct interconnection between the \( i \)th subsystem and the \( j \)th subsystem, \( j \in \{1, \cdots, M\} \setminus \{i\} \) can be represented by the smooth vector field \( h_{ij} : \mathbb{R}^{l_j} \mapsto \mathbb{R}^{n_i} \). It is worth noting that in a large-scale system, because many subsystems do not directly connect the \( i \)th subsystem, the \( h_{ij} \) function is identical zero.

In this thesis, only the abrupt (sudden) faults detection problem is considered; therefore, a step function \( \beta_i(t - T_0) = 0 \) if \( t < T_0 \) and \( \beta_i(t - T_0) = 1 \) if \( t \geq T_0 \) is used to instantiatted the time profile function \( \beta_i(.) \).

**Assumption 1.** In the system in (2.1), the fault distribution matrices \( E_i \in \mathbb{R}^{n_i \times q_i} \) satisfied the condition of 

\[
\text{rank}(C_i E_i) = q_i, \text{ for } i = 1, \cdots, M, \text{ and } q_i \leq l_i \leq n_i.
\]

As described in [19], under Assumption 1, a linear transformation of coordinates \( z_i = T_i x_i = [z_{i1}^T z_{i2}^T]^T \) with \( z_{i1} \in \mathbb{R}^{(n_i - l_i)} \) and \( z_{i2} \in \mathbb{R}^{l_i} \) exists, such that

- \( T_i A_i T_i^{-1} = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} \),
- \( T_i E_i = \begin{bmatrix} 0 \\ E_{i2} \end{bmatrix} \), where \( E_{i2} \in \mathbb{R}^{l_i \times q_i} \),
- \( C_i T_i^{-1} = [0 \quad \bar{C}_i] \), where \( \bar{C}_i \in \mathbb{R}^{l_i \times l_i} \) is orthogonal.

**Remark 1.** The transformation matrix \( T_i \) can be obtained with following procedure [21]:

\[
11
\]
• The matrix $T_C$ can be obtained as followings:

\[
T_C = \begin{bmatrix}
N_{c_i}^T \\
C_i
\end{bmatrix}
\]  

where $N_c \in \mathbb{R}^{n_i \times (n_i-l_i)}$ are the columns span the null space of $C_i$. Consequently, the transformation $C_iT_c^{-1} = [0 \ C_i]$, where $C_i \in \mathbb{R}^{n_i \times l_i}$ is orthogonal.

• The fault distribution matrix $E_i$ can be partitioned as

\[
E_i = \begin{bmatrix}
\bar{E}_{i1} \\
\bar{E}_{i2}
\end{bmatrix}
\]  

where $\bar{E}_{i1} \in \mathbb{R}^{(n_i-l_i) \times q_i}$ and $\bar{E}_{i2} \in \mathbb{R}^{l_i \times q_i}$. The left pseudo inverse $\bar{E}_{i2}^\dagger = (\bar{E}_{i2}^\top \bar{E}_{i2})^{-1} \bar{E}_{i2}^\top$ is defined and there exists an orthogonal matrix $T_o \in \mathbb{R}^{l_i \times l_i}$ such that

\[
T_o^\top \bar{E}_{i2} = \begin{bmatrix}
0 \\
E_{i2}
\end{bmatrix}
\]  

where $E_{i2} \in \mathbb{R}^{n_i \times q_i}$ is nonsingular. Consequently, the coordinate transformation matrix $T_E$ can be obtained as:

\[
T_E = \begin{bmatrix}
I_{n_i-l_i} & -\bar{E}_{i1} \bar{E}_{i2}^\dagger \\
0 & T_o^\top
\end{bmatrix}
\]  

• The coordinate transformation matrix $T_i = T_E T_C$.

Therefore, the model of the $i$th subsystem given by (2.1) in the new coordinate system...
is described by

\[
\begin{align*}
\dot{z}_i &= A_i z_i + A_i y_i + \rho_i (y_i, u_i) + \eta_i (z_i, u_i, t) + \sum_{j=1}^{M} H_{ij}^1 (z_j) \\
\dot{z}_i &= A_i z_i + A_i y_i + \rho_i (y_i, u_i) + \eta_i (z_i, u_i, t) + \beta_i (t - T_0) E_{i2} \phi_i (y_i, u_i) + \sum_{j=1}^{M} H_{ij}^2 (z_j) \\
y_i &= \bar{C}_i z_i,
\end{align*}
\]

where

\[
\begin{bmatrix}
\rho_1 (y_i, u_i) \\
\rho_2 (y_i, u_i)
\end{bmatrix} = T_i \zeta_i (y_i, u_i),
\begin{bmatrix}
\eta_1 (z_i, u_i, t) \\
\eta_2 (z_i, u_i, t)
\end{bmatrix} = T_i \varphi_i (T_i^{-1} z_i, u_i, t),
\begin{bmatrix}
H_1^1 (z_j) \\
H_1^2 (z_j)
\end{bmatrix} = T_i h_{ij} (T_j^{-1} z_j).
\]

If we consider a more general structure of the nonlinear fault model, system (2.7) can be extended to

\[
\begin{align*}
\dot{z}_i &= A_i z_i + A_i y_i + \rho_i (z_i, u_i) + \eta_i (z_i, u_i, t) + \sum_{j=1}^{M} H_{ij}^1 (z_j) \\
\dot{z}_i &= A_i z_i + A_i y_i + \rho_i (z_i, u_i) + \eta_i (z_i, u_i, t) + \beta_i (t - T_0) f_i (y_i, u_i) + \sum_{j=1}^{M} H_{ij}^2 (z_j) \\
y_i &= \bar{C}_i z_i,
\end{align*}
\]

where the nonlinear fault function in \(i\) th subsystem can be represented by the smooth vector \(f_i : \mathbb{R}^i \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^l\). It worth noting that \(f_i (y_i, u_i)\) in (2.7) can be considered as \(E_{i2} \phi_i (y_i, u_i)\) in (2.8) and \(\rho_i (y_i, u_i)\) in (2.7) can be viewed as a nonlinear function of measurable variables \(u_i\) and \(y_i\), which is a special case of the function \(\rho_i (z_i, u_i)\) in (2.8).

**Assumption 2.** The unstructured modeling uncertainty functions \(\eta_{i1} (z_i, u_i, t)\) and \(\eta_{i2} (z_i, u_i, t)\) are unknown but bounded. \(\forall (z_i, y_i, u_i) \in Z_i \times Y_i \times U_i, \forall t \geq 0,\)

\[
|\eta_{i1} (z_i, u_i, t)| \leq \bar{\eta}_{i1} (y_i, u_i, t), \ |\eta_{i2} (z_i, u_i, t)| \leq \bar{\eta}_{i2} (y_i, u_i, t),
\]

\(2.9\)
where the bounding functions $\eta_{i1}(y_i, u_i, t)$ and $\eta_{i2}(y_i, u_i, t)$ associated with $\eta_{i1}$, et $\eta_{i2}$ correspondingly, are known and uniformly bounded in $Y_i \times U_i \times \mathbb{R}^+$. Moreover, the set of state variables $Z_i \subset \mathbb{R}^{n_i}$, input variables $U_i \subset \mathbb{R}^{m_i}$ and output variables $Y_i \subset \mathbb{R}^{l_i}$ are compact sets.

**Assumption 3.** The state vector $z_i$ of the $i$th subsystem remains bounded all the time $i = 1, \cdots, M$, i.e., $z_i(t) \in L_\infty, \forall t \geq 0$.

**Assumption 4.** The nonlinear terms $\rho_{i1}(z_i, u_i)$ and $\rho_{i2}(z_i, u_i)$ in (2.8) satisfy the following inequalities: $\forall u_i \in U_i$ and $\forall z_i, \hat{z}_i \in Z_i$,

$$|\rho_{i1}(z_i, u_i) - \rho_{i1}(\hat{z}_i, u_i)| \leq \sigma_{i1}|z_i - \hat{z}_i|$$

(2.10)

$$|\rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i)| \leq \sigma_{i2}(y_i, u_i, \hat{z}_i)|z_i - \hat{z}_i| + \bar{\sigma}_{i2}(y_i, u_i, \hat{z}_i)|z_i - \hat{z}_i|^2,$$

(2.11)

where $\sigma_{i1}$ is a known Lipschitz constant, $\sigma_{i2}(\cdot)$ and $\bar{\sigma}_{i2}(\cdot)$ are known functions that are uniformly bounded.

**Assumption 5.** In the system model (2.8), the term $H^1_{ij}(z_j)$ and $H^2_{ij}(z_j)$ representing interaction between the subsystems are uniformly Lipschitz, i.e., $\forall z_j, \hat{z}_j \in Z_j$,

$$|H^1_{ij}(z_j) - H^1_{ij}(\hat{z}_j)| \leq \gamma^1_{ij}|z_j - \hat{z}_j|$$

(2.12)

$$|H^2_{ij}(z_j) - H^2_{ij}(\hat{z}_j)| \leq \gamma^2_{ij}|z_j - \hat{z}_j|,$$

(2.13)

where $\gamma^1_{ij}$ and $\gamma^2_{ij}$ are the known Lipschitz constants for the interconnection terms $H^1_{ij}(z_j)$ and $H^2_{ij}(z_j)$, respectively.

Assumption 1 characterizes the class of large-scale distributed nonlinear systems which can be transformed into (2.8). The fault distribution matrices of the system are required to satisfy the condition of $\text{rank}(C_iE_i) = q_i$ to guarantee that the faults only affect the
Assumption 2 describes the unstructured modeling uncertainty in the system model (2.8). In order to distinguish faults effects and modeling uncertainty effects to the whole system in the fault detection process, we need to know the upper bounds on the unstructured modeling uncertainty.

For instance, in the aircraft engine fault diagnosis application considered in [22], the modeling uncertainty is the deviation of the actual engine dynamics from a nominal engine model representing the dynamics of a new engine, which results from normal engine component degradation during its service life. Such normal component degradation can be modeled by small changes in certain engine component health parameters (e.g., efficiency and flow capacity parameters of the fan, compressor, and turbine). Therefore, the bounding function on the modeling uncertainty (i.e., $\bar{\eta}_1$ and $\bar{\eta}_2$) can be obtained by using the knowledge of possible normal degradation of these health parameters during a number of flights under the worst case scenario.

Compared to the classes of modeling uncertainty in the distributed fault diagnosis literature, the unstructured modeling uncertainty in the system model (2.8) is more general and applicable. The modeling uncertainty in the previous research usually assume the absence of modeling uncertainty (e.g., [18]) or structured modeling uncertainty (e.g., [19]). In the fault detection methods based on the system models with structured modeling uncertainty, it is often assumed that the uncertainty distribution matrix satisfies some certain rank conditions to achieve robustness. In addition, the proposed fault detection methods may completely decouple the fault from the modeling uncertainty by utilizing of structured uncertainty with additional assumptions on the distribution matrix.

Assumption 5 requires that the direct interconnection terms in the system model (2.8) satisfies the Lipschitz condition. In literature, the automated highway system [18, 23] and inverted pendulums [1] are introduced as the examples of large-scale distributed nonlinear systems with Lipschitz interconnection terms.
Remark 2. The objective of this thesis is to develop a distributed fault detection scheme for
the class of large-scale nonlinear uncertain systems that are transformable into (2.8). It is
worth noting that the model described by (2.8) covers a larger class of large-scale nonlinear
systems than the system model considered in [19].

Specifically, in [19] the sliding mode observer is applied in estimating of the unknown fault function. It assumes that the distribution matrices of the modeling uncertainty \( \eta_{i1}, \eta_{i2} \) and the fault function \( f_i \) satisfies some conditions.

In addition, in previous papers [24, 25], the fault diagnosis schemes are developed based on a \textit{centralized} architecture. In this research work, we focus on solving the problem of fault diagnosis for large-scale nonlinear systems based on the \textit{distributed} architecture. It worth noting that, in system model (2.8), since the interconnection terms \( H_{1j}^1(z_j) \) and \( H_{1j}^2(z_j) \) among subsystems exists, the design of distributed fault diagnosis methods is much more difficult than centralized fault diagnosis methods.
Distributed Fault Detection Method

3.1 Distributed Fault detection Estimators

According to the subsystem model (2.8), we derived the distributed fault detection estimator (FDE) for each local subsystem as followings:

\[
\begin{align*}
\dot{\hat{z}}_{i1} &= A_{i1}\hat{z}_{i1} + A_{i2}\bar{C}_{i}^{-1}y_i + \rho_{i1}(\hat{\hat{z}}, u_i) + \sum_{j=1}^{M} H_{ij}^{1}(\hat{\hat{z}}_j) \\
\dot{\hat{z}}_{i2} &= A_{i3}\hat{z}_{i1} + A_{i4}\hat{z}_{i2} + \rho_{i2}(\hat{\hat{z}}, u_i) + L_i(y_i - \hat{\hat{y}}_i) + \sum_{j=1}^{M} H_{ij}^{2}(\hat{\hat{z}}_j) \\
\hat{\hat{y}}_i &= \bar{C}_i\hat{z}_{i2},
\end{align*}
\]

(3.1)

where in the \(i\)th subsystem, \(i = 1, \cdots, M\), estimated local state is denoted by \([\hat{\hat{z}}_{i1}^T, \hat{\hat{z}}_{i2}^T]^T\), and the output variables is represented by \(\hat{\hat{y}}_i\). The matrix \(L_i \in \mathbb{R}^{l_i \times l_i}\) is defined as a design gain matrix, and \(\hat{\hat{z}}_j \triangleq [(\hat{\hat{z}}_{j1})^T (\bar{C}_{j}^{-1}y_j)^T]^T\) where \(\hat{\hat{z}}_{j1}\) represents the estimated state vector for \(z_{j1}\) in the \(j\)th interconnected subsystem. We set the initial conditions as \(\hat{\hat{z}}_{i1}(0) = 0\) and \(\hat{\hat{z}}_{i2}(0) = \bar{C}_i^{-1}y_i(0)\).

By utilizing the information of local measurements in \(i\) th subsystem and \(\hat{\hat{z}}_j\) from the FDE associated with the \(j\)th interconnected subsystem, we can establish the distributed FDE (3.1) for each local subsystem. In the literature, for example [18, 26, 19, 16, 17], we should considered the impact of limited communication among interconnected subsystems during the design procedure of distributed estimation and fault diagnosis methods.
The scheme of the distributed fault detection estimators (FDEs) is shown in Figure 3.1. For each subsystem, a fault detection estimator is established, it generates the residuals and thresholds associated with the local subsystem. For each subsystem, the fault detection decision component generates alarm if the residual exceeds the corresponding adaptive threshold.

For each local FDE, we denote \( \tilde{z}_i \triangleq z_{i1} - \hat{z}_{i1} \) and \( \tilde{z}_i \triangleq z_{i2} - \hat{z}_{i2} \) as the state estimation errors, and \( \tilde{y}_i \triangleq y_i - \hat{y}_i \) as the output estimation error. Then, before fault occurrence (i.e., for \( t < T_0 \)), we can get

\[
\begin{align*}
\dot{\tilde{z}}_{i1} &= A_{i1} \tilde{z}_{i1} + \eta_{i1} + \sum_{j=1}^{M} \left[ H^1_{ij}(z_j) - H^1_{ij}(\hat{z}_j) \right] + \rho_{i1}(z_i, u_i) - \rho_{i1}(\hat{z}_i, u_i) \quad (3.2) \\
\dot{\tilde{z}}_{i2} &= \tilde{A}_{i4} \tilde{z}_{i2} + A_{i3} \tilde{z}_{i1} + \rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i) + \eta_{i2}(z_i, u_i, t) \\
&\quad + \sum_{j=1}^{M} \left[ H^2_{ij}(z_j) - H^2_{ij}(\hat{z}_j) \right] \quad (3.4) \\
\tilde{y}_i &= C_i (z_{i2} - \hat{z}_{i2}) = \tilde{C}_i \tilde{z}_{i2}, \quad (3.5)
\end{align*}
\]

where \( \tilde{A}_{i4} \triangleq A_{i4} - L_i \tilde{C}_i \). The value of gain \( L_i \) can be proprietarily chosen to make \( \tilde{A}_{i4} \) stable.

### 3.2 Adaptive Thresholds for Distributed Fault Detection

In this section, we will investigate the design of adaptive thresholds for distributed fault detection in each subsystem. First, a bounding function on the state estimation error vector

\[
\tilde{z}_1(t) \triangleq [(\tilde{z}_{11})^\top, \cdots, (\tilde{z}_{M1})^\top]^\top
\]

before the fault occurrence (i.e., for \( 0 \leq t < T_0 \)) will be derived. Specifically, we have the following results:
Figure 3.1: Distributed fault detection estimators scheme
Lemma 1 [27]. Let \( \phi, h : [0, \infty) \mapsto \mathbb{R} \). Then

\[
\dot{h} \leq -\alpha h + \phi, \quad \forall t \geq t_0 \geq 0
\]

implies that

\[
h(t) \leq e^{-\alpha(t-t_0)}h(t_0) + \int_{t_0}^{t} e^{-\alpha(t-\tau)}\phi(\tau)d\tau, \quad \forall t \geq t_0 \geq 0
\]

for any finite constant \( \alpha \).

Lemma 2. According to the system model (2.8) and the fault detection estimator (3.1).

Assume that there exist a symmetric positive definite matrix \( P_i \in \mathbb{R}^{(n_i-l_i)\times(n_i-l_i)} \), for \( i = 1, \cdots, M \), such that,

1. the symmetric matrix

\[
R_i \triangleq -A_{i1}P_i - P_iA_{i1} - 2P_iP_i - 2\sigma_{i1}\|P_i\|I > 0, \quad (3.7)
\]

where \( I \) is the identity matrix;

2. the matrix \( Q \in \mathbb{R}^{M \times M} \), whose entries are given by

\[
Q_{ij} = \begin{cases} 
\lambda_{\min}(R_i), & i = j \\
-\|P_i\|\gamma_{ij} - \|P_j\|\gamma_{ji}, & i \neq j, \ j = 1, \cdots, M.
\end{cases} \quad (3.8)
\]

is positive definite, where \( \gamma_{ij} \) are the Lipschitz constants introduced in (2.12).

Then, for \( 0 \leq t < T_0 \), the state estimation error vector \( \tilde{z}_i(t) \) (see (3.6)) satisfies the
following inequality:

\[
|\tilde{z}_i|^2 \leq \frac{\bar{V}_0 e^{-ct}}{\lambda_{\min}(P)} + \frac{1}{2\lambda_{\min}(P)} \int_0^t e^{-c(t-\tau)} \sum_{i=1}^M |\bar{\eta}_{i1}(y_i, u_i, \tau)|^2 \, d\tau ,
\]  

(3.9)

where the constant \(c \triangleq \lambda_{\min}(Q)/\lambda_{\max}(P)\), the matrix \(P \triangleq \text{diag}\{P_1, \cdots, P_M\}\), and \(\bar{V}_0\) is a positive constant to be defined later on.

**Proof:** For the \(i\)th subsystem, a Lyapunov function candidate \(V_i = \tilde{z}_{i1}^\top P_i \tilde{z}_{i1}\) is considered. The time derivative of \(V_i\) along the solution of (3.2) is calculated as followings:

\[
\dot{V}_i = \tilde{z}_{i1}^\top (A_i^\top P_i + P_i A_i) \tilde{z}_{i1} + 2\tilde{z}_{i1}^\top P_i \bar{\eta}_{i1}(z_i, u_i, t) + 2\tilde{z}_{i1}^\top P_i \sum_{j=1}^M [H_{ij}^1(z_j) - H_{ij}^1(\hat{z}_j)] + 2\tilde{z}_{i1}^\top P_i [\rho_{i1}(z_i, u_i) - \rho_{i1}(\hat{z}_i, u_i)] .
\]  

(3.10)

Based on the (3.1), we can obtain

\[
z_j - \hat{z}_j = \begin{bmatrix} z_{j1} - \hat{z}_{j1} \\ z_{j2} - C_j^{-1} y_j \end{bmatrix} = \begin{bmatrix} \tilde{z}_{j1} \\ 0 \end{bmatrix} .
\]  

(3.11)

Based on (2.12) and (3.11), we can obtain

\[
2\tilde{z}_{i1}^\top P_i \sum_{j=1}^M [H_{ij}^1(z_j) - H_{ij}^1(\hat{z}_j)] \leq 2|\tilde{z}_{i1}| ||P_i|| \sum_{j=1}^M \gamma_{ij}^1 |z_j - \hat{z}_j| \\
= 2||P_i|| \sum_{j=1}^M \gamma_{ij}^1 |\tilde{z}_{i1}| |\tilde{z}_{j1}| .
\]  

(3.12)
Moreover, based on (2.10) and (3.11), we obtain

\[ 2\tilde{z}_i^T P_i [\rho_{i1}(z_i, u_i) - \rho_{i1}(\hat{z}_i, u_i)] \leq 2|\tilde{z}_i| |P_i| |\sigma_{i1}| |z_i - \hat{z}_i| = 2|\tilde{z}_i| |P_i| |\sigma_{i1}| |\tilde{z}_i| \]

\[ = \tilde{z}_i^T [2\sigma_{i1}|P_i||I|] \tilde{z}_i. \quad (3.13) \]

Additionally, we have

\[ 2\tilde{z}_i^T P_i \eta_{i1} \leq |2P_i \tilde{z}_i| |\eta_{i1}| \leq 2\tilde{z}_i^T P_i \tilde{z}_i + \frac{1}{2} |\eta_{i1}|^2. \quad (3.14) \]

By using (3.10), (3.12), (3.13) and (3.14), we obtain

\[ \dot{V}_i \leq \tilde{z}_i^T \left[ A_{i1}^T P_i + P_i A_{i1} + 2P_i P_i + 2\sigma_{i1}|P_i||I| \right] \tilde{z}_i + 2||P_i|| \sum_{j=1}^M \gamma_{ij}^1 |\tilde{z}_{i1}| |\tilde{z}_{j1}| \]

\[ + \frac{1}{2} |\eta_{i1}|^2. \quad (3.15) \]

Based on (3.7) and the inequality \( \tilde{z}_i^T R_i \tilde{z}_i \geq \lambda_{\min}(R_i) |\tilde{z}_i|^2 \), where \( \lambda_{\min}(R_i) \) is the minimum eigenvalue of \( R_i \), we obtain:

\[ \dot{V}_i \leq -\lambda_{\min}(R_i) |\tilde{z}_i|^2 + 2||P_i|| \sum_{j=1}^M \gamma_{ij}^1 |\tilde{z}_{i1}| |\tilde{z}_{j1}| + \frac{1}{2} |\eta_{i1}|^2. \quad (3.16) \]

Then, the following overall Lyapunov function candidate for the large-scale system: \( V = \sum_{i=1}^M V_i = \sum_{i=1}^M \tilde{z}_i^T P_i \tilde{z}_i = \tilde{z}_1^T P \tilde{z}_1 \), is analyzed where \( P = \text{diag}\{P_1, \cdots, P_M\} \). Ac-
According to (3.16) and (3.8), we have

\[
\dot{V} \leq - \sum_{i=1}^{M} \lambda_{\text{min}}(R_i) \bar{z}_{i1}^2 + \sum_{i=1}^{M} \sum_{j=1}^{M} 2||P_i|| \gamma_{ij} \bar{z}_{i1} |\bar{z}_{j1}| + \sum_{i=1}^{M} \frac{1}{2} |\bar{\eta}_{i1}|^2 \\
= - \left[ |\bar{z}_{11}|, |\bar{z}_{21}|, \cdots, |\bar{z}_{M1}| \right] Q \left[ \begin{array}{c} |\bar{z}_{11}| \\ |\bar{z}_{21}| \\ \vdots \\ |\bar{z}_{M1}| \end{array} \right] + \sum_{i=1}^{M} \frac{1}{2} |\bar{\eta}_{i1}|^2
\]

where the matrix \(Q\) is defined by (3.8). If we apply the Rayleigh principle (i.e., \(\lambda_{\text{min}}(P)|\bar{z}_1|^2 \leq V(t) \leq \lambda_{\text{max}}(P)|\bar{z}_1|^2\)) and the definition of \(V(t)\) on the above equation, we can get

\[
\dot{V} \leq -\lambda_{\text{min}}(Q)|\bar{z}_1|^2 + \sum_{i=1}^{M} \frac{1}{2} |\bar{\eta}_{i1}|^2 \leq -\frac{\lambda_{\text{min}}(Q)}{\lambda_{\text{max}}(P)} V + \sum_{i=1}^{M} \frac{1}{2} |\bar{\eta}_{i1}|^2 = -cV + \sum_{i=1}^{M} \frac{1}{2} |\bar{\eta}_{i1}|^2.
\]

Therefore, based on Lemma 1, we can achieve the following inequality:

\[
V(t) \leq V(0)e^{-ct} + \frac{1}{2} \int_0^t e^{-c(t-\tau)} \sum_{i=1}^{M} |\bar{\eta}_{i1}(y_i, u_i, \tau)|^2 d\tau.
\]

The positive constant \(\tilde{V}_0\) can be chosen to satisfy the condition \(V(0) < \tilde{V}_0\). Therefore, the proof of (3.9) can be concluded based on definition of \(V(t)\), the Rayleigh principle, (2.10) and (2.9).

\[\square\]

**Remark 3.** It is worth noting that the linear matrix inequality (LMI) toolbox can be used to find a feasibility solution to the matrix inequalities (3.7) and (3.8). Specifically, the following procedure can be adopted:

- By using the Schur complements, the nonlinear inequalities \(-A_{i1}^TP_i - P_iA_{i1} -\)
\[ 2P_i P_i - 2 \sigma_{i1} \| P_i \| I > 0 \] can be converted to a LMI form as

\[
\begin{bmatrix}
-A_i^T P_i - P_i A_i - 2 \sigma_{i1} \varsigma_i I & \sqrt{2} P_i \\
\sqrt{2} P_i & I
\end{bmatrix} > 0
\]

(3.17)

and

\[
\begin{bmatrix}
\varsigma_i I & P_i \\
P_i & \varsigma_i I
\end{bmatrix} > 0,
\]

(3.18)

where \( \varsigma_i \) is a positive constant. Then, a suitable solution of \( P_i \) can be obtained by solving (3.17) and (3.18) using the LMI toolbox.

- For the matrix \( P_i \) found in the above step, the matrix \( Q \) defined in (3.8) is verified. If \( Q \) is positive definite, the solution of \( P_i \) is valid.

The state estimation error \( \tilde{z}_{i2}(t) \) of the \( i \)th subsystem can be analyzed based on the previous conclusion. For \( 0 \leq t < T_0 \), the solution of (3.4) is shown as

\[
\tilde{z}_{i2}(t) = \int_0^t e^{\bar{A}_{i4}(t-\tau)} [A_{i3}\tilde{z}_{i1}(\tau) + \eta_{i2}(z_i, u_i, t)] d\tau + \int_0^t e^{\bar{A}_{i4}(t-\tau)} [\rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i)] d\tau \\
+ \int_0^t e^{\bar{A}_{i4}(t-\tau)} \sum_{j=1}^M [H^2_{ij}(z_j) - H^2_{ij}(\hat{z}_j)] d\tau.
\]

we apply the triangle inequality on each component of the output estimation error\( (\tilde{y}_{ip}(t) \triangleq C_{ip}\tilde{z}_{i2}(t)) \) \( p = 1, \cdots, l_i \), where \( C_{ip} \) is the \( p \)th row vector of matrix \( C_i \), and have

\[
|\tilde{y}_{ip}(t)| \leq \left| \int_0^t C_{ip} e^{\bar{A}_{i4}(t-\tau)} \sum_{j=1}^M [H^2_{ij}(z_j) - H^2_{ij}(\hat{z}_j)] d\tau \right| + \left| \int_0^t C_{ip} e^{\bar{A}_{i4}(t-\tau)} [A_{i3}\tilde{z}_{i1}(\tau) + \eta_{i2}] d\tau \right| \\
+ \left| \int_0^t C_{ip} e^{\bar{A}_{i4}(t-\tau)} [\rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i)] d\tau \right|.
\]

(3.19)
we obtain the following inequalities based on (2.11), (2.13), and (3.11),

\[
\begin{align*}
|H_{ij}^2(z_j) - H_{ij}^2(\hat{z}_j)| & \leq \gamma_{ij}^2 |\hat{z}_{j1}|. \\
|\rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i)| & \leq \sigma_{i2}(y_i, u_i, \hat{z}_{i1}) |\hat{z}_{i1}| + \sigma_{i2}(y_i, u_i, \hat{z}_{i1}) |\hat{z}_{i1}|^2.
\end{align*}
\]

(3.20)

Therefore, by using (3.19) and (3.20), we have

\[
|\tilde{y}_{ip}(t)| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ ||A_{i3}|| + \sigma_{i2}(y_i, u_i, \hat{z}_{i1}) |\hat{z}_{i1}(\tau)| + \sigma_{i2}(y_i, u_i, \hat{z}_{i1}) |\hat{z}_{i1}(\tau)|^2 + |\eta_{i2}| + \sum_{j=1}^M \gamma_{ij}^2 |\hat{z}_{j1}(\tau)| \right] d\tau,
\]

(3.21)

where \(k_{ip}\) and \(\lambda_{ip}\) are positive constants chosen such that \(|C_{ip}e^{\overline{A}_i t}| \leq k_{ip}e^{-\lambda_{ip} t}\) (since \(\overline{A}_i\) is stable, constants \(k_{ip}\) and \(\lambda_{ip}\) satisfying the above inequality always exist [27]). By letting

\[
\theta_i \triangleq \left[ \gamma_{i1}^2, \cdots, \gamma_{i(i-1)}^2, ||A_{i3}|| + \sigma_{i2}, \gamma_{i(i+1)}^2, \cdots, \gamma_{iM}^2 \right]^\top,
\]

(3.22)

(that is, the components of \(\theta_i\) include \(\theta_{ii} = ||A_{i3}|| + \sigma_{i2}\), and \(\theta_{ij} = \gamma_{ij}^2\) for \(j \neq i\)), the inequality (3.21) can be rewritten as

\[
|\tilde{y}_{ip}(t)| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ \sum_{j=1}^M \theta_{ij} |\hat{z}_{j1}(\tau)| + \tilde{\sigma}_{i2} |\hat{z}_{i1}(\tau)|^2 + |\eta_{i2}| \right] d\tau \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ |\theta_i| |\hat{z}_{i1}(\tau)| + \tilde{\sigma}_{i2} |\hat{z}_{i1}(\tau)|^2 + \tilde{\eta}_{i2} \right] d\tau.
\]

(3.23)

Now, based on (3.23) and (3.9), we have

\[
|\tilde{y}_{ip}(t)| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ |\theta_i| \chi(\tau) + \tilde{\sigma}_{i2} \chi^2(\tau) + \tilde{\eta}_{i2} \right] d\tau.
\]

(3.24)
where

\[ \chi(t) \triangleq \left\{ \bar{V}_0 e^{-ct} + \frac{1}{2\lambda_{\min}(P)} \int_{0}^{t} e^{-c(t-\tau)} \sum_{i=1}^{M} |\bar{\eta}_{ii}(y_{i}, u_{i}, \tau)|^2 d\tau \right\}^{1/2}. \] (3.25)

Therefore, based on the above analysis, we have the following

**Distributed Fault Detection Decision Scheme:** The decision on the occurrence of a fault (detection) in the \( i \)th subsystem is made when the modulus of at least one component of the output estimation error (i.e., \( \bar{y}_{ip}(t) \)) generated by the local FDE exceeds its corresponding threshold \( \nu_{ip}(t) \) given by

\[ \nu_{ip}(t) \triangleq k_{ip} \int_{0}^{t} e^{-\lambda_{ip}(t-\tau)} \left[ |\bar{\eta}_{i}| \chi(\tau) + \bar{\sigma}_{i2} \chi^2(\tau) + \bar{\eta}_{i2} \right] d\tau. \] (3.26)

The fault detection time \( T_d \) is defined as the first time instant such that \( |\bar{y}_{ip}(T_d)| > \nu_{ip}(T_d) \), for some \( T_d \geq T_0 \) and some \( p \in \{1, \ldots, l_i\} \), that is, \( T_d \triangleq \inf \{ t \geq 0 : |\bar{y}_{ip}(t)| > \nu_{ip}(t) \} \).

The above design and analysis is summarized by the following technical result:

**Theorem 1 (Robustness):** For the large-scale nonlinear uncertain system described by (2.8), the distributed fault detection method, characterized by the fault detection estimator (3.1) and adaptive thresholds (3.26) designed for each local subsystem, guarantees that each residual component \( y_{ip}(t) \) remains below its corresponding adaptive threshold \( \nu_{ip}(t) \) prior to the occurrence of a fault (i.e., for \( t < T_0 \)).

**Remark 4.** It is worth noting that \( \nu_{ip}(t) \) given by (3.26) is an adaptive fault detection threshold, which has obvious advantage over a constant one. Moreover, the threshold \( \nu_{ip}(t) \) can be easily implemented using linear filtering techniques [24]. Moreover, the constants \( \bar{V}_0 \) in (3.25) is a (possibly conservative) bound for the unknown initial conditions \( V(0) \). However, since the effect of this bound decreases exponentially (i.e., it is multiplied
by $e^{-ct}$, the practical application of such a conservative bound will not affect significantly the performance of the distributed fault detection algorithm.

### 3.3 Fault Detectability Conditions

In this section, we investigate the fault detectability conditions of the proposed distributed fault detection method.

**Theorem 2 (Fault Detectability):** For the system model (2.8), if there exist some time instant $T_d > T_0$ and some $p \in \{1, \cdots, l_i\}$, such that the fault function $f_i(z_i, u_i)$ in the $i$th subsystem satisfies

$$\left| \int_{T_0}^{T_d} C_{ip} e^{\hat{A}_{i4}(t-\tau)} f_i(z_i(\tau), u_i(\tau)) d\tau \right| \geq 2\nu_{ip}(T_d) \quad (3.27)$$

then the fault will be detected at time $t = T_d$, i.e., $|\tilde{y}_{ip}(T_d)| > \nu_{ip}(T_d)$.

**Proof:** When the fault occurs, we can find that the dynamics of the state estimation error $\tilde{z}_{i1} \triangleq z_{i1} - \hat{z}_{i1}$ and $\tilde{z}_{i2} \triangleq z_{i2} - \hat{z}_{i2}$ is satisfying the following conditions:

$$\dot{\tilde{z}}_{i1} = \mathcal{A}_{i1} \tilde{z}_{i1} + \eta_{i1}(z_i, u_i, t) + \sum_{j=1}^{M} \left[ H^1_{ij}(z_j) - H^1_{ij}(\hat{z}_j) \right] + \rho_{i1}(z_i, u_i) - \rho_{i1}(\hat{z}_i, u_i) \quad (3.28)$$

$$\dot{\tilde{z}}_{i2} = \mathcal{A}_{i4} \tilde{z}_{i2} + \mathcal{A}_{i3} \tilde{z}_{i1} + \eta_{i2}(z_i, u_i, t) + \rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i) + \sum_{j=1}^{M} \left[ H^2_{ij}(z_j) - H^2_{ij}(\hat{z}_j) \right] + \beta_i f_i(z_i, u_i) \quad (3.29)$$

Therefore, for each component of the output estimation error, i.e., $\tilde{y}_{ip}(t) \triangleq C_{ip} \tilde{z}_{i2}(t)$,
\[ p = 1, \ldots, l_i, \text{we have} \]
\[
\tilde{y}_{ip}(t) = \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} \left[ A_{i3} \tilde{z}_{i1}(\tau) + \eta_{i2} + \beta_{i} f_i(z_i, u_i) \right] d\tau \\
+ \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} [\rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i)] d\tau \\
+ \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} \sum_{j=1}^M [H_{ij}^2(z_j) - H_{ij}^2(\hat{z}_j)] d\tau.
\]

From Lemma 2, we have \[ |\tilde{z}_1(t)| \leq \chi(t) \]. Then, we obtain the following equations based on triangle inequality:
\[
|\tilde{y}_{ip}(t)| \geq \left| \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} \beta_{i} f_i(z_i, u_i) d\tau \right| - k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ ||A_{i3}|| |\tilde{z}_{i1}(\tau)| + |\eta_{i2}| \\
+ \sum_{j=1}^M \gamma_{ij}^2 |\tilde{z}_{j1}(\tau)| + \sigma_{i2} |\tilde{z}_{i1}(\tau)|^2 \right] d\tau \\
\geq \left| \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} \beta_{i} f_i(y_i, u_i) d\tau \right| - k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ \varphi_{i} |\chi(\tau) + \tilde{\eta}_{i2} \\
+ \tilde{\sigma}_{i2} \chi^2(\tau) \right] d\tau \\
\geq \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} \beta_{i} f_i(y_i, u_i) d\tau - k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ \varphi_{i} |\chi(\tau) + \tilde{\eta}_{i2} \\
+ \tilde{\sigma}_{i2} \chi^2(\tau) \right] d\tau \right| - \nu_{ip}(t) \quad (3.31)
\]

where \( \varphi_i \) is defined in (3.22). By substituting (3.26) into (3.30), we have
\[ |\tilde{y}_{ip}(t)| \geq \int_0^t C_{ip} e^{\tilde{A}_{ii}(t-\tau)} \beta_{i} f_i(y_i(\tau), u_i(\tau)) d\tau \]

Based on the property of the step function \( \beta_{i} \), if there exists \( T_d > T_0 \), such that condition (3.27) is satisfied, then it is concluded that \[ |\tilde{y}_{ip}(T_d)| > \nu_{ip}(T_d), \text{i.e., the fault is detected at time } t = T_d. \]
Application to Automated Highway Systems

4.1 Description of the Automated Highway Systems (AHS)

We have the increasing traffic congestion problem on most of the urban highways. The solution of building new highways is constrained from many factors such as limited suitable land, escalating construction costs and environment pollution. One alternative is to use the current highways system more efficiently by replacing some human operations with automation. The Automated Highway Systems (AHS) is constructed based on this principle. In AHS, the lane capacity of the highways can be increased, moreover, the vehicles are driven automatically with onboard controllers to avoid human errors that caused many of today’s automobile accidents and make driving and transportation safer.

There has been a significant interest on the development of automated highway systems (AHS). The concept of AHS was first introduced in the General Motors Pavilion at 1939 World’s Fair, and the initial research work was done in the late 1950’s by the Radio Corporation of America in cooperation with General Motors Corporation [28], [29]. Since then, various aspects of the AHS have been investigated by many researchers. For the control aspect, a lot of research work has been done on the longitudinal control of individual vehicles.
Based on the inter-vehicle communication, the longitudinal control is applied in the vehicle following scenario (See Figure 4.1). It focuses on how to maintain a close safe spacing when a vehicle follows the lead vehicle or a desired and steady velocity when a vehicle is traveling alone. In the vehicle following situation, the local control system in each vehicle can access the following information: speed and acceleration of vehicle, distance to the preceding vehicle, and acceleration and speed of the preceding vehicle in the platoon.

So, based on the characteristics of the vehicle following system under the longitudinal control, it can be considered as a suitable distributed nonlinear system for implementing the distributed fault detection proposed.

Figure 4.1: Vehicle following in a lane [1].
4.2 Simulation implementation and results of the distributed fault detection method on AHS

Spooner and Passino [23] described the car-following application in which only tracking information is available (as opposed to information about lead and other subsequent vehicles) to each following vehicle. To ensure the safe and reliable operations of AHS, the development of fault diagnosis technologies is particularly important.

Based on [23], we have the dynamics of the $i$th vehicle as follows:

\[
\begin{align*}
\dot{\psi}_i &= v_i - v_{(i-1)} \\
\dot{v}_i &= \frac{1}{m_i} \left( -D_i v_i^2 - d_i + \xi_i \right) \\
\dot{\xi}_i &= \frac{1}{\tau_i} (-\xi_i + u_i) \\
y_i &= \begin{bmatrix} \psi_i + \omega v_i \\ \xi_i \end{bmatrix},
\end{align*}
\]

where $\psi_i$ is the distance between the $i$th and the $(i-1)$th vehicle, $v_i$ denotes the $i$th vehicle’s velocity, and $\xi_i$ denotes the driving/braking force applied to the $i$th vehicle. The control input is $u_i$, $m_i$ is the vehicle mass, $D_i$ is the aerodynamic drag, $d_i$ and $\tau_i$ are the constant frictional force and the engine/brake time constant, respectively. In this simulation example, the following simulation parameters are used: $m_i = 1300$ kg, $D_i = 0.3$ Ns$^2$/m$^2$, $d_i = 100$ N, $\tau_i = 0.2$ s, and $\omega = 0.4$.

Based on the values of model parameters given above, the above model can be rewrit-
By using a linear transformation of coordinates $z_i = [z_{i1} \ z_{i2} \ z_{i3}]^\top = T_i[\psi_i \ v_i \ \xi_i]^\top$, the state space model in the new coordinate system is

$$
\begin{bmatrix}
\dot{\psi}_i \\
\dot{v}_i \\
\dot{\xi}_i
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{1}{1300} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
\psi_i \\
v_i \\
\xi_i
\end{bmatrix} +
\begin{bmatrix}
0 \\
-\frac{1}{13} - \frac{0.3}{1300}v_i^2 \\
5u_i
\end{bmatrix}
- \begin{bmatrix}
v_{(i-1)}
\end{bmatrix}
$$

$$
y_i =
\begin{bmatrix}
1 & 0.4 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\psi_i \\
v_i \\
\xi_i
\end{bmatrix}.
$$

We assume two types of the modeling uncertainty $\eta_i$. First, a disturbance signal $\vartheta_i \sin(t)$ where $|\vartheta_i| \leq 0.5$ in the state equations of $\psi_2$ and $\psi_3$, respectively; Second, up to 5% inaccuracy in the engine/brake time constant $\tau_i$ of all vehicles. Therefore, $\bar{\eta}_{i1} = 0.5$ and $\bar{\eta}_{i2} = 0.26 | - y_{i2} + u_i |$ are obtained. In the actual simulation results, $\vartheta_i = 0.4$ and 2% inaccuracy in $\tau_i$ are used. Clearly, the above system model is in the form of (2.8) with the unmeasurable state variable $z_{i1}$ and measurable state variables $[z_{i2} \ z_{i3}]^\top$. Since the
dynamics of the $i$th vehicle is only affected by $(i-1)$th vehicle, we have $\gamma_{21}^1 = \gamma_{32}^1 = 2.5$, $\gamma_{21}^2 = \gamma_{32}^2 = 0.25$, and $\gamma_{12}^1 = \gamma_{13}^1 = \gamma_{12}^2 = \gamma_{13}^2 = \gamma_{23}^1 = \gamma_{23}^2 = \gamma_{31}^1 = \gamma_{31}^2 = 0$. Additionally, for $i = 1, 2, 3$, we have $\sigma_{i1} = 0, \sigma_{i2} = 0.0058 \cdot | - 0.02y_{i1} + 0.02\hat{z}_{i1}|$, and $\sigma_{i2} = 0.00058$ (note that $(z_{i2} - 0.1z_{i1})^2 - (z_{i2} - 0.1\hat{z}_{i1})^2 = \tilde{z}_{i1}(-0.2z_{i2} + 0.02\hat{z}_{i1}) + 0.01(\hat{z}_{i1})^2$).

A local fault detection estimator for each vehicle is constructed by utilizing the method presented in Chapter 3. Based on the model (4.1) in the new coordinate system, the fault detection estimators (FDEs) can be constructed using (3.1) in chapter 3 as followings:

\[
\begin{bmatrix}
\dot{\hat{z}}_{i1} \\
\dot{\hat{z}}_{i2} \\
\dot{\hat{z}}_{i3}
\end{bmatrix} =
\begin{bmatrix}
-2.5 & 25 & 0 \\
-0.25 & 2.5 & 3.077 \times 10^{-5} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
\hat{z}_{i1} \\
\hat{z}_{i2} \\
\hat{z}_{i3}
\end{bmatrix} +
\begin{bmatrix}
0 \\
-0.0031 - 0.0058(\hat{z}_{i2} - 0.1\hat{z}_{i1})^2 \\
5u_i
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{y}_i
\end{bmatrix} =
\begin{bmatrix}
0 & 10 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{z}_i
\end{bmatrix}
\]

The parameters in the simulation are designed as followings: the gain matrix $L_i$ is chosen such that the poles of matrix $\bar{A}_{i4}$ are located at -1.5 and -1.7, respectively. The constants $k_{i1} = k_{i2}$ in local FDEs are set as 1, $\lambda_{i1} = -1.5$, and $\lambda_{i2} = -1.7$. Moreover, to determine the values of $P$ and $Q$ matrix to obtain the bound of the residuals for the unmeasurable state variables $z_1$ in the system (4.1). First, we choose $P = [0.5 \ 0 \ 0; 0 \ 0.5 \ 0; 0 \ 0 \ 0.5]$, so that corresponding to Lemma 2, for vehicle 1, since there is no uncertainty in the model of vehicle 1, the term $2P_1P_1$ (effect from the modeling uncertainty) should not be included in the $R_1$, and the nonlinear part for $z_1$ in the model (4.1) is 0. Consequently, \[R_1 = A_{i1}^TP_1P_1A_{i1} - 2\sigma_{i1}\|P_1\|I = -2.5 \times 0.5 - 0.5 \times 2.5 = 2.5 > 0.\] For vehicle 2 and 3, \[R_i = A_{i1}^TP_1P_1A_{i1} - 2\sigma_{i1}\|P_1\|I = -2.5 \times 0.5 - 0.5 \times 2.5 - 2 \times 0.5 \times 0.5 = 2 > 0\] where
\(i = 2, 3\). So the values of \(R_1, R_2\) and \(R_3\) are obtained, which guarantee that the condition (3.7) is satisfied. In this simulation of AHS, the Lipschitz constants \(\gamma_{21} = \gamma_{32} = 2.5\), and \(\gamma_{12} = \gamma_{13} = \gamma_{23} = \gamma_{24} = \gamma_{34} = 0\), then we can determine the value of \(Q\) matrix as 
\[
Q = \begin{bmatrix}
2.5 & -1.25 & 0 \\
-1.25 & 2 & -1.25 \\
0 & -1.25 & 2
\end{bmatrix}
\]
to guarantee that condition (3.8) is satisfied, and we can obtain \(c = 0.6752\) based on the \(Q\) and \(P\) matrix designed.

First, we consider an actuator fault in vehicle 1. Specifically, a simple multiplicative actuator fault is considered by letting \(u_1 = \bar{u}_1 + \theta_1 \bar{u}_1\), where \(\bar{u}_1\) is the nominal control input in the non-fault case, and \(\theta_1 \in [-1, 0]\) is the parameter characterizing the magnitude of the fault. Hence, the actuator fault can be modeled by 
\[f_1 \triangleq [0 \quad \theta_1 g_1(u_1)]^T,\]
where \(g_1 = \frac{u_1}{\gamma_1}\). Figure 4.2 and Figure 4.3 show the simulation results when an actuator fault with \(\theta_1 = -0.1\) occurs to the first vehicle at \(T_1 = 10\) second. As can be seen from Figure 4.2, all the residuals generated by the local FDEs for vehicle 2 and vehicle 3 always remain below their thresholds, while the residual associated with \(y_{12}\) generated by FDE 1 exceeds its threshold at approximately \(t = 10.05\) second (see Figure 4.3). Therefore, the actuator fault in vehicle 1 is immediately detected.
Figure 4.2: An actuator fault occurring in vehicle 1, fault detection residuals (solid and blue line) and their thresholds (dashed and red line) generated by the local FDEs for vehicle 2 and vehicle 3.

Figure 4.3: An actuator fault occurring in vehicle 1, the fault detection residuals (solid and blue line) associated with $y_{12}$ and its threshold (dashed and red line) generated by the local FDE for vehicle 1
As another illustrative example, we consider a fault causing abnormal driving/braking force in the second vehicle. Specifically, the driving/braking force $\xi_2$ decreases as a result of the fault. Thus, the fault function is in the form of $f_2 \triangleq \begin{bmatrix} \theta_2 g_2(y_2) & 0 \end{bmatrix}^T$, where $g_2 = 0.04 \frac{z_2}{m_2} = 0.04 \frac{\nu_2}{m_2}$, and $\theta_2 \in [-1, 0]$ represents the fault magnitude. Figure 4.4 and Figure 4.5 shows the simulation results when such a fault with $\theta_2 = -0.3$ occurs to the second vehicle at $T_2 = 10$ second. Again, the fault is successfully detected.

Similarly, The results of the process fault that occurs to the third vehicle is shown in Figure 4.6 and Figure 4.7 and the results of the actuator fault that occurs to the third vehicle is shown in Figure 4.8 and Figure 4.9.

The process faults in all the cases have the same structure $g_2(y_2)$ and amplitude $\theta_2$. Also, the actuator faults in all the cases have the same structure $g_1(u_1)$ and amplitude $\theta_1$. All the faults occurs at $T = 10$ second. From the results, all the residuals generated by the local FDEs for the fault free vehicles always remain below their thresholds, while the residual associated with the vehicle where the fault occurs exceeds its threshold.

Above all, based on the simulation results, we can conclude that the robust distributed fault detection method can successfully detect the two types of faults in all the three vehicles in the car-following application in AHS.
Figure 4.4: A process fault occurring in vehicle 2, the fault detection residuals (solid and blue line) and their thresholds (dashed and red line) generated by the local FDEs for vehicle 1 and vehicle 3.

Figure 4.5: A process fault occurring in vehicle 2, the fault detection residuals (solid and blue line) associated with $y_{21}$ and its threshold (dashed and red line) generated by the local FDE for vehicle 2.
Figure 4.6: A process fault occurring in vehicle 3, fault detection residuals (solid and blue line) and their thresholds (dashed and red line) generated by the local FDEs for vehicle 1 and vehicle 2.

Figure 4.7: A process fault occurring in vehicle 3, the fault detection residual (solid and blue line) associated with $y_{31}$ and its threshold (dashed and red line) generated by the local FDE for vehicle 3.
Figure 4.8: An actuator fault occurring in vehicle 3, fault detection residuals (solid and blue line) and their thresholds (dashed and red line) generated by the local FDEs for vehicle 1 and vehicle 2.

Figure 4.9: An actuator fault occurring in vehicle 3, the fault detection residuals (solid and blue line) associated with $y_{32}$ and its threshold (dashed and red line) generated by the local FDE for vehicle 3
4.3 Comparison with FDE method ignoring the interaction effect among subsystems in a distrusted system

Currently, most of the existed significant and effective fault detection methods are based on a centralized fault diagnosis architecture. However, if the centralized FD methods are applied in solving the problem of detection faults in large-scale systems without considering the interaction effect or communication among subsystems, it may be failed in estimating the unknown nominal system, generating correct residual and detecting the faults. In this comparison study section, the distributed FD method in this thesis is compared with a centralized fault detection and isolation (FDI) methodology for nonlinear uncertain systems in [24, 25]. The distributed FD scheme is same to the scheme of the centralized FDI method in [24, 25] except the estimation part of the interaction effect. So, we can apply both methods on the AHS simulation example to compare the fault detection result and see the advantage of distributed FD methods.

To do the comparison, we set all the parameters in simulation are same to the experiment above. Based on the centralized FD method in [24, 25], the FDEs for AHS can be constructed as followings:

\[
\begin{bmatrix}
\dot{z}_{i1} \\
\dot{z}_{i2} \\
\dot{z}_{i3}
\end{bmatrix} =
\begin{bmatrix}
-2.5 & 25 & 0 \\
-0.25 & 2.5 & 3.077 \times 10^{-5} \\
0 & 0 & -5
\end{bmatrix}
\begin{bmatrix}
\dot{z}_{i1} \\
\dot{z}_{i2} \\
\dot{z}_{i3}
\end{bmatrix} +
\begin{bmatrix}
0 \\
-0.0031 - 0.0058(\dot{z}_{i2} - 0.1\dot{z}_{i1})^2 \\
5u_i
\end{bmatrix}
\begin{bmatrix}
\dot{y}_i =
\begin{bmatrix}
0 & 10 & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{bmatrix} z_i.
\]

Figure 4.10 shows the residual and threshold of the FDE for the second and third vehicles when an actuator fault with \(\theta_1 = -0.1\) occurs to the first vehicle at \(T_1 = 10\) second. As can be seen from Figure 4.10, the residuals generated by the local FDEs for
vehicle 2 and vehicle 3 exceed the corresponding thresholds even the fault occurs on the first vehicle. The false alarms are generated even there is no fault in the second and third vehicle.

Figure 4.10: An actuator fault occurring in vehicle 1 in the comparison study, fault detection residuals (solid and blue line) and their thresholds (dashed and red line) generated by the local FDEs for vehicle 2 (top) and vehicle 3 (bottom).

Figure 4.11 shows the residual and threshold of the FDE for the third vehicle when a process fault with $\theta_2 = -0.3$ occurs to the second vehicle at $T_2 = 10$ second. As can be seen from Figure 4.11, the residual generated by the local FDE for vehicle 3 exceeds the threshold even the fault occurs on the second vehicle and the false alarm is generated.
Figure 4.11: An process fault occurring in vehicle 2 in the comparison study, fault detection residuals (solid and blue line) and their thresholds (dashed and red line) generated by the local FDE for vehicle 3.
Conclusions

5.1 Contributions

In this thesis, a distributed fault detection method is presented for a class of large-scale nonlinear uncertain systems. The modeling uncertainty in the systems can be unstructured. Then, under certain assumptions, adaptive thresholds are designed for distributed fault detection in each subsystem and the robustness of the detection scheme is also investigated. Furthermore, the fault detectability conditions characterizing the class of faults in each subsystem that can be detected by this approach is analyzed. Moreover, a simulation model of automated highway system is developed and is used to show the effectiveness of the distributed fault detection method. In addition, an comparison between the distributed FD scheme and centralized architecture FD scheme shows the advantage and necessariness of the distributed FD method in the area of FD of large-scale distributed systems.

5.2 Future work

The extension of the presented fault detection method to include faults occurring in the communication link between interconnected subsystems is one of interesting topics for future research. Another direction for future research is to consider large-scale nonlinear systems with more general nonlinearities and interconnection terms.
Bibliography


First appendix chapter