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An Exploratory Study of Mixed-Width Aisle Layouts for Order Picking in Distribution Centers

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AN EXPLORATORY STUDY OF MIXED-WIDTH AISLE LAYOUTS FOR ORDER PICKING IN DISTRIBUTION CENTERS

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By

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Corinne H. Mowrey ENTITLED An exploratory study of mixed-width aisle layouts for order picking in distribution centers BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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Abstract

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Order picking is arguably the most expensive operational activity for a distribution center (DC), constituting upwards of 50% of total operating costs. Designing an optimum order picking system (OPS) for a DC depends on several system parameters, such as aisle layout, storage system configuration, storage policy, picking method, and picking strategy. From an aisle layout standpoint, traditional DCs utilize either entirely wide or entirely narrow aisles in their picking systems. Wide aisles allow pickers to pass each other, reducing blocking and requiring fewer pickers. However, the space required for wide-aisle systems is high. Narrow aisles utilize less space than wide aisles, but are less efficient because of the high likelihood of congestion experienced by pickers. Space required for the picking area and labor required to perform picking are two significant costs for a DC's OPS. Traditional approaches focus on minimizing either space or minimizing labor rather than integrating the two objectives. We propose a variation to the traditional orthogonal aisle designs where both wide and narrow aisles are mixed within the system, anticipating that the mixed-width aisles may provide a compromise between space and labor. We develop analytical models for space and travel time for systems that employ randomized storage and traversal routing policies. We illustrate the use of these models by developing a cost-based optimization model to determine the optimal aisle configuration for specific OPSs. The objective of this model is to minimize the total system cost which was divided into two components, space and labor. Results indicate that mixed-aisles appear to be optimal for certain OPSs with randomized storage and traversal routing, with the resulting savings in total cost being as high as $48,000 over pure wide aisle systems. Additional benefits may be realized by using mixed-width aisles for other storage policies, such as class-based, and for semi-automated systems, both of which need further research.

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1. Introduction

A vital operation in a distribution center (DC) is order picking, or the fulfillment of customer orders by retrieving customer requested items from storage locations. Order picking is arguably the most expensive operational activity constituting upwards of 50% of a DC's total operating costs (Tompkins et al., 2003). In the ongoing quest to maximize profits, decision makers would naturally look to their order picking system (OPS) for any opportunity to increase efficiency and lower costs. One such opportunity, which ultimately leads to a cost effective OPS, comes in the form of an optimally designed picking area.

Designing an optimum OPS for a DC depends on several system parameters, such as aisle layout, storage system configuration, storage policy, picking method, and picking strategy. From an aisle layout standpoint, traditional DCs utilize either entirely wide or entirely narrow aisles in their picking systems. Wide aisles allow pickers to pass each other, reducing blocking and requiring fewer pickers to meet the required system throughput (orders/hour or items/hour). The space required for wide-aisle systems is, however, relatively high. Narrow aisles utilize less space than wide aisles, but are less efficient because of the high likelihood of congestion experienced by pickers. Space required for the picking area and labor required to perform picking are two significant costs for a DC's OPS. Traditional approaches focus on minimizing either space where the cost of land is high, or minimizing labor where the cost of land is low rather than integrating the two objectives.

In the past few years alternate aisle arrangements have been proposed that improve upon the traditional layout of the picking area. The Fishbone and Flying-V layouts designed by Gue and Meller (2009) potentially offer higher throughput or reduced costs by adding non horizontal (or vertical) cross aisles. These designs are beneficial to unit-load warehouses where only one item is picked during a pick tour, but do not offer significant improvements when picking a batch of orders resulting in multiple items per pick tour. For such OPS, we propose a variation to the traditional orthogonal aisle designs where both wide and narrow aisles are mixed within the system (see Figure 1). This specific layout incorporates both narrow and wide aisle sections in a single aisle. Could an aisle layout of this nature prove to be cost effective? We anticipate that the mixed-width aisles may provide a good compromise between space and labor; i.e., less blocking compared to pure narrow aisles due to the ability of pickers to pass each other in the wide sections and less space compared to pure wide aisles due to the inclusion of narrow sections.

Through this research we evaluate the potential savings in total cost that could be realized through the use of mixed-aisles. Our research is of significance to OPS designers and managers because it not only provides general analytical models that can be used to determine optimal aisle width (whether wide, narrow, or mixed), but it also helps compare the three alternatives to identify the optimal aisle-width.

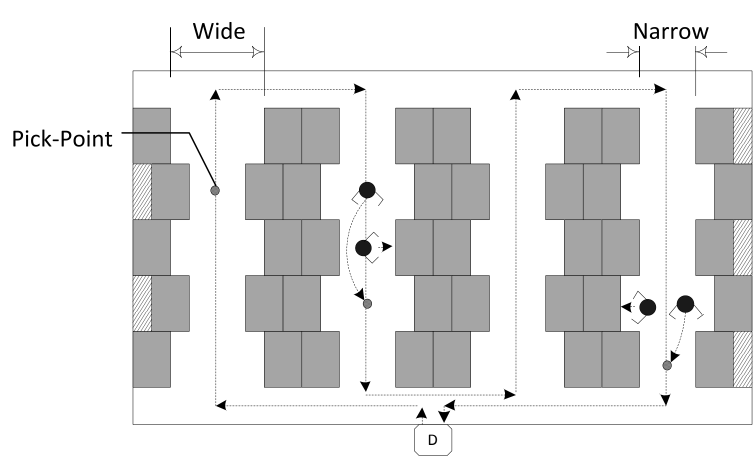


Figure 1: Mixed-width aisle system

The remainder of this thesis is organized as follows. We begin by reviewing existing research in Section 2 and develop rules to identify feasible mixed-width aisle configurations in Section 3. In Sections 4 and 5, we discuss our analytical space and throughput models. We present an optimization model and a solution approach for identifying the optimal aisle configuration in Section 6. Section 7 discusses our experiment results and offers managerial insights. We summarize our findings in Section 8.

2. Related Research

Extensive research has been performed in the area of order picking system design and operation. Rouwenhorst et al. (2000) discussed order picking design and control problems in terms of long, medium, and short term decisions such as sorting systems for long term, layout, equipment and workforce capacity for medium term, and workforce assignment for short term. In the situation where the probability of visiting every aisle for one or more picks is close to 1.0, traversal routing policy is close to optimal under randomized storage policy (Petersen and Aase, 2004). Roodbergen and Vis (2006) developed a model, which optimized the layout for a warehouse's order picking area while minimizing the average distance a picker traveled. This model was based on fixed routing policies and found that for high pick densities, the traversal routing policy was best suited for layouts with an even number of aisles. The review article by Gu et al. (2007) identified order picking planning problems relating to batching, routing and sequencing, and sorting and provided various decision support models and solution algorithms to aid in the design process. De Koster et al. (2007) indicated that most current research points to travel as the component which takes up the majority of a picker's time, and as such, continued to discuss layout designs, storage assignments, zoning, batching and routing methods in terms of minimizing distances. Roodbergen et al. (2008) considered systems which utilized cross-aisles and developed a model that minimized a picker's travel distance by optimizing the layout of one or more blocks of parallel aisles. This model was developed for systems which employed a randomized storage policy and a traversal routing policy.

A critical factor that could affect the total travel time is picker congestion, typically modeled as picker blocking. Blocking is attributed to either the inability of the pickers to pass each other in the aisle (because the aisles are narrow) or not being able to pick at a pick-column when someone else is picking there. The former is referred to as *in-the-aisle blocking*, while the latter is referred to as *pick-column blocking*. Gue et al. (2006) focused on how varying pick densities affected in-the-aisle blocking in a picking system that was comprised of pure narrow aisles. They found that as the pick density increased, or picking became busier, congestion decreased. Skufca (2005) considered the problem of in-the-aisle blocking and derived an analytical expression to estimate this blocking via a continuous loop where *k* workers traveled at an infinite speed and picked at most one stock keeping unit (SKU).

Parikh and Meller (2009) developed analytical models which estimated picker blocking in systems with aisles wide enough for passing. They considered two cases, deterministic pick time (where only one SKU is picked at a pick-column) and non-deterministic pick time (where one or more SKUs are picked at a pick-column), and concluded that blocking is significantly less in wide aisles than in narrow aisles. For narrow aisles with non-deterministic pick time, Parikh and Meller (2010) indicated that blocking experienced by pickers could actually be a concern as the system gets busier.

Hong et al. (2010) analyzed the impacts of batch picking on picker blocking for narrow aisle systems. Their study looked at both single pick and multiple pick scenarios and found that the high variation in number of picks in each aisle, which was attributed to picking one or more items at a given pick column, led to significant picker blocking as compared to picking a single item at each pick column. Building upon earlier models, Parikh and Meller (2010) derived a travel time model for semi-automated systems, which were defined as OPSs that employed person on-board order picking equipment (e.g., an order picker truck). To illustrate its significance, the model was used in a cost-based optimization model to recommend the height of a one-pallet-deep storage system. Recently, Wallace-Finney and Parikh (2011) developed a cost based model, which optimized aisle-width for a specific system configuration in both manual and semi-automated systems. Results showed a preference for wide aisles when cost of labor and required throughput were high and a preference for narrow aisles when cost of space and number of storage locations were high.

While current literature discusses optimal storage and travel policies, very little literature exists that addresses optimal aisle width, and none addresses the notion of mixed-width aisles within a single picking area. We expect to fill this gap by proposing this novel aisle configuration and developing analytical models for space and travel-time. Additionally, we present an optimization model to determine the optimal mixed-width aisle configuration for given system parameters, thus creating a valuable tool for decision makers to utilize in order to better design picking areas.

3. Mixed-Width Aisle Configurations

A mixed-width aisle configuration is characterized by two variables: the ratio of wide-aisle sections to narrow-aisle sections (*r*) and the number of consecutive pick columns in a wide-aisle section (l_w). We define a section in terms of consecutive pick-columns with the same width. Both *r* and l_w help in determining a repeatable pattern that defines the aisle layout. For example, given a system where the total number of pick columns (*s*) is 40 and the number of aisles (*a*) is 4 (10 pick columns per aisle), the combination of $r = 1$ and $l_w = 10$ would produce a wide-narrow-wide-narrow aisle configuration. If $r = 0.333$, then the aisle configuration would be 1 wide aisle followed by 3 narrow aisles. Following this logic, a system with $r = l_w = 0$ would translate to all narrow aisles, while a system with $r = \infty$ and $l_w = s$ would translate to all wide aisles in the picking area. Table 1 summarizes the notation used in our model.

Table 1: Notations used in the analytical models

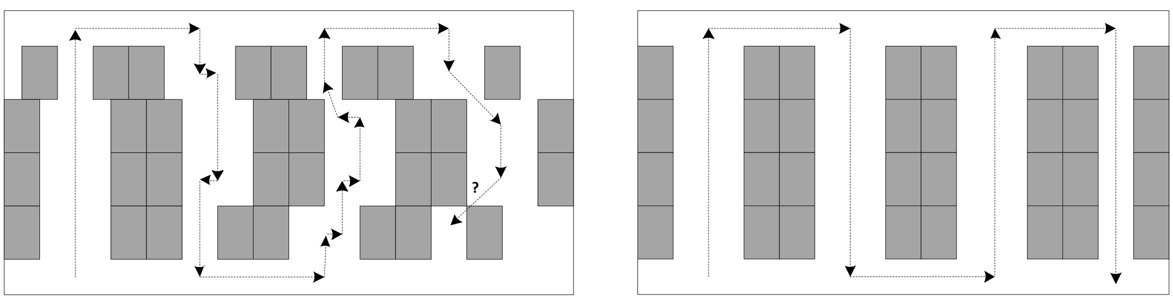
Notation	**Definition**
I	expected number of items to be picked during a pick-tour
s	total number of storage locations (pick columns) required to store all SKUs
r	ratio of wide-aisle to narrow-aisle sections
l_w	number of consecutive pick columns in a wide-aisle section
a	number of aisles
a_w^w (a_w^n)	width of a wide (narrow) aisle (ft)
a_l	length of an aisle (ft)
a_l^{max}	maximum allowable length of an aisle (ft)
p_d (p_w)	depth (width) of a pick column (ft)
c_w	width of a cross aisle (ft)
S	total space required for pick area with one storage level (ft^2)
v_h	horizontal speed of picker (fps)
t_p	time required by each picker to pick one item from a storage location (sec)
t_w	time required to travel past a pick-column (sec) = $\frac{p_w}{v_h}$
k	actual number of pickers required to achieve required throughput
u	probability of stopping at a specific pick column and picking up to 5 times
λ	theoretical pick-rate of a picker assuming no blocking (items/hr)
Λ_{req}	throughput requirement of the system (items/hr)
$B(r, l_w, k, u, t_p : t_w)$	estimated blocking experienced by each picker
$E[T]$	expected total time required for the pick-tour (sec)
$E[T_{travel}]$	expected total time to travel (sec)
$E[T_{aisles}]$	expected total time to travel the aisles (sec)
$E[T_{cross}]$	expected total time to travel the cross-aisles (sec)
$E[T_{depot}]$	expected total time to travel to and from the depot (sec)
$E[T_{circle}]$	expected total time to travel around the last aisle when *a* is odd (sec)
$E[T_{pick}]$	expected total time for a picker to pick *I* items (sec)
C_s	cost of space ($/ft)
C_k	cost of labor ($/picker)

Not all combinations of *s*, *a*, *r*, and l_w produce realistic or feasible designs. When certain combinations of *r* and l_w are used, the aisle configuration may create an erratic traversal route for pickers, which eventually becomes impossible to follow as the number of aisles increase. This can be seen in Figure 2(a) and Figure 3(a). The following rules ensure that feasible mixed-width aisle designs are achieved:

Rule 1: If $r \neq 1$, then l_w must be an integer multiple of $\frac{s}{a}$.

Rule 2: If $r = 1$, then l_w may be an integer multiple or even factor of $\frac{s}{a}$.

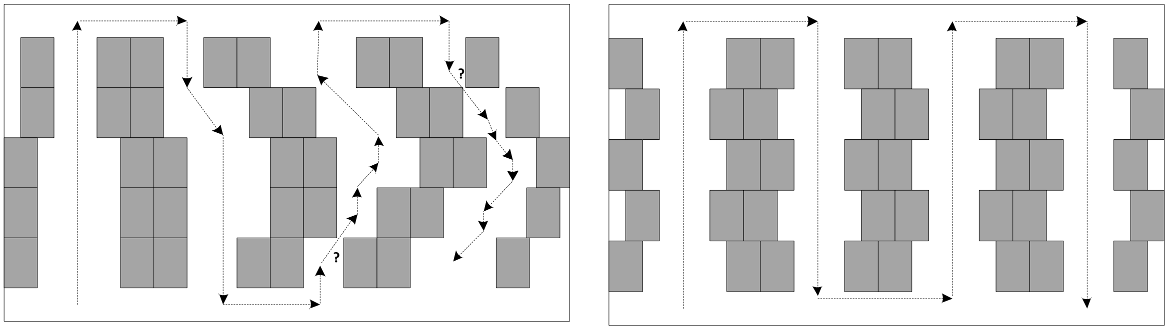
Rule 1 ensures that sections of wide aisles and subsequent sections of narrow aisles must be whole aisles when the aisle configuration is not composed of 50% wide aisles and 50% narrow aisles (i.e., $r \neq 1$). This can be visualized in Figure 2, which illustrates both feasible and infeasible systems when $r = 3 \neq 1$. According to Rule 1, l_w must then be a multiple of $s/a = 32/4 = 8$, which is not the case in Figure 2(a). As a result, the layout of the picking area progressively worsens as the number of aisles increases until the picker's path is ultimately obstructed. In Figure 2(b), where $l_w = 8$, the travel path is well-defined and unobstructed for pickers to easily traverse through the picking area.



(a) Infeasible System ($l_w = 6 \neq {}^{s}/_{a}$) **(b)** Feasible System ($l_w = 8 = {}^{s}/_{a}$)

Figure 2: Illustration of Rule 1, where $s = 32$, $a = 4$, and $r = 3$

Rule 2 allows for mixed-widths within an aisle only when $r = 1$; i.e. the configuration is 50% wide aisle and 50% narrow aisles. Figure 3 illustrates both feasible and infeasible systems when $r = 1$. According to Rule 2, l_w should be an integer multiple or even factor of $s/a = 40/4 = 10$. In Figure 3(a), $l_w = 6 \neq$ factor of 10 and so the layout of the picking area progressively worsens as the number of aisles increases until the picker's path is blocked. In Figure 3(b), $l_w = 2$ = factor of 10, resulting in a clear travel path which is easily traveled by the picker.



(a) Infeasible System ($l_w = 6$) **(b)** Feasible System ($l_w = 2$; even factor of $^{s}/_{a}$)

Figure 3: Illustration of Rule 2, where $s = 40$, $a = 4$, and $r = 1$

The above two rules provide a quick way to identify feasible designs and serve as inputs to the analytical models presented next.

4. Analytical Space Model

The analytical space model we develop in this section can be used for both single-width and mixed-width aisle configurations. Figure 4 illustrates a mixed-width aisle configuration along with the parameters used in our space calculation. An expression to determine the unused space, which results as pick-columns need to be offset in mixed-width aisles, is presented later in this section.

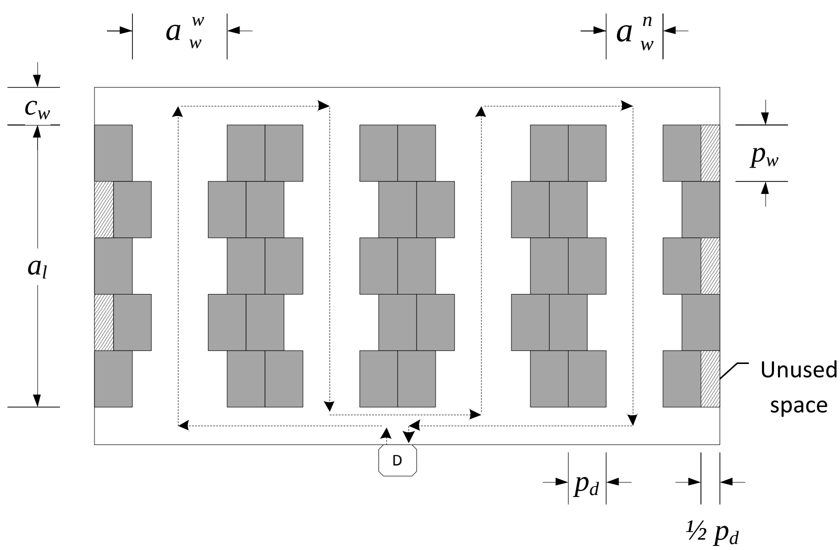


Figure 4: Mixed-width aisle configuration ($l_w < s/a$)

The total space (S) required for a single-width system (e.g., pure narrow or wide) is the sum of the space requirements for aisles, racks, and cross-aisles. The simplest approach to calculating the total space is to define the system's length and depth, and calculate the picking area as the product of these two dimensions. The elements that compose the system's length include rack depth (p_d) and aisle width (a_w). The elements that compose the system's depth include aisle length (a_l) and cross-aisle width (c_w). Using this approach, the total required space is given by

$$S = (aa_w + 2ap_d)(a_l + 2c_w), \tag{1}$$

where $a_l = (s/2a)p_w$ for a system with a single storage level (e.g., in manual systems). For a mixed-width aisle system, a_w must be replaced by an expression that accounts for both the wide aisles and narrow aisles that are present in the system. This expression, which converts aisle width for a single-width system into a weighted average of aisle widths for a mixed-width system, is given by

$$a_w = a_w^w \left(\frac{r}{r+1}\right) + a_w^n \left(1 - \frac{r}{r+1}\right). \tag{2}$$

Substituting (2) in (1) gives the following general space model applicable to wide, narrow, and mixed-aisles:

$$S = (2c_w + a_l)\left(2ap_d + a\left(a_w^w \left(\frac{r}{r+1}\right) + a_w^n \left(1 - \frac{r}{r+1}\right)\right)\right). \tag{3}$$

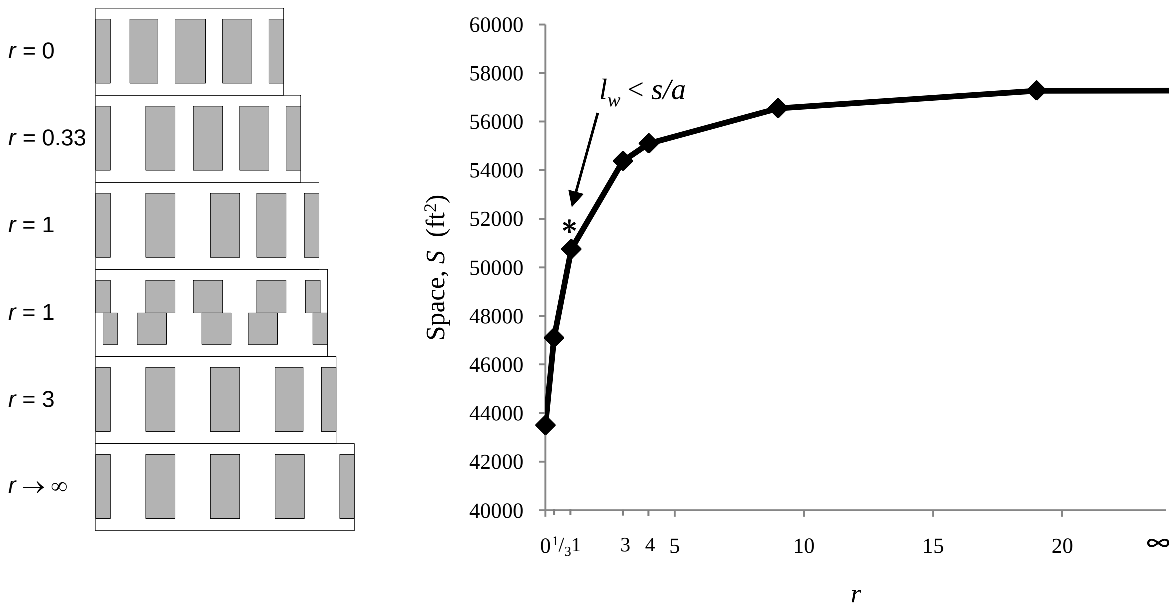
For example, in a pure narrow system where $r = 0$, $a_w = a_w^n$. Similarly, for a pure wide aisle system where $r = \infty$, $a_w = a_w^w$. Reducing and simplifying (3) for either case gives us

$$S = aa_l a_w + 2aa_l p_d + 2(aa_w c_w + 2ap_d c_w), \tag{4}$$

which is identical to the expression presented in Parikh and Meller (2010).

As shown in Figure 5, mixed-width aisle systems require more space than narrow aisle systems, but less than wide aisle systems. Also shown in Figure 5 are systems with mixed-widths within the aisles, which occur when $l_w < s/a$ or when the number of pick columns in a wide aisle section is less than the total number of pick columns in an aisle. Note that the total space for all $r = 1$

mixed-width ($l_w < s/a$) aisle systems is identical regardless of the value of l_w. As seen in (3), space is impacted by r and is independent of l_w, with the following exception: mixed-widths ($l_w < s/a$) result in offset racks, and offset racks increase the length of the picking area by half a pick column depth, p_d (see Figure 4). Due to offset racks, the space required for system with r = 1 and mixed-widths ($l_w < s/a$) is larger than mixed-width systems with whole aisles ($l_w = s/a$).



(a) System representation (b) System space comparison (s = 1,000, a = 20)

Figure 5: Mixed-width aisle systems and comparisons

Modifying (3) to account for offset racks in systems with mixed-width ($l_w < s/a$) aisles, as shown in Figure 4, gives us

$$S = \left(2c_w + \left(\frac{s}{2a}\right)p_w\right)\left(2ap_d + a\left(a_w^w\left(\frac{r}{r+1}\right) + a_w^n\left(1 - \frac{r}{r+1}\right)\right) + 0.5\alpha p_d\right), \tag{5}$$

where α is a binary parameter that may be expressed as

$$\alpha = \begin{cases} 1, \text{ if } l_w < {}^{s}/_{a}, \\ 0, \text{ if } l_w \geq {}^{s}/_{a}. \end{cases}$$

Offsetting the racks, however, results space that is unused by racks or aisles. This concept of unused space is unique to mixed-width (l_w < s/a) aisle systems and is explored in the next section.

4.1. Unused Space

Unused space occurs in systems with mixed-width (l_w < s/a) aisles and is found behind the narrow section pick columns on the first and last aisle. This was illustrated in Figure 4, which showed a system where s = 40, a = 4, r = 1, and l_w = 2.

Unused space is the product of half the pick column depth, $p_d/2$, the pick column width, p_w, and half the number of pick columns in an aisle, $s/2a$ (Figure 4). When the number of aisles in a system is odd and the number of pick columns on each side of an aisle are odd, the number of offset pick columns that create unused space is one less than half the number of pick columns in an aisle. To account for this, a binary parameter is used to subtract a "half pick column" from the equation. The unused space for a mixed-width aisle system (with l_w < s/a) can be calculated as

$$S_{unused} = \left(\frac{s}{4a} - 0.5\beta\right) p_w p_d \, , \tag{6}$$

where β is a binary parameter that may be expressed as

$$\beta = \begin{cases} 1, if\ a \text{ is odd and } {}^{s}/_{2a} \text{ is odd,} \\ 0, \text{otherwise.} \end{cases}$$

Since unused space only appears in the first and last aisle, it appears that for an r = 1 system, the most unused space would be seen in layouts with a low number of aisles, and unused space would then decrease as the total number of aisles increases. This leads us to the following theorem.

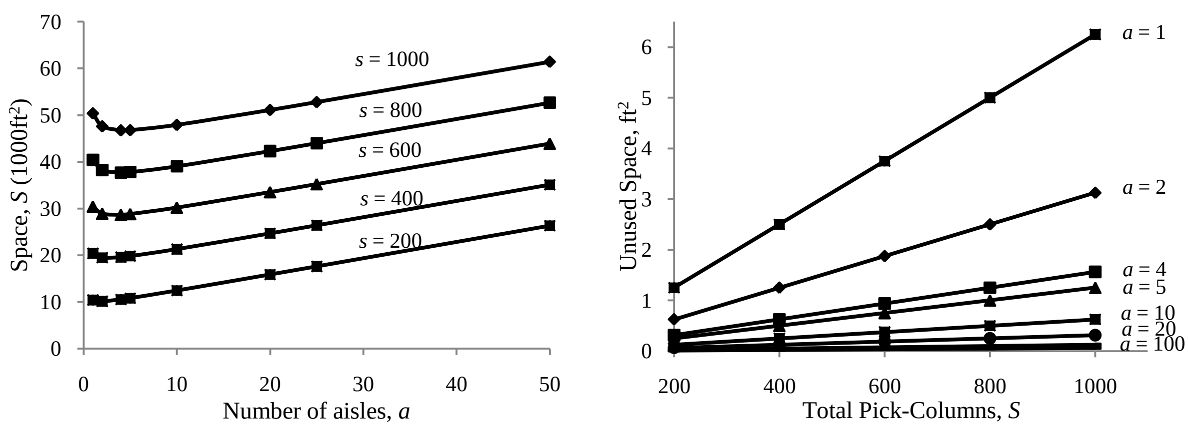
Theorem 1: The percentage of unused space for an r = 1 system converges to 0 as the number of aisles approaches infinity.

Proof: The percentage of unused space for an r = 1 system is given by dividing equation (6) by the total system space, S, and then multiplying by 100. We then take the limit of this modified equation, as the number of aisles approaches infinity, as follows:

$$\lim_{a\to\infty} \% \, S_{unused} \; = \lim_{a\to\infty} \frac{\left(\frac{s}{4a} - 0.5\beta\right) p_w p_d}{S} \times 100$$

$$\approx \lim_{a\to\infty} \frac{s}{(4a)S} = 0. \qquad \blacksquare$$

The above fact is experimentally supported in Figure 6.



(a) Total space **(b)** Unused space

Figure 6: Space comparison (r = 1, l_w = 2)

Figure 6(a) shows the relationship between the number of aisles in a system and the total amount of space required for a mixed-width system, where r = 1 and l_w = 2. Notice the slight dip in the required space for a < 10 aisles. Initially, as the number of aisles increases, the total required space decreases. This is because the reduction in aisle length leads to a substantial decrease in the overall depth of the system as compared to the increase in aisle length that is attributed to the addition of aisles. As the number of aisles increase, the reduction in system depth due to the reduction in aisle length becomes minimal in comparison to the addition in length due to the additional aisles. Figure 6(b) indicates the corresponding percentage of unused space, which tends to 0 as the number of aisles increases.

Having developed the analytical space model, we now focus on estimating the total amount of time each picker spends traversing the picking area in a single pick tour.

5. Throughput Model

The number of pickers (k) required to satisfy the required throughput for a system (Λ_{req}) is based on both the throughput of each picker as well as the blocking experienced by each picker. The throughput generated by each picker, assuming that no blocking or idle time is experienced, is referred to here as the theoretical pick rate (λ) and is given by

$$\lambda = \frac{3600I}{E[T]} \text{ items/hr.} \tag{7}$$

The expected total time required is the sum of the expected total travel time and the expected time to pick I items.

$$E[T] = E[T_{travel}] + E[T_{pick}], \text{ where } E[T_{pick}] = It_p \,. \tag{8}$$

The estimated travel time, $E[T_{travel}]$, associated with the pick tour is based on the travel path followed by a picker through the aisles. The travel path is broken into four parts as follows: (i) travel through the aisles, (ii) travel through the cross-aisles, (iii) travel to and from the depot, and (iv) time to circle back toward the depot when the system has an odd number of aisles. For a manual system with randomized storage policy and traversal routing, $E[T_{travel}]$ can be estimated through expressions (9) - (13), which generalize the expressions presented in Parikh and Meller (2010). The binary parameter γ, expressed as

$$\gamma = \begin{cases} 1, \text{if number of aisles } (a) \text{ is odd} \\ 0, \text{otherwise} \end{cases}$$

is used in the total time calculation to ensure that $E[T_{circle}]$ is included only when the number of aisles is odd. Total travel time is thus expressed as

$$E[T_{travel}] = E[T_{aisles}] + E[T_{cross}] + E[T_{depot}] + \gamma E[T_{circle}]. \tag{9}$$

We estimate $E[T_{aisles}]$ and $E[T_{cross}]$ based on the number of aisles (a), the length of an aisle (a_l), the cross-aisle width (c_w), and the walking speed of the picker (v_h); similar to Parikh and Meller (2010). That is,

$$E[T_{aisles}] = \frac{a_l a}{v_h} \text{ and} \tag{10}$$

$$E[T_{cross}] = \frac{c_w(a+1)}{v_h}. \tag{11}$$

The travel time to and from the depot is based on the number of aisles (a), the depth of a pick column (p_d), the single aisle width (a_w), and the walking speed of the picker (v_h) which is similar to Parikh and Meller (2010). Substituting (2) for a_w to account for mixed-width aisles gives

$$E[T_{depot}] = 2\left(\frac{\left(a_w^w\left(\frac{r}{r+1}\right) + a_w^n\left(1 - \frac{r}{r+1}\right)\right)(a-1) + 2p_d(a-1)}{v_h}\right). \tag{12}$$

When picking systems have an odd number of aisles, pickers exit the last aisle on the opposite side than the depot is located, thus requiring them to return to the depot via circling around the last aisle. The additional travel time required to circle around the last aisle is captured by $E[T_{circle}]$ and is given by

$$E[T_{circle}] = \frac{a_l + 2\left(a_w^w\left(\frac{r}{r+1}\right) + a_w^n\left(1 - \frac{r}{r+1}\right)\right) + 2p_d + c_w}{v_h}, \tag{13}$$

where (2) is substituted for a_w to account for mixed-width aisles.

Figure **7** illustrates that the expected total travel time required for a pick-tour increases as r increases. We had earlier indicated that as r increases the total required space increases (see Figure 5(b)). Intuitively, both these observations make sense. Recall that $r = 0$ for pure narrow systems and $r=\infty$ for pure wide systems. As r increases, the system increases in size, thus increasing the amount of time to travel through the system. Expression (7) indicates that the theoretical pick rate is inversely proportional to the total travel time, as shown in Figure 7, and so as r increases, λ decreases. Based on total system space and travel time, Figure 5 and Figure 7, maximum benefits can be realized when $r<5$.

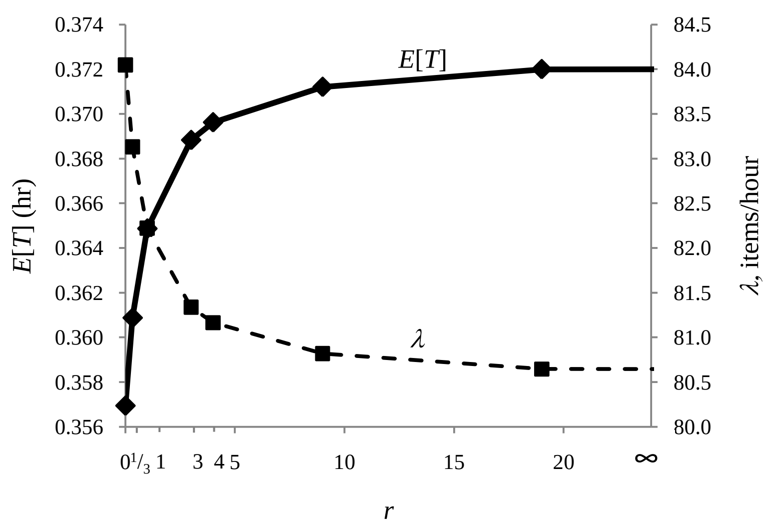


Figure 7: The effect of r on $E[T]$ and λ (s = 1,000, a = 20)

It is worth noting that picker productivity could be reduced due to blocking, which we discuss next.

5.1. Effect of Picker Blocking on Picker Throughput

A picker may experience one of two types of blocking: *pick-column blocking* and *in-the-aisle blocking*. *Pick-column blocking* occurs when two or more pickers need to pick at the same location. *In-the-aisle blocking* is only experienced in narrow aisles and occurs when one or more pickers are unable to pass someone who is actively picking (Parikh and Meller, 2009). When a picker is blocked, they experience idle time and as a result are less productive. Because blocking is prominent in high-throughput systems, its productivity-reducing effect must be accounted for when calculating a system's actual throughput. We developed a generic simulation model that can simulate any of the three aisle-widths, pure wide, pure narrow, and mixed, to estimate the average blocking experienced by pickers.

Blocking for mixed-width aisle systems is dependent on five factors: the ratio of wide to narrow sections (r), the number of consecutive pick columns in a wide section (l_w), the number of pickers in the system (k), the pick-density (u), and the pick time to walk time ratio (t_p:t_w). We refer the reader to Parikh and Meller (2009) for the procedure to estimate u for the given expected number of items picked during a pick tour (I) and the total number of pick columns for a system (s).

Our simulation model uses concepts similar to those previously researched on blocking where the picking area was modeled as one circular aisle (Gue et al., 2006; Parikh and Meller, 2009). In the simulation, pickers alternate between wide aisles and narrow aisles as defined by l_w and r. Blocking per picker was measured both in the narrow and wide aisle sections and represented as $B(r, l_w, k, u, t_p\!:t_w)$.

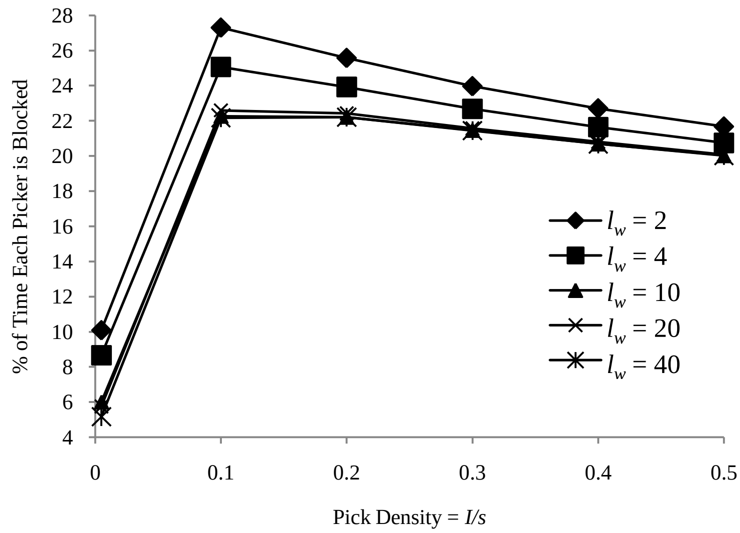


Figure 8: Blocking for less than full aisle configurations (s = 400, a = 10, k = 50)

Figure 8 shows blocking curves for varying l_w values for a system where s = 400, a = 10, and k = 50. For this system, l_w = 40 results in full aisles of a single width and all others result in mixed-widths ($l_w < s/a$) within an aisle. When l_w = 2 (= 4), wide aisle sections are very small (Figure 1), allowing pickers a very limited opportunity to pass before entering a narrow section. This limited opportunity reduces the effect of passing in wide aisles and behaves more similar to a pure narrow system, hence the higher blocking. This phenomenon is shown in Figure 8, where we see the highest blocking for l_w = 2, followed by l_w = 4 and then l_w = 10, 20, and 40. The largest l_w values (l_w = 40, 20, and 10) show blocking curves which are virtually identical where the two smallest l_w values (l_w = 4 and 2) show significantly higher blocking. We interpret this to mean that the actual number of pickers needed to meet a required throughput would be the same for l_w = 40, 20, and 10 with potentially higher numbers of pickers needed for l_w = 4 and 2. We conclude from this that the lowest cost will favor higher l_w values for this system configuration.

Recall from Section 4 that all mixed-widths ($l_w < s/a$) for a system have the same required total space. Also recall that total space for mixed-width ($l_w < s/a$) aisles is larger than the total system space required for mixed-width ($l_w = s/a$) aisles due to offset racks. If $l_w = 40$, 20, and 10 all have the same number of pickers, and $l_w = 40$ has the smallest required system space, then we can conclude that $l_w = 40$ will have the lowest total system cost. Generalizing this for all system configurations, we conclude that for systems where $r = 1$, whole aisle configurations, ($l_w = s/a$) will always be the optimum configuration (even though it might not be the optimum r value for the system).

Having determined that whole aisle configurations are optimal, we can develop the optimization model for determining the aisle layout that minimizes the total system cost.

6. Model for Optimal Aisle Configuration

The objective of our cost based optimization model is to minimize the total system cost. The following two components make up the system's cost: space and labor. Required space for an aisle configuration is based solely on physical layout, while labor is determined based on the theoretical pick rate of each picker and the amount of blocking experienced by each picker. The optimization model used to determine the optimal aisle configuration for a given system is presented below.

Minimize: $C(r, l_w)$

Subject to:
$$a = \left\lceil \frac{sp_w}{2a_l^{max}} \right\rceil \tag{14}$$

$$l_w = \frac{s}{a}, \text{ if } r < 1;\ l_w = \text{factor of } \frac{s}{a}, \text{ if } r = 1; l_w = r\left(\frac{s}{a}\right), \text{ if } r > 1 \tag{15}$$

$$S = \left(2c_w + \left(\frac{s}{2a}\right)p_w\right)\left(2ap_d + a\left(a_w^w\left(\frac{r}{r+1}\right) + a_w^n\left(1 - \frac{r}{r+1}\right)\right) + 0.5\alpha p_d\right) \tag{16}$$

$$k\lambda\left[1 - B\left(r, l_w, k, u, t_p : t_w\right)\right] \geq \Lambda_{req} \tag{17}$$

$$l_w, k, a \in I^+ \text{ and } r, S, B(.), \lambda \geq 0 \tag{18}$$

The decision variables for this model are r and l_w. The objective of the model is to minimize the total system cost $C(r, l_w)$, which is the sum of the cost of space and the cost of labor. That is

$$C(r, l_w) = SC_s + kC_k \ .$$

Through (14) we ensure that the number of aisles in the system does not exceed the maximum allowable length. Through (15) we ensure that optimal l_w designs are considered by limiting l_w to whole aisle values. The total space required for a system is calculated in (16). Constraint (17) guarantees that the required throughput for the system is met, where $B(.)$ is determined using the simulation model for estimating blocking discussed earlier. Constraints (18) indicate bounds on the decision variables in the discussion.

To solve the optimization model presented above, we present a formal solution procedure to determine the most cost-effect system layout. For all feasible r values, where $\frac{a}{r+1}$ is a positive integer, follow the steps below:

Step 1: Calculate the number of aisles in the picking area, using $a = \left\lceil \frac{sp_w}{2a_l} \right\rceil$, where $a_l \leq a_l^{max}$.

Step 2: Estimate the total travel time, $E[T]$, using (8).

Step 3: Calculate the theoretical pick rate (λ) using (7).

Step 4: Calculate the theoretical number of pickers required from the theoretical pick rate where

$$k_{theo} = \frac{\Lambda_{req}}{\lambda}.$$

Step 5: Estimate the percentage of time each picker is blocked, $B\left(r, l_w, k, u, t_p\!: t_w\right)$, using the simulation model discussed earlier.

Step 6: Calculate the actual throughput, $\Lambda_{act} = \lambda k_{theo}\left[1 - B\left(r, l_w, k, u, t_p : t_w\right)\right]$. If $\Lambda_{act} < \Lambda_{req}$, then set the new theoretical number of pickers to $k_{theo} = k_{theo} + 1$ and repeat Steps 5 and 6 until $\Lambda_{act} \geq \Lambda_{req}$.

Step 7: Calculate the total required space (S) as defined by (16).

Step 8: Calculate total system cost (labor and space).

We now present results of our experiments to illustrate the use of our proposed analytical and optimization models.

7. Experiment Design and Results

The optimization model presented earlier was used to determine the optimal aisle configuration for specific system parameters. Table 2 lists the levels and values of all the parameters used in our experiments. Approximately 810 experiments were conducted for a manual system with randomized storage and traversal routing.

The total number of possible r values for a given number of aisles (a) is equal to $a + 1$. However, for the sake of experimentation, we did not include any r values which did not result in a definitive repeatable pattern without interruptions, where the first aisle width was opposite of the last aisle width (i.e. unless pure widths, those beginning wide should have ended narrow). For example, a repeatable pattern for $a = 10$ and $r = 0.25$ would be NNNNWNNNNW, while a non-repeatable pattern for $r = 1.5$ would be NWWNWWNWWN. For $s = 1{,}000$, we evaluated 5 r values (3 where applicable), which identified the quartiles and included pure narrow ($r = 0$) and wide ($r = \infty$). The same r values were evaluated for $s = 400$ with the exception of $a = 10$. For $a = 10$ ($s = 400$) we evaluated 7 values for r (i.e., 0, 0.1111, 0.25, 1, 4, 9, and ∞) with uninterrupted repeatable patterns. The results are presented in the following tables. It is worth noting that our optimal solutions were derived considering only repeatable patterns; non-repeatable patterns exist, but we did not consider those in our search for the optimal aisle-width in this exploratory study.

Table 2: Parameter values used in experimentation

Parameter	Levels	Values Evaluated
I	1	30
s	2	400, 1,000
a	3	2, 10, 20
Λ_{req}	3	1,000, 2,500, 3,750 (items/hour)
C_S	3	1, 10, 25 ($/ft^2)
C_K	3	20,000, 30,000, 50,000 (annual loaded $/picker)
r	5	0, 0.25, 1, 4, ∞
l_w	1	2, 4, 10, 20,…, *s/a* for $r \leq 1$ or *r(s/a)* for $r > 1$

Table 3 shows the parameter values generated for each evaluated *r* configuration for a system where $s = 400$, $a = 10$, and $\Lambda = 2{,}500$. These parameters include total system space, total estimated time, the theoretical pick rate of each picker, the blocking experienced by each picker, and the total number of pickers required to meet the required system throughput considering blocking.

Table 3: Parameter values for $s = 400$, $a = 10$, $\Lambda = 2{,}500$ system

r	l_w	S (ft^2)	$E[T]$ (sec)	λ (items /hr)	$B(.)$	k
N	0	18,000	714.4	151.2	12.60%	19
0.25	40	19,200	719.8	150.0	11.06%	19
1	40	21,000	727.9	148.4	9.29%	19
4	160	22,800	736.0	146.7	3.76%	18
W	400	24,000	741.4	145.7	0.56%	18

Table 4 shows a detailed cost comparison for each evaluated *r* configuration for a system where $s = 400$, $a = 10$, and $\Lambda = 2{,}500$. Where optimal, mixed-width aisles showed cost savings of $1,200 to $48,000 over pure wide aisles. This translated into a 0.1% to 3.8% cost savings improvement in comparison to pure wide systems with the same system parameters. These cost savings could be classified as low to moderate. We speculate that mixed-width aisles might be more suitable for a class based storage policy where higher demand items would be stored in wide aisle sections and lower demand items would be stored in narrow aisle sections; this is discussed further in Section 8. Mixed-width aisle configurations were optimal in systems where s = 400 and 1,000 for both 2,500 and 3,750 throughputs. For all other combinations, pure narrow or pure wide configurations were optimal. In many cases, multiple r values required the same number of pickers to meet the required throughput. In these situations, the r value with the least amount of space was always optimum. Table 5 and Table 6 show the optimal aisle configuration as well as the total system cost for s = 400 and 1,000 systems respectively. Table 7 shows the space cost sensitivity of the optimal aisle width configuration for an s = 1,000 system.

Table 4: Detailed cost comparison for s = 400, a = 10, Λ = 2,500 system

R		Space Cost (x1,000)	$C_k = \$20{,}000$ / yr		$C_k = \$30{,}000$ / yr		$C_k = \$50{,}000$ / yr	
			Labor Cost (x1,000)	**Total Cost (x1,000)**	Labor Cost (x1,000)	**Total Cost (x1,000)**	Labor Cost (x1,000)	**Total Cost (x1,000)**
N	$C_s = \$1/ft^2$	\$18	\$380	\$398	\$570	\$588	\$950	\$968
0.25		\$19	\$380	\$399	\$570	\$589	\$950	\$969
1		\$21	\$380	\$401	\$570	\$591	\$950	\$971
4		\$23	\$360	**\$383**	\$540	**\$563**	\$900	**\$923**
W		\$24	\$360	\$384	\$540	\$564	\$900	\$924
N	$C_s = \$10/ft^2$	\$180	\$380	**\$560**	\$570	**\$750**	\$950	\$1,130
0.25		\$192	\$380	\$572	\$570	\$762	\$950	\$1,142
1		\$210	\$380	\$590	\$570	\$780	\$950	\$1,160
4		\$228	\$360	\$588	\$540	\$768	\$900	**\$1,128**
W		\$240	\$360	\$600	\$540	\$780	\$900	\$1,140
N	$C_s = \$25/ft^2$	\$450	\$380	**\$830**	\$570	**\$1,020**	\$950	**\$1,400**
0.25		\$480	\$380	\$860	\$570	\$1,050	\$950	\$1,430
1		\$525	\$380	\$905	\$570	\$1,095	\$950	\$1,475
4		\$570	\$360	\$930	\$540	\$1,110	\$900	\$1,470
W		\$600	\$360	\$960	\$540	\$1,140	\$900	\$1,500

Table 5: Optimal aisle width configuration for s = 400, I = 30 system

Λ	a	C_s	Total Cost and Optimum Configuration					
			C_k = \$20,000/yr	r	C_k = \$30,000/yr	r	C_k = \$50,000/yr	r
1,000	2	\$1/ft^2	\$135,600	W	\$195,600	N	\$315,600	N
	10		\$158,000	N	\$228,000	N	\$368,000	N
	20		\$201,000	N	\$291,000	N	\$471,000	N
	2	\$10/ft^2	\$276,000	N	\$336,000	N	\$456,000	N
	10		\$320,000	N	\$390,000	N	\$530,000	N
	20		\$390,000	N	\$480,000	N	\$660,000	N
	2	\$25/ft^2	\$510,000	N	\$570,000	N	\$690,000	N
	10		\$590,000	N	\$660,000	N	\$800,000	N
	20		\$705,000	N	\$795,000	N	\$975,000	N
2,500	2	\$1/ft^2	\$320,800	W	\$470,800	W	\$770,800	W
	10		\$382,800	**4**	\$562,800	**4**	\$922,800	**4**
	20		\$448,000	N	\$658,000	W	\$1,078,000	W
	2	\$10/ft^2	\$496,000	N	\$658,000	W	\$958,000	W
	10		\$560,000	N	\$750,000	N	\$1,128,000	**4**
	20		\$670,000	N	\$900,000	N	\$1,330,000	W
	2	\$25/ft^2	\$730,000	N	\$900,000	N	\$1,240,000	N
	10		\$830,000	N	\$1,020,000	N	\$1,400,000	N
	20		\$985,000	N	\$1,215,000	N	\$1,675,000	N
3,750	2	\$1/ft^2	\$460,800	W	\$680,800	W	\$1,120,800	W
	10		\$544,000	W	\$804,000	W	\$1,324,000	W
	20		\$668,000	W	\$988,000	W	\$1,628,000	W
	2	\$10/ft^2	\$648,000	W	\$868,000	W	\$1,308,000	W
	10		\$760,000	W	\$1,020,000	W	\$1,540,000	W
	20		\$920,000	W	\$1,240,000	W	\$1,880,000	W
	2	\$25/ft^2	\$910,000	N	\$1,170,000	N	\$1,620,000	W
	10		\$1,070,000	N	\$1,380,000	**N,0.25,W**	\$1,900,000	W
	20		\$1,265,000	N	\$1,635,000	N	\$2,300,000	W

Table 6: Optimal aisle width configuration for s = 1,000, I = 30 system

Λ	a	C_s	C_k = \$20,000/yr	r	C_k = \$30,000/yr	r	C_k = \$50,000/yr	r
			Total Cost and Optimum Configuration					
1,000	2	\$1/ft²	\$250,800	W	\$350,800	N	\$550,800	N
	10		\$280,500	N	\$400,500	N	\$640,500	N
	20		\$303,500	N	\$433,500	N	\$693,500	N
	2	\$10/ft²	\$601,000	N	\$711,000	N	\$931,000	N
	10		\$645,000	N	\$765,000	N	\$1,005,000	N
	20		\$695,000	N	\$825,000	N	\$1,085,000	N
	2	\$25/ft²	\$1,172,500	N	\$1,282,500	N	\$1,502,500	N
	10		\$1,252,500	N	\$1,372,500	N	\$1,612,500	N
	20		\$1,347,500	N	\$1,477,500	N	\$1,737,500	N
2,500	2	\$1/ft²	\$550,800	W	\$800,800	W	\$1,300,800	W
	10		\$611,300	**4**	\$891,300	**4**	\$1,451,300	**4**
	20		\$690,750	**1**	\$1,010,750	**1**	\$1,650,750	**1**
	2	\$10/ft²	\$921,000	N	\$1,191,000	N	\$1,731,000	N
	10		\$985,000	N	\$1,275,000	N	\$1,855,000	N
	20		\$1,095,000	N	\$1,425,000	N	\$2,085,000	N
	2	\$25/ft²	\$1,492,500	N	\$1,762,500	N	\$2,302,500	N
	10		\$1,592,500	N	\$1,882,500	N	\$2,462,500	N
	20		\$1,747,500	N	\$2,077,500	N	\$2,737,500	N
3,750	2	\$1/ft²	\$810,800	W	\$1,190,800	W	\$1,950,800	W
	10		\$894,000	W	\$1,314,000	W	\$2,154,000	W
	20		\$998,000	W	\$1,468,000	W	\$2,408,000	W
	2	\$10/ft²	\$1,201,000	N	\$1,611,000	N	\$2,408,000	W
	10		\$1,325,000	N	\$1,782,000	**0.25**	\$2,640,000	W
	20		\$1,455,000	N	\$1,964,000	**0.25**	\$2,930,000	W
	2	\$25/ft²	\$1,772,500	N	\$2,182,500	N	\$3,002,500	N
	10		\$1,932,500	N	\$2,392,500	N	\$3,312,500	N
	20		\$2,107,500	N	\$2,617,500	N	\$3,637,500	N

Table 7: Space cost sensitivity of optimal aisle width configuration for s = 1,000, Λ = 2,500

		C_s (\$/ft²)									
C_k = \$50,000/yr	Aisles	1	2	3	4	5	6	7	8	9	10
	2	W	W	W	W	W	W	W	W	N	N
	10	**4**	**4**	**4**	**4**	N	N	N	N	N	N
	20	**1**	**1**	**1**	**1**	**1**	**1**	N	N	N	N

7.1. Managerial Insights

The above experimental study has helped us to derive managerial insights when configuring aisles for OPSs. Wallace-Finney and Parikh (2011) found that system size (*s*), space cost (C_S), and required throughput (Λ_{req}) all had high degrees of influence, with conflicting directions of patterns (wide to narrow vs. narrow to wide), on determining optimal aisle width. This suggests that the optimum aisle configuration could change should the system size change due to aggregation or disaggregation of storage locations, the throughput change due to growth or decline, or the cost of land fluctuate due to inflation or recession. Decision makers should make sure their projections of these values are strong in order to ensure a robust OPS design.

Our evaluation of the mixed-width aisle combinations, in conjunction with pure wide and narrow under randomized storage and traversal routing, suggest the following:

- Mixed-aisles are more desirable for system that require a larger number of storage locations
- Mixed-aisles are more desirable for systems with a larger number of aisles
- Mixed-aisles are least desirable at lower throughputs
- Mixed-aisles are least desirable at higher space costs
- Mixed-aisles are least desirable at lower labor costs

8. Summary and Future Research

The main contributions of this paper are the introduction of the mixed-width aisle layout as an option to the OPS design process, and the corresponding analytical models for space and travel time. Our derived models are general enough for evaluating pure narrow and wide aisles as well. The travel time model was developed for a traversal routing policy which is best suited for systems that employ a randomized storage policy.

To illustrate the use of these models, and analyze the potential benefits of a mixed-width aisle system, we developed a cost-based optimization model to determine the optimal aisle configuration for specific OPSs. The objective of this model was to minimize the total system cost which was divided into two components, space and labor. Results indicated that while mixed-aisles are optimal for certain systems with randomized storage and traversal routing, cost improvements over pure wide aisle systems appear to be limited to 4% or less (up to $48,000) for systems where I = 30. These savings may not be viewed as considerable; however the absolute savings in dollars appear to be substantial for some of these configurations.

While further examination of varying values of I may produce higher cost savings, future research into the use of mixed-width aisles should consider other OPSs such as semi-automated systems that employ a person-onboard truck. Additional investigation into other storage policies is required to fully understand the benefits of mixed-width aisle layouts. For example, an OPS with a class based storage policy, wherein wide aisles are used for Class A items and narrow aisles for Class

B and C items, may result in additional benefits. The traversal routing policy may then be suboptimal for class based storage. Consequently, an appropriate routing policy (such as modified traversal with aisle skipping) would need to be used, for which a new travel-time model needs to be developed to estimate the theoretical throughput of pickers. Furthermore, a new blocking model for estimating average picker blocking would need to be developed. Doing so will allow the evaluation of the full potential of mixed-width aisles for order picking in distribution centers.

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