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Spectrally Modulated Spectrally Encoded Framework Based Cognitive Radio in Mobile Environment

Xue Li
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Spectrally Modulated Spectrally Encoded Framework Based Cognitive Radio in Mobile Environment

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

by

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Radio spectrum has become a precious resource, and it has long been the dream of wireless communication engineers to maximize the utilization of the radio spectrum. Dynamic Spectrum Access (DSA) and Cognitive Radio (CR) have been considered promising to enhance the efficiency and utilization of the spectrum. Since some of the spectrum bands are occupied by primary users (PUs), the available spectrum for secondary users (SUs) are non-contiguous, and multi-carrier transmission technologies become the natural solution to occupy those non-contiguous bands. Non-contiguous multi-carrier based modulations, such as NC-OFDM (non-contiguous Orthogonal Frequency Division Multiplexing), NC-MC-CDMA (non-contiguous multi-carrier code division multiple access) and NC-SC-OFDM (non-contiguous single carrier OFDM), allow the SUs to utilize the available spectrum. Spectrally Modulated Spectrally Encoded (SMSE) framework offers a general framework to generate multi-carrier based waveform for CR. However, it is well known that all multi-carrier transmission technologies suffer significant performance degradation resulting from inter-carrier interference (ICI) in high mobility environments. Current research work in cognitive radio has not sufficiently considered and addressed this issue yet. Hence, it is highly desired to study the effect of mobility on CR communication systems and how to improve the performance through affordable low-complexity signal processing techniques.

In this dissertation, we analyze the inter-carrier interference for SMSE based multi-carrier transmissions in CR, and propose multiple ICI mitigation techniques and carrier frequency offset (CFO) estimator. Specifically, (1) an ICI self-cancellation algorithm is adapted to the MC-CDMA system by designing new spreading codes to enable the system with the capability to reduce the ICI; (2) a blind ICI cancellation technique named
Total ICI Cancellation is proposed to perfectly remove the ICI effect for OFDM and MC-CDMA systems and provide the performance approximately identical to that of the systems without ICI; (3) a novel modulation scheme, called Magnitude Keyed Modulation (MKM), is proposed to combine with SC-OFDM system and provide ICI immunity feature so that the system performance is not affected by the mobility or carrier frequency offset; (4) a blind carrier frequency offset estimation algorithm is proposed to accurately estimate the CFO; (5) finally, compared to traditional ICI analysis and cancellation techniques with assumption of constant carrier frequency offset among all the subcarriers, subcarrier varying CFO scenario is considered for the wideband multi-carrier transmission and non-contiguous multi-carrier transmission for CR, and an ICI total cancellation algorithm is proposed for the multi-carrier system with subcarrier varying CFOs to entirely remove the ICI.
Contents

1 Introduction 1
  1.1 Overview of Cognitive Radio ................................. 1
    1.1.1 Dynamic Spectrum Access ............................. 2
    1.1.2 Cognitive Radio ..................................... 4
  1.2 Motivation ..................................................... 5
  1.3 Dissertation Contributions ................................... 7
  1.4 Dissertation Outline .......................................... 8

2 Overview of Multi-Carrier Transmission for SMSE based Cognitive Radio 10
  2.1 Overview of Multi Carrier Transmission ......................... 10
    2.1.1 Orthogonal Frequency Division Multiplexing (OFDM) .......... 11
    2.1.2 Multi Carrier Code Division Multiple Access (MC-CDMA) .... 11
    2.1.3 Single Carrier Orthogonal Frequency Division Multiplexing (SC-OFDM) .. 12
  2.2 SMSE Framework .................................................. 12
    2.2.1 Transmitter for SMSE Expression .......................... 14
    2.2.2 OFDM via SMSE Analytic Expression ........................ 15
    2.2.3 MC-CDMA via SMSE Analytic Expression ...................... 16
    2.2.4 SC-OFDM via SMSE Analytic Expression ...................... 17
  2.3 Conclusion ....................................................... 18

3 Inter-Carrier Interference of SMSE based Cognitive Radio 19
  3.1 OFDM System with ICI .......................................... 19
  3.2 MC-CDMA System with ICI ...................................... 21
  3.3 SC-OFDM System with ICI ...................................... 23
  3.4 Inter-Carrier Interference ..................................... 25
  3.5 Conclusion ........................................................ 27

4 Inter-Carrier Interference Reduction 28
  4.1 Overview of Self-Cancellation for OFDM System .................. 28
  4.2 ICI Self-Cancellation for MC-CDMA System without Reducing Data Rate .... 31
  4.3 Performance of ICI Self-Cancellation for MC-CDMA System .......... 35
4.3.1 AWGN Channel ......................................................... 35
4.3.2 Multipath Fading Channel ......................................... 36
4.4 Performance of ICI Self-Cancellation for CI/MC-CDMA System ... 38
  4.4.1 AWGN channel ...................................................... 40
  4.4.2 Multipath Fading Channel ......................................... 42
4.5 Conclusion ................................................................... 43

5  Total Inter-Carrier Interference Cancellation ...................... 48
  5.1 General Expression for OFDM and MC-CDMA Systems with ICI 49
  5.2 Analysis in AWGN Channels ........................................ 50
  5.3 Analysis in Multipath Fading Channel ............................. 53
  5.4 Performance of Total ICI Cancellation for OFDM System .... 54
    5.4.1 AWGN Channel ...................................................... 54
    5.4.2 Multipath Fading Channel ......................................... 56
    5.4.3 Complexity .......................................................... 57
  5.5 Performance of Total ICI Cancellation for MC-CDMA System 58
    5.5.1 AWGN Channel ...................................................... 59
    5.5.2 Multipath Fading Channel ......................................... 61
    5.5.3 Complexity .......................................................... 61
  5.6 Demonstration via Software Defined Radio ....................... 62
  5.7 Conclusion ................................................................... 66

6  ICI Immunity: SC-OFDM via Magnitude Keyed Modulation .... 67
  6.1 Interference to Carrier Power Ratio ............................... 67
  6.2 Decomposition of ICI Coefficient Matrix ........................ 70
  6.3 Analysis of SC-OFDM Performance with ICI Present ........... 71
  6.4 ICI Immune Modulation for SC-OFDM: Magnitude Keyed Modulation 72
    6.4.1 BER Performance Analysis ....................................... 73
    6.4.2 PAPR Performance Analysis ....................................... 74
    6.4.3 Analysis in Multipath Fading Channel ......................... 76
  6.5 Performance of SC-OFDM system with MKM ........................ 78
    6.5.1 AWGN Channel ...................................................... 78
    6.5.2 Multipath Fading Channel ......................................... 82
  6.6 Conclusion ................................................................... 83

7  Blind Carrier Frequency Offset Estimation ......................... 85
  7.1 High Accurate Blind CFO Estimator ............................... 86
  7.2 Carrier Frequency Offset Estimation Performance ................ 90
    7.2.1 AWGN Channel with a Constant Frequency Offset ....... 91
    7.2.2 Multipath Mobile Channel ........................................ 93
  7.3 Conclusion ................................................................... 95
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>ICI Total Cancellation with Subcarrier Varying CFO</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>8.1 System Model for Subcarrier Varying CFO</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>8.2 ICI Total Cancellation for Subcarrier Varying CFO</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>8.3 Performance of ICI Total Cancellation with Subcarrier Varying CFO</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>8.3.1 AWGN Channel</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>8.3.2 Multipath Fading Channel</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>8.4 Conclusion</td>
<td>111</td>
</tr>
<tr>
<td>9</td>
<td>Conclusion</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td>Bibliography</td>
<td>114</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Frequency Allocation Chart in United States ........................................ 2
1.2 Dynamic Spectrum Access ................................................................. 3
1.3 Three Approaches For Hierarchical Access Model .............................. 5
1.4 Dynamic Spectrum Access ................................................................. 6
3.1 Magnitude of ICI Coefficient for different NFOs ............................... 26
4.1 Magnitude of ICI Coefficient after Self-Cancellation .......................... 31
4.2 BER Performance for MC-CDMA system in AWGN channel v.s. SNR .... 36
4.3 BER Performance for MC-CDMA system in AWGN channel v.s. $\varepsilon$ .... 37
4.4 BER Performance for MC-CDMA system in multipath fading channel .... 38
4.5 The BER performance for CI/MC-CDMA in AWGN Channel (BPSK, $N_u = N_F$ and $\varepsilon = 0.2$) .................................................. 41
4.6 The BER performance for CI/MC-CDMA in AWGN Channel (QPSK, $N_u = N_F/2$ and $\varepsilon = 0.1$) .................................................. 42
4.7 The BER performance for CI/MC-CDMA in AWGN Channel (QPSK, $N_u = N_F$ and $\varepsilon = 0.1$) .................................................. 43
4.8 BER performance verse number of users for CI/MC-CDMA with and without SC in AWGN Channel (SNR=8dB and $\varepsilon = 0.3$) ................. 44
4.9 BER performance for CI/MC-CDMA in multipath fading channel, fully loaded (BPSK, $N_u = N_F$ and $\varepsilon_{\text{max}} = 0.2$) .................................. 45
4.10 BER performance for CI/MC-CDMA in multipath fading channel, half loaded (QPSK, $N_u = N_F/2$ and $\varepsilon_{\text{max}} = 0.1$) ............................. 46
4.11 BER performance for CI/MC-CDMA in multipath fading channel, fully loaded (QPSK, $N_u = N_F$ and $\varepsilon_{\text{max}} = 0.1$) .................................. 47
5.1 $||\tilde{R}_k - \bar{R}||^2$ versus $\varepsilon_k' - \varepsilon$ ............................................. 52
5.2 Block Diagram of the Total ICI Cancellation ...................................... 54
5.3 BER Performance for OFDM system in AWGN channel, $\varepsilon = 0.3$ ....... 55
5.4 BER Performance for OFDM system in AWGN channel, $\varepsilon = 0.287532$ .... 56
5.5 BER Performance for OFDM system in AWGN channel with fixed SNR .... 57
5.6 BER Performance for OFDM system in multipath fading channel .......... 58
5.7 Effect of NFO Quantization Levels ($K$) in AWGN channel, $\varepsilon = 0.1$ .... 59
8.7 BER Performance in multipath fading channel with $\varepsilon_{B_0} = 0.3$ . . . . . . . 111
8.8 Effect of NFO Quantization Levels in AWGN channel with $\frac{V_{f_0}}{c\Delta f} = 0.1$ . . . 112
List of Tables

2.1 SMSE Variable Instantiations for OFDM, MC-CDMA and SC-OFDM Signals .................................................. 15

6.1 Comparison of PAPR (dB) for Different System Configurations ................................................................. 77
Introduction

1.1 Overview of Cognitive Radio

Radio spectrum has become a precious resource, and it has long been the dream of wireless communication engineers to maximize the utilization of the radio spectrum. Figure 1.1 presents the United States radio spectrum allocation chart in 2011, which indicates that most of the frequency bands have been assigned for different users and only very small portions are available for ISM usage [1]. AT&T Mobility was the biggest buyer in the B block with 12 MHz bandwidth (i.e., 704-710 and 734-740 MHz), with 227 licenses totaling $6.6 billion [2].

Although the spectrum is crowded, the allocated spectrum has not been effectively utilized. Studies in recent years have shown that a very large portion of the radio spectrum is unused or underused for long periods of time [3, 4]. In November 2002, FCC (Federal Communications Commissions) released a report generated by the Spectrum Policy Task Force [5] that reshaped the traditional models of spectral allocation and control. Dynamic Spectrum Access (DSA) [6, 7] and Cognitive Radio (CR) [3, 4] are considered promising to enhance the efficiency and utilization of the spectrum. In CR and DSA, three platforms are proposed to enable the coexistence of primary users (PUs) and secondary users (SUs) without harmful interference to one another, namely overlay CR, underlay CR and hybrid overlay/underlay CR [8, 9].
1.1.1 Dynamic Spectrum Access

Compared to the current static spectrum management policy, the *Dynamic Spectrum Access* can encompass various approaches to spectrum reform, and the diagram of DSA with three models is shown in Figure 1.2 [7, 10].

**Dynamic Exclusive Use Model**

*Dynamic Exclusive Use Model* is the basic model indicating that the spectrum bands are licensed to services for exclusive use. One approach named “Spectrum Property Rights” allows licensees to sell and trade spectrum and to freely choose technologies [11]; the other one named “Dynamic Spectrum Allocation” tries to improve the spectrum efficiency by dynamically assigning the spectrum according to spatial and temporal traffic statistics of different services [12].
Open Sharing Model/Spectrum Common Model

Open Sharing Model/Spectrum Common Model allows peer users to share the spectrum [13]. The approaches of Open Sharing Model/Spectrum Common Model can be classified as “centralized” and “distributed”. “Centralized Spectrum Sharing” controlled by a central entity and “Distributed Spectrum Sharing” with various local policies are two strategies under this model [10].

Hierarchical Access Model

Hierarchical Access Model is most compatible with the current spectrum management which adopts a hierarchical access structure with PUs and SUs. In this model, the licensed spectrum is opened to SUs while limiting the interference to the PUs (licensees). Three approaches are proposed for spectrum sharing between PUs and SUs: Spectrum underlay, overlay and hybrid (underlay/overlay) [8, 9].

- Spectrum Underlay: a spectrum management principle by which signals with a very low power spectral density (PSD) can coexist, as a secondary user, with the primary users of the frequency band(s), e.g., Ultra Wide Band (UWB) in Figure 1.3(a).

- Spectrum Overlay: a spectrum management principle where a SU uses a channel
from a PU only when it is not occupied. Spectrum overlay techniques are based on a detect and avoid mechanism such as Dynamic Frequency Selection. The SU senses the frequency spectrum, if a PU is active, the channel will not be used as shown in Figure 1.3(b), e.g., the RLANs use a frequency band that is also used by radar systems.

- Spectrum Hybrid (Underlay/Overlay): a spectrum management principle by combining underlay and overlay techniques, which applies overlay approach on the unoccupied spectrum and applies underlay on the occupied spectrum by limiting the interference as in Figure 1.3(c).

### 1.1.2 Cognitive Radio

*Cognitive Radio* was proposed by J. Mitola in 1998 and published later in an article in 1999 [3]. It is a context-aware intelligent radio potentially capable of autonomous reconfiguration by learning from and adapting to the communication environment. It was a novel approach in wireless communications that Mitola later described as [3]:

“The point in which wireless personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to detect user communications needs as a function of use context, and to provide radio resources and wireless services most appropriate to those needs”.

CR can be summarized as a radio with function of cognition cycle, shown in Figure 1.4, which includes three functions: *Sense, Learn* and *Adapt*.

- *Sense*: Periodically sense the environment/spectrum.
- *Learn*: Learn knowledge from the spectrum sensing results.
Figure 1.3: Three Approaches For Hierarchical Access Model

- **Adapt**: Dynamically adapt the transmission according to the learned knowledge to better suit the environment.

### 1.2 Motivation

The *Adapt* process of CR aims to dynamically adapt the transmission technology corresponding to the knowledge learnt from spectrum sensing results to better suit the envi-
environment. Since some of the spectrum bands are occupied by PUs, the available spectrum for SUs becomes non-contiguous. To utilize the non-contiguous spectrum, multi-carrier transmission technologies, such as Orthogonal Frequency Division Multiplexing (OFDM), are natural choices for CR transmission. By turning off certain subcarriers to not transmit on some frequencies, OFDM and other multi-carrier transmission technologies can easily adapt their waveform to transmit on available non-contiguous spectrum.

Most of current research in CR has not sufficiently considered mobility and its effect on the performance of cognitive radio. It is well known that all multi-carrier transmission technologies suffer significant performance degradation resulting from inter-carrier interference (ICI) introduced by the carrier frequency offset (CFO) in high mobility environments. Hence, it is highly desired to study the performance of cognitive radio waveforms in mobile environment and how to improve the system performance through affordable low-complexity signal processing techniques.

Many methods in literature have been proposed to solve the carrier frequency offset problem and mitigate the inter-carrier interference for OFDM system. Most of these methods use signal processing and/or coding to reduce the sensitivity of the OFDM system to the frequency offset. For example, authors in [14] developed low-complexity minimum mean-square error (MMSE) and decision-feedback equalizer (DFE) receivers to suppress
ICI based on the fact that the ICI power mainly comes from a few neighboring subcarriers. Other researchers propose algorithms to estimate the carrier frequency offset, which can be classified as data aided estimators [15, 16], using training/pilot data to estimate the CFO, and blind estimators [17, 18, 19] utilizing the signal property without pilot training/data.

However, all existing CFO estimation or ICI cancellation methods are not without their drawbacks. For CFO estimation methods, the estimation performance of blind estimators is not as good as expected and with some restrictions for transmission; while the throughput of the data-aided estimators becomes a problem since more training data for better estimation will reduce the data rate significantly. For the ICI cancellation schemes, even though all the existing ICI cancellation methods reduce ICI and improve BER performance for the system, the performance improvement is very limited. The BER performance after ICI cancellation is still significantly worse than the original system without ICI. More important, most of the existing ICI cancellation methods achieve the ICI reduction and BER performance improvement at the cost of lowering the transmission rate and reducing the bandwidth efficiency. There do exist some methods that do not reduce the date rate, however, such methods produce even less reduction in ICI. On the other hand, only a few of these ICI cancellation methods are applied for MC-CDMA system with carrier frequency offset to improve the performance of mobile MC-CDMA system.

More importantly, none of the existing ICI mitigation techniques have been done in the context of cognitive radio. The non-contiguous and dynamic nature of the spectrum and waveform in cognitive radio makes the ICI mitigation problem even more difficult to solve.

1.3 Dissertation Contributions

In this dissertation, we analyze the inter-carrier interference for Spectrally Modulated Spectrally Encoded (SMSE) based multi-carrier transmissions for CR, and propose multiple ICI
mitigation techniques to reduce the inter-carrier interference as well as carrier frequency offset (CFO) estimation algorithm to accurately estimate the CFO. Specifically, we have

- Proposed a novel spreading code for MC-CDMA system to embed ICI self-cancellation capability without extra computation and data rate reduction.

- Proposed an ICI cancellation algorithm called Total ICI Cancellation to eliminate the ICI effect for OFDM and MC-CDMA systems, and achieve a BER performance almost identical to that of systems with no ICI. This work was implemented and demonstrated at the IEEE Globecom conference in December of 2010 and received the Best Demo Award.

- Proposed a novel modulation scheme, called *Magnitude Keyed Modulation* (MKM), to combine with SC-OFDM system and offer the system with ICI immunity.

- Proposed a blind carrier frequency offset estimator to accurately estimate the carrier frequency offset.

- Analyzed the effect of subcarrier varying carrier frequency offset scenario and proposed an ICI total cancellation algorithm to totally remove the inter-carrier interference.

1.4 Dissertation Outline

Chapter 1 introduces the cognitive radio and dynamic spectrum access and discusses the motivation of the dissertation work. In chapter 2, a brief overview of multi-carrier modulations and non-contiguous multi-carrier modulations will be presented with the general spectrally modulated spectrally encoded framework for cognitive radio. Chapter 3 presents an introduction to carrier frequency offset and inter-carrier interference for different multi-carrier transmissions. Chapter 4 presents the proposed inter-carrier interference reduction
method, called *ICI Self-Cancellation*, for MC-CDMA system. A *Total ICI Cancellation* algorithm is proposed in Chapter 5 for OFDM and MC-CDMA systems to totally remove the inter-carrier interference. Chapter 6 analyzes the ICI effect on SC-OFDM system, and proposes a novel modulation called “Magnitude Keyed Modulation” to provide the SC-OFDM system with the ICI immunity feature. A blind carrier frequency offset estimation algorithm is presented in Chapter 7, which provides accurate estimation of the CFO. Chapter 8 discusses the subcarrier varying carrier frequency offset scenario and proposes an ICI total cancellation algorithm to entirely remove the ICI effect.
Overview of Multi-Carrier Transmission

for SMSE based Cognitive Radio

In this chapter, a brief overview of multi-carrier transmissions, including OFDM, MC-CDMA, SC-OFDM, will be presented. The Spectrally Modulated Spectrally Encoded (SMSE) framework is also described to create multi-carrier waveforms for cognitive radio.

2.1 Overview of Multi Carrier Transmission

Multi carrier transmission is the principle of transmitting data by dividing the data stream into several bit streams, each of which has a much lower bit rate, and by using these sub-streams to modulate several carriers [20]. The advantages of multi-carrier transmission include relative immunity to fading caused by transmission over more than one path at a time (multipath fading), less susceptibility than single-carrier systems to interference caused by impulse noise, and enhanced immunity to inter-symbol interference [21]. After many years of further intensive research, today we appear to be on the verge of a breakthrough of these techniques. Many of the implementation problems appear solvable and multi-carrier transmission has become part of several standards.
2.1.1 Orthogonal Frequency Division Multiplexing (OFDM)

The first systems using multi-carrier transmission were military HF radio links in the late 1950s and early 1960s. A special form of multi-carrier transmission, Orthogonal Frequency Division Multiplexing (OFDM) [22], with densely spaced subcarriers with overlapping spectra of the modulating signal, was patented in the U.S. in 1970. Instead of using steep bandpass filters, OFDM time-domain waveforms are chosen such that mutual orthogonality is ensured even though spectra may overlap. It appeared that such waveform can be generated using Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) at the transmitter and receiver.

Specifically, the OFDM uses a large number of close spaced carriers that are modulated with low rate data. Normally these signals would be expected to interfere with each other, but by making the signals orthogonal to one another there is no mutual interference. This is achieved by having the carrier spacing equal to the reciprocal of the symbol period. This means that when the signals are demodulated, they will have a whole number of cycles in the symbol period and their contribution will sum to zero - in other words there is no interference from other subcarriers.

2.1.2 Multi Carrier Code Division Multiple Access (MC-CDMA)

Multi Carrier Code Division Multiple Access (MC-CDMA) is a relatively new concept [23]. Its development aimed at improved performance over multipath links. MC-CDMA combines CDMA with multi-carrier transmission together.

This scheme was first proposed at PIMRC ’93 in Yokohama by Linnartz, Yee (U. of California at Berkeley) and Fettweis (Teknekron, Berkeley, currently at U. of Dresden, Germany) [24]. Linnartz and Yee showed that MC-CDMA signals can also be detected with fairly simple receiver structures, using an FFT and a variable gain diversity combiner, in which the gain of each branch is controlled only by the channel attenuation at that sub-
carrier. Meanwhile, presentation of optimum gain control functions at PIMRC ’94 showed that a fully loaded MC-CDMA system can operate in a highly time dispersive channel with satisfactory bit error rate. These results appeared in contrast to the behavior of a fully loaded DS-CDMA link that typically does not work satisfactorily with large time dispersion. Since 1993, MC-CDMA rapidly has become an active topic of research. Around 2000, MC-CDMA has attracted tremendous attention, with entire conference sessions devoted to it.

2.1.3 Single Carrier Orthogonal Frequency Division Multiplexing (SC-OFDM)

The Single Carrier Orthogonal Frequency Division Multiplexing (SC-OFDM) [25] technique has received a lot of attention as an alternative transmission technique to the conventional OFDM due to its better performance in multipath fading channels and lower peak to average power ratio (PAPR).

SC-OFDM and similar technologies have been independently developed by multiple research groups almost simultaneously. For example, Single Carrier Frequency Domain Equalization (SCFDE) [26, 27, 28, 29] and Carrier Interferometry Orthogonal Frequency Division Multiplexing (CI/OFDM) [30, 31, 32] are essentially the same technology. They combine benefits of multi-carrier transmission with single carrier transmission using a cyclic prefix to allow frequency domain processing at receiver to exploit frequency diversity.

2.2 SMSE Framework

The general analytic Spectrally Modulated Spectrally Encoded (SMSE) framework was developed by Dr. Temple’s group [33, 34]. All the multi-carrier waveforms fall under the
SMSE framework. Specifically, the analytic framework spectral representation is a series of Hadamard vector products:

\[ S_{SMSE} = A \odot \Theta \odot F \]  \hspace{1cm} (2.1)

where \( \odot \) means Hadamard product, \( A \) is amplitude function, \( \Theta \) denotes the phase function and \( F \) presents the frequency function, and there are six waveform design variables to control these functions, including input data, coding, orthogonality, windowing, frequency component availability and frequency component use. Using vector to express these variables, we have:

- **code** \( c = [c_1, c_2, \ldots, c_{N_F}], c_i \in \mathbb{C}, \mathbb{C} \) denotes the complex number and \( N_F \) is the number of frequency samples;

- **data** \( d = [d_1, d_2, \ldots, d_{N_F}], d_i \in \mathbb{C}; \)

- **window** function \( w = [w_1, w_2, \ldots, w_{N_F}], w_i \in \mathbb{C}; \)

- **orthogonality** \( o = [o_1, o_2, \ldots, o_{N_F}], o_i \in \mathbb{C}, |o_i| = 1; \)

- **frequency assignment** \( a = [a_1, a_2, \ldots, a_{N_F}], a_i \in \{0, 1\} \) and \( P_a \equiv ||a||^2 \leq N_F; \)

- **used** frequencies \( u = [u_1, u_2, \ldots, u_{N_F}], u_i \in \{0, 1\} \) and \( P_u \equiv ||u||^2 \leq P_a \leq N_F, \)

which means users can only transmit on assigned frequencies.

- \( u \) can be modified to indicate the active/deactivated subcarriers for non-contiguous multi-carrier transmission in Cognitive Radio.
2.2.1 Transmitter for SMSE Expression

The $m^{th}$ component of the $k^{th}$ data modulated symbol $s_k[m]$ in frequency domain can be written as:

\[
s_k[m] = |c_m||d_{m,k}|w_m e^{j(\theta_{c_m} + \theta_{d_{m,k}} + \theta_{w_m})}
\]

(2.2)

where $|c_m|$, $|d_{m,k}|$ and $|w_m|$ denote the magnitude of $m^{th}$ code, data modulation and window function, and $\theta_{c_m}$, $\theta_{d_{m,k}}$ and $\theta_{w_m}$ represent the phases of them, respectively.

When further considering the orthogonality ($o$), frequency assignment ($a$) and use ($u$), $s_k[m]$ becomes:

\[
s_k[m] = a_m u_m |c_m||d_{m,k}|w_m e^{j(\theta_{c_m} + \theta_{d_{m,k}} + \theta_{w_m} + \theta_{o_{m,k}})}
\]

(2.3)

where the product $a_m u_m \in \{0, 1\}$ indicates the frequency occupation, which means the actual transmitted frequency components are those for which $a_m u_m = 1$ for all $m$. $\theta_{o_{m,k}} = [a_m u_m \sum_{i=0}^{m-1} \Delta \theta_{o_k}]$ and $\theta_{o_k} = (2\pi/P_u) \times k$, which effectively places a linear phase progression across those frequency components that are used.

Combining the magnitude, phase and frequency terms as in Eq. (2.1), the $v^{th}$ user’s $m^{th}$ component can be presented as:

\[
s^{(v)}_k[m] = A^{(v)}_{m,k} \Theta^{(v)}_{m,k} F^{(v)}_m
\]

(2.4)

where $A^{(v)}_{m,k} = |c^{(v)}_m||d^{(v)}_{m,k}|w^{(v)}_m$, $\Theta^{(v)}_{m,k} = e^{j(\theta^{(v)}_{c_m} + \theta^{(v)}_{d_{m,k}} + \theta^{(v)}_{w_m} + \theta^{(v)}_{o_{m,k}})}$ and $F^{(v)}_m = a^{(v)}_m u^{(v)}_m$.

The time domain representation of SMSE symbols with index $n$ is generated by ap-
Table 2.1: SMSE Variable Instantiations for OFDM, MC-CDMA and SC-OFDM Signals

<table>
<thead>
<tr>
<th>Operation</th>
<th>OFDM</th>
<th>MC-CDMA</th>
<th>SC-OFDM</th>
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</thead>
<tbody>
<tr>
<td>Data Modulation</td>
<td>MPSK, MQAM</td>
<td>MPSK, MQAM</td>
<td>MPSK, MQAM</td>
</tr>
<tr>
<td>Coding</td>
<td>$</td>
<td>c</td>
<td>= 1$ $\theta_c = 0$</td>
</tr>
<tr>
<td>Windowing</td>
<td>$</td>
<td>w</td>
<td>= 1$ $\theta_w = 0$</td>
</tr>
<tr>
<td>Orthogonality</td>
<td>$\theta_o = 0$</td>
<td>$\theta_o = 0$</td>
<td>$\theta_o = 0$ $\theta_{om} = \frac{2\pi mk}{P_u}$</td>
</tr>
<tr>
<td>Frequency Assignment</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
</tbody>
</table>

Applying $N_F$-point IDFT (Inverse Discrete Fourier Transform) and taking the real part:

$$s_k^{(v)}[n] = Re\left\{ \frac{1}{N_F} \sum_{m=0}^{N_F-1} A_{m,k}^{(v)} F_m^{(v)} e^{j2\pi f_m t_n} \right\}$$

$$= \frac{1}{N_F} \sum_{m=0}^{N_F-1} A_{m,k}^{(v)} F_m^{(v)} \cos(2\pi f_m t_n + \theta_{cm}^{(v)} + \theta_{dm,k}^{(v)} + \theta_{wm}^{(v)} + \theta_{om,k}^{(v)})$$ (2.5)

where $f_m = m\Delta f$, and $\Delta f$ is the frequency resolution. $\Delta t = 1/(N_F\Delta f)$ and $t_n = n\Delta t$.

Hence, simultaneous and independent operation of $N_u$ users yields to

$$s_k[n] = \frac{1}{N_F} \sum_{v=1}^{N_u} \sum_{m=0}^{N_F-1} A_{m,k}^{(v)} F_m^{(v)} \cos(2\pi f_m t_n + \theta_{cm}^{(v)} + \theta_{dm,k}^{(v)} + \theta_{wm}^{(v)} + \theta_{om,k}^{(v)})$$ (2.6)

### 2.2.2 OFDM via SMSE Analytic Expression

Table 2.1 shows the variable choice for different modulations. Since there is no windowing ($|w_m| = 1$, $\theta_{wm} = 0$) or coding ($|c_m| = 1$, $\theta_{cm} = 0$) or orthogonality control ($\theta_{om,k} = 0$) in the basic OFDM and only one user can be considered ($N_u = 1$), the $m^{th}$ spectral component for the $k^{th}$ symbol shown in Eq. (2.3) simplifies to the expression as following:

$$s_k[m] = a_m u_m |d_{m,k}| e^{j\theta_{dm,k}} = a_m u_m (j \cdot Q_{m,k} + I_{m,k})$$ (2.7)
where $d_{m,k} = |d_{m,k}|e^{j\theta_{d_{m,k}}}$, $I_{m,k}$ and $Q_{m,k}$ are the real part and imaginary part of the data constellation. For example, when QPSK is applied, $I_{m,k}, Q_{m,k} \in \{\pm 1\}$; while employing 64QAM, $I_{m,k}, Q_{m,k} \in \{\pm 1, \pm 3, \pm 5, \pm 7\}$.

The time domain signal for OFDM in Eq. (2.6) becomes

$$s_k[n] = \text{Re}\left\{\frac{1}{N_F} \sum_{m=0}^{N_F-1} a_m u_m |d_{m,k}| e^{j(2\pi f_m t_n + \theta_{d_{m,k}})}\right\}$$

$$= \frac{1}{N_F} \sum_{m=0}^{N_F-1} a_m u_m |d_{m,k}| \cos(2\pi f_m t_n + \theta_{d_{m,k}})$$

(2.8)

### 2.2.3 MC-CDMA via SMSE Analytic Expression

Compared to OFDM, multiple users are considered in MC-CDMA signal. The variables for MC-CDMA signal can be simplified as $|w_m| = 1$, $\theta_{w_m} = 0$ (no windowing) and $\theta_{\alpha_{m,k}} = 0$ (no orthogonality control). Meanwhile, the data variables only depend upon $k$ due to the spectral spreading ($d_{m,k} = d_k$). The SMSE expression in Eq. (2.3) can be simplified as:

$$s_k[m] = a_m u_m |c_m||d_k|e^{j(\theta_{c_m} + \theta_{d_k})}$$

(2.9)

The code in Hadamard Walsh Code set is $\pm 1$, so $|c_m| = 1$ and $\theta_{c_m} \in \{0, \pi\}$. The further simplifies of Eq. (2.9) is:

$$s_k[m] = a_m u_m |d_k|e^{j(\theta_{c_m} + \theta_{d_k})} = a_m u_m (j \cdot Q_k + I_k)e^{j\theta_{c_m}}$$

(2.10)

where $I_k$ and $Q_k$ depend on the modulation type. The time domain signal after IDFT
process in Eq. (2.6) is

\[ s_k[n] = Re\left\{ \frac{1}{N_F} \sum_{m=0}^{N_F-1} a_m u_m |d_k| e^{j(2\pi f_m t_n + \theta c_m + \theta d_k)} \right\} \]

\[ = \frac{1}{N_F} \sum_{m=0}^{N_F-1} a_m u_m |d_k| \cos(2\pi f_m t_n + \theta c_m + \theta d_k) \quad (2.11) \]

When consider \( N_u \) users, the \( v^{th} \) user’s contribution to SMSE component in frequency domain is:

\[ s_k^{(v)}[n] = a_m^{(v)} u_m^{(v)} |d_k^{(v)}| e^{j(\theta c_m^{(v)} + \theta d_k^{(v)})} \quad (2.12) \]

where \( ^*(v) \) means \( * \) for \( v^{th} \) user.

The time domain signal for MC-CDMA system becomes the summation of all the users’ time domain signals:

\[ s_k[n] = Re\left\{ \frac{1}{N_F} \sum_{v=1}^{N_u} \sum_{m=0}^{N_F-1} a_m^{(v)} u_m^{(v)} |d_k^{(v)}| e^{j(2\pi f_m t_n + \theta c_m^{(v)} + \theta d_k^{(v)})} \right\} \]

\[ = \frac{1}{N_F} \sum_{v=1}^{N_u} \sum_{m=0}^{N_F-1} a_m^{(v)} u_m^{(v)} |d_k^{(v)}| \cos(2\pi f_m t_n + \theta c_m^{(v)} + \theta d_k^{(v)}) \quad (2.13) \]

### 2.2.4 SC-OFDM via SMSE Analytic Expression

Compared to the OFDM signal, SC-OFDM signal is applied with a spreading code, and each data is transmitted over all the subcarriers. The orthogonality among the transmitted symbols is maintained via an orthogonal phase term: the spreading matrix is exact the normalized DFT matrix [25, 32].

Due to the spectral spreading, the data variables only depend upon \( k \) (\( d_{m,k} = d_k \)); the phase term for \( k^{th} \) symbol onto the \( m^{th} \) subcarrier is \( \theta_{o_{m,k}} = \frac{2\pi m k}{P_u} \); the coding here is orthogonal phase term that \( |c_m| = 1 \) and \( \theta_{c_m} = 0 \). Hence the \( m^{th} \) spectral component for
the $k^{th}$ symbol without windowing shown in Eq. (2.3) simplifies to:

$$s_k[m] = a_m u_m |d_k| e^{j(\theta_{d_k} + \theta_{o_m,k})}$$ (2.14)

After $N_F$ point IDFT, the time domain signal for SC-OFDM system becomes:

$$s_k[n] = \text{Re}\left\{ \frac{1}{N_F} \sum_{m=0}^{N_F-1} a_m u_m |d_k| e^{j(2\pi f_m t_n + \theta_{o_m,k} + \theta_{d_k})} \right\}$$

$$= \frac{1}{N_F} \sum_{m=0}^{N_F-1} a_m u_m |d_k| \cos(2\pi f_m t_n + \theta_{o_m,k} + \theta_{d_k})$$ (2.15)

### 2.3 Conclusion

In this chapter, we provide a brief overview of multi-carrier transmissions, including OFDM, MC-CDMA and SC-OFDM. We also present the introduction of the Spectrally Modulated Spectrally Encoded framework to represent those multi-carrier waveforms for cognitive radio.
Inter-Carrier Interference of SMSE based Cognitive Radio

In this chapter, an introduction to the carrier frequency offset and inter-carrier interference for different multi-carrier waveforms will be presented.

In multi-carrier transmission technology such as OFDM and MC-CDMA, it is crucial to maintain orthogonality among all the subcarriers. However, in a high speed mobile communication channel, the orthogonality among subcarriers is destroyed by the frequency offset introduced by Doppler shift due to high speed motion of vehicle, or by multipath fading and phase variations in the received signal due to multiple scattering, or frequency mismatch between the transmitter and receiver oscillators. The loss of orthogonality causes interference from other subcarriers which is known as inter-carrier interference (ICI), leading to significant performance degradation.

To find the solution for this problem, let’s first take a look at how the carrier frequency offset will produce the ICI and its effects on various multi-carrier transmission waveforms.

3.1 OFDM System with ICI

Assuming $N_F$ subcarriers are activated ($a_m u_m = 1, \forall m, m = 0, 1, ..., N_F - 1$), and when only focusing on one transmission (which means $N_F$ symbols ($d_{m,k}, m = 0, 1, ..., N_F - 1$)
are transmitted and index of \( k \) can be removed), at the receiver side of OFDM system, the received signal in frequency domain on the \( m^{th} \) subcarrier becomes

\[
R[m] = s[m]H_mS(0) + \sum_{l=0,l\neq m}^{N_F-1} s[l]H_lS(l - m) + n_m, \\
= d_mH_mS(0) + \sum_{l=0,l\neq m}^{N_F-1} d_lH_lS(l - m) + n_m, \quad m = 0, 1, \ldots, N_F - 1 \tag{3.1}
\]

where \( s[m] = |d_m|e^{j\theta_m} = d_m \) is the transmitted signal on the \( m^{th} \) active subcarrier in Eq. (2.7), \( H_m \) and \( n_m \) are the complex fading gain and additive white Gaussian noise sample on the \( m^{th} \) subcarrier, respectively. The sequence \( S(l - m) \) is the ICI coefficient from the \( l^{th} \) subcarrier to the \( m^{th} \) subcarrier:

\[
S(l - m) = \frac{\sin(\pi(\varepsilon + l - m))}{N_F \sin\left(\frac{\pi}{N_F}(\varepsilon + l - m)\right)} \cdot \exp\left(j\pi\left(1 - \frac{1}{N_F}\right)(\varepsilon + l - m)\right) \tag{3.2}
\]

where \( \varepsilon \) is the normalized frequency offset (NFO) given by \( \varepsilon = \frac{f_0}{\Delta f} \), \( f_0 \) is the carrier frequency offset, and \( \Delta f \) is the subcarrier spacing.

From Eq. (3.1), it is clear that the received signal on \( m^{th} \) subcarrier is not only the \( s[m] \) with amplifying variable \( S(0) \), but also includes the data on all the other subcarriers with different amplifying variables, which is known as the inter-carrier interference.

Using vector and matrix to represent the relationship between the transmitted signal and the received signal, we have

\[
\vec{R} = \vec{d}H\vec{S} + \vec{n}, \tag{3.3}
\]

\( N_F \) is the number of subcarriers.
where

\[ \vec{R} = \{ R[0], R[1], \ldots, R[N_F - 1] \} \]  
(3.4)
\[ \vec{d} = \{ d_0, d_1, \ldots, d_{N_F - 1} \} \]
\[ \vec{n} = \{ n_0, n_1, \ldots, n_{N_F - 1} \} \]
\[ \mathbf{H} = \text{diag}\{ H_0, H_1, \ldots, H_{N_F - 1} \} \]

where \( \text{diag}\{ H_0, H_1, \ldots, H_{N_F - 1} \} \) means diagonal matrix with diagonal element \( H_0, H_1, \ldots, H_{N_F - 1} \) and \( S \) is the ICI coefficient matrix having dimension \( N_F \times N_F \) with \( l^{th} \)-row and \( m^{th} \)-column elements given by \( S_{l,m} = S(l - m) \). The resultant matrix form of \( S \) is:

\[
S = \begin{bmatrix}
S(0) & S(-1) & \ldots & S(1 - N_F) \\
S(1) & S(0) & \ldots & S(2 - N_F) \\
\vdots & \vdots & \ddots & \vdots \\
S(N_F - 1) & S(N_F - 2) & \cdots & S(0)
\end{bmatrix}
\]  
(3.5)

### 3.2 MC-CDMA System with ICI

Assuming \( N_F \) subcarriers are activated \( (a_m u_m = 1, \forall m, m = 0, 1, \ldots, N_F - 1) \) and there are total \( N_u \) users in this system. When only focusing on one transmission (which means \( N_u \) symbols \( (d^{(v)}_k, v = 1, \ldots, N_u) \) are transmitted and index of \( k \) can be removed), at the receiver side of MC-CDMA system, the received signal in frequency domain on the \( m^{th} \)
subcarrier becomes

\[ R[m] = \sum_{v=1}^{N_u} s^{(v)}[m]\beta_{m}^{(v)}H_{m}S(0) + \sum_{l=0,l\neq m}^{N_F-1} \sum_{v=1}^{N_u} s^{(v)}H_{l}S(l - m) + n_{m}, \]

\[ = \sum_{v=1}^{N_u} d^{(v)}\beta_{m}^{(v)}H_{m}S(0) + \sum_{l=0,l\neq m}^{N_F-1} \sum_{v=1}^{N_u} d^{(v)}\beta_{l}^{(v)}H_{l}S(l - m) + n_{m}, \]

\[ m = 0, 1, \ldots, N_F - 1 \tag{3.6} \]

where \( s^{(v)}[m] = |d^{(v)}|e^{j(\theta_{m}^{(v)} + \theta_{d^{(v)}})} = d^{(v)}e^{j\theta_{c^{(v)}}} \) is the transmitted signal on the \( m^{th} \) active subcarrier for \( v^{th} \) user in Eq. (2.12), and \( \beta_{m}^{(v)} = e^{j\theta_{c^{(v)}}} \) is the spreading code for \( v^{th} \) user’s data onto the \( m^{th} \) subcarrier.

From Eq. (3.6), similar conclusion can be made that the received signal on the \( m^{th} \) subcarrier consists of the desired signal on \( m^{th} \) subcarrier and the signal on all the other subcarriers with a combining weight \( S(l - m) \).

Using vector and matrix to represent the relationship between the transmitted signal and the received signal, we have

\[ \vec{R} = \vec{d}\vec{C}\vec{H}\vec{S} + \vec{n}, \tag{3.7} \]

where \( \vec{C} \) is an \( N_u \times N_F \) spreading code matrix with \( \vec{C}(v, m) = \beta_{m}^{(v)} \), and

\[ \vec{d} = \{d^{(1)}, d^{(2)}, \ldots, d^{(N_u)}\} \tag{3.8} \]

At the receiver side of MC-CDMA system, to demodulate the data for each user, assuming combining weight matrix is \( \vec{W} = \text{diag}\{W_0, W_1, \ldots, W_{N_F-1}\} \), the demodulated
signal vector becomes:

\[ \vec{D} = \vec{R}WC^H = \vec{d}CHSWC^H + \vec{n}WC^H \] (3.9)

\[ = \vec{d}s' + \vec{n}' \]

where \( S' = CHSWC^H \) is an \( N_u \times N_u \) matrix. If there is no ICI (\( S = I \)) in AWGN channel (\( H = I \) and \( W = I \)), the \( S' \) reduces to be identity matrix \( I \), and the demodulated signal vector becomes \( \vec{D} = \vec{d} + \vec{n}' \).

The demodulated data for \( v^{th} \) user (\( D[v] \)) can be represented as:

\[ D[v] = \sum_{l=1}^{N_u} d^{(l)}_l s'_{l,v} + n'_v = d^{(v)}_v s'_{v,v} + \sum_{l=1,l \neq v}^{N_u} d^{(l)}_l s'_{l,v} + n'_v \] (3.10)

Eq. (3.10) indicates that the demodulated symbol for \( v^{th} \) user includes not only the desired signal \( d^{(v)} \) but also the interference from other users.

### 3.3 SC-OFDM System with ICI

Assuming \( N_F \) subcarriers are activated (\( a_m u_m = 1, \forall m, m = 0, 1, ..., N_F - 1 \)) and \( N_F \) data are spreading onto these \( N_F \) subcarriers. When only focusing on one transmission (which means \( N_F \) symbols (\( d_k, k = 0, ..., N_F - 1 \) are transmitted), at the receiver side of SC-OFDM system, the received signal in frequency domain on the \( m^{th} \) subcarrier becomes

\[ R[m] = \sum_{k=0}^{N_F-1} s_k[m] H_m S(0) + \sum_{l=0, l \neq m}^{N_F-1} \sum_{k=0}^{N_F-1} s^{(v)}_l H_l S(l - m) + n_m, \]

\[ = \sum_{k=0}^{N_F-1} d_k e^{j\theta_{m,k}} H_m S(0) + \sum_{l=0, l \neq m}^{N_F-1} \sum_{k=0}^{N_F-1} d_k e^{j\theta_{l,k}} H_l S(l - m) + n_m, \] (3.11)

\[ m = 0, 1, \ldots, N_F - 1 \]
where $s[k][m] = |d_k|e^{j\theta_k}e^{j\theta_{m,k}} = d_ke^{j\theta_{m,k}}$ is the $k^{th}$ transmitted signal on the $m^{th}$ active subcarrier in Eq. (2.14), and $e^{j\theta_{m,k}} = e^{j2\pi mk/N_F}$ is the spreading code for the $k^{th}$ symbol onto the $m^{th}$ subcarrier.

From Eq. (3.11), it is clear that the received signal on the $m^{th}$ subcarrier is not only the $s[k][m]$ with amplifying variable $S(0)$, but also includes the data on all the other subcarriers with different amplifying variables.

Using vector and matrix to represent the relationship between the transmitted signal and the received signal, we have

$$\vec{R} = \vec{d}FHS + \vec{n}$$  \hspace{1cm} (3.12)

where $F$ is an $N_F \times N_F$ spreading code matrix with $F(m, k) = \frac{1}{\sqrt{N_F}}e^{j2\pi mk/N_F}$, $k, m = 0, \cdots, N_F - 1$, which is the normalized DFT matrix, and

$$\vec{d} = \{d_0, d_1, \ldots, d_{N_F-1}\}$$ \hspace{1cm} (3.13)

For SC-OFDM system, assuming combining weight matrix is $W$, the demodulated signal vector becomes:

$$\vec{D} = \vec{R}WF^H = \vec{d}FHSWF^H + \vec{n}WF^H$$

$$= \vec{d}S' + \vec{n}'$$ \hspace{1cm} (3.14)

where $S' = FHSWF^H$ is an $N_F \times N_F$ matrix.

Hence the $k^{th}$ demodulated data ($D[k]$) can be represented as:

$$D[k] = \sum_{l=0}^{N_F-1} d^{(l)}S'_{l,k} + n'_k = d_kS'_k,k + \sum_{l=0, l\neq k}^{N_F-1} d_lS'_{l,k} + n'_k$$ \hspace{1cm} (3.15)
Eq. (3.15) illustrates that the demodulated $k^{th}$ symbol includes not only the desired signal $d_k$ but also the interference from other symbols.

### 3.4 Inter-Carrier Interference

The normalized frequency offset, $\varepsilon = f_0/\Delta f$, can contain both integer and fractional components with each having different effects on the system. The ICI coefficient in (3.2) is periodic with period $N_F$, i.e., $S_{N_F+\varepsilon(*)} = S\varepsilon(*)$, and has two responses associated with the integer and fractional parts of $\varepsilon$.

- For fractional $\varepsilon$ variation as shown in Figure 3.1(a): the energy of $S$ converges to $S(0)$ when $\varepsilon = 0$, indicating no interference from other subcarriers. However, as $\varepsilon$ increases more than zero, the energy in $S$ spreads across all subcarriers. For larger $\varepsilon$ values, there is a higher percentage of energy “leakage” across the subcarriers. It is important to note that dominant energy response of $S$ remains in the $S(0)$ component.

- For integer $\varepsilon$ as shown in Figure 3.1(b): from the definition, integer $\varepsilon$ variation corresponds to a frequency offset whereby different subcarriers are identically mistaken. Hence, when $\varepsilon = l \{l \in \mathbb{Z}\}$: $S(0)$ is not the largest component, instead, the dominant response becomes the $S(l)$; the largest weight in $R[m]$ of (3.1) will be with $d_{m+l}$; and the decision for $\hat{d}_m$ based on $R[m]$ will be unreliable with very high probability.

It is important to note that for the integer NFO, there is no interference but only mistaking. In other words, $\hat{d}_m$ can be reliably determined using $R[m+l]$ if $\varepsilon = l$. Compared with the integer NFO, the estimation or cancellation of the fractional NFO is more difficult. Hence, like most of the research work on ICI, the work of this dissertation will focus on the fractional part.
Figure 3.1: Magnitude of ICI Coefficient for different NFOs
3.5 Conclusion

In this chapter, we study the multi-carrier transmissions for cognitive radio in the mobile environment, and analyze the inter-carrier interference effect caused by the carrier frequency offset.
Inter-Carrier Interference Reduction

Most ICI cancellation methods use signal processing and/or coding to reduce the sensitivity of the system to the frequency offset. For example, in [14], authors developed low-complexity minimum mean-square error (MMSE) and decision-feedback equalizer (DFE) receivers to suppress ICI based on the fact that the ICI power mainly comes from a few neighboring subcarriers. In light of the same statement, an effective method known as the ICI self-cancellation scheme has been proposed in [35] where copies of the same data symbol are modulated on $L$ adjacent subcarriers using optimized weights. In [36], a generalized ICI self-cancellation scheme has been proposed. In [37], an ICI self-cancellation using data-conjugate method is proposed. There are few ICI cancellation technologies for MC-CDMA system, although authors in [38] provide the analysis of the multiple access interference (MAI) and ICI for MC-CDMA system with frequency offset.

4.1 Overview of Self-Cancellation for OFDM System

Considering the OFDM system with ICI in Eq. (3.1), the first term on the righthand side represents the desired signal, the second term represents the ICI. If there is no frequency offset between the transmitter and receiver, the second term drops to zero and $S(0) = 1$. The existence of the second term in Eq. (3.1) causes significant performance degradation of OFDM system.

ICI self-cancellation scheme is an effective ICI cancellation scheme proposed for
OFDM [35]. Inspired by the observation that the ICI coefficients on adjacent subcarriers are close, ICI self-cancellation scheme has been proposed to cancel the ICI in OFDM in a very simple and elegant way. In ICI self-cancellation scheme for OFDM [35], every data symbol is transmitted twice on two adjacent subcarriers. Generally, assuming there are $N_F$ subcarriers (from 0 to $N_F - 1$) and $N_F/2$ symbols (from 0 to $N_F/2 - 1$) to transmit, symbol $d_m$ is transmitted over subcarrier $2m$, and symbol $-d_m$ is transmitted over subcarrier $2m + 1$, where $m = 0, 1, ..., N_F/2 - 1$.

Hence the received signal on even ($2m^{th}$) subcarrier in AWGN channel ($H_m = 1$) becomes:

$$R[2m] = \sum_{l=0}^{N_F/2-1} [d_l S(2l - 2m) - d_l S(2l + 1 - 2m)] + n_{2m},$$

$$= d_m (S(0) - S(1)) + \sum_{l=0,l\neq m}^{N_F/2-1} d_l [S(2l - 2m) - S(2l + 1 - 2m)] + n_{2m} \quad (4.1)$$

And the received signal on odd ($(2m + 1)^{th}$) subcarrier in AWGN channel is:

$$R[2m + 1] = \sum_{l=0}^{N_F/2-1} [d_l S(2l - 2m - 1) - d_l S(2l + 1 - 2m - 1)] + n_{2m+1},$$

$$= d_m (S(-1) - S(0)) + \sum_{l=0,l\neq m}^{N_F/2-1} d_l [S(2l - 2m - 1) - S(2l - 2m)] + n_{2m+1}$$

(4.2)

At receiver side, the signal on the odd numbered subcarriers are multiplied by $-1$, then summed with the signals on the even numbered subcarriers to create the final decision.
variables. Hence the decision of $d_m$ is determined on $R'[m]$:  

$$R'[m] = (R[2m] - R[2m+1])$$

$$= \sum_{l=0}^{N_F/2-1} d_l[2S(2l-2m) - S(2l-2m-1) - S(2l-2m+1)] + n_{2m} - n_{2m+1},$$

$$= d_m[2S(0) - S(-1) - S(1)]$$

$$+ \sum_{l=0, l\neq m}^{N_F/2-1} d_l[2S(2l-2m) - S(2l-2m-1) - S(2l-2m+1)] + n_{2m} - n_{2m+1} \quad (4.3)$$

Compared to Eq. (3.1), the effective ICI coefficient in Eq. (4.3) is reduced to be $[2S(2l-2m) - S(2l-2m-1) - S(2l-2m+1)]$ and the number of summations is reduced in half. From Figure 3.1, $S(2l-2m-1) \approx S(2l-2m) \approx S(2l+1-2m)$, which means $[2S(2l-2m) - S(2l-2m-1) - S(2l-2m+1)]$ is very small. Figure 4.1 presents the reduced magnitude of new coefficient $[2S(2l-2m) - S(2l-2m-1) - S(2l-2m+1)]$ with $N_F = 64, m = 16$. Compared to the original ICI coefficient in Figure 3.1, the reduced new coefficient has much lower leakage, indicating that the interference from other subcarriers is significantly reduced.

Of course, by doing so, the data rate is cut in half since we can only use $N_F/2$ subcarriers to transmit the symbols and another $N_F/2$ subcarriers to transmit the opposite symbols. To maintain the same transmission data rate, higher modulation scheme is employed. For example, an OFDM with ICI self-cancellation with QPSK modulation provides the same data rate of a conventional OFDM with BPSK modulation. It has been shown in [35] that self-cancellation significantly reduces ICI effect and improves the BER performance of OFDM.
ICI Coefficient when $N_F=64, m=16$

**Fig. 4.1**: Magnitude of ICI Coefficient after Self-Cancellation

### 4.2 ICI Self-Cancellation for MC-CDMA System without Reducing Datarate

In this section, we apply the ICI self-cancellation concept into MC-CDMA system to reduce ICI and improve the BER performance of MC-CDMA system [39]. We show that the ICI self-cancellation in MC-CDMA is equivalent to an MC-CDMA system with a new spreading code set. Hence, the ICI self-cancellation for MC-CDMA has identical transmitter and receiver structure without any changes except using different spreading codes.

In a typical orthogonal MC-CDMA system employing Hadamard-Walsh codes, the new spreading code set happens to be a subset of the original spreading code set. Moreover, to maintain the channel capacity, we can employ two sets of orthogonal Hadamard-Walsh codes (one on the in-phase subcarrier and one on the quadrature subcarrier).
By applying the ICI self-cancellation method to MC-CDMA, we propose to spread users’ signal on only the even-numbered subcarriers and transmit the opposite signal on odd-numbered subcarriers. Just like in OFDM ICI self-cancellation, the ICI can now be cancelled out significantly.

Specifically, the \( v \)th user’s signal \( d^{(v)} \) is now spread over \( N_F/2 \) even numbered subcarriers (i.e., subcarrier 0, 2, 4, ..., \( N_F - 2 \)), the opposite signal is applied to all the odd numbered subcarriers (i.e., subcarrier 1, 3, 5, ..., \( N_F - 1 \)). The transmitted signal of \( v \)th user now corresponds to

\[
s^{(v)}[n] = \text{Re}\left\{ \frac{1}{N_F} \sum_{m=0}^{N_F/2-1} d^{(v)}(v) \beta^{(v)}(v) e^{j2\pi f_{2m} t_n} \right\} - \text{Re}\left\{ \frac{1}{N_F} \sum_{m=0}^{N_F/2-1} d^{(v)}(v) \beta^{(v)}(v) e^{j2\pi f_{2m+1} t_n} \right\} \tag{4.4}
\]

where the first term represents the signal on all the even numbered subcarriers and the second term represents the signal on odd numbered subcarriers.

The total transmitted signal of \( N_u \) users now corresponds to

\[
s[n] = \text{Re}\left\{ \frac{1}{N_F} \sum_{v=1}^{N_u} \sum_{m=0}^{N_F/2-1} d^{(v)}(v) \beta^{(v)}(v) e^{j2\pi f_{2m} t_n} \right\} - \text{Re}\left\{ \frac{1}{N_F} \sum_{v=1}^{N_u} \sum_{m=0}^{N_F/2-1} d^{(v)}(v) \beta^{(v)}(v) e^{j2\pi f_{2m+1} t_n} \right\} \tag{4.5}
\]

However, since the signal is only spread over \( N_F/2 \) subcarriers, the spreading code length is also only \( N_F/2 \). Hence, the total number of orthogonal users can be supported is \( N_F/2 \). In other words, the network capacity in terms of total number of users is cut in half. This is the direct consequence of the self-cancellation mechanism. However, we can regain the lost network capacity by introducing another set of spreading codes in the quadrature subcarriers when BPSK modulation or other modulations which only use one dimension...
(real or imaginary) to carry information is employed.

It is interesting and important to note that the MC-CDMA with ICI self-cancellation is equivalent to an MC-CDMA system employing a new spreading code set which is a subset of the original spreading code set. To explain this concept, let’s consider a simple example where \( N_F = 4 \). The original spreading code matrix is

\[
C_4 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
\end{bmatrix}
\]  

(4.6)

and each row of the matrix \( C_4 \) corresponds to one spreading code with code length \( N_F = 4 \). Now, the MC-CDMA with ICI self-cancellation only spreads the information over two even numbered subcarriers (subcarrier 0 and subcarrier 2) and the spreading code matrix is

\[
C_2 = \begin{bmatrix}
1 & 1 \\
1 & -1 \\
\end{bmatrix}
\]  

(4.7)

At the same time, the opposite of data \( d^{(v)} \) is spread over the two odd numbered subcarriers (subcarrier 1 and subcarrier 3) using the same spreading code matrix. This is equivalent to an MC-CDMA system supporting two users with a spreading code matrix:

\[
C_4' = \begin{bmatrix}
1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 \\
\end{bmatrix}
\]  

(4.8)

This new spreading code matrix can be generated from the original spreading code matrix \( C_4 \) by choosing all the odd numbered rows (row 1 and row 3).

Now, by multiplying the spreading code matrix \( C_4' \) by \( j = \sqrt{-1} \), we obtain another
spreading code matrix

$$j \cdot C'_4 = \begin{bmatrix} j & -j & j & -j \\ j & -j & -j & j \end{bmatrix}$$

(4.9)

Combine this code matrix with $C'_4$, we obtain a $N$ by $N$ spreading code matrix:

$$D_4 = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ j & -j & j & -j \\ j & -j & -j & j \end{bmatrix}$$

(4.10)

Employing the spreading code matrix $D$, we have an MC-CDMA system with ICI self-cancellation which can support $N_F$ users. Hence, the proposed MC-CDMA with ICI self-cancellation can provide the same network capacity (in terms of number of users) when one dimension modulations, e.g., BPSK modulation, is employed.

Now, we can generalize the algorithm of the proposed MC-CDMA system with ICI self-cancellation as:

- Start with a $N_F$ by $N_F$ Hadamard-Walsh matrix $C_{N_F}$.
- Create a $N_F/2$ by $N_F$ matrix $C'_{N_F/2}$ by choosing the odd rows of $C_{N_F}$.
- Multiple $j = \sqrt{-1}$ to $C'_{N_F/2}$.
- Create a $N_F$ by $N_F$ spreading code matrix

$$D_{N_F} = \begin{bmatrix} C'_{N_F/2} \\ j \cdot C'_{N_F/2} \end{bmatrix}$$

(4.11)

Now, by using matrix $D_{N_F}$ as the spreading code matrix, no change needs to be made to the MC-CDMA system transmitter and receiver while the ICI self-cancellation is embedded in the system. Meanwhile, this system can support as much as $N_F$ orthogonal users.
without sacrifice the throughput like traditional ICI self-cancellation scheme for OFDM
[35].

4.3 Performance of ICI Self-Cancellation for MC-CDMA System

In this section, we use numerical simulation results to present the effectiveness of the proposed ICI cancellation scheme. We provide BER simulation results for the proposed MC-CDMA with ICI self-cancellation in both AWGN channel and multipath fading channels compared with MC-CDMA without ICI self-cancellation. All the systems are assumed to have $N_F = 64$ subcarriers and employ BPSK modulation.

4.3.1 AWGN Channel

The simplest way to examine the effectiveness of the proposed ICI self-cancellation scheme is to transmit signals through an AWGN channel with a constant frequency offset between the transmitter and receiver. Figure 4.2 illustrates the BER versus SNR simulation results for fully loaded ($N_u = 64$) MC-CDMA system without ICI self-cancellation (legend “without SC”) and fully loaded MC-CDMA with ICI self-cancellation (legend “with SC”), when $\varepsilon = 0.1$ and $0.4$, and the BER performance of conventional MC-CDMA system without ICI is provided as the baseline. It is evident that the MC-CDMA system suffers significant performance degradation when ICI is present. On the other hand, MC-CDMA with ICI self-cancellation demonstrates a much less performance degradation.

Figure 4.3 shows the BER versus NFO $\varepsilon$ simulation results for the fully loaded MC-CDMA systems when $SNR = 5dB$ and $7dB$. It is clear that when $\varepsilon$ increases, both the performance of traditional MC-CDMA system and MC-CDMA system with ICI self system become worse, however, MC-CDMA system with ICI self-cancellation has better
BER Performance for MC–CDMA system in AWGN channel, $N_F=64$ and $N_u=64$

![Graph showing BER Performance for MC-CDMA system in AWGN channel v.s. SNR](image)

Figure 4.2: BER Performance for MC-CDMA system in AWGN channel v.s. SNR

BER performance, especially when $\varepsilon$ becomes large.

### 4.3.2 Multipath Fading Channel

In a practical mobile multipath radio channel, time-variant multipath propagation leads to Doppler frequency shift which is a random variable. Here we measure the performance of the proposed ICI cancellation method in multipath fading channels. As a measure of Doppler frequencies, we use the normalized maximum Doppler spread $\varepsilon_{\text{max}}$, which is defined as the ratio between the channel maximum Doppler spread to the subcarrier bandwidth. Here, we assume a 4-fold multipath fading channel, whose coherence bandwidth is:

$$BW = N_{\text{fold}} \cdot \Delta f = 4 \cdot (\Delta f)_c$$

(4.12)
BER Performance for MC–CDMA system in AWGN channel, $N_F=64$ and $N_u=64$

Figure 4.3: BER Performance for MC-CDMA system in AWGN channel v.s. $\varepsilon$

where $BW$ is the total bandwidth of the system and $(\Delta f)_c$ is the coherence bandwidth of the channel.

The BER performance versus SNR of traditional MC-CDMA system and MC-CDMA system with self-cancellation scheme in multipath fading channel are compared in Figure 4.4, with normalized maximum Doppler spread $\varepsilon_{max} = 0.2$ and $0.3$. Both systems are fully loaded. It is obvious that the MC-CDMA system with ICI self-cancellation outperforms the traditional MC-CDMA system in mobile environment, especially with higher mobility.
4.4 Performance of ICI Self-Cancellation for CI/MC-CDMA System

Instead of using Hadamard-Walsh code matrix, Carrier Interferometry MC-CDMA (CI/MC-CDMA) system utilizes the Carrier Interferometry (CI) code [40][41][42]. Similarly, the ICI self-cancellation can also be embedded into the CI/MC-CDMA system by simply changing the spreading code [43]. Specifically, the spreading code for CI/MC-CDMA system with ICI self-cancellation can be generated as:

1. Start with a $N_F/2$ by $N_F/2$ CI code matrix $C_{N_F/2}$:

$$C_{N_F/2}(m, n) = \exp(-j \frac{2\pi}{N_F/2} mn)$$  \hspace{1cm} (4.13)
2. Create an $N_F/2$ by $N_F$ spreading matrix $C'_N_F$ such that:

- $C'_{N_F}(m, 2n) = C_{N_F/2}(m, n)$
- $C'_{N_F}(m, 2n + 1) = (-1)C_{N_F/2}(m, n)$

3. Create an $N_F$ by $N_F$ spreading code matrix:

$$D_{N_F} = \begin{bmatrix} C'_{N_F} \\ j \cdot C'_{N_F} \end{bmatrix}$$ (4.14)

where $j = \sqrt{-1}$.

Now the spreading code from $k^{th}$ user to $i^{th}$ subcarrier, $\beta_i^{(k)}$, yields to:

1. When $k \in [0, \frac{N_F}{2} - 1]$ and $i \in [0, \frac{N_F}{2} - 1]$:

- $\beta_{2i}^{(k)} = \exp(-j \frac{2\pi}{N_F/2} ik)$;
- $\beta_{2i+1}^{(k)} = -\exp(-j \frac{2\pi}{N_F/2} ik)$;

2. When $k \in [\frac{N_F}{2}, N_F - 1]$ and $i \in [0, \frac{N_F}{2} - 1]$:

- $\beta_{2i}^{(k)} = \exp(-j (\frac{2\pi}{N_F/2} ik - \frac{\pi}{2}))$
- $\beta_{2i+1}^{(k)} = -\exp(-j (\frac{2\pi}{N_F/2} ik - \frac{\pi}{2}))$

By using matrix $D_{N_F}$ in Eq. (4.14) as the spreading code matrix, no change needs to be made to the CI/MC-CDMA system transmitter and receiver while the ICI self-cancellation is embedded in the system and total $N_F$ orthogonal users can be supported.

Now, let's use numerical simulation results to present the effectiveness of the proposed ICI cancellation scheme for CI/MC-CDMA system. We provide BER simulation results for the proposed CI/MC-CDMA ICI self-cancellation in both AWGN channel and multipath fading channels compared with traditional CI/MC-CDMA system. All the systems are assumed to have $N_F = 64$ subcarriers.
4.4.1 AWGN channel

Figure 4.5 to Figure 4.7 illustrate the BER performance versus SNR for fully/half loaded traditional CI/MC-CDMA system and the proposed CI/MC-CDMA system with self-cancellation. In these figures, the black dot line presents the traditional CI/MC-CDMA system without ICI, and the blue line marked with stars denotes the traditional CI/MC-CDMA system with ICI, while the red line with circles is the proposed CI/MC-CDMA system with self-cancellation. In Figure 4.5, fully loaded \( (N_u = N_F) \) and BPSK modulation are applied for both traditional CI/MC-CDMA system and CI/MC-CDMA system with self-cancellation when \( \varepsilon = 0.2 \), while in Figure 4.6 half loaded \( (N_u = N_F/2) \) and QPSK modulation are applied for both traditional CI/MC-CDMA system and CI/MC-CDMA system with self-cancellation when \( \varepsilon = 0.1 \). From both figures, it is evident that the proposed ICI self-cancellation scheme provides much better performance compared to the traditional CI/MC-CDMA system in ICI presence.

Figure 4.7 shows the fully loaded system \( (N_u = N_F) \) for higher throughput. Since self-cancellation can only support fully loaded system with one dimensional modulation, 4ASK modulation is applied for CI/MC-CDMA system with self-cancellation to compare with traditional CI/MC-CDMA system with QPSK when \( \varepsilon = 0.1 \). At lower SNR, the performance of self-cancellation with 4ASK modulation performs slightly worse than traditional CI/MC-CDMA system with QPSK modulation. However, when SNR increases, the performance of traditional CI/MC-CDMA system with QPSK modulation is not improved much due to the ICI, while the performance of self-cancellation with 4ASK modulation becomes much better, and larger gain is obtained for better BER requirement (e.g., 2dB gain is obtained when BER=10\(^{-2}\) in Figure 4.7).

It is important to note that the performance of the proposed system depends significantly on the number of users, since when \( N_u > N_F/2 \), only the modulations which use one dimension to carrier information can be applied to the proposed these \( K \) orthogonal users in the CI/MC-CDMA system with self-cancellation. The BER performance versus number
Figure 4.5: The BER performance for CI/MC-CDMA in AWGN Channel (BPSK, \(N_u = N_F\) and \(\varepsilon = 0.2\)).

The BER performance for CI/MC-CDMA in AWGN Channel (BPSK, \(N_u = N_F\) and \(\varepsilon = 0.2\)).

of users \((N_u)\) is shown in Figure 4.8 with the fixed SNR=8dB and \(\varepsilon = 0.3\). The blue line marked with stars and green line with triangles present the performance for CI/MC-CDMA system with BPSK and QPSK modulation; while the red line marked with circle and purple line with diamond illustrate the performance for CI/MC-CDMA system with BPSK and 4-ary modulation (QPSK is applied if \(N_u \leq N_F/2\), 4ASK is applied if \(N_u > N_F/2\)). It is evident that when \(N_u\) increases, performances of both systems with BPSK modulation degrades, but the self-cancellation provides much significant gain over the traditional CI/MC-CDMA system. When \(N_u \leq N_F/2\), QPSK modulation is applied to the proposed system. When \(N_u > N_F/2\), the proposed system changes to use 4ASK modulation which experiences a jump in BER performance. For different number of users \((N_u)\) illustrated in Figure 4.8, ICI self-cancellation with 4ASK provides much better performance compared to the traditional CI/MC-CDMA system with QPSK modulation.
4.4.2 Multipath Fading Channel

Figure 4.9 to Figure 4.11 present the BER performance versus SNR for fully/half loaded traditional CI/MC-CDMA system and the proposed CI/MC-CDMA system with self-cancellation in multipath fading channel. Figure 4.9 shows fully loaded \((N_u = N_f)\) system, and BPSK modulation is applied to both benchmark CI/MC-CDMA system and ICI self-cancellation one with \(\varepsilon_{\text{max}} = 0.2\). Figure 4.10 illustrates half loaded \((N_u = N_f/2)\) system, and QPSK modulation is applied to both systems with \(\varepsilon_{\text{max}} = 0.1\). From both figures, it is evident that the proposed ICI self-cancellation scheme provides significant performance gain in multipath fading channel with ICI.

Figure 4.11 shows the fully loaded system \((N_u = N_f)\) for higher throughput. Here, 4ASK modulation is applied to CI/MC-CDMA system with self-cancellation since \(N_u > \)
Figure 4.7: The BER performance for CI/MC-CDMA in AWGN Channel (QPSK, $N_u = N_F$ and $\varepsilon = 0.1$)

$N_F/2$ to compare with traditional CI/MC-CDMA system with QPSK when $\varepsilon_{\text{max}} = 0.1$. At lower SNR, the performance of self-cancellation with 4ASK modulation performs worse than traditional CI/MC-CDMA. However, when SNR increases, traditional CI/MC-CDMA system with QPSK modulation observes an error floor due to the ICI, while self-cancellation system has no such floor and eventually outperforms traditional system.

### 4.5 Conclusion

In this chapter, we apply the ICI self-cancellation scheme into MC-CDMA system to reduce inter-carrier interference and improve BER performance. By spreading users’ signal over even numbered subcarriers, and transmitting the opposite signal on adjacent odd num-
bered subcarriers, the inter-carrier interference is greatly diminished and the BER performance is significantly improved. Furthermore, two sets of orthogonal codes are applied to maintain the same network capacity coupled with one dimensional modulations. We find that the proposed MC-CDMA with ICI self-cancellation system is equivalent to an MC-CDMA system with a new set of orthogonal spreading codes. Therefore, the proposed ICI self-cancellation scheme is easy to implement without changing the transmitter and receiver design or increasing system complexity. Simulation results in AWGN channel and multipath fading channel confirm the effectiveness and efficiency of the proposed ICI self-cancellation for MC-CDMA systems.

Figure 4.8: BER performance verse number of users for CI/MC-CDMA with and without SC in AWGN Channel (SNR=8dB and $\varepsilon = 0.3$)
Figure 4.9: BER performance for CI/MC-CDMA in multipath fading channel, fully loaded (BPSK, $N_u = N_F$ and $\varepsilon_{\text{max}} = 0.2$)
Figure 4.10: BER performance for CI/MC-CDMA in multipath fading channel, half loaded (QPSK, $N_u = N_F/2$ and $\varepsilon_{max} = 0.1$)
Figure 4.11: BER performance for CI/MC-CDMA in multipath fading channel, fully loaded (QPSK, $N_u = N_F$ and $\varepsilon_{\text{max}} = 0.1$)
Total Inter-Carrier Interference Cancellation

In the literature, all existing ICI cancellation methods are not without their drawbacks. Even though all the existing ICI cancellation methods reduce ICI and improve BER performance for the system, the performance improvement is very limited. The BER performance after ICI cancellation is still significantly worse than that of the original system without ICI. More important, most of the existing ICI cancellation methods achieve the ICI reduction and BER performance improvement at the cost of lowering the transmission rate and reducing the bandwidth efficiency. There do exist some methods that do not reduce the date rate, however, such methods produce even less reduction in ICI. Nevertheless, only a few of these ICI cancellation methods are applied for MC-CDMA system with carrier frequency offset to improve the performance of mobile MC-CDMA system. Hence it is imperative to propose a new general inter-carrier interference cancellation for both OFDM and MC-CDMA systems which can cancel the effect of ICI perfectly without sacrificing the data rate.
5.1 General Expression for OFDM and MC-CDMA Systems with ICI

For more general case, let’s generate the universe expression for the OFDM and MC-CDMA systems. According to Eq. (3.3) and Eq. (3.7), the received signal vector in AWGN channel of OFDM and MC-CDMA systems can be presented as:

$$\vec{R} = \vec{d} \mathbf{C} \mathbf{S} + \vec{n},$$

where $\mathbf{C}$ is $N_F \times N_F$ identity matrix for OFDM system, while $\mathbf{C}$ becomes $N_u \times N_F$ spreading code matrix for MC-CDMA system, e.g., $N_u$ rows of $N_F \times N_F$ size Hadamard-Walsh code matrix.

From Eq. (5.1), it is obvious that the received signal on all the subcarriers can be viewed as a new MC-CDMA signal with a new spreading code matrix $\mathbf{C} \mathbf{S}$. Now, it is important to note that the ICI coefficient matrix $\mathbf{S}$ is an orthonormal matrix, i.e.,

$$\mathbf{S} \mathbf{S}^H = \mathbf{I}$$

(5.2)

where $\mathbf{S}^H$ is the conjugate transpose of matrix $\mathbf{S}$ and $\mathbf{I}$ is identity matrix.

Hence, the OFDM or MC-CDMA signal with ICI at receiver side can be considered as a new orthogonal MC-CDMA system with spreading code matrix $\mathbf{C} \mathbf{S}$. As a direct result, the ICI can be totally removed from the new MC-CDMA signal if we apply a matrix multiplication of $(\mathbf{C} \mathbf{S})^H$ to $\vec{R}$ in Eq. (5.1), yielding:

$$\vec{Y} = \vec{R} \mathbf{S}^H \mathbf{C}^H = \vec{d} + \vec{n} \mathbf{S}^H \mathbf{C}^H$$

(5.3)

Next, we can simply make decision of $\vec{d}$ based on the sign of $\vec{Y}$. Since $\mathbf{S}^H$ is also an
orthonormal matrix, the noise vector $\vec{n}S^H$ in $\vec{Y}$ has the same covariance matrix as that of $\vec{n}$. Hence, the entire ICI is eliminated and the BER performance would be the same of an MC-CDMA system without ICI at all.

Now the problem is: the receiver does not know the spreading code matrix $S$ because the normalized frequency offset $\varepsilon$ is unknown. Hence, it has been proposed to estimate the normalized frequency offset $\varepsilon$ through some training symbols. Of course, by doing so, some bandwidth needs to be allocated for the training symbols, and sophisticated frequency offset estimation algorithms need to be implemented at receiver side.

## 5.2 Analysis in AWGN Channels

Here, we propose the Total ICI Cancellation scheme to eliminate ICI on mobile OFDM and MC-CDMA systems without transmitting any training symbols (and reducing data rate) [44][45]. While the normalized frequency offset $\varepsilon$ is unknown to the receiver, we can quantize $\varepsilon$ into $K$ equally spaced values:

$$\varepsilon'_k = k \cdot \Delta \varepsilon, k = 0, 1, \ldots, K - 1$$  \hspace{1cm} (5.4)

where $\Delta \varepsilon$ is the quantization level of normalized frequency offset, and $K$ is the number of quantization levels:

$$\Delta \varepsilon = \frac{1}{K}$$  \hspace{1cm} (5.5)

One of these $K$ quantized $\varepsilon'$s is the closest to the true $\varepsilon$.

Now, we can build $K$ parallel branches at the receiver. Each branch uses one of the $K$ quantized $\varepsilon'$s to create the corresponding ICI coefficient matrix $\tilde{S}$. Hence, we have $K$ ICI
coefficient matrices $\tilde{S}_0, \tilde{S}_1, \cdots, \tilde{S}_{K-1}$ where the $k^{th}$ matrix corresponds to:

$$\tilde{S}_k = \begin{bmatrix}
S_k(0) & S_k(-1) & \cdots & S_k(1-N_F) \\
S_k(1) & S_k(0) & \cdots & S_k(2-N_F) \\
\vdots & \vdots & \ddots & \vdots \\
S_k(N_F-1) & S_k(N_F-2) & \cdots & S_k(0)
\end{bmatrix} \tag{5.6}$$

and

$$S_k(l-m) = \frac{\sin(\pi(\varepsilon'_k + l-m))}{N_F \sin(\frac{\pi}{N_F}(\varepsilon'_k + l-m))} \cdot \exp\left(j\pi\left(1 - \frac{1}{N_F}\right)(\varepsilon'_k + l-m)\right) \tag{5.7}$$

Using these $K$ matrices, we can have $K$ decisions on the transmitted data vector $\vec{d}$ where the $k^{th}$ branch will make decision on the estimation of $\hat{\vec{d}}$.

$$\hat{\vec{d}}_k = \text{Decision}\{\bar{R}\tilde{S}_k^H \bar{C}^H\} \tag{5.8}$$

For example, $\text{Decision}\{x\}$ can simply be the sign of the real part of $x$ if BPSK modulation is applied.

Next, with the data vector estimation $\hat{\vec{d}}_k$, each branch can reproduce the received signal $\hat{R}_k$ by using the data vector estimation $\hat{\vec{d}}_k$ and the ICI coefficient matrix of that branch $\tilde{S}_k$:

$$\hat{R}_k = \hat{d}_k \bar{C}\tilde{S}_k \tag{5.9}$$

It is easy to understand that the one branch whose $\varepsilon'_k$ is the closest to the true value of $\varepsilon$ should reproduce the received signal $\hat{R}_k$ also being the closest to the received signal vector $\bar{R}$. Hence, we only need to calculate and compare the Euclidean distances between the $K$ reproduced received signal vectors $\hat{R}_k$ and the truly received signal vector $\bar{R}$, and pick the
one with the minimum distance to be the best branch and use that branch’s estimated data vector as the final decision:

\[
\hat{d} = \arg\min \{||\hat{R}_k - \hat{R}||^2\} \tag{5.10}
\]

where \(||\hat{R}_k - \hat{R}||^2\) represents the Euclidean distance between vector \(\hat{R}_k\) and vector \(\hat{R}\).

Figure 5.1 shows the \(||\hat{R}_k - \hat{R}||^2\) versus \((\varepsilon' - \varepsilon)\) for different SNR, and it is clear that when there is no noise, \(||\hat{R}_k - \hat{R}||^2\) reaches the optimum when \((\varepsilon' - \varepsilon) = 0\). Meanwhile, at high SNR, the optimum also occurs when \(|\varepsilon'_k - \varepsilon|\) is very small.

It is important to note that the complexity of the proposed Total ICI Cancellation method increases linearly with the quantization level \(K\), keeping the computational complexity at a reasonable range. The increased complexity is not significant, especially when \(K\) is small.
5.3 Analysis in Multipath Fading Channel

Similar to the analysis in AWGN channel, the received signal of OFDM/MC-CDMA system can be represented as

\[
\vec{R} = \vec{d}CHS + \vec{n},
\]

(5.11)

where \(C\) is similar to Eq. (5.1). The received signal \(\vec{R}\) in Eq. (5.11) can also be viewed as a new MC-CDMA system. Hence, if the spreading code matrix \(S\) is known, we can eliminate the ICI by multiplying \(S^H\) to the received vector \(\vec{R}\).

So the Total ICI Cancellation scheme works the same way as in AWGN channel with only one exception: the fading channel characteristics \(H\) needs to be estimated at the receiver side (which is required for OFDM and MC-CDMA transmissions). Consider the fading effects, we can use a combining technique to improve the estimation performance of \(d\):

\[
\hat{d}_k = \text{Decision}\{\tilde{R}\tilde{S}_k^HW^H\}
\]

(5.12)

where \(W\) is a diagonal matrix containing the combining weights, e.g., \(W\) becomes an identity matrix if equal gain combining (EGC) is applied. The reproduced received signal vector now also has to consider the fading effects:

\[
\hat{R}_k = \hat{d}_kCH\tilde{S}_k
\]

(5.13)

The block diagram of the proposed Total ICI Cancellation scheme is shown in Figure 5.2.
5.4 Performance of Total ICI Cancellation for OFDM System

In this section, we use numerical simulation results to present the effectiveness of the proposed Total ICI Cancellation scheme for OFDM system [44]. We provide BER simulation results for the proposed Total ICI Cancellation scheme in both AWGN channel and multipath fading channels. All the systems are assumed to have $N_F = 32$ subcarriers and employ BPSK modulation.

5.4.1 AWGN Channel

First, we analyze the BER performance in AWGN channel with constant carrier frequency offset. Figure 5.3 illustrates the simulation result when the NFO $\varepsilon = 0.3$. In the Total ICI Cancellation scheme, we use $K = 10$. In both figures, the blue line shows the BER performance of OFDM without ICI, the green line marked with circles represents the performance of OFDM with ICI, and the red line marked with stars represents that of our proposed Total ICI Cancellation scheme. Since we chose $K = 10$, the quantization level of normalized frequency offset ($\Delta \varepsilon$) becomes 0.1. Hence, one of the $K$ branches actually
has the perfect ICI coefficient matrix to work with.

To prove the effectiveness of our Total ICI Cancellation method in all scenarios, Figure 5.4 presents the results when $\varepsilon = 0.287532$, so none of the $K$ branches matches the actual $\varepsilon$. It is obvious from these figures that the BER performance of OFDM significantly degrades due to ICI, but the Total ICI Cancellation scheme effectively eliminates the ICI and provides the same BER performance as that of an OFDM without ICI.

![BER Performance of OFDM system in AWGN channel, $\varepsilon = 0.3$](image)

**Figure 5.3:** BER Performance for OFDM system in AWGN channel, $\varepsilon = 0.3$

Figure 5.5 shows the BER performance versus $\varepsilon$. It is evident that with the increase of $\varepsilon$, the BER performance of OFDM significantly degrades while the OFDM with Total ICI Cancellation keeps the same performance despite the carrier frequency offset.
BER Performance of OFDM system in AWGN channel, $N_f=32$, $\varepsilon=0.287532$.

Figure 5.4: BER Performance for OFDM system in AWGN channel, $\varepsilon = 0.287532$

5.4.2 Multipath Fading Channel

As a measure of Doppler frequencies, we use the normalized maximum Doppler spread $\varepsilon_{\text{max}}$, which is defined as the ratio between the channel maximum Doppler spread to the subcarrier bandwidth. We use the Hilly Terrain (HT) channel models defined by the GSM standard as our channel model.

Figure 5.6 shows the case when $\varepsilon_{\text{max}} = 0.3$. In the Total ICI Cancellation scheme, we use $K = 10$. It is clear from this figure that the proposed Total ICI Cancellation entirely eliminates the effect of ICI and matches the performance of the OFDM without ICI in fading channels as well.
5.4.3 Complexity

Figure 5.7 illustrates the effect of the number of normalized frequency offset quantization levels $K$ on the performance of the proposed Total ICI Cancellation scheme. In Figure 5.7, three BER versus $K$ curves of different SNRs are shown. It is easy to understand that when $K$ increases, more quantization levels are used and better ICI coefficient matrix estimation is achieved, so the performance of the proposed scheme also improves. As shown in Figure 5.7, when $K$ is very small, the proposed Total ICI Cancellation scheme actually offers pretty bad performance due to the large quantization error. However, when $K$ increases, the Total ICI Cancellation converges fast and provides ICI cancellation and BER improvement quickly. When $K$ is larger than 5, there is no noticeable performance gain to increase the quantization level. This can be explained as follows: when the quantization step $\Delta \varepsilon$ is small enough, the Total ICI Cancellation’s ICI cancellation capability is sufficient to remove all the inter-carrier interference and there is no need to decrease $\Delta \varepsilon$ anymore. It is
evident from Figure 5.7 that the computational complexity of the proposed scheme is very reasonable.

5.5 Performance of Total ICI Cancellation for MC-CDMA System

In this section, we use numerical simulation results to present the effectiveness of the proposed Total ICI Cancellation scheme for MC-CDMA system [45]. We provide BER simulation results for the proposed Total ICI Cancellation scheme in both AWGN channel and multipath fading channels compared with Self-Cancellation schemes for MC-CDMA system. All the systems are assumed to have $N_F = 64$ subcarriers, full loaded and employ BPSK modulation.
Figure 5.7: Effect of NFO Quantization Levels ($K$) in AWGN channel, $\varepsilon = 0.1$

5.5.1 AWGN Channel

Figure 5.8 illustrates the simulation result when $\varepsilon = 0.1$. In the Total ICI Cancellation scheme, we use $K = 10$. In this figure, the blue line shows the BER performance of MC-CDMA without ICI, the magenta line marked with circles represents the performance of MC-CDMA with ICI, the red line marked with square represents that of our proposed Total ICI Cancellation technology, and the green line with triangle shows the performance of self-cancellation scheme. Again, since we choose $K = 10$, so the quantization level of normalized frequency offset $\Delta \varepsilon = 0.1$. Hence, one of the $K$ branches actually has the perfect ICI coefficient matrix to work with.

To prove the effectiveness of our Total ICI Cancellation method in all scenarios, Figure 5.9 presents the results when $\varepsilon = 0.485372$, so none of the $K$ branches matches the actual $\varepsilon$, and the BER performance of Total ICI Cancellation scheme also matches with the MC-CDMA system without ICI.
From these figures, it is evident that the proposed Total ICI Cancellation technology provides MC-CDMA system the same BER performance as that of a MC-CDMA without ICI no matter what $\varepsilon$ is, while other two systems degrade when $\varepsilon$ increases. Meanwhile, the Total ICI Cancellation scheme provides much more gain compared to the other two systems when NFO increases.

Figure 5.10 shows the BER performance versus normalized frequency offset $\varepsilon$ when $\text{SNR}= 5dB$. It is obvious that when the normalized frequency offset $\varepsilon$ increases, the BER performance of MC-CDMA significantly degrades, and that of MC-CDMA with self-cancellation scheme also becomes worse, but the Total ICI Cancellation scheme eliminates the ICI and keeps the same performance despite the frequency offset.
BER Performance of MC-CDMA system in multipath fading channel, $N_F=64$, $\varepsilon=0.485372$

Figure 5.9: BER Performance for MC-CDMA system in AWGN channel, $\varepsilon = 0.485372$

5.5.2 Multipath Fading Channel

Figure 5.11 shows the case when $\varepsilon_{max} = 0.4$. In the Total ICI Cancellation scheme, we use $K = 10$. The legend is the same as in AWGN channel. It is clear from these figures that the proposed Total ICI Cancellation entirely eliminates the effect of ICI and matches the performance of the MC-CDMA without ICI in fading channels as well, no matter what NFO is; while other two benchmarks have much worse performance, especially when NFO becomes large.

5.5.3 Complexity

Figure 5.12 illustrates the effect of the number of normalized frequency offset quantization levels $K$ on the performance of the proposed Total ICI Cancellation scheme. In Figure
BER Performance of MC–CDMA system in AWGN channel, $N_F=64, SNR=5dB$

![BER Performance](image)

Figure 5.10: BER Performance for MC-CDMA system in AWGN channel, $SNR = 5dB$

5.12, four BER versus $K$ curves of different SNRs are shown. It is easy to understand that when $K$ increases, more quantization levels are used and better ICI coefficient matrix estimation is achieved, so the performance of the proposed scheme also improves. As shown in Figure 5.12, when $K$ increases, the Total ICI Cancellation converges fast and provides ICI cancellation and BER improvement quickly. When $K$ is larger than 7, there is no noticeable performance gain to increase the quantization level. It is evident from Figure 5.12 that the computational complexity of the proposed scheme is very reasonable.

### 5.6 Demonstration via Software Defined Radio

To illustrate the effectiveness of the proposed Total ICI cancellation method, we implement this algorithm on a software defined radio (SDR) [46] and integrate it into our SDR
Theoretical BPSK without ICI
Total Cancellation
Self Cancellation
BPSK with ICI

Figure 5.11: BER Performance for MC-CDMA system in multipath fading channel

based cognitive radio demonstration [47]. In the demonstration, we use the Universal Software Radio Peripheral (USRP) [48] SDR platform to implement an OFDM/NC-OFDM (non-contiguous OFDM) transmitter and receiver. We then use a Spirent SR5500 wireless channel emulator to emulate the multipath fading channel and the Doppler shift between the transmitter and receiver. The demonstration setup is shown in Figure 5.13(a).

In the demonstration, the carrier frequency is $515 \text{MHz}$, the total transmission bandwidth is $2 \text{MHz}$. The total number of subcarriers is 64, the frequency separation between adjacent subcarriers is $\Delta f = 31.25 \text{KHz}$. Figure 5.13(b) compares the spectrum. The spectrum in red is the transmission, while the spectrum in blue is the received spectrum with normalized carrier frequency offset $\varepsilon$ at 0.3. Obviously, the receiver needs to eliminate this frequency offset to perfectly demodulate the data.

From Figure 5.13(a), we can see on the two laptop screens that real-time video and
audio transmission over the cognitive radios is supported seamlessly. Part of this work was demonstrated in IEEE Globecom 2010 and received the Best Demo Award [47].

To statistically show the benefit of the proposed algorithm, we conducted thorough tests and experiments in environments with and without carrier frequency offset.

- **Case 1**: In this case, no Doppler shift is introduced, and there is no ICI effect. 98.13% of the transmitted packets are successfully received and 99.4% of the received packets have no error.

- **Case 2**: In this case, a normalized frequency offset $\varepsilon = 0.3$ is introduced into the system along with the multipath fading, leading to the ICI effect. No packet can be received (100% packet loss), and the system breaks down.

- **Case 3**: In this case, a normalized frequency offset $\varepsilon = 0.3$ is introduced into the
system along with the multipath fading, and the proposed Total ICI cancellation algorithm is employed in the receiver. 98.1% of the transmitted packets can be successfully received and 99.26% of them have no error.

The demonstration vividly shows the effectiveness of the proposed scheme.

![Demonstration Setup and Spectrum Comparison](image)

(a) Software Defined Radio Based Demonstration Setup

(b) Spectrum Comparison with and without frequency offset, Captured by USRP

Figure 5.13: Demonstration Setup and Spectrum Comparison
5.7 Conclusion

In this chapter, we proposed a novel inter-carrier interference cancellation scheme called Total ICI Cancellation for OFDM and MC-CDMA systems in mobile environments. Taking advantage of the orthogonality of the ICI coefficient matrix, the proposed ICI cancellation scheme can eliminate the ICI experienced in mobile OFDM and MC-CDMA systems entirely and provide significant BER improvement which matches the BER performance of OFDM and MC-CDMA systems without ICI at all. The proposed Total ICI Cancellation scheme provides perfect performance, and it doesn’t reduce the bandwidth efficiency of the system like many existing ICI cancellation methods. Simulations over AWGN channel and multipath fading channel, as well as the demonstration, confirm the effectiveness of the proposed scheme. Finally, the Total ICI Cancellation scheme achieves superb performance at a very reasonable computational complexity which linearly grows with the number of normalized frequency offset quantization.
ICI Immunity: SC-OFDM via Magnitude Keyed Modulation

Let’s revisit the ICI Self-Cancellation scheme here. It is better to treat this scheme as a modulation which is a simple MC-CDMA system with a new spreading code set, rather than an ICI cancellation scheme, because there is no extra computation required to cancel the ICI effect. However, the ICI self cancellation cannot entirely remove the ICI and the performance of this new modulation scheme is not as good as when there is no ICI. Hence, it is highly desired to design a modulation which is immune to the ICI. In other words, no matter what the CFO is, the performance of the proposed modulation will maintain the same.

6.1 Interference to Carrier Power Ratio

To provide a further understanding of how the ICI coefficient impacts system performance, we pay more attention to an AWGN channel \( H_m = 1, \forall m \). For the analysis we must determine the ICI power. This can be done using the Carrier-to-Interference Power Ratio (CIR), defined as [35, 49]:

\[
CIR = \frac{\text{Desired Signal Power}}{\text{ICI Power}}.
\]

(6.1)
However, when no ICI presents, i.e., \( \varepsilon \to 0 \), the CIR approaches infinity which cannot be shown in a figure. As an alternative approach, the ICI power can be estimated using the Interference-to-Carrier Power Ratio (ICR), defined as:

\[
ICR = CIR^{-1} = \frac{ICI \quad Power}{Desired \quad Signal \quad Power}.
\] (6.2)

The ICR of OFDM signal in Eq. (3.1) can be computed as

\[
ICR_{OFDM} = \text{avg} \left\{ \frac{\left| \sum_{l=0,l \neq m}^{N_F-1} d_l S(l - m) \right|^2}{|d_m S(0)|^2} \right\}_{m=0}
\] (6.3)

The ICR of MC-CDMA signal in Eq. (3.10) is:

\[
ICR_{MC-CDMA} = \text{avg} \left\{ \frac{\left| \sum_{l=1,l \neq v}^{N_u} d^{(l)} S'_{l,v} \right|^2}{|d^{(v)} S'_{v,v}|^2} \right\}_{v=1}
\] (6.4)

The ICR of SC-OFDM signal in Eq. (3.15) becomes:

\[
ICR_{SC-OFDM} = \text{avg} \left\{ \frac{\left| \sum_{l=0,l \neq k}^{N_F-1} d_l S'_{l,k} \right|^2}{|d_k S'_{k,k}|^2} \right\}_{k=0}
\] (6.5)

The definition in Eq. (6.2) implies that the ICR becomes smaller as the desired signal power to ICI power ratio increases, and our goal is to reduce the ICR. It is evident that ICR is system dependent and thus critical for us to consider several possible cases. To
statistically analyze the $ICR$ feature for different modulations, we average the $ICR$ across all subcarriers and all data, which represents a more reliable approach relative to what was used in [35]. Specifically, ICR on $k^{th}$ subcarrier can be represented as $ICR(k) = \frac{\text{ICI power from non-}k^{th}\text{ subcarrier}}{\text{Signal power on } k^{th}\text{ subcarrier}}$. The average $ICR$, which is represented as $\frac{1}{N} \sum_{k=0}^{N-1} ICR(k)$, is compared here; while $ICR(0)$ or $CIR(0)$ is compared in literature [35].

![Comparison of ICR for OFDM, MC−CDMA and SC−OFDM systems with 64 subcarriers](image)

**Figure 6.1:** ICR Comparison for OFDM, MC-CDMA and SC-OFDM Systems.

Results in Figure 6.1 show $ICR$ versus $\varepsilon$ for OFDM, MC-CDMA and SC-OFDM systems using $N_F = 64$ subcarriers with ICI present. It is interesting to notice that $ICR$ of SC-OFDM is zero for all $\varepsilon$ values; in other words, the power of desired signal component used for data estimation is virtually unaffected by ICI. Given the ICI of SC-OFDM system is much lower than that of the OFDM and MC-CDMA systems, the benefit of using SC-OFDM under conditions with ICI present are clearly evident by comparison to traditional OFDM and MC-CDMA under similar conditions. The following analysis of ICI coefficient matrix will show how ICI affects the overall performance and helps explain why the SC-
OFDM system experiences zero ICR.

### 6.2 Decomposition of ICI Coefficient Matrix

It is difficult to tell the effect of ICI directly from Eq. (3.2) and Eq. (3.5). To simplify matrix $\mathbf{S}$, it is crucial to analyze ICI coefficient $S(l,m)$ and the three parameters therein: $l$, $m$ and $\varepsilon$. After decomposition, the ICI coefficient can be expressed as [50]:

$$S(l,m,\varepsilon) = \frac{\sin[\pi(l-m+\varepsilon)]}{NF \sin[\frac{\pi}{NF}(l-m+\varepsilon)]} \exp[j\pi \left(1 - \frac{1}{NF}\right)(l-m+\varepsilon)]$$

$$= \frac{1}{NF} \left[1 - \cos \left[2\pi(l-m+\varepsilon)\right] - j \sin \left[2\pi(l-m+\varepsilon)\right]\right]$$

$$= \frac{1}{NF} \left[1 - \exp[j2\pi(l-m+\varepsilon)]\right]$$

$$= \frac{1}{NF} \sum_{n=0}^{NF-1} \exp\left[j\frac{2\pi}{NF}(l-m+\varepsilon)n\right]$$

$$= \frac{1}{NF} \sum_{n=0}^{NF-1} \exp\left(j\frac{2\pi}{NF}ln\right) \exp\left(j\frac{2\pi}{NF}\varepsilon n\right) \exp\left(-j\frac{2\pi}{NF}mn\right) \quad (6.6)$$

It is now clear in (6.6) that $S(l,m,\varepsilon)$ is a summation of exponential products with each exponent only being a function of a single parameter of interest. Using vector and matrix notation, $S(l,m,\varepsilon)$ can be expressed as

$$S(l,m,\varepsilon) = \mathbf{F}^H(l,:) \cdot \mathbf{\Psi}(\varepsilon) \cdot \mathbf{F}(;m), \quad (6.7)$$

where

$$\mathbf{F}^H(k,:) = \frac{1}{\sqrt{NF}} \left[e^{j\frac{2\pi}{NF}k0}, e^{j\frac{2\pi}{NF}k1}, \ldots, e^{j\frac{2\pi}{NF}k(NF-1)}\right]_{1\times NF}, \quad (6.8)$$
\[
\Psi(\varepsilon) = \begin{bmatrix}
e^{j\frac{2\pi}{N_F}\varepsilon_0} \\
e^{j\frac{2\pi}{N_F}\varepsilon_1} \\
\vdots \\
e^{j\frac{2\pi}{N_F}\varepsilon(N_F-1)}
\end{bmatrix}_{N_F \times N_F},
\]

where \( F^H(k,:)= F(:,l) \) denotes the \( k^{th} \) row of \( F^H \), and \( F(:,l) \) is the \( l^{th} \) column of \( F \). It is clear that \( F \) is the normalized DFT matrix, while \( F^H \) is the normalized IDFT matrix. \( \Psi \) is an diagonal matrix \( \Psi = \text{diag}\{\psi_0, \psi_1, \ldots, \psi_{N_F-1}\} \) with diagonal elements (eigenvalues of matrix \( S \)) given by \( \psi_k = \exp\left(j\frac{2\pi k}{N_F}\right) \). It is important to note that \( |\psi_k| = 1 \) for all \( k \).

Therefore, the ICI coefficient matrix \( S \) can be written in the well-known eigen decomposition form as

\[
S = F^H \Psi F \tag{6.8}
\]

Now, the ICI coefficient matrix is decomposed to be the product of normalized IDFT matrix, a diagonal matrix and normalized DTF matrix. The unknown parameter \( \varepsilon \) only exists in the diagonal matrix \( \Psi \).

### 6.3 Analysis of SC-OFDM Performance with ICI Present

Due to the perfect ICR performance of the SC-OFDM system, we next analyze its performance with ICI present. Using the ICI coefficient matrix \( S = F^H \Psi F \), we revisit the expression in Eq. (3.14) in AWGN channel (\( H = I \) and \( W = I \)), and rewrite the received
SC-OFDM signal vector as

\[
\vec{D} = \vec{d} \mathbf{S} \mathbf{F}^H + \vec{n} \mathbf{F}^H \\
= \vec{d} \mathbf{F} (\mathbf{F}^H \Psi \mathbf{F}) \mathbf{F}^H + \vec{n} \mathbf{F}^H \\
= \vec{d} \Psi + \vec{n}'
\]  

(6.9)

with the received signal according to the \(k^{th}\) symbol \(d_k\) corresponds to:

\[
D[k] = d_k \psi_k + n'_k = d_k \exp(j \frac{2\pi}{N_F} \varepsilon k) + n'_k
\]  

(6.10)

Recall that \(|\psi_k| = 1\) for all \(k\), it is noted that the ICI effect on SC-OFDM data symbols \(\vec{d}\) is simply a (different) phase offset on each and every data symbol \(d_k\). Compared with an OFDM system under similar ICI conditions, SC-OFDM provides significantly better performance. This is due to the received OFDM signal vectors being a combination of subcarrier data symbols and shifted responses thereof, while the subcarrier data symbols in the SC-OFDM signal vector given by Eq. (6.10) only experience a phase offset—this is why we observe zero ICR for all \(\varepsilon\) and the benefits of SC-OFDM are realized.

### 6.4 ICI Immune Modulation for SC-OFDM: Magnitude

#### Keyed Modulation

After observing the ICI coefficient property, we find that \(\mathbf{F} \mathbf{S} \mathbf{F}^H\) is a diagonal matrix with each diagonal element having unit magnitude. Hence, the ICI has no effect on the magnitude of each and every SC-OFDM data symbol. Therefore, when there is no noise present. Eq. (6.10) shows that \(|D[k]| = |d_k|\) independent of \(\varepsilon\). To fully exploit the inherent ICI immunity in SC-OFDM, we introduce a novel digital modulation scheme called Magni-
tude Keyed Modulation (MKM) [51][50]. Specifically, MKM will only use the magnitude to carry digital symbols. For example, binary MKM (2MKM) is equivalent to binary On-Off Keying (OOK). Note that MKM is different than Amplitude Shift Keying (ASK) using antipodal signal pairs given that MKM is a non-coherent modulation scheme and doesn’t require phase reference.

According to Eq. (6.10), the decision of the \( k \)-th data symbol can be easily made for SC-OFDM using MKM:

\[
\hat{d}_k = |D[k]| \tag{6.11}
\]

### 6.4.1 BER Performance Analysis

For 2MKM, the BER performance is exactly the same as OOK with non-coherent detection given by

\[
BER = P(\hat{x}_k = 1|x_k = 0)P(x_k = 0) + P(\hat{x}_k = 0|x_k = 1)P(x_k = 1)
= \left[ Q_1 \left( 0, \sqrt{SNR} \right) + 1 - Q_1 \left( 2\sqrt{SNR}, \sqrt{SNR} \right) \right] / 2 \tag{6.12}
\]

where \( Q_1 \) is the Marcum \( Q \)-Function [52] defined as

\[
Q_M(\alpha, \beta) = \frac{1}{\alpha^{M-1}} \int_{\beta}^{\infty} x^M e^{-(x^2+\alpha^2)/2} I_{M-1}(\alpha x) dx \tag{6.13}
\]

where \( I_n(x) \) is a modified Bessel function of the first kind [53]. It can also be written in series form as

\[
Q_M(\alpha, \beta) = e^{-(\alpha^2+\beta^2)/2} \sum_{k=1-M}^{\infty} \left( \frac{\alpha}{\beta} \right)^k I_k(\alpha \beta) . \tag{6.14}
\]

For \( L \)-MKM, the same process is used to derive the Symbol Error Ratio (SER) per-
formance as:

\[
SER = \sum_{m=0}^{L-1} P(\hat{x}_k \neq m | x_k = m) P(x_k = m)
\]

\[
= \frac{1}{L} \sum_{m=0}^{L-1} P(\hat{x}_k \neq m | x_k = m)
\]

\[
= Q_1(0, 0.5\lambda)/L
\]

\[
+ \frac{1}{L-2} \sum_{m=1}^{L-2} \{ (1 - Q_1 [m\lambda, (m - 0.5)\lambda] + Q_1 [m\lambda, (m + 0.5)\lambda]) / L
\]

\[
+ \{1 - Q_1 [(L-1)\lambda, (L-1 - 0.5)\lambda]\} / L
\]

(6.15)

where \( \lambda = \frac{\text{Signal Amplitude}}{\sqrt{\text{Noise Power}}} = \sqrt{\frac{12 \cdot \text{SNR} \cdot \log_2(L)}{(L-1)(2L-1)}} \), and the resultant MKM BER can be approximated by Eq. (6.16) when assuming Gray Code symbol assignment [54] [55]:

\[
BER \approx \frac{SER}{\log_2(L)}
\]

(6.16)

### 6.4.2 PAPR Performance Analysis

Since one of the important benefits of an SC-OFDM system is a much lower peak to average power ratio (PAPR) when compared with conventional OFDM, it is necessary to analyze the PAPR performance for SC-OFDM with MKM. This is done using one particular definition of discrete PAPR of an OFDM or SC-OFDM symbol: the maximum amplitude squared divided by the mean power of discrete symbols in the time domain [56].

Given time domain symbol vector \( \vec{s} = [s_0, s_1, ..., s_{N-1}] \), with maximum amplitude of \( \| \vec{s} \|_\infty = max(|s_0|, |s_1|, ..., |s_{N-1}|) \) and mean power of \( \| \vec{s} \|_2^2 = (|s_0|^2 + |s_1|^2 + ... + |s_{N-1}|^2)/N \), the PAPR of \( \vec{s} \) is

\[
PAPR = \frac{\| \vec{s} \|_\infty^2}{\| \vec{s} \|_2^2}.
\]

(6.17)

For an OFDM system with BPSK modulation, when the signal in time domain con-
verges to one peak (e.g., in frequency domain \(x_k = (-1)^k\)), the worst PAPR is obtained and equals \(N\). However, for single carrier systems such as SC-OFDM with MPSK (BPSK, QPSK, etc.) modulation, the maximum amplitude squared equals to the mean power in the time domain and therefore \(\text{PAPR} = 1 \ll N\). Unlike SC-OFDM with MPSK modulation, the SC-OFDM system with MKM cannot retain the \(\text{PAPR} = 1\) feature since the magnitude (amplitude) varies for different symbols in time domain. However, as shown next the SC-OFDM system with MKM has a much lower PAPR than an OFDM system with either PSK or MKM.

To compare the PAPR for different systems, we analyze the Cumulative Distribution Function (CDF) of the PAPR defined in Eq. (6.17) and given by

\[
P(PAPR \leq z) = CDF(z)
\]

and provide simulated CDF plots of PAPR in Figure 6.2, where PAPR of OFDM with QPSK is overlapped with PAPR of OFDM with 8PSK. These results are based on Monte Carlo simulation with \(10^5\) trials using \(N_F = 256\) total subcarriers and configurations that included OFDM and SC-OFDM systems with various combinations of BPSK, 2MKM, QPSK, 4MKM, 8PSK, and 8MKM modulations as indicated. The minimum and maximum values in the plots, along with average PAPR, are presented in Table 6.1. The metrics “Minimum”, “Maximum” and “Average” in Table 6.1 indicate the smallest, largest and average observed PAPR in the simulation, respectively. In Figure 6.2, the “Minimum” PAPR denotes the largest value for CDF is zero, the “Maximum” PAPR denotes the smallest value for the CDF is one.

The results in Table 6.1 clearly show that the SC-OFDM system consistently has the lowest PAPR, and that all combinations of SC-OFDM with MPSK modulation maintain a \(\text{PAPR} = 0\) dB for all \(M\). For combinations with higher modulation order \((M = 4\) and \(M = 8\)), i.e., SC-OFDM with MKM and OFDM with both MKM and PSK, PAPR is non-
zero and the OFDM systems always produce a higher PAPR for any given modulation type and order. Comparing results for a given modulation order, SC-OFDM with MKM always results in a lower PAPR relative to the corresponding OFDM system using either PSK or MKM.

### 6.4.3 Analysis in Multipath Fading Channel

In multipath fading channel, reconsider the demodulated signal vector $\vec{D}$:

$$\vec{D} = \vec{d} F HF^H + \vec{n} F^H$$

$$= \vec{d} F H (F^H \Psi F) F^H + \vec{n} F^H$$

$$= \vec{d} F H F^H \Psi + \vec{n}'$$

(6.19)
Table 6.1: Comparison of PAPR (dB) for Different System Configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM+BPSK</td>
<td>13.76</td>
<td>4.38</td>
<td>7.34</td>
</tr>
<tr>
<td>OFDM+2MKM</td>
<td>22.15</td>
<td>19.78</td>
<td>21.06</td>
</tr>
<tr>
<td>SC-OFDM+BPSK</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SC-OFDM+2MKM</td>
<td>4.59</td>
<td>2.01</td>
<td>3.02</td>
</tr>
<tr>
<td>OFDM+QPSK</td>
<td>12.54</td>
<td>5.29</td>
<td>7.81</td>
</tr>
<tr>
<td>OFDM+4MKM</td>
<td>22.80</td>
<td>21.27</td>
<td>22.16</td>
</tr>
<tr>
<td>SC-OFDM+QPSK</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SC-OFDM+4MKM</td>
<td>5.42</td>
<td>3.07</td>
<td>4.11</td>
</tr>
<tr>
<td>OFDM+8PSK</td>
<td>12.24</td>
<td>5.29</td>
<td>7.81</td>
</tr>
<tr>
<td>OFDM+8MKM</td>
<td>23.03</td>
<td>21.92</td>
<td>22.53</td>
</tr>
<tr>
<td>SC-OFDM+8PSK</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SC-OFDM+8MKM</td>
<td>5.70</td>
<td>3.45</td>
<td>4.48</td>
</tr>
</tbody>
</table>

Hence, using the same property of ICI coefficient matrix, the decision of the data symbols $\hat{\vec{d}}$ can be determined by

$$
\hat{\vec{d}} = \arg\min_{\hat{\vec{d}}} (||\hat{\vec{d}}FHF^{H}| - |\vec{D}||^2)
$$

which means the optimal $\hat{\vec{d}}$ is that which can minimize the cost function $||\hat{\vec{d}}FHF^{H}|| - ||\vec{D}||^2$. Since this procedure is similar to a multi user detection (MUD) and exhausted search algorithm is applied, the complexity is much higher compared the decision procedure in Eq. (6.11) for AWGN channel.

It is obvious that since the digital information is only carried on the magnitude, and the ICI effect does not change the magnitude of the received signal, an SC-OFDM with MSK modulation is immune to ICI. In other words, no matter how large the ICI is, the SC-OFDM with MSK will provide identical performance at the same SNR. Although the
BER performance of the SC-OFDM system with MSK is worse than that of the SC-OFDM system with PSK (phase shift keying) modulation when there is no ICI, the proposed MSK modulation offers SC-OFDM significantly better performance in a severe ICI environment.

6.5 Performance of SC-OFDM system with MKM

The BER performance of the proposed SC-OFDM system with MKM modulation is examined. Specifically, we compare performance of 1) SC-OFDM with binary MKM versus OFDM/SC-OFDM/MC-CDMA with BPSK modulation, 2) SC-OFDM with 4MKM versus OFDM/SC-OFDM/MC-CDMA with QPSK, and 3) SC-OFDM with 8MKM versus OFDM/SC-OFDM/MC-CDMA with 8PSK/8QAM, under conditions consistent with a high speed mobile environment [50, 51].

6.5.1 AWGN Channel

Firstly, we analyze the BER performance in AWGN channel with constant carrier frequency offset. The labeling convention for plotted BER results in the following figures is as follows: green line with circle markers–OFDM with PSK modulation; blue line with triangle markers–SC-OFDM with PSK modulation; purple line with diamond markers–MC-CDMA with PSK modulation; red rectangular–SC-OFDM system with proposed MKM modulation; cyan dot line–analytical performance for SC-OFDM system with proposed MKM modulation. Yellow line with stars–OFDM with 8QAM modulation; blue line with diamonds–SC-OFDM with 8 QAM modulation; black line with triangles–MC-CDMA with 8QAM modulation in Figure 6.4(b). Performance of the baseline OFDM system using PSK modulation without ICI present is shown as the black line with dot markers, where theoretical BER performance is illustrated as baseline in Figure 6.3 and Figure 6.4 [55][56], and simulation BER performance is illustrated as baseline in Figure 6.6.
Figure 6.3: AWGN Channel: BER vs. SNR for OFDM & SC-OFDM with binary modulations, \( N_F = 64 \) subcarriers, and \( \epsilon = 0.3 \).

Figure 6.3 shows simulated BER versus SNR for OFDM, SC-OFDM and MC-CDMA systems with binary modulation, \( N_F = 64 \) subcarriers, and AWGN channel conditions. These results were generated for normalized CFO of \( \epsilon = 0.3 \). With high NFO \( \epsilon = 0.3 \), OFDM/SC-OFDM/MC-CDMA systems with BPSK modulation break down, and the proposed system outperforms these benchmarks significantly when SNR is high (\( \geq 4dB \)).

Figure 6.4 shows simulated BER versus SNR for OFDM, SC-OFDM and MC-CDMA systems with 4MKM, QPSK, 8MKM, 8PSK and 8QAM modulations, \( N_F = 64 \) subcarriers, AWGN channel conditions, and NFO values of \( \epsilon \in [0.1, 0.2] \). When compared with Figure 6.3 results which show that the benefit of SC-OFDM with 2MKM is realized for \( \epsilon = 0.3 \) at all SNR, results in Figure 6.4(a) show that SC-OFDM with 4MKM outperforms other configurations when \( \epsilon = 0.2 \) and SNR \( \geq 6.0 \) dB (the other two systems are virtually unusable under these same conditions). A similar trend is observed in Figure 6.4(b).
Figure 6.4: AWGN Channel: BER vs. $\varepsilon$ for OFDM & SC-OFDM using $N_F = 64$ subcarriers with indicated modulation order and $\varepsilon$ values.
results which show that SC-OFDM with 8MKM provides an advantage for $\varepsilon = 0.1$ and $SNR \geq 10.0 \text{ dB}$ while the other systems are again unusable.

![Figure 6.5: AWGN Channel: BER vs. $\varepsilon$ for OFDM & SC-OFDM using $N_F = 64$ subcarriers, $SNR = 10 \text{ dB}$, and indicated modulation order.](image)

Final simulated AWGN results are presented in Figure 6.5 which shows BER versus NFO $\varepsilon$ for OFDM/SC-OFDM/MC-CDMA using $N_F = 64$ subcarriers, $SNR = 10 \text{ dB}$, $\varepsilon \in [0.1, 1.1]$ (including fractional and integer components), and binary, 4-ary, and 8-ary modulation orders. These results illustrate that the BER performance of SC-OFDM with 2MKM, 4MKM, and 8MKM remains constant as $\varepsilon$ increases, while the BER performances of traditional OFDM/SC-OFDM/MC-CDMA systems with BPSK, QPSK, 8PSK and 8QAM modulations degrade significantly and catastrophically (approaches 0.5 in the worst cases).

By comparing Figure 6.3 to Figure 6.5 results, it is apparent that when $\varepsilon$ increases or
higher order modulation is used, the SC-OFDM system with the newly proposed MKM modulation significantly outperforms all OFDM/SC-OFDM/MC-CDMA systems using conventional PSK/QAM modulations. More specifically, SC-OFDM with MKM maintains nearly identical BER performance independent of $\varepsilon$ variation while OFDM/SC-OFDM/MC-CDMA with PSK/QAM are very sensitive to changes, especially when using higher order modulations.

It is important to note that in these simulations we assumed that $\varepsilon$ was a small fractional number, consistent with a residual CFO contribution that may remain after some types of cancellation or estimation processing have been applied in a PSK/QAM system. It is obvious from our results that OFDM/SC-OFDM/MC-CDMA systems using PSK/QAM are virtually useless when this residual $\varepsilon$ exists. However, we have demonstrated that the SC-OFDM system with MKM modulation maintains nearly constant performance regardless of the fractional $\varepsilon$ value and without requiring any additional processing. As a final note of validation, Figure 6.3 and Figure 6.4 provide theoretical BER performance for comparison with simulated results for proposed system (SC-OFDM with MKM). As evident in both figures, theoretical and simulated performances are equivalent, which validates the analytic BER results for SC-OFDM with MKM, specifically, SC-OFDM with binary MKM in Eq. (6.12) and SC-OFDM with L-ary MKM in Eq. (6.16).

6.5.2 Multipath Fading Channel

As a measure of Doppler frequencies, we use the normalized maximum Doppler spread $\varepsilon_{\text{max}}$, which is defined as the ratio between the channel maximum Doppler spread to the subcarrier bandwidth. We use 4-fold multipath fading channel in Eq. (4.12) as our channel model.

Simulated BER performances for a multipath fading channel are provided in Figure 6.6 for OFDM/SC-OFDM/MC-CDMA systems with binary modulation, $N_F = 16$ subcarriers, and $\varepsilon_{\text{max}} = 0.4$. The observations here are consistent with previous AWGN results:
BER performance in multipath fading channel when $\epsilon_{\text{max}} = 0.4$

Figure 6.6: Multipath Fading Channel: BER vs. SNR for OFDM & SC-OFDM with binary modulation, $N_F = 16$ subcarriers, and $\epsilon_{\text{max}} = 0.4$.

1) SC-OFDM with the newly proposed 2MKM modulation is the most robust combination and virtually unaffected by $\epsilon_{\text{max}}$, and 2) when SNR is high ($\geq 18\, \text{dB}$), performance for the OFDM/SC-OFDM/MC-CDMA systems with conventional BPSK modulation is poorer and a BER floor ($\approx 10^{-2}$) is observed.

### 6.6 Conclusion

In this chapter, we analyze the effect of ICI on an SC-OFDM receiver and propose a novel modulation scheme called Magnitude-Keyed Modulation (MKM) to use with an SC-OFDM system. Taking advantage of unique ICI coefficient matrix properties, we showed that the ICI effect on a received SC-OFDM signal is simply a phase offset on each and every data symbol, while the magnitude of the data symbol is unaffected. Hence, by trans-
mitting digital information only on the SC-OFDM signal magnitude, we develop a novel modulation scheme called MKM and apply it to an SC-OFDM system. The resultant SC-OFDM system with MKM modulation experiences a boost in ICI immunity and significantly outperforms traditional OFDM, SC-OFDM and MC-CDMA systems using Phase Shift Keying (PSK) modulation and Quadrature Amplitude Modulation (QAM) in severe ICI environments. Simulation results are presented for SC-OFDM with binary, 4-ary, and 8-ary MKM modulations and the performance of each configuration compared with traditional OFDM/SC-OFDM/MC-CDMA using PSK/QAM modulation. Results for both AWGN and multipath fading channels clearly demonstrate that SC-OFDM with MKM is superior and much less BER degradation is observed as normalized carrier frequency offset and normalized Doppler spread increase.
Blind Carrier Frequency Offset Estimation

Besides the ICI cancellation schemes and ICI immune modulation design, we can also compensate the carrier frequency offset after estimating the carrier frequency offset (CFO) to improve the performance of the system. Meanwhile, the estimated CFO can be directly used as the compensation for a few consecutive blocks in a slow changing environment, e.g., the CFO does not change very fast.

Many methods in literature have been proposed to estimate the CFO for OFDM system. Generally speaking, existing CFO estimation schemes can be classified as the data-aided estimators and the blind ones. Recently, blind estimators have received a lot of attentions due to the system power and the high bandwidth efficiency. The blind estimator in [57] proposes an estimation algorithm based on maximum likelihood criterion, which exploits the cyclic prefix preceding the OFDM symbols to estimate the CFO. The MOV estimator [19] utilizes minimum output variance to estimate CFO. [58] also presents a non-data-aided CFO estimator according to minimum power of the received symbol. In [59] and [18], they estimate CFO by exploiting the key idea of smoothing power spectrum (SPS). The subspace method in [60] employs the correlation of the channel. Authors in [17] derive a kurtosis CFO estimator by measuring the non-Gaussian property of the received signal.

However, all existing blind CFO estimation schemes are not without their drawbacks.
Some of these estimators are proposed for constant modulus (CM) constellation, some of them require large number of OFDM blocks to reach acceptable estimation performance, some others need the channel order much less than the number of subcarriers and also the performances of most blind estimators are not as good as desired.

Recall *ICI Total Cancellation* technique discussed in Chapter 5, which treats an OFDM or MC-CDMA system with ICI as a new MC-CDMA system with unknown spreading code that is related to or determined by the carrier frequency offset $\varepsilon$. We propose to quantize the normalized frequency offset region into $M$ discrete values, leading to $M$ spreading code matrices as candidates [61]. Next, by decoding the received signal using these $M$ spreading code matrices, $M$ decisions are made on the data symbols. Using these $M$ data symbols to recreate the received signal with ICI and measuring the Euclidean distance of the $M$ recreated signals with the actual received signal, the best normalized frequency offset is chosen and the best corresponding data symbols are determined. By iteratively decreasing the possible normalized frequency offset region, we can refine the estimation accuracy.

### 7.1 High Accurate Blind CFO Estimator

Recall the discrete system model, the received signal vector can be expressed as:

$$\tilde{R} = \tilde{d}HS + \tilde{n}$$

where $\tilde{R}$ denotes the received signal vector; $\tilde{d}$ is the transmitted symbol vector; fading matrix $H$ is a diagonal matrix $H = \text{diag}(H_0, H_1, ..., H_{N_F-1})$; matrix $S$ is the ICI coefficient matrix defined in Eq. (3.5).

Let $\hat{\varepsilon}$ denote a candidate estimate of the CFO $\varepsilon$. Using $S^H(\hat{\varepsilon})$ to compensate the CFO,
and the de-correlated signal vector $\tilde{Y}$ in Eq. (5.3) can be represented as:

$$\tilde{Y} = \tilde{R}S^H(\tilde{\varepsilon})H^{-1} = \tilde{d}HS(\varepsilon - \tilde{\varepsilon})H^{-1} + \tilde{n}S^H(\tilde{\varepsilon})H^{-1} \quad (7.1)$$

Since $\tilde{Y}$ can be reconstructed by $\tilde{\varepsilon}$, we can make decision of $\hat{X}$ based on $\tilde{Y}$: $\hat{d} = \text{Decision}(\tilde{Y})$. For example when BPSK modulation used,

$$\hat{d}(k) = \text{Decision}(Y(k)) = \begin{cases} 1 & \text{when } \text{Re}[Y(k)] > 0 \\ -1 & \text{when } \text{Re}[Y(k)] \leq 0 \end{cases} \quad (7.2)$$

Next, we can reproduce the received signal $\hat{R}$ by using the data vector estimation $\hat{d}$ and the ICI coefficient matrix $S(\tilde{\varepsilon})$:

$$\hat{R} = \hat{d}HS(\tilde{\varepsilon}) \quad (7.3)$$

It is easy to understand that when there is no noise and if the CFO is compensated perfectly, we should have $\hat{R} = \tilde{R}$. Simply speaking, when $\tilde{\varepsilon}$ is the closest to the true value of $\varepsilon$, the reproduced the received signal $\hat{R}$ should also be the closest to the received signal vector $\tilde{R}$. Hence, the cost function is [61]:

$$J(\tilde{\varepsilon}) = ||\hat{R} - \tilde{R}]||^2$$

$$= ||\hat{d}HS(\tilde{\varepsilon}) - \tilde{R}||^2$$

$$= ||\text{Decision}(\tilde{Y})HS(\tilde{\varepsilon}) - \tilde{R}||^2$$

$$= ||\text{Decision}(\tilde{R}S^H(\tilde{\varepsilon})H^{-1})HS(\tilde{\varepsilon}) - \tilde{R}||^2 \quad (7.4)$$

where $||\hat{R} - \tilde{R}||^2$ represents the Euclidean distance between vector $\hat{R}$ and vector $\tilde{R}$.

Different cases for the cost function $J(\tilde{\varepsilon})$ versus estimation error $(\tilde{\varepsilon} - \varepsilon)$ are shown in
Figure 7.1: Cost Function $J(\hat{\varepsilon})$ versus $(\hat{\varepsilon} - \varepsilon)$ when BPSK applied

Figure 7.1, and we can notice that, the minimum cost function can be achieved when the estimation error $(\hat{\varepsilon} - \varepsilon) = 0$ when there is no noise, and in the reasonable SNR region, the minimum also occurs when estimation error $(\hat{\varepsilon} - \varepsilon)$ is pretty small. Hence, by minimizing the cost function $J(\hat{\varepsilon})$, the CFO estimation $\hat{\varepsilon}$ can be achieved.

To search the optimum $\hat{\varepsilon}$ to minimize the cost function $J(\hat{\varepsilon})$, we can quantize the CFO reasonable region into $M$ discrete values at one iteration and search for the minimum region, and then iteratively using $M$ quantization values to search for more accurate minimum region. Specifically,

1. Set initial $\varepsilon$ range for the $it^{th}$ iteration, $[L, U]$, e.g., $L^0 = 0$ and $U^0 = 1$;

2. Compute $M$ branches:

$$\varepsilon_m = L^it + m \frac{U^it - L^it}{M - 1}, \quad m = 0, 1, \ldots, (M - 1) \quad (7.5)$$
Figure 7.2: Estimation MSE v.s. number of iterations when $SNR = 10dB$ and $M$ fixed

3. Find the optimal $\varepsilon_l$ which produces the minimal cost function in Eq. (7.4):

$$J(\varepsilon_l) \leq J(\varepsilon_k) \quad k = 0, 1, \ldots, (M - 1)$$ (7.6)

4. $it = it + 1;$

5. Renew the minimal range:
   - If $l = 0$, $L^{it} = \varepsilon_0$ and $U^{it} = \varepsilon_1$
   - If $l = M - 1$, $L^{it} = \varepsilon_{M-2}$ and $U^{it} = \varepsilon_{M-1}$
   - If $0 < l < M - 1$, $L^{it} = \varepsilon_{l-1}$ and $U^{it} = \varepsilon_{l+1}$

6. Stopping criteria:
If the range has already reach the predefined precision $\delta$, e.g., $U_{it} - L_{it} \leq \delta$, where $\delta > 0$, then stop and $\hat{\varepsilon} = (U_{it} + L_{it}) / 2$;

- Otherwise, go back to step 2.

It is important to note that the complexity of the proposed CFO estimator is linearly growing with the quantization level $M$, keeping the computational complexity at reasonable range. For each OFDM symbol, this estimation algorithm requires $2M$ matrix multiplications and $M$ comparisons in one refining iteration. The increased complexity is not significant, especially when $M$ is small. Figure 7.2 illustrates the mean square error (MSE) of CFO estimator for different $M$ and different number of iterations. It is clear that we don’t need to use a huge $M$ or a lot of iterations to achieve the best performance. In all the cases, $M = 20$ with 3 iterations is good enough to provide pretty small estimation MSE. The proposed CFO estimator aims to minimize the reconstructed error $\|\hat{R} - \tilde{R}\|^2$ and this estimator is named as “Minimal Reconstructed Error” (MRE) estimator.

### 7.2 Carrier Frequency Offset Estimation Performance

Now, we use numerical simulation results to illustrate the effectiveness of the proposed CFO estimation scheme [61]. We compare mean square error (MSE) simulation results defined as following in both AWGN channel and multipath fading channel.

$$MSE = E[|\hat{\varepsilon} - \varepsilon|^2] \approx \frac{1}{P} \sum_{p=1}^{P} |\hat{\varepsilon}_p - \varepsilon|^2$$ (7.7)

where $\hat{\varepsilon}_p$ is the estimated CFO and $P$ is number of Monte Carlo runs.

In the simulation results, we compare our proposed MRE estimation scheme with Kurtosis [17], MOV [19] and SPS [59][18] estimation schemes. All the systems are assumed to have $N_F = 64$ subcarriers and simulation runs $P = 1000$, and in all simulations we use
$M = 20$ and 3 iterations to achieve the optimization. The number of OFDM blocks is set to be 1.

### 7.2.1 AWGN Channel with a Constant Frequency Offset

First, we will examine the estimation performance in AWGN channel with constant frequency offset. Figure 7.3 illustrates the simulation result when the normalized frequency offset (NFO) $\varepsilon = 0.3$ in AWGN channel, and the modulation scheme for all these estimators is QPSK. Figure 7.4 shows the case when $\varepsilon = 0.4$ with 16QAM modulation. In Figure 7.5 and Figure 7.6, the estimation MSE versus $\varepsilon$ is shown for QPSK with $SNR = 10dB$ and 16QAM with $SNR = 15dB$ respectively.

![Figure 7.3: Estimation Performance v.s. SNR with QPSK modulation in AWGN channel ($\varepsilon = 0.3$)](image)

In all these figures, the blue line shows the MSE for Kurtosis estimation scheme, the green line marked with circles represents the performance of SPS estimator, the magenta
Figure 7.4: Estimation Performance v.s. SNR with 16QAM modulation in AWGN channel ($\varepsilon = 0.4$)

line marked with diamond represents the MSE for MOV estimation and the red line marked with stars represents that of our proposed MRE estimation algorithm.

From Figure 7.3, it is evident that when QPSK modulation is applied, the proposed MRE estimation outperforms all the other estimators. While employing 16QAM modulation in Figure 7.4, the benefit of MRE estimation is much more significant. When we fix the $SNR$ at a reasonable high level and plot the MSE performance versusing the normalized CFO $\varepsilon$, e.g., in Figure 7.5 and Figure 7.6, the gain from MRE schemes is apparent and the MSE keeps the same for different $\varepsilon$. Hence, it is obvious that the proposed MRE estimation algorithm outperforms the Kurtosis, MOV and SPS estimation schemes in AWGN channel, and MRE estimator can be used for non CM constellation modulation schemes (e.g., 16QAM).
7.2.2 Multipath Mobile Channel

In a practical mobile multipath radio channel, time-variant multipath propagation leads to Doppler frequency shift which is a random variable. Here we measure the performance of the proposed blind CFO estimation scheme in multipath fading channels. As a measure of Doppler frequencies, we use the normalized maximum Doppler spread $\varepsilon_{\text{max}}$, which is defined as the ratio between the channel maximum Doppler spread to the subcarrier bandwidth. We use the Hilly Terrain (HT) channel models defined by the GSM standard as our channel model. The total number of subcarriers $N_F$ is also assumed to be 64.

Figure 7.7 illustrates the simulation result when the $\varepsilon_{\text{max}} = 0.3$ in multipath fading channel. The modulation scheme for all these estimators is QPSK. Figure 7.8 shows the case when $\varepsilon_{\text{max}} = 0.4$ with 16QAM modulation. In Figure 7.9 and Figure 7.10, the MSE versus $\varepsilon_{\text{max}}$ is shown for QPSK with $SNR = 10dB$ and 16QAM with $SNR = 15dB$. 

Figure 7.5: Estimation Performance v.s. $\varepsilon$ with QPSK modulation in AWGN channel ($SNR = 10dB$)
Figure 7.6: Estimation Performance v.s. $\varepsilon$ with 16QAM modulation in AWGN channel ($SNR = 15dB$) respectively.

The legends in these figures are the same as in AWGN channel. Figure 7.7 compares MRE scheme with other three estimators, and it is evident that the proposed MRE estimator significantly outperforms the other three estimators, especially at high SNR. At high SNR region, the MSE continues to decrease for MRE scheme, while the MSE of other estimation schemes keep flat indicating a performance floor at $MSE \approx 10^{-4}$. In Figure 7.8 when 16QAM modulation is applied, the MSEs of three benchmark estimators suffer performance floor at $MSE = 10^{-2}$, while the performance of proposed MRE scheme outperforms all of the other three schemes significantly and reaches $10^{-7}$ MSE when $SNR = 30dB$. The performance gain can also be noticed in Figure 7.9 and Figure 7.10 when $SNR$ is fixed, and the estimation performance of MRE scheme does not change while $\varepsilon$ changes.
Figure 7.7: Estimation Performance v.s. SNR with QPSK modulation in multipath fading channel ($\varepsilon_{\text{max}} = 0.3$)

7.3 Conclusion

In this chapter, we proposed a novel accurate blind CFO estimation scheme based on minimum reconstruction error (MRE) for mobile OFDM systems. Taking advantage of the orthogonality of the ICI coefficient matrix, the proposed estimation scheme for OFDM system can be applied as MC-CDMA system and provide accurate CFO estimation by minimizing the reconstruction error. The proposed blind CFO estimator provides pretty good estimation performance compared to the estimators in the literature, and it doesn’t have some requirements like other blind estimation schemes (e.g., constant modulus, large number of blocks and smaller channel order). Finally, proposed CFO estimation scheme achieves such superb performance at a reasonable computational complexity which linearly grows with the number of normalized frequency offset quantization.
Figure 7.8: Estimation Performance v.s. SNR with 16QAM modulation in multipath fading channel ($\varepsilon_{max} = 0.4$)
Figure 7.9: Estimation Performance v.s. $\varepsilon_{\text{max}}$ with QPSK modulation in multipath fading channel ($SNR = 10dB$)
Figure 7.10: Estimation Performance v.s. $\varepsilon_{max}$ with 16QAM modulation in multipath fading channel ($SNR = 15dB$)
ICI Total Cancellation with Subcarrier

Varying Carrier Frequency Offset

All existing ICI mitigation methods assume the frequency offsets on all subcarriers are the same. This is a valid assumption for narrowband multi-carrier transmission, since the Doppler shift on one subcarrier is almost the same as that on another subcarrier and the difference in Doppler shifts is negligible. But this assumption is no longer true for a wideband or non-contiguous multi-carrier system where the frequency variance is so large that the Doppler shift on the lowest frequency subcarrier is significantly different from the Doppler shift on the highest frequency subcarrier. This is particularly important for applications with very high mobility such as aerial vehicle communication [62].

Doppler frequency shift is defined as

$$\Delta F = \frac{v}{c}(f_c) \cos \theta \quad (8.1)$$

where $v$ denotes the speed and $\theta$ represents relative angle between transmitter and receiver, $c = 3 \times 10^8 m/s$ is the speed of light. It is important to note that different subcarriers are operating at different frequencies. Hence, different Doppler frequency shifts happen on different subcarriers, and varying carrier frequency offset has to be considered for the wideband or non-contiguous multi-carrier transmission, which will have large frequency
difference for these subcarriers.

8.1 System Model for Subcarrier Varying Carrier Frequency Offset

Figure 8.1: Normalized Frequency Offsets for Different Subcarriers
For wideband or non-contiguous OFDM systems in high mobility environment, it is necessary to consider the scenario that different subcarriers suffer different carrier frequency offsets according to the frequency of each subcarrier. Figure 8.1 shows the NFO for different subcarriers. The NFO is a linear function of the frequency, and the slope is proportional to the relative speed between the transmitter and receiver. It is also easy to understand that in a narrowband OFDM system, the NFO can be assumed to be a constant. For example, in a 64 subcarriers narrowband OFDM system shown in Figure 8.1(a), the NFO varies from 0.1 to 0.1012 over all 64 subcarriers, and the difference is negligible. Hence, a constant NFO 0.1 is assumed in current OFDM ICI mitigation techniques. However, for the wideband OFDM system, e.g., an OFDM with 512 subcarriers shown in Figure 8.1(b), the NFO changes 10% from 0.1 to 0.11 which cannot be ignored. More importantly, although the effective bandwidth of NC-OFDM for CR might not be large, the active subcarriers in NC-OFDM can spread over extremely large bandwidth to harness multiple spectrum holes together. For such NC-OFDM systems, the NFO in one subcarrier could be significantly different from that in another subcarrier. For example in Figure 8.1(c), when there are only 64 subcarriers in the NC-OFDM system occupying four non-contiguous spectrum holes, the NFO changes 20% from 0.1 to 0.12, although the effective bandwidth is as small as that in Figure 8.1(a).

Considering the subcarrier varying CFO scenario, the received wideband OFDM signal on subcarrier \( k \) in AWGN channel is [62]:

\[
R[k] = d_k S_{\varepsilon_k}(0) + \sum_{l=0, l \neq k}^{N-1} d_l S_{\varepsilon_l}(l - k) + n_k, \quad k = 0, 1, \ldots, N_F - 1
\]  

(8.2)

where \( S_{\varepsilon_l}(l - k) \) is defined as

\[
S_{\varepsilon_l}(l - k) = \frac{\sin(\pi(\varepsilon_l + l - k))}{N_F \sin(\pi/2 \varepsilon_l + l - k))} \cdot \exp \left( j \pi \left( 1 - \frac{1}{N_F} \right) (\varepsilon_l + l - k) \right)
\]  

(8.3)
ε_l denotes the normalized carrier frequency offset on l^{th} subcarrier, where ε_l = \frac{ΔF_l}{Δf}, and
ΔF_l = \frac{v}{c}(f_c + lΔf) \cos θ is the frequency offset on l^{th} subcarrier.

Now, denote vector \vec{d} as the transmitted symbol \vec{d} = \{d_0, d_1, \ldots, d_{N-1}\}, vector \vec{R} as the received signal vector \vec{R} = \{R[0], R[1], \ldots, R[N_F-1]\}, and \vec{n} = \{n_0, n_1, \ldots, n_{N_F-1}\}, we have:
\vec{R} = \vec{d}S + \vec{n} \tag{8.4}

where S is the ICI coefficient matrix, and the p^{th} row and q^{th} column element of N × N matrix S is
S_{p,q} = S_{ε_l}(p - q) \tag{8.5}

and the matrix S corresponds to
\[
S = \begin{bmatrix}
S_{ε_0}(0) & S_{ε_0}(-1) & \ldots & S_{ε_0}(1-N_F) \\
S_{ε_1}(1) & S_{ε_1}(0) & \ldots & S_{ε_1}(2-N_F) \\
\vdots & \vdots & \ddots & \vdots \\
S_{ε_{N_F-1}}(N_F - 1) & S_{ε_{N_F-1}}(N_F - 2) & \ldots & S_{ε_{N_F-1}}(0)
\end{bmatrix} \tag{8.6}

From Eq. (8.4), it is obvious that the received signal can be viewed as an MC-CDMA signal with N_F users, the k^{th} user’s information symbol is d_k, and the k^{th} user’s spreading code is the k^{th} row of matrix S.

It is important to note that, if there is no difference between the NFO on each subcarrier, S is an orthonormal matrix (SS^H = I, where S^H denotes the transpose conjugate of S and I is an identity matrix); with the existence of the difference between NFO on each subcarrier, S is no longer orthonormal matrix, however, it is still invertible. Hence, the OFDM signal with ICI at receiver side can be considered as an MC-CDMA system with spreading code matrix S. As a direct result, the ICI can be totally removed from the OFDM
signal if we apply a matrix multiplication to the received signal vector $\vec{R}$:

$$\vec{Y} = \vec{R}S^{-1} = \vec{d} + \vec{n}S^{-1} \quad (8.7)$$

where $S^{-1}$ presents the inverse of matrix $S$. Next, we can simply make decision of $\vec{d}$ based on the $\vec{Y}$.

Of course, the problem is: the receiver does not know the spreading code matrix $S$ because the normalized frequency offsets $\epsilon_k (k = 0, 1, ..., N_F - 1)$ are unknown.

### 8.2 ICI Total Cancellation for Subcarrier Varying CFO

Here, we propose to use parallel processing with speed quantization to eliminate ICI on wideband OFDM [62]. Compared to the Total ICI cancellation algorithms discussed in Section 5, we have to quantize the relative speed instead of the normalized frequency offset.

While the normalized frequency offsets $\epsilon_k (k = 0, 1, ..., N_F - 1)$ are unknown to the receiver, these $N_F$ unknown variables are determined by only one parameter: relative speed $V = v \cos \theta$ between the transmitter and receiver. Hence, we can quantize the relative speed $V$ in a typical range $[V_{\min}, V_{\max}]$ into $M$ equally spaced values:

$$V'_m = V_{\min} + m \cdot \Delta V, m = 0, 1, ..., M - 1 \quad (8.8)$$

where $\Delta V$ is the quantization level of relative speed $\Delta V = \frac{1}{M}(V_{\max} - V_{\min})$, and $M$ is the number of quantization levels. Since the typical $\epsilon_0 \in [0, 1]$, the typical speed range can be easily computed since $\epsilon_0 = \frac{Vf_c}{c\Delta f}$; meanwhile, with the support from navigation systems, e.g., GPS, the rough speed estimation helps to shorten the searching range. One of these $M$ quantized $V'$s is the closest to the true relative speed $V$.

Now, let’s build $M$ parallel branches at the receiver. Each branch uses one of the
quantized $V'$s to create the corresponding NFOs for all the subcarriers, and generate ICI coefficient matrix $\tilde{S}$ in Eq. (8.6). Hence, we have $M$ ICI coefficient matrices $\tilde{S}_0, \tilde{S}_1, \ldots, \tilde{S}_{M-1}$.

Using these $M$ matrices, we can have $M$ decisions on the transmitted data vector $\tilde{d}$ where the $m^{th}$ branch will make decision on the estimation of $\tilde{d}$, e.g., BPSK modulation:

$$\hat{d}_m = \text{sgn}\{\text{Re}[\tilde{R}\tilde{S}_m^{-1}]\} \tag{8.9}$$

where $\text{sgn}(d)$ presents the sign of $d$, and $\text{Re}[\cdot]$ denotes the real part of the complex number.

Next, with the data vector estimation $\hat{d}_m$, each branch can reproduce the received signal $\hat{R}_m$ by using the data vector estimation $\hat{d}_m$ and the ICI coefficient matrix of that branch $\tilde{S}_m$:

$$\hat{R}_m = \hat{d}_m \tilde{S}_m \tag{8.10}$$

It is easy to understand that the one branch whose $V'_m$ is the closest to the true value of $V$ should reproduce the received signal $\hat{R}_m$ also the closest to the received signal vector $\tilde{R}$. Hence, we only need to calculate and compare the Euclidean distances between the $M$ reproduced received signal vectors $\hat{R}_m$ and the received signal vector $\tilde{R}$ and pick the one with the minimum distance to be the best branch and use that branch’s estimated data vector as the final decision:

$$\hat{d} = \left\{ \hat{R}_p | \text{Dis}(p) \leq \text{Dis}(m); \forall \, m \neq p \right\} \tag{8.11}$$

where $\text{Dis}(m) = ||\hat{R}_m - \tilde{R}||^2$ represents the Euclidean distance between vector $\hat{R}_m$ and vector $\tilde{R}$. Figure 8.2 illustrates the Euclidean distance versus the relative error $(V' - V)\frac{\varepsilon_f}{\Delta f} = (\varepsilon'_0 - \varepsilon_0)$ for different SNR, and it is clear that when there is no noise, $||\hat{R}_m - \tilde{R}||^2$ reaches to the optimum when $V' = V$ or $\varepsilon'_0 = \varepsilon_0$. Meanwhile, at high SNR, the optimum also occurs when $|\varepsilon'_0 - \varepsilon_0|$ is very small.
It is important to note that the complexity of the proposed ICI cancellation method is linearly growing with the quantization level $M$, keeping the computational complexity reasonable. Furthermore, the $M$ ICI coefficient matrix $\tilde{S}_m (m = 0, 1, \ldots, M - 1)$ and the inverse of these matrices only depend on the quantized speed which can be pre-computed and stored. For each OFDM symbol, the proposed ICI cancellation scheme requires $2M$ matrix multiplications and $M$ comparisons. The increased complexity is insignificant, especially when $M$ is small.

The block diagram of the proposed ICI cancellation scheme is shown in Figure 8.3.

In a multipath fading channel, let’s denote the complex fading gain on the $k^{th}$ sub-carrier is $H_k$. Then the received OFDM signal after transmission through such a fading channel with frequency offset is:

$$\tilde{R} = \tilde{d}H\tilde{S} + \tilde{n}$$ (8.12)
where $H$ is a diagonal matrix $H = \text{diag}\{H_0, H_1, \ldots, H_{N_F-1}\}$.

Similar to the analysis in AWGN channel, the received OFDM signal represented in equation (8.12) can be viewed as an $N_F$ user MC-CDMA system with spreading code matrix $S$ and the $k^{th}$ user’s data symbol is $H_k d_k$. Hence, if the spreading code matrix $S$ is known, we can eliminate the ICI by multiplying $S^{-1}$ to the received vector $\hat{\mathbf{r}}$.

So the ICI cancellation scheme works the same way as in AWGN channel with only one exception: the fading channel characteristics $H$ needs to be estimated at the receiver side (which is required for OFDM transmission) and the reproduced received signal vector now has to consider the fading effects:

$$\hat{\mathbf{r}}_m = \hat{d}_m H \hat{\mathbf{S}}_m \quad (8.13)$$
8.3 Performance of ICI Total Cancellation with Subcarrier Varying CFO

Now, we will use numerical simulation results to show the effectiveness of the proposed scheme [62]. We provide BER simulation results for the proposed ICI cancellation scheme for wideband OFDM system in both AWGN channel and multipath fading channel. All system are assumed to have $N_F = 512$ subcarriers and employ BPSK modulation.

8.3.1 AWGN Channel

The simplest way to examine the effectiveness of the proposed scheme is to transmit signals through an AWGN channel with carrier frequency offsets ($\varepsilon_k$ maintains the same and $\varepsilon_k \neq \varepsilon_l$ when $k \neq l$). Figure 8.4 illustrates the simulation result when the normalized carrier frequency offset $\varepsilon_k$ varies from $\varepsilon_{min} = 0.1$ to $\varepsilon_{max} = 0.1998$ for different subcarriers and Figure 8.5 shows the case when $\varepsilon_k$ varies from $\varepsilon_{min} = 0.1498$ to $\varepsilon_{max} = 0.3494$. $M = 20$ quantization levels are applied for the ICI Total Cancellation scheme. In both figures, the black dots correspond to the BER performance of wideband OFDM without ICI, the blue line is the performance for wideband OFDM with $\varepsilon_k$ varying from $\varepsilon_{min}$ to $\varepsilon_{max}$ for different subcarriers; the green line marked with triangles represents the performance for the case $\varepsilon_k = \varepsilon_{max}, (k = 0, 1, ..., N_F - 1)$, and the red line marked with rectangles represents the performance for the case $\varepsilon_k = \varepsilon_{min}, (k = 0, 1, ..., N_F - 1)$; the purple line marked with circle is that of our proposed ICI Total Cancellation scheme.

It is clear that the BER performance for wideband OFDM with $\varepsilon_k$ varying from $\varepsilon_{min}$ to $\varepsilon_{max}$ has better performance than that of the OFDM with all the subcarriers suffering the same NFO $\varepsilon_{max}$, and worse performance than that of OFDM with all the subcarriers suffering the same NFO $\varepsilon_{min}$. Meanwhile, when the $\varepsilon_{min}$ increases or the range ($\varepsilon_{max} - \varepsilon_{min}$) increases, the BER performance of OFDM significantly degrades, but the ICI Total
Cancellation scheme eliminates the ICI and provides the same BER performance as that of an OFDM without ICI.

Figure 8.4: BER Performance in AWGN channel with \( \varepsilon \) varies from 0.1 to 0.1998 among subcarriers.

Figure 8.6 shows the BER performance versus NFO on the first subcarrier \( \varepsilon_0 = \frac{V_f c}{\Delta f} \). When \( \varepsilon_0 = 0 \) (no ICI), the proposed system has the same performance as traditional OFDM system. However, when the speed increases or carrier frequency offset increases, the performance of traditional OFDM system degrades significantly, while the ICI Total Cancellation algorithm helps the OFDM system maintain as good as there is no ICI.

8.3.2 Multipath Fading Channel

Here we measure the performance of the proposed ICI Total Cancellation method in multipath fading channels. As a measure of Doppler frequencies, we use the normalized maximum Doppler spread \( \varepsilon_{B_0} \), which is defined as the ratio between the channel maximum...
Doppler spread on the first subcarrier \((V f / c)\) to the subcarrier bandwidth \(\Delta f\).

Figure 8.7 shows the case when \(\varepsilon_{B_0} = 0.3\). In the ICI Total Cancellation scheme, we use \(M = 20\). In this figure, the black dots represent the BER performance of OFDM without ICI, the blue line represents the performance of OFDM with ICI, and the purple line marked with circle represents that of our proposed ICI Total Cancellation scheme. It is evident from this figure that the proposed ICI Total Cancellation technique entirely eliminates the effect of ICI and matches the performance of the OFDM without ICI in fading channels as well.

Figure 8.8 illustrates the effect of the number of quantization levels \(M\) on the performance of the proposed ICI Total Cancellation scheme. In this figure different quantization levels \((M = 1, 2, 4, 8, 16, 32)\) are applied and NFO varies from 0.15 to 0.3496 among these 512 subcarriers. In Figure 8.8, three BER versus \(M\) curves of different SNRs are shown. It
is easy to understand that when \( M \) increases, more quantization levels are used and better ICI coefficient matrix estimation is achieved, so the performance of the proposed scheme also improves. As shown in Figure 8.8, when \( M \) increases, the ICI Total Cancellation converges fast and provides ICI cancellation and BER improvement quickly. When \( M \) is larger than 8, there is no noticeable performance gain to increase the quantization level. This can be explained as the following: when the quantization step \( \Delta V \) is small enough, the ICI Total Cancellation’s ICI cancellation capability is enough to remove all the inter-carrier interference and there is no need to decrease \( \Delta V \) anymore. It is evident from Figure 8.8 that the computational complexity of the proposed scheme is reasonable.
8.4 Conclusion

In this chapter, we consider subcarrier varying carrier frequency offset scenario for wide-band multi-carrier transmission and non-contiguous multi-carrier transmission of CR in high mobility environment. We propose an effective algorithm to eliminate the ICI effect. To our knowledge, this work is the first to address such issue. Specifically, we apply the algorithm, called *ICI Total Cancellation*, for mobile wideband OFDM systems to mitigate the ICI effect and improve the BER performance. Taking invertible property of the ICI coefficient matrix, the proposed ICI cancellation scheme can eliminate the ICI experienced in wideband OFDM systems and provide significant BER improvement. The proposed scheme provides excellent performance without reducing data rate and bandwidth efficiency. Simulations over AWGN channel and multipath fading channel confirm the effectiveness of the proposed scheme. We also show that the proposed scheme achieves
superb performance at reasonable computational complexity which linearly grows with the number of normalized frequency offset quantization.
Conclusion

While multi-carrier transmission technologies become the natural candidates for the cognitive radio, most of current research in cognitive radio has not considered mobility. It is well known that all multi-carrier transmission technologies suffer significant performance degradation resulting from inter-carrier interference introduced by the carrier frequency offset in high mobility environments. It is highly desired to study the performance of cognitive radio waveforms in mobile environment and improve the performance through affordable low-complexity signal processing techniques.

In this dissertation, we analyze the ICI for SMSE based multi-carrier transmissions for CR, and propose multiple ICI mitigation techniques to reduce the inter-carrier interference and carrier frequency offset estimation algorithm to accurately estimate the CFO. Specifically, we have (1) proposed a novel spreading code for MC-CDMA system to embed ICI self-cancellation capability without extra computation and data rate reduction; (2) proposed a total ICI cancellation algorithm to entirely eliminate the ICI effect for OFDM and MC-CDMA systems, and recover the system performance as good as there is no ICI effect; (3) proposed a novel modulation, called Magnitude Keyed Modulation, to combine with SC-OFDM system and offers the system with ICI immunity feature, and the system performance will not be affected by the carrier frequency offset or inter-carrier interference; (4) proposed a blind carrier frequency offset estimator to accurately estimate the carrier frequency offset; and (5) analyzed the effect of subcarrier varying CFO scenario and proposed an ICI total cancellation algorithm to totally remove the inter-carrier interference.
Bibliography


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