2013

Shock Correlation Investigation in a Gaseous Fueled Axisymmetric Scramjet Flowpath

Jaime Omar Larios-Barbosa
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SHOCK CORRELATION INVESTIGATION IN A GASEOUS FUELED
AXISYMMETRIC SCRAMJET FLOWPATH

A thesis submitted in partial fulfillment of the
requirements for the degree of
Masters of Science in Engineering

By

JAIME O. LARIOS-BARBOSA
B.S., CALIFORNIA STATE UNIVERSITY LONG BEACH, 2009

2013
Wright State University
WRIGHT STATE UNIVERSITY
GRADUATE SCHOOL

July 29, 2013

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Jaime Omar Larios-Barbosa ENTITLED Shock Correlation Investigation in a Gaseous Fueled Axisymmetric Scramjet Flowpath BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering

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ABSTRACT

Larios-Barbosa, Jaime Omar. M.S. Egr, Wright State University, 2013. Shock Correlation Investigation in a Gaseous Fueled Axisymmetric Scramjet Flowpath

A pressure-based investigation was conducted on a gaseous fueled axisymmetric scramjet flowpath in Research Cell 22 located at Wright Patterson AFB. The investigation followed the work performed by Waltrup and Billig using combustion data to validate the form of the correlation in an attempt to achieve a good predictive tool for isolator length. Two years of axial wall pressure data using Mach values of 1.8 and 2.2 were analyzed in this investigation. The combustion data was shown to follow the same quadratic distribution show in Waltrup and Billig’s work with the data being shifted to the right relative to the correlation implying that the original correlation would have under predicted the length of the isolator. The data was then curve fitted to be able to generate new coefficients while maintaining the form of the quadratic equation developed by Waltrup and Billig resulting in a new equation that was in good agreement with the compiled data where the first order coefficient has a value of ~247 and the second order term a value of ~149. The conclusion being that the shock train length is independent of the process or mechanism used to backpressure the duct but significantly sensitive to the axial area distribution.
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M Mach number of inlet flow
St axial distance of pressure rise spread
Re₀ momentum thickness based Reynolds
D diameter at isolator inlet
Dₜ Hydraulic Diameter
Θ undisturbed flow momentum thickness
Pᵥ Isolator exit pressure
Pᵦ Isolator entrance pressure
Pₘₐₙₐ Maximum pressure
Tᵥ Isolator exit temperature
Tₐ Isolator entrance temperature
P₁ Static Pressure Upstream of Shock
P₂ Static Pressure Downstream of Shock
Pₜ Wall pressure
Pₜₒ Total Pressure
Pₕ Pressure Rise Fraction
ρₒ free stream density
uₒ free stream velocity
ρ(y) density as a function of wall distance
u(y) velocity as a function of wall distance
γ ratio of specific heats
R² Coefficient of Determination
σ variance
f(xi) theoretical data value
yᵢ actual data value
L Duct Length
L₀.₈ Length to reach 80% Pₕ
x axial distance
h facility nozzle exit height
Cᵣ₀ Skin Friction Coefficient
x̄ mean of the calculated values
xᵢ calculated values
ȳ mean of the actual values
yᵢ actual value
I. INTRODUCTION

Author’s Note

Fresh out of college, new second lieutenant, and newlywed all happening at back to back weekends and to add to the madness with my assignment to the Wright Patterson Air Force in hand, my wife and I head east to Dayton, OH. Upon my arrival to the then AFRL/RZAS Propulsion Directorate I hit the proverbial “fork on the road” and am given the option to choose between working on the widely encompassing turbine engines or the not so widely known “scramjet” engines. Being an aerospace engineer, and having favored space, I could not pass up the opportunity to work on the elusive scramjet. Extremely delighted to get some hands on work on scramjets, I find out that I am assigned to Research Cell 18 whereupon I realize the magnitude of the learning curve I have to overcome before I could be of any help to the researchers working on these wonderful ground test scramjet engines. After discovering that a scramjet is a supersonic combustion ramjet and being present at several experimental runs my breath of knowledge expands and I am moved to a bigger ground testing facility referred to as Research Cell 22 (RC-22) which is the source of the experimental data being analyzed in this thesis, all of which will be described in more extensive detail later in this thesis.
Background

A supersonic combustion ramjet (scramjet) is an air breathing engine, which unlike the ramjet where the airflow is subsonic, relies on combustion taking place in supersonic airflow. This in turn allows it to be a great propulsion system for air vehicles wanting to travel at hypersonic speeds (>Mach 5). Scramjets have several distinct advantages over other air breathing propulsion systems like a turbojet engines for example. One of those advantages is that scramjet engines have no moving parts which minimize the mechanical complexity and failure points. Also because it is an air breathing engine, though it travels at speeds greater than Mach 5, it does not have to carry its own oxidizer like a conventional rocket would. Lastly scramjets have a higher specific impulse or ISP than a conventional rocket engine making something like a single stage to orbit a plausible venture. For all of its great advantages it does suffer from one big drawback which is the requirement for supersonic airflow, which implies that it must be moving supersonically to begin with. Currently rocket boosters (like the ATCAM on the X-51) and dual mode engines have all been used to overcome this requirement of achieving supersonic speeds to function.

![Figure 1.1 Scramjet Components](image)
As Fig 1.1 illustrates a scramjet engine consists of an inlet, isolator, combustor and a divergent nozzle. The primary function of the inlet is to capture the air efficiently, compress the air and decelerate the incoming air to a suitable supersonic velocity. The combustor is where the fuel is injected and ignited to heat up and accelerate the air downstream to the nozzle where upon the air is further accelerated creating the thrust required for hypersonic flight. In the space between the inlet and the combustor lies the isolator which serves to contain the shock train that is created due to the back pressure the combustor is generating. This isolator and the resulting shock train is very important because if the isolator is too short for the length of the shock train then the shock will move forward into the inlet and cause an “unstart”. If the isolator is too long it will cause inadvertent weight and drag to the system. An unstart occurs when the shock train or pre-combustion shock train is moved too far forward as a result of heat release or back pressurization that the flow becomes choked and the flow diffuses. An unstart event is characterized by mass capture loss resulting in decreases thrust, combustor blow out and engine failure. The dynamics and isolator pressure rise will be further discussed as it is the main topic of this thesis.

Applications

As one may already imagine the benefits of scramjet engine technology can apply to various aerospace applications. There are many examples of the efforts being made by researchers across the world to understand and develop this technology because of the implications of maturing this capability. The first and most obvious application of this technology is its use for national defense purposes, more specifically missiles. In the
1960’s the Navy began looking into an exploratory program to develop and demonstrate scramjet technology necessary for flight in an internally ducted scramjet powered missile. The program became known as the Supersonic Combustion Ramjet Missile or SCRAM shown in Figure 1.2. SCRAM was supposed reach Mach 7.5 at an altitude of 100,000ft with liquid HiCal 3-D as its fuel. Due to the fact that it needed a large active seeker to acquire and intercept target autonomously at long ranges the program was terminated in 1977\textsuperscript{18}. The next concept was devised by J.L Keirsey of Applied Physics Laboratory (APL) which was a dual combustor ramjet capable of meeting what the SCRAM could not. One of the dual combustor ramjet powered missile concepts was called the Wide-

![Figure 1.2 Supersonic Combustion Missile System](image1)

Area Defense missile (WADM) is shown in Figure 1.3. Unfortunately the WADM program was terminated by congress in 1986. Recently in 2004 NASA made history by getting 10 seconds of hypersonic data when they tested and flew the X-43. The X-43 was a hydrogen fueled scramjet which was boosted up to speed by the Pegasus rocket that
reached Mach 10 in its voyage flight\textsuperscript{2}. In May of 2010 an experimental test vehicle named the X-51 WaveRider, a JP-7 fueled scramjet vehicle, flew over the pacific for 300 seconds at hypersonic speed on its own power which shattered the X-43’s 10 sec record holding flight\textsuperscript{2}. There was a total of 4 flights with one experiencing an unstart event and another having a broken control fin. The greater than Mach 5 speeds along with the transportability of a scramjet make the concept of a scramjet missile alluring and a highly desirable capability in the front lines. Even if the scramjet missile were to just use the kinetic energy, with no payload, it would still be a formidable capability to any adversary.

**Scope**

At Wright Patterson AFB there is a direct connect wind tunnel in RC-22 that has been conducting scramjet research for several years now and has generated a significant amount of data from an axisymmetric combustor with the significant difference from that found in the Waltrup & Billig\textsuperscript{19} paper being that all of the data attained was using a gaseous fueled combustor and not a backpressure valve. In this thesis, the RC-22 axisymmetric combustion data will be analyzed in a similar manner to that done by Waltrup and Billig\textsuperscript{19} in an attempt to validate that correlation for a combusting direct-connect facility. The objective is to verify the correlation that Waltrup and Billig developed in their work and apply it to the data attained using the gaseous fueled combustor in TC-22. If the correlation fits then combustion has no influence on the correlation and their work can be validated with in a combustion process but if it does not fit then the results will be analyzed to determine the differences and to determine if a quadratic distribution still is applicable.
II. PREVIOUS WORK

Previous Correlations

The idea of developing a correlation for the pressure rise in an isolator has been attempted for rectangular as well as round duct geometries due to the alluring simplicity of a quadratic equation being able to describe something as complex as the shock-boundary layer interaction in a scramjet. Though Waltrup & Billig might have pioneered this line of thinking, their work in developing the correlation only applied to round constant area ducts. As Eq.2 demonstrates their equation was modified slightly for applications toward constant area rectangular ducts, where $h$ is the facility nozzle exit height and $L_{1.0}$ is the at 100% pressure rise fraction.$^7$ The difference between the original equation and equation 2 is the power to which the Reynolds number is taken which is 0.2 versus 0.25. Also with regards to the geometry difference the boundary layer

$$P_f = \frac{P - P_1}{P_{max} - P_1}$$

Equation 3

in a rectangular duct has to deal with corners and the boundary layer properties like

\[
\frac{h_t (M^2 - 1) Re_{\theta}^{1/4}}{D^{1/2} \theta^{1/2}} = 50 \left( \frac{P_f}{P_a} - 1 \right) + 170 \left( \frac{P_f}{P_a} - 1 \right)^2
\]

Equation 1

\[
\left( L_{1.0} / h \right) \left( M_{1}^2 - 1 \right) Re_{\theta}^{0.20} / \left( \theta / h \right)^{0.5} = 50 \left( \frac{P_{max}}{P_1} - 1 \right) + 170 \left( \frac{P_{max}}{P_1} - 1 \right)^2
\]

Equation 2
thickness and momentum thickness differs depending on the wall you are observing. McLafferty\textsuperscript{16} came up with another correlation represented here by Eq. 3 & Eq. 4 based on the pressure rise fraction and the length to reach the 80% pressure rise fraction

\[ P = P_1 + 1.196 \left( P_{\text{max}} - P_1 \right) \left[ 1 - \exp \left( -1.3 \frac{x}{L_{0.8}} \right) - 0.05 \frac{x}{L_{0.8}} \right] \]

**Equation 4**

Sullins found that the length for the 80% pressure rise was better defined for a rectangular duct with aspect ratio of 2.5 by Eq. 5. When the Sullins and McLaffety\textsuperscript{16} along with the modified Waltrup and Billig equations were applied in a combustion back pressurization process fed by hydrocarbon fuel it was found that they both over predict the length of the isolator\textsuperscript{7}.

\[ L_{0.8} \int h = 5.3(M - 1) \]

**Equation 5**

An effort was made to develop a correlation that would be insensitive to duct geometry in TC-18 at the Air Force Research Lab. Three distinct isolators with equivalent cross-sectional areas were used along with a throttling valve for back pressurization in their investigation. The result of their investigation was the following a pressure and Mach number based formulation \( M_o^2 \left( P_{s-P_{s'}} / P_o \right) / (P_o / P_1) \). Interestingly enough the formulation that they developed does not explicitly include any boundary layer information just the nominal Mach number for the facility nozzle and pressures at specific locations in the isolator. The formulation described collapses the shock train pressure profiles of all three isolators, yet it had a bit of a dependence on isolator
configuration. Just like in this report the investigation done in RC-18 was conducted for two Mach numbers.

With regards to the pressure gradient in a duct, based upon large amounts of experimental data, Ortwerth found that the rate of pressure rise is directly proportional to the dynamic pressure and the skin friction coefficient at the initial point of separation in the duct, and inversely proportional to the duct hydraulic diameter as shown in Eq.6\(^{13}\). This diffuser model provides the user with the ability to determine the length scale over which the pressure rise is spread. Equation 6 became part of a much more complex quasi-one dimensional diffuser model for a real combustion flow involving a set of ODE’s and roughly eight different unknowns\(^{14}\). The mentioned correlations in this section all make an effort to describe the flow dynamics axially across the isolator section by taking into account as little information about the flow as possible. Which in some circumstances make the correlation unique to that specific geometry or isolator configuration, of which is useful for the users of the particular duct as a predictive tool and it helps describe the relationships between some parameters of the flow. Even with these advantages to the user these correlations hold little avail in the pursuit of developing a “universal” construct of the flow physics involved in the interworking of scramjets yet the hope is that the gained understanding, small as it may be, becomes a piece of a puzzle which is a fundamental understanding of scramjet physics.

\[
\frac{dp}{dx} \approx \frac{89}{D_h} C_{f0} \left( \frac{\rho V^2}{2} \right)
\]

Equation 6
Constant Area Duct

The idea to conduct this investigation came from the “Future work” section of an AFRL AIAA paper titled “Development and Calibration of an Axisymmetric Direct-Connect Supersonic Combustion Flowpath”\textsuperscript{15} which used the same ground testing facility used in this investigation. The AFRL researchers recognized the similarity to a paper by P.J Waltrup and F.S Billig (Waltrup-Billig) and the significance of validating the correlation resulting from Waltrup-Billig’s paper with real combustion data. In an AFRL journal paper, the objective was to characterize the newly built axisymmetric direct connect tunnel in which it was concluded that a constantly divergent isolator performed better than a constant area divergent isolator. This is the reason that the data presented in this investigation is of the constantly diverging isolator only. In the Waltrup-Billig paper, they investigated the structure of shock waves in a cylindrical duct for the purpose of understanding the dynamics in the isolator. The similarity between both investigations is the fact that they both used axisymmetric flow paths in their experimental set up. The key difference between them being the simple fact that one used combusting fuel to backpressure the isolator section and the other did not. Fortunately Waltrup & Billig developed a simple quadratic correlation based on inlet and exit isolator pressure ratio which at the time seemed like a simple universal approach to size an isolator for the predicted inlet conditions. Their approach was to use the direct connect ground testing
facility shown as a schematic in Fig 2.1. They applied two experimental techniques with the first being that the flow was allowed to exhaust to the atmosphere and the total pressure was adjusted to set the ratio of exhaust to inlet pressure and hence the shock location. The second technique fixed the total pressure and adjusted the valve in steps with data being acquired at each step until the shock moved so far upstream it resided in the facility nozzle. The resulting axial pressure profiles for a pre-determined Mach number at each step are shown in Figure 2.2. Each symbol in Figure 2.2 represents a
different total pressure condition along the axial direction. Waltrup & Billig did this for several inlet conditions shown in Table 1, but the reader’s attention should be directed to the column on farthest right of Table 1. It will either show atm or throttled where in the latter they used a butterfly valve to backpressure the isolator and in the former the total pressure conditions where changed according to the third column from the right in Table 1 labeled $P_{to}$ psia. The nozzle was operated below its designed stagnation conditions so they used a pitot pressure survey with a rake made up of seven equally spaced tubes to determine the inlet Mach number. With the above information Waltrup and Billig performed a detailed investigation into the flow physics within their constant area duct. Interestingly enough one of the conclusions of their research is a correlation of the inlet pressure over the backpressure to a normalized distance, shown in the previous section as Equation 1. This equation has been used extensively by other researchers to determine the required size of the isolator for a particular case study or design. Equation 1 is really at the core of this investigation and is the basis upon which the combustion data analyzed in this thesis will be compared to for congruency. The parameters of Equation 1 will be described in a later section and for future reference the left hand side of the equation (the

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<th>$D_{in}$</th>
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<th>$\theta_1$</th>
<th>$10^5$ ft</th>
<th>$Re_{p} \times 10^{-4}$</th>
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<td>0.51 – 1.04</td>
<td>3.13</td>
<td>0.655</td>
</tr>
</tbody>
</table>

Table 1 Waltrup-Billig Inlet Conditions
one with the boundary layer parameters) will be referred to as the correlation unless explicitly stated.

**Shock Train**

The name isolator arises from the fact that it serves the purpose of “isolating” the inlet operations from the downstream pressure rise due to combustion in the range of operational Mach numbers. The pressure rise through the isolator occurs gradually through a series of oblique shocks which are the result of a complex interaction between the strong normal shock wave and the boundary layer along the isolator walls. If the normal shock is strong enough to separate the boundary layer then the shock is bifurcated and a series of shocks occur downstream of the bifurcated shock and this is what is referred to as a shock train. Figure 2.3 shows a shadowgraph of a shock train in a constant area isolator where the darker lines running vertically are the shocks making up this shock train. Looking closely at the top and bottom walls of the isolator in Figure 2.3, the turbulent boundary layer and how the shocks reside between the two boundary layers is shown. From Figure 2.3, it is evident that the axial distribution of these shock waves and thus pressure rise determines the size of the isolator in a scramjet. Waltrup & Billig’s investigated the wave structure in a duct in more detail using pressure
measurements to obtain an understanding of the wave structure shown in Figure 2.4 depicting case 3 from Table 2. As Figure 2.4 shows, the flow at the end of the duct is still supersonic and the oblique shocks are bounded by the boundary layer. Waltrup & Billig then avoided finding a correlation which would reconstruct the shock structure in the duct and through some separation model output the initial wave angle because they felt it was more meaningful to use the static pressure data directly in a correlation while using other normalizing parameters consistent with the flow structure. They then
normalized the final static pressure or duct exit pressure, $P_f$, with the initial pressure before the pressure rise, $P_a$, and plotted them vs the total distance, $s_t$, over which the pressure rise occurs represented here in Figure 2.5. Each line in Figure 2.5 represents a different inlet Mach number in Waltrup & Billig’s constant area duct which is why they conducted a statistical analysis of the data in Figure 2.5 and concluded that $(M_a^2 - 1)^{-1}$ was the simplest functions which would collapse all of the data. Subsequently for the same $P_f/P_a$, $s_t$ was found to vary inversely with $Re_\theta$ and directly to the diameter and momentum thickness. Figure 2.6 represents the end goal of Waltrup & Billig’s research which was to come up with a simple quadratic equation based on the normalized static pressure data for different inlet condition and duct diameters. Figure 2.6 shows Equation 1 which was used to collapse all of the data into a more uniform distribution upon which a trend line was added based on the normalized static pressure. This same approach will be taken in this thesis to analyze the combustion data taken from the axisymmetric rig in RC-22 to investigate the effect of using a combustor to back pressure the direct connect wind tunnel to answer the question of the applicability of the Waltrup-Billig correlation in this environment. 

![Figure 2.6 Waltrup-Billig Correlation](image)
III. EXPERIMENTAL SET UP

Test Facility

Due to the high altitudes and extreme conditions scramjet vehicles experience, it is very difficult to conduct ground tests on a fully integrated system. Therefore what many researchers have done in the past to be able to simulate the conditions that scramjet would experience is to build either a semi free-jet, free-jet, or a direct connect wind tunnel. Figure 3.1 demonstrates the differences between each of the previously mentioned wind tunnels and as shown in Figure 3.1 each wind tunnel type encompass different aspects of the total system. Unlike typical wind tunnels where the test article is placed in a cross flow like in a free jet or semi free jet, in a direct connect wind tunnel 100% percent of flow resides within the simulated system where the inlet conditions are being provided by the facility nozzle Since a direct connect wind tunnel is used by AFRL in RC-22, its design will be discussed in greater detail. In a direct connect wind tunnel, the
inlet conditions are simulated but the isolator and combustor are physically reproduced. This means that there is no inlet to be able to raise the temperature and pressure like a real scramjet would do. To make up for this fact some direct connect facilities have something called a vitiator. The vitiator serves as an air preparation section by using a “torch” like device that uses compressed natural gas to heat up the incoming air. Just a bit downstream of this vitiator oxygen is added to the core flow to make up for the burnt O2 the torch used up. This process creates a bit of “dirty” air which mean that this “vitiated” inlet air is not 100% representative of the air a scramjet would see but that is part of the penalty researchers need to take in order to be able to reach the correct flight enthalpy conditions. Once the air is at the temperature and mass flow rate desired it moves downstream to a convergent divergent nozzle which accelerates the air isentropically to the desired flight Mach number at the isolator entrance.

At Wright-Patterson Air Force Base, an axisymmetric direct connect wind tunnel was modified from a rectangular cross sectional flow path to a round flow path at RC-22 and this axisymmetric scramjet combustor was investigated to evaluate its performance. The facility is a continuous-flow supersonic combustion research facility that is capable of simulating flight conditions from Mach 3 to Mach 7 by changing the stagnation temperature and pressure along with the converging-diverging facility nozzle. At
maximum capacity the facility can supply air at up to 30 lbm/sec, 750 psia, and 1660°F while employing a 3.0-psia continuous exhaust. Figure 3.2 illustrates the dimensions and physical component in this direct connect wind tunnel and in this figure the flow is from left to right. Compressed natural gas is used in the vitiator for the vitiation process and a liquid-oxygen system provides oxygen to the combustion-heated air flow. A cooling-water system provides 2500 gpm at 70 psia to aid in the thermal management of the components making up the rig. The typical run times in this ground testing facility range between 30-60 seconds due to the fact that combustor temperatures can reach up to 4,000 °F. The entire flowpath is secured to a thrust stand which directly measured the thrust generated by this direct connect wind tunnel. Figure 3.3 shows the complexity and distribution of the instrumentation involved in health monitoring and data acquisition during testing. The facility is operated from the safety of control room where personnel can monitor the tunnel using cameras and real time feeds from the instrumentation. Due to the fact that the hardware is cycled many times the facility is modular and can interchange sections relatively quickly, opening the possibility for a wide array of component level testing in this facility.
Experimental Layout

During the period of 2009-2011 AFRL conducted hundreds of tests in their efforts to attain a fundamental understanding of the complex physics involved in scramjet operations. Different fuel injection techniques and ignition methods were studied with some employing flow visualization techniques and other runs trying to use laser absorption methods to analyze the combustion species in real time. Yet the investigation conducted in this thesis focuses on the pressure distribution contained within the isolator section of the ground test facility as shown in Figure 3.4 by the red bracket. Figure 3.4 shows a facility Nozzle, which through the time span encompassed by the data, was interchanged between a Mach 1.8 and 2.2 nozzle. The specifics of the nozzles used are shown in Table 2. To attain the pressure data, wall static pressure taps at 0.5 inch spacing at 0, 120, and 240 degrees circumferentially were used spanning the length of the rig starting at the end of the optical calibrator and ending at the end of the 24’’ isolator section. It has been demonstrated that the pressure values are indeed symmetric.

<table>
<thead>
<tr>
<th>Facility Nozzle</th>
<th>M</th>
<th>D_throat</th>
<th>X_P_thrt</th>
<th>D_exit</th>
<th>X_P_exit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.8</td>
<td>3.4948</td>
<td>-15.800</td>
<td>4.370</td>
<td>-9.051</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>2.8824</td>
<td>-17.951</td>
<td>4.370</td>
<td>-9.078</td>
</tr>
</tbody>
</table>

Table 2 Nozzle Specifications
regardless of the circumferential position simplifying the choice of pressure location.\(^8\).

The combustor of the experimental facility is Thermal Barrier Coated (TBC) to be able to sustain the heat load experienced. This investigation focuses on looking at data from a combusting backpressure source, therefore though two combustors were used between the span of this data, the approach is to just look at the shock location and the pressure ratio between the min and max pressure in the isolator. At first glance the facility isolator section of interests (labeled B-D in Figure 3.4) seems to be of constant area but the truth is that the section in question is divergent as Figure 3.5 shows. This isolator section has a 0.25 degree divergence from the beginning to the end of the 12” section and then diverges again 0.25 degrees all the way to the beginning of the combustor, thus 250_250 isolator was set as a label to refer to this isolator layout. The only component that is of
constant area in this inlet is the optical calibrator or the section from A to B in Figure 3.5. Downstream of the 24” section is a circular combustor with injectors in the circumference shown in Figure 3.6. Lastly downstream from that are two possibilities with regards to flowpaths as shown in Figure 3.7. Either the flowpath will have an abrupt step then have a constant area or it will continuously diverge to the start of the exhaust adapter section. Though this investigation focuses on the events happening upstream of the combustor, the underlying assumption is that the flow dynamics happening in the isolator are governed by the value of the backpressure without any consideration of the physical mechanism being implemented to impose that backpressure. As this investigation will show in general the aforementioned holds true and it is for this reason that the comparison between combustion and valve as backpressure “devices” can be made, but through this investigation some examples showed that this was not universally true and it perhaps pertains to the option between a step or a constant divergence immediately downstream of the combustor. This will be further discussed in a later section.

![Figure 3.7 Downstream Combustor Options](image-url)
IV. PROCEDURE
   Approach

Figure 4.1 shows the pressure profiles for data obtained in the RC-22 tunnel in 2009 and the resemblance to that of Figure 2.2 is quite evident. The legend of Figure 4.1 shows the run identification given for that specific run and the appropriate symbols for that run. Unlike Figure 2.2 where the pressure distribution was set by a backpressure valve that was stepped closed creating each pressure profile, Figure 4.1 used a combustion process to set that pressure rise. To establish a way to quantify this backpressure generated by the combustor we use the fuel to air equivalence ratio (ER). The ER is the ratio of actual fuel to air to the stoichiometric ratio for a given fuel. In the case of TC-22 the direct connect facility used ethylene fuel through the time that t
used ethylene fuel through the time that the presented data was collected.

An initial attempt was made to reproduce Figure 2.5 by taking any pressure profile’s maximum pressure as Pf and the pressure right before the shock as Pa and placing that data point with its respective St value. This was a key step to determining the correlation since the curves in Figure 2.5 were collapse to achieve Figure 2.6. After conducting this for just a handful of runs the analysis hit a small problem which was that the curves in Figure 2.5 each represent a different Mach number but the data being used were all of the same Mach number, in this case Mach 1.8 and 2.2. Figure 4.2 shows a similar trend to Figure 2.5 of a “fanning” out effect form the origin caused by the fact that the plot represents different stagnation conditions though the same Mach nozzle is used.

<table>
<thead>
<tr>
<th>Case#</th>
<th>Mach#</th>
<th>To (R)</th>
<th>Po (Psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case#1</td>
<td>2.2</td>
<td>1950</td>
<td>80</td>
</tr>
<tr>
<td>Case#2</td>
<td>2.2</td>
<td>1950</td>
<td>105</td>
</tr>
<tr>
<td>Case#3</td>
<td>2.2</td>
<td>2200</td>
<td>105</td>
</tr>
<tr>
<td>Case#4</td>
<td>1.8</td>
<td>1500</td>
<td>55</td>
</tr>
<tr>
<td>Case#5</td>
<td>1.8</td>
<td>1800</td>
<td>55</td>
</tr>
</tbody>
</table>

Table 3 Inlet conditions

![M 1.8 & M 2.2 Data](image)
The cause of this wide dispersion in the data, though the Mach number is the same, is due to the fact that there are three different inlet conditions for the Mach 2.2 nozzle tabulated in Table 3 which affect the isolator entrance pressure.

The nature of this investigation was to analyze the data that had already been acquired through several years of experimental testing in an axisymmetric direct connect ground testing facility, but before that could be accomplished an understanding of the process needed to be attained. Therefore the best way to understand the process is to take the data Waltrup & Billig used and process it to get the same correlation output. Since all the actual tabulated data is not available to process directly, Figure 2.5 was blown up and every point available was extrapolated manually for its Pf/Pa values and its corresponding St values. The values from this process are shown in Tables 4 and 5 where the headings are the case number and the associated Mach number. Once the data was tabulated the next step was to apply Equation 1 to this data. However other parameters were needed to evaluate the data in the same manner as Waltrup & Billig. Fortunately Table 1 had the momentum thickness, momentum thickness based Reynolds
number, and the diameter for each case. This data was tabulated using excel and the value of the correlation was calculated for each point. Using Excel Pf/Pa vs the Waltrup-Billig Correlation (WB_Corr) was plotted and the outcome is shown in Figure 4.3. This is the same plot as Figure 2.6, which validates the normalization process for the Waltrup-Billig correlation. With the process now established and validated, all that was left was to process the combustion data in the same manner.

The combustion data comes from a series of tests conducted from 2009 to 2011 and it is available in its raw format until it is post processed after a test series. A brief explanation of the run identification is required for the readers benefit. An example of a run identification is F09062 where the red # represents the year in this case 2009 and the blue # is the 62nd day of the 2009 year which in this case was March 3rd. Each run night conducts a series of tests which start with the run identification of AA through which
ever run identification is needed for that night. For example if during a run night we get to the run identification AZ then 26 tests have been conducted in this run night series. The variables needed from the data were outputted into a user friendly excel spreadsheet format which could then be processed.

There was one minor hurdle, which did not become evident until it came time to apply the correlation to the combustion data, which was that there was no data for the momentum thickness and subsequently the momentum thickness based Reynolds Number. The momentum thickness for compressible flow is described by Equation 7.

The free stream velocity and density can be calculated from the instrumentation in the rig and inlet conditions, but the velocity and density profiles were not proved experimentally. Taking a look at how Waltrup & Billig attained the momentum thickness for their data it was made clear that a boundary layer code was used to determine these values. In this study Computational Fluid Dynamics (CFD) was used to provide the density and velocity profile. The inflow conditions such as the total temperature, total pressure and species concentration at the facility nozzle were used as the inputs for the CFD modeling. As previously mentioned, all of the data taken used the 250_250 isolator so the rest of the inputs to the CFD are listed in Tables 5 and 6 by case number where the Mach # column in Table 3 is the nozzle being used in the tunnel as shown in Figure 3.4. The underlying assumption in the CFD model is that the inlet nozzle has adiabatic walls with Table 3 as inputs for each case. The CFD model outputs the momentum thickness and various other
key variables at point A shown in Figure 3.4. Table 7 lays out all of the CFD variable outputs for each case. The velocity and density of the air could have been attained without the use of CFD but the variable that could not be attained without the use of CFD is the viscosity. Note that the viscosity is a turbulent viscosity and is essential to attain one of the key parameters of the correlation which is the momentum thickness based Reynolds number. Attempts were made to try to attain a fair estimate of the viscosity without having to rely on CFD but the high temperatures of up to 2,200°F made using Sutherland’s Formula an ineffective approach. Therefore, for consistency the density and velocity associated with the viscosity attained by CFD were used to ensure that the appropriate values are being used, therefore minimizing the error in the calculations. Due to the fact that Waltrup & Billig’s work also took advantage of the aid of computational resources, the use of CFD in this investigation does not skew the validation process. Having attained the value of the momentum thickness most of the key parameters to perform the arithmetic are now available.
Method

With the aid of CFD, most of the parameters needed to apply the correlation are at hand except for those in red shown here in Equation 8, which is essentially Equation 1 rearranged. Due to the fact that the typical run duration of the testing facility is roughly 30-60 seconds per test and the data collection begins before and ends after the tunnel is on condition, the data needed to be visually inspected to determine the distance over which the pressure rise occurred. The maximum pressure during a particular run was taken as \( P_f \) which for every run occurred at the exit of the isolator. Before being able to determine the value for \( P_a \) the shock location needed to be determined. Since there is no shadowgraph equipment for any of these runs to locate the shock visually like that seen in Figure 2.3 the need to rely on the instrumentation was necessary. During each run a

\[
\left( \frac{S_t}{D} \right) \left( \frac{M^2 - 1}{\text{Re}_\theta} \right)^{1/2} \left( \frac{P_f}{P_a} - 1 \right) + 170 \left( \frac{P_f}{P_a} - 1 \right)^2 = 50
\]

Equation 8

![Figure 4.4 Baseline Example](image)

which for every run occurred at the exit of the isolator. Before being able to determine the value for \( P_a \)
baseline pressure profile is attained, as depicted in Figure 4.4 and superimposed on to the baseline are the pressure profiles for a given run to give a graph like Figure 4.5. To extract the distance over which the pressure rise occurs, St, the characteristic shock pressure rise needs to be located. The problem of locating the exact point where the pressure begins to separate from the baseline would be the location of the shock which is not as simple as one would think. The data is post processed into a similar plot as Figure 4.5 with a line at 10% above the baseline pressure and the plot is visually inspected by making an imaginary line between the last point on the baseline and the following points in the pressure rise then seeing where this imaginary line between points intersects the 10% line and in that way the shock location is selected in this study. The choice for the 10% pressure rise to determine the shock location was one selected by the researchers in RC-22. There is no agreed upon nor widely accepted threshold found in the literature for the percent pressure rise to determine shock location but it was determined that 5% was
to susceptible to noise which would give rise to false positives and 15% was too high
given the distribution of data.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Iso/1950 To 105 Po</th>
<th>Iso/1950 To 80 Po</th>
<th>Iso/2200 To 105 Po</th>
<th>Iso/1500 To 55 Po</th>
<th>Iso/1800 To 55 Po</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach 2.2/250-250</td>
<td>2.29E+04</td>
<td>3.33E+04</td>
<td>2.16E+04</td>
<td>4.47E+03</td>
<td>5.58E+03</td>
</tr>
</tbody>
</table>

Equation 8 can be a bit misleading because it represents the result of the work
carried out by Waltrup & Billig for their specific tunnel, which makes the coefficients on
the right hand side of Equation 8 a result of applying the correlation. To achieve the right
hand side of Equation 8 each case (shown in Table 3) was considered independently and
the Pf/Pa was plotted versus the boundary layer parameters of the left hand side as will be
discussed in the next section. It is worth mentioning that in the time span of 3 years there
were 23 different runs that had the inlet conditions specified in Table 3 yet the inlet
conditions were not exclusive to each run. Meaning that other than the Mach number
(determined by the facility nozzle) the total pressure and temperature were changed
during a run. This is worth mentioning because as the reader will see in the next section
run identifications are repeated for separate inlet conditions thus will appear in different
graphs. The last parameter which has not been explicitly expressed so far is the
momentum thickness based Reynolds number which is shown in Table 8.
V. DATA

Results

The first case being presented is Case #1 (according to Table 3) which consists of a Mach 2.2 facility nozzle with the total temperature of 1950 °R and a total pressure of 80 psia seen here in Figure 5.1. The plot shows the normalized pressure in the Y-axis and the X-axis is the left hand side of Equation 8, which after doing some dimensional analysis

\[
\frac{P_2}{P_1} = \frac{2\gamma M^2 - (\gamma - 1)}{(\gamma + 1)}
\]

Equation 9

represents a normalized distance. The red line represents the maximum pressure ratio possible for the given Mach number (given by Equation 9). The \(P_2\) & \(P_1\) in Equation 9 are
the pressure after the shock and before the shock respectively which when calculated, using a gamma of 1.4, give a value of 5.48 for the Mach 2.2 case. The significance of Equation 10 and subsequently the red dashed line in Figure 5.1 is that it represents the pressure rise across a normal shock. With these inlet conditions the pressure ratio did not get close to the maximum in any of the Mach 2.2 runs. The solid black line, shown in the legend as WB_Corr, represents the quadratic given by the right hand side of Equation 9. If the data points were to fall on that line this would validate the correlation developed by
Watlrup and Billig with combustion data in an axisymmetric direct connect tunnel.

Figure 5.2 shows Case #5 which makes use of the Mach 1.8 facility nozzle and has the same format as Figure 5.1 but with a different maximum pressure ratio line shown here as a dashed line. The Max Pf/Pa for the Mach 1.8 facility nozzle is 3.61, which shows the reason for this analysis to determine why for the deviation from the WB_Corr line is happening. The reader might be wondering why there are some points from run F09175 that seem to fall off the distribution the other data points have taken, but this observation will be addressed later in this thesis. Next is Case #2 and as Figure 5.3 shows this.
condition was widely used through the time span the tests were conducted and therefore has the highest quantity of data points when compared with the other four cases. The last of the Mach 2.2 cases is Case #3 shown in Figure 5.4, which has the highest temperature out of all of the cases being presented in this report. Analyzing Figures 5.1, 5.3, and 5.4, it becomes evident that while using the Mach 2.2 facility nozzle the combustor never provides enough backpressure to reach the max pressure ratio represented by the red line in the figures. The last of the Mach 1.8 cases is shown in Figure 5.5. Having now presented the processed data for all of the conditions set forth earlier in Table 5 it is very encouraging to note that just by simply taking a quick glance at the data they all follow a similar distribution profile and by the comparison to the WB_Corr line it is safe to say that the distribution falls within a quadratic equation.
The fact that the distribution amongst all of the plotted data falls below the WB_Corr line for each case suggests that Waltrup & Billig’s correlation under predicts the size of the isolator. As previously established, the X-axis in the plots shown in the previous section describe a normalize distance and for comparison it represents a shock location where by as the X value increases the shock would be further upstream.

Therefore, for a specific Pf/Pa value the WB_Corr predicts that the shock will be farther downstream than it is in reality. For example in Case #2, the WB_Corr under predicts the size of the isolator section by an average of 19%, which means that if the WB_Corr were to be used to predict the size of the isolator used in the TC-22 wind tunnel for Case #2 it would have been 19% too small. The previous example demonstrates the specificity of the data presented in the previous section. To be able to speak about the data more
generically the data needs to be compiled in some way so that it will represent a more general case. Analyzing how Waltrup & Billig originally collapsed their data suggests that compiling the combustion data by Mach number would produce a better result.

Following this path, the Mach 2.2 data was analyzed which turned out to be a good choice as Figure 5.6 demonstrates. The data in Cases #1, 2, & 3 are represented in Figure 5.6 and the data closely follows the general distribution trend. Due to the fact that there

![Figure 5.6 Compiled Mach 2.2 Data](image)

are different cases being represented, Figure 5.6 implies that the boundary layer parameters normalize the data fairly well considering the only parameters that held their value throughout are not just the respective Mach number per data set but also the
diameter of the duct. Analyzing the compiled Mach 1.8 data in Figure 5.7, the previous point is validated. In general, the data follows the same trend as mentioned before, which is a quadratic distribution, but there are a couple of points that maybe outliers since they don’t fall within the distribution. Fortunately they are actually not outliers as will be explained later. Now that the Mach 2.2 and Mach 1.8 data have been compiled independently, it is logical that we compile all of this data into one plot and evaluate if the trend continues to be consistent to the one that has been seen with all of the previous plots. Figure 5.8 has the Mach 1.8 and Mach 2.2 data in one plot and it shows that the distribution trend still continues with variation in Mach #. The Mach 1.8 curve in Figure
5.8 shown in red, bows away from the WB_Corr curve and its own distribution profile at an X value of approximately 1600, but it was an expected effect of having reached the natural plateau of the maximum pressure distribution. This effect in the Mach 1.8 data makes it a bit difficult to place a best fit curve with all of the data because it will also try to fit the points at the high end of the Mach 1.8 data set. The initial approach then is to derive a best fit curve for the data set individually and see how far the coefficients are from each other. This will help evaluate the need to make a best fit curve for the entire
data set or if the evaluation of the data should remain as two separate equations, one for each Mach number. In an effort to remain consistent with the Waltrup & Billig correlation, it was desired to be able to place a line of best fit with the same form as Equation 5. The coefficients $C_1$ and $C_2$ for the Waltrup & Billig equation are 50 and 170, respectively; therefore having the ability to only vary these coefficients for the combustion data would allow a one to one comparison and a unique equation for the data in this thesis. To achieve this task the use of a powerful plotting program called KaleidaGraph was used because of its unique capability to place a trend line on a plot with a user specified format. Figure 5.9 shows the Mach 2.2 data with the trend line in the

$$\left(\frac{S_t}{D}\right) \left(\frac{M^2 - 1}{Re_x}\right)^{1/4} = C_1 \left(\frac{P_f}{Pa} - 1\right) + C_2 \left(\frac{P_f}{Pa} - 1\right)^2$$

Equation 10

Figure 5.9 Mach 2.2 Best Fit Curve
same format as Equation 10. The red box in the lower right hand corner of the plot
displays the format of the equation at the top like the following $Y = m_1*(M0-1)+m_2*(M0-1)^2$ where the $M0$ is the independent variable of $Pf/Pa$. The manner in which the data is plotted is reversed to what the actual relationship is according to the normal format of a quadratic equation, meaning that the axis are reversed to remain consistent with the original correlation found in Figure 2.6. The values of $m_1$ and $m_2$ correspond to the coefficients $C_1$ and $C_2$ of Equation 10 and lastly the program also outputs some statistical information like the $R^2$ value also known as the coefficient of determination. Having an $R^2$ value of $0.8668$ for the Mach 2.2 data gives good confidence for the trend line and subsequently the coefficients set by the KaleidaGraph program. The same process as the Mach 2.2 data was followed for the Mach 1.8 data and the result is shown in Figure 5.10. For the purpose of representing the proper data the outliers found in Figures 5.2 and 5.7 were taken out so that the trend line represents
Figure 5.11 Compiled Data Best Fit Curve

Figure 5.12 Data Sets Coefficients
the proper data and is not skewed by the present “outliers”. Lastly, Figure 5.11 demonstrates the trend line imposed to all of the compiled combustion data, which is the goal of this report. Having now placed a best fit curve in the three different compiled data sets shown here in Figure 5.12 it is worthwhile comparing the outputs to the original reworked data shown in Figure 4.3. For the original correlation, the statistical $R^2$ value are shown in the bottom right corner of Figure 5.13. Interestingly the coefficient of determination or $R^2$ value is 0.93574 and given that a $R^2$ value of 1.0 means that the trend line lies right on the data point, this value expresses a good fit. When looking at the three distinct data sets the absolute value for the C1 coefficient is actually an order of magnitude higher than the original one set by Waltrup & Billig, which makes sense since

![Graph](image)

**Figure 5.13 Waltrup & Billigs Original Data Correlation**

<table>
<thead>
<tr>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chisq</td>
<td>5.2359e6</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.93574</td>
</tr>
</tbody>
</table>
it is not the coefficient for the second order term.

Taking the coefficients in Figure 5.11 the equation that satisfies the combustion data for the axisymmetric direct connect tunnel in RC-22 is Equation 12. Though

$$s_t(M^2 - 1)Re\theta^\frac{1}{4} = \frac{Pf}{Pa} - 1 + 149.03\left(\frac{Pf}{Pa} - 1\right)^2$$

Equation 11

Equation 11 only has an approximate $R^2$ value 0.87, which is a good value for this data set, the error in the coefficients from the outputted value is acceptable. The error for the $C_1$ coefficient is 10.3% which seems pretty large but because it is associated with the first order term in the quadratic its impact is not very significant. The $C_2$ coefficient has an error of 5.9% which is just shy of the desired 5% error since it’s associated with the second order term. Looking at all of the information together the $R^2$ along with the error column the value of the coefficients agrees with the data. Since the goodness of fit is determined by the $R^2$ value it is worth defining this statistical parameter. KaleideGraph defines the $R$ (correlation coefficient) by equation 12. The square of the correlation coefficient is displayed in the previous graphs.

$$\sum \frac{(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sqrt{\sum (y_i - \bar{y})^2}}$$

Equation 12
Model Characteristics
The model described in this thesis has advantages and disadvantages associated to both its application and the flow physics it is representing. Though the method might seem fairly straightforward, it makes use of a significant amount of information. The best example would be the use of CFD to attain boundary layer information and viscosity. Though CFD is a powerful tool in aerodynamics it still has to make an effort to simplify the number of equations and variables it solves for by assuming values to certain parameters or processes. Other than the adiabatic wall assumption, the surface roughness distribution varies both circumferentially and along the axial direction which makes a true representation difficult to achieve. Even if at some point the surface roughness is mapped out for the duct as soon as the subsequent experiment is carried out the map would no longer be valid since the heat and air addition would cause an inadvertent minute change in the surface roughness thus changing the value of the viscosity which in turn would change the output value of the correlation.

The quasi one dimensional analysis presented here is possible due to the geometry of the duct and the circumferential agreement in the pressure data along the axial direction. This allows for the representation of the average axial pressure to be meaningful when being applied in the correlation. Alternatively the correlation would need to be modified, as mentioned in the second section of this thesis, to represent a rectangular cross sectional flowpath. This implies that the presented correlation is geometry dependent which does not limit the value of the correlation since the literature suggests that the more efficient and better performing geometry is a round one. As has
been shown, the correlation is applicable to diverging ducts since the inputs account for the divergence by the values of the parameters.

Even though Waltrup and Billig created this correlation over 40 years ago it still applies today. Due to the ability to post process the pressure data, researchers should make this correlation a part of their characterization because it not only provides the researcher with boundary layer information but it also accounts for a wide range of Mach numbers. The disadvantage is if there is no CFD support the facility would need to attain boundary layer information experimentally which would require more time and energy to probe the flow radially at different axial location to attain the boundary layer velocity profile. Also, the intent of the correlation is to be able to use it in a scramjet ground testing facility design but not necessarily a fully integrated scramjet. The inlet of an actual scramjet vehicle has a design Mach number and contraction ratio due to the shock wave structure at the inlet making any ground testing facility not a true representation of the flow characteristics the scramjet vehicles experienced. This implies that users should be warned that this type of correlation has not been conducted on actual flight data, yet its application is not limited to only ground testing facilities. If the data were at hand for a flight tested scramjet it would be interesting to see the agreement to this correlation.

There is one problem that arises with the construct of one equation to represent the presented data set which is that as soon as the value of Pf/Pa is approximately greater than 3.4 the Mach 1.8 data set splits and Equation 12 is no longer representative of those data points which is approximately 6% of that data set. This means that if the Mach 2.2 data were to continue to its maximum Pf/Pa value then it too would taper and the same
problem would arise which is that the quadratic would no longer represent the full data set. Therefore, it is up to the user to understand that if the value is within 10% of the maximum value (or red line in the above plots) the user is better served by going with the correlation quadratic for that individual data set then the correlation for the aggregate combustion data.

Discussion

Though the objective of this investigation was met by Equation 11, a fundamental question arises which is why doesn’t Equation 11 match Equation 8? To answer this question it is necessary to evaluate the differences between the backpressure valve tunnel Waltrup and Billig used to the combustion tunnel RC-22 uses. The first and most obvious observation is the fact that the backpressure methods are being compared but from the data it is evident that combustion does a good job at holding the back pressure even if it is a dynamic process. The combustion process can be eliminated as a culprit for the two equation coefficients not matching and furthermore due to the normalization of the duct diameter with the boundary layer parameters, the physical size of the duct can also be eliminated. The next step in the investigation leads us to examine the tunnel geometry along the axial direction. The tunnel used in Waltrup and Billig’s investigation was of constant cross sectional area along the axial direction but the RC-22 tunnel had a constant divergence of 0.25 degree from the inlet to the combustor which increased the cross sectional area 8.9 in² when it reaches the combustor. This divergence in the RC-22 tunnel creates a significant difference in the boundary layer and the correlation is heavily dependent upon the characteristics of the boundary layer. Figure 2.4, in the shock train
section, depicts what happened in the isolator to maintain a supersonic air stream at the end of the isolator as well as the cause for that shock train, which is boundary layer separation. The shock structure in the tunnel depends on the initial conditions necessary to separate the boundary layer meaning that if the conditions for boundary layer separation were to change then the correlation would look significantly different, which seems to be the case for the RC-22 data. A very pertinent conclusion of Waltrup & Billig is the fact that for a given Pf/Pa the distance over which the pressure rise is spread or better yet the shock location is inversely proportional to the \((M^2-1)Re_\theta^{1/4}\) and directly proportional to \(D^{1/2} \theta^{1/2}\). But, the experimental set up in the presented data called for a vitiator which raised the temperature of the incoming air flow. The higher temperature for the inlet conditions lowers the effective air density and subsequently the momentum thickness value increases since the free stream density is the denominator in Equation 3. This means that for a given Pf/Pa value the distance over which pressure rise occurs will be greater. This implies that for the constant area duct used by Waltrup and Billig, the shock location for a given Pf/Pa would be farther downstream when compared to that of the vitiated rig in RC-22. This can be seen in the data by the fact that combustion data falls below the WB_Corr line which means that for a given Pf/Pa ratio the WB_Corr line predicts a smaller correlation value than the combustion data describes. The resulting effect of the vitiation causes the distribution seen in the previous plots and answers the question of why the coefficients in Equation 12 differ from Equation 7. The other pertinent question concerns the outliers found in Figures 5.2 and 5.7, which significantly diverged from the quadratic trend set by the rest of the data. In the analysis section and
for the purpose of attaining a trend line for the compiled data these outlier points were disregarded since they would have skewed the analysis. The unedited version of the data as it was processed for the Mach 1.8 cases is shown in Figure 5.14. From a quick inspection of Equation 10, it is evident that the equation is only valid for a Pf/Pa>1 and a correlation value greater than zero. In Figure 5.14 the outliers lie below the quadratic distribution and have the form of solid symbols to tell them apart from the rest of the data. There is too much data not following the trend to suggest that these points are just outliers, especially with the care that is taken to set up each and every test. To understand what is happening it is necessary to turn to the pressure profiles created for each run during a run night. The run identifications that corresponds to the outlier data are F09194, F11174, and one data point belongs to F11286. The approach taken to understand what is
happening is first to identify two point having the same X-axis value but two different Pf/Pa values. Figure 5.15 is just a blown up section of Figure 5.14 that identifies two sets of points belonging to the same run night, F09194 that represent just the desired case to perform the investigation. The first task was to identify the specific runs that belong to the two data points which in this particular case are labeled “1” in Figure 5.15. The data point with the lower Pf/Pa value belongs to run F09194AD and the point with the higher Pf/Pa value belongs to the run F09194BS(3). With that information it was time to find the post processed pressure profiles to determine pertinent information. Figure 5.16 shows one full plot and part of the other plot being compared along with a lot of new information in one concise plot. Beginning at the top the plot has a rough schematic of the geometric representation of the tunnel flow path to give the reader an idea of where the pressure in the plot is located. With that being said the red upper triangles between the x/D values of 8 and 12 represent the combustion section which is not represented in any previous part of this report because this investigation was only concerned with the events occurring up to the end of the isolator and not the combustor. The Y-axis is a form of normalized pressure and the X-axis is a normalized axial distance by the duct diameter. Lastly the data distribution below the peaks in the plot represents the baseline pressure distribution upon which the shock location is determined. Below the phrase “AFRL F09194AD” in the legend of Figure 5.16 there is a set of information that is very pertinent to this investigation. PF/CI/SF stands for Primary Fueling, Cavity Injector, and Secondary Fueling and the fuel splits are shown right below. In this case 100% of the fuel was distributed between the primary and secondary
fueling sites which resided just upstream and downstream of the cavity respectively. The curve with the higher pressure distribution belongs to the F09194BS(3) run which has the following PF/CI/SF values 63%/5%/32%. The dashed line denotes the shock location. The shock locations for the top curve and bottom curve are the same yet the distance over which the pressure rise occurred is significantly different for both run I.D’s. This explains the difference in Figure 5.15 in example one since the pressure ratios are different yet the shock location is the same the correlation sets relatively the same length normalizations though the pressure ratios differ. The next step is to verify that the phenomenon that is happening is not just a fluke in the data. Applying the same process for example number two as that for the first example the two data run I.D’s for the second set of points are F09194AH and F0919BS(4). From Figure 5.17 and 5.18 it is clear that the same phenomenon is occurring between the two sets of data where the shock locations and distance over which the pressure rise occurs match but the pressure ratio differs. With these two examples and examination of the rest of the data this phenomenon is confirmed.
to be happening but what is causing this difference? From the conclusions of Waltrup and Billig’s work and the premise upon which this investigation is based on, for a specific length and duct geometry the relationship of Pf/Pa to shock location should be a one to one relationship. Meaning that for every Pf/Pa value there should be a unique shock location associated with it which makes the comparison of combustion data and a
backpressure valve possible since it is assumed that what is causing the backpressure should not matter as much as the value of the backpressure that is being provided. Figures 5.17 and 5.18 seem to be evidence against this premise but there is another aspect of this experimental set up that has not been discussed and that is the geometry of the flow path downstream of the combustor. As the reader may recall from Figure 3.6, the data was taken with only the 250_250 isolator but with either the step then constant area right downstream of the combustor or the constantly divergent flow path. The outlier data only happens when the step then constant area option is used downstream of the combustor, yet interestingly enough there are data points within this option that lie in the distribution set by the rest of the data as shown in Figure 5.19. Having now narrowed down the culprits involved with the outliers in the data, a more detailed observation is required to isolate the cause for the observed phenomenon. When the details of F09194AD and F09194BS(3) are compared side by side the only difference other than the value of Pf/Pa is the fueling distribution. This implies that when the fueling is concentrated or distributed to the secondary fueling site the inlet pressure and backpressure ratio will be lower for the same equivalence ratio but with a fuel distribution concentrated in the primary fueling site. Going any further would require a complete new line of investigation and would be out of the scope of this report.
Figure 5.19 Trend vs Outliers
VI. CONCLUSION

Waltrup & Billig conducted their work in a time where computational resources were not readily available to produce plots and trend line like those that were presented in this thesis. This explains why the coefficients for their equation were very good round numbers. The values for their correlation have an $R^2$ value of approximately 0.94 which is impressive since even with modern computational resources the correlation for the combustion data developed in this thesis only has a 0.86 $R^2$ value. It was observed in this report that the effect of vitiation or preheating the air flow has a significant effect on the boundary layer and subsequently the shock location. This in turn changed the distribution of the data points and as a result the coefficients in the equation. It can then be concluded the Waltrup & Billig quadratic equation does not apply to the combustion data acquired in the RC-22 direct connect facility. Interestingly enough though, taking the actual value of the coefficients aside, the fundamental relationships observed by Waltrup & Billig still apply to a vitiated constantly diverging direct connect wind tunnel. Also, though the coefficients are not the same, the combustion data is collapsible and the distribution of data follows the same trend Waltrup & Billig observed which resulted their relationship.

Future Work

From the Discussion section of this thesis the outlier data seems to suggest that the correlation is not complete and that the value of the shock location is not solely dependent on the inlet pressure to backpressure ratio value. The rapid area increase of the flowpath section immediately downstream of the combustor seems to not only help with pressure relief of the combustion process seen by the somewhat linear pressure rise
versus the bulge like pressure rise profile overlaid in Figure 5.16, but also with the amount of heat released to the combustion section. This suggests that the fuel distribution and physical geometry of the section immediately downstream of the combustor has a significant effect on the flow dynamics in the isolator section found upstream of the combustor. Furthermore if the shock location is associated with the heat released to the combustor this would suggest that the quadratic correlation requires another parameter that would be constant depending on the value of the heat released represented by the C3 term in Equation 13. The suggestion of a form similar to Equation 13 is merely conjecture since this would be the simplest way to differentiate between the “Normal” or trend observed here and the outlier data without changing the quadratic distribution.

Lastly the future of scramjets is headed in the direction of liquid fuels for its main fuel source and if the shock location is dependent on heat release it would be a good to have a similar investigation with liquid fuel data.

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VII. REFERENCES

   [Accessed 02 July 2013].

   [Accessed 02 July 2013].


