Computational and Experimental Investigations into Aerospace Plasmas

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Computational and Experimental Investigations into Aerospace Plasmas

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

William T. Bennett
B.S., Wright State University, 2006

2008
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION
BY William T. Bennett ENTITLED Computational and Experimental Investigations into
Aerospace Plasmas BE ACCEPTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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Investigations into two different fields of plasma research are presented here. These include the study of ion engine performance and the use of plasma discharges for flow control. In the area of ion engine performance, optimizing electron confinement is the primary goal of this work. The work of prior researchers was expanded through the study of the cathode emission location and the energy of the primary electrons. Cathode position was shown to have minimal effect on confinement length. For electron energy values greater than 20eV the effect on confinement length was also found to be very small. The strength of the magnetic field was also tested and compared with results from prior researchers. The results showed that for a magnet circuit that is already optimized, increasing the magnetic field strength through adding more magnets or using stronger magnets only decreases the confinement.

In the area of plasma actuators for flow control, the objective is to garner a qualitative understanding of both heating and the addition of forces to subsonic and hypersonic flows. This was done through the use of a commercially available CFD package. Results showed that for plasma discharges the dominant effect on surface pressure in the hypersonic regime is that of heating. Representative force sources showed some effect but were smaller. Subsonic computational studies showed that heating had no significant effect on the pressure distribution. Results for the force sources show that it is possible to get some small changes in the surface pressure through the use of a
sufficiently large force. Experimental results conducted in a subsonic wind tunnel confirmed the minimal influence of the heating effect. Long range Lorentz forces were obtained by placing magnets within the plate. The resulting forces on the plate match well with the Lorentz force law, but due to limitations in power, the plasma discharge did not reach the desired length.
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Nomenclature

\( A_g \)  Area of grid
\( A_{\theta} \)  Magnetic vector potential in azimuthal direction
\( A_{\theta}^* \)  Nondimensional magnetic vector potential in azimuthal direction
\( \vec{B} \)  Magnetic induction vector
\( B_r \)  Magnetic flux density in radial direction
\( B_z \)  Magnetic flux density in axial direction
\( C_{\text{plate}} \)  Plate specific heat (J/kg-K)
\( \Delta C_p \)  Change in pressure coefficient
\( \vec{D} \)  Electric displacement vector
\( d_{ch} \)  Chamber diameter (cm)
\( \vec{E} \)  Electric field vector
\( |e| \)  Magnitude of charge of an electron
\( \vec{F}_{LR} \)  Lorentz force vector (N)
\( g \)  Gravitational constant of acceleration (m/s^2)
\( \vec{H} \)  Magnetic field vector
\( H_{cr} \)  Coercive force component in radial direction
\( H_{cz} \) Coercive force component in axial direction

\( h_s \) Source height (cm)

\( I_{sp} \) Specific Impulse (sec)

\( I \) Current (Amps)

\( \rightarrow J_f \) Free current

\( k_{plate} \) Plate thermal conductivity (W/m-K)

\( \rightarrow L \) Current length vector (m)

\( l_{avg} \) Average chamber confinement length (cm)

\( l_{avg}^* \) Nondimensional average chamber confinement length

\( L_e \) Length from leading edge to source (cm)

\( L_p \) Plate length (cm)

\( L_r \) Larmor radius

\( L_s \) Source length (cm)

\( m \) Mass

\( \cdot m_p \) Propellant mass flow rate (kg/sec)

\( M \) Mach number

\( M_\theta \) Angular momentum

\( M_\theta^* \) Nondimensional angular momentum

\( N \) Number of particles tracked

\( n_i \) Number density of ions
$n_n$  Number density of neutral atoms

$n_{pe}^*$  Nondimensional primary electron number density

$P$  Energy source magnitude per cm in spanwise direction (P/cm)

$p$  Pressure (Pa)

$P_{u}$  Probability primary electron does not have a collision

$r$  Radial Position

$r^*$  Nondimensional radial Position

$S_{F_x}$  X-Direction Force Source Term (N/m$^3$)

$S_{F_y}$  Y-Direction Force Source Term (N/m$^3$)

$S_H$  Energy Source Term (W/m$^3$)

$t$  Time (sec)

$t^*$  Nondimensional confinement time

$t^*$  Nondimensional time

$T$  Temperature (K)

$u_e$  Thruster Exit Velocity (m/s)

$u$  X-direction component of velocity (m/s)

$v$  Y-direction component of velocity (m/s)

$V_r$  Radial component of velocity

$V_\theta$  Azimuthal component of velocity

$V_z$  Axial component of velocity

$V_r^*$  Nondimensional radial component of velocity
$V_{\theta}$  Nondimensional azimuthal component of velocity

$V_{z}^{*}$  Nondimensional axial component of velocity

$V$  Primary electron velocity magnitude

$V_d$  Discharge voltage

$V_i$  Velocity magnitude of ions

$V_n$  Velocity magnitude of neutral atoms

$\vec{V}$  Velocity of primary electron

$x$  X-direction location (m)

$Y$  Vertical location of centerline of source term (cm)

$y$  Y-direction location (m)

$y^+$  Y-Plus value at the wall

$z$  Axial Position

$z^{*}$  Nondimensional axial Position

$\alpha$  Wedge angle (degrees)

$\lambda_u$  Mean free path

$\mu$  Dynamic Viscosity (N•s/m²)

$\mu_r$  Permeability in radial direction

$\mu_z$  Permeability in axial direction

$\phi_n$  Grid transparency of neutral atoms

$\phi_i$  Grid transparency of ions

$\Phi$  Viscous Dissipation Term (W/m³)
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Dedicated to

My mother and father. They have made great sacrifices in their own lives to support me in my endeavors. Without the support I have received from them, continuing my education would have been a monumental task. For this reason it is only fitting that this work be dedicated to them.
I. Introduction

The study of plasmas represents a wealth of opportunities to the aerospace community. The work presented here covers a broad range of applications of plasmas in modern engineering practice. The two regimes this work deals with are in space propulsion and in air vehicle control. These regimes operate in very different physical conditions, from the near vacuum conditions within an ion engine thruster to arc plasma actuators in atmospheric pressure. The distinct challenges of operating in either environment will be discussed later.

In both space propulsion and in air vehicle control, plasma devices represent the potential for significant improvement over the current systems being employed. For long-range deep space missions, ion engines embody many advantages over traditional chemical rocket propulsion. An ion engine does not have the ability to reach orbit on its own and currently requires a booster rocket to achieve this. Once in orbit the ion engine can operate over an incredibly long duration before exhausting its supply of fuel. In the cases studied here the fuel for the engine is xenon gas. While the thrust of an ion engine does not come close to approaching that of a traditional chemical rocket, the ability to run for extremely long periods combined with the fact that the engines fuel is a gas makes for distinct benefits for certain types of space missions. The work presented here attempts to build upon developments in ion engine design practices that have been put forth by
prior research at Wright State. Specifically this research addresses areas that were assumed to have little importance in previous work.

The use of an electrically generated plasma arc for air vehicle flight control has gained considerable attention for use on military aircraft and weapon systems. The use of a plasma arc would eliminate the needs for traditional fin style control surfaces. Traditional control surfaces function by changing their orientation to the flow, in turn changing the pressure distribution. This change in pressure corresponds to a change in the force subjected to the vehicle. This method has a distinct disadvantage when compared to plasma actuators. The aerodynamic drag associated with traditional control surfaces can be significant; especially as vehicle speed continues to increase. Plasma actuators would be mounted within the skin of a vehicle, greatly reducing the aerodynamic drag. Plasma actuators would likely not require the same level of mechanical complexity currently necessary for fin type control surfaces.

Within this area of study, the goal of this work is to decouple and isolate fundamental effects that are present in plasma discharges in both sub-sonic and supersonic flows. These effects are studied through both computational and experimental methods, with the goal of gaining insight into the effectiveness of a plasma actuator for use as a control device on an air vehicle.
II. Computational Modeling of an Ion Engine Discharge Chamber

1. Introduction

1.1. Electrical Propulsion Fundamentals

Electrical propulsion encompasses a large array of devices that utilize the electrostatic acceleration of positively charged heavy atomic particles to produce thrust. The history behind the various techniques for electrical propulsion can be seen in Ogunjobi and Menart (2006) and Jahn (1996) and will not be discussed at great length in this work. The specific type of electric propulsion studied here is the xenon ion propulsion system (XIPS). This device uses xenon, a heavy noble gas, as the fuel to create the heavy charged particles needed to develop thrust. These positively charged particles, or ions, are produced through the creation of plasma within the discharge chamber of the engine. By inducing a voltage potential between the positively charged anode wall and the negatively charged cathode, electrons will be drawn from the cathode to the anode. During this process the electrons collide with the neutral xenon particles that are introduced to the chamber. When these collisions take place, the neutral xenon atom can lose an electron and thus have a net positive charge. This newly formed ion is then thrust
out of the chamber through a large negative electric field produced by a grid located at
the rear of the engine. Finally electrons are reintroduced to the exhausted ions to
neutralize them. Figure 1 depicts how this device operates. This process can create
enormous exit velocities from the engine; however, due to the small mass associated with
the xenon ion, the thrust produced is very small.

![Schematic of an ion engine](Ref. nasa.gov)

Figure 1.1: Schematic of an ion engine (Ref. nasa.gov).

Another important element of the ion engine is the permanent magnet rings that
encircle the chamber. The magnetic field that is created by the proper placement of the
poles of the magnets creates a barrier to the negatively charged electrons in front of the
positively charged chamber wall. If this magnetic field were not present the electrons
would run to the anode wall without creating a significant number of ions. The
arrangement of these magnets is of great importance. Recent work by Ogunjobi and
Menart (2006) has sought to find optimum magnet configurations which will maximize
the amount of time the primary electrons are contained within the chamber.

1.2. Performance Metrics
One of the distinct advantages of electric propulsion over traditional chemical rockets is the dramatically higher gas exit velocities that can be obtained through the electrostatic acceleration of particles. One method of judging this distinction is in the comparison of the specific impulse, $I_{sp}$. Specific impulse is defined to be the ratio of thrust produced to the weight of the propellant used,

$$I_{sp} = \frac{\dot{m}_p u_e}{m_p g} = \frac{u_e}{g} \text{ (sec)},$$  \hspace{1cm} (1.1)

where $\dot{m}_p$ is the mass flow rate of propellant, $u_e$ is the exit velocity of the propellant and $g$ is the gravitational acceleration constant. Devices with a high specific impulse have a better utilization of propellant than devices with a lower specific impulse. Chemical rockets can yield values of specific impulse ranging from 170 – 450 seconds where electric propulsion thrusters can deliver a specific impulse from 2000 – 20,000 seconds. For certain applications this can provide up to a 90% decrease in the weight of the propulsion system. However, due to the low thrust produced, ion thrusters are currently being used on deep space missions and in satellite maneuvering.

1.3. **Scope of Work**

In recent years basic preliminary design rules have been put forth by researchers at Wright State University. Ogunjobi and Menart (2006) have specified design principles concerning the use of permanent magnet rings for the confinement of the primary electrons emitted by the cathode. The studies put forth by Ogunjobi and Menart
considered magnet spacing and magnet orientation and their effect on the confinement of
the primary electrons within the chamber. The confinement length $l_{avg}$ is defined as,

$$l_{avg} = d_{ch} \left( \frac{1}{N} \sum_{j=1}^{N} t_{\text{conf},j}^* \right) \text{ (cm)},$$  \hspace{1cm} (1.2)

where, $d_{ch}$ is the chamber diameter in centimeters, $N$ represents the total number of
electrons tracked, and $t_{\text{conf},j}^*$ is the nondimensional confinement time. By increasing the
confinement length, the primary electrons will have a greater chance of colliding with
neutral xenon atoms, creating xenon ions which are used to produce thrust. Confinement
length was the primary measure of the effectiveness of a magnetic circuit configuration
for the work presented by Ogunjobi and Menart. The primary results presented by
Ogunjobi and Menart dealt with changing the axial and radial spacing of the magnets for
both perpendicular magnet poles and parallel magnet poles. These configurations are
depicted in Figures 1.2 and 1.3.

Figure 1.2: Perpendicular magnet configuration.
Due to the immense parameter space associated with optimizing the confinement length for a given chamber Ogunjobi and Menart (2006) neglected to study the effects of: cathode emission location and electron energy. The studies conducted for this work considered the optimal configurations for both parallel and perpendicular magnet configurations and considered changing the radial and axial spacing of the magnets.

### 1.3.1. Parameters Studied

#### 1.3.1.1. Cathode Emission Location

In the studies conducted by Ogunjobi and Menart (2006) the cathode emission point was defined to be 3cm from the front wall of the chamber as depicted in Figures 1.2 and 1.3. The goal of the work being presented here is to determine whether the cathode location has an impact on the confinement length. For the sake of this study the emission location was varied from a distance of 1cm to a distance of 13cm from the front wall of the chamber. The optimal magnet configurations for chamber diameters of 10, 20 and 30 centimeters were considered for the perpendicular magnet configuration. Likewise,
optimal magnet placement for chamber diameters of 10, 20 and 40 centimeters were studied for the parallel magnet configuration.

1.3.1.2. **Electron Energy**

The energy at which a primary electron is propelled throughout the discharge chamber was also studied in this work. It had been suggested that increasing the electron energy may increase the confinement length of the primary electrons. In order to see what affects the electron energy may have, the primary electron energy was adjusted from 1eV up to 50eV. These cases were conducted using an optimally configured 20cm diameter, perpendicular chamber configuration as put forth by Ogunjobi (2006). These studies were conducted for two independent cases: the first included particle collisions between the primary electrons and the heavy neutrals and singly charged ions and the second case considered no particle collisions.

1.3.1.3. **Magnetic Field Strength**

The field generated by the permanent magnets is essential for proper operation of an ion engine. The ability of the field to contain the primary electrons within the discharge chamber greatly improves ionization and therefore performance. Given this knowledge it would stand to reason that increasing the magnetic field strength would increase the confinement length of electrons within the chamber. For this investigation two methods were used to increase the field strength.

The first method was to increase the thickness of the permanent magnet rings that encircle the chamber. While this does not increase the field strength at the surface of the magnets, it does increase the strength of the field at longer distances. These studies only considered a perpendicular magnet pair as shown in Fig. 1.2. The optimal magnet spacing
for a 20cm diameter chamber as presented by Ogunjobi and Menart (2006) was used for this study. Three different scenarios were investigated. Magnet thickness was taken relative to the standard thickness used in previous studies. This work considered magnet thicknesses of 0.5, 1, 2 and 3 times the previous standard value. As stated before this chamber configuration consisted of a pair of magnets, one on the front wall and one on the side wall of the chamber. Studies are conducted changing the thickness of both magnets, varying only the side wall magnet and finally varying only the front wall magnet.

The second method employed for increasing the magnetic field strength was to increase the number of magnet rings on the chamber. Again, for this case an optimum initial magnet spacing as put forth by Ogunjobi (2006) was utilized. Other researchers have suggested that to achieve increased electron confinement the largest gauss line should be closed (Wirz and Goebel (2006)). By increasing the total number of magnets within the magnetic circuit greater magnitude field lines will close on themselves. Using a perpendicular magnet configuration the total number of magnets was increased from 2 to 8. These magnets were added such that the final chamber had three magnets on the front wall and five magnets on the side wall.
2. Computational Modeling Tools

The computational modeling of the ion engine discharge chamber is conducted in two steps. First the magnetic field generated by the configuration of the permanent magnet rings is solved. The information from the magnetic field solver is then input into the primary electron tracking solver to determine the trajectories of the electrons within the chamber. To accomplish this task two numerical tools are required. MAXWELL2D is utilized to solve for the magnetic field in the chamber. An overview of MAXWELL2D and the mathematical models that it solves will be presented in Sections 2.1.1 and 2.1.2. The primary electron tracking code PRIMA is used in these studies to determine the trajectories of the electrons in the chamber. A brief discussion of PRIMA and its history are discussed in Sections 2.2.1 and 2.2.2 and the mathematical model used to represent the electron tracking is presented in Section 2.2.3.

2.1. Magnetic Field Modeling

2.1.1. MAXWELL 2D

As discussed in the previous section MAXWELL 2D is used to solve for the static magnetic field generated by the permanent magnets that encircle the chamber.

MAXWELL 2D was developed by the Ansoft Corporation (MAXWELL 2D, 2007). The software is capable of simulating electrostatic and magneto-static conditions within a
two-dimensional solution domain. This domain can be either in Cartesian or polar coordinates. For the purpose of this study a two-dimensional, axisymmetric domain is chosen. This formulation is valid due to the fact that in the radial and axial plane, the magnetic vector potential in the circumferential direction is constant. This eliminates the need to model the magnetic field in three dimensions.

MAXWELL 2D uses the finite element method to solve Maxwell’s Equations in two dimensions. The mathematical model used for these cases will be discussed in Section 2.1.2. The software uses an unstructured triangular mesh within the domain. Given boundary conditions and material properties the solver then refines the size of the mesh until subsequent calculations for the magnetic vector potential yield results within a given percentage. Based on previous convergence studies conducted by Deshpande (2004) this value is defined to be 0.6%. This feature ensures that a well converged solution is obtained without requiring a mesh convergence study for each case selected. Due to the significant changes that can occur between different magnet configurations, the ability of the software to adjust mesh density in regions where it is required is of significant advantage over a manual mesh fit. This is accomplished by tracking the residuals at each nodal location. If the residual is larger than the specified value, then more mesh elements are added until the calculated residual falls within the proper criteria. MAXWELL 2D is capable of handling a very large array of two-dimensional geometries; however, for these studies only domains with straight lines are considered. This is due to a restriction imposed by PRIMA, the primary electron tracking program. A more detailed presentation of the modeling methods used by MAXWELL 2D is given by Deshpande and Menart (2004).
2.1.2. Magnetic Field Model

MAXWELL 2D models the magnetic field within the chamber by solving Maxwell’s equations. These equations relate the spatial derivatives of the magnetic and electric fields to their respective time derivatives, as well as both internal and external sources. In this case the sources are permanent magnets. The general form of Maxwell’s equations are:

\[ \nabla \cdot \vec{D} = \rho_f \]  
(2.1)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  
(2.2)

\[ \nabla \cdot \vec{B} = 0 \]  
(2.3)

and

\[ \nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \]  
(2.4)

where \( \vec{D} \) is the electric displacement, \( \rho_f \) is the free charge density, \( \vec{E} \) is the electric field, \( \vec{H} \) is the magnetic field, \( t \) is time, \( \vec{B} \) is the magnetic induction and \( \vec{J}_f \) is the free current. For this work electric fields, free currents, and transient operation are not considered. Given these assumptions Maxwell’s equations in a two-dimensional axisymmetric coordinate system reduce to the following:

\[ \left( \frac{\partial H_{ez}}{\partial z} - \frac{\partial H_{er}}{\partial r} \right) = \frac{\partial}{\partial r} \left( \frac{1}{r \mu_z} \frac{\partial (r A_{r})}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu_r} \frac{\partial (A_{z})}{\partial z} \right) \]  
(2.5)

where \( H_{er} \) is the coercive force of the magnets in the radial direction, \( H_{ez} \) is the coercive force of the magnets in the axial direction, \( \mu_r \) is the permeability in the radial direction,
\( \mu_z \) is the permeability in the axial direction, and \( A_\theta \) is the magnetic vector potential in the circumferential direction. The required boundary conditions for Eqn. (2.5) are

\[
A_\theta \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty \quad \quad A_\theta \rightarrow 0 \quad \text{as} \quad z \rightarrow -\infty
\]

\[
\frac{\partial A_\theta}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad \quad A_\theta \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty .
\]  

(2.6)

A detailed derivation of Maxwell’s equations to the reduced form of Eqn. 2.5 is presented by Deshpande and Menart (2004).

Prior to being input into PRIMA, the magnetic vector potential, \( A_\theta \), must be converted into a magnetic flux density, \( B \). This conversion is accomplished by taking the appropriate derivatives as seen in Eqns. 2.7 and 2.8,

\[
B_r = \frac{1}{r} \frac{\partial(r \partial A_\theta)}{\partial r}
\]  

(2.7)

and

\[
B_z = \frac{\partial A_\theta}{\partial z}.
\]  

(2.8)

A computer program was written by Deshpande (2004) that performs these derivatives and formats the data for input into PRIMA.
2.2. Primary Electron Tracking Program

2.2.1. PRIMA

PRIMA uses a particle-in-cell (PIC) technique to track the primary electrons within an axisymmetric discharge chamber. This code was originally developed by Arakawa and Ishihara (1991) and has undergone significant modification and improvement. The version of PRIMA used for these studies was modified by Mahalingam and Menart (2002). PRIMA employs the PIC simulation method by tracking representative macro particles. Each of these macro particles represents a given number of actual particles. PRIMA only tracks primary electrons, as these are the most important constituents for the initiation and continued generation of the xenon ions. Also PRIMA considers all primary electron collisions to be elastic and that the electric field generated by the plasma is negligible.

Within the discharge of an ion engine there are a large number of different types of particles present. These can include: primary electrons, secondary electrons, neutral xenon atoms, singly charged xenon ions, doubly charged xenon ions and others. Tracking all of these particles requires a substantial amount of computing power and time. Mahalingam (2007) has conducted research in using PIC simulations to track the above-mentioned particles within an axisymmetric ion engine discharge chamber. The computational complexity of modeling the discharge is immense. In order to reduce these complexities PRIMA only tracks the primary electron trajectories. Due to the fact that primary electrons are the greatest contributors to ionization, some insight into the engine operation can be garnered from this simplified modeling. In addition, Deshpande and Menart (2004) show that the results from PRIMA match closely to experimental data.
collected by Hiatt and Wilbur (1986) as well as the theoretical formulation put forth by Brophy and Wilbur (1985)

As stated previously all primary electron collisions are considered to be elastic. This implies that kinetic energy is conserved and that collisions only result in a change of direction. In actuality ionization collisions, between the electrons and the neutral xenon atoms are inelastic, meaning there is some quantity of energy that is absorbed. PRIMA is capable of modeling inelastic collisions but for the sake of this work where electron confinement is the primary parameter of measure, only elastic collisions are considered. Brophy and Wilbur (1985) stated that inelastic collisions should not be included in the calculation of electron confinement length. The inclusion of such collisions would have a tendency to skew results and make it difficult to compare differing magnetic field configurations. Only considering elastic collisions also removes a level of computational complexity.

To achieve another degree of computational simplicity the electric fields generated by the plasma sheath are neglected. In order to determine these electric fields all particle constituents within the chamber must be determined. As mentioned before, tracking these particles is a daunting task. Recent work by Mahalingam (2007) has shown that the electric fields do have an influence on the confinement length of primary electrons.

### 2.2.2. Literature Survey of PRIMA

Over the past decade PRIMA has been used to optimize the electron confinement within an ion engine discharge chamber. This work first began at Wright State University and
was carried out by Mahalingam and Menart (2002). This work consisted of correcting errors in the original code developed by Arakawa and Ishihara (1991). Mahalingam made modifications to the code that include: greater flexibility regarding geometry, parallelization, corrected the collision probability function, and corrected other small errors. The improved version of PRIMA can handle any straight wall geometry the user desires. The parallelization of the code allows for significant reductions in computational time. By correcting the collision probability function the accuracy of the results from PRIMA were greatly improved. Mahalingam also determined the appropriate input parameters for the various numerical techniques used within PRIMA.

Validation of PRIMA with experimental results was conducted by Deshpande and Menart (2004). The result shows that for the plasma ion energy cost the difference between PRIMA and experimental values was a maximum of 18%. This enabled further work to be conducted to study the affects of the magnetic circuit on electron confinement. Deshpande also utilized a new magnetic field solver, MAXWELL 2D. This solver is more user friendly and gave better options for increasing mesh density in regions of large magnetic field gradients. Deshpande determined the proper operating parameters for MAXWELL 2D and developed an interface between MAXWELL 2D and PRIMA.

Deshpande also considered the effects of the cusp region on the electrons as well as what affect the magnet spacing might have. A significant study into the spacing and orientation of the magnet rings was conducted by Ogunjobi and Menart (2006). The purpose of this work was to develop a set of general ‘rules of thumb’ regarding the placement of magnets around the chamber. Magnet pairs were considered in both a front-side as well as a front-rear configuration. The spacing between each magnet was varied.
and the angle between the magnet pairs was also studied. The resulting guidelines are summarized below:

1. One magnet ring must be placed on the front wall with a radius no greater than 4cm.
2. Axial and Radial Spacing between magnet rings should be between 11cm and 16cm.
3. For chambers greater than 25cm in diameter two magnetic rings are not sufficient for optimum confinement.
4. Magnets should be placed such that axis of polarization is placed parallel to the normal of the wall.
5. A minimum number of magnetic rings should be used to reduce the number of cusps while not violating rule #2.

In the work presented by Ogunjobi and Menart (2006) the electron energy and cathode emission location were assumed to have little influence on electron confinement and therefore were held constant. It is these assumptions that are to be studied in the work presented here.

2.2.3. Primary Electron Tracking Model

In order to track the primary electrons within the discharge chamber PRIMA solves the non-dimensional equations of motion for an electron in the presence of a magnetic field. PRIMA must also use other mathematical formulations to determine the number densities of various particles, as well as the probability function of different particle collisions. A
more detailed derivation of these equations is presented by Deshpande and Menart (2004) and Mahalingam and Menart (2002).

For this model it is often easier to derive the equations of motion using a Lagrangian formulation as opposed to a Newtonian one. The Lagrange equation approach determines the trajectory of a particle by solving the path that minimizes the action. The sum of the Lagrangian over time defines the trajectory of the particle being tracked. In this formulation of the equations of motion the motion of the electron due to the magnetic field is considered as potential energy (Greenwood 1997). The following are the equations of motion in an axisymetric coordinate system for an electron in a magnetic field:

\[
\frac{m}{r} \frac{dV_r}{dt} = m \frac{V_{\theta}^2}{r} - |e| \frac{V_{\theta}}{r} \frac{\partial (rA_{\theta})}{\partial r} \quad (2.9)
\]

\[
\frac{m}{r} \frac{dV_z}{dt} = -|e| \frac{V_{\theta}}{r} \frac{\partial (rA_{\theta})}{\partial z} \quad (2.10)
\]

and

\[
M_\theta = mr V_{\theta} - |e| r A_{\theta} \quad (2.11)
\]

Equations (2.9) and (2.10) represent Newton’s Law in the radial and axial directions. Equation (2.11) shows that angular momentum is conserved. These equations predict the movement of an electron and how its path is affected by collisions. To include the effect of the magnetic field one must first understand the influence of the Lorentz force on the electron. The Lorentz force acts perpendicular to the direction of the electron. This force will change the direction of the electron when the electron encounters a magnetic field. These Lorentz force affects are evident in the terms involving \( A_{\theta} \) in Eqns. (2.9) through (2.11).
Initially the primary electrons are given a velocity magnitude based on the cathode discharge voltage, $V_d$,

$$V = \sqrt{\frac{2|e|V_d}{m}}$$  \hspace{1cm} (2.12)

where, $V$ represent the velocity magnitude, $|e|$ electron charge magnitude and $m$ is the mass of the electron. The direction of this initial velocity is determined using a Monte Carlo technique. This technique randomizes the direction between angles of 0 and 90 degrees but yields a uniform electron distribution. The initial position is defined as the cathode emission location.

To determine the primary electron collisions with neutral atoms, as well as ions, a number density of these constituents in the chamber must be found. These number densities can be found from the following equations:

$$n_n = \frac{4 m_p (1 - \eta)}{|e| V_n \phi_n A_g}$$  \hspace{1cm} (2.13)

and

$$n_i = \frac{4 m_p \eta}{|e| V_i \phi_i A_g}$$  \hspace{1cm} (2.14)

where, $n_n$ and $n_i$ are the neutral and ion number densities respectively which are determined from the propellant mass flow rate, $m_p$, the utilization efficiency, $\eta$, the grid transparency to neutrals or ions, $\phi_n$ and $\phi_i$, the area of the grids, $A_g$, and the neutral and ion velocities, $V_n$ and $V_i$.

PRIMA uses a probability function to determine if a collision takes place between a primary electron and a neutral atom or an ion. This probability is given by
\[ P_u = e^{-\lambda_u} \]  

(2.15)

where, \( \lambda_u \) is the mean free path of the electrons given by

\[ \lambda_u = \frac{1}{n_n \sigma_n} \]  

(2.16)

The distance an electron moves in a single time step, \( l \), is determined using a Runge-Kutta technique. \( \lambda_u \) is the mean free path length, \( n_n \) is the neutral atom number density (Eqn. (2.13)), and \( \sigma_n \) is the collision cross section of the neutral atom to the primary electron. The determination of this probability function is discussed in greater detail by Deshpande and Menart (2004), as well as Mahalingam and Menart (2002).

PRIMA utilizes the normalized versions of the equations of motion shown in Eqns. (2.9 to 2.11). These normalized equations are

\[
\frac{dV_r^*}{dt^*} = \frac{V_{\theta}^*}{r^*} \frac{\partial (r^* A_{\theta}^*)}{\partial r^*} \\
\frac{dV_{\theta}^*}{dt^*} = -V_{\theta}^* \frac{\partial (r^* A_{\theta}^*)}{\partial \theta^*} \\
\frac{dV_z^*}{dt^*} = -V_{\theta}^* \frac{\partial (r^* A_{\theta}^*)}{\partial \theta^*}
\]  

(2.17)

(2.18)

and

\[ M_{\theta}^* = r^* V_{\theta}^* - r^* A_{\theta}^* \]  

(2.19)

The variables in Eqns. (2.17 to 2.19) are obtained by dividing the dimensional forms of the equations (Eqns. (2.9 to 2.11)) by a maximum reference value for that variable. These reference values as well as the derivation of these normalized equations of motion are presented by Mahalingam and Menart (2002), as well as by Deshpande and Menart (2004). PRIMA uses a finite element method to solve for the spatial derivatives in Eqns. (2.17) and (2.18). This finite element method utilizes rectangular bilinear elements. By applying shape functions and knowing the particle’s axial and radial position, the
magnetic vector potential, $A^\theta_0$, can be interpolated. These values are obtained from the nodal magnetic vector potential values obtained from MAXWELL 2D. To solve for the first order temporal derivatives a fourth order Runge-Kutta method is employed. A Monte Carlo technique is used to provide initial values for this solution. To maintain the stability of the Runge-Kutta scheme very small non-dimensional time steps must be used. A Monte Carlo technique is also used to predict the likelihood that a collision between particles will occur. A random value is compared with the probability function of Eqn. (2.15). If this random value is greater than the probability function a collision will occur and vice versa if the random value is less than the probability function value. An explanation of the finite element, Runge-Kutta and the Monte Carlo techniques utilized by PRIMA are discussed in much greater detail by Mahalingam and Menart (2002) and by Deshpande and Menart (2004).

PRIMA generates a very large quantity of output data, but for the sake of this work only two quantities will be discussed. The normalized confinement length and the relative number density of primary electrons are the prominent quantities of interest for this work. The electron confinement length was discussed earlier in Section 1.3 and will be discussed in slightly more detail here. Due to the normalization of the velocity and the assumption of elastic collisions, the normalized velocity of an electron is held at a value of 1.0. This leads to the average normalized confinement length being defined as the sum of the confinement times for all primary electrons as

$$l^*_{\text{avg}} = \frac{1}{N} \sum_{j=1}^{N} l^*_{\text{conf},j}.$$  \hspace{1cm} (2.20)

Results for the confinement length are obtained by multiplying the normalized confinement length by the chamber diameter.
Lastly the normalized relative number density is represented by

\[
\frac{n^*_{pe}}{n^*_{pe,max}} = \frac{\sum_{j=1}^{N} \Delta t^*_{element,j}}{\sum_{j=1}^{N} \Delta t^*_{element,j \, max}}.
\]  

Eqn. (2.21) indicates how the primary electrons are distributed within the discharge chamber. This quantity represents the ratio of the primary electron density at a certain location to the maximum number density within the chamber.
3. Results of Parametric Studies

Three different parameters for the confinement of electrons within an ion engine discharge chamber are considered for this work. These parameters are: the cathode emission location, the energy of the primary electrons, and the magnetic field strength. The goal of this work is to determine the significance of these parameters on the confinement length. By doing this the general design rules developed in prior work will be strengthened or weakened depending on the results. Prior work that developed the design rules only considered one electron energy and one cathode position. In addition only one strength of magnet was considered. These results only represent the relative changes to the confinement length of the primary electrons and do not make any assertions as to the stable operation of the plasma within the discharge chamber. If the confinement of the electrons is too good then there will be no current flow through the plasma. If this occurs then the engine will not run. Also if the magnetic field causes the electrons to be clustered in one region, non-uniformities will exist in the beam profile. These non-uniformities have adverse effects on the operation and life expectancy of the engine. The goal of the results presented here are to give an initial configuration for optimal confinement. If necessary those initial conditions can then be changed to reduce confinement until stable engine operation and a more uniform beam profile are achieved.

3.1. Cathode Emission Location
3.1.1. **Perpendicular Magnet Configuration**

The results from this section represent test cases on a perpendicular magnet pair configuration for varying cathode emission locations. The chamber geometries selected for these cases were selected from the optimal configurations as determined by Ogunjobi and Menart (2006). In the work of Ogunjobi and Menart (2006) the cathode emission location was specified to be 3 cm from the inside of the front wall of the chamber as depicted in Figure 1.2. In this work the cathode was moved from a position of 1 cm to a position of 13 cm in increments of 2 cm. All chambers in these cases are 14 cm in length and diameters of 10, 20, and 30 cm are considered. Figure 3.1 shows the confinement length versus the cathode location for these three chamber diameters.

![Figure 3.1: Effect of cathode position on confinement length for 10, 20, and 30 cm diameter chambers with perpendicular magnets.](image)

The case where the cathode position is located at 3 cm is considered to be the baseline for other comparisons for each chamber diameter. All of the tests represented in
Fig. 3.1, except one, show less than 4% difference to the baseline. The only case that falls outside that range is the 10 cm diameter chamber with the cathode located at 13 cm, this configuration shows only a 7% deviation from the base case. These results show that for a range of chamber diameters the confinement length of the primary electrons is not a strong function of the cathode emission location.

### 3.1.2. Parallel Magnet Configuration

Presented here are the results for parallel magnet pair configurations with a varying cathode emission point (see Fig. 1.3). As was done for the perpendicular magnet cases, the optimal results from Ogunjobi and Menart (2006) were selected for study. Three chamber geometries were considered. All chambers were 16cm in length and 10, 20, and 40cm in diameter, respectively. For the 10 and 20 cm diameter cases the front wall magnet is located at 3.5cm in the radial direction and for the 40cm diameter case this magnet is located at 5.5 cm in the radial direction. Other information regarding the chamber geometry can be seen in Fig. 1.3. The cathode position is varied in the same manner as was done for the perpendicular magnet tests. The confinement lengths results are shown in Fig. 3.2.
Figure 3.2: Effects of cathode location of confinement length for 10, 20, and 40cm diameter chambers with parallel magnets.

It should be noted that for the 10 and 20cm diameter cases the deviation in the confinement length from the base case is less than 4%. The 40cm diameter chamber shows significant changes over the range of cathode locations. These changes are as high as 13% for the case where the cathode is located at 1cm. The greater sensitivity to cathode position exhibited by the 40cm diameter chamber is likely due to the actual configuration of the magnet pairs. In order to minimize the loss of electrons through magnetic field gaps on the front wall, the front magnet is moved upwards to 5.5cm. This in turn results in a gap being opened between the cathode and the front magnet. It is this gap that causes the significantly lower confinement length, as well as the sensitivity to the cathode position. A 40cm diameter chamber of this nature is too large to adequately cover with only two magnets. Ogunjobi and Menart (2006) state that this case requires 3 magnets for optimum confinement. Figure 3.3 shows the relative electron number density.
as well as the magnetic vector potential lines for the case where the chamber diameter is 40cm and the cathode is located at 7 cm.

Figure 3.3: Relative number density and magnetic vector potential lines for a 40cm diameter chamber with the cathode emission point located at 7cm.

These results show that for most cases the electron confinement length is not a strong function of the cathode emission point. The results for the 40cm diameter chamber are skewed due to the large aspect ratio and inadequate magnetic circuit considered. This further validates the design guidelines this work was intended to confirm.

### 3.2. Electron Energy Study

As mentioned in previous sections the energy of the electrons leaving the cathode was held constant in previous studies conducted by Ogunjobi and Menart (2006), as well as
those by Deshpande and Menart (2004). As another step of validating the Wright State University design guidelines this work seeks to determine the effect of electron energy on the confinement of these primary electrons. To conduct this study a 20cm diameter chamber was considered. This chamber used a pair of perpendicular magnets spaced in the optimal configuration as determined by Ogunjobi and Menart (2006). Previous work considered the electron energy to be held at 30eV. This work considered cases where the electron energy ranges from 1eV up to 50eV. Studies were conducted with and without elastic collisions with heavy particles. The results of these cases are shown in Figure 3.4.

![Figure 3.4: Effect of electron energy on confinement length, both including and excluding collisions.](image)

It is apparent from Fig. 3.4 that for the cases where electron collisions are considered the confinement length decreases with increasing electron energy. This can be explained by considering the value of the Larmor radius
where \( m \) is the mass of an electron, \( \vec{V} \) is the electron velocity, \( \vec{B} \) is the magnetic flux density, and \( |e| \) is the magnitude of the charge of the electron. The Larmor radius defines the radius of the circular motion of an electron in the presence of a magnetic field. As the electron energy increases the particle velocity also increases. Therefore as the electron energy increases the Larmor radius increases.

When no collisions are considered the larger Larmor radius increases the ability of the magnet cusp to reflect the electron back into the chamber. As the Larmor radius decreases the electron can pass through the cusp region easier. This phenomenon explains the continual increase in the confinement length as electron energy increases for the no collision cases considered. When collisions are considered the electron energy has a reduced affect on the confinement length. The only cases that represent an increase in confinement length are the 5eV and the 10eV cases. The results presented here indicate that the collisions between the primary electrons and the heavy particles reduce the effect that electron energy has on the confinement length of the electrons. This indicates that the assumption regarding the electron energy made by previous researchers as Wright State University is a valid one.

### 3.3. Magnetic Field Strength Study

In recent years a significant amount of work has gone into the design of the magnetic field circuit for use on ion engines. Work presented by Ogunjobi and Menart (2006) has
presented initial guidelines for the placement and orientation of the magnet rings on the chamber. These guidelines are based on the premise that electron confinement is a principle factor in the performance of an ion thruster. These rules are to be used as a starting point and do not require computational modeling to establish an initial starting point. Recent work presented by Wirz and Goebel (2006), suggests that increasing the magnetic field such that the largest possible gauss line is closed will improve ion engine performance. This method requires modeling of the chamber to determine the configuration that will have the largest closed gauss line.

Starting with an optimized perpendicular magnet configuration for a 20cm diameter chamber two methods were employed to determine the affect of increasing magnetic field strength on the electron confinement. The first method was simply to increase the number of magnets on the chamber. By placing the magnets closer together the short range field strength is increased. That is the field strength close to the discharge chamber walls is increased and that further away from the walls it is decreased. It should be noted that this does not change the magnetic field strength of the surface of the magnet to any significant degree. The second method used was to increase the thickness of the magnets. Making the magnets longer increased the field strength away from the walls of the discharge chamber. Two magnets were considered and placed in the optimal location.

### 3.3.1. Number of Magnets

The goal of this work is to determine the effect of increasing the value of the largest closed gauss line on the confinement length of the primary electrons. This is achieved by increasing the number of magnet rings encircling the chamber. As this distance between the magnets within the circuit is decreased the wall magnetic field will increase. This will
result in the closing of ever higher gauss lines. Wirz and Goebel (2006) have stated that closing the highest gauss line that will maintain stable operation of the discharge yields the greatest performance. This proposed increase in performance is partly due to better electron confinement within the chamber. Figure 3.5 shows that as the number of magnets is increased and the subsequent largest closed gauss line increases the electron confinement decreases. This is in contradiction to the statement of Wirz and Goebel. Figure 3.5 does show an increase in confinement length between the case for 4 magnets and 5 magnets respectively.

![Figure 3.5: Effect of the number of magnets and the open cusp regions on electron confinement](image)

The increase between 4 and 5 magnets can be explained by the magnetic circuit geometry used for this case. The magnets were placed in such a way that they closed off the electrons from entering the cusp region of either magnet. This can be seen in Fig. 3.6.
It can be seen that the magnets in the upper left hand corner of the chamber are blocking electrons from entering. For this reason the second line of Fig. 3.5 was produced. This curve relates the number of open cusps to the confinement length. This open cusp line shows a decrease in confinement length as the number of open cusps is increased. This result would indicate that there is an increase in the electrons lost through the magnet cusps.

![Graph](image.png)

Figure 3.6: Relative electron number density, magnetic field lines (gauss-cm), and magnetic flux density lines (gauss) for a 5 magnet configuration.

Primary electrons can be lost either through a weak area in the magnetic field, referred to as a hole, or through the cusp region of the magnets. The initial magnetic field circuit had been optimized using the Wright State design guidelines for two magnets.
Figure 3.7 indicates that very few electrons can make it to the wall in the region between the magnets. This region is the hole in the magnetic field for the two-ring case. This means that for a 20cm diameter chamber the holes are sufficiently small. As the number of magnets increases, so does the number of cusp regions where the electrons may be lost (see Fig. 3.8). Figures 3.7 and 3.8 show relative number density, magnetic field, and magnetic flux density for a two magnet and a six magnet case respectively. The relative number density plots indicate the regions of the magnetic field that electrons are penetrating substantially.

Figure 3.7: Relative electron number density, magnetic field lines (gauss-cm), and magnetic flux density lines (gauss) for a 2 magnet configuration.
Figure 3.8: Relative electron number density, magnetic field lines (gauss-cm), and magnetic flux density lines (gauss) for a 6 magnet configuration.

It should be noted that for the six magnet case the 100 gauss line has been closed (see Fig. 3.8) whereas for the two magnet case the 25 gauss line has not been closed (see Fig. 3.7). However, referring back to Fig. 3.5 there is a decrease in confinement of almost 1000 cm from the two magnet case to the six magnet case. This result shows that more than just the closure of the largest gauss line should be considered when designing the magnetic circuit of the discharge chamber. The baseline case utilizing two magnets, optimized using Wright State’s guidelines, far outperforms other configurations considered in terms of electron confinement.

3.3.2. Magnet Thickness
The second method used for testing the affect of magnetic field strength on electron confinement was to increase the thickness of the magnet. Again, a standard 20 cm diameter chamber with a pair of perpendicular magnets was used. Three different conditions were considered. These include: changing the side magnet while leaving the rear magnet constant, changing the rear magnet and leaving the side magnet constant, and finally both magnets were changed. The magnet thickness was altered based on the original values used in the work presented by Ogunjobi and Menart (2006). The three conditions were each run for cases where magnet thickness was changed to one half, two and three times the baseline configuration. Work presented by Wirz and Goebel (2006) suggests that increasing the magnet thickness will increase the confinement of primary electrons by increasing the magnetic field strength.

Figure 3.9: Effect of varying magnet thickness on primary electron confinement length.
The results from Fig. 3.9 show that increasing the magnetic field strength by increasing the magnet thickness has a negative affect on confinement length. By increasing the value of the largest gauss line the electrons are kept away from the walls of the chamber; however, as these electrons are held in this region they are drawn into the cusp of one of the magnet rings.

Figure 3.10: Relative electron number density, magnetic field lines (gauss-cm), and magnetic flux density lines (gauss) for a magnet thickness 3 times greater than baseline.

Figure 3.10 shows the funneling affect that is present when the electrons are forced further down into the chamber. It is this funneling into the cusps that decreases the confinement length when compared to the baseline magnet thickness case. Varying either
the front or the rear magnet had the same type of affect as varying both magnets, but to a lesser degree.

In addition to increasing the magnet thickness, the case where the magnet thickness was reduced by one half was considered. As seen in Fig. 3.9 the confinement length is reduced by a factor of two when both magnets are shortened. This reduction is caused by a large hole formed in the magnetic field.

![Figure 3.11: Relative electron number density, magnetic field lines (gauss-cm), and magnetic flux density lines (gauss) for a magnet thickness 0.5 times greater than baseline.](image)

As evident in Fig. 3.11, the magnetic field is not strong enough to deter the primary electrons from being able to reach the anode biased side wall of the chamber. When Fig 3.7 is compared to Fig 3.10 and Fig 3.11 it is evident that there is a fine line
that must be walked to achieve optimal electron confinement. If the field is too strong, then electrons are funneled into the cusps. If the field is too weak, then holes open and allow primary electrons to be lost to the wall. Given this information it is determined that merely increasing the thickness of the magnets will not necessarily increase confinement length. Making modifications to the locations of the magnetic rings as set forth by Ogunjobi and Menart (2006) is a better approach than merely increasing magnet strength.
4. Conclusions

Throughout the past decade research in primary electron confinement conducted at Wright State University has yielded many significant contributions to the area of ion engine design. PRIMA was modified and parallelized by Mahalingam (2002) which enabled more accurate and faster results. Deshpande (2004) conducted a significant number of convergence studies to determine the optimal conditions for convergence of PRIMA. Ogunjobi (2006) was able to take the previous work conducted by Deshpande and Mahalingam and perform a vast parametric study. The goal of this parametric study was to develop guidelines for the placement of the magnet rings on the discharge chamber that would yield the greatest confinement of primary electrons.

The work presented here is a continuation of the work mentioned above. Ogunjobi (2006) assumed that the electron confinement was not a strong function of either the cathode emission location or the energy of the primary electron. This work sought to confirm these assumptions. Results presented in Chapter 3 indicate that these assumptions are valid for most of the cases considered. Cathode position has a very small influence on the confinement length of the primary electrons.

The electron energy study was conducted for the cases where electron collisions with heavy particles were included and excluded. The results show that for the cases where particle collisions are excluded the confinement length continues to increase as the electron energy increases. When collisions are included the confinement length is mostly
constant for electron energy values ranging from 20\text{eV} to 50\text{eV}. There is an increase in the confinement length at both 5\text{eV} and 10\text{eV}. At the time of this publication the reasoning for this increase at the lower energy levels is not fully understood. The results presented by Ogunjobi (2006) took the electron energy to be constant and equal to 30\text{eV}. Over the range specified above the electron energy has minimal influence on the confinement length of the primary electrons within the discharge chamber. This combined with the results of the cathode emission location study further validate the results for the preliminary magnet circuit design guidelines presented by Ogunjobi (2006).

Lastly the affect of the magnetic field strength was studied. Recent work by Wirz and Goebel (2006) suggest that to increase ion engine performance and electron confinement the largest possible gauss line should be closed. To test this statement two conditions were considered. First the number of magnet rings on the chamber was increased. Second the thickness of the magnet rings was changed. For both cases an optimally configured chamber using the guidelines present by Ogunjobi (2006) were used as the base case for comparison. The results show that as the number of magnets is increased, as well as increasing the magnet thickness, the confinement length of the primary electrons decreases. In the case of increasing the number of magnets this drop in confinement length is due to an increase in the number of cusp regions where electrons can escape to the anode biased walls. When the magnet thickness is increased the electrons are forced towards the center of the chamber and are more easily funneled into the cusps of the magnets. These results indicate that merely trying to close the largest
gauss line will not necessarily result in greater confinement. A more detailed design
guideline must be employed, such as the one presented by Ogunjobi (2006).

In conclusion, the work presented here has further validated the magnetic circuit
design guidelines developed at Wright State University. Previous assumptions have been
confirmed to be correct and new insight was gained on the effects of increasing magnetic
field strength on electron confinement. It is the hope of the author that this work will help
to further advance the understanding and design practices associated with ion propulsion.
III. Effects of Energy and Force Sources on Supersonic and Sub-Sonic Flows

5. Methodologies and Scope of Work

5.1. Energy and Force Sources to Imitate Plasma Discharge

In recent years significant work has gone into the use of energy deposition in high speed and low speed flows to serve as active flow control. These include but are not limited to: DC arc discharges, dielectric barrier discharges, corona discharges, RF discharges, microwaves, lasers, and electric resistance heating. The goal of the work presented here is to determine some qualitative effects of adding energy and force sources to the flow independently. Much of the recent work in DC arc discharges has been conducted at speeds greater than supersonic. In addition to investigating the affects on supersonic flows, the research presented in this work also looks into the abilities of these force and energy sources to modify low speed, sub-sonic flows.

5.1.1. Energy Sources

Using various types of energy sources has been shown to be an effective method for changing the surface forces on a body in a high-speed flow. Menart et al. (2004) studied the affects of surface heating via an electric resistance heater and volumetric heating through the use of a DC plasma discharge. Computational and experimental work
demonstrated the ability to cause large changes in the surface pressure at conditions greater than Mach 5. The goal of the work presented here is to build on the experimental results that have been obtained and conduct a large parametric study to determine the effects of adding energy sources to supersonic and sub-sonic flows. Work has been presented by Shang et al. (2004) that utilizes a magneto-aerodynamic version of the conservation equations to determine the affects of plasma sources on supersonic flows. The goal of this work is to perform a large scale parametric study using a commercially available Navier-Stokes solver. FLUENT was chosen to conduct these studies and the energy deposition was achieved by defining an additional source term in the energy equation for the region of study. The study considered a half-wedge plate as seen in Fig. 5.1.

Figure 5.1: Half-wedge model used for supersonic and subsonic flow investigations.
The plate length is represented by $L_p$, the source length by $L_s$, the distance between the leading edge and the source equals $L_e$, and the wedge angle is $\alpha$. The source region is located on the top surface of the plate and was moved in both the vertical and in the flow direction. The thickness of the source was changed dependant upon the direction in which the source was being moved. In addition to moving the source region, the magnitude of the energy deposition was varied using a source region of a constant size and position. The specifics of the supersonic configurations can be seen in Section 7.1 and the sub-sonic configurations can be seen in Section 8.1.

### 5.1.2. Lorentz Force Sources

The Lorentz force defines the force exerted on a moving charged particle subject to a magnetic field. A DC plasma contains moving charged particles and is therefore subject to a Lorentz force when a magnetic field is present. The Lorentz force is defined as

$$\vec{F}_{LR} = I \vec{L} \times \vec{B},$$

(5.1)

where $I$ is the current in amps, $\vec{L}$ is the direction vector of the current in meters, $\vec{B}$ is the magnetic field strength vector in tesla, and $\vec{F}_{LR}$ is the long range Lorentz force in newtons. Menart et al. (2007) has shown that a plasma discharge in a hypersonic wind tunnel can be moved when in the presence of a magnetic field. Examples of effect of the magnetic field on the discharge can be seen in Fig 5.2.
Figure 5.2: Experimentally demonstrated effect of magnetic field on discharge (ref Menart et al., 2007).

It is clearly evident that the magnetic field has a significant influence on the location of the plasma. Surface pressure changes were seen based on the new location of the plasma. The difficulty in interpreting these experimental investigations is the fact that there are multiple mechanisms contributing to the observed changes. Menart and Shang (2005) have identified four of these mechanisms for flow modification. The first is a direct heating effect, which is a volumetric heating of the air by the plasma discharge as discussed in Section 5.1.1. The second is a direct Lorentz force effect where the magnetic field directly effects the bulk movement of the charged particles present in the plasma discharge and this force is directly transferred to the neutral particles. The third mechanism is an indirect Lorentz force. This occurs when the heating location is changed due to the presence of a magnetic field. As the plasma is moved, the region in which the heating is occurring is changing. Lastly indirect heating occurs when the plasma heats the surface of the model, which in turn heats the air. Indirect heating also includes the energy
that is absorbed into the model without being transferred back to the air. The difficulty comes from separating the effects of the direct Lorentz force from the indirect Lorentz forces. The work presented here decouples these effects through the use of computational fluid dynamics. As discussed in Section 5.1.1 FLUENT, a Navier Stokes based flow solver, is used to investigate the effects of these Lorentz forces. The forces are input as source terms within the momentum equation in both the x and y directions. The magnitude of the force was calculated using the Lorentz force equation (Eqn. 5.1). Details regarding the configurations for supersonic and subsonic cases can be seen in Section 7.1 and Section 8.1 respectively.
6. Computational Tools

6.1. FLUENT

As discussed in Chapter 5 the computational tool utilized in this work is the software package FLUENT. FLUENT is a computational fluid dynamics code that gives the user many options for solving the Navier-Stokes conservation equations. These equations are shown in Eqn. 6.1 through Eqn. 6.4, in two-dimensional, unsteady form,

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0, \tag{6.1}
\]

\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho uu)}{\partial x} + \frac{\partial (\rho vu)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[ 2\mu \left( \frac{\partial u}{\partial x} - \frac{2}{3} \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) \right] + \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial x} \left( \mu \left( \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right) \right) \right] + S_{F_x}, \tag{6.2}
\]

\[
\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho vv)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left[ 2\mu \left( \frac{\partial v}{\partial y} - \frac{2}{3} \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) \right] + \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial y} \left( \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right) \right] + S_{F_y}, \tag{6.3}
\]

and

\[
\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho u e)}{\partial x} + \frac{\partial (\rho v e)}{\partial y} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) - p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \Phi + uS_{F_x} + vS_{F_y} + S_T. \tag{6.4}
\]

Equation 6.1 is the continuity equation where \( \rho \) is the air density, \( u \) is the velocity in the \( x \)-direction, \( v \) is the velocity in the \( y \)-direction, and \( t \) is time. Equations (6.2) and (6.3) above are the momentum equations in the \( x \)-direction and \( y \)-direction respectively. In these equations \( \mu \) is the dynamic viscosity, \( p \) is pressure, \( S_{F_x} \) is a force per unit volume...
per time in the $x$-direction, and $S_{F_y}$ is a force per unit volume per time in the $y$-direction.

Lastly, Eqn. (6.4) above is the energy equation where $e$ is the internal energy of the air, $T$ is temperature, $k$ is the thermal conductivity, $\Phi$ is the viscous dissipation term, and $S_H$ is the heat source per unit volume per unit time. FLUENT discretizes the conservation equations above through the use of the finite volume method. This method involves integrating the governing transport equations over every control volume and yields a discrete equation expressing the conservation law on a control-volume basis (FLUENT 2006).

Two general methods are used to solve the discretized conservation equations. These two methods are a pressure-based solver and a density-based solver. For the work presented here the pressure-based solver is used for all sub-sonic work and the density-based solver is used for all supersonic studies. The pressure-based solver used for this work employs the SIMPLE algorithm to decouple the conservation equations. This method solves for the velocities sequentially, followed by a pressure correction step. Once the pressure correction step is completed the mass flux, pressure and velocity are solved for again. Finally the energy, species and turbulence scalars are solved for. This process is repeated until a converged solution is reached. Since the equations are solved sequentially the memory requirements for this method are not as high; however, it is an iterative solution and therefore requires some additional processor time (FLUENT 2006).

The density-based solver solves the discretized governing equations simultaneously. The density-based solver can utilize either an implicit discretization or an explicit discretization. The implicit method uses information from the surrounding cells that is a mixture of known and unknown quantities. This requires that the equations be
solved simultaneously for each iteration. The explicit formulation only uses neighboring values that are already known. This causes only a single unknown to appear in the equation and the solutions may be computed for each governing equation one at a time for each cell. Traditionally the density-based solver has been used in high-speed applications making it the appropriate choice for the supersonic cases considered for this study.

For the supersonic cases a first order upwind scheme is used for spatial discretization. This method assumes that the cell center values of a variable are representative of the cell-average value and are valid throughout the entire cell. This method is first order accurate in space. For the baseline case where there is no energy or force sources the percent change in the average surface pressure between a first order and a second order upwind scheme is 3.3%. The first order scheme has a tendency to over predict the thickness of the shock when compared to the second order scheme; however, given the very small changes in the surface pressure this fact was determined to be of little concern. For several cases, where larger thermal gradients exist due to the large magnitude of the energy source, second order accurate schemes showed significant oscillations in the solution or large divergence. The first order upwind scheme shows minimal problems converging and yielded small errors when compared to the second order scheme and thus was determined to be a better choice. For this reason the first order upwind scheme was selected as the standard spatial discretization method for all supersonic cases.

The subsonic cases were conducted using a second order upwind spatial discretization. For the sub-sonic cases this method proved to be stable and there was no
appreciable increase in computational time. The percent change compared to the average surface pressure for the baseline case was found to be 0.01%. Given this very small change and the fact that no stability problems were encountered, the second order upwind scheme was selected for all subsonic cases.

For the majority of the work the plate was considered to be adiabatic, therefore all the energy deposited by the source was transferred only to the air. To determine approximately how much energy the plate absorbed, a select number of cases were conducted with a thermally conducting plate. To draw comparisons between the results and work presented by Kimmel et al (2006), the thermally conducting plate cases were run transiently for only a few seconds. All other cases were considered to be steady state and therefore the time derivatives of the conservation equations are set to zero. For the unsteady cases considered an implicit time stepping formulation is used. This method employs a pseudo-time step $\Delta \tau$, which is determined by the specified CFL condition. This pseudo-time step is driven to zero for each physical time step through an iterative process (FLUENT 2006). The distinct advantage of this method is that a desired physical time and CFL condition can be specified and the subsequent pseudo-time step used in the formulation is determined within the software.

Boundary conditions are defined using a pressure far field approach. This approach attempts to model the free stream conditions at infinity. The free stream Mach number, static pressure, temperature, flow direction and turbulence parameters are specified at the far field. From these inputs, free stream velocity can be calculated from Mach number and the speed of sound. The density throughout the domain is calculated using the ideal gas law. The resulting free stream boundary conditions are as follows:
\[ u(x, -\infty, t) = u_\infty, \]
\[ u(x, \infty, t) = u_\infty, \]
\[ u(-\infty, y, t) = u_\infty, \]
\[ v(x, -\infty, t) = 0, \]
\[ v(x, \infty, t) = 0, \]
\[ v(-\infty, y, t) = 0, \]
\[ v(\infty, y, t) = 0, \]
\[ p(x, -\infty, t) = p_\infty, \]
\[ p(x, \infty, t) = p_\infty, \]
\[ p(-\infty, y, t) = p_\infty, \]
\[ p(\infty, y, t) = p_\infty, \]
\[ T(x, -\infty, t) = T_\infty, \]
\[ T(x, \infty, t) = T_\infty, \]
\[ T(-\infty, y, t) = T_\infty, \]
\[ T(\infty, y, t) = T_\infty, \]  

(6.5)

where \( u_\infty, p_\infty, \) and \( T_\infty \) represent the specified u-velocity, static pressure and static temperature at infinity respectively.

The four edges of the plate are all handled with adiabatic, no-slip wall boundary conditions for the non-thermally conducting plate. The adiabatic wall is defined such that the heat flux into the wall is zero. The no-slip condition defines the fluid velocity parallel to the wall to be zero at the surface. For the thermally conducting plate, the no-slip condition is maintained. No boundary conditions are required for the energy equation as the static temperatures throughout the plate are calculated. The only item that needs to be included is the different thermal conductivities of the air and the plate.

6.2. Gambit

Gambit is the meshing software that accompanies FLUENT. Physical geometries, boundary conditions, and the meshed solution domain are all created in Gambit and they
exported for use in FLUENT. Various mesh types are available within Gambit. In two-dimensions these are quadrilateral and triangular mesh elements. There is also a function that allows for a combination of these two types of mesh elements. The quadrilateral elements are referred to as a structured mesh and the triangular elements as an unstructured mesh. Structured elements are used for the majority of the solution domain. Any regions that are modeling air are meshed using structured elements. When the thermally conducting plate was considered the plate itself was modeled with an unstructured mesh.

Mesh density was increased near the surface and the leading edge of the plate. This is done to capture boundary layer features as well as the shock formed along the leading edge in the supersonic cases. Mesh height above the surface of the plate was maintained at a sufficiently small value to keep wall $y^+$ values equal to or less than one. This constraint is based on the stable operation of the Spalart-Allmaras turbulence model employed (Roy 2006). The cases where no energy or force sources are present are considered to be the baseline. A mesh convergence study for both supersonic and subsonic baseline cases will be presented in section 6.3.

6.3. Model Validation and Mesh Convergence

Due to the nature of the work, model validation for the cases where energy and force sources are present is difficult to achieve. However, with that said the baseline cases for both the subsonic and the supersonic studies can be investigated and compared to experimental, theoretical, and computational results in the literature. The validation of the supersonic model will be considered first, followed by the subsonic validation.
The primary means of comparison between the model used in this work and previous studies is the static pressure along the top surface of the half-wedge plate. Shang et al. (2004) presents results for surface pressure over a flat plate for high speed flow. These results were based on computational modeling as well as pressure interaction theory. Figure 6.1 shows these results.

![Figure 6.1: Nondimensional surface pressure over a flat plate (ref Shang et al. 2004).](image)

The results from the model used for the cases presented in this work are shown in Fig. 6.2.
Comparisons between the work conducted by Shang and the work presented here were taken at locations of X/L equal to 0.3, 0.5, and 0.7 for both M=5.15 and M=10. For the case of M=5.15 the error between the two sets of results are approximately 12%, 7%, and 6% for the three locations specified. For the M=10 cases the errors are 6%, 5%, and 4% respectively for the three locations. The results presented by Shang et al. (2004) show the same general trend of an asymptotic decay; however the model used for this work tends to decay slightly quicker in the leading edge region. Given the fact that this is a first order accurate scheme the reasonable agreement with the work presented by Shang is encouraging.

Validating the subsonic work proved to be more difficult. Classical results such as the Blasius solution for flow over a flat plate are often used draw comparisons with acquired results. The difficulty in this situation is that fact that the Blasius solution
considers an infinitely thin plate. Due to the geometry presented in this work that assumption is not valid, therefore a comparison between the computational results gathered for this model and the Blasius solution can not be drawn. While significant information is available for traditional airfoil shapes, symmetric wedges, and cylinders, little information is available for the half-wedge geometry in the subsonic flow regime.

By the nature of the subsonic flow regime, it is possible for the computational domain boundary to have an influence on the solution. If the boundary is located too close to the object being studied then reflections and perturbations from the boundary can be felt by the object. To alleviate this problem the boundaries must be placed sufficiently far such that this influence is not felt. A domain size study was conducted to confirm that the original selected domain size was large enough. The original domain was set such that the distance to the boundary in all directions is five times the chord length of the plate. This distance was then doubled to ten times the chord length and the results were compared. Results of surface pressure distribution are shown followed by the integral of the surface pressure with respect to the computational iteration number.
Figure 6.3: Surface pressure distribution for increasing domain size.

Figure 6.4: Surface pressure integral vs. iteration number for differing domain sizes.
The results from both Fig. 6.3 and Fig. 6.4 show an incredibly small effect of increasing the domain size on the plate surface pressure. It should be noted that while there appears to be a difference between the two cases in Fig. 6.4, given the scales this difference is less than 0.01%. With these results it was determined that the baseline domain size was sufficiently large to not have an effect on the plate pressure.

Finally a grid independence study was conducted for both the supersonic and the subsonic cases. For both studies the plate surface pressure is used to determine if there are any effects of mesh density on the solution. The initial mesh used for the baseline is defined such that the wall $y^+$ values fall within an acceptable range based on the Spalart-Allmaras turbulence model (Roy 2006). The size of the solution domain was held constant and the number of total cells within the domain was increased. For both the supersonic and subsonic studies the mesh cell density was doubled and quadrupled from the baseline case. Figures 6.5 and 6.6 show the effects on non-dimensional surface pressure for the supersonic and sub-sonic mesh studies respectively.
Figure 6.5: Nondimensional surface pressure results for supersonic mesh density study.

Figure 6.6: Nondimensional surface pressure results for subsonic mesh density study.
When making comparisons between the baseline and the higher mesh density cases the average nondimensional surface pressure over the plate is considered. For the subsonic results both the 2X and the 4X mesh density cases show less than 0.25% change from the baseline case. The results of the supersonic study show that the percent change for the 2X and 4X cases are less than 1.5%. Given these results it is shown that for the cases considered there is no significant dependence of the mesh density on the surface pressure given the stipulation mentioned previously regarding the wall $y^+$ value.
7. **Supersonic Investigations**

7.1. **Test Configuration and Parameters of Study**

This section will discuss the flow field and model constants as well as the various parameters that were studied for free-stream conditions of Mach 5. For all cases the farfield conditions are specified as: \( M_\infty = 5 \), \( p_\infty = 80 \text{ Pa} \), \( T_\infty = 45 \text{ K} \) and the flow is aligned parallel with the horizontal axis. For adiabatic wall cases no heat flux is allowed into the walls. The thermally conducting plate has the following properties:

\[
k_{\text{plate}} = 36 \frac{W}{m - K}, \quad C_{\text{plate}} = 765 \frac{J}{kg - K}, \quad \rho_{\text{plate}} = 3970 \frac{kg}{m^3},
\]

where \( k_{\text{plate}} \) is the plate thermal conductivity, \( C_{\text{plate}} \) is the plate specific heat, and \( \rho_{\text{plate}} \) is the plate density. For the supersonic investigations three cases were considered and are as follows:

Case 1. Vary the energy magnitude using a fixed location of the source

Case 2. Vary the location of the source using a constant energy input

Case 3. Vary the source location and apply a constant Lorentz Force

Case 1 uses a source height, \( h_s \), equal to 1.02 cm, the source length, \( L_s \), equal to 9.52 cm, and the distance from the leading edge, \( L_c \), is equal to 2.0 cm. The bottom edge of the source is located at the surface of the plate at \( y = 0 \). The energy input per centimeter in the spanwise direction, \( P \), was set to 1, 5, 10, 50, and 100 W/cm respectively for an adiabatic wall. Cases of \( P=5 \text{ W/cm} \) and \( P=50 \text{ W/cm} \) were conducted using the thermally conducting plate.
Case 2 considers three different methods for varying the source location. First the source height was set at 0.25cm. The source length and leading edge distance remained the same as in Case 1. The distance to the centerline of the source was moved vertically up through and out of the boundary layer. The energy input was held constant at \( P = 30 \text{W/cm} \). Secondly the source height was defined as 0.13cm and the distance between the centerline of the source and that plate surface was set at 0.06cm, 0.19cm, and 0.32cm. The energy input was maintained at \( P = 30 \text{W/cm} \). Lastly the source height is set to 1.02cm and the source length is set to 2cm. The bottom edge of the source is placed on the top surface of the plate and the source is moved in the flow-wise direction along the plate. The energy input for each location is held at \( P = 10 \text{W/cm} \).

Case 3 uses the same source locations and configurations as the second part of Case 2 whereby \( L_s = 9.52 \text{cm} \), \( L_c = 2 \text{cm} \), and \( h_s = 0.13 \text{cm} \). The source location was moved vertically such that the centerline distance to the plate surface was 0.06cm, 0.19cm, and 0.32cm respectively. The Lorentz force, \( F_{LR} \), equals 4.25 \( \frac{mN}{cm} \) which corresponds to a current of \( I = 0.05A \) and \( B = 0.9T \). These conditions were selected to match experimental work conducted by Kimmel et al. (2006).

Contour and streamline plots of temperature and density for the baseline, no source case are shown in Figs. 7.1 and 7.2 respectively. These plots can be compared to see the effects of adding energy and force sources.
Figure 7.1: Temperature and streamline contours for Mach = 5 with no source terms.

Figure 7.2: Density and streamline contours for Mach = 5 with no source terms.

7.2. Supersonic Results

7.2.1. Energy Study Results
The energy magnitude study was conducted using a constant volume energy source and a fixed location. This energy source term is considered to be volumetric. Since this work was conducted in two dimensions a spanwise reference length must be specified for the calculation of volumetric terms. By default FLUENT defines this spanwise length to be 1 m. For this reason the energy input must be scaled accordingly. All the results here are scaled such that represent they energy associated with 1 cm in the spanwise direction.

![Figure 7.3: Energy magnitude results for supersonic study.](image)

The results from Fig 7.3 show the surface pressure on the top of the plate divided by the free-stream pressure. The energy sources range from 1W/cm up to 100W/cm. Thermally conducting plate cases are shown for 5W/cm and for 50W/cm. The important information garnered from this data is the extremely large changes associated with fairly small energy inputs. An energy input of 100W/cm into the flow yields a 247% change in
the surface pressure when compared to the no source term case. Even a source term of only 5W/cm yields a change of 21% in the surface pressure. This phenomenon can best be explained through inviscid-viscous interaction (Shang et al. 2005). The presence of the boundary layer acts to change the actual shape of the plate as seen by the free-stream flow (Anderson 2000). The increase in the displacement thickness at the leading edge of the plate deflects the flow outward and compression waves form over the surface of the plate. This outward deflection coalesces into an oblique shock. The addition of an energy source lowers the local density through an increase in the air temperature. The boundary layer displacement thickness is increased in this region of lower density and this greater displacement thickness creates a greater change in the surface pressure through the inviscid-viscous interaction (Shang et al. 2005). This effect can be seen in Figs. 7.4 and 7.5.

Figure 7.4: Density and streamline contours for P=100W/cm.
Figure 7.5: Density and streamline contours for P=5W/cm.

Both figures show an order of magnitude decrease in the density at the surface of the plate compared to the free stream value. They also show the increase in the density across the oblique shock. In can be seen in Fig. 7.4 that a second shock has formed in response to the larger region of lower density gas.

The other important information presented by Fig 7.3 is the effect of a thermally conducting plate on the surface pressure. To achieve these results the solver is run to steady state prior to the energy source being enabled. The plate acts as a heat sink, absorbing energy from the source. This in turn reduces the temperature of the air over the plate. This reduction in temperature increases the density, shrinking the displacement thickness, thereby reducing the pressure increase associated with the inviscid-viscous interaction. For the 50W/cm case the average pressure change between the conducting and the adiabatic plate is –6% over the whole length. However, if only the source region is considered this pressure change increases to –13%. Figures 7.6 and 7.7 show the density contours for the adiabatic and the thermally conducting plate respectively. It
should be noted that the thermally conducting plate cases were run transiently to a time of five seconds.

Figure 7.6: Density and streamline contours for P=50W/cm.

Figure 7.7: Density and streamline contours for P=50W/cm with a thermally conducting plate at t=5s.

Figures 7.6 and 7.7 clearly show the changes associated with the plate absorbing energy from the source. The secondary shock is still formed for the conducting plate case, but is
not as pronounced as the adiabatic plate case. The absolute maximum temperature between these two cases decreases by approximately 50% from the adiabatic to the conducting plate case.

Three experimental data points are presented in Fig. 7.3. These data points were taken from surface pressure data presented by Kimmel et al. (2004). The data presented by Kimmel relates the surface pressure above the plate for a case where there is air flow and the discharge are related to the no flow and no discharge static pressure. These results were recalculated to compare the surface pressure to the free-stream pressure. The experimental results show a change of approximately 10% for a 60W case. Based on the size of the discharge, the experimental power input has to be scaled to be appropriately compared to the computational results which are based on so many watts per cm width of plate. The resulting experimental energy input is \( P = 19 \text{W/cm} \). The experimental results lie between the 5W/cm and the 10W/cm computational results. This is indicative of the fact that a substantial amount of energy is not being used to heat the air. The development of the plasma excites vibrational energy states that do not result in temperature increases until much further downstream of the plate. This energy lost to other processes is not considered in the modeling conducted in this work.

Menart et al. (2005) has experimentally demonstrated the ability of a magnetic field to change the location of a plasma generated over the surface of a flat plate in a Mach 5 flow. The results presented here seek to determine the effects of moving the energy source vertically from the surface of the plate up through the boundary layer. This was undertaken for two sets of conditions. The first considered a source thickness of 0.25cm and the second used a source of 0.13cm thickness. The 0.25cm cases will be
presented first. The pressure distributions of the plate for the various cases are depicted in Fig. 7.8. The y-location values taken at the centerline of the source region are defined by, Y. The energy input is equal to 30W/cm for all cases. The Y=0.13cm case has the bottom edge placed at the surface and shows a similar trend to those results presented in Fig. 7.3. As expected the surface pressure decreases as the source is moved further away from the plate. This again is due to the sources reduced ability to modify the boundary layer displacement thickness. Much of the influence on the pressure is based on the presence of additional oblique shocks that are formed as a result of the density gradients created by the source region. This can be seen in Fig 7.9.

![Figure 7.8: Effect of vertical location of source on surface pressure distribution in Mach 5 flow, P=30W/cm, h_s=0.25cm.](image-url)
In Fig. 7.9 a second oblique shock can be seen above the source region. In addition the shock created at the leading edge of the plate collides with the shock coming off of the lower portion of the source region. This is the cause of the higher density in the region between the plate and the source. Figure 7.6 shows that the effect on the pressure for y=1.40cm is not felt by the plate until further downstream. This is due to the way a shock propagates in a flowing gas.

In order to better understand the effect of heating within the boundary layer a thinner source region was used. The source height was reduced to 0.13cm and was moved vertically to a maximum height of y=0.32cm. Again these cases were conducted for an energy input of 30W/cm. The same configuration was run for both an adiabatic and a conducting plate. Figures 7.10 and 7.11 show the pressure distribution for the adiabatic case and the thermally conducting cases respectively.
Figure 7.10: Effect of vertical location of source on surface pressure distribution in Mach 5 flow, $P=30W/cm$, adiabatic plate, $h_s=0.13\text{cm}$.

Figure 7.11: Effect of vertical location of surface pressure distribution in Mach 5 flow, $P=30W/cm$, thermally conducting plate, $h_s=0.13\text{cm}$, $t=3s$. 
Maximum temperatures on the order of 1600K were observed in flow for the adiabatic wall cases. An energy balance was conducted using average velocities and properties within the source region. Given the 30W/cm heat input the energy balance yielded results that are reasonably close to the results from the computational simulation. As shown previously in the energy magnitude studies the thermally conducting plate has the effect of decreasing the surface pressure. This is again due to the absorption of energy by the plate and the subsequent decrease in the air temperature. One interesting observation is the effect of the vertical location of the source on the pressure. Even within a very small region, once the source is lifted off the surface the heat sink effect of the plate is drastically reduced. Once the source is 0.32cm off the surface the difference between the adiabatic and the conducting wall results are very small. This shows that when considering the thermal effects of the energy source on the flow, moving the heated region a few millimeters off the surface can have significant impact on the effectiveness of the actuator.

Lastly the energy source was placed on the surface of the plate and moved downstream in the flow direction. This is done to simulate the moving of the electrodes used to generate the plasma. Menart et al. (2006) has shown that the lift over a flat plate in a Mach 5 flow can be affected by the location of the cathode. Their results showed that the optimum location for cathode placement was near the leading edge of the plate. The work presented here seeks to replicate a similar scenario to be studied computationally. The results for pressure and temperature distribution on the top surface can be seen in Figs. 7.12 and 7.13 respectively.
Figure 7.12: Pressure distributions for changing flow direction energy source.

Figure 7.13: Temperature distributions for changing flow direction energy source.
The pressure distributions shown in Fig. 7.12 show approximately the same magnitude for the peak pressure. However the x=1 cm has a slightly larger peak pressure and has a greater effect downstream. When compared to the baseline no source case, the x=1 cm case shows an 11.2% increase in the average pressure over the plate. By comparison the x=7 cm case only exhibits a 9.7% change. Given the fact that these cases were studied using a 10W/cm source term, these pressure changes are significant. The tendency for higher overall pressure changes with the heating source at the leading edge again is explained by taking advantage of the viscous-inviscid interaction.

The temperature distributions show that as the source is moved downstream the temperature along the surface of the plate increases. This is due to the thickening of the boundary layer along the length of the plate. As the boundary layer thickens the average fluid velocity within the source region decreases. The change in temperature is inversely proportional to the fluid velocity. Therefore as the velocity of the air is decreased due to the thickening boundary layer, the temperature will increase in that region for a constant heat input.

The results presented here have shown that significant pressure changes can be realized through the addition of energy sources in a supersonic flow. By modifying the displacement thickness it is evident that one can take advantage of the viscous-inviscid interaction to aid in the increase in the surface pressure. It is shown that the plate itself absorbs significant amounts of energy from the source when the source is placed at the surface, but this effect decreases as the separation between the source and the plate increases. One final comment must be made; being a purely Navier-Stokes based solver, chemical reactions, dissociation, ionization, vibrational excitation, rotational excitation,
electronic excitation, etc. are not considered within this model. Each of these processes absorbs and releases energy. Some of these processes trap the energy in excited states that do not manifest as temperature until far downstream of the plate. The results presented here are intended to give qualitative trends, not quantitative results. To achieve more accurate results higher level methods of plasma modeling must be employed.

7.2.2. Lorentz Force Results

As mentioned in Section 7.1 the Lorentz force for the supersonic studies was held constant and the vertical location of the source region was varied. For each location four scenarios were investigated. The force vector was oriented in both the positive and negative x-direction and in both the positive and negative y-direction. Since the Lorentz force is a function of the cross product of the magnetic field vector and the length vector (see Eqn. 5.1) the direction of the force can be modified by changing the magnetic field orientation, the direction of current flow, or both. Figures 7.14 through 7.17 show the effects of vertical source position on surface pressure distribution for each of the four directions.
Figure 7.14: Surface pressure distributions for a negative y-direction force of 4.25mN/cm.
Figures 7.14 and 7.15 show the effects of positive and negative vertical forces on the flow as a function of the vertical location of the source. As one would expect the negative y-direction forces result in an increase in surface pressure and the positive y-direction forces result in a decrease in surface pressure. Kimmel et al. (2006) show that as a positive Lorentz force acts on the plasma the pressure increases and when a negative force is applied the pressure decreases. This is contrary to the results presented here for Lorentz force effects. The primary influence on the experimental results presented by Kimmel et al. (2006) is heating location. In the experimental cases the Lorentz force effect is very small when compared to the effect of heating. If it were possible to remove the heating from the experimental test it is the belief of the author that the trends would match the computational results. Figure 7.11 shows that as the heating location is moved
off of the plate the pressure increases due to a reduction in the influence of the wall reducing gas temperature. It is believed that these wall effects have an even greater influence on the experimental results than the computational results.

Figure 7.16: Surface pressure distributions for a negative x-direction force of 4.25mN/cm
Figures 7.16 and 7.17 show the effects of negative and positive x-direction forces on the pressure distribution over the surface of the plate. One of the most interesting results of this work is that applying a force in the counter-flow direction has the greatest influence on the surface pressure. It was a surprise to the author that given the large inertial force associated with Mach 5 flow that a very small counter-flow force could have such a large effect. It is the belief of the author that this force caused a local increase in the displacement thickness, thus causing stronger shocks to be formed. The positive x-direction force term acts to accelerate the flow over the plate. This higher speed flow reduces the boundary layer thickness and also decreases the pressure along the surface.

It has been shown experimentally by Menart et al. (2005) and by Kimmel et al. (2006) that the application of a magnetic field has an effect on surface plasma discharges.
This work attempted to isolate the effect of the Lorentz force on the plasma. It has been shown that when compared to the heating effect, Lorentz forces are very small. Given the results presented here it is the opinion of the author that the dominating effect on the surface pressure is the location of the volumetric heating associated with the plasma discharge. This location was changed by an externally applied magnetic field. As the discharge is pushed away from the plate, the pressure increases due to an increase in the temperature. When the discharge is driven downward toward the plate, the plate absorbs substantial amounts of energy, decreasing the temperature of the gas. With this information the author has shown that the dominant effect on surface pressure in a Mach 5 flow is the location of the heating source, not the effect of the Lorentz force.
8. **Subsonic Investigations**

The objective of investigating the effects of arc plasmas in the subsonic regime is two fold. First the author seeks to decouple the combined effects of heating and Lorentz force seen in previous hypersonic experimental work. By considering cases in a low flow speed regime the inviscid-viscous interaction is no longer present. In addition to this fact, the effects of viscous heating on the plate are negligible. For these reason the movement of the discharge due to the application of a Lorentz force should have minimal thermal effect on the pressure distribution. Secondly the author wishes to determine if a DC arc discharge is a valid means of flow control at low speed.

### 8.1. Test Configuration and Parameters of Study

The test conditions for the sub-sonic investigations were chosen to match conditions for experimental work that followed. The farfield parameters were specified to correspond with the subsonic wind tunnel facility housed at Wright State University. These conditions are as follows: \( M_{\infty} = 0.058, \ p_{\infty} = 96259 \ Pa, \ T_{\infty} = 293 \ K \). \( M_{\infty}=0.058 \) corresponds to a flow velocity of 20 m/s for the specified free-stream conditions.

The plate geometry is defined as follows: \( L_p = 10.12 \ cm, \ L_s = 7 \ cm, \) and \( \alpha = 30^\circ \) (see Fig. 5.1 for plate geometry). Again as done in the supersonic investigation the study here is broken down into the addition of energy sources and forces sources. The energy source cases considered both the magnitude of the energy source using a fixed source region and the effect of source location using a fixed magnitude. The study of the addition of forces
only considered the magnitude of the force in both positive and negative $x$-directions as well as positive and negative $y$-directions. The magnitude studies for both energy and force were conducted using a source region defined by $h_s=1.02$ cm and $L_s=7$ cm. The energy source was varied from $P=100\text{W/cm}$ up to $P=2000\text{W/cm}$. The force source ranged from $0.14\text{N/cm}$ up to $140\text{N/cm}$. To study the effect of the energy location on the surface pressure, the energy source magnitude was selected to be $1000\text{W/cm}$ and the source was moved from the surface up to a location of $Y=1.4$ cm. The source thickness for the energy location studies was selected as $h_s=0.25$ cm.

For sake of comparison density, temperature, velocity, and streamline contours for some of the various cases will be shown. These contours for the baseline case where no source terms are present can be seen in Fig. 8.1, Fig. 8.2, and Fig. 8.3 respectively.

![Figure 8.1: Density and streamline contours for baseline subsonic flow case with no source.](image)

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8.2. Results for Sub-Sonic Investigations

8.2.1 Energy Source Results
The effect of the magnitude of the energy source will be discussed first, followed by the effect of energy source location. To determine the effect of the energy source on the plate two quantities were considered. The first being the ratio of the surface pressure with the free-stream pressure. The second considered the change in the pressure coefficient over the top surface of the plate. This change in the pressure coefficient is defined as

\[ \Delta C_p = \frac{p_{source} - p_{no\_source}}{\frac{1}{2} \rho_\infty u_\infty^2}, \quad (8.1) \]

where \( p_{source} \) represents the static pressure with a source term present, \( p_{no\_source} \) is the static pressure with no source term present, \( \rho_\infty \) is the free-stream density, and \( u_\infty \) is the free-stream velocity.

Figure 8.4: Pressure distributions for various energy source magnitudes.
Figures 8.4 and 8.5 show the effect of the magnitude of the energy source on the pressure distribution and the changes in the pressure coefficient respectively. The rise in the pressure in a region around X/L=0.2 is due to a slight stagnating of the flow due to the presence of the energy source. Over the surface of the plate the density can change by two orders of magnitude for the P=2000W/cm case. The large density gradients act as a flow obstruction. The reduction in the pressure towards the aft end of the plate is due to the lower density of the flow. This explains why the higher power cases show higher pressures towards the leading edge of the plate and lower values towards the trailing edge. Density and x-velocity contours for the P=2000W case are shown in Fig. 8.6 and Fig. 8.7.
It should be noted that while the effects on pressure exist, they are very small. The maximum change in average pressure when compared to the baseline no source case is approximately 0.07%. The changes in the pressure coefficient are shown to be larger. When comparing cases of $P=100\text{W/cm}$ and $P=2000\text{W/cm}$, there is a 43% change in the
average pressure coefficient along the plate. It should also be noted that these changes require a significant amount of energy to be developed, especially when compared to changes seen in the Mach 5 cases discussed previously. The lack of the viscous-inviscid interaction effect present at Mach 5 greatly reduces the effectiveness of using density changes to modify the pressure distribution.

In addition to studying the effects of energy magnitude on subsonic flows, the effect of energy location is also studied. For this work a source of P=1000W/cm was selected and moved vertically above the surface of the plate. The results for the surface pressure distribution can been seen in Fig. 8.8.

![Figure 8.8: Pressure distributions for various source locations P=1000W/cm.](image)

As with the cases where energy magnitude was considered, changing the location of the energy source has very little effect on the surface pressure. Changes are on the order of 0.2% when compared to the baseline case. The temperatures were as high as
5000K and the local density in that region is an order of magnitude smaller. This again indicates that while significant changes to the flow field can be created with the use of an energy source, the ability to use these changes to create modifications to the surface forces on the plate is negligible.

8.2.2. Lorentz Force Results

The goal of the work presented here is to decouple the combined heating and force effects as was done in the Mach 5 results. By moving to a subsonic regime the heating effects on surface pressure are shown to be very small. These results attempt to show the possible potential of an actuator that is independent of thermal effects. As with the cases studied at Mach 5, the Lorentz force source term is calculated from Eqn. 5.1. This equation is used to determine the magnitude of the force. The force is then oriented in all four directions mentioned in section 8.1.
Figure 8.9: Pressure distributions for various magnitude forces oriented in the positive y-direction.

Figure 8.10: Pressure distributions for various magnitude forces oriented in the negative y-direction.
The pressure distribution results for the positive and negative y-direction force sources are shown in Fig. 8.9 and Fig. 8.10 respectively. As one would expect, as the force is directed into the plate the pressure increases and when the force is directed away from the plate the pressure decreases. The results show that while the sign of the change is different, the magnitude is approximately equal for positive and negative y-direction cases. The largest changes in the pressure are approximately 2%. Figure 8.9 shows some variance in the pressure towards the trailing edge of the plate for the larger force cases. The forces here are large enough to cause significant flow field modification resulting in a jet like effect that manifests itself around X/L=0.8. This is clearly seen in the y-velocity contours and streamlines shown in Fig. 8.11. Figure 8.10 shows a similar non-uniformity towards the leading edge of the plate. When the force directed downwards into the plate is large enough, a plume begins to develop. This plume can be strong enough to overcome the momentum of the free-stream flow and cause a flow reversal. This phenomenon is easily depicted in the x-velocity contour and streamline plot shown in Fig. 8.12.
Figure 8.11: Y-velocity and streamline contours for F=70N/cm oriented in the positive y-direction.

Figure 8.12: X-velocity and streamline contours for F=70N/cm oriented in the negative y-direction.

The resulting pressure changes associated with the F=70N/cm cases are approximately 1%. Using a constant magnetic field strength of B=0.2 T and source length
of $L_s=7 \text{cm}$, the current required to generate a force of 70N/cm is 50 amps. This is a large amount of current to achieve such a small change in surface force.

Lastly the results for positive and negative x-direction forces are presented. The pressure changes in these cases are smaller than those presented for the y-direction force studies; however, the flow field modifications can be much greater for the higher force cases. Pressure distributions for positive and negative x-direction force sources are shown in Fig. 8.13 and Fig. 8.14 respectively.

![Figure 8.13: Pressure distributions for various magnitude forces oriented in the positive x-direction.](image_url)
Figure 8.14: Pressure distributions for various magnitude forces oriented in the negative x-direction.

The positive x-direction forces result in an acceleration of the flow over the surface of the plate. This results in an x-velocity approaching 50 m/s for a $F=14 \text{N/cm}$ case. This acceleration leads to an increase in the pressure towards the trailing edge of the plate as seen in Fig. 8.13. X-velocity and streamline contours for the $F=14 \text{N/cm}$ case are shown in Fig. 8.15. The negative x-direction force study provided some of the largest surprises to the author. When the force is equal to $14 \text{N/m}$ and directed opposite the flow direction a large vortex bubble forms over the majority of the plate. This relatively small force is able to completely overcome the inertia of the flow near the surface of the plate. As the force is increased this effect is even greater resulting in the reversal of very large portions of the domain. The x-velocity and streamlines contours for a force directed in the negative x-direction with a magnitude of $14 \text{N/cm}$ are shown in Fig. 8.16.
The results shown in this section clearly show that for subsonic flow the use of energy sources associated with a DC arc plasma offer little ability to create changes in the surface pressure over a flat plate. Force sources show much larger changes in surface
pressures but they are less than a couple percent. However, even a couple percent change at atmospheric pressure is a significant change in the force on the plate. Large flow field changes are also associated with the source forces.
9. Experimental Subsonic Study

The objective of studying the effects of magnetic fields in subsonic conditions is to attempt to isolate the Lorentz force effects independent from the heating effects that are so prevalent in the hypersonic regime. In addition to isolating the Lorentz force effects on the bulk air flow, the author wishes to investigate the potential of taking advantage of the long range forces that exist between the current within the plasma and magnets located in the plate. A description of the experimental facility and apparatus will be discussed in Section 9.1 followed by the results in Section 9.2.

9.1. Experimental Facility and Testing Procedures

All of the experimental work presented here took place in the low speed wind tunnel at Wright State University. The tunnel is an open circuit design capable of delivering air speeds in the test section from 0.6 m/s up to 36 m/s. The inlet pressure is considered to be atmospheric, while the pressure in the test section is 0.91 atmospheres for an air speed of 36 m/s. Figure 9.1 and Fig. 9.2 show a schematic diagram and a picture of the tunnel respectively.
Figure 9.1: Schematic diagram of low speed wind tunnel at Wright State University.

Figure 9.2: Wright State University low speed wind tunnel facility.

The flow enters the inlet and passes through a grid of hexagonally shaped flow straighteners. The tunnel undergoes an area reduction of 6.25:1 from the inlet to the test section. The test section is square perpendicular to the flow direction and has side wall lengths of 0.6 m. Downstream of the test section is the diffuser. The diffuser has a nominal wall angle of 6 degrees. Connected to the diffuser via a flexible coupling is an
axial flow fan driven by a 20 hp, 3-phase electric motor. The fan speed is controlled via a variable frequency drive unit. This unit varies the frequency of the supply voltage which in turn changes the operating speed of the motor. The inlet and diffuser sections are constructed from fiberglass reinforced plastic, while the test section is constructed from Plexiglas for easy viewing.

A flat plate, half-wedge model was chosen to conduct the investigations into the effects of magnetic fields on plasmas in subsonic, atmospheric conditions. This type of model was chosen to draw comparisons to experimental results conducted at Mach 5. The model is 7.6 cm in length in the flow-wise direction and 5.3 cm in length in the spanwise direction. The model is 1.3 cm in thickness and has a half-wedge leading edge angle of 43 degrees. Figure 9.3 shows the model mounted in the test section of the wind tunnel. Two parallel pockets were cut into the bottom of the model to house the permanent magnets utilized to test the long range Lorentz force interaction between the magnets in the plate and the current. The permanent magnets are 5.3 cm in length and approximately 1.1 cm in thickness.
Figure 9.3: Half-wedge model mounted in test section.

The model is attached to the side wall of the test section by an aluminum rod. In order to determine the changes in the forces on the model a laser displacement meter is located below the model. Since aluminum deforms elastically for the ranges of stress considered, the deflection in the rod is proportional to the force exerted to the end of the rod. This ratio of force as a function of deflection was calibrated using a series of weights. Each time the model was moved a new calibration constant was determined. For all cases the calibration yielded a model sensitivity of approximately $0.0035 \frac{N}{\mu m}$.

During the course of testing the discharge, displacements were observed that required a significant amount of time to decay. These observations were most prevalent in the cases where there was no air flow through the tunnel. Due to the incredibly high temperatures experienced by the model when the plasma is being generated, it is the belief of the author that thermal expansion was playing a significant role. In the original configuration the laser displacement meter was placed directly under the far edge of the
model. When the plasma is activated on the top surface of the plate, very large temperature gradients exist between the top surface and the bottom surface. These gradients result in non-uniform thermal expansion, causing the plate to curve slightly downward. To help alleviate this problem the laser displacement meter was moved to a point on the cantilever rod itself. This greatly reduced the influence of thermal expansion on the displacement results.

The greatest difficulty in conducting arc plasma work in atmospheric conditions is the voltage required to initiate and to maintain the arc. In order to get around this problem the arc is struck by bringing the carbon electrodes into contact and gradually drawing them apart. Data was collected over the entire time the electrodes were being drawn apart. Therefore the discharge length is increasing with time. The limiting factor in these experiments proved to be the power supply used. A Miller XMT 304 welding power supply was used. This unit is a current driven supply capable of delivering up to 100 amps. Based on the resistance of the arc the power supply will vary the supply voltage in order to maintain the specified current. In practice the maximum voltage supplied is approximately 60 volts. This was insufficient to achieve a discharge length equal to the length of the magnets. When the magnets are placed in the model the discharge length is significantly reduced. Therefore the results presented for the Lorentz force include the effects of both the current in the discharge as well as the current flowing through the electrodes. The configuration of the model and the electrodes can be seen in Fig. 9.4. The upstream electrode is held in a fixed location and the downstream electrode is attached to a translator capable of moving in all three directions.
9.2. Experimental Results

This section will present results for three specific cases. These cases will include the following: no air flow and no magnets, 23 m/s airflow without magnets, and finally 23 m/s airflow with magnets in place. The cases presented for no air flow and no magnets will indicate the effects on displacement by heating of the plate. For all cases conducted with the air flow on, the tunnel was turned on and the model was allowed to reach a stable, steady-state condition prior to initiating the arc. A positive force is oriented upward and a negative force is directed downward.

For all results presented in this chapter current is represented by the blue line, voltage by the green line, and force by the red line. Current and voltage are measured off of the left vertical axis and the force off the right vertical axis. All results are presented as a function of time. All data sets were taken for 30 seconds.
The first results presented are for the case where there is no flow and the magnets are not in the model. Figure 9.5 shows the results for current, voltage, and force. The change in the force here is purely a function of the thermal expansion of the plate. If the change in force were associated with a volumetric heating of the air then the force should have diminished when the discharge was extinguished. The residence time of the air over the surface of the plate is 3.3 ms. Therefore any changes associated with heating of the air should be seen on that same time scale. The effects shown in Figure 9.5 are on the order of seconds, indicating that this cannot be attributed to the heating of the air. This heating corresponds to a maximum deflection of approximately 35 µm. It should be noted that the effects when no magnets are in place are of the same order of magnitude whether the cathode is placed upstream or downstream.

Figure 9.5: Current, voltage, and force measurements for high current setting with no air flow and no magnets with cathode downstream.
To show the influence of air flow on the plate heating, Fig. 9.6 shows current, voltage, and force measurements for a 23 m/s flow with no magnets in place and a downstream cathode. Figure 9.6 shows an 80% reduction in deflection from the case with no airflow. It will be shown in following results that this deflection associated with plate heating is insignificant when compared to the long range Lorentz force.

![Figure 9.6: Current, Voltage, and Force measurements for high current setting with 23 m/s air flow, no magnets with cathode downstream.](image)

Lastly the results for a 23 m/s airflow with the magnets in place are presented. A series of tests were conducted with the cathode located upstream and with the cathode located downstream. Referring back to Eqn. 5.1 it should be noted that the Lorentz force is a cross product of the magnetic field and the current-length vectors. The magnetic field vector is maintained constant. The length vector is taken to be in the direction of the current flow. By changing the cathode from the upstream to the downstream position the direction of the current flow is reversed, therefore the direction of the force is reversed.
Figure 9.7: Current, voltage and force measurements for high current setting with 23 m/s air flow, with a 0.8 T magnetic field at the electrode location with cathode downstream.
Figures 9.7 and 9.8 both show that as the current changes so does the magnitude of the force. When the cathode is located in the downstream position the resulting Lorentz force is in the positive direction and when it is placed in the upstream position the force is negative. The nominal power supply current was set to 50 amps. It is evident that there is some change in this current over time. This is largely a result of the process of drawing the electrodes apart as well as electrode wear. For both of these cases there is excellent agreement between the nominal current settings and the magnitude of the force. It should also be noted that as soon as the discharge extinguishes the force returns to approximately zero.
Figure 9.9: Current, voltage, and force measurements for high current setting with 23 m/s air flow, with a 0.8 tesla magnetic field at the electrode location with cathode downstream pulsing the current.

To further reinforce this point Fig. 9.9 shows a case where the current is pulsed on and off. This figure shows that the force follows directly with the current indicating that the resulting deflection is purely a function of the Lorentz force. The discharge was only on between 2 and 3 seconds. This is not a sufficient amount of time to cause significant plate heating and thermal expansion. Also at the initiation of the current there is a strong current spike. The resulting force approaches 1 N.
Figures 9.10 and 9.11 show the effects of positive and negative Lorentz forces on the discharge. A positive force acts to push the discharge out away from the model. Conversely a negative force drives the discharge down into the plate. It became apparent that the magnetic field strength was great enough to overcome the electric field strength.
between the electrodes. This resulted in the discharge being extinguished after the electrodes are separated only a small amount. In order to overcome this problem a power supply capable of delivering much higher voltages while delivering approximately 60 amps is required.

Figure 9.12 shows a comparison of the measured force values and compares it to a calculation of the Lorentz force as a function of current. A magnetic field strength of 0.08 Tesla and a length of 0.053 m were used to calculate the Lorentz Force. For most of the experimental points there is good agreement with the Lorentz force calculation corresponding to a magnetic field strength equal to 0.08 tesla. However, there are groupings that show either more negative or more positive force. The relative changes in the magnitude for both regions are similar. These other groupings indicate that the magnetic field strength was increased to 0.13 tesla. It is the belief of the author that this increase in the magnitude of the force is due to the deflection of the electrodes. One must remember that the magnetic field strength is inversely proportional to the square of the distance. Therefore a small reduction in the distance can yield a large change in the magnetic field in that region. A larger magnetic field will result in a greater force. These forces associated with the electrodes changing positions are always in the direction of greater magnitude, and for a constant current this can only be explained by an increase in the magnetic field strength.
It has been shown that volumetric heating associated with a plasma discharge has little to no effect on the lift forces generated by a flat plate in low speed flow. These experimental results match well with the computational studies presented earlier. The most significant forces are due to the long range Lorentz force of the current interacting with the permanent magnets located in the plate. The changes attributed to the Lorentz force can not be due to changes in surface pressure. The forces measured experimentally act in the opposite direction of those witnessed in the computational results (see Figs. 8.9 and 8.10). In order to match the computational results a discharge much larger than the one achieved in the experiments would need to be present and that discharge would need to be closer to the surface of the plate. The majority of the Lorentz force measured can be attributed to the current in the electrodes. Due to the weak electric field the magnetic field acted to extinguish the discharge after only a very short gap was achieved. If this is
considered to be a viable method of actuation, power supplies with higher voltage capacities at high currents are required.
10. **Energy and Force Sources Conclusions**

Through the course of this work a large parametric study was conducted computationally to determine the effects of energy and force sources on both supersonic and subsonic air flows. Due to the nature of the solution method these results are meant to show trends as opposed to showing precise quantitative results for an actual situation using plasma discharges. The complex physics associated with the plasma is not taken into consideration in this work. However a better understanding of the types of effects that can be accomplished with plasmas under different operating conditions has been achieved.

In the supersonic investigations it was shown that relatively small energy sources can cause significant changes to the surface pressure on the plate. This is done by taking advantage of the viscous-inviscid interaction and thereby changing the shock location or angle. It was also shown that the location of the source has a large impact on its effect on the plate. Sources located on the surface of the plate were less effective due to the amount of energy that is absorbed by the plate itself. It was shown that moving this source only a few millimeters off the surface drastically reduces the energy absorption by the plate. Force sources showed similar abilities to cause changes to the surface pressure; however, when compared to the effect of heating on the supersonic flow the force effects were small.

Subsonic results showed that with very large energy sources which produced incredibly high temperatures within the flow yielded little to no effect on the surface
pressure on the plate. This result was independent of the magnitude of the energy source or its location. The same can be said for the force sources but it must be remembered that a 1% change in the value of $\frac{p}{p_\infty}$ results in a sizeable change in the force on the plate. When one considers that freestream pressure to be defined as 96259 Pa, a 1% change is significant. It has been shown that force sources of 14 N/cm can produce complete reversal in the flow field above the plate. It should be noted that in the sub-sonic regime, the oblique shocks that form in the supersonic regime are not present and therefore can not be taken advantage of to cause pressure changes.

Previous experimental work conducted by Kimmel et al. (2006) used an externally applied magnetic field to cause changes in a supersonic flow field. The authors wanted to investigate the potential of taking advantage of the Lorentz force interaction between the current and magnets housed within the plate. These magnets would push off of the current running through the plasma causing deflection. The experimental work did confirm that heating had minimal effect on the plate forces when compared to the long range Lorentz forces. While the Lorentz force effect was witnessed, the length of the discharge obtained was very small. If a more suitable power supply were used then longer discharges could be achieved. It is the opinion of the author that given the necessary power supplies this method could prove effective; however, the power requirements are going to be very large if this device is to be used in atmospheric pressure conditions.
11. References


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