Model and Solution Approaches for the Equipment Scheduling under Disruption Problems in USPS Mail Processing and Distribution Centers

Arvindkumar Ravi Chakravarthy

Wright State University

Follow this and additional works at: http://corescholar.libraries.wright.edu/etd_all

Part of the Engineering Commons

Repository Citation
MODEL AND SOLUTION APPROACHES FOR THE EQUIPMENT SCHEDULING
UNDER DISRUPTION PROBLEMS IN USPS MAIL PROCESSING AND
DISTRIBUTION CENTERS

A dissertation submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

By

ARVINDKUMAR RAVI CHAKRAVARTHY
M.S., Wright State University, 2002

2008
Wright State University

Xinhui Zhang, Ph.D.
Dissertation Director

Ramana V. Grandhi, Ph.D.
Director, Ph.D. in Engineering Program

Joseph F. Thomas, Jr., Ph.D.
Dean, School of Graduate Studies

Committee on Final Examination

Xinhui Zhang, Ph.D.

S. Narayanan, Ph.D., P.E.

George G. Polak, Ph.D.

Yan Liu, Ph.D.

James T. Moore, Ph.D.
ABSTRACT


This research addresses the equipment scheduling problem under disruptions in United States Postal Service mail processing and distribution centers. These facilities contain a large variety of equipment and employ a non-homogeneous workforce that work on shifts of various lengths and start times. The scheduling of equipment (the determination of the configuration and usage of equipment to match mail arrivals) and the scheduling of workforce (the determination of the optimal size and composition of the workforce, their days off / lunch assignments, and overtime usage) to meet processing service commitment with a constantly changing demand are some of the most challenging problems.

Over the years, there have been many research studies that focused on solution of the postal equipment and staff scheduling problems. A comprehensive review of these studies is conducted. In the most general sense, each of the equipment and staff schedule problems can be decomposed temporally so and hierarchical analytic approaches have been adopted. Along the time axis, these studies can be classified into strategic, tactical and operational levels.

This thesis focuses on the operational equipment scheduling problem or equipment scheduling under disruptions and addresses the adjustment of production plans and workforce schedules through the use of overtime and flexible employees in the face of disruptions such as demand fluctuation and absenteeism that happen on a
daily basis and may significantly change demand and the size of workforce. This problem is modeled as a large-scale integer program, which contains equipment scheduling, shift scheduling and overtime management, and break assignment modules. Comprehensive experiments have been designed to investigate the effects of the use of overtime, the control of absenteeism, and the importance of integrating equipment and workforce scheduling simultaneously. The model integrates seamlessly with other research studies and provides the necessary tools to manage the resources in a facility on a routine basis.

To improve computational time, an efficient LP based decomposition algorithm has been developed. The algorithm uses linear programming solutions as target solutions to construct a local search process to examine neighboring integer solutions. The heuristic was first proposed for the equipment scheduling under disruptions and then extended to the staff scheduling problem where multiple diverse initial solutions were generated to cover the solution landscape. These heuristics were computational efficient and were able to quickly obtain high-quality feasible solutions and deliver final solutions on par with the state of the art branch and bound algorithm in the solution of integer programs.

**Keywords:** Postal Operations, Equipment Scheduling, Workforce Scheduling, Overtime Management, Integer Programming
TABLE OF CONTENTS

1. INTRODUCTION .................................................................................................................. 1

1.1 Introduction to Scheduling Problems at Mail Processing and Distribution Center .............................................................. 1

1.2 The Contribution of the Dissertation Research .............................................................. 3

1.3 Organization of the Dissertation Research ....................................................................... 4

2. REVIEW OF MODELS AND METHODOLOGY FOR SCHEDULING PROBLEMS IN

USPS MAIL PROCESSING AND DISTRIBUTION CENTERS ........................................... 5

2.1 Overview of the Research Problems and Models ............................................................... 5

2.2 Literature Review ........................................................................................................... 12

2.2.1 Production Planning and Staff Scheduling Models in Practice ...................................... 12

2.2.2 P&DC Staff Scheduling at Strategic Planning Level ....................................................... 14

2.2.3 P&DC Equipment Selection and Scheduling at Strategic Planning Level ....................................................... 15

2.2.4 P&DC Equipment and Staff Scheduling Under Disruptions at Tactical Planning Level ....................................................... 16

3. EQUIPMENT SCHEDULING PROBLEM UNDER DISRUPTION IN P&DCs ....... 18

3.1 Problem Statement ......................................................................................................... 18

3.1.1 Mail Processing Activities and Equipment .................................................................. 18

3.1.2 The Use of Non-Standard Shifts and the Assignment of Breaks .................................. 20

3.1.3 Disruptions -- Demand Fluctuation and Absenteeism ............................................... 21
3.1.4 The Use of Overtime and Called-in Casuals to Handle Fluctuations .. 24

3.2 The Problem ....................................................................................................... 24

3.2.1 Mathematical Models ........................................................................... 25

3.3 Computational Improvements ........................................................................... 34

3.4 Experimental Results .......................................................................................... 38

3.4.1 Experiment 1 - The Impact of Absenteeism ........................................... 39

3.4.2 Experiment 2 - The Impact of Overtime ................................................. 42

3.4.3 Experiment 3 - Integrated (Holistic) Modeling of Equipment and Staff Schedule ........................................................................................................ 46

4. THE STAFF SCHEDULING MODEL AND COMPUTATIONAL ALGORITHM ............ 49

4.1 The Baseline Model for Staff Scheduling ......................................................... 50

4.2 Techniques to Generate Alternative Solutions to Sample the Solution Space ................................................................................................................. 55

4.3 Linear Programming Based Neighborhood Structures for Integer Programs........................................................................................................... 59

4.3.1 LP Based Neighborhood In the literature ................................................. 59

4.3.2 LP Based Neighborhood for the Postal Staff Scheduling Problem .......... 61

4.4 Computational Results ....................................................................................... 67

5. SUMMARY ....................................................................................................... 72

APPENDIX A ..................................................................................................... 74

APPENDIX B ...................................................................................................... 78
LIST OF FIGURES

Figure 2.1: Relationships among the models under discussion ......................................... 8

Figure 3.1: Major Operations and mail flow for letter processing

at Dallas facility ........................................................................................................... 20

Figure 3.2: Total Arrival for Each Day of the Week for Four Weeks

from Dallas Facility ..................................................................................................... 22

Figure 3.3: Arrival patterns for Monday (dotted line) and

the average Monday (solid line) .............................................................................. 23

Figure 4.1: Solution Process of B&B and Heuristic ..................................................... 70

Figure A-1: Arrival Profiles of four Mondays at Dallas Facility .................................... 74

Figure A-2: Arrival Profiles of four Tuesdays at Dallas Facility .................................... 75

Figure A-3: Arrival Profiles of four Wednesdays at Dallas Facility ............................... 75

Figure A-4: Arrival Profiles of four Thursdays at Dallas Facility .................................. 76

Figure A-5: Arrival Profiles of four Fridays at Dallas Facility ...................................... 76

Figure A-6: Arrival Profiles of four Saturdays at Dallas Facility .................................. 77

Figure A-7: Arrival Profiles of four Sundays at Dallas Facility ..................................... 77
# LIST OF TABLES

Table 3.1: Full-time and part-time shift types included in the model ........................................ 21
Table 3.2: Overtime costs under different absenteeism ratios .................................................. 40
Table 3.3: Cost under different overtime ratios ........................................................................ 42
Table 3.4: Cost under different maximum overtime hours ...................................................... 43
Table 3.5: The use of overtime to reduce workforce size and increase productivity .......... 45
Table 3.6: Comparison of cost obtained from two models ...................................................... 47
Table 4.1: The size of the problem (I) - Variables .................................................................. 53
Table 4.2: The size of the Problem (II) -- Constraints ............................................................. 54
Table 4.3: Complete set of optimal solutions to the sample problem ..................................... 58
Table 4.4: Fractional Solution for Full Time Shifts ................................................................. 61
Table 4.5: Integer Solution for Full Time Shifts ...................................................................... 62
Table 4.6: Fractional Solution for Part Time Shifts ................................................................. 62
Table 4.7: Integer Solution for Part Time Shifts ................................................................... 63
Table 4.8: Distance between LP and IP for Full Time Shifts .................................................. 63
Table 4.9: Distance between LP and IP for Part Time Shifts .................................................. 64
Table 4.10: The reduced Problem under NHF(P) ................................................................. 65
Table 4.11: Comparison between B&B and the Heuristic Algorithm ...................................... 68
Table B-1: Absenteeism Ratio for Dallas P&DC ................................................................. 78
Table C-1: Overtime Cost under Different Absenteeism Ratios for Week 1 ......................... 79
Table C-2: Overtime Cost under Different Absenteeism Ratios for Week 2 ......................... 80
Table C-3: Overtime Cost under Different Absenteeism Ratios for Week 3 ......................... 81
Table C-4: Overtime Cost under Different Absenteeism Ratios for Week 4 ................. 82
Table D-1: Comparison of cost obtained from two models for Week 1....................... 83
Table D-2: Comparison of cost obtained from two models for Week 2....................... 84
Table D-3: Comparison of cost obtained from two models for Week 3....................... 85
Table D-4: Comparison of cost obtained from two models for Week 4....................... 86
Table E-1: Alternative LP Solution 1 for Full Time Shifts ........................................ 87
Table E-2: Alternative LP Solution 1 for Part Time Shifts ........................................ 88
Table E-3: Alternative LP Solution 2 for Full Time Shifts ........................................ 89
Table E-4: Alternative LP Solution 2 for Part Time Shifts ........................................ 90
Table E-5: Alternative LP Solution 3 for Full Time Shifts ........................................ 91
Table E-6: Alternative LP Solution 3 for Part Time Shifts ........................................ 92
ACKNOWLEDGEMENTS

I am extremely thankful to Dr. Xinhui Zhang for giving me the opportunity to conduct research under his supervision. He has guided me all through my research by pointing my efforts in the right direction, and giving me the freedom to explore different research strategies. I am grateful for his consent to conduct my research remotely that allowed me to gain valuable experience working as a consultant for Honda of America Manufacturing in Sidney, Ohio and Valassis in Livonia, Michigan. He boosted my confidence in being able to perform my tasks by lauding me on good work and giving critical reviews in areas where I could improve.

I also express my gratitude to Dr. S. Narayanan, Dr. James T. Moore, Dr. George Polak and Dr. Yan Liu for reviewing my thesis and for being a part of my thesis committee.

Finally, I would like to take the opportunity to thank my parents Mrs. Indra Ravi and Mr. Ravi Chakravarthy, my brother Mukund Chakravarthy and my wife Sobhana for all their love, moral support and encouraging me to pursue my Ph.D. Their inspiration and affection helped me overcome challenging moments in my graduate school experience.
Dedicated to:

My Parents, Brother and Wife
1. INTRODUCTION

1.1 Introduction to Scheduling Problems at Mail Processing and Distribution Center

The United States Postal Service (USPS) is in the business of delivering mail to every household in the United States in a timely fashion. In the year 2004, it delivered 206 billion mail pieces to more than 142 million homes and businesses. The success of this large operation relies on a large network of approximately 275 major behind-the-scenes processing facilities which sort, barcode and sequence mail or packages through large-capital equipment and large labor pools. These processing facilities are complex manufacturing systems with a network of production structures, constant changing of exogenous demands and complicated workforce composition. Though various studies have been conducted on the logistics design in these industries and certain aspects of the processing analysis, the key to the operational success of the delivery logistics network with high efficiency and low costs i.e. the planning and management of resources in these facilities, has largely been overlooked.

These facilities run 24 hours a day, 7 days a week and operate a complicated manufacturing system – disassemble mail arrivals, sort and dispatch them to other facilities. To ensure timely processing, the facilities contain a large variety of advanced equipment in the form of optical character readers, automated facer cancellers and barcode sorters for automated mail processing and employ a non-homogeneous
workforce composed of full-time, part-time and casual employees that work on shifts with various lengths and start times. The scheduling of equipment (the determination of the configuration and usage of equipment to match mail arrivals) and the scheduling of workforce (the determination of the optimal size and composition of the workforce, their days off/lunch assignments, and overtime usage) to meet processing service commitment with a constantly changing demand are some of the most challenging problems and their solution is critical to the success of USPS operations.

Over the years, there have been many research studies that focused on the solution of the postal equipment and staff scheduling problems. On the equipment scheduling side, the early studies on the equipment scheduling side include the facility design model by Bard et al. (1993) and the equipment selection model by Jarrah et al. (1994a) in an effort to configure a facility and decide its capacity. Zhang and Bard (2005) addressed the equipment scheduling problem for use with workforce scheduling models in an effort to derive an optimal permanent workforce.

On a daily basis, however, various disruptions such as demand fluctuation and absenteeism could happen and could significantly change demand and the size of workforce. How to adjust production plans and workforce schedules through the use of overtime and flexible employees in the face of these disruptions to meet the service commitment is a challenging problem that has not yet been solved. This equipment scheduling under disruption problem is the focus of this study.
1.2 The Contribution of the Dissertation Research

This equipment scheduling under disruptions is to adjust production plans and workforce schedules through the use of overtime and flexible employees in the face of disruptions such as demand fluctuation and absenteeism to meet the service commitment. The problem is complicated by stringent service commitment, complicated mail processing process, the various rules governing the assignment of breaks as well the overtime usage. To solve this problem, an integrated approach that addresses both equipment scheduling and staff scheduling is necessary. In view of this, a large-scale integer program, which contains equipment scheduling, shift scheduling and overtime management, and break assignment modules is proposed, each addressing a different part of the problem.

Comprehensive experiments have been designed to investigate the effects of the use of overtime, the control of absenteeism, and the importance of integrating equipment and workforce scheduling simultaneously. The model integrates seamlessly with other research studies and provides the necessary tools to manage the resources in a facility on a routine basis. Results from the Dallas facility suggest financial savings in the tens of millions of dollars annually could be achieved when the model is implemented nationwide.

To improve computational time, an efficient LP based decomposition algorithm has been developed. The algorithm uses linear programming solutions as target solutions to construct a local search process to examine neighboring integer solutions. A decomposition algorithm was first proposed for the equipment scheduling under
disruptions and then extended to the staff scheduling problem where multiple diverse initial solutions were generated. Combined multiple initial solution generation with neighborhood search, could lead to novel neighborhood search to the solution of general integer programs.

1.3 Organization of the Dissertation Research

The remainder of the proposal is organized as follows. Chapter 2 provides an overview of the scheduling problems in the P&DC, the various models developed and the relationships among them. This is followed by the mathematical model and computational improvement for the equipment scheduling under disruption problem in Chapter 3. The computational algorithm based on linear programming relaxation is further extended in Chapter 4 where the development of an effective neighborhood search to generate multiple diverse linear programming solutions that could be used together with LP based neighborhood search for general integer programs and its application to staff scheduling model, is presented. Finally, concluding remarks are given in Chapter 5.
2. REVIEW OF MODELS AND METHODOLOGY FOR SCHEDULING PROBLEMS IN USPS MAIL PROCESSING AND DISTRIBUTION CENTERS

This chapter presents an overview of the scheduling problems in the literature. The models developed for these problems are classified into strategic, tactical and operations models with the relationships among them illustrated. The review provides an overview picture of the models currently implemented in practice and points out the need to address the equipment scheduling problem under disruptions.

2.1 Overview of the Research Problems and Models

The postal equipment and workforce scheduling problems are some of the most challenging problems seen in industry. To better understand the complexity of the problems, several terms used to describe the characteristics of the facilities are first defined.

*Mail Arrival Profile:* Mail arrives throughout the day and an arrival profile stipulates the amount of mail received during a specific time of the day and its characteristics. The arrivals follow a highly fluctuating pattern that varies from hour to hour, and the total volume could be anywhere from 3 to 5 million pieces.
Operation, Equipment and Equipment Scheduling: Depending on a letter’s characteristics, upon arrival, it may require several operations before it is finally dispatched. An operation is performed with a piece of equipment; the equipment, however, is capable of processing several operations. Equipment scheduling determines the optimal size, and use of equipment to ensure the prompt processing of mail with the least labor cost.

Shifts and Shift Scheduling: To operate these machines, a non-homogeneous workforce is employed; each member of the workforce could work on many of the possible shifts with various lengths and start times. Shift scheduling finds the optimal crew size and their assignments to satisfy demand during each time period of the day.

Days Off and Its Assignment: To construct an employee’s weekly schedule, it is necessary to specify the days off and as such, sufficient slack must be provided through the week so that the days-off requirement is satisfied for every worker. Typically, two consecutive days off is preferable to an employee, but there is no strict restriction for it.

Break and Its Assignment: A lunch break is required for all shifts that exceed a certain length. For the USPS, the practice is to create a break window -- a set of consecutive periods for every shift during which a break can be given. Because an employee is off the clock, there should be sufficient resources to cover for him.

Staff Scheduling: The staff scheduling problem determines the optimal size and composition of the workforce and their assignments to make sure that the demand (determined by machine activities) in each time period of the week is satisfied. In the
most comprehensive form, staff scheduling includes shift scheduling, days-off assignments and break assignments.

**Disruptions (I) – Demand Fluctuation**: A major disruption that affects the equipment and workforce schedule is departure from normal demand. Historic data show that dramatic seasonal and daily variations in the total amount of mail exist, and the arrival patterns could also differ significantly from hour to hour within a day.

**Disruptions (II) – Absenteeism**: Another major disruption that affects the equipment and workforce schedule is employee absenteeism. For the USPS, absenteeism could vary anywhere between 6% and 21% and significantly reduce the size of the workforce.

**Schedule Adjustment and Overtime Management**: To handle disruptions, equipment schedules have to be adjusted to ensure processing commitments and additional labor resources such as called-in workers and overtime, must be scheduled to complement the workforce schedule. Overtime management is used to optimally assign overtime to employees while observing contractual and union rules to match adjustment processing activities.

With these terms defined, the various models developed can be examined. Along temporal lines, these studies can be classified into three levels: strategic planning, tactical planning and operational planning levels. These models are listed under each level below and the relationships among them are shown in Figure 2.1. Here, the blocks represent the models; the solid arcs represent the dependence relations and the dashed arcs represent the extension relations. The studies on the equipment scheduling side
The models are listed under each level as follows.

**Figure 2.1: Relationships among the models under discussion**

**Strategic Planning Level:** The problem at this level is how to design a facility, to determine the capacity and makes of the equipment, and to evaluate the impact of various workforce policies.

**Facility Design:** Bard et al. (1993) studied the facility design problem and presented a two-level approach that started with a large-scale mixed integer program. The solution was then used as input to a simulation model that was used to investigate operational issues related to service standards, growth in mail volume, and the use of new equipment.

**Equipment Selection:** Jarrah et al. (1994a) studied the equipment selection problem. The problem was to make a choice among multiple machine makes. A mixed integer linear program was first solved to select equipment and propose a tentative schedule. This was followed by a linear program to compress or eliminate the idle time of the machines.
**Workforce Composition:** Berman et al. (1997) studied the workflow management and workforce composition problem. The problem was modeled as a queuing network and linear programming was used in the analysis. This study examined various policies such as full time-to-part time ratio, the switching of jobs during a day, etc.

**Tactical Planning Level:** The problem here is how to generate an equipment schedule that matches mail arrivals and to determine the optimal size and composition of a permanent workforce. Supplement issues include how to select the best arrival profile to use in these analyses and how to address uncertainty in the demand while configuring the workforce.

**Equipment Scheduling:** Zhang and Bard (2005) studied the equipment scheduling problem. The problem took as the input the average arrival profile, whose selection was proposed in the arrival profile selection model (Bard, 2004a) and tried to determine the optimal use of equipment. The problem was modeled as an integer program and a surrogate shift covering constraint was used to capture labor costs to provide a link to combine equipment with workforce scheduling.

**Staff Scheduling:** Jarrah et al. (1994b) were the first to study the staff scheduling problem. The problem was modeled as an integer program that combined shift scheduling and days-off scheduling in a unified manner. These ideas were expanded in Bard et al. (2003) with several new features incorporated. The staff scheduling problem was to find the optimal size and composition of a permanent workforce to meet the
demand (generated from the equipment scheduling) and was the kernel of tactical planning.

**Staff Scheduling with Down Grading:** Bard (2004b) extended the staff schedule problem to a multi-skilled workforce. Demand was specified by skill type, and in the downgrading analysis, a person in a higher skill category could be assigned a job in a lower skill category, but at the higher rate of pay. A mixed-integer linear programming model for this problem was developed based on staffing requirements.

**Stochastic Staff Scheduling:** Bard et al. (2007) extended the staff scheduling problem that took demand uncertainty into consideration. They proposed a two-stage stochastic integer program with recourse for the analysis. In the first stage, before the demand was known, the number of employees was determined. In the second stage, demand was revealed and workers were assigned to specific shifts over the week. When necessary, overtime and casual workers were used to satisfy the demand.

**Arrival Profile Selection:** Bard (2004a) studied the selection of the best arrival profile when running the staff scheduling model. Because demand varied throughout the year, the choice of the input data was crucial. If a week of low volume was selected, the solution might call forth an insufficient number of workers; if a week of high volume is chosen, excessive idle time might result. The selection of the best “average” arrival profiles in these analyses was solved using an efficient trial and error approach to find the lowest volume whose slack is sufficient to cover all weeks of greater volume without exceeding the guidelines for use of overtime, part timers and casuals.
**Operation Planning Level:** The problem here is how to adjust the equipment and employee schedules to meet the service commitment in the face of various disruptions such as demand fluctuations and employee absenteeism.

**Weekly Staff Scheduling under Disruptions:** Bard and Wan (2005) studied the weekly staff scheduling problem under disruptions. The problem here was how to adjust employee schedules by overtime assignment, slight modification of employee configuration such as increasing the number of part-time hours, and calling in temporary workers in response to an updated demand (generated from equipment schedule under disruption).

**Equipment Scheduling under Disruptions:** Zhang et al (2008b) studied the equipment scheduling under disruptions and proposed an integer program for the analysis. The model takes as input the actual arrival profile and workforce and attempts to make the optimal adjustments to equipment and staff schedules to meet processing commitment. The model is solved for each day of the coming week. Several analyses were conducted to evaluate the effects of the use of overtime and the controlling of absenteeism.

**Staff Scheduling with Movement Restriction:** Bard and Wan (2008) later extended the weekly staff scheduling problem under disruptions to a multi-skilled workforce when movement restrictions exist between workstation groups. A new model is proposed that integrates WSG restrictions with the shift scheduling and task assignment constraints. The model takes the form of a large-scale integer program and is solved with one of the decomposition heuristics.
Other Models: Several other problems have also been studied. Particularly, Judice et al. (2004) proposed an integer program for a lot-sizing and workforce problem in these facilities. Wang et al. (2005) studied sequencing the processing of incoming mail in order to match a given outbound truck delivery schedule. Qi and Bard (2006) proposed a simulation and optimization technique to generate the staff requirements.

In the following sections, a brief overview of the production planning and staff scheduling Models in practice is presented and a detailed description of the three kernel models in this framework is given, namely, the equipment scheduling model by Zhang and Bard (2005), the staff scheduling model by Bard et al. (2003), and the weekly workforce scheduling model by Bard and Wan (2005), that are currently implemented in the USPS and have brought millions of savings to the USPS. The focus here is to present the various aspects of postal staff scheduling problems as well as the role equipment scheduling plays in the management of demand for staff scheduling models. Much technical and managerial insight is presented with latest results on significant cost reduction, especially when equipment and staff scheduling are modeled in an integrated manner.

2.2 Literature Review

2.2.1 Production Planning and Staff Scheduling Models in Practice

Equipment and staff scheduling have received much attention in the operations research literature due to their vast applications in practice. The equipment scheduling
problem for Processing and Distribution Center (P&DC) is essentially a production planning problem and can be modeled as a lot sizing problem. Models for production planning have evolved from single-level un-capacitated lot sizing problems to complicated multi-level lot sizing problems with setup times and multiple resource constraints (workforce is typically incorporated in production planning as a flexible resource which can be resized dynamically through hires, fires and overtime usage). The staff scheduling problem includes shift scheduling and assignment of days-off in a week, but the problem can get more complicated with the inclusion of flexible workforce with different shift lengths, multiple skill categories, assignment of breaks, overtime limitations, and managerial considerations such as start time rules. Though there have been some studies on the schedule of production planning and workforce under disruptions (Yang et al. 2005, Easton and Goodale 2005), these studies focus only on one part of the problem and have not provided a holistic model that would address the complete problem yet. For surveys on production planning, please see Shapiro (1993), on staff scheduling, please see Ernst et al. (2004).

Now let us look at the studies on equipment and staff scheduling at P&DCs. Most of these studies began in the 1990s when the USPS sponsored a series of studies of its operations to become financially self sufficient. These studies address the different aspects of the problems (Chakravarthy et al, 2008) and along temporal lines can be classified into strategic and tactical planning models.
2.2.2 P&DC Staff Scheduling at Strategic Planning Level

For the scheduling problems on the staff side, Berman et al. (1997) studied the workforce composition problem. The problem was modeled as a queuing network and linear programming was used in the analysis to find the best full-time to part-time ratio and various labor policies.

Jarrah et al. (1994b) were the first to study the staff scheduling problem. This problem was modeled as an integer program that combined shift scheduling and days-off scheduling in a unified manner. These ideas were expanded in Bard et al. (2003) with several new features incorporated. In essence, these two models attempted to find the optimal size and composition of a permanent workforce to satisfy an average demand profile and to construct weekly tours. Because demand varied throughout the year, the choice of the arrival profile becomes crucial. The selection of the best “average” arrival profiles in these analyses was later addressed by Bard (2004a) where an efficient trial and error approach to find the lowest volume whose slack was sufficient to cover all weeks of greater volume without exceeding the guidelines for use of overtime, part timers and casuals. These models are being tested and will be implemented nationwide in the next few years.

Several complementary staff scheduling models were recently studied with the staff scheduling of multi-skill workforce with downgrading by Bard (2004b), the staff scheduling with workforce movement restriction by Wan and Bard (2007), and the robust staff scheduling by Bard et al. (2006).
2.2.3 P&DC Equipment Selection and Scheduling at Strategic Planning Level

To introduce the scheduling problems on the equipment side, one has to bear in mind that equipment scheduling and staff scheduling are closely related – equipment schedule determines machine activities and machines need to be operated by a workforce; the former as a front-end optimizer and determines the demand for workers of the latter. This suggests that to derive good equipment schedules, staff requirements must be included in an equipment scheduling model to avoid suboptimal solutions that would otherwise occur when they are solved separately.

The early studies on the equipment scheduling side include the facility design model by Bard et al. (1993) and the equipment selection model by Jarrah et al. (1994a) in an effort to configure a facility and decide its capacity. In these models, staff requirements were roughly estimated as a portion or workload. Though this technique might be adequate for these models, it is not suited for an equipment scheduling model where equipment schedule is used to generate demand for the optimization of multiple shifts in staff scheduling.

Zhang and Bard (2005) were the first to address the equipment scheduling problem. They proposed a lot-sizing model with shift covering constraints. The novelty of the model is the inclusion of a surrogate for labor costs in the form of shift covering constraints to provide a link to combine equipment scheduling with workforce scheduling models. Results show the cost of running the facility can be dramatically reduced -- savings in the order of $1.6 million on staffing costs per facility were achieved with the use of the equipment scheduling system.
2.2.4 P&DC Equipment and Staff Scheduling Under Disruptions at Tactical Planning Level

On a daily basis, however, the schedules from the above long term equipment and staff scheduling models are not applicable, let alone optimal, due to various disruptions such as demand fluctuation and absenteeism. In the face of these disruptions, new models have to be developed and the goal here is to find the optimal adjustments to the established equipment and workforce schedules to meet the postal service commitment.

The study of weekly staff schedule (adjustment) under disruption was recently proposed in Bard and Wan (2005). Their model began with a set of permanent employees whose work patterns were nominally fixed and attempted to make adjustments in their schedules by altering shift starting times, assigning overtime, and calling in temporary employees in a way that minimized the marginal labor costs. An integer program was used in the analysis to determine daily shifts and a post-processor is used to construct weekly tours for part-timers and casuals.

The critical requirement of the weekly staff adjustment model, the demand of workers, however, has not been addressed in their study. As the previous experience shown in Zhang and Bard (2005), the generation of demand is critical to the overall quality of solution obtained. The proper generation of the adjusted demand could only be solved through an integrated optimization of equipment and staff scheduling under disruptions -- the focus of this research.
Despite the abundant research on production planning and workforce scheduling, holistic models that address production and staff scheduling under disruptions is rare. This research could offer deep insights in understanding the interplay of equipment and staff schedules that goes beyond P&DC operations and represent a measurable improvement over production and staff scheduling procedures used in manufacturing and service industries.
3. EQUIPMENT SCHEDULING PROBLEM UNDER DISRUPTION IN P&DCs

This chapter formally presents the equipment scheduling problem under disruptions, the mathematical models, computational improvements and the results.

3.1 Problem Statement

Equipment scheduling under disruption is complicated in that it has to address the stringent service commitment, complicated mail processing activities, the rules on the use of overtime and break assignments that exist in practice. To understand the complexity of the problem, it is necessary to look at the processing activities, the fluctuations in demand, the extent of absenteeism, the use of flexible shifts, and the rules on overtime usage and break assignments.

3.1.1 Mail Processing Activities and Equipment

Mail arriving at a P&DC can be categorized as letters, flats and bundles. For illustration purposes, only letter processing is presented in this paper. The automated processing of a letter follows three steps: (1) canceling the stamp (if one exists), (2) reading the address and identifying the destination with a barcode, and (3) sorting the letter to its destination.
Four types of equipment are currently deployed in P&DCs for letter processing: (1) advanced face-canceller systems (AFCSs), (2) multi-line optical character readers (MLOCRs), (3) remote barcode sorters (RBCSs), and (4) delivery barcode sorters (DBCSSs).

The processing or production of mail is referred to as an operation which is typically performed on specific types of machines. To meet the service commitment and to allow timely dispatch of mail to other facilities, an end time is usually specified for an operation; similarly, a start time can be assigned to an operation to allow the accumulation of mail. This defines the time window of an operation. To prepare an operation, a certain setup time is required; so is the clearance time required to clean the machines for the next operation.

To give a complete structure of the mail flow, the major operations and the activities for letter processing at the Dallas facility are shown in Figure 3.1. Here, arrows represent mail flows and nodes represent the processing operations (the identification numbers and the equipment used for these operations are also included).
3.1.2 The Use of Non-Standard Shifts and the Assignment of Breaks

To cope with non-uniform mail arrival, P&DCs employ a non-homogeneous workforce that is composed of full-time regulars, part-time regulars, part-time flexibles, and casuals. A regular employee has a predetermined start time for every working day. Flexible employees and casuals are not necessarily given a 5-day a week schedule, but are called in when needed.

These employees work on shifts of various lengths and start times. A full-timer works either a standard shift (SS) of 8½ consecutive hours and can work on one overtime shift (OS): 9½, 10½, 11½, or 12½ hours. A part-timer, on the other hand, may
be assigned one of five different shift lengths from 4 to 8 ½ hours. For reference, all the possible shifts for a full-time and part-time employee are adopted from the study by Bard et al. (2003) and are shown in Table 3.1.

Furthermore, for USPS, a ½ hour lunch break is required for all shifts that last 6 or more hours. The practice here is to create a break window -- a set of consecutive periods (between the 9th and 12th period) during which a break must be given. Because an employee is off the clock during the break, there must be sufficient resources to cover for him or her.

Table 3.1: Full-time and part-time shift types included in the model

<table>
<thead>
<tr>
<th>Shift type</th>
<th>Start times</th>
<th>Shift lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-time</td>
<td>7:00, 8:00, 9:00 (a.m.)</td>
<td>SS: 8½ hours (17 periods)</td>
</tr>
<tr>
<td></td>
<td>3:00, 4:00, 5:00 (p.m.)</td>
<td>OS: 9½, 10½, 11½ and 12½ hours</td>
</tr>
<tr>
<td></td>
<td>8:30, 9:30, 10:30 (p.m.)</td>
<td>(19, 21, 23, 25 periods, respectively)</td>
</tr>
<tr>
<td>Part-time</td>
<td>7:00, 8:00, 10:00, 11:00 (a.m.)</td>
<td>4, 5, 6½, 7½ and 8½ hours</td>
</tr>
<tr>
<td></td>
<td>1:00, 2:00, 4:00, 5:00 (p.m.)</td>
<td>(8, 10, 13, 15 and 17 periods, respectively)</td>
</tr>
<tr>
<td></td>
<td>7:00, 8:00, 9:30, 10:30 (p.m.)</td>
<td></td>
</tr>
</tbody>
</table>

3.1.3 Disruptions -- Demand Fluctuation and Absenteeism

On a daily basis, various disruptions occur. The two most significant disruptions are 1) departure from normal demand and 2) absenteeism such as vacations and sick leave.

P&DCs face dramatic fluctuations in the amount of mail being processed. The amount of mail arrives could vary dramatically depending on a daily basis. Figure 3.2
shows the total arrivals on different days for a period of four consecutive weeks in the Dallas facility. As we can see, the average total number of mail pieces in a day over these four weeks is around 5.0 million. However, the lowest could be nearly 60% below of the average demand, and the highest 32% above the average demand.

Figure 3.2: Total Arrival for Each Day of the Week for Four Weeks from Dallas Facility

Further, the arrival patterns can also vary dramatically on a daily basis. Figure 3.3 plots the arrivals (dotted line) of a particular Monday and the average arrivals (solid line) of the four Mondays from the above four-week arrival profile. Though a general trend of arrival can be observed, arrivals could be significantly different from the average values. While the average number of mail pieces received on these four Mondays is 4.8 million, the actual numbers for these weeks are 4.3, 6.2, 4.9 and 4.0 million mail pieces, respectively, with the lowest being 19% below of the average, and the highest being 27% above the average.
The arrival patterns for each day of the week for these four consecutive weeks from the Dallas facility is presented in Appendix A under Figure A-1, Figure A-2, Figure A-3, Figure A-4, Figure A-5, Figure A-6, Figure A-7, respectively.

The other type of disruption is absenteeism. Absenteeism can occur due to a vacation, known as annual leave (AL), or a sick leave (SL) and the sum of them is the total leave (TL). For USPS, absenteeism ranges anywhere between 6% and 21% and varies from year to year. (To see the extent of variation of absenteeism, in Appendix B, Table B-1 presents the percentage of sick leave, annual leave and the absenteeism for the Dallas facility in the year 1999, 2000 and 2001. A fiscal year is divided into 13 equal accounting periods (AP) comprising of 4 weeks each. As can be seen, the absenteeism ratios for the three years can be as high as 12.92%, 13.38% and 13.69% respectively. On an average, this absenteeism ratio comprises of 4.23% and 8.69% in 1999, 4.37% and 9.01% in 2000, and 4.41% and 9.28% in 2001 in SL and AL respectively).
3.1.4 The Use of Overtime and Called-in Casuals to Handle Fluctuations

To meet the service commitment to process and dispatch mail in a timely manner, these facilities resort to the use of overtime and called-in casuals in the face of these disruptions. Though overtime usually requires premium pay, it cuts the capita expenses such as pensions, retirement benefits, health care benefits if a permanent workforce has to be hired and thus is widely used. Casual employees are called in only when needed and are primarily used when the demand sharply exceeds the production capacity.

Rules for assigning overtime are somewhat complicated but, in general, the USPS limits it to no more than 6% of the total hours worked by a full time regular employee. Notice that part timers and called-in casuals do not get the overtime premium and thus are not restricted by these rules. The 6% ratio does not refer to a maximum in any day or week but to an annual average; however, this value is used as the default for computations. Imposing an annual value would require a much larger model to keep track of employees’ yearly activities, is computationally intractable and thus is not adopted.

3.2 The Problem

The equipment scheduling (adjustment) problem is to adjust the established equipment and workforce in the face of disruptions. The problem takes as input the forecast arrival profile and the available workforce and attempts to find the most efficient adjustment to equipment and employees’ schedules by assigning overtime and increasing casual
employees to match the machine processing activities and to process the mail in a timely fashion.

### 3.2.1 Mathematical Models

To solve this problem, it is necessary to consider simultaneously the scheduling of equipment and workforce and address union rules on the use of overtime and the assignment of lunch breaks. The mathematical model proposed here consists of three modules: equipment scheduling, shift scheduling, overtime management, and break assignments. Each module deals with a different aspect of the problem and is discussed separately below.

#### 3.2.1.1 Module 1: Equipment Scheduling - A Multi-level Lot Sizing Problem

The first module is the scheduling of equipment. An equipment schedule is the specification of the start times for each operation to be performed in a facility, the machine type to be used for that operation, and the corresponding end times. To keep track of the specific activities throughout the day, the day is divided into 48 half-hour periods. The following notation is used in the development of this module.

**Indices**

- $i, o$ indices for input and output mail streams
- $p, n$ indices for operations
- $m$ index for machine groups
\( t \) index for time periods; \( t \in T = \{1,\ldots,48\} \)

**Sets**

- \( I, O \) input and output mail streams
- \( M, M(n) \) all machine groups and machine groups capable of performing operation \( n \)
- \( N, N(m) \) all operations and operations performed by machine group \( m \)
- \( P(n) \) operations immediately preceding operation \( n \)
- \( I(n), O(n) \) input mail streams to and output mail stream from operation \( n \)
- \( T(n) \) periods during which operation \( n \) can be performed
- \( T(i), T(o) \) periods during which input mail \( i \) is accepted or output stream \( o \) is processed.

- \( a_i(t) \) amount of external mail of stream \( i \) arriving in period \( t \); \( t \in T(i) \)
- \( q_m(t) \) number of machines available in group \( m \) during period \( t \)
- \( \rho_n \) processing rate for operation \( n \)
- \( f_{pn} \) fraction of mail processed at predecessor operation \( p \) that is sent to operation \( n \)
- \( \tau_1, \tau_2 \) time required to start up or clear a machine

**Decision Variables**

- \( v_n(t) \) inventory level of operation \( n \) at the end of period \( t \)
- \( w_{mn}(t) \) amount of mail processed for operation \( n \) by machine group \( m \) during period \( t \); \( t \in T(n) \)
- \( Y_{mn}(t) \) number of machines devoted to operation \( n \) by machine group \( m \) during period \( t \); \( t \in T(n) \)
$Z^1_{mn}(t)$ number of startups at the beginning of period $t$; $t \in T(n)$

$Z^2_{mn}(t)$ number of clearance at the end of period $t$; $t \in T(n)$

The mathematical model for the equipment scheduling module is as follows.

\[ v_n(t-1) + \sum_{i \in I(n), i \in T(i)} a_i(t) + \sum_{m \in M} \sum_{n \in N} f_{pn} w_{mp}(t-1) - \sum_{m \in M} w_{mn}(t) = v_n(t) \]

\[ \forall n \in N, t \in T \] (1)

\[ w_{mn}(t) + \frac{\tau_1}{30} \rho_n Z^1_{mn}(t) + \frac{\tau_2}{30} \rho_n Z^2_{mn}(t) \leq \rho_n Y_{mn}(t) \]

\[ \forall m \in M(n), t \in T(n), n \in N \] (2)

\[ Z^1_{mn}(t) - Z^2_{mn}(t-1) = Y_{mn}(t) - Y_{mn}(t-1) \]

\[ \forall t \in T, m \in M, n \in N \] (3)

\[ \sum_{m \in N(m)} Y_{mn}(t) \leq q_m(t) \]

\[ \forall t \in T, m \in M \] (4)

$Y_{mn}(t)$ is integer

\[ \forall t \in T(n), m \in M(n), n \in N \] (5)

Constraint (1) keeps track of the inventory balance for each node $n$ and stipulates that for any period $t$, the summation of starting inventory, external arrivals, and mail transferred from predecessor operations, minus the sum of mail processed in this period, equals the ending inventory. Constraint (2) states that in period $t$, the workload at node $n$ allocated to group $m$, plus the lost capacity during startup and clearance, must not exceed the processing capacity of the machines in group $m$ assigned to operation $n$ at time $t$. Constraint (3) defines the startup and clearance activities. These activities take $\tau_1$ and $\tau_2$ minutes and the lost production capacity is taken into account in constraint (2). Constraint (4) ensures that the number of machines in group $m$ operating in time period $t$ cannot exceed the number of machines available.
This equipment scheduling module is essentially a multi-level lot sizing problem. Here, multilevel refers to the fact that the input of an operation depends not only on external arrivals, but also on flows from upstream operations. The multilevel lot sizing problem with capacity constraints has been proven to be NP-hard (see Shapiro 1993) and can be extremely hard to solve. To match staff activities with machine activities, two issues must be addressed: the first one is the scheduling of overtime shifts and the second one is the assignment of breaks to shifts.

3.2.1.2 Module 2: Shift Scheduling and Overtime Management

The shift scheduling and overtime assignment management module is designed to match the machine activities with staff availability and provides a link between equipment scheduling and staff scheduling sub-problems.

For USPS, a shift is defined as a set of continuous periods with a lunch break in-between when needed. A shift could have different start and end times and be of various lengths and could have overtime appended to increase the length of a worker’s shift. For simplicity, it is assumed that overtime only occurs at the end of an employee’s scheduled shift. Employees who have just completed their duty are a convenient source of overtime labor since they are already on-site and therefore incur minimal managerial difficulties. In the development of this aspect of the problem, the following additional notation is used.

Indices and Sets

$F$ set for regular shifts (without overtime) that employees are assigned
set for full time shifts (without overtime) that employees are assigned, $F' \subset F$

set of overtime shifts where an employee working on shift $f$ can take, $f \in S(f)$

indices for shifts

Parameter

$h_{st}$ whether period $t$ lies within the start and end periods of shift $s$ or not (1=yes, 0=no)

$r_m$ number of employees required to run machine $m$

$n_f$ number of employees reported for duty to work on shift $f$

$l_f$ length in periods of shift $f$

$\alpha_{fs}$ number of overtime periods of an overtime shift $s$ extended from shift $f$, $s \in S(f)$, $f \in F$

Decision Variables

$x_s$ number of employees assigned to overtime shift $s$, $s \in S(f)$, $f \in F$

$\omega_s$ number of extra workers called to work on shift $s$, $s \in S(f)$, $f \in F$

$\beta_t$ number of breaks in period $t$.

The constraints for the shift scheduling and overtime management module can be stated as follows.

$$\sum_{m \in M(n)} \sum_{m \in M(n)} r_{m} Y_{m,n}(t) \leq \sum_{f \in F} \sum_{s \in S(f)} h_{st} x_{s} + \sum_{f \in F} \sum_{s \in S(f)} h_{st} \omega_{s} - \beta_t \quad \forall t \in T$$  \hfill (6)

$$\sum_{s \in S(f)} x_{s} = n_f \quad \forall f \in F$$  \hfill (7)

$$\sum_{f \in F} \sum_{s \in S(f)} o_{fs} x_{fs} \leq 0.06 \sum_{f \in F} l_f n_f \quad \forall f \in F'$$  \hfill (8)
Constraint (6) states that in any time period \( t \), the total number of active employees (shifts) including employees reported for duty and called-in casuals, minus the number of breaks incurred in the period, denoted by \( \beta_t \), should be adequate to cover the requirement of workers to run the machines. This constraint serves as the link between equipment scheduling and staff scheduling sub problems. Constraint (7) states that of the \( n_f \) employees reported for duty on \( f \), exactly \( n_f \) overtime shifts can be assigned. Finally, constraint (8) states that for any full time employee, the total overtime cannot exceed a certain percentage (set at 6%) of the total work hours. Here \( o_{fs} \) is the number of overtime periods in shift \( s \) and is computed as \( o_{fs} = l_s - l_f \), where \( l_s \) is the length of the overtime shift \( s \) and \( l_f \) is the length of the original shift \( f \).

3.2.1.3 Module 3: The Implicit Modeling of Breaks

Finally, to complete the whole problem, the third module is necessary to address the assignment of breaks in a shift and to define \( \beta_t \) the number of breaks used in constraint (6) in the previous module. To model the breaks in a single break window, the methodology proposed by Bechtold and Jacobs (1990) was adopted. In the development of this part of the problem, more parameters and constraints are necessary and are defined below.

**Index**

\( k \) indices for time periods where a break can occur

**Parameters**

\( e \) the earliest period a break can begin for any of the permissible shifts
\( I \) the latest period a break can begin for any of the permissible shifts
\( P \) the set of initial periods of the break windows, in ascending order
\( Q \) the set of final periods of the break windows, in ascending order
\( B_{bf} \) the set of shifts whose break window lies entirely between \( e \) and \( k \)
\( F_{bf} \) the set of shifts whose break window lies entirely between \( k \) and \( l \)

The following three constraints are used to implicitly model the breaks.

\[
\sum_{i=0}^{k} \beta_{i} - \sum_{x \in F_{bf}} x_{i} - \sum_{x \in F_{bf}} \omega_{j} \geq 0 \quad \forall k \in Q
\] (9)

\[
\sum_{i=k}^{l} \beta_{i} - \sum_{x \in B_{bf}} x_{i} - \sum_{x \in B_{bf}} \omega_{j} \geq 0 \quad \forall k \in P
\] (10)

\[
\sum_{i=e}^{l} \beta_{i} - \sum_{f \in F \cap S(f)} \sum_{x \in F \cap S(f)} x_{i} - \sum_{f \in F \cap S(f)} \sum_{x \in F \cap S(f)} \omega_{j} = 0
\] (11)

The first constraint (9) is referred as the *forward pass* constraint. It assures that the total number of breaks initiated from period \( e \) up to a given period \( r \) exceeds the total number of employees who should have taken their breaks by that period. The employees included in the constraint are those whose break windows are fully covered through \( r \), but not the ones who have the option of a break in some future period.

The second constraint (10) is referred to as the *backward pass* constraint and ensures that the total number of breaks that are initiated from some specific period \( r \) through the end of the day (or until period \( q \), the last period that can be taken as a break) exceeds the number of employees who are entitled to a break during this interval. In other words, there should be sufficient breaks in the future to satisfy the break requirement for the rest of the day. These two constraints are needed to provide
every employee with a 1-period break, but they are not sufficient to enforce the requirement that exactly one break be assigned to each shift entitled to one. Furthermore, they do not limit the break assignments to their respective ranges.

The last constraint (11) is the balance equation, which is needed to ensure that every shift is assigned one break that is within its permitted window.

### 3.2.1.4 Optimization Criteria

Several criteria can be identified in the solution of the equipment schedule (adjustment) problem. The first criterion is to ensure the on-time dispatching of mail (service commitment). In case arrivals exceed capacity, one wishes to process as much mail as possible. This can be achieved by minimizing the ending inventory at the end of the day which can be written as follows.

$$
\sum_{n \in N} v_n \quad (48) \tag{Objective 1}
$$

The second criterion is to minimize the total overtime and casual costs, which can be written as follows.

$$
\sum_{f \in F} \sum_{s \in S(f)} c_s x_s + \sum_{f \in F} \sum_{s \in S(f)} \bar{c}_s \omega_s \quad (Objective 2)
$$

where $c_s$ is the overtime cost of extending employees standard shift to overtime shift $s$ and $\bar{c}_s$ is the cost of a call-in casual that works on shift $s$. The first term represents the overtime cost while the second term represents the cost of called-in casuals. The cost structure of $c_s$ and $\bar{c}_s$ is defined as follows: for the first two hours of overtime, the overtime cost is $30/hr; for the third and fourth overtime hours, the cost is $40/hr; for
casual employees, the cost is $30/hr. These values are suggested by Berman and Larsson (1993).

To refine the equipment schedule, a third criterion is introduced. This criterion is to minimize the total number of startups and weighted working periods and can be written as follows.

\[
\sum_{n \in N} \sum_{m \in M(n)} \sum_{t \in T} Z_{mn}^1(t) + \sum_{n \in N} \sum_{m \in M(n)} \sum_{t \in T} (1-0.01t)Y_{mn}(t)
\]  

(Objective 3)

The first term minimizes the total number of startups. Minimizing the sum of startups or clearance reduces the lost capacity and thus is desirable in an equipment schedule. The second term minimizes the weighted sum of working periods. Intuitively, given the volume of mail associated with an operation, the smaller the value of the total number (the un-weighted sum) of working periods, the more likely that a machine will be running close to its capacity. The weighted sum is obtained by multiplying by the coefficient \((1 - 0.01t)\) which decreases with time. When \(t\) is small, the coefficient is relatively large, so processing in earlier periods is penalized more than processing in later periods. This pushes an operation to as late as possible and shortens the working intervals of an operation, a quality preferred by the managers in the facility.

3.2.1.5 Solution Framework

The above problem is essentially a multi-criteria mixed integer optimization problem and can be solved in three stages using a pre-emptive approach based on the priorities
of the criteria. Let \( \theta_1 \geq 0, \theta_2 \geq 0 \) be the relaxation parameters and \( \theta_3 \) be a weight parameter between the two terms of objective 3, \( 0 \leq \theta_3 \leq 1 \).

Stage 1: Minimize \( \psi_1 = \sum_{n \in N} \nu_n (48) \), subject to constraints (1) – (5) (the equipment scheduling module). The objective is to minimize the ending inventory.

Stage 2: Minimize \( \psi_2 = \sum_{f \in F} \sum_{s \in S(f)} c_s x_s + \sum_{f \in F} \sum_{s \in S(f)} \bar{c}_s w_s \omega_s \), subject to (1) – (11) and

\[
\nu_n (48) \leq \psi_{1n} + \theta_1 \rho_n \quad \forall \ n \in N \tag{12}
\]

Here, \( \psi_{1n} \) is the ending inventory of operation \( n \) from the solution \( \psi_1 \) found in Stage 1. The optimization problem includes all the original constraints in all three modules. The objective is to minimize the total overtime and casual costs.

Stage 3:  Minimize \( \sum_{n \in N} \sum_{m \in M} \sum_{t \in T} Z_{mn}^1 (t) + \theta_3 \sum_{n \in N} \sum_{m \in M} \sum_{t \in T} (1 - 0.1 t) Y_{mn} (t) \),

subject to (1) – (12) and

\[
\sum_{f \in F} \sum_{s \in S(f)} c_s x_s + \sum_{f \in F} \sum_{s \in S(f)} \bar{c}_s w_s \omega_s \leq \psi_2 + \theta_2 \psi_2 \tag{13}
\]

Here, \( \psi_2 \) is the optimal cost found in Stage 2. The objective here is to minimize a weighted sum of the total number of startups and working periods to further refine the schedule.

3.3 Computational Improvements

The above models are large scale mixed integer programs and could contain as many as 9,955 variables and 4,760 constraints. However, computation results with Xpress (Dash Optimization Inc, 2002) are mostly successful. Problems of practical size are usually solved in an hour or are able to converge to within 1% of optimality prior to the time
limit for the optimization problems in stages 1 and 2. The optimization problem in Stage 3, however, is much harder to solve. The reason seems to be the inclusion of constraint (13) which sets a tight limit on workforce capacity. For this optimization problem, depending on the value of relaxation parameter \( \theta_2 \), the optimality gap could remain as high as 5%~7% even after one hour of computation.

To solve this problem efficiently, a LP based heuristic is adopted. The basic idea behind the heuristic is that, for general integer programs that are not combinatorial in nature, linear programming relaxation usually gives enough information on the solution. This leads to the idea of using the LP solution as a target and attempts to find an integer solution that is as close to it as possible by minimizing the absolute deviation from the LP solution. The solution process is divided into three parts where the first part is to solve the LP relaxation. Then, two additional integer programs are solved in sequence, each with an aim to refine the solution obtained from the previous one. For details of the algorithm, see Zhang and Bard (2006).

To apply the above algorithm to solve the optimization problem in Stage 3, let us define three additional parameters -- \( \beta_1 \) “small” weight associated with the original objective function in Stage 3; \( \beta_1 \geq 0 \) and \( \beta_2 \): “small” weight associated with the objective of minimizing the ending inventory; \( \beta_2 \geq 0 \) and \( \theta_3 \): weight parameter associated with the working periods term in objective function of Stage 3; \( 0 < \theta_3 < 1 \). The details algorithm is in three steps and can be stated below.

Step 1: Solve the LP relaxation of the original problem in stage 3 with constraints (1) – (13). Denote the solution for the production variables \( Y_{mn}(t) \) by \( Y_{mn}^{LP}(t) \).
Step 2: Solve the following MIP.

\[ \text{Minimize} \sum_{t \in T} \sum_{n \in N} \sum_{m \in M (n)} d_{mn} (t) + \beta_1 \sum_{n \in N} \sum_{m \in M (n)} \sum_{t \in T} Z_{mn}^1 (t) + \theta_2 \sum_{n \in N} \sum_{m \in M (n)} \sum_{t \in T} \left(1 - 0.01 r \right) Y_{mn} (t) \]

subject to \quad (3) – (11), (13) and

\[ d_{mn} (t) \geq Y_{mn} (t) - Y_{mn}^{LP} (t) \quad \forall \ m \in M(n), \ t \in T(n), \ n \in N \quad (14) \]

\[ d_{mn} (t) \geq Y_{mn}^{LP} (t) - Y_{mn} (t) \quad \forall \ m \in M(n), \ t \in T(n), \ n \in N \quad (15) \]

\[ d_{ng} (t) \geq 0 \quad \forall \ m \in M(n), \ t \in T(n), \ n \in N \quad (16) \]

Constraints (14) and (15) define \( d_{ng} (t) = \left| Y_{ng} (t) - Y_{ng}^{LP} (t) \right| \), the \( L_1 \)-norm. The objective is to minimize a combination of the deviations (term 1, the primary objective) and the original objective (term 2, the secondary objective). It is important to note that only constraints (3) – (11) and (13) are included in the model. Denote the integer solution by \( Y_{ng}^{IP} (t) \).

Step 3: Solve the following MIP.

\[ \text{Minimize} \sum_{t \in T} \sum_{n \in N} \sum_{m \in M (n)} d_{mn} (t) + \beta_2 \sum_{n \in N} v_n (48) / \rho (n) \]

subject to \quad (1) – (11), (13) and

\[ d_{mn} (t) \geq Y_{mn} (t) - Y_{mn}^{IP} (t) \quad \forall \ m \in M(n), \ t \in T(n), \ n \in N \quad (17) \]

\[ d_{mn} (t) \geq Y_{mn}^{IP} (t) - Y_{mn} (t) \quad \forall \ m \in M(n), \ t \in T(n), \ n \in N \quad (18) \]

\[ d_{mn} (t) \geq 0 \quad \forall \ m \in M(n), \ t \in T(n), \ n \in N \quad (19) \]

Similarly, constraints (17) and (18) define the deviation from the target, in this case, the integer solution obtained in Step 2. The objective is to minimize a weighted sum of the
deviation (term 1, primary objective) and the inventory at the end of the day (term 2, secondary objective).

In Step 1, the linear programming relaxation of the original problem is solved. The solution is used as a basis for constructing integer solutions. In Step 2, a reduced MIP is solved to find an integer solution as close to the LP solution as possible. This is achieved by minimizing a combination of the deviation and the original objective. The original objective is included to select among alternative optima the one that is closest to the original objective value. Since the first objective is the primary, $\beta_1$ is set at a small value, 0.1 in the implementation.

Notice that in Step 2, constraints (1), (2) and (12) as well as variables $v_n(t)$ and $w_{mn}(t)$ are all dropped from the model. This is permissible because they have no effect on the objective function. Consequently, the model is reduced to nearly half of its original size and without the complicating multi-level lot sizing constraints. As such, the model can be solved quickly, usually in one to two minutes when a 1% optimality gap is specified. By constructing an IP solution close to the LP solution, the expectation is that the production, inventory levels and the flow of mail will remain close to the solution found in Step 1. However, due to the absence of constraints (1), (2) and (12), there is no guarantee that this solution is feasible to the original problem.

In Step 3, another optimization problem is solved to resolve this issue. In this model, constraints (1), (2) and (12) are added back to account for the inventory and processed volume. The goal is to find a feasible solution that is close as possible to the
infeasible integer solution found in Step 2, but with less ending inventory. The integer solution to this problem is then reported as the final solution to the original problem. High quality solutions to the optimization problem in Stage 3 are usually obtained in minutes and these solutions are able to process more than 99.5% of the mail on the same day with small optimality gaps. These results are consistent with what was observed in the early study and thus are not reported. The reason for the success seems to be that the feasible region being searched is limited to a neighborhood of the target solution. Similar ideas are being used in the development of heuristic algorithms, such as local branching (Fischetti and Lodi 2003) where promising results are obtained.

3.4 Experimental Results

To evaluate the effectiveness of the proposed model and to gain managerial insights, extensive experiments are conducted. All these models are implemented using XPress’s modeling language and solved using its general linear and integer solver (Dash Optimization Inc, 2002). The computation was run on a 2.8 GHZ computer with 512 MB RAM and the computation time limit is set at one hour or an optimality gap of 1% has been reached.

Two inputs are necessary to run the equipment scheduling (adjustment) model. The first one is the arrival profiles. The second is the actual staff level and planned schedules. For the first requirement, four-week arrival profiles were provided by the Dallas P&DC. Each week’s profile contains the arrivals for the seven days and an average arrival of a Monday is then calculated as the average of the four Mondays. While the
four Monday profiles are used as the forecast arrivals, the average profile is used to calculate the permanent workforce for the week. The permanent staff schedule is perturbed to generate an actual workforce -- the second requirement of the experiments. This perturbation process is described as follows.

First, the average arrival profile of the seven days is sent to the equipment scheduling model by Zhang and Bard (2005) to generate the demand for workers for the week which is input to the staff scheduling model by Bard et al. (2003) to calculate the size of the workforce. Then, as suggested in Bard et al. (2003), to account for absenteeism $\alpha$, the size of the permanent workforce is randomly augmented to $T\beta$ where $\beta = 1/(1-\alpha)$. The reason is that if a certain number of workers, say $A$, are needed, and if, on average, $\alpha$ percentage of the workforce is on leave, then the size of the permanent workforce $T$ should be $T-\alpha T = A$ or $T = A/(1-\alpha) = T\beta$. Finally, this permanent workforce is randomly reduced to $T$ to generate the actual workforce.

3.4.1 Experiment 1 - The Impact of Absenteeism

The first experiment studies the impact of absenteeism on the overtime cost. In this experiment, the absenteeism ratio $\alpha$ is varied and set at 0%, 3%, 6%, 9%, 12%, 15%, 18%, and 21% and for each of these values, the perturbation procedure is run five times with different randomization seeds to get different perturbed staff schedules. The average overtime and casual costs from the optimization problems under these five schedules for each day of the week under the four arrival profiles are reported in.
Table 3.2. The actual overtime and casual worker costs obtained for each week are presented in Appendix C under Table C-1, Table C-2, Table C-3, Table C-4, respectively.

<table>
<thead>
<tr>
<th>Ratio (α)</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
<th>12%</th>
<th>15%</th>
<th>18%</th>
<th>21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>0.0</td>
<td>28.8</td>
<td>191.9</td>
<td>159.4</td>
<td>249.8</td>
<td>361.8</td>
<td>402.0</td>
<td>662.1</td>
</tr>
<tr>
<td>Tuesday</td>
<td>89.3</td>
<td>180.0</td>
<td>260.1</td>
<td>513.5</td>
<td>545.0</td>
<td>689.6</td>
<td>677.3</td>
<td>864.3</td>
</tr>
<tr>
<td>Wednesday</td>
<td>216.8</td>
<td>207.1</td>
<td>163.4</td>
<td>336.3</td>
<td>338.0</td>
<td>605.4</td>
<td>519.0</td>
<td>687.3</td>
</tr>
<tr>
<td>Thursday</td>
<td>0.0</td>
<td>61.7</td>
<td>52.4</td>
<td>66.3</td>
<td>213.3</td>
<td>252.7</td>
<td>326.5</td>
<td>288.6</td>
</tr>
<tr>
<td>Friday</td>
<td>0.0</td>
<td>96.0</td>
<td>206.9</td>
<td>162.2</td>
<td>393.3</td>
<td>277.5</td>
<td>665.4</td>
<td>741.7</td>
</tr>
<tr>
<td>Saturday</td>
<td>307.5</td>
<td>352.43</td>
<td>365.0</td>
<td>346.0</td>
<td>311.8</td>
<td>356.3</td>
<td>368.4</td>
<td>358.7</td>
</tr>
<tr>
<td>Sunday</td>
<td>0.0</td>
<td>58.4</td>
<td>60.5</td>
<td>110.0</td>
<td>119.1</td>
<td>157.3</td>
<td>119.1</td>
<td>222.5</td>
</tr>
<tr>
<td>Cost (Weekly)</td>
<td>572</td>
<td>939</td>
<td>1,259</td>
<td>1,669</td>
<td>2,277</td>
<td>2,619</td>
<td>3,249</td>
<td>3,605</td>
</tr>
<tr>
<td>Cost (Yearly)</td>
<td>29,777</td>
<td>48,874</td>
<td>65,486</td>
<td>86,807</td>
<td>118,397</td>
<td>136,203</td>
<td>168,931</td>
<td>187,481</td>
</tr>
<tr>
<td>% Increase</td>
<td>--</td>
<td>64.1%</td>
<td>34.0%</td>
<td>32.6%</td>
<td>36.4%</td>
<td>15.0%</td>
<td>24.0%</td>
<td>10.1%</td>
</tr>
<tr>
<td>Times (x)</td>
<td>1.00</td>
<td>1.64</td>
<td>2.21</td>
<td>2.92</td>
<td>3.98</td>
<td>4.58</td>
<td>5.67</td>
<td>6.30</td>
</tr>
</tbody>
</table>

In this table, the values along the “Cost (Weekly)” and “Cost (Yearly)” rows represent the average overtime cost for the four different arrival profiles. The weekly costs are computed by adding the overtime cost of each day of the week, and the yearly cost is obtained by multiplying the weekly cost by 52, the number of weeks in a year. The “% Increase” row reports the percentage increase in overtime cost compared with the costs.
under the previous ratio. The “Times (×)” row reports number of times the overtime cost under the corresponding absenteeism ratio is compared with that under a 0% ratio. As can be seen, when the absenteeism increases, the cost of running the facility increases. The existence of absenteeism destroys the optimal staff schedule and as such overtime has to be used to complement a worker’s schedule to match the arrivals. A high absenteeism ratio means a large perturbation and results in a higher overtime cost. Though this result is not surprising, the magnitude of increase in cost is so dramatic that it has never been reported in the literature.

If the absenteeism ratio is increased from 0% to 3%, the overtime cost will increase by 60%; from 3% to 6%, an additional increase of 34%; from 6% to 9%, yet another increase of 32%. The overtime cost under 6%, 12%, 18% absenteeism are about 2.21, 3.98 and 5.67 times the overtime cost under 0% absenteeism, respectively.

For USPS, absenteeism could lie anywhere between 6% and 21% and varies from facility to facility. If a comparison of two facilities running at 6% and 21% absenteeism is made, then the overtime cost almost triples from $65,486 under 6% absenteeism to $187,481 under 21% absenteeism. In other words, a facility that runs under 21% absenteeism could save as much as $121,995 annually if the absenteeism can be reduced to 6%. Considering the fact that there are 275 facilities nationwide, policies and studies such as cross training and downgrading to reduce the effect of absenteeism could lead to tens of millions in financial savings for USPS.
3.4.2 Experiment 2 - The Impact of Overtime

The second experiment studies the impact of various overtime policies on the cost of running a facility. This experiment is composed of two parts. The first part investigates the impact of overtime ratio (the percentage of overtime allowed in the total number of regular hours) and the second part the impact of the maximum overtime hours allowed during a day.

Table 3.3: Cost under different overtime ratios

<table>
<thead>
<tr>
<th>Overtime ratio</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>645.0</td>
<td>475.2</td>
<td>475.2</td>
<td>475.2</td>
</tr>
<tr>
<td>Tuesday</td>
<td>528.0</td>
<td>379.5</td>
<td>379.5</td>
<td>379.5</td>
</tr>
<tr>
<td>Wednesday</td>
<td>561.0</td>
<td>412.4</td>
<td>406.4</td>
<td>387.8</td>
</tr>
<tr>
<td>Thursday</td>
<td>546.0</td>
<td>308.2</td>
<td>308.2</td>
<td>308.2</td>
</tr>
<tr>
<td>Friday</td>
<td>396.0</td>
<td>352.2</td>
<td>352.2</td>
<td>352.2</td>
</tr>
<tr>
<td>Saturday</td>
<td>287.3</td>
<td>335.3</td>
<td>339.3</td>
<td>327.8</td>
</tr>
<tr>
<td>Sunday</td>
<td>541.5</td>
<td>139.3</td>
<td>139.3</td>
<td>139.3</td>
</tr>
<tr>
<td>Weekly Cost</td>
<td>3505</td>
<td>2402</td>
<td>2400</td>
<td>2370</td>
</tr>
<tr>
<td>Annual Cost</td>
<td>182,247</td>
<td>124,907</td>
<td>124,801</td>
<td>123,241</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>31.5%</td>
<td>0.08%</td>
<td>1.25%</td>
</tr>
</tbody>
</table>
### Table 3.4: Cost under different maximum overtime hours

<table>
<thead>
<tr>
<th>Max. Overtime</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>504.0</td>
<td>504.0</td>
<td>240.0</td>
<td>249.0</td>
<td>253.2</td>
</tr>
<tr>
<td>Tuesday</td>
<td>684.0</td>
<td>592.1</td>
<td>560.2</td>
<td>560.2</td>
<td>559.6</td>
</tr>
<tr>
<td>Wednesday</td>
<td>670.5</td>
<td>619.0</td>
<td>619.4</td>
<td>611.0</td>
<td>632.5</td>
</tr>
<tr>
<td>Thursday</td>
<td>420.0</td>
<td>356.9</td>
<td>276.7</td>
<td>276.7</td>
<td>275.5</td>
</tr>
<tr>
<td>Friday</td>
<td>880.0</td>
<td>743.3</td>
<td>676.7</td>
<td>676.7</td>
<td>676.3</td>
</tr>
<tr>
<td>Saturday</td>
<td>301.5</td>
<td>222.7</td>
<td>287.7</td>
<td>313.5</td>
<td>292.0</td>
</tr>
<tr>
<td>Sunday</td>
<td>276.0</td>
<td>234.3</td>
<td>172.4</td>
<td>172.4</td>
<td>172.4</td>
</tr>
<tr>
<td>Weekly Cost</td>
<td>3,736</td>
<td>3,272</td>
<td>2,833</td>
<td>2,859</td>
<td>2,861</td>
</tr>
<tr>
<td>Annual Cost</td>
<td>194,272</td>
<td>170,154</td>
<td>147,319</td>
<td>147,319</td>
<td>147,319</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>12.4%</td>
<td>13.4%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

In the first part of the experiment, the absenteeism ratio is fixed at 12% and the maximum overtime hours at 4 hours. The overtime ratio is set at 0%, 3%, 6%, and 9%. In the second part of the experiment, the absenteeism ratio is fixed again at 12% and the overtime ratio at 6%. The maximum overtime allowed during a day by a full time employee, however, is set at different values of 0, 1, 2, 3 and 4 hours. The overtime costs for these two experiments are reported in Table 3.3 and Table 3.4, respectively.

The first observation is the significant reduction in cost of called-in workers that overtime can bring in the face of disruptions. In the first experiment as seen in Table 3.3, when the overtime ratio is set to 0%, the highest cost of $182,247 is observed. When
the overtime ratio is increased from 0% to 3%, a cost reduction as much as 31.5% is observed and the cost is reduced to $124,907. Similarly, in the second experiment, as seen in Table 3.4, when no overtime is allowed, the highest cost of $194,272 results. As the maximum number of overtime hours allowed is increased from 0 to 1 hour, a decrease of 12.4% is observed and the cost is $170,154. If the maximum overtime is further increased from 1 to 2 hours, another decrease of 13.4% is observed and the cost is $147,317. That is, a total reduction of as much as 24.2% is achieved. As it can be seen, the use of overtime increases the overall utilization of workers, eliminates unnecessary called-in workers, and ultimately reduces sharply the overtime costs.

To illustrate, consider the following sample excerpt from a solution (Table 3.5). The demand for workers over a certain set of periods is listed under the “requirement of workers” row. If no overtime is allowed, a total of 7 workers are needed with 3 on shift 1 and 4 on shift 2. One of the workers on shift 1 remains idle for periods 21 and 22. However, if overtime is allowed, then only 6 workers are needed, 2 on shift 1 and 4 on shift 2. With only one of the workers on shift 2 working on 2-period (one hour) overtime, the total number of workers required has dropped from 7 to 6, and no employee is idle in their assigned duty periods.
Table 3.5: The use of overtime to reduce workforce size and increase productivity

<table>
<thead>
<tr>
<th>Time period</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand for workers</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Without Overtime</td>
<td>Shift 1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shift 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>With Overtime</td>
<td>Shift 1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shift 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>OT1</td>
<td>OT1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

The second observation, however, is somewhat surprising. When the overtime ratio is further increased or maximum number of overtime hours allowed, no significant reduction in cost is observed. In Table 3.3, when the overtime ratio is increased from 3% to 6%, the decrease is only 0.08% and from 6% to 9%, the decrease is only 1.25%. In Table 3.4, no significant decrease is observed when the maximum overtime hours is increased to 2 or more hours. The reason for these results seems to be that even when extra excess overtime is allowed, such assignment might extend overtime to the 3rd and 4th hours beyond an employee’s shift and incurs penalty pay, so instead of using expensive overtime, the use of called-in employee at 1½ times regular pay seems more cost effective. While a moderate use of overtime could significantly decrease the cost of running a facility by as much as 25-30%, excess use of overtime should be used with caution because it may induce fatigue, lower the productivity of an employee, and seems to have no effect on the reduction of overtime cost.
3.4.3 Experiment 3 - Integrated (Holistic) Modeling of Equipment and Staff Schedule

The third experiment is designed to show the importance of modeling equipment and staff in a holistic model. Traditionally, machine scheduling and staff scheduling are solved separate from each other. Though this might be adequate in many applications, it fails to consider the impact a machine schedule imposes on the staff; a schedule from a pure machine scheduling point of view may not necessarily be good from a staff scheduling point of view.

If such a traditional approach was adopted, it would translate into a procedure that attempts to solve the problem in two sequential steps. First, the equipment scheduling module is solved with the objectives to 1) minimize the number of unprocessed mail pieces at the end of the day, and then 2) minimize the weighted sum of startups and working periods. Second, the shift scheduling and break assignment modules are solved using the result from machine scheduling.
Table 3.6: Comparison of cost obtained from two models

<table>
<thead>
<tr>
<th>Absenteeism Ratio</th>
<th>6%</th>
<th></th>
<th>12%</th>
<th></th>
<th>18%</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sequential</td>
<td>Integrated</td>
<td>Sequential</td>
<td>Integrated</td>
<td>Sequential</td>
<td>Integrated</td>
</tr>
<tr>
<td>Monday</td>
<td>659.5</td>
<td>0.0</td>
<td>811.5</td>
<td>281.5</td>
<td>1039.9</td>
<td>30.0</td>
</tr>
<tr>
<td>Tuesday</td>
<td>270.4</td>
<td>36.8</td>
<td>966.8</td>
<td>846.8</td>
<td>1396.5</td>
<td>1328.5</td>
</tr>
<tr>
<td>Wednesday</td>
<td>354.0</td>
<td>31.5</td>
<td>494.5</td>
<td>390.0</td>
<td>1462.5</td>
<td>1387.5</td>
</tr>
<tr>
<td>Thursday</td>
<td>899.5</td>
<td>539.5</td>
<td>1,178</td>
<td>868.0</td>
<td>2,379.5</td>
<td>2,069.5</td>
</tr>
<tr>
<td>Friday</td>
<td>400.0</td>
<td>0.0</td>
<td>812.5</td>
<td>614.5</td>
<td>180.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Saturday</td>
<td>875.5</td>
<td>544.5</td>
<td>936.4</td>
<td>498.0</td>
<td>1029.8</td>
<td>604.9</td>
</tr>
<tr>
<td>Sunday</td>
<td>307.5</td>
<td>157.5</td>
<td>244.5</td>
<td>94.5</td>
<td>273.0</td>
<td>63.0</td>
</tr>
<tr>
<td>Weekly Cost</td>
<td>3,765.9</td>
<td>1,309.8</td>
<td>5,443.9</td>
<td>3,593</td>
<td>7,761.1</td>
<td>5,483.1</td>
</tr>
<tr>
<td>Annual Cost</td>
<td>195,826</td>
<td>68,107</td>
<td>283,082</td>
<td>186,836</td>
<td>403,579</td>
<td>285,123</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>65.22</td>
<td>--</td>
<td>33.99</td>
<td>--</td>
<td>29.35</td>
</tr>
</tbody>
</table>

Table 3.6 shows the average overtime and casual worker costs for the four weeks, obtained from the two approaches (the sequential and holistic models), under different absenteeism ratios. As can be seen, the total overtime and casual worker cost obtained from the integrated approach is significantly smaller than that obtained from the sequential approach. For the 6% absenteeism ratio, the annual overtime and casual worker cost is $195,826 using the sequential approach, but is only $68,107 using the integrated approach; a 65% cost reduction is observed. For the 12% and 18% absenteeism ratios, cost reductions of 34% and 29% are observed if the integrated
approach is used. These results suggest that additional financial savings of nearly $40 million annually could be achieved if the system is implemented nationwide.

The actual overtime and casual worker costs for the four weeks, obtained from the sequential and holistic models, under different absenteeism ratios are quite similar and presented in Appendix D. (The decrease is in overtime and casual worker cost, for week 1, from Table D-1, are approximately 54%, 31% and 24%; for week 2, from Table D-2, 75%, 46%, and 42%; for week 3, Table D-3, 61%, 28%, and 26%; and for week 4, Table D-4 61%, 23% and 23%, from 6%, to 12%, and to 18% absenteeism ratio respectively.)

These results clearly demonstrate the superiority of a holistic integrated model versus a sequential model in production and staff scheduling problems. For P&DCs, though exogenous mail arrivals are imposed by customers and are outside the control of a facility, proper equipment scheduling serves as a demand management tool that smoothes the demand for workers and leads to superior solutions from a staff scheduling perspective. The results of this study as well the previous experience in equipment scheduling with staff consideration (Zhang and Bard 2005), where nearly 1.6 million dollars were saved per facility or nearly 400 million nationwide, clearly demonstrate the necessity to build integrated models in production and staff scheduling models and both in planning and disruption phases of the solution of the problem and applies in manufacturing and service industries that go beyond P&DCs.
4. THE STAFF SCHEDULING MODEL AND COMPUTATIONAL ALGORITHM

In Chapter 3, a LP based heuristic is proposed to solve the equipment scheduling under disruption problem. The heuristic uses the LP solution as a target and attempts to find a feasible integer solution that is as close to it as possible. The idea behind this heuristic is that, for general integer programs that are not combinatorial in nature, LP solution usually gives sufficient information on the integer solution. The procedure to search integer solutions around the neighborhood of the LP solution is referred to as LP based neighborhood search. (Notice that for the equipment scheduling under disruption, in order to improve computation efficiency, the linear programming based procedure is future divided into three parts where the first part is to solve the LP relaxation. Then, two additional integer programs are solved in sequence, each with an aim to refine the solution obtained from the previous one).

This chapter extends the linear programming based neighborhood search and examines meta-heuristic procedures that could apply to the solution of general integer programs. While a linear programming based neighborhood search provides an intensification procedure similar to what a swap or or-opt move provides to a combinational optimization problem, it could be trapped into a local optimal solution. To escape from a local optimal solution and to cover the solution landscape,
diversification schemes have to be developed. To address this issue, algorithms to generate diverse linear programming solutions has been proposed and applied to the solution of a general integer program, the postal staff scheduling problem.

4.1 The Baseline Model for Staff Scheduling

The postal staff scheduling problem 1) finds the optimal size and composition of a permanent workforce, and 2) constructs weekly tours for all employees that comply with union and contractual rules to satisfy a given demand. The postal staff scheduling problem presented here was finalized by Bard et al. (2003).

The postal staff scheduling problem is divided into three components where the first component is the shift scheduling problem. The shift scheduling problem begins with the definition of all the possible shifts for both part-time and full-time employees and concludes with the number of employees that should be assigned to each shift to satisfy daily demand. The second component of the weekly schedule requires the specification of days off. The optimal workforce size should be provided with sufficient slack throughout the week to satisfy the days off requirement for each employee. The last component of the weekly schedule is to accommodate lunch breaks, assigned to be within a specific break window for each shift. For the USPS, a full time worker works 8½ consecutive hours, which includes a half-hour lunch break. A part-timer, on the other hand, may be assigned to one of the many possible-length shifts. All employees working more than 6 hours per day must be given a ½ hour lunch break. The breaks are to be assigned between the 9th and the 12th periods. Each worker must be given two days off, preferably two consecutive days or at least one Saturday or Sunday off, per week.
Given these requirements, the staff scheduling problem is modeled as an integer program as follows. The following notation is used in the development of the model.

Indices

\( d \) index for the days of the week; \( d = 1,\ldots,7 \)

\( t \) index for time periods during a day; \( t = 1,\ldots,48 \)

\( f \) index for the full-time shift types; \( f = 1,\ldots,n^F \)

\( p \) index for the part-time shift types; \( p = 1,\ldots,n^P \)

Parameters

\( c_f \) prorated weekly cost of full-time shift \( f \)

\( c_p \) prorated weekly cost of part-time shift \( p \)

\( G_{ft} \) 1 if full-time shift type \( f \) covers period \( t \); 0 otherwise

\( P_{pt} \) 1 if part-time shift type \( p \) covers period \( t \); 0 otherwise

\( D_{dt} \) demand for period \( t \) on day \( d \)

\( n^F \) number of full-time shifts

\( n^P \) number of part-time shifts

\( \rho \) full-time to part-time labor ratio

Decision Variables

\( x_{fd} \) number of employees assigned to full-time shift type \( f \) on day \( d \)

\( y_{pd} \) number of employees assigned to part-time shift type \( p \) on day \( d \)

\( \beta_{dt} \) total number of breaks in period \( t \) on day \( d \)

\( w_f \) total number of full-time employees needed for shift type \( f \)
The Staff Scheduling Problem

Minimize  \[ z = \sum_{f=1}^{n^f} c_f w_f + \sum_{p=1}^{n^p} c_p v_p \]  

subject to

\[ \sum_{f=1}^{n^f} G_f^d x_{fd} + \sum_{p=1}^{n^p} P_p^d y_{pd} - \beta_{dt}^d \geq D_{dt}^d, \quad d = 1, \ldots, 7; \quad t = 1, \ldots, 48 \]  

\[ \sum_{f=1}^{n^f} w_f \geq \rho \sum_{p=1}^{n^p} v_p \]  

\[ w_f \geq \frac{1}{7} \sum_{d=1}^{7} x_{fd}, \quad f = 1, \ldots, n^f \]  

\[ w_f \geq x_{fd}, \quad f = 1, \ldots, n^f; \quad d = 1, \ldots, 7 \]  

\[ v_{p} \geq \frac{1}{7} \sum_{d=1}^{7} y_{pd}, \quad p = 1, \ldots, n^p \]  

\[ v_{p} \geq y_{pd}, \quad p = 1, \ldots, n^p; \quad d = 1, \ldots, 7 \]  

\[ w_{f} \geq 0, \quad v_{p} \geq 0, \quad \beta_{dt}^d \geq 0, \quad x_{fd} \geq 0, \quad y_{pd} \geq 0 \quad \text{and integer, } \forall t, k, p, d \]  

\[ \beta_{dt}^d : \text{modeled through implicit modeling of breaks} \]

The objective function (20) minimizes the total weekly cost of the workforce. Constraint (21), assures that the net workforce is sufficient to cover the demand. The net workforce is the total number of part-time and full-time employees whose shift definitions cover that specific period, less those who have a break during that period. The latter is modeled using a methodology proposed by Bechtold and Jacobs (1990) (for details, please see module 3 of the equipment scheduling under disruption model).
0-1 matrices \((G \text{ and } P)\) filter out shifts that do not cover the period under consideration.

Constraint (22) limits the number of part-time employees. Constraints (23) - (26) are used to calculate lower bounds on the number of workers required to meet the daily demand. The first of these bounds, \(L1\), is needed to assure that there is enough coverage so that every worker can take two days off per week. Constraints (23) and (25) correspond to \(L1\), which equals \(\frac{1}{5} \sum_{d=1}^{7} x_{fd}\) for the full-timers and \(\frac{1}{5} \sum_{d=1}^{7} y_{pd}\) for the part-timers, respectively. The second lower bound, \(L2\), is necessary to assure that a sufficient number of workers exist to cover the day with the highest demand. Constraints (24) and (26) correspond to \(L2\), which equals \(\max\{x_{fd} : d = 1, ..., 7\}\) for full-timer and part-timer, respectively.

The problem includes 9 full time shifts, 60 part time shifts, as listed in Table 3.1 and a week is divided into 48 time period per day with a total of 336 time periods. As such, the problem has a total of 1,092 constraints and 888 general integer variables.

The breakdown of the constraints and variables are listed below.

Table 4.1: The size of the problem (I) - Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimension</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w)</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>(v)</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>(x)</td>
<td>9 x7</td>
<td>63</td>
</tr>
<tr>
<td>(y)</td>
<td>60x7</td>
<td>630</td>
</tr>
<tr>
<td>(b)</td>
<td>48x7</td>
<td>336</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>888</td>
</tr>
</tbody>
</table>
Table 4.2: The size of the Problem (II) -- Constraints

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Dimension</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(21)</td>
<td>48x7</td>
<td>336</td>
</tr>
<tr>
<td>(22)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(23)</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>(24)</td>
<td>9x7</td>
<td>63</td>
</tr>
<tr>
<td>(25)</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>(26)</td>
<td>60x7</td>
<td>420</td>
</tr>
<tr>
<td>Forward Pass</td>
<td>14x7</td>
<td>98</td>
</tr>
<tr>
<td>Backward Pass</td>
<td>14x7</td>
<td>98</td>
</tr>
<tr>
<td>Balance</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1092</td>
</tr>
</tbody>
</table>

The postal staff scheduling problem is a general integer program and has limited structures that can be used to derive customized algorithms. Most of the state of the art algorithms, such as column generation, cutting planes and swap based local search do not apply to the problem. An initial attempt to develop a column generation by decomposing the problem into days does not seem promising and a very slow convergence was observed. Thus, the solution of the general integer optimization problems relies almost solely on the branch and bound algorithm, which could require extensive computation time to solve the problem.
However the linear programming relaxation for the staff scheduling problem
does provide reasonable information about the final integer solution. For example, if the
linear programming relaxation to a shift gives a fractional solution of 11.40, it is likely to
assume that the final solution could be somewhere near that number. On the other
hand, if a linear programming solution to a shift returns a solution of 0.0, it is unlikely
that that shift provides any use in covering the demand. These observations prompt to
investigate the use of neighborhood search to get integer solutions around the linear
programming solutions. However, as mentioned, while linear programming is an
intensification scheme, a diversification scheme has to be developed to escape from a
local optimal solution. The algorithm developed in this chapter borrows much from
meta-heuristics and is mainly composed of two phases. The first part is a diversification
scheme to generate multiple and diverse linear programming solutions to cover the
solution landscape and the second part is to conduct local search to examine
neighboring integer solutions of the diverse linear programming solutions. The following
section examines the generation of diverse linear programming solutions.

4.2 Techniques to Generate Alternative Solutions to Sample the Solution Space

The generation of multiple diverse alternative solutions, however, is not an easy task.
Typical approaches using linear programming techniques to search for alternative
solutions in the final simplex tableau, or changing coefficients in the objective functions,
are computationally prohibitive and there is no guarantee that the solutions are
sufficiently diverse to cover the solution landscape.
To achieve the goal of sufficient solution landscape coverage, the following procedure developed by Brill et al. (1982) is utilized to generate multiple alternative solutions of maximal differences.

For an integer program defined by Equations. (28)–(31),

\[
\begin{align*}
\text{Maximize} & \quad z = cx, \\
\text{subject to} & \quad Ax = b, \\
\text{where} & \quad x_i \geq 0 \quad \text{for all } i \text{ in set } N, \\
\text{and} & \quad x_i \text{ is integer} \quad \text{for } i \subseteq I, \text{ and } I \subseteq N,
\end{align*}
\]

the procedure can be summarized as follows:

Step 1: Obtain the linear programming solution; denote the optimal solution value as $\psi^*$. 

Step 2: To generate alternative solutions, solve the following Multiple Generations of Alternatives (MGA) optimization problem:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{k \in \mathcal{K}} x_k \text{ subject to Eq. (28) – (30) and } cx \geq \psi^* - \theta \psi^* \\
\end{align*}
\]

where $\theta$ is a small relaxation ratio. $\mathcal{K}$ is the set of indices of the variables that are nonzero in the initial set of solutions or generated during the process thus far.

Step 3: Stop when no new variables appear in the optimization problem.

The procedure works as follows. While Step 1 provides an initial linear programming solution, Step 2 aims to provide an alternative solution that is sufficiently different from previous solutions. This is done by minimizing the sum of decision variables that are nonzero in the previous solution or solutions. Constraints $cx \geq \psi^* - \theta \psi^*$ ensure that the alternative solution is close with respect to the model.
objective. If $\theta$ is set to 0, the solutions will have the same objective as in Step 1. The above procedure involves iteratively solving a series of linear programs and upon completion; it provides a set of LP solutions that are diverse and will serve as the starting points or the target solutions for neighborhood search.

**Coverage Property for a sample problem:** A key question of course is whether or not the above procedure could provide alternative solutions and these solutions represent a good sampling of the solution landscape. To do so, a simple staff scheduling problem is solved using the above procedure. The problem is to find the minimum number of workers to satisfy the demand in a day. Here, a day is divided into six four-hour periods and a worker starts at the beginning of a time period and works two periods. For a demand of $(4, 8, 10, 7, 12, 4)$ for the six periods, the problem can be stated as follows:

Minimize: $x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

subject to $x_6 + x_1 \geq 4; x_1 + x_2 \geq 8; x_2 + x_3 \geq 10; x_3 + x_4 \geq 7; x_4 + x_5 \geq 12; x_6 + x_1 \geq 4$ and $x_i$ integer, where $x_i$ are the decision variables representing the number of workers that start their shifts at the beginning of time period $i$.

To compare the solution coverage, all its optimal solutions are enumerated (by trying all possible combinations) and compared all its optimal solutions with those produced from the above procedure. The complete set of optimal solutions to the sample problem and the alternative solutions, marked in bold, are presented in the following table.
As can be seen, there are a total of 11 optimal solutions for this integer program with the optimal solution value of 26 (because of the total unimodularity, the LP solutions to this problem are all integers); however, some of these solutions differ slightly and thus can be divided into 3 clusters, shown above. This procedure to generate alternative solutions returned 4 diverse solutions (labeled in bold), at least one in each cluster, and does seem to provide a good sample of the solution space.

For the staff scheduling problem, however, alternative solutions will not be integer solutions, but rather, fractional linear programming solutions around which neighborhood search can be performed.
4.3 Linear Programming Based Neighborhood Structures for Integer Programs

4.3.1 LP Based Neighborhood In the literature

The LP-based neighborhood for an integer program defines the set of integer solutions $x$ within a certain close distance, denoted as $\Delta (x, \bar{x})$, to the LP or fractional solution $\bar{x}$. In the literature, these neighborhoods are usually defined in various procedures that transform a fractional solution to nearly integer solutions.

The most common procedure to transform a fractional solution is the rounding procedure. The procedure iteratively selects a portion of variables with fractional values in the LP solution, rounds them to their nearest integer values, and solves the remaining problem until an integer solution is obtained or the reduced problem becomes infeasible. Thinking $\Delta (x, \bar{x}) = \sum_j \Delta_j$ as the summation over all $j$, the above procedure is essentially a heuristic way to obtain solution with minimum distance from a fraction solution -- If $\bar{x}_j$ is close to $\lceil \bar{x}_j \rceil$, set it at $\lceil \bar{x}_j \rceil$ and $\Delta_j = x_j - \lceil \bar{x}_j \rceil$; if $\bar{x}_j$ is close to $\lfloor \bar{x}_j \rfloor$, set it at $\lfloor \bar{x}_j \rfloor$ and $\Delta_j = \lfloor \bar{x}_j \rfloor - x_j$. The problem with such a rounding scheme is the proper selection of variables at each step or iteration. If a variable is wrongly selected to be fixed at its upper or lower value at an early stage, it is possible to lead to inferior or infeasible solutions that are difficult to recover in later stages. Several approaches under the name of random rounding have been developed that randomly select variables to be fixed to overcome this problem. For details, please see Raghavan and Thompson (1987), Asratian and Kuzjurin (2001) and Chudak and Shmoy (2003).
Rather than explicitly selecting and fixing variables, another approach is to add either a constraint or an objective term to limit the summation of distance $\Delta_j$ for all $j$ and let the optimization procedure select the variables to be set or rounded.

For a 0-1 integer program, the distance can be easily defined as

$$
\Delta(x, \bar{x}) = \sum_{j \in S} (1 - x_j) + \sum_{j \in N \setminus S} x_j
$$

where $S$ is the set of variables $j$ such that $\bar{x}_j$ is close to $x_j$ and $N \setminus S$ defines the set of variables $j$ such that $\bar{x}_j$ is close to $\lceil \bar{x}_j \rceil$. This distance can easily be extended to general integer programs where

$$
\Delta(x, \bar{x}) = \sum_{j=1}^n (d_j^+ + d_j^-)
$$

where $d_j^+, d_j^-$ represent the positive and negative deviations from the LP solution and $d_j^+ - d_j^- = x_j - \bar{x}_j$.

The distance measure can be used either as a constraint, $\Delta(x, \bar{x}) \leq k$, or as an objective term to be minimized. Fischetti and Lodi (2003) use the constraint form under the name of local branching. While $\Delta(x, \bar{x}) \leq k$ intensifies the search, constraint $\Delta(x, \bar{x}) \geq k$ is used to diversify search. The heuristic is embedded in a branch and bound algorithm and promising results for several integer programs were reported. In the previous chapter, the heuristic uses the objective term form to get solutions that are as “close” to the LP solution as possible.
4.3.2 LP Based Neighborhood for the Postal Staff Scheduling Problem

The comparison of LP and IP solutions

To find the appropriate neighborhood, let us first look at the similarity of the LP relaxation solution and potential optimal or near optimal integer solutions. In the following tables, the LP solution (Table 4.4 and Table 4.6 for full time and part shift, respectively) and an integer solution (Table 4.5 and Table 4.7 for full time and part shift, respectively) obtained from B&B after 3600 seconds of computation are presented.

Table 4.4: Fractional Solution for Full Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
<th>w(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.00</td>
<td>5.00</td>
<td>8.00</td>
<td>8.00</td>
<td>11.00</td>
<td>10.00</td>
<td>8.00</td>
<td>11.40</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.32</td>
<td>1.00</td>
<td>0.68</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>1.32</td>
<td>1.32</td>
<td>1.00</td>
<td>1.32</td>
<td>1.32</td>
<td>0.00</td>
<td>1.32</td>
</tr>
<tr>
<td>4</td>
<td>9.00</td>
<td>5.00</td>
<td>7.68</td>
<td>10.32</td>
<td>14.00</td>
<td>14.00</td>
<td>10.00</td>
<td>14.00</td>
</tr>
<tr>
<td>5</td>
<td>3.42</td>
<td>3.42</td>
<td>2.32</td>
<td>1.10</td>
<td>3.42</td>
<td>3.42</td>
<td>0.00</td>
<td>3.42</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>3.58</td>
<td>3.58</td>
<td>2.57</td>
<td>3.58</td>
<td>1.92</td>
<td>2.68</td>
<td>3.58</td>
</tr>
<tr>
<td>7</td>
<td>10.82</td>
<td>0.00</td>
<td>16.98</td>
<td>19.25</td>
<td>19.00</td>
<td>21.90</td>
<td>21.55</td>
<td>21.90</td>
</tr>
<tr>
<td>8</td>
<td>6.67</td>
<td>5.00</td>
<td>6.67</td>
<td>3.73</td>
<td>5.00</td>
<td>3.83</td>
<td>2.43</td>
<td>6.67</td>
</tr>
<tr>
<td>9</td>
<td>26.33</td>
<td>7.00</td>
<td>31.00</td>
<td>34.00</td>
<td>35.00</td>
<td>36.00</td>
<td>33.00</td>
<td>40.47</td>
</tr>
</tbody>
</table>
### Table 4.5: Integer Solution for Full Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>13</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>0</td>
<td>17</td>
<td>22</td>
<td>18</td>
<td>22</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>27</td>
<td>7</td>
<td>32</td>
<td>34</td>
<td>36</td>
<td>36</td>
<td>33</td>
</tr>
</tbody>
</table>

### Table 4.6: Fractional Solution for Part Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>17</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>20</td>
<td>2.68</td>
<td>1.68</td>
<td>2.68</td>
<td>2.68</td>
<td>0.68</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>24</td>
<td>1.32</td>
<td>1.00</td>
<td>3.00</td>
<td>0.68</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>28</td>
<td>1.68</td>
<td>0.00</td>
<td>5.32</td>
<td>6.00</td>
<td>5.00</td>
<td>6.00</td>
<td>6.00</td>
</tr>
<tr>
<td>31</td>
<td>0.77</td>
<td>0.00</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
<td>0.00</td>
<td>0.77</td>
</tr>
<tr>
<td>32</td>
<td>4.82</td>
<td>0.00</td>
<td>7.92</td>
<td>7.57</td>
<td>7.92</td>
<td>5.58</td>
<td>8.23</td>
</tr>
<tr>
<td>36</td>
<td>3.32</td>
<td>0.00</td>
<td>0.00</td>
<td>3.32</td>
<td>3.32</td>
<td>3.32</td>
<td>3.32</td>
</tr>
<tr>
<td>50</td>
<td>0.77</td>
<td>0.00</td>
<td>0.77</td>
<td>0.77</td>
<td>0.00</td>
<td>0.77</td>
<td>0.77</td>
</tr>
</tbody>
</table>
Table 4.7: Integer Solution for Part Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>36</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

As can be seen, for the postal staff scheduling problem, the LP fractional solution bears similarity to the final integer solutions obtained. For example, the LP solution for the full time shift is \((11.40, 1.00, 1.32, 14.00, 3.42, 3.58, 21.90, 6.67, 40.47)\) while the final integer solution for the full time shift is \((11, 2, 1, 13, 5, 3, 22, 6, 41)\). The total absolute distance is 6.18 with an average absolute distance of 0.66 and maximum individual distance of 1.58 (shift 5).

Table 4.8: Distance between LP and IP for Full Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>11.4</td>
<td>1.32</td>
<td>14</td>
<td>3.42</td>
<td>3.58</td>
<td>21.9</td>
<td>6.67</td>
<td>40.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IP</td>
<td>11</td>
<td>2</td>
<td>1</td>
<td>13</td>
<td>5</td>
<td>3</td>
<td>22</td>
<td>6</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>Dist</td>
<td>0.4</td>
<td>-1</td>
<td>0.32</td>
<td>1</td>
<td>-1.58</td>
<td>0.58</td>
<td>-0.1</td>
<td>0.67</td>
<td>-0.53</td>
<td>-0.04</td>
</tr>
<tr>
<td>Abs Dist</td>
<td>0.4</td>
<td>1</td>
<td>0.32</td>
<td>1</td>
<td>1.58</td>
<td>0.58</td>
<td>0.1</td>
<td>0.67</td>
<td>0.53</td>
<td>0.66</td>
</tr>
</tbody>
</table>
Table 4.9: Distance between LP and IP for Part Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>16</th>
<th>17</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>31</th>
<th>32</th>
<th>36</th>
<th>50</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>1</td>
<td>0</td>
<td>2.68</td>
<td>3</td>
<td>6</td>
<td>0.77</td>
<td>8.41</td>
<td>3.32</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>IP</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Dist</td>
<td>1</td>
<td>-1</td>
<td>-0.32</td>
<td>1</td>
<td>0</td>
<td>0.77</td>
<td>-0.59</td>
<td>-0.68</td>
<td>-0.23</td>
<td>-0.0056</td>
</tr>
<tr>
<td>Abs Dist</td>
<td>1</td>
<td>1</td>
<td>0.32</td>
<td>1</td>
<td>0</td>
<td>0.77</td>
<td>0.59</td>
<td>0.68</td>
<td>0.23</td>
<td>0.62</td>
</tr>
</tbody>
</table>

The similarity between the LP solution and IP solution for part time shifts is more apparent. Of the 60 part time shifts, 52 of them have a LP solution of 0.0, and 52 of them with an IP solution of 0.0. Most of part time shifts have IP solutions that are quite close to their linear programming solution value with the only difference being shifts 16 and 17 – for shift 16, the LP solution is 1.00 and the IP solution is 0; for shift 17, the LP solution is 0.0 and the IP solution is 1. Notice, if both the LP and IP solutions are 0, they are not reported in Table 4.6 and Table 4.7. The LP solution for the part time shift which are not zero is (1, 0, 2.68, 3, 6, 0.77, 8.41, 3.32, 0.77) while the final integer solution for these shifts is (0, 1, 3, 2, 6, 0, 9, 4, 1). The total absolute distance is 5.59 with an average absolute distance of 0.62 and maximum individual distance of 1.

Besides the above LP solution, three alternative LP solutions are obtained from the procedure to generate alternative solution and are presented in Appendix E under Table E-1 and Table E-2, Table E-3 and Table E-4, Table E-5 and Table E-6, for full time and part time shifts respectively.
The Neighborhood Definition: Based on these observations, the following neighborhoods can be defined.

1) NHF(P): Explicit Fixing that applies to part time shifts: If a part time shift has a LP solution of 0, it seems to indicate that the shift offers little contribution to the solutions and should be set to zero. Otherwise, still include it in the model.

2) NSF(P) or NSF(F): Soft Fixing for Part Time or Full Time Shift, respectively. Here, constraint $\Delta(x, \bar{x}) = \sum_{j=1}^{n}(d_j^+ + d_j^-) \leq k$ is added to the model to limit the distance between the LP and IP solution for full time and part time variables.

For the above example, if neighborhood 1) is used, there are only 9 instead of 60 part time shifts and the total number of variables can be reduced by nearly 50% to 480, compared with 888 in the original model. This reduction could significantly reduce the computational time associate with the model.

Table 4.10: The reduced Problem under NHF(P)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimension</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>v</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>x</td>
<td>9x7</td>
<td>63</td>
</tr>
<tr>
<td>y</td>
<td>9x7</td>
<td>63</td>
</tr>
<tr>
<td>b</td>
<td>48x7</td>
<td>336</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>480</td>
</tr>
</tbody>
</table>

Finally, several parameters setting used in the heuristic algorithms are discussed below.
1) The Relaxation ratio $\theta$. $\theta$ was initially set at 0.001, to get alternative LP optimal solutions. Every time an improved IP solution is obtained, we compute $\alpha$, the optimality gap, between the LP and IP solution, and update $\theta$ to be $0.5\alpha$ – the alternative LP solution is between the LP solution and the current best solution.

2) Distance $k$ used in the NSF(F) and NSF(P) definition. For soft fixing around a LP solution, the distance $k$ is set at $\frac{1}{2}$ of the distance between the current LP solution and its nearest alternative LP solutions to eliminate redundant computation around the alternative LP solutions.

3) The use of neighborhood in the search process. NHF (P): By setting part time shift variable to 0, explicit Fixing for part time variable significantly reduce the size of the model and was initially invoked to get a good solution around each alternative LP solution.

4) However, as seen above, it is possible that a part time variable could still appear in the final IP solution. To get better optimal solutions, we invoked NSF(P), the soft fixing neighborhood, when the NHF(P), the hard fixing for part time shifts, is performed for all the alternative LP solutions. All the shifts that have non-zero values in these solutions are included in the model as an elite solution analysis.

The final algorithm, called neighborhood search for integer program (NSIP), can be summarized as follows.

*Step 1:* Generating an initial LP and an initial IP, using the embedded B&B heuristic, Let $\alpha$, be the optimality gap, generate multiple alternative solutions with $\theta = 0.5\alpha$. Calculate the distance between each LP solution components.
Step 2: For each fraction solution \( i \), conduct local search -- NHP(P) + NSF(F), to find integer solutions around it. \( k \) is set at the half of the minimum distance between fraction solution \( i \) and its neighborhood LPs. Record the best solution.

Step 3: Elite Solution Analysis: Perform an NSF(P) + NSF(F) with all the part time shifts that appear in the IP solutions included. Record the best solution.

Step 4: Stop and report the best solution found.

4.4 Computational Results

The algorithm was implemented using XPRESS’ modeling language and solved using its general linear and integer solver (Dash Optimization Inc, 2002). The computation was run on a 2.8 GHZ computer with 512 MB RAM.

To evaluate the performance of the heuristic, in the following table, we present the comparison results of the B&B algorithm and the heuristic. The baseline demand data was obtained from the Oklahoma City Facility and is perturbed similar to the procedure used in section 3.4 to get different test cases. Again, we randomly augmented the size of permanent workforce to \( T \beta \). Here \( \beta \) equals \( 1/(1-\alpha) \) where \( \alpha \) is the perturbation ratio. Then this permanent workforce is randomly decreased to \( T \) to generate the actual demand of workforce in each day of the week. Five cases were generated for each perturbation ration where \( \alpha \) is set 0.03, 0.06, 0.09, 0.12, 0.15, 0.18, respectively and a total of 30 instances were obtained.
<table>
<thead>
<tr>
<th>Ratio</th>
<th>Case</th>
<th>Sol</th>
<th>Time</th>
<th>Overall Time</th>
<th># of Sol</th>
<th>Sol</th>
<th>Time</th>
<th>Overall Time</th>
<th>Ratio BB /HA</th>
<th>Time in BB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>1</td>
<td>96600</td>
<td>587</td>
<td>3805</td>
<td>4 96600</td>
<td>58</td>
<td>225</td>
<td>1.0000</td>
<td>587</td>
<td>1</td>
</tr>
<tr>
<td>0.03</td>
<td>2</td>
<td>97400</td>
<td>535</td>
<td>3763</td>
<td>4 97480</td>
<td>84</td>
<td>219</td>
<td>0.9992</td>
<td>352</td>
<td>2</td>
</tr>
<tr>
<td>0.03</td>
<td>3</td>
<td>97560</td>
<td>23</td>
<td>1938</td>
<td>4 97560</td>
<td>5</td>
<td>130</td>
<td>1.0000</td>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>0.03</td>
<td>4</td>
<td>98640</td>
<td>36</td>
<td>94</td>
<td>4 98640</td>
<td>17</td>
<td>64</td>
<td>1.0000</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>0.03</td>
<td>5</td>
<td>99040</td>
<td>27</td>
<td>1073</td>
<td>4 99040</td>
<td>31</td>
<td>178</td>
<td>1.0000</td>
<td>27</td>
<td>1</td>
</tr>
<tr>
<td>0.06</td>
<td>1</td>
<td>100840</td>
<td>196</td>
<td>3711</td>
<td>3 100840</td>
<td>53</td>
<td>160</td>
<td>1.0000</td>
<td>196</td>
<td>1</td>
</tr>
<tr>
<td>0.06</td>
<td>2</td>
<td>100880</td>
<td>375</td>
<td>3859</td>
<td>4 100880</td>
<td>72</td>
<td>218</td>
<td>1.0000</td>
<td>375</td>
<td>2</td>
</tr>
<tr>
<td>0.06</td>
<td>3</td>
<td>102680</td>
<td>112</td>
<td>267</td>
<td>4 102760</td>
<td>54</td>
<td>205</td>
<td>0.9992</td>
<td>128</td>
<td>2</td>
</tr>
<tr>
<td>0.06</td>
<td>4</td>
<td>104680</td>
<td>2459</td>
<td>3840</td>
<td>6 104720</td>
<td>108</td>
<td>317</td>
<td>0.9996</td>
<td>421</td>
<td>4</td>
</tr>
<tr>
<td>0.06</td>
<td>5</td>
<td>106840</td>
<td>53</td>
<td>296</td>
<td>4 106880</td>
<td>33</td>
<td>225</td>
<td>0.9996</td>
<td>44</td>
<td>4</td>
</tr>
<tr>
<td>0.09</td>
<td>1</td>
<td>108240</td>
<td>223</td>
<td>1536</td>
<td>4 108280</td>
<td>44</td>
<td>149</td>
<td>0.9996</td>
<td>45</td>
<td>2</td>
</tr>
<tr>
<td>0.09</td>
<td>2</td>
<td>111000</td>
<td>12</td>
<td>12</td>
<td>3 111040</td>
<td>12</td>
<td>60</td>
<td>0.9996</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>0.09</td>
<td>3</td>
<td>113640</td>
<td>79</td>
<td>156</td>
<td>4 113660</td>
<td>20</td>
<td>127</td>
<td>0.9998</td>
<td>48</td>
<td>3</td>
</tr>
<tr>
<td>0.09</td>
<td>4</td>
<td>118680</td>
<td>1507</td>
<td>3600</td>
<td>5 118720</td>
<td>59</td>
<td>297</td>
<td>0.9997</td>
<td>878</td>
<td>3</td>
</tr>
<tr>
<td>0.09</td>
<td>5</td>
<td>123560</td>
<td>30</td>
<td>30</td>
<td>4 123580</td>
<td>21</td>
<td>21</td>
<td>0.9999</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>0.12</td>
<td>1</td>
<td>132840</td>
<td>324</td>
<td>3877</td>
<td>5 132920</td>
<td>80</td>
<td>297</td>
<td>0.9994</td>
<td>546</td>
<td>4</td>
</tr>
<tr>
<td>0.12</td>
<td>2</td>
<td>139320</td>
<td>16</td>
<td>144</td>
<td>3 139320</td>
<td>5</td>
<td>8</td>
<td>1.0000</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>0.12</td>
<td>3</td>
<td>149480</td>
<td>10</td>
<td>9</td>
<td>7 149520</td>
<td>4</td>
<td>8</td>
<td>0.9984</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>0.12</td>
<td>4</td>
<td>159760</td>
<td>39</td>
<td>3971</td>
<td>3 159720</td>
<td>7</td>
<td>19</td>
<td>1.0000</td>
<td>39</td>
<td>2</td>
</tr>
<tr>
<td>0.12</td>
<td>5</td>
<td>169880</td>
<td>11</td>
<td>120</td>
<td>3 169880</td>
<td>2</td>
<td>3</td>
<td>1.0000</td>
<td>35</td>
<td>2</td>
</tr>
<tr>
<td>0.15</td>
<td>1</td>
<td>183320</td>
<td>34</td>
<td>24</td>
<td>4 183360</td>
<td>11</td>
<td>39</td>
<td>0.9998</td>
<td>34</td>
<td>2</td>
</tr>
<tr>
<td>0.15</td>
<td>2</td>
<td>199480</td>
<td>18</td>
<td>19</td>
<td>3 199520</td>
<td>2</td>
<td>2</td>
<td>0.9998</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>0.15</td>
<td>3</td>
<td>215800</td>
<td>6</td>
<td>7</td>
<td>5 215800</td>
<td>4</td>
<td>6</td>
<td>1.0000</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>0.15</td>
<td>4</td>
<td>238200</td>
<td>8</td>
<td>8</td>
<td>4 238200</td>
<td>1</td>
<td>15</td>
<td>1.0000</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>0.15</td>
<td>5</td>
<td>269880</td>
<td>17</td>
<td>17</td>
<td>4 269880</td>
<td>2</td>
<td>6</td>
<td>1.0000</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>0.18</td>
<td>1</td>
<td>306720</td>
<td>1</td>
<td>1</td>
<td>4 306720</td>
<td>1</td>
<td>2</td>
<td>1.0000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.18</td>
<td>2</td>
<td>333480</td>
<td>20</td>
<td>244</td>
<td>4 333480</td>
<td>1</td>
<td>8</td>
<td>1.0000</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>0.18</td>
<td>3</td>
<td>371680</td>
<td>29</td>
<td>4048</td>
<td>4 371720</td>
<td>3</td>
<td>4</td>
<td>0.9998</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>0.18</td>
<td>4</td>
<td>403680</td>
<td>24</td>
<td>4075</td>
<td>4 403680</td>
<td>15</td>
<td>15</td>
<td>1.0000</td>
<td>24</td>
<td>4</td>
</tr>
<tr>
<td>0.18</td>
<td>5</td>
<td>434440</td>
<td>35</td>
<td>4253</td>
<td>4 434440</td>
<td>1</td>
<td>8</td>
<td>1.0000</td>
<td>35</td>
<td>5</td>
</tr>
</tbody>
</table>

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>228</td>
<td>1625</td>
<td>27</td>
<td>102</td>
<td>0.9998</td>
<td>134</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.11: Comparison between B&B and the Heuristic Algorithm
In Table 4.11, the “Ratio” and “Case” columns represent the ratio and case numbers, the “Sol”, “Time” and “Overall Time “ columns represent the best objective value, the time to achieve this best solution, and the overall time under the branch and bound and Heuristic algorithm. The “# of Sols” represents the number of different diverse LP solutions generated. The “Ratio BB/HA” represents the Ratio of the best solution from B&B to the best solution obtained from the Heuristic. For a fair comparison, we also report the time when the best heuristic solution is obtained from the B&B process under column “Time in BB”. Notice that to see what the potential optimal solution could be, the B&B algorithm was allowed to run 3,600 seconds after the last integer solution was obtained or the solution is proved optimal.

As can be seen, the heuristic was able to obtain comparable solutions quickly. The overall time for the heuristic to complete was 102 seconds and the average time to get the best solution was 27 seconds. The average time to get the corresponding solution from the branch and bound algorithm was 134 seconds, which is almost 5 times more than that of the heuristic. The solution is on average within 99.98% of the best solution obtained from the branch and bound algorithm.

The reason for this improved performance seems to be as follows. The diverse linear programming solutions provide good starting points where promising regions are located. The local search efficiently reduced the size of the problem to be optimized around the local region and leads to overall faster algorithm.
To demonstrate the effectiveness of the heuristic, the following figure shows the solution processes for the original (unperturbed demand) of the heuristic and that of the branch and bound algorithm. Here, the x-axis represents the solution time and y-axis the solution values.

![Figure 4.1: Solution Process of B&B and Heuristic](image)

As can be seen, the performance of the NSIP algorithm is quite satisfactory. It quickly obtains high-quality feasible solution, converges faster, and delivers final solutions on par with the B&B algorithm. For this unperturbed staff scheduling case, the NSIP algorithm obtains a solution of 95,000 in 30 seconds; the corresponding solution was obtained from B&B in 167 seconds.

This algorithm is also tested for several other well known problems in the literature, an airline scheduling problem, and a set covering problem (Beasley, 1990). On average, the algorithm gets multiple good or better solutions five times faster than the branch and bound algorithm. Furthermore, the algorithm is able to generate more
than one solution that is within 1% of optimal. This could be of critical importance for decision makers in complex and dynamic environments. The detailed results are beyond the scope of this thesis and are not presented here.

The algorithm still leaves much room for improvement and other schemes can be designed. For example, randomness can be easily introduced into the initial solution generation to sample the solution landscape. Some studies along these directions have been studied and procedures to prevent revisiting solutions have been proposed. The initial results seem to suggest that the algorithm is robust, efficient, and is able to obtain multiple high quality solutions.
5. SUMMARY

This research investigates model and solution approaches to operational equipment and staff scheduling problems in manufacturing, specifically its applications in USPS mail processing and distribution centers (P&DCs). This research consists of two objectives. The first objective is to build mathematical models to solve the equipment and staff scheduling problems in USPS P&DC and investigate the impact of disruptions such as workforce absenteeism, sick leave and workload fluctuations to equipment and staff schedules in manufacturing. The second objective is to investigate an advanced algorithm, which utilizes linear programming relaxations as targets to find good solutions, to solve these large scale staff scheduling problems and investigate its application to general integer programs, specifically the general set covering problem.

For the first objective, a mathematical model has been proposed and solved through a holistic model that considers simultaneously the scheduling of equipment and daily workforce, and to address union rules on the use of overtime and lunch breaks. The mathematical model proposed consists of several modules: equipment scheduling, shift scheduling and overtime management, and break assignments. The model has provided a necessary decision tool and to gain managerial insights in manufacturing that goes beyond mail processing facilities, specifically investigates the effects of the use of
overtime, the controlling of absenteeism, and the importance of scheduling equipment
and workforce simultaneously.

For the second objective, advanced solution approaches for equipment and staff
Scheduling Problem has been developed. Computational approximation algorithms for
the operational equipment scheduling have been proposed and the results are rather
promising. The algorithm is computational efficient and was able to quickly obtains high-
quality feasible solutions, converges faster, and delivers final solutions on par with the
B&B algorithm.

To further this research, it would be necessary to investigate the stochastic
equipment scheduling problem and to continue to investigate the LP based
neighborhood search approach to the solution of general integer optimization
problems, such as derive new models to methods to generate sufficiently diverse,
multiple linear programming solutions; study the characteristics of the sample solution
landscape space provided by these diverse solutions; analyze the characteristics of the
solutions obtained via the local search and provide methods to generate new target
solutions to intensify and diversify the search;
APPENDIX A

ARRIVAL PROFILES FOR THE DIFFERENT DAYS OF THE FOUR WEEKS

Figure A-1: Arrival Profiles of four Mondays at Dallas Facility
Figure A-2: Arrival Profiles of four Tuesdays at Dallas Facility

Figure A-3: Arrival Profiles of four Wednesdays at Dallas Facility
Figure A-4: Arrival Profiles of four Thursdays at Dallas Facility

Figure A-5: Arrival Profiles of four Fridays at Dallas Facility
Figure A-6: Arrival Profiles of four Saturdays at Dallas Facility

Figure A-7: Arrival Profiles of four Sundays at Dallas Facility
# APPENDIX B

## ABSENTEEISM VARIATION FOR DIFFERENT YEARS FOR DALLAS FACILITY

Table B-1: Absenteeism Ratio for Dallas P&DC

<table>
<thead>
<tr>
<th>YEAR</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SL %</td>
<td>AL %</td>
<td>TL %</td>
</tr>
<tr>
<td>1</td>
<td>4.23</td>
<td>5.44</td>
<td>9.67</td>
</tr>
<tr>
<td>2</td>
<td>3.37</td>
<td>4.77</td>
<td>8.14</td>
</tr>
<tr>
<td>3</td>
<td>3.26</td>
<td>6.18</td>
<td>9.44</td>
</tr>
<tr>
<td>5</td>
<td>4.16</td>
<td>9.09</td>
<td>13.24</td>
</tr>
<tr>
<td>7</td>
<td>4.38</td>
<td>11.66</td>
<td>16.04</td>
</tr>
<tr>
<td>8</td>
<td>4.35</td>
<td>9.76</td>
<td>14.11</td>
</tr>
<tr>
<td>10</td>
<td>4.34</td>
<td>10.92</td>
<td>15.26</td>
</tr>
<tr>
<td>12</td>
<td>5.11</td>
<td>10.36</td>
<td>15.48</td>
</tr>
<tr>
<td>13</td>
<td>4.47</td>
<td>7.54</td>
<td>12.02</td>
</tr>
<tr>
<td>AVG</td>
<td>4.23</td>
<td>8.69</td>
<td>12.92</td>
</tr>
</tbody>
</table>
APPENDIX C

OVERTIME COSTS FOR THE FOUR WEEKS UNDER DIFFERENT ABSENTEEISM RATIOS

Table C-1: Overtime Cost under Different Absenteeism Ratios for Week 1

<table>
<thead>
<tr>
<th>Ratio (α)</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
<th>12%</th>
<th>15%</th>
<th>18%</th>
<th>21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>0.00</td>
<td>90.00</td>
<td>294.6</td>
<td>63.00</td>
<td>295.8</td>
<td>552.0</td>
<td>414.6</td>
<td>466.2</td>
</tr>
<tr>
<td>Tuesday</td>
<td>0.00</td>
<td>168.0</td>
<td>511.8</td>
<td>269.7</td>
<td>420.9</td>
<td>531.0</td>
<td>271.5</td>
<td>953.7</td>
</tr>
<tr>
<td>Wednesday</td>
<td>147.0</td>
<td>185.4</td>
<td>131.7</td>
<td>594.9</td>
<td>714.9</td>
<td>887.4</td>
<td>649.5</td>
<td>835.2</td>
</tr>
<tr>
<td>Thursday</td>
<td>0.00</td>
<td>71.70</td>
<td>77.70</td>
<td>96.3</td>
<td>90.3</td>
<td>346.8</td>
<td>200.1</td>
<td>67.2</td>
</tr>
<tr>
<td>Friday</td>
<td>0.00</td>
<td>120.0</td>
<td>329.4</td>
<td>109.5</td>
<td>384.6</td>
<td>300.0</td>
<td>895.2</td>
<td>920.4</td>
</tr>
<tr>
<td>Saturday</td>
<td>63.00</td>
<td>256.8</td>
<td>282.9</td>
<td>343.8</td>
<td>212.4</td>
<td>345.9</td>
<td>231.0</td>
<td>372.3</td>
</tr>
<tr>
<td>Sunday</td>
<td>0.00</td>
<td>12.60</td>
<td>18.90</td>
<td>44.1</td>
<td>182.7</td>
<td>69.3</td>
<td>50.4</td>
<td>144.9</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Weekly)</td>
<td>210</td>
<td>905</td>
<td>1,647</td>
<td>1,521</td>
<td>2,301</td>
<td>3,032</td>
<td>2,712</td>
<td>3,760</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Yearly)</td>
<td>10,920</td>
<td>47,034</td>
<td>85,644</td>
<td>79,107</td>
<td>119,683</td>
<td>157,684</td>
<td>141,039</td>
<td>195,514</td>
</tr>
<tr>
<td>% Increase</td>
<td>--</td>
<td>330.71</td>
<td>82.09</td>
<td>-7.63</td>
<td>51.29</td>
<td>31.75</td>
<td>-10.56</td>
<td>38.62</td>
</tr>
<tr>
<td>Times (x)</td>
<td>1.00</td>
<td>4.31</td>
<td>7.84</td>
<td>7.24</td>
<td>10.96</td>
<td>14.44</td>
<td>12.92</td>
<td>17.90</td>
</tr>
</tbody>
</table>
Table C-2: Overtime Cost under Different Absenteeism Ratios for Week 2

<table>
<thead>
<tr>
<th>Ratio (α)</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
<th>12%</th>
<th>15%</th>
<th>18%</th>
<th>21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>0</td>
<td>0</td>
<td>277.8</td>
<td>121.5</td>
<td>77.7</td>
<td>514.2</td>
<td>603.6</td>
<td>977.4</td>
</tr>
<tr>
<td>Tuesday</td>
<td>147</td>
<td>288</td>
<td>127.2</td>
<td>552.9</td>
<td>517.2</td>
<td>564.3</td>
<td>432</td>
<td>599.1</td>
</tr>
<tr>
<td>Wednesday</td>
<td>720</td>
<td>337.5</td>
<td>197.1</td>
<td>443.1</td>
<td>426</td>
<td>1032.3</td>
<td>717</td>
<td>965.4</td>
</tr>
<tr>
<td>Thursday</td>
<td>0</td>
<td>35.7</td>
<td>6.3</td>
<td>35.7</td>
<td>379.8</td>
<td>132.6</td>
<td>330.6</td>
<td>414.6</td>
</tr>
<tr>
<td>Friday</td>
<td>0</td>
<td>35.7</td>
<td>169.5</td>
<td>121.8</td>
<td>313.2</td>
<td>260.4</td>
<td>469.8</td>
<td>996.3</td>
</tr>
<tr>
<td>Saturday</td>
<td>712.5</td>
<td>577.2</td>
<td>627.3</td>
<td>516.6</td>
<td>513.3</td>
<td>555.9</td>
<td>611.1</td>
<td>555.9</td>
</tr>
<tr>
<td>Sunday</td>
<td>0</td>
<td>6.3</td>
<td>50.4</td>
<td>56.7</td>
<td>37.8</td>
<td>50.4</td>
<td>18.9</td>
<td>132.3</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Weekly)</td>
<td>1,580</td>
<td>1,280</td>
<td>1,456</td>
<td>1,848</td>
<td>2,265</td>
<td>3,110</td>
<td>3,183</td>
<td>4,641</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Yearly)</td>
<td>82,134</td>
<td>66,581</td>
<td>75,691</td>
<td>96,112</td>
<td>117,780</td>
<td>161,725</td>
<td>165,516</td>
<td>241,332</td>
</tr>
<tr>
<td>% Increase</td>
<td>--</td>
<td>-18.94</td>
<td>13.68</td>
<td>26.98</td>
<td>22.55</td>
<td>37.31</td>
<td>2.34</td>
<td>45.81</td>
</tr>
<tr>
<td>Times (x)</td>
<td>1.00</td>
<td>0.81</td>
<td>0.92</td>
<td>1.17</td>
<td>1.43</td>
<td>1.97</td>
<td>2.02</td>
<td>2.94</td>
</tr>
</tbody>
</table>
Table C-3: Overtime Cost under Different Absenteeism Ratios for Week 3

<table>
<thead>
<tr>
<th>Ratio (α)</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
<th>12%</th>
<th>15%</th>
<th>18%</th>
<th>21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>0</td>
<td>0</td>
<td>132</td>
<td>289.2</td>
<td>449.4</td>
<td>255</td>
<td>426</td>
<td>952.8</td>
</tr>
<tr>
<td>Tuesday</td>
<td>0</td>
<td>156.3</td>
<td>96.3</td>
<td>630.9</td>
<td>58.8</td>
<td>744.3</td>
<td>522.6</td>
<td>832.2</td>
</tr>
<tr>
<td>Wednesday</td>
<td>0</td>
<td>263.4</td>
<td>144.6</td>
<td>271.5</td>
<td>6.3</td>
<td>223.2</td>
<td>127.5</td>
<td>506.1</td>
</tr>
<tr>
<td>Thursday</td>
<td>0</td>
<td>77.7</td>
<td>73.2</td>
<td>66.9</td>
<td>169.8</td>
<td>278.7</td>
<td>448.8</td>
<td>384</td>
</tr>
<tr>
<td>Friday</td>
<td>0</td>
<td>132.3</td>
<td>121.8</td>
<td>255.3</td>
<td>482.1</td>
<td>272.1</td>
<td>631.2</td>
<td>308.4</td>
</tr>
<tr>
<td>Saturday</td>
<td>214.5</td>
<td>284.1</td>
<td>243.3</td>
<td>249.3</td>
<td>285.3</td>
<td>285</td>
<td>361.5</td>
<td>237.6</td>
</tr>
<tr>
<td>Sunday</td>
<td>0</td>
<td>84.6</td>
<td>85.8</td>
<td>138.9</td>
<td>111</td>
<td>246</td>
<td>224.1</td>
<td>394.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Weekly)</td>
</tr>
<tr>
<td>(Yearly)</td>
</tr>
<tr>
<td>% Increase</td>
</tr>
<tr>
<td>Times (x)</td>
</tr>
</tbody>
</table>
Table C-4: Overtime Cost under Different Absenteeism Ratios for Week 4

<table>
<thead>
<tr>
<th>Ratio (α)</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
<th>12%</th>
<th>15%</th>
<th>18%</th>
<th>21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>0</td>
<td>25.2</td>
<td>63</td>
<td>163.8</td>
<td>176.4</td>
<td>126</td>
<td>163.8</td>
<td>252</td>
</tr>
<tr>
<td>Tuesday</td>
<td>210</td>
<td>107.7</td>
<td>305.1</td>
<td>600.3</td>
<td>1182.9</td>
<td>918.6</td>
<td>1482.9</td>
<td>1072.2</td>
</tr>
<tr>
<td>Wednesday</td>
<td>0</td>
<td>42</td>
<td>180</td>
<td>35.7</td>
<td>204.6</td>
<td>278.7</td>
<td>582</td>
<td>442.2</td>
</tr>
<tr>
<td>Thursday</td>
<td>0</td>
<td>61.7</td>
<td>51.4</td>
<td>66.3</td>
<td>213.3</td>
<td>252.7</td>
<td>326.5</td>
<td>288.6</td>
</tr>
<tr>
<td>Friday</td>
<td>0</td>
<td>96</td>
<td>206.9</td>
<td>162.2</td>
<td>393.3</td>
<td>277.5</td>
<td>665.4</td>
<td>741.7</td>
</tr>
<tr>
<td>Saturday</td>
<td>240</td>
<td>291.6</td>
<td>306.6</td>
<td>274.2</td>
<td>236.1</td>
<td>238.2</td>
<td>270</td>
<td>269.1</td>
</tr>
<tr>
<td>Sunday</td>
<td>0</td>
<td>129.9</td>
<td>87</td>
<td>200.4</td>
<td>144.9</td>
<td>263.4</td>
<td>183</td>
<td>218.1</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Weekly)</td>
<td>450</td>
<td>754</td>
<td>1,200</td>
<td>1,503</td>
<td>2,552</td>
<td>2,355</td>
<td>3,674</td>
<td>3,284</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Yearly)</td>
<td>23,400</td>
<td>39,213</td>
<td>62,400</td>
<td>78,151</td>
<td>132,678</td>
<td>122,465</td>
<td>191,027</td>
<td>170,763</td>
</tr>
<tr>
<td>% Increase</td>
<td></td>
<td>67.58</td>
<td>59.13</td>
<td>25.24</td>
<td>69.77</td>
<td>-7.70</td>
<td>55.98</td>
<td>-10.61</td>
</tr>
<tr>
<td>Times (x)</td>
<td>1.00</td>
<td>1.68</td>
<td>2.67</td>
<td>3.34</td>
<td>5.67</td>
<td>5.23</td>
<td>8.16</td>
<td>7.30</td>
</tr>
</tbody>
</table>
## APPENDIX D

### COMPARISON OF COST OBTAINED FROM TWO MODELS

Table D-1: Comparison of cost obtained from two models for Week 1

<table>
<thead>
<tr>
<th>Absenteeism</th>
<th>6%</th>
<th>12%</th>
<th>18%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sequential</td>
<td>Integrated</td>
<td>Sequential</td>
</tr>
<tr>
<td>Monday</td>
<td>612</td>
<td>0</td>
<td>870</td>
</tr>
<tr>
<td>Tuesday</td>
<td>0</td>
<td>0</td>
<td>930</td>
</tr>
<tr>
<td>Wednesday</td>
<td>151.5</td>
<td>31.5</td>
<td>300</td>
</tr>
<tr>
<td>Thursday</td>
<td>511.5</td>
<td>448.5</td>
<td>840</td>
</tr>
<tr>
<td>Friday</td>
<td>240</td>
<td>0</td>
<td>781.5</td>
</tr>
<tr>
<td>Saturday</td>
<td>762</td>
<td>478.5</td>
<td>883.5</td>
</tr>
<tr>
<td>Sunday</td>
<td>157.5</td>
<td>157.5</td>
<td>94.5</td>
</tr>
<tr>
<td>Cost ($) (Weekly)</td>
<td>2,434.5</td>
<td>1,116</td>
<td>4,699.5</td>
</tr>
<tr>
<td>Cost ($) (Annual)</td>
<td>126,594</td>
<td>58,032</td>
<td>244,374</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>54.16</td>
<td>--</td>
</tr>
</tbody>
</table>

83
Table D-2: Comparison of cost obtained from two models for Week 2

<table>
<thead>
<tr>
<th>Absenteeism Ratio</th>
<th>6%</th>
<th>12%</th>
<th>18%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sequential</td>
<td>Integrated</td>
<td>Sequential</td>
</tr>
<tr>
<td>Monday</td>
<td>1860</td>
<td>0</td>
<td>1917</td>
</tr>
<tr>
<td>Tuesday</td>
<td>31.5</td>
<td>0</td>
<td>957</td>
</tr>
<tr>
<td>Wednesday</td>
<td>931.5</td>
<td>31.5</td>
<td>1050</td>
</tr>
<tr>
<td>Thursday</td>
<td>898.5</td>
<td>511.5</td>
<td>1227</td>
</tr>
<tr>
<td>Friday</td>
<td>240</td>
<td>0</td>
<td>634.5</td>
</tr>
<tr>
<td>Saturday</td>
<td>1407</td>
<td>790.5</td>
<td>1617</td>
</tr>
<tr>
<td>Sunday</td>
<td>517.5</td>
<td>157.5</td>
<td>454.5</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Weekly)</td>
<td>5,886</td>
<td>1,491</td>
<td>7,857</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Yearly)</td>
<td>306,072</td>
<td>77,532</td>
<td>408,564</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>74.67</td>
<td>--</td>
</tr>
</tbody>
</table>
Table D-3: Comparison of cost obtained from two models for Week 3

<table>
<thead>
<tr>
<th>Absenteeism Ratio</th>
<th>6%</th>
<th>12%</th>
<th>18%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sequential</td>
<td>Integrated</td>
<td>Sequential</td>
</tr>
<tr>
<td>Monday</td>
<td>165</td>
<td>0</td>
<td>333</td>
</tr>
<tr>
<td>Tuesday</td>
<td>720</td>
<td>0</td>
<td>840</td>
</tr>
<tr>
<td>Wednesday</td>
<td>181.5</td>
<td>31.5</td>
<td>447</td>
</tr>
<tr>
<td>Thursday</td>
<td>1288.5</td>
<td>658.5</td>
<td>1467</td>
</tr>
<tr>
<td>Friday</td>
<td>720</td>
<td>0</td>
<td>1021.5</td>
</tr>
<tr>
<td>Saturday</td>
<td>391.5</td>
<td>579</td>
<td>511.5</td>
</tr>
<tr>
<td>Sunday</td>
<td>157.5</td>
<td>157.5</td>
<td>94.5</td>
</tr>
<tr>
<td>Cost ($) (Weekly)</td>
<td>3,624</td>
<td>1,426.5</td>
<td>4,714.5</td>
</tr>
<tr>
<td>Cost ($) (Yearly)</td>
<td>188,448</td>
<td>74,178</td>
<td>245,154</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>60.64</td>
<td>--</td>
</tr>
</tbody>
</table>
Table D-4: Comparison of cost obtained from two models for Week 4

<table>
<thead>
<tr>
<th>Absenteeism Ratio</th>
<th>6%</th>
<th>12%</th>
<th>18%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sequential</td>
<td>Integrated</td>
<td>Sequential</td>
</tr>
<tr>
<td>Monday</td>
<td>0</td>
<td>0</td>
<td>126</td>
</tr>
<tr>
<td>Tuesday</td>
<td>330</td>
<td>147</td>
<td>1140</td>
</tr>
<tr>
<td>Wednesday</td>
<td>151.5</td>
<td>31.5</td>
<td>180</td>
</tr>
<tr>
<td>Thursday</td>
<td>899.5</td>
<td>539.5</td>
<td>1178</td>
</tr>
<tr>
<td>Friday</td>
<td>400</td>
<td>0</td>
<td>812.5</td>
</tr>
<tr>
<td>Saturday</td>
<td>940.5</td>
<td>330</td>
<td>733.5</td>
</tr>
<tr>
<td>Sunday</td>
<td>397.5</td>
<td>157.5</td>
<td>334.5</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Weekly)</td>
<td>3,119</td>
<td>1,205.5</td>
<td>4,504.5</td>
</tr>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Yearly)</td>
<td>162,188</td>
<td>62,686</td>
<td>234,234</td>
</tr>
<tr>
<td>% Decrease</td>
<td>--</td>
<td>61.35</td>
<td>--</td>
</tr>
</tbody>
</table>
## APPENDIX E

### ALTERNATIVE LP SOLUTIONS FOR THE POSTAL STAFF SCHEDULING PROBLEM

Table E-1: Alternative LP Solution 1 for Full Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>10.96</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.21</td>
<td>0.42</td>
<td>0.42</td>
<td>0.21</td>
<td>0.42</td>
<td>0.42</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>11.96</td>
<td>12.24</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>3.77</td>
<td>2.96</td>
<td>4.94</td>
<td>4.03</td>
<td>6.65</td>
<td>6.65</td>
<td>4.25</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
<td>1.15</td>
<td>1.77</td>
</tr>
<tr>
<td>7</td>
<td>8.42</td>
<td>0.92</td>
<td>13.96</td>
<td>16.62</td>
<td>16.89</td>
<td>17.94</td>
<td>14.96</td>
</tr>
<tr>
<td>8</td>
<td>9.58</td>
<td>5</td>
<td>8.48</td>
<td>4.69</td>
<td>6.9</td>
<td>6.02</td>
<td>7.24</td>
</tr>
<tr>
<td>9</td>
<td>23.42</td>
<td>7</td>
<td>31</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td>33</td>
</tr>
</tbody>
</table>
### Table E-2: Alternative LP Solution 1 for Part Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.79</td>
<td>1.58</td>
<td>3.58</td>
<td>3.79</td>
<td>1.62</td>
<td>1.58</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>0.62</td>
<td>1</td>
<td>4.62</td>
<td>4</td>
<td>4.62</td>
<td>4.62</td>
<td>3.62</td>
</tr>
<tr>
<td>29</td>
<td>1.38</td>
<td>0</td>
<td>1.38</td>
<td>0</td>
<td>1.38</td>
<td>1.38</td>
<td>1.38</td>
</tr>
<tr>
<td>31</td>
<td>1.65</td>
<td>0</td>
<td>1.65</td>
<td>1.65</td>
<td>1.65</td>
<td>0</td>
<td>1.65</td>
</tr>
<tr>
<td>32</td>
<td>3.58</td>
<td>0</td>
<td>5.41</td>
<td>5.41</td>
<td>5.41</td>
<td>3.11</td>
<td>4.13</td>
</tr>
<tr>
<td>36</td>
<td>3.21</td>
<td>0</td>
<td>0</td>
<td>3.21</td>
<td>3.21</td>
<td>3.21</td>
<td>3.21</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>0.21</td>
<td>0.21</td>
<td>0</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>50</td>
<td>2.44</td>
<td>0</td>
<td>2.44</td>
<td>2.44</td>
<td>0</td>
<td>2.44</td>
<td>2.44</td>
</tr>
<tr>
<td>Shift</td>
<td>Monday</td>
<td>Tuesday</td>
<td>Wednesday</td>
<td>Thursday</td>
<td>Friday</td>
<td>Saturday</td>
<td>Sunday</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>---------</td>
<td>-----------</td>
<td>----------</td>
<td>--------</td>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>10.89</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.22</td>
<td>0.44</td>
<td>0.44</td>
<td>0.22</td>
<td>0.44</td>
<td>0.44</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>11.89</td>
<td>12.22</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>3.67</td>
<td>2.98</td>
<td>4.84</td>
<td>6.34</td>
<td>6.56</td>
<td>6.56</td>
<td>1.84</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>2.94</td>
<td>2.94</td>
<td>1.88</td>
<td>2.94</td>
<td>1.23</td>
<td>2.78</td>
</tr>
<tr>
<td>7</td>
<td>8.5</td>
<td>0.85</td>
<td>13.73</td>
<td>14.45</td>
<td>16.89</td>
<td>17.94</td>
<td>17.34</td>
</tr>
<tr>
<td>8</td>
<td>9.68</td>
<td>5</td>
<td>8.78</td>
<td>6.97</td>
<td>6.89</td>
<td>6.12</td>
<td>4.97</td>
</tr>
<tr>
<td>9</td>
<td>23.32</td>
<td>7</td>
<td>31</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td>33</td>
</tr>
</tbody>
</table>
Table E-4: Alternative LP Solution 2 for Part Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
<th>y(p,d)</th>
<th>v(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.78</td>
<td>1.56</td>
<td>3.56</td>
<td>3.78</td>
<td>1.66</td>
<td>1.56</td>
<td>3</td>
<td>3.78</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.78</td>
<td>0</td>
<td>3</td>
<td>2.22</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.72</td>
<td>1</td>
<td>4.5</td>
<td>2.78</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>1.5</td>
<td>0</td>
<td>1.5</td>
<td>1</td>
<td>1.5</td>
<td>1.5</td>
<td>0.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>1.55</td>
<td>0</td>
<td>1.55</td>
<td>1.55</td>
<td>1.55</td>
<td>0</td>
<td>1.55</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>3.78</td>
<td>0</td>
<td>5.61</td>
<td>4.23</td>
<td>5.61</td>
<td>3.22</td>
<td>5.61</td>
<td>5.61</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>3.22</td>
<td>0</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>0.22</td>
<td>0.22</td>
<td>0</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.33</td>
<td>0</td>
<td>2.33</td>
<td>2.33</td>
<td>0</td>
<td>2.33</td>
<td>2.33</td>
<td>2.33</td>
<td></td>
</tr>
</tbody>
</table>
Table E-5: Alternative LP Solution 3 for Full Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>10.89</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.22</td>
<td>0.44</td>
<td>0.44</td>
<td>0.22</td>
<td>0.44</td>
<td>0.44</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>11.89</td>
<td>12.22</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>3.67</td>
<td>2.98</td>
<td>4.84</td>
<td>3.9</td>
<td>6.56</td>
<td>6.56</td>
<td>4.29</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>2.94</td>
<td>2.94</td>
<td>2.94</td>
<td>2.94</td>
<td>1.23</td>
<td>1.71</td>
</tr>
<tr>
<td>7</td>
<td>8.5</td>
<td>0.85</td>
<td>13.73</td>
<td>16.84</td>
<td>16.89</td>
<td>17.94</td>
<td>14.96</td>
</tr>
<tr>
<td>8</td>
<td>9.68</td>
<td>5</td>
<td>8.78</td>
<td>4.59</td>
<td>6.89</td>
<td>6.12</td>
<td>7.36</td>
</tr>
<tr>
<td>9</td>
<td>23.32</td>
<td>7</td>
<td>31</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td>33</td>
</tr>
</tbody>
</table>
### Table E-6: Alternative LP Solution 3 for Part Time Shifts

<table>
<thead>
<tr>
<th>Shift</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.78</td>
<td>1.56</td>
<td>3.56</td>
<td>3.78</td>
<td>1.66</td>
<td>1.56</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>0.78</td>
<td>0</td>
<td>3</td>
<td>2.22</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>0.72</td>
<td>1</td>
<td>4.5</td>
<td>3.78</td>
<td>4.5</td>
<td>4.5</td>
<td>3.5</td>
</tr>
<tr>
<td>29</td>
<td>1.5</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>31</td>
<td>1.55</td>
<td>0</td>
<td>1.55</td>
<td>1.55</td>
<td>1.55</td>
<td>0</td>
<td>1.55</td>
</tr>
<tr>
<td>32</td>
<td>3.78</td>
<td>0</td>
<td>5.61</td>
<td>5.61</td>
<td>5.61</td>
<td>3.22</td>
<td>4.23</td>
</tr>
<tr>
<td>36</td>
<td>3.22</td>
<td>0</td>
<td>0</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
<td>3.22</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>0.22</td>
<td>0.22</td>
<td>0</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>50</td>
<td>2.33</td>
<td>0</td>
<td>2.33</td>
<td>2.33</td>
<td>0</td>
<td>2.33</td>
<td>2.33</td>
</tr>
</tbody>
</table>
REFERENCES


VITA

Arvindkumar Ravi Chakravarthy was born in Chennai, India on October 9, 1977. He is the eldest son of Indra Ravi and Ravi Chakravarthy. He earned his M.S. in Engineering (Electrical Engineering) from Wright State University in June 2002 and his B.E. in Electronics and Instrumentation from University of Madras, Chennai, India in April 1999. In September 2003, he entered into the PhD in Engineering at Wright State University in Dayton, Ohio. He passed the Candidacy Examination in April 2006 and the Research Proposal in April 2007. His research focuses on Mathematical Modeling and Optimization in the College of Engineering at Wright State University, Dayton, Ohio. He has worked on optimization problems in VLSI testing and optimization, crew scheduling in airlines and equipment and workforce scheduling in USPS mail processing facilities. He is currently consulting with Valassis Inc. as a Senior Software Engineer in the Information Technology Department.

Permanent Address: 35428 Highview Ct., Apt 207, Farmington Hills, MI 48335

This dissertation was typed by the author.