Identifying Structurally Significant Items Using Matrix Reanalysis Techniques

Bhushan M. Kable
Wright State University

Follow this and additional works at: http://corescholar.libraries.wright.edu/etd_all
Part of the Mechanical Engineering Commons

Repository Citation

This Thesis is brought to you for free and open access by the Theses and Dissertations at CORE Scholar. It has been accepted for inclusion in Browse all Theses and Dissertations by an authorized administrator of CORE Scholar. For more information, please contact corescholar@www.libraries.wright.edu.
Identifying Structurally Significant Items using

Matrix Reanalysis Techniques

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Engineering

By

BHUSHAN KABLE
B.S., Nagpur University, INDIA

2009
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Bhushan Kable ENTITLED Identifying Structurally Significant Items using Matrix Reanalysis Techniques BE ACCEPTED IN PARTIAL FULLFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

Ravi C. Penmetsa, Ph.D.
Thesis Director

George P. G. Huang, Ph.D.
Department Chair

Committee on Final Examination

Ravi C. Penmetsa, Ph.D.

Eric Tuegel, Ph.D.

Nathan W. Klingbeil, Ph.D.

Joseph F. Thomas, Jr., Ph.D.
Dean, School of Graduate Studies
Abstract

Kable, Bhushan. M.S.E, Department of Mechanical and Materials Engineering, Wright State University, 2009
Identifying Structurally Significant Items using Matrix Reanalysis Techniques

Knowledge of critical structural items for an aircraft structural system is crucial for any risk integrated design and maintenance procedure. These critical items are those whose failure can cause catastrophic damage to the entire structure or result in loss of availability. For example, failure of the fuselage longeron of an F-15 aircraft resulted in the separation of the aircraft cockpit from the rest of the structure, resulting in a complete loss of the aircraft. This is clearly a critical structural item that was identified during the design process but did not have appropriate design, manufacturing, or maintenance controls that could have prevented the accident through early detection of manufacturing flaws. While this failure is catastrophic, there can be other damage scenarios that are not catastrophic but they could lower aircraft availability due to maintenance and repair requirements.

Moreover, these critical structural items can be in areas of the aircraft that require extensive teardown in order to assess their condition. Therefore, along with the criticality of the structural failure, the location of the component also becomes important. In this research, Failure Modes Effects and Criticality Analysis (FMECA) will be used to integrate, event criticality, event frequency, and damage detection capability into one metric. This process enables integration of structural sizing and maintenance planning to minimize the operational cost while maximizing the aircraft availability. This process can
also be used to quantify the impact of structural health monitoring system on the overall risk of failure of the structure.

In this research, a Boeing 707 lower wing skin with stiffeners is used to demonstrate the process of developing an FMECA procedure for structural systems. In order to make this process applicable for large scale systems efficient structural re-analysis methods that minimize the analysis cost are also implemented. This FMECA process can be used to develop design, manufacturing, and maintenance controls that ensure quality and health of the critical structural items.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. Failure Mode Effect and Criticality Analysis (FMECA)</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Severity</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Matrix Reanalysis</td>
<td>8</td>
</tr>
<tr>
<td>2.2.1 Numerical Example</td>
<td>12</td>
</tr>
<tr>
<td>2.2.2 Reduced Basis (RB) Method</td>
<td>14</td>
</tr>
<tr>
<td>2.2.3 Successive Matrix Inversion (SMI) Technique</td>
<td>19</td>
</tr>
<tr>
<td>2.2.4 Inversion of Perturbed Matrix (IPM) Method</td>
<td>24</td>
</tr>
<tr>
<td>2.3 Occurrence</td>
<td>30</td>
</tr>
<tr>
<td>2.4 Detection</td>
<td>31</td>
</tr>
<tr>
<td>2.5 Risk Priority Number (RPN)</td>
<td>32</td>
</tr>
<tr>
<td>3. Numerical Example</td>
<td>34</td>
</tr>
<tr>
<td>3.1 Boeing 707 Lower Wing</td>
<td>34</td>
</tr>
<tr>
<td>3.1.1 Severity</td>
<td>40</td>
</tr>
<tr>
<td>3.1.2 Occurrence</td>
<td>43</td>
</tr>
<tr>
<td>3.1.3 Detection</td>
<td>45</td>
</tr>
<tr>
<td>3.2 FMECA for the Lower Wing Model</td>
<td>46</td>
</tr>
<tr>
<td>4. Summary</td>
<td>48</td>
</tr>
<tr>
<td>Appendices</td>
<td>49</td>
</tr>
<tr>
<td>References</td>
<td>60</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Figure 1.1</td>
<td>Fighter Aircraft Model</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Ten Bar Truss</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>Boeing 707 Lower Wing Skin</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Boeing 707 Lower Wing Skin Meshed Model</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Boeing 707 Lower Wing Skin with Spring</td>
</tr>
<tr>
<td>Figure 3.4</td>
<td>Dimensions of the Stringer (inches)</td>
</tr>
<tr>
<td>Figure 3.5</td>
<td>Spacing of Fastener in Stringer</td>
</tr>
<tr>
<td>Figure 3.6</td>
<td>Boeing 707 Lower Wing Skin with Boundary Conditions</td>
</tr>
<tr>
<td>Figure 3.7</td>
<td>Boeing 707 Lower Wing Skin with Spring Boundary Conditions</td>
</tr>
<tr>
<td>Figure 3.8</td>
<td>Severity Calculation Flowchart</td>
</tr>
<tr>
<td>Figure 3.9</td>
<td>Histogram comparing shift in margin of safety for stringer 6 and 4 failure individually</td>
</tr>
<tr>
<td>Figure 3.10</td>
<td>PDF of Crack Length</td>
</tr>
<tr>
<td>Figure 3.11</td>
<td>Location of the Stiffened Panel in the Wing</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1</td>
<td>FMECA Layout</td>
<td>6</td>
</tr>
<tr>
<td>Table 2.2</td>
<td>Displacement Results for the compared Re-analysis Method</td>
<td>30</td>
</tr>
<tr>
<td>Table 2.3</td>
<td>Detection Rating Criteria</td>
<td>32</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Shear and Axial Stiffness of the Fastener</td>
<td>37</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Skin Thicknesses</td>
<td>38</td>
</tr>
<tr>
<td>Table 3.3</td>
<td>Severity values for the stringer based on fracture at wing attached point</td>
<td>43</td>
</tr>
<tr>
<td>Table 3.4</td>
<td>Occurrence Value for the Wing Model</td>
<td>44</td>
</tr>
<tr>
<td>Table 3.5</td>
<td>FMECA for the Wing Model</td>
<td>47</td>
</tr>
</tbody>
</table>
Acknowledgements

The author gratefully acknowledges the financial support of the Midwest Structural Sciences Center (MSSC). MSSC is supported by the U.S. Air Force Research Laboratory Air Vehicles Directorate under the contract number FA8650-06-2-3620.
Dedications

To all who have showed me support and had faith in me:

My parents, Dr. Ravi C. Penmetsa, Dr. Eric Tuegel, My friends, and many others
Chapter 1: Introduction

Design engineers in the aerospace industry have been working with the traditional metallic structural systems for decades and have lots of test and in-service data for aircraft strengths and limitations. However, future platforms will have new material systems and they will be flown in combined thermal-mechanical-acoustic environments that have limited data. For these new systems, relying entirely on the engineering judgment to determine structurally significant items can be impractical. While these experts might have limited information about future platforms they have tremendous experience with existing platforms. Therefore, a quantitative process will be developed in this research to capture the “expert opinion” about the importance of a structural detail.

In a traditional design process, a structure is sized in order to have a positive margin of safety when subject to limit load conditions [1-2]. This criterion ensures reliability of the structural detail in the presence of variations in load and material properties. This was sufficient information when dealing with material systems that had large test data sets and flight conditions that were gathered through years of flight data recording. This design paradigm cannot ensure the same level of reliability for future platforms. Therefore, a risk integrated design process needs to be developed in order to model and propagate uncertainties in input parameters through the design.

System reliability methods that are available in the literature [3-7] depend on the failure tree for the structural system. This failure tree captures the sequence of events that need to occur for the collapse of a structure. The initiating events for this failure tree are the
highly probable events. Starting from these events a progressive failure analysis is performed until collapse and the sequence of failure that occurred before collapse constitute a failure path or branch. The shorter the path the more severe is the main initiating event. These reliability methods are based on bounding criteria that are quantitative measures used to determine the number of initiating events to be considered. Therefore, these methods are capable of determining high failure probability events using efficient algorithms. However, since these methods terminate low probability events they have a tendency to ignore high severity events that are designed for high reliability. For example, locations like the wing attachment points and fuselage longeron (Figure 1.1) are critical structural elements that will typically be designed using a high margin of safety. These are the types of components that will not be represented in the structural failure tree. And even if they are included they do not impact the system failure probability.

Figure 1.1: Fighter Aircraft Model (Courtesy: NASTRAN)

Severity of failure can be determined using techniques similar to structural vulnerability analysis. Structural vulnerability has been applied extensively to truss like structures in the civil engineering discipline [8-10]. Most of these methods focused on loss of load
carrying capacity and also loss of form of the truss structure due to loss of a structural element. While these captured the most vulnerable elements that would lead to structural failure they rarely involved probabilistic information. Ref [11] looked into the probabilistic aspect of the vulnerability analysis and developed a framework for identifying vulnerability and damage tolerance of a structure based on its system reliability. Conceptually this vulnerability represents change in failure probability of the damaged structure with respect to the intact structure. For large-scale problems performing system reliability analysis for each damage state can become prohibitively expensive. Therefore, in this research a new severity assessment method is presented that would eliminate the need for repeated system reliability analysis.

The proposed severity analysis requires analysis of the damaged structure for all the failure states. These analyses can become prohibitively expensive for large scale structures and thereby make the proposed process impractical. Therefore in this research structural reanalysis techniques [13-26] will be discussed that can be used to minimize the cost of re-analysis. These methods use information from the original structural response and the change in structural stiffness due to the damage to predict the state of the damaged structure. When the damage zone is a small percentage of the overall structure these methods provide highly accurate estimates with negligible computational cost. An example demonstrating the implementation of this algorithm is presented in the following chapter.
Along with probability of failure and severity, the ability to detect damage before it becomes catastrophic is also an important aspect of any risk based design process. Therefore, for any structural component all these three aspects need to be investigated to determine its risk. One of the techniques available in the literature to handle these three quantities is the Failure Mode Effects and Criticality Analysis (FMECA) [12]. In this research, a Boeing 707 lower wing skin section is used to demonstrate the application of FMECA to structural problems.

The following chapter 2 will first introduce the concept of FMECA and identify its three components, Severity, Occurrence, and Detection. Since severity calculations require multiple finite element simulations details of three most commonly used structural re-analysis techniques will be provided. This re-analysis methodology improves the efficiency of the proposed FMECA process. A simple truss example is used to demonstrate the implementation of each of these methods and also to compare and discuss their accuracy issues.

In chapter 3 a Boeing 707 lower wing panel is used to demonstrate the implementation of a FMECA process for a structural system. Details of the severity, occurrence and detection with respect to this particular example are discussed in detail.
Chapter 2: Failure Mode Effect and Criticality Analysis (FMECA)

The focus of a FMECA is to identify and/or analyze risk which may be due to a variety of reasons. The fundamental purpose of risk management is to answer what, when, where and how components can fail and what will be the risk involved with these events. FMECA is a specific methodology to evaluate systems using all the possible failure modes and along with risk associated with those. The FMECA layout is given in Table 2.1. It is a proactive technique which studies the cause and effect of failure before the design is finalized. It also provides a systematic approach to investigate all the potential failure modes that can occur. For each flaw identified an estimate of its severity ‘S’, detect actability ’D’ and likelihood of occurrence ‘O’ is made. The estimates of all these three parameters are made on the scale range from 1 to 10. Furthermore the product of these three parameters (S*O*D) is used to calculate a Risk Priority Number (RPN). This number unifies all the structural components into a single comparison metric irrespective of their failure mode and analysis discipline. For a situation where the failure leads to loss of life, loss of entire structure, etc. then the severity is rated 10 whereas for the situation where the failure can be addressed during the regular maintenance schedule is rated lower. Similarly, if the failure is likely to occurs too frequently then the occurrence ‘O’ is rated as 10, whereas for low failure rates lower values are assigned. Finally, the detection criteria is based on the fact that if detection of a flaw is only possible after complete teardown of the structure then it is rated 10, if the flaw is detected visually without any instrument it is rated as 1. Moreover, Failure Mode Effect Criticality Analysis (FMECA) provides information that can be readily used in root cause analysis in the unlikely event of an un-expected failure occurring in service. The computational
cost incurred in calculating severity due to multiple runs of FEA makes current FMECA computationally expensive. In this research, re-analysis based methods are explored and implemented for efficient calculation of severity for large-scale models. Traditionally, severity and occurrence have been assigned values based on expert opinion; therefore, in this research new equations are developed to assign these values using analytical results. The three important parameter of FMECA i.e. Severity, Occurrence and Detection are discuss in detail the following sub-section.

<table>
<thead>
<tr>
<th>Part</th>
<th>Potential Failure Mode</th>
<th>Severity “S”</th>
<th>Occurrence “O”</th>
<th>Detection “D”</th>
<th>RPN (O<em>S</em>D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- Damaged wing stiffener</td>
<td>Calculated Using Severity Factor</td>
<td>Calculated By Using Probability of Occurrence of the event</td>
<td>Assigned based on the location of part</td>
<td>Product Of Severity Occurrence Detection</td>
</tr>
<tr>
<td>2</td>
<td>- Damaged fuselage Longeron</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>- Skin cracking</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>- Rib failure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>- Etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: FMECA Layout
2.1 Severity

Severity is a measure relative change in structural response characteristics when certain damage is introduced into the structure. In this research, severity is assessed by using MS_Shift shown in Eq. 2.1, which represents shift in the Margin of Safety (MS) of damage structure from the margin of safety of the original structure.

\[
MS\ Shift = Damage\ State\ MS - Original\ State\ MS \quad (2.1)
\]

When the structure has built in redundancy to certain damage state, the severity factor turns out to be positive because the structural elements are capable or redistributing the load and thereby avoiding collapse. In such situations, even if the entire structure has undergone a significant change in the response the severity is assigned a value of 1 because none of the elements have exceeded their design limits. Therefore, for 0 to 100% of the elements experiencing response variations that are below the design limits, the severity of that particular damage is assigned 1.

However, when the structural response exceeds design limits, severity of damage is determined as a measure of the spread of its influence and magnitude of change in response. Therefore, in this research, a weighted average approach is used to combine number of elements affected and percentage of violation using the following equation

\[
Severity = 1 + 9 \times \left( \frac{\sum_{Negative \ MS\ Shift} (MS\ Shift \times \#\ of\ Elements\ Affected)}{\sum_{All\ MS\ Shift} (MS\ Shift \times \#\ of\ Elements\ Affected)} \right) \quad (2.2)
\]
For the above equation, a histogram needs to be constructed using all the negative changes in margin of safety. This enables the determination of weighted average of change and spread of damage.

In order to determine the severity factor that eventually enables assignment of values for severity of damage, multiple structural analyses are necessary based on each of the damage states being investigated. Therefore, in this research multiple matrix reanalysis based techniques have been explored. The Inversion of a Perturbed Matrix (IPM) method was identified as the best technique. It was implemented to minimize the computational effort associated with repeated analyses. The following subsection provides details about the IPM and also discusses two other most commonly used methods, reduced basis method and successive matrix inversion. The advantages of the IPM over these two methods are discussed using a Ten Bar truss as an example.

2.2 Matrix Reanalysis

Severity analysis requires multiple FEA runs to analyze structural response after removing certain elements to represent a failure mode. The matrix recalculation and solution process is carried out at all failure modes. In such process large matrices are recalculated and the large numbers of simultaneous equilibrium equations are formed as in equation (2.3) and that equation is solved as equation (2.4) to get modified response vector in FEA. The computational cost is wasted in recalculating these matrices at all failure modes as most of the degrees of freedom of these matrices remain unmodified. Besides, most of the computational cost is incurred in inverting or decomposing the
stiffness matrix to solve the equilibrium equations. Therefore, any method that requires multiple stiffness matrix inverses will become impractical for large-scale problems unless a more efficient process is available for handling matrix inversion. In order to eliminate the need for this inversion, several re-analysis techniques like Reduced Basis (RB) Method, Successive Matrix Inversion (SMI) technique and Inversion of a Perturbed Matrix (IPM) method were explored which involve simple matrix operations that require a fraction of the cost of inverting the entire stiffness matrix. The reanalysis techniques are categorized as approximate reanalysis technique and exact reanalysis techniques. The approximate reanalysis techniques quickly solve the iterative problems where we have small modification in actual structure as compared to exact reanalysis technique. All of these techniques provided accurate estimates for small changes in element properties but most failed for situations where complete element removal was considered. Only, one method that was applicable for complete element removal was Inversion of a Perturbed Matrix (IPM) method [24]. This section will cover the comparative study of the reanalysis methods: Reduced Basis (RB) Method, Successive Matrix Inversion (SMI) technique and Inversion of a Perturbed Matrix (IPM) method.

Assume that the first simulation using FEA is given by Eq. (2.3)

\[ \{K_o\} \{d_o\} = \{f\} \]  \hspace{1cm} (2.3)

To solve for the displacement response vector the Eq. (2.3) is modified as

\[ \{d_o\} = \{K_o\}^{-1} \{f\} \]  \hspace{1cm} (2.4)

Where,

\[ \{K_o\} \] -- Initial stiffness matrix in global coordinate,

\[ \{f\} \] -- Force vector,
\{d_0\} -- Initial response vector.

\{K_0\}^{-1} -- Inversion of the stiffness matrix

After solving simultaneous equilibrium Eq. (2.4) the information of inversion of the stiffness matrix and initial response vector will be available and can be useful for calculating the modified response vector. The actual computational cost is incurred in solving for stiffness inverse \(\{K_0\}^{-1}\).

When a damage state is introduced into the structure then its stiffness is modified by \(\Delta K\) and the Eq. (2.3) becomes

\((\{K_0\} - \{\Delta K\})\{d\} = \{f\}\) \hspace{1cm} (2.5)

To solve for the modified response vector \(\{d\}\) Eq. (2.5) is modified as

\(\{d\} = (\{K_0\} - \{\Delta K\})^{-1} \ast \{f\}\) \hspace{1cm} (2.6)

The iterative process of analysis using actual FEA will solve Eq. (2.5) to calculate the modified response vector by again formulating \((\{K_0\} - \{\Delta K\})\), \(\{f\}\) and recalculating \((\{K_0\} - \{\Delta K\})^{-1}\) while most of the information is unmodified and available from initial iteration. This iterative step makes the modified response computational expensive. The same Eq. (2.5) can be further simplified for reanalysis. If we multiply Eq. (2.5) by \(\{K_0\}^{-1}\) on both sides then the Eq. (2.5) will become

\((I - \{K_0\}^{-1} \ast \{\Delta K\})\{d\} = \{K_0\}^{-1} \ast \{f\}\) \hspace{1cm} (2.7)

Rearranging Eq. (2.7) gives

\(\{d\} = (I - \{K_0\}^{-1} \ast \{\Delta K\})^{-1} \ast \{d_0\}\) \hspace{1cm} (2.8)

Eq. (2.8) can also be written as
\( \{ \mathbf{d} \} = ([\mathbf{I}] + [\mathbf{B}])^{-1} \ast \{ \mathbf{d}_0 \} \) \hspace{1cm} (2.9)

Where,

\( [\mathbf{B}] = -[\mathbf{K}_0]^{-1} \ast [\mathbf{\Delta K}] \)

\( [\mathbf{\Delta K}] \) -- Elemental stiffness matrix in global coordinate of a modified structure

\( \{ \mathbf{d} \} \) -- Modified response vector due to the modification in the structure

\([\mathbf{I}] \) -- Identity matrix same as size of \( \{ \mathbf{K}_0 \} \)

Modified response vector in Eq. (2.8), (2.9) is given in terms of known quantities such as inverse of stiffness matrix from previous iteration \( [\mathbf{K}_0]^{-1} \), initial response vector \( \{ \mathbf{d}_0 \} \), stiffness matrix of a modified structure in global co-ordinate \( [\mathbf{\Delta K}] \) but the only challenge is to calculate inverse of \( ([\mathbf{I}] + [\mathbf{B}]) \) matrix efficiently. Exact reanalysis techniques generate the exact solution of the modified response vector using matrix operations. The methods under this category are Virtual Distortion Method [17], Sherman Morrison Woodbury Formulas (SMW) [14, 15, 16], Inversion of a Perturbed Matrix (IPM) method [24], etc. Approximate re-analysis techniques generate approximate solutions of the modified response vector. The two main factors concerning the approximate re-analysis are accuracy of the solution and convergence speed. There is always a compromise between these two. If an attempt is made to improve the accuracy of the response vector the convergence rate is very slow or may diverge so that the actual finite element method becomes faster than the re-analysis itself. If on the other hand efficiency is of interest then the accuracy will be lost considerably. The compromise has to be made according to the given engineering problem. The approximate methods include the use of sensitivity vectors, Taylor series expansions in terms of design
variables, and reduced basis and iterative techniques [26]. The methods under this category are Reduced Basis Method (RB) [19, 20], Successive Matrix Inversion Method (SMI) [26], Combined Approximations Method (CA) [21, 22] etc. Among the available reanalysis techniques only three reanalysis techniques are compared in this section. These reanalysis techniques are: Reduced Basis (RB) Method, Successive Matrix Inversion (SMI) technique and Inversion of a Perturbed Matrix (IPM) method. All these methods are implemented and discussed in detail in the following sub-section using a ten bar truss example.

2.2.1 Numerical example

The demonstration of all the three methods to compute the modified response is explained by using ten bar truss example. The horizontal and vertical bars are 360 in length. Material is considered as Aluminum and its modulus of elasticity is considered as 10e6 psi. From the initial FEA analysis the initial stiffness matrix \( [K_0] \), inverse of initial stiffness matrix \( [K_0]^{-1} \), force vector \( \{f\} \), initial displacement vector \( \{d_0\} \) and damage state stiffness matrix \( [\Delta K] \) are known. All the three methods were tested considering failure of the element connecting nodes 2 and 3, shown in Figure 2.2, to simulate the modification in structure. \( U_i, V_i \) are the x and y displacement respectively corresponding to \( i^{th} \) node number and \( F_{xi}, F_{yi} \) are the x and y force respectively corresponding to \( i^{th} \) node number. The following are the stiffness matrix, its inverse, change in stiffness due to loss of an element, the force vector, and the displacement vector for the Ten Bar truss example.
**Figure 2.2:** Ten Bar Truss

\[
\begin{bmatrix}
U_2 & V_2 & U_3 & V_3 & U_5 & V_5 & U_6 & V_6 \\
3.76 & 0.982 & 0 & 0 & -2.778 & 0 & -0.982 & -0.982 \\
0.982 & 3.76 & 0 & -2.778 & 0 & 0 & -0.982 & -0.982 \\
0 & 0 & 3.76 & -0.982 & -0.982 & 0.982 & -0.982 & -2.778 \\
0 & -2.778 & -0.982 & 3.76 & 0.982 & -0.982 & 0 & 0 \\
-2.778 & 0 & -0.982 & 0.982 & 7.52 & 0 & 0 & 0 \\
0 & 0 & 0.982 & -0.982 & 0 & 4.742 & 0 & -2.778 \\
-0.982 & -0.982 & -2.778 & 0 & 0 & 0 & 7.52 & 0 \\
-0.982 & -0.982 & 0 & 0 & 0 & -2.778 & 0 & 4.742 \\
\end{bmatrix} = 10^4 \]

\[
\begin{bmatrix}
6.489 & -7.367 & -0.711 & -7.033 & 3.223 & -2.156 & -0.377 & -1.444 \\
-0.711 & 7.033 & 6.489 & 7.367 & -0.377 & 1.444 & 3.223 & 2.156 \\
3.223 & -5.378 & -0.377 & -5.422 & 3.179 & -1.989 & -0.421 & -1.611 \\
-0.377 & 5.422 & 3.223 & 5.378 & -0.421 & 1.611 & 3.179 & 1.989 \\
\end{bmatrix} = 10^{-5} \]
Using the above information we compute the modified response vector for the three reanalysis method and the results are compared to find the most effective method for our application.

2.2.2 Reduced Basis (RB) Method [19, 20]

RB method is an approximation method which uses binomial series expansion to simulate the inverse of modified stiffness and also the modified response vector. Binomial series can be expressed as

\[(I + A)^{-1} = I - A + A^2 - A^3 + A^4 \ldots\]  \hspace{1cm} (2.10)

\([A]\) can be any arbitrary nonsingular matrix. This series expansion is also known as Geometric Series expansion or Neumann Series expansion. Since it is an approximation method it is applicable for small changes in few members of the structure. RB method uses Eq. (2.6), (2.7) to calculate the modified response vector.
Eq. (2.9) can also be written in terms of series expansion form shown in Eq. 2.10 by replacing $A$ with $B$.

$$\{d\} = \left\{ \left[ I \right] - \left[ B \right] + \left[ B \right]^2 - \left[ B \right]^3 + \left[ B \right]^4 \ldots \right\} \{d_0\}$$

(2.11)

Since we know the stiffness of modified element $[\Delta K]$ and the inverse of the initial stiffness matrix $[K_0]^{-1}$, it is easy to compute the product of these two matrices to calculate $[B]$ matrix.

$$[B] = -[K_0]^{-1} \ast [\Delta K] = \begin{bmatrix}
0 & 0 & 0.109 & 0 & -0.109 & 0 & 0 & 0 \\
0 & 0 & -0.345 & 0 & 0.345 & 0 & 0 & 0 \\
0 & 0 & -0.191 & 0 & 0.191 & 0 & 0 & 0 \\
0 & 0 & -0.355 & 0 & 0.355 & 0 & 0 & 0 \\
0 & 0 & 0.099 & 0 & -0.099 & 0 & 0 & 0 \\
0 & 0 & -0.095 & 0 & 0.095 & 0 & 0 & 0 \\
0 & 0 & -0.101 & 0 & 0.101 & 0 & 0 & 0 \\
0 & 0 & -0.105 & 0 & 0.105 & 0 & 0 & 0
\end{bmatrix}$$

The RB method is based on the evaluation of the modified response vector using the reduced set of the basis vectors which in turn reduces the size of the simultaneous equilibrium equations to be solved. The reduced size decreases the computational cost and makes the RB method computationally cost effective. The first step in the RB method is to calculate its basis vectors. The number of basis vector needed to compute modified response vector depends of level of accuracy in response and computational efficiency. If accuracy in the modified response is desired then more number of basis vectors are used to compute modified response. If computational cost is required then accuracy is compromised and less number of basis vectors are used. Assume the modified response vector can be written as

$$\{d\} = r_1 \ast d_1 - r_2 \ast d_2 + r_3 \ast d_3 - \ldots - r_n \ast d_n = \sum_{i=1}^{n} r_i \ast d_i$$

(2.12)
Where,

\[ \{ r_i \} \quad \text{--- Set of Basis Vector} \]

\[ \{ d_r \} \quad \text{--- Vector of coefficient to be determined} \]

\[ \{ d \} \quad \text{--- Modified Response Vector} \]

\[ \{ r_i \} = [ r_1, r_2, r_3, \ldots, r_n ] \]

We select the basis vectors using the following approach,

\[ \{ r_i \} = \{ d_0 \} \quad (2.13) \]

\[ \{ r_2 \} = [ B ] * \{ r_1 \} \quad (2.14) \]

\[ \{ r_3 \} = [ B ] * \{ r_2 \} \quad (2.15) \]

\[ \{ r_n \} = [ B ] * \{ r_{n-1} \} \quad (2.16) \]

After using Eq. 2.13, 2.14 and 2.15 the values for \( \{ r_i \} \), \( \{ r_2 \} \) and \( \{ r_3 \} \) are

\[
\begin{align*}
\{ r_1 \} &= \left[ \begin{array}{cccc}
0.848 \\
-3.795 \\
-0.952 \\
-3.94 \\
0.703 \\
-1.674 \\
-0.737 \\
-1.802
\end{array} \right] \\
\{ r_2 \} &= \left[ \begin{array}{cccc}
-0.181 \\
0.571 \\
0.316 \\
0.588 \\
-0.164 \\
-0.158 \\
0.168 \\
0.173
\end{array} \right] \\
\{ r_3 \} &= \left[ \begin{array}{cccc}
0.052 \\
-0.165 \\
-0.091 \\
-0.17 \\
0.047 \\
-0.046 \\
-0.049 \\
-0.05
\end{array} \right]
\end{align*}
\]

After getting the values for the individual basis vector it is then combined together to form a set of basis vector
Substituting Eq. (2.12) in Eq. (2.3) and multiplying it by \( [r_b]^T \) on both sides gives:

\[
[r_b]^T [K_0 - \Delta K] [d_r] = [r_b]^T \{ f_r \}
\]  

(2.17)

The Eq. (2.17) can also be written as

\[
[K_r] \{d_r\} = \{ f_r \}
\]  

(2.18)

Where,

\( [r_b]^T \) -- Transpose of basis vector

\( [K_r] \) -- Reduced order stiffness matrix

\( \{ f_r \} \) -- Reduce order force vector

\[
[K_r] = [r_b]^T [K_0 - \Delta K] [r_b]
\]

\[
[K_r] = 10^3 \begin{bmatrix}
49.803 & -5.409 & 1.566 \\
-5.409 & 1.566 & -0.453 \\
1.566 & -0.453 & 0.131
\end{bmatrix}
\]

\[
\{f_r\} = [r_b]^T \{f\}
\]

\[
\{f_r\} = 10^3 \begin{bmatrix}
57.417 \\
-7.613 \\
2.204
\end{bmatrix}
\]

The reduce order modified response vector can be solved by solving Eq. (2.18) as
\[ \{ d_r \} = [ K_r ]^{-1} \{ f_r \} \]  

(2.19)

\[ \{ d_r \} = \begin{bmatrix} 1 \\ -34 \\ -120 \end{bmatrix} \]

Here we solved for inverse of 3*3 \([ K_r ]\) matrixes rather solving for 8*8 \([ K_0 - \Delta K ]\). This method saves the computational cost for structural problems with higher number of Degree of Freedom (DOF).

Using Eq. 2.12 we determine the modified response vector

\[ \{ d_{RBMethod} \} = \{ d \} = \begin{bmatrix} 0.713 \\ -3.371 \\ -0.718 \\ -3.503 \\ 0.582 \\ -1.557 \\ -0.612 \\ -1.674 \end{bmatrix} \]

As the size of inverting matrix \([ K_r ]\) is reduced due to basis vector to n * n, the computational cost is reduced and the process becomes efficient. However, despite the reduced computational cost there are drawbacks to the RB method:

1. The convergence of the actual modified vector could be slow as the convergence depends on the number of basic vectors used. This sets a bound on design modification criteria.

2. More basic variables increase the computational cost and fewer basic variables compromise accuracy.

3. The sufficient condition for series type of reanalysis is that the spectral radius of the matrix should be less than unity. Spectral radius means the every single value within the
matrix should be less than 1. If the value in the matrix is greater than 1, the solution of the modified response vector corresponding to that location begins to diverge and we end up getting erroneous results. If the value in the matrix is close to 1 then more basis vectors are needed to compute accurate results for the modified response vector corresponding to the location where matrix value is close to 1.

Even though the RB method is a good approximate reanalysis method its application is limited due to these drawbacks.

### 2.2.3 Successive Matrix Inversion (SMI) technique [26]

The Successive Matrix Inversion (SMI) technique is improved version of the SMW technique but it also uses binomial series expansion. It is capable of calculating stiffness inverse and modified response vector. It is better than the RB method based on the series expansion as it overcomes one of the drawbacks, convergence. The SMI technique yields the exact solution for the stiffness matrix inverse and the response vector when there is a small modification in structure. The ability of SMI to simulate the exact solution can be used in many sequential reanalysis techniques to make the iterative process computationally inexpensive. The SMI technique uses Eq. (2.6) just as the RB method. These Eq. (2.6) can be written in series expansion form as

\[
\{d\} = \left( \{ I \} - \{ B \} + \{ B \}^2 - \{ B \}^3 \ldots \ldots \ldots - \{ B \}^n \right) \{d_o\} 
\]

\[
(2.20)
\]

A new matrix called \( \{ P \} \) matrix is introduced in this method which is defined as shown below.

\[
\{ P \} = -\{ B \} + \{ B \}^2 - \{ B \}^3 \ldots \ldots \ldots - \{ B \}^n
\]

\[
(2.21)
\]
After computing \([ B ] \ [ B ]^2 \ [ B ]^3 \ldots \ [ B ]^n\) we substitute all of these into Eq. (2.21) and compute \([ P ]\). In this case \(n\) of 8 was selected to determine \(P\), which does not represent a converged solution. More terms would be required for the series to converge to the actual value of the \(P\) matrix.

\[
[ P ] = \begin{bmatrix}
0 & 0 & -0.0985 & 0 & 0.1227 & 0 & 0 & 0 \\
0 & 0 & 0.5261 & 0 & 0.2564 & 0 & 0 & 0 \\
0 & 0 & 0.2357 & 0 & -0.1602 & 0 & 0 & 0 \\
0 & 0 & 0.5509 & 0 & -0.2621 & 0 & 0 & 0 \\
0 & 0 & -0.0899 & 0 & 0.1096 & 0 & 0 & 0 \\
0 & 0 & 0.1054 & 0 & -0.0871 & 0 & 0 & 0 \\
0 & 0 & 0.1126 & 0 & -0.0919 & 0 & 0 & 0 \\
0 & 0 & 0.1169 & 0 & -0.0947 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The \(P\) matrix above was determined using square, cube and higher order multiples of \(B\) matrix. Since this can be computationally expensive for problems with slow convergence an element by element power approach has been published in Ref.[26]. Based on these individual elements of \(B\), the elements of \([ P ]\) can be written as follows,

\[
P_{ij} = -B_{ij} + B_{ij}^2 - B_{ij}^3 \ldots \ldots \ldots \ldots B_{ij}^n
\]  \hspace{1cm} (2.22)

\(B_{ij}^n\) is the \((i, j)^{th}\) element of \(B^n\) and ‘\(n\)’ is the highest power of series expansion (\(n=8\) in this case).

\([ P ]\) is computed again based on the individual values of \([ B ]\) where square, cube, etc. of the individual term is calculated and substituted into Eq. (2.22).
\[ I \begin{bmatrix} P_{ij} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.0985 & 0 & 0.1227 & 0 & 0 & 0 \\ 0 & 0 & 0.5261 & 0 & 0.2564 & 0 & 0 & 0 \\ 0 & 0 & 0.2357 & 0 & -0.1602 & 0 & 0 & 0 \\ 0 & 0 & 0.5509 & 0 & -0.2621 & 0 & 0 & 0 \\ 0 & 0 & -0.0899 & 0 & 0.1096 & 0 & 0 & 0 \\ 0 & 0 & 0.1054 & 0 & -0.0871 & 0 & 0 & 0 \\ 0 & 0 & 0.1126 & 0 & -0.0919 & 0 & 0 & 0 \\ 0 & 0 & 0.1169 & 0 & -0.0947 & 0 & 0 & 0 \end{bmatrix} \]

Since \( B_{ij} \) keeps repeating in the series an attempt was made to make \( B_{ij} \) common multiplier by introducing a new series in terms of \( r_{ij} \). This new series would enable faster convergence compared to the equation in 2.22. The term \( r_{ij} \) is given as follows,

\[
 r_{ij}^n = \frac{B_{ij}^{n+1}}{B_{ij}} 
\]

(2.23)

Where \( n \) is the power of \( r_{ij} \).

Then Eq. (2.22) can be represented in terms of \( r_{ij} \) as

\[
P_{ij} = -B_{ij} \left( 1 - r_{ij}^2 + r_{ij}^3 \ldots \ldots \ldots r_{ij}^n \right)
\]

(2.24)

The right hand side of Eq. (2.24) is itself a new series expansion equation in terms of \( r_{ij} \) and it can be written as

\[
P_{ij} = \frac{-B_{ij}}{1 + r_{ij}}
\]

(2.25)

After converting new expanded series in term \( r_{ij} \) to the unexpanded series term it is calculated as
However this expansion is possible only for a few select cases where the recursive term remains constant. In order to apply the process for a general case, the issue with the variability in the recursive term can be handled by decomposing the modified stiffness matrix into separate matrices [26].

\[
[ \Delta K ] = \sum_{j=1}^{N} [ \Delta K^j ]
\]  \hspace{1cm} (2.26)

i.e.

\( N \) -- Total degrees of freedom in a structural model

\( [ \Delta K^j ] \) -- Matrix which has non-zero elements only in the \( j^{th} \) column.

After calculating the series terms with the B matrix, it has been suggested by author in Ref.[26] that the recursive term for the B matrix is nothing but the \( (j, j)^{th} \) element of the B matrix, as a constant value.

\( r = B_{jj} \)  \hspace{1cm} (2.27)

Finally the Eq. (2.25) can be written as

\[
P_{ij} = \frac{B_{ij}}{-(I + r)}
\]  \hspace{1cm} (2.28)
\[
[P_y] = \begin{bmatrix}
0 & 0 & -0.135 & 0 & 0.1212 & 0 & 0 & 0 \\
0 & 0 & 0.426 & 0 & -0.3826 & 0 & 0 & 0 \\
0 & 0 & 0.236 & 0 & -0.2116 & 0 & 0 & 0 \\
0 & 0 & 0.439 & 0 & -0.3942 & 0 & 0 & 0 \\
0 & 0 & -0.122 & 0 & 0.1096 & 0 & 0 & 0 \\
0 & 0 & 0.118 & 0 & -0.1058 & 0 & 0 & 0 \\
0 & 0 & 0.125 & 0 & -0.1123 & 0 & 0 & 0 \\
0 & 0 & 0.129 & 0 & -0.1161 & 0 & 0 & 0 \\
\end{bmatrix}
\]

By substituting Eq. (2.28), which is the approximation of the P matrix based on the expansion series, into Eq. (2.21), (2.20) the modified response vector is calculated. This approximate solution is suggested for improved efficiency in an optimization process presented in Ref. [26].

\[
\{d_{\text{SMI}}\} = \{d\} = \begin{bmatrix}
1.0616 \\
4.4698 \\
-1.3255 \\
-4.6348 \\
0.8967 \\
-1.8610 \\
-0.9348 \\
-2.0069 \\
\end{bmatrix}
\]

The improved feature in SMI is that the convergence criteria is eliminated by introducing a new term \(r_y\) which corresponds to the n number of basis vectors which improve the accuracy of SMI analysis. As mentioned earlier in RB method the reanalysis technique incorporating series expansion is cost effective and the same is true for the SMI method. As we are solving Eq. (2.28) and (2.20) the need of fresh analysis at iterations can be eliminated to calculate modified response vector. However despite of the benefits there are drawbacks which restrict the use of this method. These drawbacks are:

1. The sufficient condition for series type of reanalysis is that the spectral radius of the matrix should be less than unity.

2. It is not accurate for larger structural modifications.
2.2.3 Inversion of a Perturbed Matrix (IPM) method [24]

This is an exact analysis to calculate inverse of a modified structure. It is the extension of SMW formulae [24]. The IPM is more versatile and can be implemented for a wider array of problems. The best feature of this technique is that it can be applied to the problems where perturbed entities are single elements, a row of elements, a column of elements, a block of elements, or even scattered element without any restriction [24].

This process uses the Eq. (2.3), (2.4) to calculate modified stiffness or the modified response vector. This method computes the modified response vector by using SMW formulae [14], [15], [16] to compute the inverse of the modified stiffness matrix as

\[
\{d\} = [K_o - \Delta K]^{-1} \cdot \{f\} = \left[K_o^{-1} + K_o^{-1} \left(K_o^{-1} + \Delta K^{-1}\right)^{-1} \cdot \Delta K \cdot K_o^{-1}\right] \cdot \{f\}
\] (2.29)

As we know the stiffness matrix \( \Delta K \) might be singular, so it is not possible to find the inverse of that matrix. Therefore, we multiply Eq. (2.29) by \( \Delta K \cdot \Delta K^{-1} \) to eliminate the singularity in matrix. The Eq. 2.29 can be rearranged as

\[
[K_o - \Delta K]^{-1} = \left[K_o^{-1} - K_o^{-1} \cdot \left(K_o^{-1} + \Delta K^{-1}\right)^{-1} \cdot \Delta K \cdot K_o^{-1}\right]
\]

\[
= \left[K_o^{-1} - K_o^{-1} \cdot \left(\Delta K \cdot K_o^{-1} + I\right)^{-1} \cdot \Delta K \cdot K_o^{-1}\right]
\]

\[
= \left[K_o^{-1} - K_o^{-1} \cdot \left(I + \Delta K \cdot K_o^{-1}\right)^{-1} \cdot \Delta K \cdot K_o^{-1}\right]
\]

(2.30)

\[
\{d\} = [K_o - \Delta K]^{-1} \cdot \{f\} = \left[K_o^{-1} - K_o^{-1} \cdot \left(I + \Delta K \cdot K_o^{-1}\right)^{-1} \cdot \Delta K \cdot K_o^{-1}\right] \cdot \{f\}
\]

\[
[K_o - \Delta K]^{-1} = \left[K_o^{-1} - K_o^{-1} \cdot \Delta K \cdot \Delta K^{-1} \cdot \left(K_o^{-1} + \Delta K^{-1}\right)^{-1} \cdot K_o^{-1}\right]
\]

\[
= \left[K_o^{-1} - K_o^{-1} \cdot \Delta K \cdot \left(K_o^{-1} + \Delta K^{-1}\right)^{-1} \cdot K_o^{-1}\right]
\]

\[
= \left[K_o^{-1} - K_o^{-1} \cdot \Delta K \cdot \left(I + K_o^{-1} \cdot \Delta K^{-1}\right)^{-1} \cdot K_o^{-1}\right]
\]

(2.31)

\[
\{d\} = [K_o - \Delta K]^{-1} \cdot \{f\} = \left[K_o^{-1} - K_o^{-1} \cdot \Delta K \cdot \left(I + K_o^{-1} \cdot \Delta K^{-1}\right)^{-1} \cdot K_o^{-1}\right] \cdot \{f\}
\]
Computing modified response vector based on Eq. 2.30, 2.31 and 2.6 it can be seen that the singularity can be avoided by modifying the Eq. 2.29 while maintaining the accuracy of the solution

\[
\{d\} = \begin{bmatrix}
1.102 & 1.102 & 1.102 \\
-4.598 & -4.598 & -4.598 \\
-1.397 & -1.397 & -1.397 \\
-4.767 & -4.767 & -4.767 \\
0.934 & 0.934 & 0.934 \\
-1.897 & -1.897 & -1.897 \\
-0.973 & -0.973 & -0.973 \\
-2.046 & -2.046 & -2.046
\end{bmatrix}
\]

The idea behind using this Eq. (2.30) or Eq. (2.31) is to reduce the size of stiffness matrix to be inverted. The actual size of the inverting stiffness matrix \( (I - K_0^{-1} * \Delta K)^{-1} \) is 8x8 in our case but by using IPM method the size of inverting matrix becomes equal to the size of the modified stiffness matrix and thus reducing the cost to compute the inverted matrix and the modified response. In its current form the Eq. 2.31 requires inversion of an 8x8 matrix thereby eliminating any benefits of reformulating the modified stiffness matrix. The benefits are derived from the following process that evaluated the Eq. 2.31 using an efficient process that does not require inversion of the 8x8 matrix.

The Eq. (2.30) or Eq. (2.31) can be restated as

\[
(K_0^{-1} - \Delta K)^{-1} = K_0^{-1} + H
\]

where

\[
H = K_3 * (I - K_1 * K_2)^{-1} * K_1 * K_4
\]

or

\[
H = K_3 * K_1 * (I - K_2 * K_1)^{-1} * K_4
\]

(2.32)

(2.33)
The new matrices $K_1, K_2, K_3, K_4$ are called partition matrices and are defined using $K_0^{-1}$ and $\Delta K$. Actual form of these matrices is discussed below. It has been observed that only those rows and columns are used which are having non zero values in the modified matrix $\Delta K$. Those rows and columns which corresponding to non zero value in $\Delta K$ matrix is used to partition $K_0^{-1}$ and $\Delta K$ as $K_1 K_2 K_3 K_4$ and further used for the calculation of modified response vector.

$$[\Delta K] \Leftrightarrow \begin{bmatrix} K_1 & 0 \\ 0 & 0 \end{bmatrix}$$

(2.34)

$K_1$ is modified stiffness matrix. The location of row and column is based on DOF of the failing element. In our case the link 2 – 3 is considered to fail and hence the DOF correspond to those node number are row = [3 4 5 6] and column = [3 4 5 6]. We keep only those elements corresponding to row and column number to formulate the partition matrices.

$$[\Delta K] = 10^4 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.278 & 0 & -0.278 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.278 & 0 & 0.278 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K_1] = 10^4 \begin{bmatrix} 0.278 & 0 & -0.278 & 0 \\ 0 & 0 & 0 & 0 \\ -0.278 & 0 & 0.278 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
It is the actual modified stiffness matrix in the local co-ordinate. It is not necessary that it
has to be at the same position where it is shown. It can be anywhere within the matrix.

$$[K_0]^{-1} \Leftrightarrow \begin{bmatrix} K_2 & K_{02} \\ K_{02} & K_{03} \end{bmatrix} \quad K_2 = position(K_1^T) \quad (2.35)$$

$K_2$ is the element positions corresponding to the transpose of element positions of $K_1$ and
it is the sub matrix within $K_0^{-1}$. $K_{02}$ $K_{03}$ are rest of the values in the $K_0^{-1}$ matrix. The
transpose of rows and columns corresponding to $\Delta K$ are used in order to find $K_2$. Since
the rows and columns in this problem are symmetric the rows [3, 4, 5, 6] and columns [3, 
4, 5, 6] are retained as shown below.


$$[K_2] = 10^{-5} \ast \begin{bmatrix} 6.489 & 7.367 & -0.377 & 1.444 \\ 7.367 & 28.988 & -5.422 & 10.575 \\ -0.377 & -5.422 & 3.179 & -1.989 \\ 1.444 & 10.575 & -1.989 & 7.613 \end{bmatrix}$$

Now $K_3$ and $K_4$ can be determined as follows,

$$K_3 = \begin{bmatrix} K_2 \\ K_{02} \end{bmatrix} \quad (2.36)$$

$K_3$ is column vector of $K_0^{-1}$ correspond to the row number of $K_1$
\[
\begin{bmatrix}
6.489 & -7.367 & -0.711 & -7.033 & 3.223 & -2.156 & -0.377 & -1.444 \\
-0.711 & 7.033 & 6.489 & 7.367 & -0.377 & 1.444 & 3.223 & 2.156 \\
3.223 & -5.378 & -0.377 & -5.422 & 3.179 & -1.989 & -0.421 & -1.611 \\
-0.377 & 5.422 & 3.223 & 5.378 & -0.421 & 1.611 & 3.179 & 1.989 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-0.711 & -7.033 & 3.223 & -2.156 \\
7.033 & 27.377 & -5.378 & 10.408 \\
6.489 & 7.367 & -0.377 & 1.444 \\
7.367 & 28.988 & -5.422 & 10.575 \\
-0.377 & -5.422 & 3.179 & -1.989 \\
1.444 & 10.575 & -1.989 & 7.613 \\
3.223 & 5.378 & -0.421 & 1.611 \\
2.156 & 10.408 & -1.611 & 6.169 \\
\end{bmatrix}
\]

\[
K_4 = \begin{bmatrix} K_1 \ K_02 \end{bmatrix} = 10^{-5} \begin{bmatrix} 3.223 \ 2.156 \ 0.377 \ 1.444 \ 2.156 \end{bmatrix}
\]

\[
K_4 \text{ is row vector of } K_0^{-1} \text{ correspond to the column number of } K_1
\]

\[
\begin{bmatrix}
6.489 & -7.367 & -0.711 & -7.033 & 3.223 & -2.156 & -0.377 & -1.444 \\
-0.711 & 7.033 & 6.489 & 7.367 & -0.377 & 1.444 & 3.223 & 2.156 \\
3.223 & -5.378 & -0.377 & -5.422 & 3.179 & -1.989 & -0.421 & -1.611 \\
-0.377 & 5.422 & 3.223 & 5.378 & -0.421 & 1.611 & 3.179 & 1.989 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-0.711 & 7.033 & 6.489 & 7.367 & -0.377 & 1.444 & 3.223 & 2.156 \\
3.223 & -5.378 & -0.377 & -5.422 & 3.179 & -1.989 & -0.421 & -1.611 \\
\end{bmatrix}
\]
After computing $K_1 K_2 K_3 K_4$ the values are substituted back to any one of the Eq. (2.33) and while calculating $(I - K_1 * K_2)^{-1}$ the size of the inverting matrix has been reduced. 

The $[H]$ can now be easily computed using any of the Eq. (2.33) as

$$[H] = 10^{-6} * \begin{bmatrix}
0.605 & -1.909 & -1.056 & -1.967 & 0.547 & -0.528 & -0.56 & -0.579 \\
-1.056 & 3.332 & 1.843 & 3.433 & -0.955 & 0.922 & 0.978 & 1.011 \\
0.547 & -1.726 & -0.955 & -1.778 & 0.495 & -0.477 & -0.507 & -0.524 \\
-0.528 & 1.666 & 0.922 & 1.717 & -0.477 & 0.461 & 0.489 & 0.506 \\
-0.56 & 1.768 & 0.978 & 1.822 & -0.507 & 0.489 & 0.519 & 0.537 \\
-0.579 & 1.828 & 1.011 & 1.883 & -0.524 & 0.506 & 0.537 & 0.555 
\end{bmatrix}$$

The modified response vector is then calculated as:

$$\{d\} = \left[K_0^{-1} + H\right] \ast \{f\} \tag{2.38}$$

$$\{d_{IPM}\} = \{d\} = \begin{bmatrix}
1.102 \\
-4.598 \\
-1.397 \\
-4.767 \\
0.934 \\
-1.897 \\
-0.973 \\
-2.046
\end{bmatrix}$$

This reanalysis technique is the most effective as this is direct matrix manipulation method without any restrictions. It uses all the information available without creating any other information of its own. The computational cost involved in calculating $H$ matrix is minimal as the size of the inverting matrix is equal to the size of the modified matrix. This makes this reanalysis technique computationally efficient and as the results indicates, in Table 2.2., it is accurate compared to the other two methods.
Element No. | Actual FEA | IPM method | RB method | SMI method  
--- | --- | --- | --- | ---  
1 | 1.1024 | 1.1024 | 0.7135 | 1.0616  
2 | -4.5985 | -4.5985 | -3.3714 | -4.4698  
3 | -1.3967 | -1.3967 | -0.7178 | -1.3255  
4 | -4.7674 | -4.7674 | -3.503 | -4.6348  
5 | 0.9335 | 0.9335 | 0.5819 | 0.8967  
6 | -1.8966 | -1.8966 | -1.5572 | -1.861  
7 | -0.9725 | -0.9725 | -0.6123 | -0.9348  
8 | -2.0459 | -2.0459 | -1.6735 | -2.0069  

Table 2.2: Displacement Result for the compared Re-analysis Method

2.3 Occurrence

Occurrence rating value is an estimate of the number of times failure could occur due to a given failure mode. To quantify this phenomenon failure probability of a particular failure event is considered in this research. Occurrence rating is assigned based on magnitude of the probability of failure due to a given failure mode. This rating is calculated for multiple failure modes. Occurrence rating is assigned values from 1 to 10 where 10 correspond to the highest probability of occurrence and 1 corresponds to the lowest. Since occurrence is a relative parameter that needs to be assigned for all the damage events, a baseline value for probability of occurrence is identified and all other probabilities are scaled to this parameter. In this research, $10^{-7}$ is selected as the reference probability value. Any value of probability of occurrence for an event below
this value should result in a negative occurrence value which corresponds to the lower end of probability rating which is assigned 1 for occurrence. For any other value that is higher than the reference value the following approach is selected to assign the occurrence rating. If the estimated Occurrence value from Eq. (2.3.1) is zero or less than 1 then it is assigned 1.

\[
Occurrence = \frac{10}{7} \ast \log_{10}\left(\frac{Probability\ of\ Occurrence > 10^{-7}}{10^{-7}}\right) - - - (2.3.1)
\]

In the above equation, the quantity \(\frac{10}{7}\) is used to ensure that the maximum possible value for occurrence is 10 based on the maximum possible value of the equation which is \(\log_{10}(10^7)\) which is 7.

2.4 Detection

The objective of the detection criteria is to find out the deficiency in the design as early as possible. Early detection provides effective control and ensures safe operation of the structural component. The detection parameter represents the ability to detect damage before it becomes catastrophic or requires unscheduled maintenance. The detection criteria is assigned based on the level of difficulty in detecting the failure, e.g., visual ground inspection, visual inspection with minimal support equipment, inspections requiring sophisticated tools, tear down analysis, etc. The Detection Rating is similar to the rating of the Severity and Occurrence to maintain the uniformity in the rating process throughout the FMECA. The Detection Rating is scaled from 1 to 10, where 10 correspond to difficulty to detect and 1 corresponds to ease to detect. In this report, the
Detection Rating is assigned following the guidelines presented below which have been obtained from [29].

<table>
<thead>
<tr>
<th>Inspection Requirements Detection</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Special Detail Inspection</td>
<td>9 – 10</td>
</tr>
<tr>
<td>➢ Intensive check using special technique such as NDE</td>
<td></td>
</tr>
<tr>
<td>2. Detail Inspection</td>
<td>7 – 8</td>
</tr>
<tr>
<td>➢ Intensive visual inspection with elaborate access procedures</td>
<td></td>
</tr>
<tr>
<td>3. Internal Surveillance</td>
<td>5 – 6</td>
</tr>
<tr>
<td>➢ Visual check of internally visible discrepancies</td>
<td></td>
</tr>
<tr>
<td>4. External Surveillance</td>
<td>3 – 4</td>
</tr>
<tr>
<td>➢ A visual check of externally visible discrepancies</td>
<td></td>
</tr>
<tr>
<td>5. Walk Around Check</td>
<td>1 – 2</td>
</tr>
<tr>
<td>➢ A visual check performed from ground level</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.3: Detection Rating Criteria**

While the inspection requirement for detection presents bounds for certain situations anything that falls within these bounds can be scaled accordingly.

2.5 Risk Priority Number (RPN)

The Risk Priority Number ranks the failures using a single metric even when the individual failure modes are from different disciplines. It uses the information of all the three parameters, Severity (S), Occurrence (O), and Detection (D). The Risk priority number is the product of these three parameters and can be calculated as RPN=S*O*D. Since the Severity, Occurrence and Detection Ratings are unitless, the product of these three is also a unitless quantity. RPN can be used to assess various failure modes to
determine which of one requires corrective action based on a combination of the likelihood of damage and the severity. While severity can identify safety criticality, occurrence can identify maintenance criticality, RPN can identify situations that are a combination of maintenance critical and safety critical. Therefore, a constraint on RPN can ensure that the designs do not have maintenance critical items that are also safety critical. These can be situations like cracking near bolt holes that happen more often than fracture of a longeron. The design should ensure that the cracks near the bolt hole do not lead to catastrophic damage.
Chapter 3: Numerical Examples [28]

In this research, the Boeing 707 lower wing skin example is considered and the details of the implementation of FMECA are explained using stringer cracking as a failure mode.

3.1 Boeing 707 Lower Wing

The wing model shown in Figure 3.1 is a stiffened panel with symmetric stringers about the centerline. The configuration of these stringers is shown in Figure 3.4. Stringers 2 and 3 are identical and stringers 9 and 10 are mirror image of Stringer 2 and 3. The spacing between each stringer is 7 in. The skin panel is 70 in. wide and 100 in. long. The stringers are attached to skin using rivets. The spacing of rivets in stringers 4, 5, 7 and 8 was according to the rivet pitch shown in Figure 3.5. The single row of rivets in stringers 2, 3, 6, 9 and 10 were spaced 1.2 in. apart. The skin thickness is not uniform throughout the panel and the thickness information of the lower skin is given in Table 3.2.

The skin and stringers are modeled as shell elements and the stringers are attached to the skin using rivets as shown in figure 3.2. The rivets are modeled as spring elements using rivet material properties as shown in figure 3.3. These spring elements translate forces between the stringer and skin in all three directions to represent the shear and axial stiffness of the rivets. The values of the shear in the X and the Z direction and axial stiffness in the Y direction for the fastener for a given stringer are given in Table 3.1. The skin and the base of the stringers were separated by 0.01 in. Spring elements joined nodes at the locations of rivets.
Figure 3.1: Boeing 707 Lower Wing Skin

Figure 3.2: Boeing 707 Lower Wing Skin Meshed Model
Figure 3.3: Boeing 707 Lower Wing Skin with Springs
Figure 3.4: Dimensions of the Stringers (inches)

Figure 3.5: Spacing of Rivets in Stringers

<table>
<thead>
<tr>
<th>Stringer Number</th>
<th>Shear Stiffness (lb/in)</th>
<th>Axial Stiffness (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 and 8</td>
<td>686000</td>
<td>3160000</td>
</tr>
<tr>
<td>5 and 7</td>
<td>978000</td>
<td>1540000</td>
</tr>
<tr>
<td>6</td>
<td>612000</td>
<td>4190000</td>
</tr>
<tr>
<td>2, 3, 9 and 10</td>
<td>591000</td>
<td>4450000</td>
</tr>
</tbody>
</table>

Table 3.1: Shear and Axial Stiffness of the Rivets
Table 3.2 - Skin Thicknesses

<table>
<thead>
<tr>
<th>Skin Section</th>
<th>Thickness (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Stringer 5 and 7</td>
<td>0.18</td>
</tr>
<tr>
<td>Under Stringer 5 and 7</td>
<td>0.375</td>
</tr>
<tr>
<td>Forward and aft of Stringer 5 and 7</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The skin and all stringers were made of 7075 aluminum alloy and possess the material properties: Young’s Modulus $10.3 \times 10^3$ ksi and Poisson’s ratio of 0.33. In this example, the boundary conditions were selected to represent a uniform stress state in both skin and stringers. This was accomplished using a constant displacement of 0.25 in. as in figure 3.6 for all the nodes on the skin and the stringer at one end of the panel, while the opposite nodes of the stringers and the skin were restrained using high stiffness spring element as in figure 3.7 to simulate fixed boundary conditions. The springs are given a very high stiffness magnitude ($9e35$ lb/in) to simulate fixed boundary condition. The idea behind using the spring to model boundary condition is to simulate stringer failure at the wing attached point by reducing the stiffness of the spring to zero at the failure location. Some of the nodes on the stringer are also restricted at 4 locations along the length of the plate to represent rib attachment as shown in figure 3.6. Those nodes are restrained at 13.5in, 40in, 67in, 100in from fixed end of the panel as shown in figure 3.6. Based on these boundary conditions, the stress state in the skin and stringer elements was used to determine various parameters that are required for the FMECA. In this example, fracture of individual stringers due to cracking are considered as the failure modes at the wing attached point where springs are used to model as fixed boundary condition. The FMECA
is then modeled and Severity, Occurrence and Detection rating are calculated based on the formulae presented earlier.

**Figure 3.6:** Boeing 707 Lower Wing Skin with Boundary Conditions

**Figure 3.7:** Boeing 707 Lower Wing Skin with Spring Boundary Condition
3.1.1 Severity

For the severity assignment a shift in the margin of safety is calculated for each of the failure conditions. The process of computing severity is in figure 3.8. First we compute the original margin of safety of all the elements in a given structure using OMS as in figure 3.8. Then we recalculate damage state margin of safety DMS for all elements as in figure 3.8. Once we have the original and the damage state margin of safety we compute the difference between the margins of safety of individual element to compute the shift in margins of safety for all elements. The histogram is then formulated to find the number of element with negative shift in margin of safety and positive shift in margin of safety. The element with negative shift in margin of safety is considered to move toward the failure zone as they trim down the margin of safety after damage and number of element with positive shift in margin of safety is considered to move away from failure zone as they improvise on their margin of safety after damage. The shift in margin of safety for individual failure of stringer 4 and stringer 6 is compared as in figure 3.9. The histogram is plotted based on the shift in margin of safety vs. number of element correspond to the shift in margin of safety. In the case where there is redundancy in the structure or failure is not prominent there the failure will result in redistribution of load within the structure with less number of elements with higher magnitude of shift in margin of safety and more number of elements with low magnitude of shift in margin of safety as in failure of stringer 6 in figure 3.9. On the other hand if the failure is severe and can cause series of failure events then in that case the load redistribution due to failure will result in higher number of element with higher shift in margin of safety and less number of elements with low magnitude in shift in margin of safety as in failure of stringer 4 in figure 3.9. The
histogram of the shift in margin of safety for all the elements for individual failure was constructed and then Eq. 2.2 was used to determine the severity value that needs to be assigned for each of the failure modes. Table 3.3 shows the severity values for all the stringers in the current model where fracture at the wing attached point due to cracking is considered as failure mode. Here both the criteria i.e. number of element involved and magnitude of failure is considered before computing severity. From the result it can be clearly seen that stringer 4, 5, 7, and 8 are more critical than the other stringer based on their severity rating. This is due to fact that the amount of load carried by the failing stringer for example stringer 4 i.e. 60309 psi (maximum stress carried by stringer 4) gets redistributed to the neighboring elements and the load carrying capacity of the stringer 4 is 68000 psi is lost due to damage. The failure of stringer 4 will have more adverse effect than the failure of other stringer for example stringer 6 i.e. 34532 psi (maximum stress carried by stringer 6) which get redistributed to the neighboring elements and still the same load carrying capacity of the stringer 6 i.e. 68000 psi is lost due to damage. Here we can say that higher the load carried and the capacity of the failing member higher is the severity and lower the load carried and higher the capacity of the failing member lower is the severity.
Figure 3.8: Severity Calculation Flowchart

\[ MS_{\text{Shift}} = (DMS_i)_j - (OMS)_j \]

Figure 3.9: Histogram comparing shift in margin of safety for stringer 6 and 4 failure individually
3.1.2 Occurrence

In order to determine occurrence values for this failure modes probability of fracture of a stringer is determined using the following Eq. (3.1). In this equation, \( f(a) \) is the PDF of crack length and it is modeled as a log-normal distribution with mean \( \mu = -3.307 \) and standard deviation \( \sigma = 0.860 \). Fracture toughness \( K_{cr} \) is modeled as a normal distribution with mean \( \mu = 22.8 \text{ ksi (in)}^{0.5} \) and coefficient of variation of 10%. Applied stress was considered as a deterministic variable along with the geometry. Eq. (3.1) can be easily implemented using numerical integration scheme to determine the probability of fracture. Once POF is obtained it is substituted into Eq. (2.3-1) to determine the value of occurrence for that particular stringer. Table 3.4 shows the occurrence values obtained for the current wing panel.

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Severity ‘S’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stringer 2</td>
<td>2.5</td>
</tr>
<tr>
<td>Stringer 3</td>
<td>2.8</td>
</tr>
<tr>
<td>Stringer 4</td>
<td>3.8</td>
</tr>
<tr>
<td>Stringer 5</td>
<td>3.9</td>
</tr>
<tr>
<td>Stringer 6</td>
<td>2.5</td>
</tr>
<tr>
<td>Stringer 7</td>
<td>3.6</td>
</tr>
<tr>
<td>Stringer 8</td>
<td>3.8</td>
</tr>
<tr>
<td>Stringer 9</td>
<td>3.0</td>
</tr>
<tr>
<td>Stringer 10</td>
<td>2.7</td>
</tr>
</tbody>
</table>

**Table 3.3:** Severity values for the stringer based on fracture at wing attached point
\[ \text{Probability of Fracture} (POf) = \int_{0}^{a_{c}} f(a) P \left[ \frac{K_{cr}}{\sqrt{\pi a \beta}} - \sigma_{\text{applied}} \right] da \] (3.1)

**Figure 3.10:** PDF of Crack Length

<table>
<thead>
<tr>
<th>Stringer</th>
<th>Probability of Fracture</th>
<th>Occurrence ‘O’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stringer 2</td>
<td>6.23e-7</td>
<td>3.1</td>
</tr>
<tr>
<td>Stringer 3</td>
<td>6.23e-7</td>
<td>3.1</td>
</tr>
<tr>
<td>Stringer 4</td>
<td>2.80e-17</td>
<td>1.0</td>
</tr>
<tr>
<td>Stringer 5</td>
<td>1.34e-13</td>
<td>1.0</td>
</tr>
<tr>
<td>Stringer 6</td>
<td>7.16e-3</td>
<td>7.6</td>
</tr>
<tr>
<td>Stringer 7</td>
<td>1.34e-13</td>
<td>1.0</td>
</tr>
<tr>
<td>Stringer 8</td>
<td>2.80e-17</td>
<td>1.0</td>
</tr>
<tr>
<td>Stringer 9</td>
<td>6.23e-7</td>
<td>3.1</td>
</tr>
<tr>
<td>Stringer 10</td>
<td>6.23e-7</td>
<td>3.1</td>
</tr>
</tbody>
</table>

**Table 3.4:** Occurrence Values for the Wing Model
3.1.3 Detection

The wing box section represented by the box “Model” in the figure below was used to perform the FMECA. As shown in Figure 3.11, there is no access hole for any of the stringer that provides easy access to the root of the wing. Therefore, based on the rules described previously a common detection rating of 7 is assigned to all the stringer fractures. This is the only parameter that is relatively subjective and the other two parameters are determined through analysis.

Figure 3.11: Location of the Stiffened Panel in the Wing (Ref. [28])
3.2 FMECA for the Lower Wing Model

Using the above three parameters a FMECA table is constructed for stringer failures of the skin panel subject to pure tension as represented in Table 3.5. Here only single failure mode is considered for all the stringers where they fail individually. Multiple failure mode and/or multiple loading conditions can also be incorporated into the same table to determine the risk priority number for those situations. This process gives the comprehensive information about the structural behavior which might be neglected while considering severity or probabilistic approaches individually. Moreover, it can be considered as an outline for a timely maintenance to avoid any further catastrophic events. The basic framework for the process remains the same as described in this example of skin panel.
<table>
<thead>
<tr>
<th>Component</th>
<th>Potential Failure Mode</th>
<th>Severity “S”</th>
<th>Occurrence “O”</th>
<th>Detection “D”</th>
<th>RPN = S<em>O</em>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Wing Stringer of Boeing 707</td>
<td>Fracture of Stringer 2</td>
<td>2.5</td>
<td>3.1</td>
<td>7</td>
<td>54.25</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 3</td>
<td>2.8</td>
<td>3.1</td>
<td>7</td>
<td>60.76</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 4</td>
<td>3.8</td>
<td>1.0</td>
<td>7</td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 5</td>
<td>3.9</td>
<td>1.0</td>
<td>7</td>
<td>27.3</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 6</td>
<td>2.5</td>
<td>7.6</td>
<td>7</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 7</td>
<td>3.6</td>
<td>1.0</td>
<td>7</td>
<td>25.2</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 8</td>
<td>3.8</td>
<td>1.0</td>
<td>7</td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 9</td>
<td>3.0</td>
<td>3.1</td>
<td>7</td>
<td>65.1</td>
</tr>
<tr>
<td></td>
<td>Fracture of Stringer 10</td>
<td>2.7</td>
<td>3.1</td>
<td>7</td>
<td>58.59</td>
</tr>
</tbody>
</table>

Table 3.5: FMECA for the Wing Model
Chapter 4: Summary

In this research, a process for identifying structurally significant items has been presented. This is a formal process based on analytical simulations rather than pure engineering judgment, which could be biased based on the individual performing the analysis. FMECA provides the information required for appropriate design and manufacturing by determining critical events, frequent events and providing the information regarding the detection of the impending flaws. Since this process involves significant analyses of damaged states, using reanalysis or perturbation-based methods will make the process practical for large-scale structural systems. The FMECA process enables identification of critical components based on reliability issues, maintenance issues, or a combination of the two. This RPN can then be used to design structural systems to meet both the reliability and maintenance issues using a single metric. Since the design criteria for aircraft structures involve more than just fracture, additional failure modes need to be incorporated into FMECA. This research is an attempt to develop a process that can be applied to other failure modes. Using the methodology developed, various other failure criteria can be examined.
Appendices I

Nastran Input File to extract Global Stiffness Matrix

RESTART VERSION=LAST,KEEP $ 70K DBS=70K
assign master='bar10_truss.MASTER' $ Change the file name which you need the
stiffness matrix with file extension MASTER
ASSIGN OUTPUT4='matfile', UNIT=12, FORM=FORMATTED, DELETE
ASSIGN OUTPUT4='rhsfile', UNIT=13, FORM=FORMATTED, DELETE
ID MSC,UM531 $ EXAMPLE
TIME 600
DIAG 8 $ PRINT MATRIX TRAILERS AND RECOVERED DATA BLOCKS
DIAG 31 $ PRINT MODULE PROPERTIES LIST (MPL)
SOL 100 ` MALTER 'MALTER:USERDMAP'
TYPE PARM,NDDL,I,N,PEID,MPC,SPC,LOAD,LUSETS,SEID $
TYPE DB,CSTM,PG,KLL,PL,EQEXINS,GPDT,ECTS,GPECT,SIL,SILS,GPLS $
TYPE DB,EST,KGG,BGPDT,ECTS,GPECT,SILS,GPLS $ TYPEDB,ETT,KFS,KIJ,KELM,KAA,BGPDT,CSTM,SIL,K2GG $
PEID=0 $ SEID=0 $ MPC=0 $ SPC=1 $ LOAD=10 $
MATPCH KLL// $ MATPRT OF KGG
OUTPUT4 KLL,,-/-1/12/0/TRUE/9 $ Unit 12, may need FMS statement
OUTPUT4 PL,,-/-1/13/0/TRUE/9 $ Unit 13, may need FMS statement
$MATPCH PL//0/V,Y,NOPRT=1 $ OPTIONALLY PRINT PL BY COLUMNS
ENDALTER
LINK USERDMAP,INCL=MSCOBJ $
CEND

TITLE = THIS ILLUSTRATES THE OUTPUT TYPES UM531
LABEL = DMAP DOES NOT USE MUCH FROM CASE CONTROL DECK

BEGIN BULK
PARAM,NOPRT,1 $ PRINT PG THIS TIME
PARAM,UNUSED,1 $ UNUSED PARAMETER
ENDDATA
Appendices II

Matlab code to formulate Global Stiffness matrix for Nastran result file

Clc
clear all
close all
fid=fopen('matfile','r')
tline=fgetl(fid)
input_size=sscanf(tline,'%e %e %e %s %s ')
row_size=input_size(1)
column_size=norm(input_size(2))
Stiffness_Matrix=zeros(row_size,column_size)
tline=fgetl(fid)
while(feof(fid)~=1)
  if (size(tline,2)==24)
    input_size=sscanf(tline,'%e %e %e ')
    row_number=input_size(1)
    first_column_num=input_size(2)
    last_column_num=input_size(3)
  end
  if (size(tline,2)> 24)
    while(last_column_num ~= 0 )
      input_value=sscanf(tline,'%e %e %e %e %e ')
      for j=1:size(input_value,1)
        Stiffness_Matrix(row_number,first_column_num)=
        input_value(j)+Stiffness_Matrix(row_number,first_column_num)
        first_column_num = first_column_num+1;
        last_column_num = last_column_num - 1
      end
      if(last_column_num ~= 0)
        tline=fgetl(fid)
      end
    end
  end
  tline=fgetl(fid)
end
fclose(fid);
Matlab code to compare Reanalysis Methods (RB method, SMI method, IPM method) displacement solution with Actual displacement solution

clear all
load M_10bar.txt
load Kglobal.txt
load K1_3.txt
format long
K0=Kglobal
xyz=0.1
New_K(3,3)=K0(3,3)-xyz*K1_3(1,1);
New_K(3,4)=K0(3,4)-xyz*K1_3(1,2);
New_K(4,3)=K0(4,3)-xyz*K1_3(2,1);
New_K(4,4)=K0(4,4)-xyz*K1_3(2,2);
New_K(3,5)=K0(3,5)-xyz*K1_3(1,3);
New_K(3,6)=K0(3,6)-xyz*K1_3(1,4);
New_K(4,5)=K0(4,5)-xyz*K1_3(2,3);
New_K(4,6)=K0(4,6)-xyz*K1_3(2,4);
New_K(5,3)=K0(5,3)-xyz*K1_3(3,1);
New_K(5,4)=K0(5,4)-xyz*K1_3(3,2);
New_K(6,3)=K0(6,3)-xyz*K1_3(4,1);
New_K(6,4)=K0(6,4)-xyz*K1_3(4,2);
New_K(5,5)=K0(5,5)-xyz*K1_3(3,3);
New_K(5,6)=K0(5,6)-xyz*K1_3(3,4);
New_K(6,5)=K0(6,5)-xyz*K1_3(4,3);
New_K(6,6)=K0(6,6)-xyz*K1_3(4,4);

Del_K=K0-New_K
K0_inv=inv(K0)
B=-(K0_inv*Del_K)
f1=[0 0 0 -10000 0 0 0 -10000]'

d0=K0_inv*f1

d1=inv(New_K)*f1

51
%% RB Method : Series Expansion  

r1=d0  
r2=B*r1  
r3=B*r2  
rb=[r1 r2 r3]  
K=K0-Del_K  
Kr=rb*(K)*rb  
fr=rb*f1  
dd=inv(Kr)*fr  
z=0  
for i=1:size(rb,2)  
dseries=z+dd(i)*rb(:,i)  
z=dseries  
end  
[d0 d1 dseries]  

%% SMI Method : Series Expansion  

%%%%% STEP 1  
P=-B+B.^2-B.^3+B.^4-B.^5+B.^6-B.^7+B.^8-B.^9  
dseries1=(eye(size(B))+P)*d0  
[d0 d1 dseries dseries1]  

%%%%% STEP 2  
for i=1:length(B)  
    for j=1:length(B)  
        T=B(i,j)  
        P1(i,j)= -T+T^2-T^3+T^4-T^5+T^6-T^7+T^8-T^9  
    end  
end  
dseries2=(eye(size(B))+P1)*d0  
[d0 d1 dseries dseries1 dseries2]  

%%%%% STEP 3  
column = find(B(1,:))  
ni=0  
l=1  
for m=1:length(B)  
    Number of column in the B matrix  
end
if m==column(l)
    for n=1:length(B)%% Number of rows in the B matrix
        for k=1:8 % since we are taking k = 9
            if (mod(k,2)==0)
                r(n,m)=ini-B(n,m)^(k+1)/B(n,m) % Rij matrix
            else
                r(n,m)=ini+B(n,m)^(k+1)/B(n,m) % Rij matrix
            end
        end
        ini=r(n,m)
    end
    ini=0
    P2(n,m)=B(n,m)*(-1+r(n,m))
    if n==length(B)
        if l==length(column)
            break;
        else
            l=l+1
        end
    end
    end
else
    r(:,m)=B(:,m)
    P2(:,m)=B(:,m)
end
end
dseries3=(eye(size(B))+P2)*d0
[d1 dseries dseries2 dseries3] % Comparing result for modified response vector

%%% STEP 4 %%%
%% Computing Pij matrix based on Rij matrix

for i=1:length(B)
    for j=1:length(B)
        P3(i,j)=B(i,j)/(1+r(i,j))
    end
dseries4=(eye(size(B))+P3)*d0
[d0 d1 dseries dseries1 dseries2 dseries3 dseries4]

%%% STEP 5 %%%
%% Computing Pij matrix based on single R value

for i=1:length(B)
    for j=1:length(B)
        r=B(j,j)
        P4(i,j)=B(i,j)/(1+r)
\begin{align*}
\text{dsmi} &= (\text{eye}(\text{size}(B)) + P4) * d0 \\
\begin{bmatrix} d0 & d1 & \text{dseries} & \text{dsmi} \end{bmatrix}
\end{align*}

\% % % % % % % IPM Method % % % % % % % %
%
%%% Comparing equation to prove its validity
IPM1 = inv(K0 - \text{Del}_K) * f1 \\
IPM2 = (K0_inv + K0_inv * (\text{inv}(\text{eye}(8,8) - \text{Del}_K * K0_inv)) * \text{Del}_K * K0_inv) * f1 \\
IPM3 = (K0_inv + (K0_inv * \text{Del}_K * (\text{inv}(\text{eye}(8,8) - K0_inv * \text{Del}_K)) * K0_inv)) * f1

[IPM1 IPM2 IPM3]

i = 1;  
for x = 1:size(Del_K,2)  
\quad \text{if } \max(\text{abs}(\text{Del}_K(x,:))) > 0  
\quad \quad \text{row}(i) = x \ % \ \text{Looking for which column has non zero values}  
\quad \quad \text{row}(i+1) = x+1  
\quad i = i+2;  
\end{align*}

\% Stiffness matrix of a Single Element "Change"

\begin{align*}
\text{for } i = 1:\text{size}(\text{row},2)  
\quad \text{for } j = 1:\text{size}(\text{column},2)  
\quad \quad K1(i,j) = \text{Del}_K(\text{row}(i),\text{column}(j))  
\quad \end{align*}

\begin{align*}
\text{for } i = 1:\text{size}(\text{row},2)  
\quad \text{for } j = 1:\text{size}(\text{column},2)  
\quad \quad K2(i,j) = K0_inv(\text{column}(j),\text{row}(i))  
\end{align*}

K2 = K2';
for i=1:size(row,2)
    K3(i,:)=K0_inv(:,row(i))
end
K3 = K3';
for i=1:size(column,2)
    K4(i,:)=K0_inv(column(i),:)
end
I=eye(size(K1,1),size(K2,2))
Z=inv(I - (K1*K2))
H = K3*Z*K1*K4
NewK_inv = K0_inv + H
dipm = NewK_inv * f1

[d0 d1 dseries dsmi dipm] Comparing all result together to find best fit
Appendices IV

Matlab code for writing spring information in the Wing Box Model input file.

clc
close all
clear all
load Set1; load Set2; load Set3; load Set4; load Set5; load Set6; load Set7; load Set8; load Set9; load Set10; load Set11; load Set12; load Set13; load Set14; load Set15; load Set16; load Set17;
tfid = fopen('spring_info.inp','w');
a=1;
b=1;
for i=1:17
  if i==1
    Set= Set1;
    connect1=[1 1];
    connect2=[4 1];
    axial=4450000;
    shear=591000;
  end
  if i==2
    Set= Set2;
    connect1=[1 1]
    connect2=[4 2]
    axial=4450000;
    shear=591000;
  end
  if i==3
    Set= Set3;
    connect1=[1 1]
    connect2=[5 1]
    axial=3160000;
    shear=686000;
  end
  if i==4
    Set= Set4;
    connect1=[1 1]
    connect2=[5 1]
    axial=3160000;
    shear=686000;
  end
  if i==5
    Set= Set5;
    connect1=[2 1]
    connect2=[5 1]
axial=3160000;
shear=686000;
end
if i==6
    Set= Set6;
    connect1=[2 1]
    connect2=[5 1]
    axial=3160000;
    shear=686000;
end
if i==7
    Set= Set7;
    connect1=[2 1]
    connect2=[6 1]
    axial=1540000;
    shear=978000;
end
if i==8
    Set= Set8;
    connect1=[2 1]
    connect2=[6 1]
    axial=1540000;
    shear=978000;
end
if i==9
    Set= Set9;
    connect1=[2 1]
    connect2=[7 1]
    axial=4190000;
    shear=612000;
end
if i==10
    Set= Set10;
    connect1=[2 1]
    connect2=[6 2]
    axial=1540000;
    shear=978000;
end
if i==11
    Set= Set11;
    connect1=[2 1]
    connect2=[6 2]
    axial=1540000;
    shear=978000;
end
if i==12
Set = Set12;
connect1 = [2 1]
connect2 = [5 2]
axial = 3160000;
shear = 686000;
end
if i == 13
  Set = Set13;
  connect1 = [2 1]
  connect2 = [5 2]
  axial = 3160000;
  shear = 686000;
end
if i == 14
  Set = Set14;
  connect1 = [3 1]
  connect2 = [5 2]
  axial = 3160000;
  shear = 686000;
end
if i == 15
  Set = Set15;
  connect1 = [3 1]
  connect2 = [5 2]
  axial = 3160000;
  shear = 686000;
end
if i == 16
  Set = Set16;
  connect1 = [3 1]
  connect2 = [8 1]
  axial = 4450000;
  shear = 591000;
end
if i == 17
  Set = Set17;
  connect1 = [3 1]
  connect2 = [8 2]
  axial = 4450000;
  shear = 591000;
end
for j = 1:3
  tline = fprintf(tfid, '%Element, type=Spring2, elset=Springs/Dashpots-%d-spring', a);
for k = 1: (size(Set, 1))
  tline = fprintf(tfid, '\n');
tline=fopen(tfid, '%d, PART-%d-%d.%d, PART-%d-
%d.%d', b, connect1(1,1), connect1(1,2), Set(k,1), connect2(1,1), connect2(1,2), Set(k,5))
b=b+1
end
tline=fopen(tfid, \n');
tline=fopen(tfid, '*Spring, elset=Springs/Dashpots-%d-spring', a);
tline=fopen(tfid, \n');
if j==1
  tline=fopen(tfid, '%d, %d', 1, 1);
tline=fopen(tfid, \n');
tline=fopen(tfid, '%d. ', shear);
end
if j==2
  tline=fopen(tfid, '%d, %d', 2, 2);
tline=fopen(tfid, \n');
tline=fopen(tfid, '%d. ', axial);
end
if j==3
  tline=fopen(tfid, '%d, %d', 3, 3);
tline=fopen(tfid, \n');
tline=fopen(tfid, '%d. ', shear);
end
tline=fopen(tfid, \n');
a=a+1
end
end
fclose(tfid)
REFERENCES


