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Stochastic Modeling of Geometric Mistuning and Application to Fleet Response Prediction

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Stochastic Modeling of Geometric Mistuning and Application to Fleet Response Prediction

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Engineering

by

Emily B. Henry
B.S.M.E., Wright State University, 2012

2014
Wright State University
I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Emily B. Henry ENTITLED Stochastic Modeling of Geometric Mistuning and Application to Fleet Response Prediction BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering.

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ABSTRACT


An improved spatial statistical approach and probabilistic prediction method for mistuned integrally bladed rotors is proposed and validated with a large population of rotors. Prior work utilized blade-alone principal component analysis to model spatial variation arising from geometric deviations contributing to forced response mistuning amplification. Often, these studies considered a single rotor measured by contact probe coordinate measurement machines to assess the predictive capabilities of spatial statistics through principal component analysis. The validity of the approach has not yet been demonstrated on a large population of mistuned rotors representative of operating fleets, a shortcoming addressed in this work. Furthermore, this work improves the existing predictions by applying principal component methods to sets of airfoil (rotor) measurements, thus effectively capturing blade-to-blade spatial correlations. In conjunction with bootstrap sampling, the method is validated with a set of 40 rotors and quantifies the subset size needed to characterize the population. The work combines a novel statistical representation of rotor geometric mistuning with that of probabilistic techniques to predict the known distribution of forced response amplitudes.
List of Symbols

\begin{itemize}
  \item \textbf{C} \quad \text{Engine Order Excitation Number}
  \item \textbf{D} \quad \text{K-S Test Statistic}
  \item \textbf{f} \quad \text{Generalized Force}
  \item \textbf{F} \quad \text{Forcing Function}
  \item \textbf{F}_a \quad \text{Empirical Distribution Function}
  \item \textbf{G}_b \quad \text{Empirical Distribution Function}
  \item \textit{m} \quad \text{Number of Retained PCs}
  \item \textit{n} \quad \text{Number of Components Measured}
  \item \textit{N} \quad \text{Number of Sectors per Rotor}
  \item \textit{N}_F \quad \text{Number of Natural Frequencies Solved}
  \item \textit{p} \quad \text{Number of Surface Coordinates}
  \item \textit{sf} \quad \text{Modal Scatter Fraction}
  \item \textit{t} \quad \text{Time}
  \item \textbf{u} \quad \text{Mode Shape Matrix}
  \item \textbf{U} \quad \text{Displacement}
  \item \textit{x} \quad \text{Vector of } \textit{p d}-\text{Dimensional Coordinates}
  \item \textit{\bar{x}} \quad \text{Vector of Mean Coordinates}
  \item \textbf{X} \quad \text{Original Data Set Matrix}
  \item \textbf{\Delta X} \quad \text{Matrix of Measured Deviations}
  \item \textbf{Z} \quad \text{Principal Component Score}

\end{itemize}

\textbf{Greek Symbols}
\begin{itemize}
  \item \textbf{\alpha} \quad \text{Confidence Level}
  \item \textbf{\Lambda} \quad \text{Eigenvalue Matrix}
  \item \textbf{\Omega} \quad \text{Imposed Forcing Frequency}
  \item \textit{\omega} \quad \text{Natural Frequency}
  \item \textbf{\Phi} \quad \text{Phase}
  \item \textbf{\Psi} \quad \text{Eigenvector Matrix and Uncorrelated PCs}
  \item \textbf{\Sigma} \quad \text{Covariance Matrix of } \textbf{\Delta X}
  \item \textit{\zeta} \quad \text{Damping Coefficient}
\end{itemize}

\textbf{Latin Superscripts}
\begin{itemize}
  \item \textit{d} \quad \text{Number of Dimensions Number}
  \item \textit{s} \quad \text{Sector Number}
\end{itemize}

\textbf{Latin Subscripts}
\begin{itemize}
  \item \textit{c} \quad \text{Complex Component}
  \item \textit{i} \quad \text{Distinct Set of Measured Points}
  \item \textit{j} \quad \text{Surface Point with } \textit{d}-\text{Dimensional Data}
  \item \textit{k} \quad \text{Summation Index}
  \item \textit{mo} \quad \text{Mode number}
\end{itemize}
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Dedicated to

Olivia Jayne
Introduction

Turbine engines are comprised of sequential fan and compressor stages which aerodynam-ically compress free-stream air flow to produce thrust [5]. Air enters the engine and passes through the compressor, which has alternating stationary (stator) and rotating (rotor) components. Rotors are constructed from airfoils attached to a hub integrally or as inserted units. Once the compressor stage increases the air pressure and temperature, the resulting flow is passed to the combustor where it is mixed with fuel and ignited. This further increases the pressure, temperature, and velocity of the air moving through the system. Flow is then directed through the turbine section, where the pressure performs work on the turbine rotors. The action of the turbine powers the compressor, while the exhaust gases accelerate through the nozzle to produce thrust. It is the structural dynamics of these rotors – components that sustain multi-axis thermo-mechanical fatigue loading – that is of principal interest in this thesis.

Rotors are subjected to thermal, centrifugal, and bending loads, resulting stresses that may lead to fatigue damage. Particularly insidious is high cycle fatigue (HCF), which reduces structural integrity through repeated loading, inducing catastrophic failure. High cycle fatigue was responsible for 56% of class A engine-related failures between 1982 and 1996, incidents distinguished by fatalities or a minimum of one million dollars in damage [6]. These failures manifest because of inability of prior generation design systems to properly account for airfoil vibration. Predicting airfoil vibration is challenging due to the complexities of unsteady aerodynamics and structural dynamic response. Therefore, the
primary safeguard against fatigue failure is improving the ability to understand and predict vibratory response of airfoils and rotors.

A primary cause of HCF failure is forced vibration amplification resulting from a phenomenon known as mistuning. Mistuning arises from cyclic asymmetry, a product of inevitable random sector deviations caused by a multitude of factors, including uneven wear patterns, material property variation, and precision of manufacturing [7, 8]. The impact of these perturbations on structural response is disproportionate to their size [9–11].

Previous efforts quantified HCF fleet risk of mistuned rotors by incorporating frequency-based approaches to capture vibratory characteristics [12–17]. These methods assume geometric perturbations change blade natural frequencies without impacting the corresponding mode shapes. Subsequently, the forced response is approximated as a linear combination of tuned blade modes. The linear approximation was later compared to geometry-based mistuning models, considering both frequency and mode shape variation as a product of mistuning. This significantly improved probabilistic predictions of forced response and fleet risk calculations [3].

It is desirable to use these more accurate geometry-based mistuning approaches for probabilistic simulations; therefore, a statistical method is applied to model airfoil surface geometry variations. A particular approach utilizes principal component analysis (PCA) as a tool to reduce the required geometric parameters while maintaining accurate mistuning response fidelity [18–25]. Principal component analysis represents the variation in a set of data as the linear combination of principal components (PCs) calculated by eigen-analysis of its covariance matrix. Decomposition of airfoil geometries through PCA and retention of limited PCs can accurately simulate modal properties for small sets of rotors [18, 19]. Various sources postulate that more than three times as many PCs are required to accurately predict aerodynamic performance responses than are necessary to converge to within a single percent of geometric variation, indicating that geometry explained by the retained PCs does not directly correlate to accurate estimation of system responses.
Furthermore, approaches utilizing this technique experience compounding error during reconstruction; work expanding on these initial findings indicated that 3% variation in natural frequencies propagated to greater error in modal response, possibly limiting the effectiveness of its application for modeling geometric mistuning [25]. Rote utilization of PCA fails to represent the target population with limited geometric variation, implying the requirement of intelligent application.

The work that follows builds upon these prior efforts by describing the convergence of PC retention on geometry, frequency, and forced response of statistically decomposed airfoil measurements. This study utilizes airfoil PCA while expanding application to rotor geometries, where each rotor set of airfoils is treated as a single measurement observation in order to effectively capture blade-to-blade correlation. Airfoil geometry correlations are essential for prediction of mistuned response behavior in the population. This work adds to the body of knowledge by assessing convergence on a large population of mistuned, in-service rotors. In addition, it demonstrates the ability of the defined approach to model the population with a small sample of rotors. The novelty of this study lies in the probabilistic framework based upon decomposition through PCA, which extends the science of mistuning prediction from individual airfoil use to holistically consider the airfoil sets.
Mistuning Research Review

2.1 Basic Vocabulary

Integrally bladed rotors (IBRs) are manufactured monolithically using computer numerical control (CNC) milling or welding [26]. IBR blades are irreplaceable yet increase engine efficiency through considerable weight reduction. Despite design for cyclical symmetry, in-service IBRs display inevitable random geometric deviations caused by a multitude of factors [7, 8]. The impact of these perturbations on structural response is disproportionate to their size, generating forced response amplitude magnifications severe enough to instigate HCF failure [9–11].

The primary method of HCF mitigation is a design approach known as Resonance Avoidance. Component design is governed by the Campbell diagram exemplified in Figure 2.1. These diagrams unify measurements of natural frequencies with corresponding wheel speeds for circumferential (nodal diameter) and radial (nodal circle) wave numbers. The intersections of these characteristic descriptors with engine order excitation lines define the resonant speeds at which forced vibration is undesirable [4]. As such, it is necessary to avoid these crossings at engine idle, cruise, or maximum RPM; in fact, such an occurrence at low engine order would necessitate redesign of the rotor.

Fatigue crack inspection practices traditionally utilize the propagation rate of observed cracks as the metric determining removal from service, a method known as the damage...
tolerant approach [27]. This method is exceptionally challenging to implement into fleet assessment, as the detection rate of failure flaws is unreliable. The inherent uncertainty cuts in both directions, causing early removal of in-service hardware to ensure safety without mitigating the risk of an undetected flaw leading to catastrophic failure. As such, the better solution is to better manage the vibratory responses of the engine components through careful design or improved simulated fleet assessment techniques.

Unfortunately, design practices focused on avoiding HCF, including damage tolerant approaches and resonance avoidance, produce overly conservative structures that reduce engine performance due to excess weight or inefficient geometries. Thus, it is essential to examine improved fleet risk assessment techniques capable of accurately predicting the forced response distributions of engine hardware using limited information. The approach will, by necessity, be stochastic in nature, as forced response is heavily dependent on the distribution of small variations in structural geometry.

The most common types of vibration problems are (1) resonant vibration at an integral order (i.e. multiple of rotation speed) and (2) flutter, which is instability occurring at a non-integral order vibration [4]. It is therefore of great need to design around these failure regions in order to minimize their effects on the system. Rotors experience system modes of vibration influenced by: cantilevered blade vibration; support structure vibration; engine speed; temperature; damping; and the extent of mistuning. Therefore, not only are the frequencies describing the system of interest, the corresponding mode shapes are also
required in order to fully describe the structural dynamics of a given structure and predict forced response values.

2.2 Vibration Characteristics of Tuned Rotors

Tuned IBRs are inherently cyclically symmetric; thus, the response of the entire system may be ascertained from the response of a single blade and rotor segment [11, 28].

2.2.1 Classification of Single Blade Modes

A variety of system modes are observed during vibration of a cantilevered blade, including torsional (T), bending (B), chordwise bending (CWB), and edgewise bending (EWB). Examples of cantilever plate bending and torsional modes are presented in Figure 2.2, where the parenthetical numbers respectively denote the number of nodal lines and the modal response type. Although blade mode shapes rarely appear so simplistic, they are in fact composites of these basic shapes.

2.2.2 Classification of Single Disk Modes

Cyclically symmetric structures exhibit identical mode shapes across each sector with the exception of phase differences in sector-to-sector interfacing. This phase difference is represented by the inter-blade phase angle and causes nodal lines to appear across the disk. System modes are thus referred to as nodal diameter (ND) modes [11]. These lines of zero displacement are equally spaced and diametric. The response can be tracked in identical locations sector-to-sector to form sinusoidal response curves. The number of periods present within the sinusoidal displacement diagram indicates the number of ND modes present at the excited natural frequency.
A sequence of ND lines ranging from zero to three is depicted in Figure 2.3. Since the displacement of the leading edge tip results in a circumferential sinusoidal response, the response extrema necessarily occurs halfway through each sector space delineated by the nodal lines. Such a response is illustrated in Figure 2.4, where each stem represents the vibratory response of a leading edge tip on a 22-bladed rotor. The plot indicates a period of three for the sinusoidal response with six locations of zero response; therefore, the response is excited at a ND of three.
A further representation of ND responses is provided in Figure 2.5 from [2]. The total number of NDs is limited by the number of sectors $N$ as $\frac{1}{2}N$ for an even number of blades or $\frac{1}{2}(N - 1)$ for an odd number of blades. For a limited number of blades, nodal diameters may appear curved, straightening diametrically as the number of blades increases to cause a more distributed response. The figure describes responses at NDs of one through five.

In addition, when the nodal diameter is zero or at its maximum value, the corresponding mode shapes are singular; comparatively, all other nodal diameters arise in pairs with equivalent, orthogonal mode shapes at equal natural frequencies. Finally, natural frequencies tend to occur in clusters corresponding to particular modal families but at different ND responses.

Figure 2.4: Sinusoidal Response of Tip Displacement with Nodal Diameter 3.
2.2.3 Forcing Fields and Modal Response

Natural frequencies are calculated with finite element analysis and plotted as a function of the number of NDs. Lines are drawn to connect specific blade modes (e.g. 1B, 2B, 1T, etc.) as the number of ND modes increase from zero to one-half the total number of sectors. This is done in an effort to visualize blade-dominated and disk-dominated modes as shown in Figure 2.6. Modes dominated by blade motion appear as horizontal lines, revealing the near constancy of the natural frequencies over a range of NDs. Disk-dominated modes appear as slanted lines in the diagram, since disk modal stiffness increases rapidly with the number of nodal diameters. These diagrams are referred to as tuned natural frequency curves or nodal diameter plots [23].

Engine order (EO) excitation describes the perturbations experienced by a rotor due to flow field disturbances encountered during rotation [11]. Rotor excitation at EO-4 implies passage of each blade through four evenly spaced forcing peaks per revolution. This type of forcing excites only modes with a harmonic index matching the engine order – for example, the mode resulting in a ND of four is likewise excited by EO-4.
Figure 2.6: Example Natural Frequency vs. Nodal Diameter Plot, Adapted from [3].

Dynamic loading arises from the interaction between rotor and stator stages. As air flows through these regions, flow field distortion causes equally spaced high and low pressure regions to form circumferentially [4]. These regions produce harmonic structural responses as a function of rotational speed, with variable loading simultaneously applied to all blades. As the driving frequency achieves parity with the resonant condition, the vibratory response of the rotor dramatically increases. This effect is worsened with the introduction of geometric mistuning, as the forced response amplitudes exhibit hypersensitivity to rotor variations.

2.2.4 Frequency Veering

Frequency veering describes the non-intersecting convergence and divergence of loci eigenvalues with respect to NDs [29]. In Figure 2.6, this phenomenon is evident at a ND of two and a frequency of 3,500 Hz. It is associated with interaction or coupling of two or more system vibration modes and arises in a variety of structural dynamics problems [29–31]. Representations of the veering phenomenon were developed by Balmés using a three degree-of-freedom lumped parameter model, wherein interacting modes within the veering region were shown to exchange shape through stiffness element variation in the lumped
parameter model, such that the mode shape corresponding to the lower natural frequency becomes the mode shape of the higher natural frequency and vice-versa. This phenomenon was also found to exist within cases of rotor dynamics [32]. Locations of veering are highly sensitive to mistuning and can cause extreme amplification in forced response analyses, especially in regions where disk-dominated modes of strong interblade coupling interact with blade-dominated modes of weak interblade coupling. Therefore, it is necessary to narrow response analyses to these veering locations with the intent of capturing the most aggressive phenomena causing forced response amplification [23].

2.3 Mistuning Mechanisms

2.3.1 Introduction

Geometric mistuning causes deviations from nominal physical responses and resonant frequencies, with system mode shapes exhibiting distorted ND responses caused by excitation at multiple EOs, no longer pure ND modes at a single EO excitation. This magnifies forced response peak amplitudes and has the potential to localize the vibration energy catastrophically to a single blade. As a result, HCF life is decreased through accelerated rotor fatigue [11, 33]. Further, frequency veering regions feature significant vibratory response at the intersection of relevant modal excitation and the existence of energy transference mechanisms between blade and disk, resulting in strong disk-blade dynamic interactions with significant mistuning effects [23, 34–36]. Mistuning harmonically excites modes off-resonance, further disrupting system dynamics which is worsened by operating the engine in veering regions.
2.3.2 Frequency Splitting

Tuned systems have repeating natural frequencies with phase shifts resulting in ND rotation, although the shape remains unchanged. Mistuning disrupts these reoccurrences, resulting in a phenomenon known as frequency splitting. Instead of repeated eigenvalues, mistuned systems develop a band around the tuned natural frequency and respond above and/or below the tuned value. Subsequently, nodal diameters are no longer accurately produced on the rotor and mistuned modes tend to localize in certain blades. An example of frequency splitting is shown in Figure 2.7.

![Figure 2.7](image.png)

Figure 2.7: Example of Frequency Splitting.

2.3.3 Mode Localization

For a periodic structure, the model is composed of dynamically coupled identical substructures. With the introduction of mistuning, periodicity is broken and mode shapes that were once characterized by sinusoids extended through the structure are now confined to a small region – an effect known as mode localization[11]. Figure 2.8 shows mode localization by displaying the relative displacement of each blade. In the mistuned case (Figure 2.8, lower) the responses are localized and amplified in certain regions, whereas the tuned response is shown as cyclically symmetric following a sinusoidal pattern for amplitudes of responses.
Another example of this phenomenon shows a tuned rotor in Figure 2.9(a) with ND-2 occurring at 90° intervals [37]. The resulting mode shape mapped onto the geometry of the rotor is symmetric and equally spaced, where the amplitude of displacement ranges from low (blue) to high (red). The introduction of mistuning alters the mode shape to that of Figure 2.9(b), where the modal response is localized to the 9 o’clock blade with resulting response deformation and subsequent magnification.

In addition, mode localization is affected by blade-to-blade coupling and rotation speed. Blade-to-blade coupling is the ratio of mistuning strength to coupling strength and has been identified as the key parameter for determining mode localization [38]. As interblade coupling decreases, mode localization increases. Mode localization can be mitigated through the increase of rotation speed [39].

### 2.3.4 Amplitude Magnification

As a result of mistuning, forced response amplitudes of certain blades are much larger than those predicted by analysis of the tuned design [15]. This likewise carries through to stresses within the rotor. In a tuned case, engine orders excite modes with matching nodal diameter indices (e.g. when EO-2 is excited, the mode shape causes a nodal diameter
of equivalent value). Mistuned cases have higher harmonic content resulting in multiple modes excited by the engine order excitation. The modes that retain a significant portion of harmonic content matching the excitation number will highly respond.

Responses of mistuned structures are denoted as amplitude magnifications, which is the ratio of the mistuned peak response to that of the tuned peak response in a given driving frequency range. As mistuning increases, the frequency at which the response occurs increases in span, which is caused by multiple modes becoming excited with a single EO excitation. Additionally, the responses – which once occurred at a single frequency and amplitude for a tuned rotor – become varied around the rotor. The magnitudes of these responses are further increased when considering frequency veering locations due to their high sensitivity to mistuning.

For mistuned structures, responses can be much greater than what is expected of a tuned disk, driving the need for conservative stress predictions. Amplitude magnification is affected by geometric perturbations, operating conditions, material variations, the strength of blade to blade coupling, mistuning, blade mode shapes, and damping.
2.4 Statistical Techniques

This variability in blade responses can be visualize through response distributions. In order to predict a fleet response to harmonic loading, a corresponding distribution is predicted and is compared to the known true distribution of mistuned responses. There are two basic approaches: parametric versus non-parametric, where parametric approaches rely on a standard distribution (e.g. normal) while non-parametric approaches do not. For an unspecified, non-normal distribution, the two-sample Kolmogorov-Smirnov (K-S) test is of use. It tests two empirical cumulative distribution functions (CDFs) to determine if they could be sampled from the same parent distribution.

Advantages of a K-S test include its ability to be used with no underlying parent distribution being defined for comparison, but is limited by increased sensitivity near the center of the distribution as compared to the tails. The K-S test can be used with discretized data and non-parametric distributions, which is very important for this work. Previous works with forced response distributions of turbine engines have applied this approach for determining accuracy of distributions and found it a useful comparison technique [40, 41].

Additionally, Monte Carlo (MC) simulations are used to model fleet responses with limited data [42]. In these simulations, mistuning is randomly introduced through various mechanisms and the statistics are calculated of the forced response amplitudes. Generally, this requires thousands of iterations in order to obtain converged solutions, a process that is computationally expensive.

A demonstration of statistical approaches is provided in [43] with the combination of a first-order statistical forced response perturbation method and MC simulations. It leverages MC simulations through sampling within a distribution for modal properties accompanied by a modal analysis to obtain the forced response values.
2.5 Fleet Response Review

Srinivasan in [4] expands the understanding of resonant vibration characteristics of engine blades in his IGTI scholar paper. His work outlines the need of comprehension of this topic as there are industry-wide incidents of rotor blade failures due to vibration-induced high cycle fatigue. He outlines six ways to account for vibratory characteristics: (1) assessment of flow variations, both origin and transportation along the flow field; (2) unsteady aerodynamics of cascades under various flow conditions expected; (3) vibration natural frequencies and corresponding mode shapes for the operating range; (4) non-aerodynamic damping quantification; (5) material property variation under operation and the influence on structural integrity; and (6) dissimilarities in aerodynamic and structural parameters and their influence on the response.

This work focuses on approaches (3) and (6) and how to utilize this method for fleet response predictions. Structural dynamics account for both frequency and mode shape responses of a rotor and provide insight into resonant vibration levels. Prediction of forced response amplitudes can then be quantified depending on the accuracy of the structural characteristics. Thus, the limitation is based on accurately capturing mistuning system response characteristics of the model. While work can further progress to focus on the inconsistencies of material properties throughout a rotor as in approach (5) and the consequential change in forced response, this work merely considers geometric causes of mistuning.

The survey of literature in [9] outlines statistical and deterministic approaches to mistuned bladed disk analysis. These approaches are used to design around natural frequency avoidance, determine the sensitivities of mistuning effects, and presents work regarding forced response amplitudes. The literature presented concludes mistuning greatly impacts forced response results and accounting for these deviations requires detailed finite element analysis.
2.5.1 Frequency Mistuning

Previous efforts have relied on frequency mistuning to determine rotor amplification due to forced response [12–17]. While this approach is effective for some cases, it is limited in that it does not account for any mode shape variation as it assumes equal mode shapes for frequency perturbations. Accurate response representation is thereby limited, as both frequency splitting and mode shape localization provide stark evidence of mode shape variation with the introduction of geometric mistuning. The following works show the results of these approaches regarding investigation of mistuned response.

Works investigated the deviations in mass and stiffness matrices for understanding mistuned behavior with the presumption that mode shapes remain unchanged [44]. It applied variations to the arrangement of these matrices in order to determine a pattern recognition technique for controllers of mistuned response; by manipulating the variations in these matrices, it was presumed that mistuned response could be predicted and applied as an HCF mitigation technique. The work validates the importance of coupling and traveling wave direction with respect to response and determined optimal sequencing of mistuning and blades to further reduce mistuning and its detrimental effects, producing a purposely mistuned structure as proposed in [9].

This approach continued in the Bladh et al. study [15] to develop and test reduced order models of mistuned systems using component mode synthesis (CMS) as the primary technique and non-CMS for comparison. With frequency mistuning as the basis of the reduced order models, responses were predicted through the following methods. The reduced order models tested in this study included projection, standard and SMART Craig-Bampton (CB) methods compared against REDUCE, a standard technique for reduced order modeling of mistuned structures. The non-CMS approach uses mistuning projections and operates under the assumption that mistuned modes of a bladed disk are linear combinations of tuned modes, projecting mistuned data onto tuned modes. REDUCE is a
disk-induced blade constraint and is a way to truncate the modes for data reduction. The SMART CB-CMS approach is a secondary modal analysis performed on an already reduced CMS model; it is introduced in the modal domain instead of the physical, which reduces computational expense compared to other methods applied in the study. Each method was compared using a theoretical count of floating point operations during set up and use in forced response predictions. The work concludes that SMART works with improved accuracy and efficiency, but is still reliant on the linear combination assumption for mistuned mode shapes [45]. Though these methods appear to work for reduced order modeling, they rely on the false basis that only natural frequencies change. Further investigation is required to mend these shortcomings to gain understanding of how mistuning affects the system.

The works conducted on the basis of frequency mistuning effects do not account for the variations in the mode shapes except by assuming the mistuned mode shapes are a linear combination of tuned shapes. This limits their applicability to mistuned structures for probabilistically determining vibratory response amplitudes. While effective for reducing data and assessing natural frequencies of the system, the lack of mode shape predicability requires further work for appropriately assessing the consequences of mistuning.

2.5.2 Geometric Mistuning

The next iteration of work focuses to address this need by considering the propagated effects of geometric mistuning in place of frequency mistuning. This wraps in changes to both structural dynamic properties through geometric variations in the model. Garzon and Darmofal explored principal component analysis (PCA) as a way to quantify geometric variation of blades caused by mistuning on the performance of a compressor IBR [18]. This was done to reduce the order of the model and likewise, probabilistically assess mistuning from geometric deviations. The analyses considered include: fluid flow analysis for passage loss and turning and quantification of efficiency and pressure loss with respect to
geometric variations. The work found that although only 5 of 150 principal components generated by four rotors are required to model 99% of geometric scatter, though 15 are necessary to model 99% of the overall aerodynamic response impact. Geometric mistuning is therefore a large contributor to the response of rotors and without sufficient retention of information (more than 99% in this case), the appropriate dynamic responses cannot be accurately predicted. It is then of interest to determine the convergence of the predicted responses based upon the retained geometric variance.

The work presented by Brown and Grandhi evaluates the effects of geometric mistuning on blade-alone forced response by simulating measured data for a single rotor, representative of actual manufacturing deviations [21]. This work utilized reduced order modeling techniques to determine the detriment of geometric mistuning on forced response results. Statistics from the simulated measured rotor were used in conjunction with MC sampling to generate probabilistic fleet responses. These statistics are determined by applying PCA to the airfoil geometric data and sampling from within them assuming a uniform normal distribution. It further states that geometric variation on the order of 5 mils leads to response deviations from 5-40%, although sampling from PCA information can conservatively predict these ranges of forced response. It concludes that probabilistic analysis of airfoil responses is the next step for improving geometric mistuning and its effects in response and reliability.

High cycle fatigue assessment is performed probabilistically in [22] for an IBR; it suggests an alternative approach with autoregressive models to account for spatial sector correlation. The approach of a probabilistic blade HCF assessment begins by measuring and mapping surface geometry of a test rotor through CMM data. PCA is then applied to the airfoil geometry in order to obtain a reduced order FEM. Next, natural frequencies, their corresponding mode shapes, and modal stresses are predicted. This leads to a tuned forced response prediction and with the aid of deterministic input such as mechanical and aero damping, a mistuning calculation can simultaneously be obtained. Then, with the
tuned and mistuned calculations and the application of probabilistic Goodman assessment, a probabilistic fleet reliability can be calculated. While appealing, these models require further research for validation and use in fleet risk assessment.

The work presented in Baik, et al. identifies an alternative method to flow assessment entitled power flow analysis [23], service as a less expensive alternative to Monte Carlo simulation. Power flow analysis assesses mistuning effects based on tuned information and characterizes the dynamic interaction between the blades and disks of the tuned structure. It allows the computation of energy propagation through modes in order to estimate a mistuned response. Power flow analysis explains the sensitivity of forced response amplitudes to geometric mistuning as a result of disk-blade coupling. Though this method is efficient for design qualifications and parameters, it is used for the proposed work of this thesis.

The work presented by Duffner expands upon PCA application with the objective to: characterize manufacturing variability and replicate it for analysis; calculate performance scatter with respect to limited manufacturing variations; and quantify distributions of flow sensitivity to geometric mistuning values [24]. This is done by application of PCA on vane measurements to decompose geometric variations into uncorrelated mode shapes or principal components (PCs). The combination of these modes account for iterative geometric variation explanations from mistuning as each mode is responsible for a different and decreasing amount of geometric scatter. As the mode evaluated increases, computational fluid dynamics (CFD) analysis reveals that flow field disruption is much greater for dominating modes retaining most of the geometric variation. Understanding the implications of early modes is required to accurately capture the dynamic responses of a population.

Brown and Grandhi [25] utilize the limitations of frequency mistuning in their physics-based reduced order modeling of modal and forced responses in airfoils to account for geometric deviations. The work applies PCA to the blade geometries of an advanced damping low-aspect ratio fan (ADLARF) rotor. It considers the natural frequencies of the geometrically mistuned predictions from PCA. The results show that although the model is not ac-
curately capturing the true mistuning, frequencies are only subtly changed and within ±3% but occur in locations of increased amplitude magnification. Next, the work focused on the effects of PCA on modal force results of blade geometries, as the deviation in mode shapes with geometric mistuning had not been as thoroughly investigated. Brown et al. found that the error and spread of modal force is much larger than that of the frequency plot, narrowing that it is much more challenging to capture mistuned mode shapes than it is to obtain the natural frequencies.

Finally, Brown and Grandhi considered the effects of PCA on forced response [25]. Since forced response is highly dependent on modal force, the forced response results were semi-resemblant of these deviations. Future work should consider how PCA application affects the actual results since this work reveals some of its limitations. This study demonstrated the impact of geometric mistuning on modal and forced response behaviors of airfoils by tracking deviations in natural frequencies, mode shapes, and forced response amplitudes and suggested to apply a linear regression model to quantify and reduce error caused by PCA. With progression of this work, it is necessary to track the deviations in these responses in order to assess the abilities of PCA on probabilistic fleet risk prediction.

Sinha, et al. considered the application of PCA onto blade geometries of a population of eleven rotors with 54 blades per rotor [8]. This was done to model geometric variations between blades for application of geometric mistuning models. The work progressively retains airfoil PC modes to investigate the impact of increasing geometric variation on natural frequencies and mode shapes. With small deviations of blade geometries from the average blade, it is expected that the relationship between a modal parameter and principal components can be linearized without any significant loss of accuracy. If the linearization is valid, then partial retention of principal components are sufficient to describe the variations in modal parameters of blades due to geometric mistuning. As this work builds upon the findings of Sinha et al. in [8], appropriate understanding of these relationships can be utilized for probabilistic methods.
The work of Sinha, et al. in [8] was expanded in [19] with reference to the modified modal domain approach (MMDA). Under this approach, blade surface geometry was collected using a CMM device. This surface information shows geometric mistuning, known to propagate to deviations in mass and stiffness matrices from a tuned rotor. The work of [8] shows that vibratory parameters can be extracted through use of proper orthogonal decomposition (POD, similar to PCA) analyses. The work presented shows that an MMDA algorithm can accurately produce a reduced order model with geometric mistuning, and further, it can predict frequencies, mistuned mode shapes, and forced response of a bladed disk with geometric mistuning on an academic rotor with twenty four blades. MMDA is likewise shown to work using a second order Taylor series approximation of perturbations to mass and stiffness matrices. This is of great importance as it suggests that reduced models created by retaining limited POD information can accurately represent part specific responses in an academic rotor case through manipulation of geometric mistuning on the modal domain. This case, however, is very academic and has not been extensively utilized on flown rotors. The work for MMDA was further expanded in [46], where the approach was shown to be useful when given results and calculating the perturbations to the mass and stiffness matrices. This reversed process highlights the usefulness of MMDA and the underlying POD analysis with an academically mistuned rotor.

The work presented in [20] by Lange, et al. describes how PCA can be applied to compressor blades for probabilistic CDF simulation. It models the variation on a population of 136 blades and utilizes limited retention of modes to investigate the impact of manufacturing variability on a blade’s performance levels. This work is limited in the geometric variation retained to the first 20 modes of PCA since they explain 99% of the geometric scatter. Results of this work show that responses of the blades are sensitive to the input values; while 20 modes explain 99% of geometry scatter, it only explains 93% of performance scatter; an additional 40 modes are required to explain 99% of the performance scatter. PCA, while informative, can inaccurately predict physical performance results with
limited geometric variation, which is the general motivation of PCA.

### 2.5.3 Summary

Previous efforts leveraged PCA to stochastically model airfoil geometries of small populations, with application to a larger model outstanding. Their approach was to perform PCA on the set of airfoil data and use a statistical model on the variation in PCA coefficients to generate random models, however, when applied to mistuned probabilistics, only limited geometry was considered in the construction of the probabilistic model.

This work that ensues builds upon the prior work and first shows the convergence of PCA mode retention on geometry, frequency, and forced response. Unlike the prior works, this work adds to the body of knowledge by assessing convergence on a large population of production mistuned IBRs. Results show that to accurately predict response of a specific IBR requires nearly all calculated PCA modes. However, when looking at the results from the population of rotors as a whole, far fewer PCA modes are needed. This suggests that for probabilistic predictions of population response, a reduced basis set of PCA modes may be acceptable.

This suggestion is pursued in this work’s probabilistic simulation of the large population of rotors using the PCA statistics and bootstrap sampling. It initially shows that the prior methods of airfoil PCA are ineffective for predicting the population of rotors response distribution. This effort then introduces PCA of the IBR, in other words, each set of IBR airfoils is treated as a single set of measurements. This approach effectively captures the blade-to-blade correlation of airfoil geometry deviations and leads to accurate prediction of mistuned response behavior in the population. The last contribution of this thesis is demonstrating the ability of the defined approach to model the IBR population with a small sample of rotors. It is shown that with as few as eight IBRs, the population results can be accurately predicted.
Theory

The IBRs first undergo principal component analysis followed by forced response analysis. Physical metrological data is analyzed by PCA so as to measure surface deviations and the vibratory responses of simulated IBRs are evaluated with finite element analysis.

3.1 Principal Component Analysis

Principal component analysis is a statistical procedure designed to reduce the order of a model through decomposition of the data into a set of uncorrelated modes and corresponding scores. This is performed through an orthogonal transformation, which converts a set of observations of measurements, \( n \), with unknown correlations into a set of linearly uncorrelated mode shapes known as principal components (PCs). Retention of \( m \) components results in partial explanation of the full model; as \( m \) increases to total modal retention, the original model is fully recovered.

Application of PCA to geometric information begins with \( x \), a matrix of \( i \) observations (e.g. blades or rotor geometries) of \( j \) variables (e.g. surface coordinates) such that \( x_{i,j} \in \mathbb{R}^d \), \( i = 1, \ldots, n, \ j = 1, \ldots, p \) where \( d \) represents the dimensionality of the geometry measurements, \( n \) represents the number observations evaluated, and \( p \) represents the number of coordinate measurements as generated from a finite element model (FEM) [18]. The set \( x \) is organized to align like measurements and is mean-centered using Equations (3.1) and (3.2), producing the deviation matrix \( \Delta X \). Note that for a population of perfectly
tuned IBRs, the mean-centered matrix $\Delta X$ would be trivial.

$$\bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} X_{i,j} \text{ where } j = 1, \ldots, dp$$  \hspace{1cm} (3.1)

$$\Delta X_i = x_{i,j} - \bar{x}_j$$  \hspace{1cm} (3.2)

Next, the covariance matrix $\Sigma$ is calculated using Eq. (3.3). The covariance matrix is a square symmetric matrix of size $dp \times dp$, with diagonal terms representing the variance of a particular node in a single degree of freedom and off-diagonal terms describing the covariance between different nodal locations within a single sector.

$$\Sigma = \frac{1}{n - 1} \Delta X^T \Delta X$$  \hspace{1cm} (3.3)

The next step is to perform an eigenanalysis of $\Sigma$, where the resulting eigenvalues are the PC contributions ($\Lambda$) and the eigenvectors are the PC modes of the system ($\Psi$). When $n \ll dp$, eigenvalues indexed higher than $n$ do not contribute to the description of the model; thus the number of PCs and relevant scores equals $n - 1$.

$$\Sigma \Psi = \Psi \Lambda$$  \hspace{1cm} (3.4)

Scores $Z$ are then calculated by transformation of the deviations $\Delta X$ to principal component space using Eq. (3.5). The score matrix is an $n \times m$ matrix where $m$ is the number of retained PCs and is no larger than $(n - 1)$ when $n \ll dp$. Each score corresponds to a principal component mode and explains the participation factor of each value in the PC mode shape.
\[ Z = \Delta X \Psi \]  \hspace{1cm} (3.5)

Eq. 3.6 is evaluated in order to determine the contribution of each PC with reference to the total explanation of geometry. Eigenvalues are sorted in decreasing order to determine how well the data is explained by a single mode as described by the modal scatter fraction, \( s f_k \). This explanation is used cumulatively to determine the percentage of geometry explained for sequential mode retention.

\[ s f_k = \frac{\Lambda_{k,k}}{\sum_{i=1}^{dp} \Lambda} \]  \hspace{1cm} (3.6)

A reduced basis set matrix is then calculated using the first \( m \) PCs using Eq. (3.7), resulting in a new converging geometry data set.

\[ X_{new} = Z_{n \times m} \Psi_{dp \times m}^T + \bar{x}_{j,1 \times m} \]  \hspace{1cm} (3.7)

### 3.2 Forced Response

With a finite element analysis (FEA) based eigenanalysis of the scanned population complete, the forced response of each rotor is then determined. This is performed by applying modal superposition, a technique utilizing the eigensolutions to characterize the dynamic response of the rotor at harmonic engine order (EO) excitations. First, the equation of motion is converted to modal form,

\[ \ddot{u}_{mo} + 2\omega_{mo}\zeta \dot{u}_{mo} + \omega_{mo}^2 u_{mo} = f_{mo} \quad mo = 1, 2, \ldots, N_F \]  \hspace{1cm} (3.8)
where \( mo \) is the mode of interest of the \( N_F \) modes retained, \( u_{mo} \) is the mode shape in physical space, \( \omega_{mo} \) is the natural frequency of mode \( mo \), \( \zeta \) is the damping for the and \( f_{mo} \) is the modal force. For sinusoidal vibration, the modal force as a function of time, \( t \), has the complex form

\[
f_{mo} = f_{mo,c} e^{j\Omega t}
\]

(3.9)

where the complex modal force is defined as \( f_{mo,c} \) for mode \( mo \) and \( \Omega \) is the imposed forcing frequency defined across a range of values. The forcing function on each blade is calculated using

\[
F^{(s)} = F_{max} \cos \left[ \frac{2\pi C(s - 1)}{N} \right] + jF_{max} \sin \left[ \frac{2\pi C(s - 1)}{N} \right] \quad s = 1, 2, \ldots, N
\]

(3.10)

where \( F_{max} \) is the amplitude of the force applied, \( C \) is the EO excitation number, \( s \) denotes the blade number and serves to apply the appropriate force phase shift, and \( N \) is the number of blades. The phase at blade \( s \) is written as [33]

\[
\phi^{(s)} = \frac{2\pi C(s - 1)}{N}
\]

(3.11)

The modal force in physical degrees of freedom on the \( s \)th blade is calculated using the product of the loading and mode shape vectors according to Eq. (3.12).

\[
f_{mo,c} = \{ u_{mo}^T \} \{ F \} \quad mo = 1, 2, \ldots, N_F
\]

(3.12)

Similar to Eq. (3.9), \( u_{mo} \) has a similar complex form,

\[
u_{mo} = u_{mo,c} e^{j\Omega t}
\]

(3.13)
as it is assumed to be harmonic, with \( u_{m_o,c} \) representing the complex displacement amplitude vector. Differentiating and applying this to Eq. (3.8) results in the complex amplitudes of the modal coordinates,

\[
u_{m_o,c} = \frac{f_{m_o,c}}{(\omega^2 - \Omega^2) + i2\omega\Omega\zeta}
\]

which is a function of the modal loading, excitation frequency, and damping. Finally, the total complex displacement is found to be

\[
\mathcal{U}_c = \sum_{m_o=1}^{N_F} [\{u_{m_o}\}{u_{m_o,c}}]
\]

representing the summation of the contribution of the forced response amplitude of each mode [27].

### 3.3 Bootstrapping

Bootstrapping is a stochastic method of resampling introduced by B. Efron in [47]. The approach utilizes a random sample from an unspecified probability distribution; this sample is called a bootstrap sample and the data populating it is selected with replacement from the original set of data. The unspecified probability distribution is then approximated by the bootstrap distribution. The theoretical, approximated distribution is observed to be equivalent to the actual distribution. How well it actually describes the physical distribution, however, is dependent upon the sample used in the method.

To determine the resulting bootstrapped population, Monte Carlo simulations are repeated for various bootstrapping samples. This repetition provides a better gauge to the physical distribution and the resulting distribution after an iterative number of simulations is then taken as an approximation.

Bootstrapping repeatedly draws samples from the population of interest multiple times.
with analysis using Monte Carlo Simulation. It uses the known population as to approximate the larger population distribution that is not known. From these results, it produces a large number of sample statistics as computed from the bootstrap samples. In short, it is a technique that resamples known data points with replacement to create a larger sample denoted as the bootstrap sample, the purpose of which is to approximate the sampling distribution by filling gaps of known information with approximated, bootstrapped information [48].

The data can be sampled in clusters of spatially correlated information or selected individually from within the data set. As bootstrapping samples are collected, rotors can be built consequentially for fleet response prediction.

### 3.4 Kolmogorov-Smirnov Test

There are many tests utilized to compare distributions; the testing utilized in this document is that of a Two-Sample Kolmogorov-Smirnov (K-S) Test. The K-S test is applied to non-parametric data to compare the resulting cumulative distribution functions (CDFs) of two compared models, that of a tested case against the corresponding known distribution.

This test was introduced by Smirnov in 1939 and continued in his work of [49]. It uses the test statistic of

\[
D_{a,b} = \sup_x |F_a(x) - G_b(x)|
\]  

(3.16)

where \( F_a \) and \( G_b \) are empirical distribution functions (EDFs) associated with two respective random variables. This test is applied to determine whether two data-sample EDFs come from the same parent theoretical distribution, where the parent distribution is unknown and not identified. The exact parent distribution is not limited to a standard distribution type, therefore it can be used on non-parametrically defined distributions [50].
The null hypothesis \((H_o)\) states that \(F_a\) and \(G_b\) are from the same parent distribution. If the null hypothesis is rejected, the test results that \(F_a\) and \(G_b\) are from different distributions and if it fails to reject the null hypothesis, \(F_a\) and \(G_b\) could have come from the same parent distribution. This is determined by a set confidence level \(\alpha\).

The test is popular due to the distribution-free nature of the governing distribution as it is sensitive to both shape and location of the compared empirical cumulative distribution functions. It finds the maximum vertical difference between the two tested CDFs and retains this as the K-S Statistic, \(D\). If this value is greater than the corresponding critical value for a specific \(\alpha\), the null hypothesis is rejected.
Common Modeling and Approach

4.1 Objectives and Approach

The objective of this work is to progress the science of fleet risk assessment through evaluation of the effects of geometric mistuning. Both statistical and probabilistic tools will be employed to isolate the mistuning phenomenon, primarily observed through its magnification of nominal (tuned) response amplitudes. The following predictions of a variety of system parameters will be compared to the responses of the actual population; furthermore, prediction quality will be summarized with an assessment of fleet failures per engine flight hour.

The schematic for this work is presented in Figure 4.1, a process which is applied to an as-manufactured population of 40 rotors removed from service exhibiting geometric deviation from the nominal design due to a combination of engine wear and manufacturing variation. Additionally, two distinct subsets with different serial prefixes exist within this set: review of these subsets reveals considerable differences in the airfoil-to-platform transition fillet and the airfoil thickness. Examination of geometric variation from the nominal begins with digital representation of each rotor using optical metrology, a procedure resulting in a highly accurate array of three-dimensional points representing visible surfaces. These point clouds are utilized by finite element analysis (FEA), the computational method widely used to determine the structural response of intricate geometries to complex loading. Construction of finite element models (FEMs) is handled by MORPH, an
automated optimization process which consistently translates the discretized structure and boundary conditions of a prepared model onto each point cloud. The resulting model is then geometrically decomposed with PCA, which concludes the preparatory phase of this study.

The next step – described by the Identify Test decision within the flowchart – represents multiple methodologies of modeling, both statistically and probabilistically, geometric mistuning. The first method investigates the harmonic response convergence of partially reconstructed FEMs to the unmodified structure, where reconstruction is governed by selective retention of PC mode subsets. This method, henceforth known as PCA Convergence, considers the capability of PCA to assess the effects of mistuning through examination of geometry, modal properties, rotor-specific peak airfoil and peak rotor response, and total population peak airfoil and peak rotor response distributions. The second method probabilistically resamples (bootstraps) the geometries, including those decomposed by PCA, to simulate large notional rotor populations with limited geometric input. The structural responses of this notional population are compared to the MORPHed baseline distribution. When applied to PC modes, it is known hereafter as PCA Bootstrapping. This method describes the efficacy of probabilistic PCA resampling in the prediction of total population peak airfoil and peak rotor response distributions.
Figure 4.1: Flow Diagram of Presented Work.
4.2 Geometry Modeling

Investigation of geometric mistuning requires accurate representation of the physical rotor population; thus, modeling occurs as a two-step process. First, IBRs are digitized (Section 4.2.1) to form high-fidelity arrays of three-dimensional coordinates (point clouds) representing the visible surfaces. A random point cloud is then reduced to a representative sector, discretized, and swept about the center axis to form the seed model. The resulting tuned seed IBR model serves as the base model for application of mesh morphology to measured IBR point clouds through application of the MORPH technique (Section 4.2.2) [51].

4.2.1 Capture

A considerably sized rotor population (40 units) necessitates careful selection of a digitization technique. One approach is to use a coordinate measurement machine (CMM), which collects spatial data through contact between a multi-axis sensor and the component at regular intervals [25, 52]. Although highly accurate at the measured locations, CMM digitization suffers several drawbacks, including: slow data acquisition rate; lack of fidelity in highly-curved regions; and the low density of geometry information. An alternative approach is the application of optical inspection techniques, which combine photogrammetry with structured light to rapidly acquire dense, highly-accurate coordinate measurements of the target geometry [53, 54].

For these reasons, optical metrology with the Advanced Topographic Optical Sensor (ATOS) was chosen for digitization. The ATOS system is comprised of several components, including: (1) stereographic charged-coupled device (CCD) cameras within a housing unit capable of translation and rotation in five axes; (2) a blue-light projector; (3) a rotation table providing the sixth axis of motion; and (4), a control computer for scan op-
eration and data reduction [55]. As the CCD cameras are of fixed focal length, multiple lenses are needed to provide different measuring volumes. Note that image resolution does not change with measuring volume; thus, point cloud density increases with decreasing measuring volume size. This study uses a measuring volume of 320 mm x 240 mm x 240 mm.

The acquired point cloud is meshed to form a dense set of facets and vertices, which is exported in stereolithographic (.stl) format to the reverse-engineering computer-aided drafting (CAD) program Rapidform. Within Rapidform, the ATOS-generated mesh guides creation of a representative solid model. It is this model that is imported into ANSYS and discretized to form the seed FEM. Although powerful, this process of digitization to a manually developed reverse-engineered model is time consuming, prompting application of the MORPH procedure.

4.2.2 MORPH

The scan-CAD-FEM method results in models that can fail FEA due to meshing errors; furthermore, this approach is time consuming and difficult to automate [54]. This method is likewise limited by its inability to definitively capture challenging blade contours (e.g. blade-root fillets, blade-tip fillets, etc.). MORPH removes these limitations by intelligently modifying surface nodes of the FEM to match those of point cloud data scans.

The MORPH process is as follows: first, the seed model generated using the traditional method of scan-CAD-FEM provides general nodal locations and connectivity matrices. Next, normal vectors are determined for all external nodes. Ray tracking algorithms compute the intersections between the nodal normal vectors and scan data. Once the program determines the surface to which the model should be MORPHed, it moves exterior nodes using an iterative spring analogy. This allows for the capture of challenging geometries that were difficult or impossible to obtain and replicate with previous methods. The resulting FEM is required to pass element shape checking standards imposed for a solution.
The final MORPHed model accurately resembles the physical rotors without corrupting elements, maintains node and element continuity, and reduces user-CAD interaction through systematic reconstruction of each FEM. This alleviates computational requirements and increases model fidelity when transferring point cloud information directly to a finite element model.

Applying MORPH requires concentricity and blade-alignment between the seed model and point cloud information to produce a systematic and comparable fleet of rotors. Since the blades are of interest for this fleet risk assessment, only the blades and rim of the disk are MORPHed. Internal nodes, not modified during the MORPHing process, are moved to optimized locations to ensure element shapes are within analysis tolerance. The movement of these internal nodes is small but produces elements that have reduced skew angles and nearly equal aspect ratios, both metrics which correlate to the accuracy of the findings. Note, the accuracy of the MORPHed models never exceeds that of the originating scan.

### 4.3 Overview of Finite Element Conditions

The seed model is constructed of 56,432 nodes meshed with 40,040 linear hexahedral elements, resulting in 169,026 degrees of freedom (DOF). Discretization of the IBR sector segments the model into two sub-geometries denoted as blade-rim geometries and disk geometries; blade-rim geometries include the airfoil, airfoil-to-platform transition, and rim surface while disk geometries include the remaining nodes as shown in Figure 4.2 and further details of the structure of the FE model are presented in Table 4.1. Titanium 6Al-4V material properties and a damping ratio of 0.002 are assigned globally to the FEM [56]. Fixed displacement boundary conditions are applied to constrain all DOF at the rotor bore. Results are obtained for the first 110 dynamic modes, spanning first bend (1B) through second chordwise bend (2CWB). Airfoils are geometrically perturbed by modifying nodal locations of the blade-rim surface. This process is achieved through scripted application of
spatial statistics techniques for both subpopulations described by unique serial identifiers. The number of rotors within each subpopulation is expanded upon in Table 4.2.

Table 4.1: Details of Discretized Model.

<table>
<thead>
<tr>
<th>Component</th>
<th>Elements</th>
<th>Nodes</th>
<th>DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade-Rim Sector</td>
<td>1280</td>
<td>1905</td>
<td>5715</td>
</tr>
<tr>
<td>Blade-Rim Sector (External Surface)</td>
<td>n/a</td>
<td>1274</td>
<td>3822</td>
</tr>
<tr>
<td>Disk</td>
<td>11880</td>
<td>14432</td>
<td>43296</td>
</tr>
<tr>
<td>Full Model</td>
<td>40040</td>
<td>56342</td>
<td>169026</td>
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</tbody>
</table>

For the forced response analysis, it is desired to mimic the conditions of a Traveling Wave Excitation test, an experimental test that uses multiple magnets to excite each airfoil at different phase values to replicate EO loading [57]. Each blade is excited at the trailing edge tip with a force of unit magnitude at EO-3. Loading follows the phase shift formulation identified in Equation 3.10, with out-of-plane displacement responses recorded at the leading edge tip. Input (excitation) and output (response) locations are measured across a range of frequencies outlined in Figure 4.3 between first torsion (1T) and third bend (3B), a region affected by frequency veering for ND 3. As forced response analyses are sensitive to geometric mistuning, additional EOs are excited within this frequency band.

Table 4.2: Subpopulations Sizes for Analysis.

<table>
<thead>
<tr>
<th>Population</th>
<th>Number of Rotors</th>
<th>Number of Blades</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>32</td>
<td>704</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>176</td>
</tr>
<tr>
<td>Mixed</td>
<td>40</td>
<td>880</td>
</tr>
</tbody>
</table>
Figure 4.2: Blade-Rim and Disk Sector (Distorted Geometry).

Figure 4.3: Natural Frequency vs. Nodal Diameter Plot of the Tuned Model.
4.4 Importance of Results

4.4.1 Peak Airfoil Response

Small variations in rotor blade geometry lead to blade-specific perturbations of the nominal harmonic response, resulting in a complex structural response distribution innate to each IBR. Thus, examination of the IBR-specific harmonic responses provides effective insight into not only geometric deviations between rotors, but also quantifiable variations in fatigue life risk.

Current models make conservative judgments on fatigue risk based solely on the peak responding blade of the rotor. While an approximation, this approach does permit creation of conservative rotor response distributions that provide valuable insight into the effects of geometric mistuning. However, this method fails to consider blade-to-blade geometric variations affecting the forced response. Examination of these variations elicits a more accurate assessment of risk.

An example of blade response mistuning is presented in Figure 4.4, where the response of each blade is denoted by a different color. From this figure it is evident that each blade responds at differing frequencies and magnitudes; additionally, some responses exhibit multiple peaks over the driving frequency range caused by bleed from other EOs within the same natural frequency cluster. The skyline denoted by the black line describes the highest response in the rotor per driven natural frequency and changes blade indices throughout. Note that for the tuned case, the skyline would appear as a monolithic peak.

4.4.2 Peak Rotor Response

The maximum resonant response amplitude of an IBR within a frequency range of interest – the peak rotor response – represents the likely location of failure [58]. Past efforts
conservatively predicted HCF rotor life by approximating the response of each blade as equivalent to the peak rotor response; however, this results in significant decrease of useful rotor life. A better understanding of peak rotor response will extend rotor life while maintaining acceptable safety margins.

The methods applied in the following sections will probabilistically predict peak rotor responses representative of the physical rotors. Furthermore, the evaluated responses will be used to create distributions that accurately represent a notional population of rotors far exceeding that available.
Test Definitions

The following methods modify the geometry of the nominal tuned model with the addition of geometric mistuning. Nodes modified on the FEM are referred to as blade-rim geometries, but correspond to the surface of the blade, root, and outer circumferential face of the rotor (i.e. sector of the annulus). It is necessary to accurately capture root variations in order to represent the physical rotor and produce geometries solvable with FEA. When a blade is mentioned in this document, it refers to a single sector of these surface geometries.

5.1 Principal Component Analysis Convergence Study

As discussed in Chapter 3.1, PCA establishes a modal description of known geometry $\bar{x}$ with perturbations $\Delta X$. A subset of these modes may be utilized to reduce the order of the model, with parity between the approximation and the full model reached as the number of retained modes increases. Selecting a subset of modes has the benefit of reducing computational expense, the cost of which represents a significant hurdle to application of FEA techniques to fleet management of in-service IBRs. However, the rate of convergence between the approximation and the full model has not been studied for system parameters and forced response amplitudes. Thus, investigation of the convergence rate is a necessary portion of this work.

For this study, $m$ PCs of the perturbed model are utilized to construct the corresponding geometry. This geometry is passed to ANSYS, where it is discretized and assigned
material properties and boundary conditions as denoted in Chapter 4. The blade-to-blade and peak rotor response to the prescribed boundary conditions over the entire range of driving frequencies is recorded and compared with respect to the amount of geometry explained by the number of retained modes \( \sum_{k=1}^{m} s f_k \). As the number of retained modes \( m \) approaches the total number of PCs \( n - 1 \), parameters of the reduced-order model forced response should converge on the IBR-specific baseline data.

### 5.1.1 Blade Analysis

The approach described in Section 5.1 is applied to two subpopulations consisting of 32 and 8 rotors with 22 blades per rotor belonging to Subpopulations A and B respectively, for a total of 704 and 176 blades. Digitization of the IBR is performed in three steps; first, a point cloud of each rotor is captured using the ATOS; then, the geometry data is translated to an FEM using MORPH; finally, each blade is aligned to the same physical space to permit PCA. Since each FEM is built off of a tuned IBR with structured geometry, the coordinate data points are easily comparable. The data is structured within a matrix of size \( n \times dp \), where \( n \) is the number of observations (blades) in the given population, \( d \) is the dimensionality of the model (three), and \( p \) is the number of nodes on the external face of a single blade of the seed model (1274 from Table 4.1). Note that although the IBR models are aligned radially to the rotor bore, each IBR is angularly aligned to the blades without reference to the manufacturing order. However, the analytical process necessarily detaches the blades from the rotor, rendering the lack of angular indexing inconsequential.

Figure 5.1 shows the data matrix for \( X \) of Equation 3.1. The first \( p \) columns are the \( x \)-coordinate of nodes one through \( p \) of the surface geometry data. The second and third sets of \( X \) represent the \( y \)- and \( z \)-coordinates respectively. Each coordinate \( (x_{i,j}) \) is identified by \( i \), the blade index ranging from one to \( N \), and \( j \), the node number one through \( p \). Each row of the data set represents a single blade and each set of \( N \) rows defines the aligned set of blades belonging to a physical rotor. A single rotor is highlighted in red. The bold \( R \)
identifies a single rotor ranging from one to $\ell$, corresponding to 32 or 8 depending on the subpopulation.

Note, each row has nodes $1 : p$, but in the FEM, these node numbers vary from sector to sector. Principal component analysis requires the nodes to be structured such that they align in the same space across each measurement (i.e. the trailing edge tip node is defined in all blades (rows) as columns $7, p + 7$, and $2p + 7$ for $x$-, $y$-, and $z$- coordinate locations).

With all information aligned in each column, the deviations can be readily calculated from the nominal average geometry.

For each subpopulation, a nominal model is created using the average data as described in Equation 3.1. Recall, this method takes all $dp$ columns from $n$ rows of data and solves for the mean of each column, resulting in a single “nominal” blade with average geometry representing the subset of data. The nominal blade is used to create a tuned rotor for measuring amplitude magnifications in the mistuned population. Principal component analysis is then performed on the deviations between the nominal model and the scanned rotors using the blade-only approach for both subpopulations. With the addition of retained PCs, the MORPHed model is identical to the models created retaining $(n - 1)$ principal components where $n$ is 704 or 176 for populations A and B respectively.

In this case, $m$ is selected based on the first ten modes followed by successive com-

\[
X_{n,dp} = \begin{bmatrix}
R_1: & x_{1,1} & x_{2,2} & \cdots & x_{1,p} & y_{1,1} & y_{1,2} & \cdots & y_{1,p} & z_{1,1} & z_{1,2} & \cdots & z_{1,p} \\
R_2: & x_{2,1} & x_{2,2} & \cdots & x_{2,p} & y_{2,1} & y_{2,2} & \cdots & y_{2,p} & z_{2,1} & z_{2,2} & \cdots & z_{2,p} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
R_l: & x_{N,1} & x_{N,2} & \cdots & x_{N,p} & y_{N,1} & y_{N,2} & \cdots & y_{N,p} & z_{N,1} & z_{N,2} & \cdots & z_{N,p} \\
\end{bmatrix}
\]

Figure 5.1: Blade PCA Geometry Data.
ponents new whole intervals of explained geometric variance by the cumulative sum of
Equation 3.6. The new $\Delta X$ of Equation 3.7 is separated back into a subset of IBR sec-
tor models (where the first $N$ rows belong to Rotor 1, the second to Rotor 2, etc.) and
the blades are transformed back into the radial orientation and placed on the hub. The
nodal locations are modified by the $\Delta X_{new}$ matrix on the seed model and reanalyzed with
FEA under the global conditions of Chapter 4. The results of all analyses performed are
compared to the baseline model created using all Principal Components. This baseline is
equivalent to using the initial data fed into PCA as produced by MORPH, meaning baseline
values change from rotor to rotor.

5.1.2 Rotor Analysis

Similarly, PCA is performed on the full IBR geometry. While this method relies on ori-
entation of the first manufactured blade to be in the same sector of each rotor (i.e. first
manufactured blade oriented to the 12 o’clock position), it will better predict correlations
between the blades than the previous method by maintaining sector correlations measured
as a set. In this case, the first manufactured blade is unknown, so work progresses by
aligning circumferential orientation of blades. As before, the cumulative geometric vari-
ance explained by summing the first $m$ retained PCs is evaluated and the corresponding
geometries are mapped to an FEM and evaluated with the same consistent conditions as
prescribed by Chapter 4.

Using the same notation, $X$ is a three dimensional set of data points of size $n \times dNp$,
where $p$ is 1274, $N$ is 22 (the number of blades per IBR), and $n$ is the number of rotors in
the given subpopulation. As before and in all cases, Subpopulations A and B are evaluated
individually due to large dissimilarities in geometric primitives along the fillet and surface
contours. Thus, $n$ is described as 32 and 8 for A and B respectively.

This matrix, shown in Figure 5.2, is similar to that of Figure 5.1. Each set of $Np$
columns represents the $x$-, $y$-, and $z$- coordinates of full rotor geometries, built such that
all blade sets are retained within a single measured observation. All blades are kept to their respective sectors instead of collapsing all blade geometries to a single sector location. The indices of each coordinate \((x_{i,j})\) correspond to the rotor number and node number respectively. As before, the geometries are structured such that each column represents the same node from rotor to rotor and the red indicates nodal locations belonging to a single rotor.

\[
X_{n,dNp} = \begin{bmatrix}
R_1: x_{1,1} & x_{2,1} & \cdots & x_{1,Np} & y_{1,1} & y_{1,2} & \cdots & y_{1,Np} & z_{1,1} & z_{1,2} & \cdots & z_{1,Np} \\
R_2: x_{2,1} & x_{2,2} & \cdots & x_{2,Np} & y_{2,1} & y_{2,2} & \cdots & y_{2,Np} & z_{2,1} & z_{2,2} & \cdots & z_{2,Np} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
R_n: x_{n,1} & x_{n,2} & \cdots & x_{n,Np} & y_{n,1} & y_{n,2} & \cdots & y_{n,Np} & z_{n,1} & z_{n,2} & \cdots & z_{n,Np}
\end{bmatrix}
\]

Figure 5.2: Rotor PCA Geometry Data.

Each subpopulation creates a mistuned model using the average data from the entire IBR. Due to the geometric variation in this average model, the amplitude magnifications of the iterative geometries are compared to the nominal model as created by blade PCA. A single PCA feature generated carries blade to blade correlations, where sets of features are required with application of PCA to airfoils.

Once PCA of the IBR is performed, each converging rotor is analyzed at specified intervals of retained modes, \(m\), that describe up to 90% of the data followed by full percent intervals up to 99% of the geometry. This is followed by the addition of 99.5%, 99.9%, and 100% as before. Note, few of these increments will be met due to the small observation sizes of both subpopulations. The rotors created by retaining limited geometric variation are evaluated under the same conditions as before for ease of comparison. The results of modal and harmonic analyses are recorded and reviewed in later sections.
5.1.3 Results

In order to validate the results of the reduced geometry with varying geometric mistuning, the following items are evaluated:

1. Natural Frequency Convergence

2. Accurate Representation of Peak Airfoil Responses by Tracking Indexed Blades

3. Convergence of Peak Rotor Response

4. Representation of Overall Response Distribution

Principal component analysis is tested for its accuracy by determining the convergence of natural frequencies, peak airfoil responses per blade, and peak rotor responses where each rotor is compared to its converged counterpart.

Beyond geometry convergence, the first metric to consider is the convergence of natural frequency. Since the region of interest is at EO-3 between 1T and 3B, these are the selected frequencies tracked for convergence criteria. Recall that in a tuned rotor, natural frequencies repeat for NDs of 1:10 and only appear alone at ND 0 and 11. With a tuned rotor, the region of interest occurs in the 50\textsuperscript{th}/51\textsuperscript{st} natural frequency for 1T and the 72\textsuperscript{nd}/73\textsuperscript{rd} frequency for 3B. The introduction of mistuning causes frequency splitting; as such, two frequencies are tracked for the convergence study although neither may experience a ND of 3 in this region. The frequencies tracked are the 51\textsuperscript{st} and 73\textsuperscript{rd} frequencies from the eigenanalysis as to provide consistency in the values traced.

These frequencies are then normalized to the baseline value for each rotor – the 51\textsuperscript{st} and 73\textsuperscript{rd} natural frequency of the MORPHed model. As geometric mistuning converges with PC retention, these indexed frequencies are compared to the baseline value to test their convergence. This is plotted as natural frequency versus explained geometric variance where actual levels of geometric mistuning is obtained at 100%.
Next is to examine the peak airfoil response convergence percent error between converging and baseline responses with respect to explained geometric variance. Each line in the resulting plot represents an individual blade, where each plot represents a single rotor. Mapped to this plot are the peak rotor responses for each iteration as the peak responding airfoil varies from sector to sector through each iteration of geometric mistuning convergence.

Since peak rotor response produces conservative estimates of rotor life, peak rotor response convergence is traced versus the number of retained PCs. This is done by evaluating the percent error between the iterative peak rotor response and its corresponding baseline value for successive retention of geometry from PCA based on the corresponding physical response from MORPH. The peak rotor response is not limited to the blade in which it occurs, as it is likely that the peak rotor response will shift from blade to blade as geometric mistuning increases.

Geometry, natural frequency, peak airfoil and peak rotor response convergence are examined to determine how well limited geometric mistuning can represent the physical models. Each are compared to their respective physical rotor results as a baseline, however, this limits the amount of information gained from PCA. A more comprehensive approach examines the overall distributions of fleet-wide responses over convergence of rotor-specific values. For this, peak airfoil and peak rotor response distributions of each iterative retention of PC modes. Each response is evaluated as a CDF, with each iteration tested against the true MORPHed baseline, equivalent to the retaining 100% of the geometric variation. Application of the K-S test to both distributions will identify the amount of modal retention required to accurately represent the baseline model. All of these metrics will provide understanding of the capabilities of PCA.
5.2 Probabilistic Fleet Prediction

Bootstrapping is a Monte Carlo approach used to create random rotors realizations through resampling of known data. The information applied to the method is outlined in the following sections; for each of the following applications of bootstrapping, 1000 iterations (or the greatest number of permutations possible) are performed and checked for a converged solution. Table 5.1 outlines the tests performed and the iterations for which the test is performed for different bootstrapping simulations. The first test performed is by randomly selecting rotors from Subpopulation A or B, detaching the blades, and reorganizing them onto a new random rotor realization. This is done for increasing random rotors selected as shown in the “Cases” columns for 1000 iterations in order to obtain an accurate and converged response distribution. The second application of bootstrapping is to analyze resampling of PCA scores created from blade geometries for Subpopulation A and only considers the retention of the first 8 and 217 of 880 columns of scores. A similar test is then performed on the rotor geometry scores, but with retention of 1, 8, 16, 24, and 31 columns of scores in Subpopulation A and 1, 2, 4, 6, and 7 for Subpopulation B. Finally, a test is performed on a subset of randomly selected rotors from both populations, where the number of rotors selected is shown in the Cases column. The full matrix of scores is bootstrapped (with no limited retention as before) for the subset of rotors and bootstrapping is performed on the corresponding score matrix, but the number of possible iterations performable is by the number of rotors analyzed for each case.

In addition, a mixed population is created by compiling results from Subpopulations A and B. Because Subpopulation A is 80% of the entire set, 80% of the simulations are represented by Subpopulation A bootstrapped data. As such, the minimum configuration utilizes a single rotor from each subpopulation, scaled to the 80/20 configuration of the true population. Note that no rotors are formed with a mixture of blades from both subpopulations. This is due to the gross geometric differences between the subpopulations and the
likelihood that these differences would not exist simultaneously on any rotor in the fleet. Therefore, 80% of the results come directly from Subpopulation A bootstrapping with the remaining 20% stemming from Subpopulation B.

Table 5.1: Test Matrix of Cases for Experimental Bootstrapping Simulations.

<table>
<thead>
<tr>
<th>Test Population Cases</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random IBR^{1} A</td>
<td>1-8, 29, 32</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>Blade PCA Scores^{3}</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
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<td>A</td>
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<td></td>
<td>A</td>
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</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
</tbody>
</table>

^{1}Cases represent number of rotors selected for reassembly of blade orientation. See Section 5.2.1 for more information.

^{2}Case with three rotors selected performed for three different samples.

^{3}Cases evaluate number of retained PC modes (and corresponding explained geometric variance). See Section 5.2.2 for more information.

^{4}Cases selected based on number of modes retained (and corresponding percentage of variation explained). See Section 5.2.3 for more information.

^{5}Cases represent selected number of rotors.

^{6}Performed for three random samples of eight rotors.
5.2.1 Bootstrapping Rotor Geometry

For this analysis, a random sample of rotors from each subpopulation (A or B, individually) is selected. Peak airfoil and peak rotor response distributions for increasing rotor sample sizes are evaluated against respective MORPH baseline distributions. Randomly selected rotor samples are segmented into detached blades; then, $N$ blades are drawn from this pool and attached to the hub in random order. A single rotor with $N$ blades can be bootstrapped in this manner $N!$ times. Therefore, it is important to verify that the number of simulations performed results in a converged solution for both peak airfoil and peak rotor distributions as the number of cases simulated is only 1000. It is expected that as the sampling pool increases to match the subpopulation size, the rotor response distribution will attain parity with the MORPHed baseline.

Since the Subpopulation B is much smaller, it is analyzed more completely, meaning that the number of IBRs in the resampling population increases from one to eight without omission, while Subpopulation A is bootstrapped using 1, 2, . . . , 8, 29, and 32 rotors. The selection of rotors is shown in Figure 5.3. The peak rotor responses from MORPH are shown in blue and the randomly selected peak rotor response is circled in red. As the number of rotors increase, the peak rotor response distribution of the selected rotors becomes more and more representative of all peak rotor responses, with each sample representing a varied subpopulation distribution. As the sampled rotors span the range of MORPHed responses, it is assumed that the bootstrapped subset will accurately represent the target distribution. This method of data prediction – applied to a random subset of the target population – represents a powerful and novel method of fleet response prediction.
5.2.2 Bootstrapping Blade PCA Scores

The second type of bootstrapping applied to the population is to create rotors from random blade PCA scores for MC simulation. Recall Equation 3.7 uses $Z$, an $n \times m$ matrix multiplied by the transpose of $\Psi$, a $dp \times m$ matrix, summed with the columnar mean to create rotor realizations after application of PCA with limited retention of $m$ modes. This application of bootstrapping creates a single blade instead of $n$ blades by randomly selecting one value from each of the retained $m$ columns. This random score then is a $1 \times m$ vector resulting in a $1 \times dp$ vector after Equation 3.7 is applied, such that a random blade is created using a single blade PCA score contribution from $m$ columns. The number of possible blades created is $n^m$, where $m$ is the number of retained PCs and $n$ is the number of measured blades.

Figure 5.4 shows an example of how the bootstrapping technique is applied to the $Z$-score matrix. The score matrix is of dimensions $n \times m$. The matrix is populated by $z_{i,j}$ where $i$ and $j$ vary from 1 to $n$ and 1 to $m$, respectively. A random $Z_{1,m}^{(r)}$ vector with length
$m$ is defined as $[z_{r_1,1}, \ z_{r_1,2}, \ z_{r_1,3}, \ \cdots \ z_{r_1,m}]$ where the row selected $r_i \in (1:n)$ and varies from column to column. In the example, rows three, one, and two of the first three retained modes are selected for use in Equation 3.7.

Because no rotor in either population is completely tuned, performing a MC simulation on such a rotor is not useful in predicting the fleet forced response amplitudes. Therefore, in order to create the $N$ blades required per IBR simulated, this process repeats $N$ times with selection from the $m$ modes. Note, the maximum allowance for $m$ is $n - 1$ where $n$ is the number of blades in the given population.

This application of bootstrapping is applied to Subpopulation A for two different iterations of retained modes: retention of the first eight modes and retention of the modes explaining 99.5\% of the geometric variation. First, the seed rotor is MORPHed to the generated geometry; then, FEA is applied with the same parent constraints as previously outlined. The peak airfoil and peak rotor responses for each of the 1000 rotors created is retained for post-processing.

### 5.2.3 Bootstrapping Rotor PCA Scores

#### 5.2.3.1 Full Analysis

The final test for fleet forced response prediction is to utilize IBR PCA scores in place of blade PCA scores. Rotor PCA stores this sector to sector variation information, so it is of

$$Z_{n,m} = \begin{bmatrix} z_{1,1} & \boxed{z_{1,2}} & z_{1,3} & \cdots & z_{1,m} \\ z_{2,1} & \boxed{z_{2,2}} & \boxed{z_{2,3}} & \cdots & z_{2,m} \\ \boxed{z_{3,1}} & z_{3,2} & \boxed{z_{3,3}} & \cdots & z_{3,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{n,1} & z_{n,2} & z_{n,3} & \cdots & \boxed{z_{n,m}} \end{bmatrix} \implies Z_{1,m}^{(r)} = \begin{bmatrix} \boxed{z_{3,1}} & z_{1,2} & z_{2,3} & \cdots & z_{n,m} \end{bmatrix}$$

Figure 5.4: Selection of Random $Z$-scores for PCA Score Bootstrapping Methods.
interest to resample this data with bootstrapping. This is done using $m$ retained modes for
the entire IBR population of size $n$, 32 or 8 for A and B respectively. Each population is
analyzed using 25%, 50%, 75%, and 100% of the available $n$ modes. Additionally, both
populations were evaluated by retaining a single mode.

This corresponds to Subpopulation A being evaluated at modes that contain 58.0%,
84.2%, 92.6%, 97.4% and 100% of the geometric variation introduced through geometry
mistuning. With the lack of information in retaining a single mode, only 32 rotors are
generated, which match the input rotor geometries. This is due to the equation introduced
previously as $n^m$, where $n$ is 32 for Subpopulation A and $m$ is one for retaining and resam-
pling from within one retained mode.

Subpopulation B is then evaluated at 79.7%, 89.8%, 95.4% 98.9% and 100% of the
explained geometric variance. The number of iterations is again limited by $n^m$, where the
number of testable different cases is 8 and 64 for the retention of 1 and 2 modes. Other than
these identified cases, where the number generated rotors is limited by the input data, cases
are evaluated for 1000 iterations at which the distribution converges and only changes in
minimally.

The full population is then modeled for all combinations of Subpopulations A and
B, where 80% of the full population distribution comes from Subpopulation A results and
20% stems from Subpopulation B results. These results will describe how much geometric
variation is required in order to accurately model the full mixed population of first stage
compressor rotors.

5.2.3.2 Subset Analysis

This same approach of bootstrapping from PCA scores of rotor geometry is applied to sub-
sets of the A and B subpopulations as outlined previously in Table 5.1. For Subpopulation
A, 8, 16, and all 32 rotors are randomly selected, with the subset of eight chosen three
times. The distribution of these responses can be observed in Figure 5.5. As before, blue
indicates the peak rotor responses of the subpopulations. The selected rotors for subset analysis are circled in red for both populations.

![Graph](image)

Figure 5.5: Peak Rotor Responses for Two Subpopulations with Responses Highlighted for the Selected Rotors for Subset Rotor PCA Bootstrapping Cases.

It is hypothesized that the forced response distributions of the bootstrapped population will converge on the MORPHed baseline as the number of rotors sampled reaches the total population size. Although most tests are conducted for 1000 rotor realizations, subsets of two, three, and four drawn from Subpopulation B are restricted to two, nine and 64 realizations as defined by the equations governing the possible permutations, $n^{n−1}$. Once the rotors are selected, PCA is performed on the geometries of the subset population. The score matrix of this limited PCA application is then bootstrapped to generate new geometries as per Equation 3.7. After the realized geometries are created, the seed FEM is modified for each case and analyzed under consistent FEA boundary conditions to determine the forced response. The peak airfoil and peak rotor responses are then recorded for post-processing.

As before, the entire population distribution is then created for all combinations of A and B subset rotor retention. With the same approach, 80% of the distribution is modeled directly from the response distribution of Subpopulation A with the remaining 20% governed by B.
5.2.4 Results

Unlike the convergence analysis of the PCA models, bootstrapped rotors can only be studied with respect to fleet-wide distributions as a measure of their probabilistic abilities. As such, the K-S test will be applied to examine the prediction capabilities of the bootstrapped distributions. These cases are evaluated against each subpopulation, as well as the mixed population.
Principal Component Analysis

Convergence Study

Although modest in magnitude, geometric mistuning produces considerable deviation to modal properties of the system. Thus, the effectiveness of PCA as a fleet risk assessment tool pivots upon its ability to accurately converge on the measured response through partial reconstruction of the available population. Figure 6.1 graphically depicts geometric convergence to physical geometric surface measurements through retention of increasing PCs. For this example, 99.9% of the geometry is explained by 50% of the PC modes; however, the veracity of this limited reconstruction with respect to forced response must be systematically established.

The following process is utilized to address this challenge: first, PCA is performed on the rotors of both populations; then, models of increasing geometry explanation are formed from iterative manipulation of FEM nodal locations, thereby inducing geometric mistuning. Finally, these iterations are evaluated using FEA, producing response amplitudes for the driving frequency range. This process is applied to both blade-alone (Section 6.1) and rotor (Section 6.3) geometries. Following sections detail results for:

1. Natural Frequency Convergence

2. Accurate Representation of Peak Airfoil Responses by Tracking Indexed Blades
3. Convergence of Peak Rotor Response

4. Representation of Overall Response Distribution

5. PCA Mode Shapes

### 6.1 Blade Analysis

#### 6.1.1 Subpopulation A

Principal component analysis is first performed on Subpopulation A blade geometries. The Pareto chart of Figure 6.2 depicts both the fractional and cumulative geometric variation contained within the first ten PC modes, essentially representing spatial differences in blade nodal locations across the fleet. The first mode contains 57% of the information on geometric variation, with subsequent modes rapidly diminishing in content. Principal components beyond the tenth – greater than 98% of the total modes – explain only 13% of the total geometric variation, indicating the nearly asymptotic convergence of geometric reconstruction. Thus, in an effort to reduce computational expense, modal retention beyond the tenth PC is
restricted to those sets nearest to whole percent values of geometry explained within 10% of the nominal model (Table 6.1).

Table 6.1: Geometric Variation Explained by Blade PCs of Subpopulation A.

<table>
<thead>
<tr>
<th>PCs</th>
<th>Cumulative $s f_k$ (%)</th>
<th>PCs</th>
<th>Cumulative $s f_k$ (%)</th>
<th>PCs</th>
<th>Cumulative $s f_k$ (%)</th>
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<td>1</td>
<td>57.3</td>
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6.1.1.1 Natural Frequency

Geometric mistuning inherently disrupts the nominal structure, resulting in measurable changes to modal properties. As natural frequencies are fundamental indicators of system response, it is advantageous to examine the rotor-specific convergence trend for increasing PC retention. This is done by tracking the index of mode shapes of interest (originally identified within the tuned model) through successive analyses of progressive PC retention. Figure 6.3 details the poorest-performing trend for 1T and 3B – blade-dominated modes located within the veering region of Figure 4.3 – across the entire subpopulation. In each case, retention of only a single mode results in error not exceeding 2%, with reduction to 0.1% error at 93% explanation of geometric variance. These results indicate conclusively that natural frequencies are accurately captured with limited PC retention.

6.1.1.2 Blade Response Capture of Physical Rotors

With the natural frequencies determined, the corresponding peak blade amplitudes are recorded for the subpopulation. A composite representation of peak blade response is developed in the skyline plot of Figure 6.4, where the ordinate has been normalized to the peak response per rotor for retention of all PCs. For a tuned rotor, this plot would collapse
upon a single natural frequency and amplitude; however, geometric mistuning results in the disharmony evident within the figure. The goal is to accurately represent the peak airfoil responses of this skyline plot with limited PC modal retention.

In order to better understand how each peak within the skyline converges upon its final value, the stem plot is constructed for every iteration of retained geometry (Figure 6.5). Each subplot compares the forced response of a converging geometry to that of the corresponding physical rotor model, with the abscissa of each describing the blade index and the ordinate detailing the amplitude magnification with respect to the physical peak rotor response. From these results, it is evident that limited retention of geometric mistuning results in roughly homogeneous (tuned) rotor response. As mistuning is progressively applied, the apparent homogeneity vanishes as the responses begin to align with their target as indicated by crosses.

Within a single rotor, the index of peak blade response varies with progressive mistuning as described in Table 6.2. The four columns of Table 6.2 describe the retained PCs, the
Figure 6.3: Natural Frequency Convergence for Blade PCA on Subpopulation A, Worst Case.

geometric variance explained, the blade index of the peak responding blade per iteration, and the indexed error – the percent error of the particular indexed blade with respect to its physical model baseline response. The poor performance of blade response PCA is illustrated in Figure 6.6, which details the convergence trend of each blade within a single rotor as geometric variation increases. The peak responding blade is highlighted as a circle for each iteration equivalent to the blades mentioned in Table 6.2. Here, it is shown that even 99.5% geometry explanation results in error spanning 40% with respect to the baseline response of each blade. Thus, PCA cannot be used as a reduced order model to accurately
Figure 6.4: Iterative Skyline for Increased Geometric Variation Explained after Application of Blade PCA on Subpopulation A.

predict physical blade response.

Table 6.2: Peak Rotor Index in Rotor 11 of Subpopulation A with Increased Blade PC Mode Retention.

<table>
<thead>
<tr>
<th>PCs</th>
<th>Geometry (%)</th>
<th>Blade Index</th>
<th>Indexed Error (%)</th>
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6.1.1.3 Peak Rotor Response Capture of Physical Rotors

Next, consider the peak rotor response error with respect to the full physical model geometry obtained from MORPH, iteratively tracked for the same rotor of 6.1.1.2 in Figure 6.7. The error across the entire range of evaluated geometries spans 60%, a considerable improvement on the 210% error span of the blade response comparison. With regards to convergence, the rotor amplitude response remains outside the acceptable 5% band over most of the evaluated range, attaining 20% error even at 98% of geometry explained due
Figure 6.5: Rotor 11 of Subpopulation A with Various Total Geometry Explained, Amplitude Magnification vs. Blade Index Plots.

to the sensitivity of responses to geometric mistuning and the chaotic nature of these perturbations. Comparably, the natural frequencies converged more quickly and remained, on
average, within a 2% band for all quantities of retained PCs as shown by Figure 6.3.

The peak rotor convergence trend is consistently poor across the measured subpopulation, although the convergence behavior varies widely between rotors. Similar to blade response, the majority of PCs must be incorporated into the model to accurately capture peak rotor response on individual rotors; thus, PCA shows little capacity to represent vibratory responses of the target subpopulation with respect to blade geometry deviations.
6.1.1.4 Distribution Evaluation

Reconstruction of full rotor responses represents an alternative method of evaluating PCA as a mistuning assessment tool. Recall that Subpopulation A is comprised of 704 blades distributed equally across 32 rotors, with system responses measured for progressive retention of geometric variation. The following figures detail the variation in response accuracy with geometric convergence through the lens of the cumulative distribution function (CDF), wherein the ordinate denotes the cumulative probability and the abscissa describes the amplitude magnification with respect to the nominal rotor representing the average sector. Progressive CDFs are compared to the physical model distribution from MORPH using the two-sample K-S test outlined in Section 3.4, where distributions rejecting the null hypothesis are identified as dotted lines. This indicates the test case distribution does not come from the parent distribution of MORPH.

For the frequency range discussed in Section 5.1, Figure 6.8 shows the distribution of peak airfoil responses for increasing geometry explanation compared to that of the nominal MORPHed model. Note that this distribution evaluates the performance of all blades in the population independent of specific rotors. As shown in the figure, the overall blade
response converges at 79% geometry explanation, with consistent under-prediction beyond this amount of explained geometry. It is evident from these results that PCA applied to blade geometries readily represents fleet-wide forced response distributions with limited retention of PC modes. This examination provides insight on the application of blade PCA to overall fleet responses, a wider perspective than physical blade representation.

Figure 6.8: Peak Airfoil Response Distribution using Blade PCA on Subpopulation A.

Alternatively, the peak rotor distribution can be mapped as shown in Figure 6.9. The jagged appearance of each CDF is a result of the small peak rotor response population available for evaluation – 32 for Subpopulation A. As indicated by the figure, peak rotor response distributions behave similarly to those of peak airfoil response, converging upon the baseline distribution developed from full geometry retention. Furthermore, retention of only 8 PC modes, corresponding to 84.5% geometry retention, results in passing of the 5% significance K-S test. This indicates that limited retention of PC modes accurately represents the actual distribution of physical peak rotor responses. Compounded with the
results of the peak airfoil distribution, it is clear that PCA performed on blade geometries results in representative response distributions fleet-wide.

![Empirical CDF](image)

Figure 6.9: Peak Rotor Response Distribution using Blade PCA on Subpopulation A.

### 6.1.2 Subpopulation B

Analysis of Subpopulation B proceeded in parallel to that of Subpopulation A with examination of the capability of blade PCA in approximating natural frequencies and peak airfoil/rotor responses. To begin, the contribution of successive iterations of explained geometry are plotted in Figure 6.10, which indicates that 74% of the geometry is explained within the first PC mode. The contribution of remaining modes diminishes rapidly, with the second PC describing only 8.1% of the geometry. Retention of the first ten modes of 175 results in explanation of 94% of the geometric mistuning variation, readily describing the rapid spatial convergence of the PCA process.
Figure 6.10: Fractional Variance Explained and Cumulative Variation Explained versus Principal Component Mode for Subpopulation B for Blade PCA.

Careful paring of available PCs is necessary to permit a tractable study, with the resulting reduced set described in Table 6.3. These iterations form the geometrically perturbed models used to evaluate the performance of fleet risk assessment using PCA-based reduced order models.

6.1.2.1 Natural Frequency

With respect to natural frequency, considerable diversity is observed in the convergence trends of geometric explanation iterations. Figure 6.11 describes these trends for 1T and 3B natural frequencies corresponding to a tuned response at ND-3. With respect to 1T (Figure 6.11(a)), the approximated natural frequency deviates by 1.5% from the baseline value

Table 6.3: Geometric Variation Explained by Blade PCs of Subpopulation B.

<table>
<thead>
<tr>
<th>PCs</th>
<th>Cumulative $s_{f_k}$ (%)</th>
<th>PCs</th>
<th>Cumulative $s_{f_k}$ (%)</th>
<th>PCs</th>
<th>Cumulative $s_{f_k}$ (%)</th>
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</tbody>
</table>
for retention of only a single mode; as retention increases to 95% of geometric variance explained, the error reduces to 0.1%, where the baseline model represents the corresponding response from the MORPHed model. Comparatively, the 3B natural frequency of Figure 6.11(b) is underestimated by 3% until 94% of the geometric variation has been incorporated. In both cases the error remains below 5% for all iterations of PC retention; thus, reduced order models from PCA accurately represent subpopulation B natural frequencies. However, replication of natural frequencies is a necessary but insufficient metric for full response prediction – to fully explore the veracity of this approach, the physical responses must be duplicated.

Figure 6.11: Natural Frequency Convergence for Blade PCA on Subpopulation B, Worst Case.

6.1.2.2 Blade Response Capture of Physical Rotors

The first physical response to investigate is that of the peak airfoil. Peak airfoil responses for a single rotor representative of the subpopulation are presented in Figure 6.12, where each line represents the convergence in error of a single blade. For 99.9% geometric variation explanation, the error per blade spans nearly 30%; thus, as found for Subpopulation A, the convergence trend of sequentially retained PC modes is too slow to accurately represent physical blade response.
6.1.2.3 Peak Rotor Response Capture of Physical Rotors

The second physical response to examine is that of the peak rotor. Here, the response of each blade denoted by black circles in Figure 6.12 is compared to the known peak rotor amplitude magnification at 100% geometry explanation. Figure 6.13 showcases the convergence of the percent error between the approximated and actual peak rotor amplitude with respect to geometric variance explained as the peak response changes locations around the IBR. The convergence trend depicted in this figure is poor, resulting in an under-prediction.
of the actual response by 10% at 99.5% variation explained. These results match those of Subpopulation A, where nearly every PC mode is required to accurately represent peak rotor response and again shows the sensitivity of harmonic responses to small geometric perturbations.

Figure 6.13: Convergence Trend of Peak Rotor Response around a Single Rotor of Subpopulation B after Application of Blade PCA, Rotor 5.

### 6.1.2.4 Distribution Evaluation

Principal component analysis of the Subpopulation B blade geometries results in the response distributions of Figure 6.14. Each distribution is created from the 176 peak airfoil responses comprising the 8-rotor population. Figure 6.14(a) indicates rapid convergence to the physical distribution according to the 5% significance K-S test, marred only by systemic under-prediction of the actual response.

Figure 6.14(b) documents the peak rotor response distribution. The jagged nature of this figure is a result of the small nature of Subpopulation B, which contains only 8 rotors. Similar to the peak airfoil distribution, peak rotor responses rapidly converge to the physical distribution according to the 5% significance K-S test. These results further support the
application of PCA in representing the overall peak airfoil and peak rotor responses using limited retention of PC modes.

### 6.2 Correlation Capture

Both subpopulations feature considerable correlation in the geometry variation between sectors, which must be quantified prior to further fleet response prediction. Several methods are employed to isolate these correlations, including examination of component volumes, natural frequencies, and mode shapes.

Volumetric correlation has previously been shown to stem from tool wear, with increasing volumes linked to the progressive degradation of the tool work surface [51]. Mismatch of the component volumes resulted in correlated geometric mistuning, thereby making tool wear a likely candidate for the observed correlations of the population. Figure 6.15 depicts the blade volumes versus their indexed position within the rotor. Unfortunately, identifiable patterns across the fleet in the MORPHed blade volumes are not evident. Furthermore, although 11 of 32 Subpopulation A rotors exhibit a steady volumetric increase with blade index, this trend is not representative of the fleet and cannot be utilized as a
generalized statement. Thus, volumetric considerations hint at sector correlation but fail to provide quantifiable evidence applicable to the fleet.

![Blade Volume for AG Population](image)

**Figure 6.15: Volumetric Source of Correlation.**

From the study of blade volumes, it is found that the element volumes of the blade-rim sectors of Subpopulation A are on average 5% larger than those found in Subpopulation B. Recall that these sectors account for blade, fillet, root, and annulus; however, the subdermal elements of the annulus do not move with MORPH, meaning that the face in which the boundary conditions are applied are always in the same location. Therefore, this volume gives an overall representation of the differences between the sectors in A and B. This aids in accounting for the higher amplitude magnifications of the peak rotor responses of Subpopulation B.

Sector correlations may also be explained through patterns in modal response, specifically through natural frequencies. Thus, PCA-based blade geometries are analyzed under cantilever-beam boundary conditions to generate the respective natural frequencies. A random response is shown in Figure 6.16, where the natural frequency is plotted against the blade index for a randomly selected rotor of Subpopulation A. There is no discernible pattern for responses that account for the correlations existent in the population, however, there is evidence of frequency decrease with volumetric increase, showing that sequence of blade is important to predict actual responses.

Next is to consider the PCA scores as a possible source of correlation shown in Figure
These plots consider the corresponding scores for single PC mode shapes and plot one versus another to determine the existence of correlation. The plots map score density and show the actual score values in white. The first mode in blade and IBR PCA show major deviations in three rotors. When considering 22 sequential scores corresponding to a single rotor in airfoil application, the clustered information is captured accurately by IBR PCA. Though correlation seems to exist in 1 vs. 2 and 3 vs. 2, there is not a readily trackable and applied correlation for monitoring blade to blade correlation.

The final method of identifying sector correlations requires analysis of PCA mode shapes, represented as contour plots of surface deviations. Each contour plot is developed from a set of patches, generated from the averaged deviation of the respective nodes within the PC mode matrix. These patches are scaled against the total set and mapped to the corresponding geometry. Figures 6.18 and 6.19 detail representative deviation profiles for blades and rotors drawn from both subpopulations. In each case, strong deviation patterns visible on the blade (a) are replicated about the rotor for the corresponding mode shape from IBR PCA. However, the magnitude of the deviations vary between blades, seen in (b), indicating the existence of spatial correlations not captured by blade PCA. Thus, application of PCA to blade-alone geometries detrimentally affects fleet response approximation by generalizing the rotor response to the highest responding blade and negating spatial correlation.
Figure 6.17: Score vs. Score Density.
Figure 6.18: Map of PC Mode Deviations of Subpopulation A, Single Mode.

Figure 6.19: Map of PC Mode Deviations of Subpopulation B, Single Mode.
6.3 Rotor Analysis

To preserve spatial correlation, PCA is next applied directly to full rotor geometries. Thus, the number of modes for evaluation is limited by the number of rotors in each subpopulation, just as Blade PCA is limited by the number of blades.

6.3.1 Subpopulation A

Principal component analysis is first performed on Subpopulation A blade geometries. The Pareto chart of Figure 6.20 depicts both the fractional and cumulative geometric variation contained within the first ten PC modes, representing spatial differences in blade nodal locations across the fleet. PCs beyond the tenth – greater than 67% of the total modes – explain only 13% of the total geometric variation, indicating the nearly asymptotic convergence of geometric reconstruction. Thus, in an effort to reduce computational expense, modal retention beyond the tenth PC is restricted to those sets nearest whole percent values of geometry explained within 10% of the nominal model (Table 6.4), where the number of modes reduced out of the analysis are much less in quantity compared to the blade analysis approach.

Table 6.4: Geometric Variation Explained by Rotor PCs of Subpopulation A.

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<thead>
<tr>
<th>PCs</th>
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6.3.1.1 Natural Frequency

Geometric mistuning inherently disrupts the nominal structure, resulting in measurable changes to modal properties. Therefore, it is advantageous to examine the rotor-specific convergence trend for increasing PC retention. This is done by tracking the index of mode shapes of interest (originally identified within the tuned model) through successive analyses of progressive PC retention. Figure 6.21 details the poorest-performing trend for 1T and 3B across the entire subpopulation. In each case, retention of only a single mode results in error not exceeding 5%, with reduction to 0.1% error at 91% explanation of geometric variance. These results indicate conclusively that natural frequencies are accurately captured with restricted PC retention.

Another method of natural frequency evaluation is to compare the frequency deviations at the peak response locations. Figure 6.22 shows these deviations for application of blade and IBR PCA for a side-by-side comparison. Retaining the first mode (57% and 58% of the geometric variance explained by blade and IBR PCA respectively) share a limited
Figure 6.21: Natural Frequency Convergence for IBR PCA on Subpopulation A, Worst Case.

range of values, or spread. This was illustrated in the skyline plot of Figure 6.4, where the skyline trace exhibits close peaks with little spread in natural frequency. It is understood that as geometric mistuning is introduced into the system, the final skyline trace of 100% geometry retention spreads in peak location and magnitude. Therefore, as the geometric mistuning introduced increases from 58% to 100%, the frequency spread should increase, with the goal to accurately capture this frequency mistuning as well as the amplitude mistuning with limited retention of PC modes. Both applications of PCA to blade and rotor geometries result in similar spreads of natural frequency, but the resulting error in forced response in considered next.

Figure 6.22: Natural Frequency Variation of Subpopulation A for Iterative Geometric Variations Explained by the Retained Modes.

Both cases of blade and IBR PCA of Figures 6.22(a) and 6.22(b) show that the con-
vergence on mistuning spread is slow. Although PCA performs well in producing a narrow range and fast convergence trend while tracking indexed natural frequencies, the frequency range responsible for the forced response locations is much slower to match the true spread with reduced models.

6.3.1.2 Blade Response Capture of Physical Rotors

The peak airfoil responses for a single rotor are shown in Figure 6.23, where each colored line tracks a single blade. The values in the ordinate correspond to the percent error of the indexed blade with respect to the physical blade response, indicating that at 100% geometry retention the resulting error for all blades is 0%. The highlighted blades change location with the increase of explained geometric variance. At 99.9% geometric variation explanation, the error per blade spans more than 160%. While this extremely large band is not a good representation of the entire population, it does highlight the limitations of PCA. Although retention of 30/31 modes results in 99.9% geometry explanation, other response metrics still exhibit 40-80% error. This follows the same conclusions as drawn by application of blade PCA, which shows that the convergence trend is too slow to limit the number of retained modes in order to accurately represent the physical blade responses. In order to capture these physical blade responses within a small error band, all modes must be retained.

6.3.1.3 Peak Rotor Response Capture of Physical Rotors

Next, consider the same rotor of the previous section for peak rotor response error of each retention interval with respect to the known peak rotor response of the physical model baseline in Figure 6.24. These are the same blades as those circled in Figure 6.23. The peak rotor response has a very poor convergence trend, under-predicting the actual response by nearly 10% even with 99.5% of the geometric variance explained and rarely remaining
within the ±5% bounds shown by dotted lines in the figure. The overall convergence trend is so poor and lacking a distinguishable pattern that limitation of modes retained would promote error of an unacceptable level. This reiterates the findings of blade PCA application to both subpopulations, wherein all PC modes are required in order to accurately represent both blade response and peak rotor responses with a small bounding error. Without the retention of all modes, the forced response inaccurately represents the physical responses.
6.3.1.4 Distribution Evaluation

Principal component analysis of the IBR population results in the blade distribution shown in Figure 6.25. These results show little variation in all distributions calculated for progressive geometry explanation percentages; additionally, almost all distributions are identical to the known solution for Subpopulation A. The peak airfoil distribution of Figure 6.25(a) shows a nearly converged distribution with limited information, where cases 58.0-82.5% fall outside of the significance band as applied by using the K-S. Further, Figure 6.25(b) shows that all tested distributions are from the same parent distribution of 100% geometry retained within a 5% significance of the K-S test. The ability to capture the profile of the distribution is much stronger with the application of PCA to rotor geometry versus blade geometries.

These findings further indicate that although PCA cannot be applied to accurately capture physical results by calling out individual blades or the peak rotor counterpart as represented with limited geometry, it can be used to accurately capture response results fleet wide.
6.3.2 Subpopulation B

Analysis of Subpopulation B proceeded in parallel to that of Subpopulation A with examination of the capability of IBR PCA in approximating natural frequencies and peak airfoil/rotor responses. To begin, the contribution of successive iterations of explained geometry are plotted in Figure 6.10, which indicates that 79.7% of the geometry is explained within the first PC mode. The contribution of remaining modes diminishes rapidly, with the second PC describing only 10.1% of the geometry, as explained by Table 6.5. Since the number of modes calculated by PCA is limited by the number of observations, seven modes describe the geometric variation rotor to rotor experienced in Subpopulation B. Therefore, all iterations are tested to create geometrically perturbed models for assessment of PCA capabilities with limited retention of PC modes.

Table 6.5: Geometric Variation Explained by Rotor PCs of Subpopulation B.

<table>
<thead>
<tr>
<th>PCs</th>
<th>Cumulative $s_k$ (%)</th>
<th>PCs</th>
<th>Cumulative $s_k$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79.7</td>
<td>5</td>
<td>97.4</td>
</tr>
<tr>
<td>2</td>
<td>89.8</td>
<td>6</td>
<td>98.9</td>
</tr>
<tr>
<td>3</td>
<td>93.2</td>
<td>7</td>
<td>100.0</td>
</tr>
<tr>
<td>4</td>
<td>95.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.3.2.1 Natural Frequency

Each of the geometrically perturbed models identifying nearly 80% through 100% of the geometric mistuning have different convergence trends for the resulting natural frequencies. Figure 6.27 shows poorest convergence trends for 1T and 3B natural frequencies corresponding to a tuned ND-3 response. While 3B is slow to converge and has the widest disparity of 3B natural frequencies across all eight rotors, all frequencies produced by the iterative geometries are within 5% of the actual value for both 1T and 3B, indicating that PCA applied to rotors can accurately capture natural frequencies within 5% for this operating window.

6.3.2.2 Blade Response Capture of Physical Rotors

The peak airfoil responses of individual blades on a single rotor are shown in Figure 6.28 as a percentage error as compared to the respective response on the physical baseline. With the explanation of 98.9% geometric variation utilizing six of the seven total modes, the error
per blade spans more than 60% and follows no predictable trend. This error band is the largest of the population at this iteration and highlights the limitations of PCA. This follows the same conclusions as blade PCA as well as application of IBR PCA on Subpopulation A, which shows that the convergence trend is too slow to limit the number of retained modes in order to accurately represent the physical blade responses. All modes must be retained in order to capture these physical blade responses within a small error band.

6.3.2.3 Peak Rotor Response Capture of Physical Rotors

The second physical response to examine is that of the peak rotor. Here, the response of each blade denoted by black circles in Figure 6.28 is compared to the known peak rotor amplitude magnification at 100% geometry explanation. Figure 6.29 showcases the convergence of the percent error between the approximated and actual peak rotor amplitude with respect to geometric variance explained as the peak response changes locations around the IBR. The convergence trend depicted in this figure is poor, resulting in an over-prediction of the actual response by 20% at 98.9% variation explained with the retention of 6 of 7 total modes. These results match those of Subpopulation A, where nearly every PC mode is required to accurately represent peak rotor response.
Figure 6.28: Convergence Trend of Peak Airfoil Responses around a Single Rotor of Subpopulation B after Application of IBR PCA, Rotor 5.

Figure 6.29: Convergence Trend of Peak Rotor Response of a Single Rotor in Subpopulation B with Application of IBR PCA, Rotor 5

6.3.2.4 Distribution Evaluation

PCA of the Subpopulation B blade geometries results in the response distributions of Figure 6.30. Each distribution is created from the 176 peak airfoil responses comprising the 8-
rotor population. Figure 6.30(a) indicates rapid convergence to the physical distribution according to the 5% significance K-S test, marred only by systemic under-prediction of the actual response.

![Graph](image)

(a) Peak Airfoil

(b) Peak Rotor

Figure 6.30: Distributions using IBR PCA on Subpopulation B.

Figure 6.30(b) documents the peak rotor response distribution. The jagged nature of this figure is a result of the small nature of Subpopulation B, which contains only 8 rotors. Similar to the peak airfoil distribution, peak rotor responses rapidly converge to the physical distribution according to the 5% significance K-S test. These results further support the application of PCA in representing the overall peak airfoil and peak rotor responses using limited retention of PC modes.

### 6.4 Continued Discussion and Relative Conclusions

The work covering application of PCA focuses on the amount of frequency mistuning and amplitude magnification that occurs by geometrically perturbing the model by only retaining limited geometric mistuning information. The previous sections outline the results for:
1. Natural Frequency Convergence

2. Accurate Representation of Peak Airfoil Responses by Tracking Indexed Blades

3. Convergence of Peak Rotor Response

4. Representation of Overall Response Distribution

5. PCA Mode Shapes

for two subpopulations of the compressor rotors. The natural frequency convergence trend is poor, but it does predict the natural frequencies within 5% accuracy for all tested geometric variations for blade and rotor analyses. Although PCA can recreate the natural frequencies with limited retention of mistuning through modes, the accuracy does not extend through forced response amplitude magnifications in either peak airfoil or rotor responses.

What PCA does provide is a platform for obtaining a fleet-wide response distribution with a limited retention of modes. While it cannot obtain specific blade-indexed responses, it can capture an overall picture of the response for a single rotor and moreover the entire tested subpopulation. Wielding this information will provide a more useful application of PCA in the future.

Additionally, PCA should be applied to measure rotor variations over blade variations since these populations experience sector correlations exposed by mapping the mode shapes onto the geometry. The previous sections expand that although the convergence is poor for both applications, the resulting response distributions are much more accurate for a limited number of retained modes for both subpopulations. Continued work should consider application of PCA to rotor geometry over blade geometry so long as there are enough rotors to do so.
Probabilistic Fleet Prediction

Variation in the average geometry and response distributions of the two subpopulations make direct analysis of a combined population infeasible. However, implementation of a weighting schema apportioning the response distribution by population composition (80%/20% for Subpopulation A vs. B) correctly models the fleet-wide response, with the remaining ability to analyze individual subpopulations. The following methods compare blade and peak rotor distribution prediction confidence through application of the K-S test (Section 3.4).

7.1 Bootstrapping Rotor Geometry

Bootstrapping is first applied to extend Subpopulations A and B, as well as the comprehensive population consisting of a mixture of rotor geometries. The process of generating extended fleet response distributions is as follows: first, random rotors are selected from within the target population and the corresponding blade geometries are detached from the hub; then, the randomly drawn blades are reoriented around the disk to form a new rotor geometry. Although the blades selected for reorientation are sampled with replacement, computer algorithms ensure none of the rotors are identical. Boundary conditions matching previous FEA efforts are then applied to the bootstrapped rotors, and the forced response is developed in the following figures.

Subpopulation A is evaluated first, with peak airfoil and peak rotor responses depicted
in Figure 7.1. Results are rendered for a progression of randomly bootstrapped rotors, with the available pool of blades ranging from \( N-32N \) as denoted in the figure as the number of selected rotors. Furthermore, each test is evaluated 1000 times, creating a fleet of rotors of identical dimension containing 22,000 blades with corresponding forced response distributions. It is initially assumed that the predicted rotor response would converge upon the MORPHed distribution as the number of rotors modeled attained parity with the total rotor population. Yet, 7.1(a) belied this assumption; in fact, only two iterations – those utilizing four and five rotors during resampling – passed the K-S test, and each severely under-predicted the target response amplitudes generated by the MORPHed population. Similar results were found for 7.1(b), with severe over-prediction of the response amplitude for at least 50\% of the cumulative probability and a failure of any distribution to pass the K-S test.

Figure 7.1: Distribution by Bootstrapping Random Rotor Blades of Subpopulation A.

Although random bootstrapping of Subpopulation A fails to accurately represent the corresponding fleet, it is necessary to examine the response of Subpopulation B to the same analysis. Test parameters were identical except for the available pool of blades, which ranged from \( N-8N \) for Subpopulation B. Figure 7.1 details the results for both peak rotor and peak airfoil response distributions. Similar to Subpopulation A, the peak rotor predic-
tions of Subpopulation B consistently under-predict the physical distributions as shown in Figure 7.2(a); however, in this case five subsets passed the K-S test, including those drawn from three and five to eight randomly selected rotors. The peak airfoil distributions of Figure 7.2(b) perform more poorly than those of the peak rotor, with only the subset created from two random rotors passing the K-S test. The over-prediction trend evident in the peak airfoil distributions of Subpopulation A is mirrored in the results of Subpopulation B.

The comparative performance of bootstrapped Subpopulation B results provides additional evidence of considerable sector correlation within Subpopulation A. Thus, further analysis is warranted of a successful Subpopulation B design case – the use of three randomized rotors. Figure 7.3 depicts the peak rotor responses of the eight rotors comprising Subpopulation B, with the randomly selected responses visibly distinguished to highlight those drawn for re-analysis. Examination reveals that the randomized selections capture the gamut of high-responding rotors, low-responding rotors, and a balance of the extrema.

Figure 7.2: Distribution by Bootstrapping Random Rotor Blades of Subpopulation B.

Each case is analyzed under the same set of conditions used previously, with the resulting response distributions presented in Figure 7.4. The peak rotor results of Figure 7.4(a) indicate that only the first case passes the K-S test with respect to the morphed results.
Figure 7.3: Peak Rotor Responses of Rotors Selected for Bootstrapping of Selected Rotors.

These results are unsurprising, as this case most closely matches the target distribution. Conversely, the peak airfoil distribution of Figure 7.4(b) identifies cases two and three as passing the K-S test. Therefore, the response of the rotors selected for analysis significantly impacts the performance of the distribution prediction.

Figure 7.4: Distribution by Bootstrapping Random Rotor Blades of Subpopulation B, 3 Rotors Selected Multiple Times.

The capability of this methodology to predict the responses of Subpopulation B led to consideration of its ability with regard to the entire population, 80% of which belongs to Subpopulation A. Figure 7.5 portrays evaluations developed from the best subset of three rotors from Subpopulation B combined with progressive subsets of Subpopulation A.
The peak rotor distributions of Figure 7.5(a) are considerably more accurate than previous predictions, with combined A/B subsets of 4/3 and 5/3 passing the 5% significance K-S test.

Figure 7.5: Distribution by Bootstrapping Random Rotor Blades of the Entire Population.

The successes achieved in total population peak rotor prediction are not sustained in analysis of peak airfoil response. Figure 7.5(b) shows that for all Subpopulation A/B subset combinations, the resulting peak airfoil distribution is not representative of the expected physical distributions, with considerable deviation between 0% and 80% of the CDF probability. The totality of these results indicate that bootstrapping is a powerful tool, albeit one that requires intelligent application to capture sector correlations intrinsic to the available populations.

### 7.2 Bootstrapping PCA Scores

Next is to consider application of bootstrapping to the resampling of PCA scores as outlined in Sections 5.2.2 and 5.2.3.
7.2.1 Blade Analysis

Bootstrapping of resampled PCA scores is first applied to blade analysis of Subpopulation A. The results of bootstrapping PCA scores of two cases are displayed in Figure 7.6, corresponding to the retention of the first 8 and 217 blade modes (84.5% and 99.5% geometry variance explained, respectively). These modes were selected based on the response distributions of Figures 6.8 and 6.9, where 84.5% variation explanation was shown to pass the K-S test with respect to both peak airfoil and peak rotor distributions.

These figures provide ample evidence that bootstrapping of PC scores accurately represent neither the peak rotor distribution (Figure 7.6(a)) nor the peak airfoil distribution (Figure 7.6(b)) with limited retention of geometric variance. With respect to estimation of the peak rotor response, the predicted CDF matches the shape of the expected response but spans an incorrect range of amplitude magnifications. In comparison, the peak airfoil responses span the correct range of amplitude magnifications but fail to accurately capture the shape of the expected response. Therefore, resampling of PC scores is shown to be an inaccurate method of fleet prediction assessment; furthermore, the results of this study preclude investigation of PC score bootstrapping for Subpopulation B or the comprehensive rotor fleet.
7.2.2 Rotor Analysis

The results presented in Section 6.2 indicate that PCA applied to full rotor geometries better captures sector correlations, which strongly influence response distributions. As such, it is of interest to attempt bootstrapping of rotor PCA scores to further improve the quality of the resulting predictions. As Subpopulation A features considerable sector correlations, it serves as the primary test of this method. Limitation of information content was achieved by resampling from within the first 1, 8, 16, 24, and 31 rotor PC modes pertaining to geometric variance explanation of 58.0%, 84.2%, 92.6%, 97.4%, and 100%, respectively. Peak rotor and peak airfoil distribution predictions are presented in Figures 7.7 and 7.8.

First, consider the peak rotor distributions of Figure 7.7(a), where each prediction except that formed from a single retained mode passes the K-S test. As the single mode retains only 58% of the geometric variance – and resamples from only 32 rotors – it is unsurprising that this prediction fails. Subsequent predictions exhibit under-prediction from onset to 70% cumulative probability and over-prediction from thereon. Figure 7.7(b) compares the prediction developed from bootstrapping of all PC modes and the baseline MORPHed
response, demonstrating the disparity between the two despite passage of the K-S test.

Next, examine the peak airfoil distributions presented in Figure 7.8, where again each prediction, except for that formed from a single PC mode, passes the K-S test. As before, the loss of nearly 50% of geometry variance is a likely culprit for this failure. Figure 7.8(b) magnifies the discrepancies between the prediction generated from all PC modes and the baseline MORPHed case – from this plot, it is seen that although the prediction is close, the actual response distribution oscillates between amplitude magnifications of 0.4 to 0.7. This oscillation is not captured by the predicted data in blue, but instead is leveled off through that region. Nevertheless, application of PC mode bootstrapping effectively models Subpopulation A rotor dynamics.

This methodology – verified on Subpopulation A – is likewise applied to Subpopulation B. PC retention spanned modes one, two, four, six, and seven, corresponding to retained geometry of 79.7%, 89.8%, 95.4%, 98.9%, and 100%, respectively. The small size of Subpopulation B limits the first two cases to 8 and 64 realized rotors; however, further cases utilize 1000 Monte Carlo simulations. Figure 7.9 displays the results of peak rotor and peak airfoil distributions for this subpopulation. From Figure 7.9(a), it is evident
Figure 7.8: Peak Airfoil Distribution by Bootstrapping IBR PCA Scores of Subpopulation A.

that all five test cases pass the K-S test.

Although the peak rotor distributions of Figure 7.9(a) developed from one and two resampled PC modes reside stray from the baseline comparator, the remaining test cases highly resemble smoothed facsimiles of the MORPHed distribution. These results signify the applicability of the bootstrapped approach to peak rotor prediction of both subpopulations.

Figure 7.9: Distribution by Bootstrapping IBR PCA Scores of Subpopulation B.

Next is considered the peak airfoil distribution of Figure 7.9(b). The first two test
cases – those generated for resampling of one and two PC modes – fail the K-S test. Nevertheless, distributions generated from resampling of four, six, and seven modes are good representations of the actual distribution. Again, this case supports the application of this bootstrapping approach to peak airfoil prediction of both subpopulations.

The final step is to consider application to a mixed population constructed from both subpopulations, with the rotors apportioned such that 80% of the data is drawn from Subpopulation A. Figure 7.10 describes these results for retention of a progression of PC modes of Subpopulation A with complete retention of all modes in Subpopulation B, while Appendix A contains the response distributions for other amounts of Subpopulation B retention.

Figure 7.10(a) shows the quality of the peak rotor distribution prediction, with only the case of a single retained Subpopulation A PC mode matched with the seven retained Subpopulation B modes failing the K-S test. With this mixture, the corresponding response distribution is generated with the 80%-20% split, though only a subset of the results from Subpopulation B are utilized. Each remaining distribution matches well with the baseline behavior, with some small over-prediction between amplitude magnifications of 1 and 1.5 and slight under-prediction beyond amplitude magnification of approximately 1.75. Similar results are found in Figure 7.10(b), which describes the peak airfoil distribution predictions. Again, only the case of a single Subpopulation A mode combined with seven Subpopulation B modes rotor realizations fails the K-S test, with the remaining predictions closely matching the baseline mixed distribution.

The conclusion drawn from these results is the powerful predictive nature of the bootstrapping approach when coupled with PCA, even with limited retention of PC modes, produces usable and accurate response prediction distributions.
Figure 7.10: Distribution by Bootstrapping IBR PCA Scores of the Entire Population.

7.2.3 Subset Population Rotor Analysis

With the potential of PCA bootstrapping well established, interest exists in ascertaining the performance of this approach using a limited subset of rotors. This new method is evaluated on each subpopulation, beginning with the rotors of Subpopulation A. Subsampling within Subpopulation A is performed three times for eight rotors and once for sixteen rotors, with the full subpopulation set representing complete sampling. Construction of these data sets is followed by PCA of the rotor geometries selected, bootstrapping of the PC scores through full modal retention, and FEA analysis of the resulting 1000 rotors. Peak airfoil and peak rotor distributions for this case are presented in Figures 7.11 and 7.12.

Consider first the peak rotor distribution of Figure 7.11(a), where each tested case passes the 5% K-S test. The distributions presented here behave similarly to those of Figure 7.7, which detailed application of bootstrapping to limited PC modal retention using the full subpopulation. Of particular note are the under-prediction between amplitude magnifications of 1.3 and 1.75, as well as the over-prediction occurring beyond 0.8 cumulative probability. Figure 7.11(b) examines the second sampling of eight rotors as compared to the baseline MORPHed distribution. The predicted distribution matches well with the baseline
comparator, indicating successful simulation of the available data.

Next, consider the peak airfoil response distributions of Figure 7.12(a). Only two cases – iterations one and three of the eight randomly selected rotors – exist outside the 5% significance level of the K-S test. While these iterations under- or over-predict the desired result, the overall shape of each CDF varies little from that of the baseline distribution. Thus, any of these cases could be applied to the task of peak airfoil prediction with limited error – a conclusion further supported by the results of Figure 7.12(b), which denotes the near complete accuracy of the second sampling of eight rotors in representing the MORPHed distribution.

Successful application of this technique to Subpopulation A encourages similar use with Supopulation B as depicted in Figure 7.13, which displays both peak rotor and peak airfoil distributions for subsets of two, three, four, six, and eight randomly sampled rotors. As before, PCA is performed on the rotor geometries, and all resulting PC modes are bootstrapped to create new rotors. Note that subsets of two, three, and four rotors are limited in the number of available non-repeated new rotors to two, four, and nine, respectively. The remaining cases employ the 1000 rotor MC simulation as previously applied.
Figure 7.12: Peak Airfoil Response Distribution by Bootstrapping IBR PCA Scores of a Subset of Subpopulation A.

Every peak rotor distribution of Figure 7.13(a) is shown to pass the K-S test; furthermore, as the number of rotors within the subset $n$ increases, the smoothness of the resulting distribution increases since the number of MC simulations increases. Similar results are found in Figure 7.13(b), where again each case has passed the K-S test with the progressive trend previously mentioned. As this study progresses, the necessity of obtaining sufficient data to construct a smooth CDF is found to be crucial in developing the appropriate shape of the resulting distribution.

Figure 7.13: Distribution by Bootstrapping Subset IBR PCA Scores of Subpopulation B.
Finally, this methodology is applied to the entire mixed population, with results presented in Figures 7.14 and 7.15. Figure 7.14 describes the resulting peak rotor distributions for a subset of two rotors from Subpopulation B mixed with all subsets of Subpopulation A. The CDFs of Figure 7.14(a) appear jagged due to the mathematics of the population composition: as two Subpopulation B rotors must represent 20% of the total population, only eight Subpopulation A peak rotor responses are available for mixed analysis to form the remaining 80%. Regardless, the peak rotor response distribution does pass the K-S test for all tested subsets of Subpopulation A, albeit with considerable deviation from the baseline curve beyond a cumulative probability of 90%. Peak airfoil results presented in Figure 7.14(b) are similar, although the third subset of eight Subpopulation A subsamples fails the K-S test. The combination of these results shows that this technique performs very well even with limited geometry retention; in fact, the population of 40 rotors is captured with near complete accuracy using only 25% of each subpopulation.

Figure 7.14: Distribution by Bootstrapping IBR PCA Scores of the Entire Population, 2 Rotors from B.

Now, consider a mixed population constructed from a complete resampling of Subpopulation B, with results presented for progressive subsets of Subpopulation A in Figure 7.15. Not only does each peak rotor prediction (Figure 7.15(a)) more closely match the
expected distribution, but also all subsets of Subpopulation A pass the K-S test for representing the mixed population. The limitations of these predictions directly match those observed with predictions constructed from limited modal retention for resampling of both subpopulations. These results are corroborated by Figure 7.15(b), which investigates peak airfoil prediction. Each of these predictions closely matches the baseline comparator, although subsets one and three exist outside the 5% significance bound of the K-S test. Although the predicted distributions are uniformly conservative, any of the passing resampled subsets would accurately represent the geometry despite limited information.

![Graphs showing cumulative probability distributions](image)

Figure 7.15: Distribution by Bootstrapping IBR PCA Scores of the Entire Population, 8 Rotors from B.

The previous case was evaluated with the entirety of Subpopulation B; nevertheless, the small size of this subpopulation (eight rotors) severely limits the number of testing iterations. Convergence to the actual population distribution requires further iterations; yet, even with these constraints, this approach has proven valuable in accurately predicting the existing fleet with limited data retention. The quality of these predictions would be improved through further study of those qualities governing rotor selection for inclusion within the tested subset.

As a final measure, it is instructive to review the response distributions in a variety
of other ways. Figure 7.16 employs two methods to better visualize the predictive performance of this approach for a mixed set composed of eight Subpopulation B rotors and various subsets of Subpopulation A; Figure 7.16(a) directly compares the predicted distribution against the baseline CDF, while Figure 7.16(b) normalizes the predicted distribution against the actual distribution using the amplitude magnification values at iterative progressions of cumulative probability percentiles. The results of Figure 7.16(a) describe the chronic under-prediction of each distribution compared to the MORPHed baseline, while Figure 7.16(b) shows that the crafted predictions generally reside within a ±5% band centered about unity. Part (b) further highlights the inaccuracies present between 0-5% and 95-100% cumulative probability values.

![Figure 7.16: Peak Rotor Distribution by Bootstrapping IBR PCA Scores of the Entire Population, 8 Rotors from B.](image)

### 7.2.4 Fleet Risk Assessment

The culmination of these predictions is an evaluation of fleet risk, which is performed using a mixed population of eight rotors drawn from both subpopulations (Subpopulation A subset two, which passed the K-S test for both peak rotor and peak airfoil). As a notional example, failure is specified to occur at a magnification factor of 1.75, highlighted in Figure
7.17 for the peak rotor distribution predicted by the subsets. This figure identifies 25.80% of rotors at risk of failure due to geometric mistuning. For an actual population of 5000, 1289 would thus be at risk. Assuming each rotor is rated for 4000 hours of service, the total number of fleet service hours is 20,000,000. Thus, the number of failures per engine flight hour is 6.445E-05. Compare this to the true distribution, which indicates 21.95% of rotors will be at risk for geometric mistuning failure. This results in 5.484E-05 failures per engine flight hour – the predicted distribution is 17.52% more conservative than the actual distribution. Considering the computational cost savings of limited data retention through PCA in addition to utilizing subsets of subpopulation, a conservative estimate deviating from the actual distribution by less than 20% is highly valuable.

Figure 7.17: Peak Rotor Distribution as Predicted with Failure Region Highlighted.
Conclusions

The work presented within this manuscript indicates that despite some limitations in accurate prediction of forced response amplitudes, Principal Component Analysis offers an effective approach for modeling geometric mistuning. Although difficulties in convergence were encountered in modeling blade or rotor geometries separately using PCA, it was shown through extensive evaluation of peak rotor and peak airfoil response distributions of a fleet of rotors across a veering range of excitation frequencies that prediction of fleet-wide responses was readily achievable.

Furthermore, it was found that PCA performance is improved through application to rotor geometries, permitting capture of blade-to-blade correlations existing in the test subpopulations. This was leveraged during bootstrapping of retained PC scores, where limited retention of resampled rotor geometric variation accurately predicted peak rotor and peak airfoil response distributions of each subpopulation, as well as the comprehensive set.

These successes can be compounded with additional work. For example, this approach considers a single excitation location and a single response location selected based on available experimental data. A more powerful approach would employ an algorithm to identify the locations of highest response on each blade; then, the analysis could track how well these various nodes are predicted using IBR PCA bootstrapping. Additionally, only the out-of-plane displacements have been considered within this work, whereas future implementations of this research would observe the response in all degrees of freedom.
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Appendix A

Cumulative Distribution Functions

A.1 Bootstrapping Rotor PCA Scores

Figure A.1: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 1 Modes Resampled in Subpopulation B and Various Modes from Subpopulation A.
Figure A.2: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 2 Modes Resampled in Subpopulation B and Various Modes from Subpopulation A.

Figure A.3: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 4 Modes Resampled in Subpopulation B and Various Modes from Subpopulation A.
Figure A.4: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 6 Modes Resampled in Subpopulation B and Various Modes from Subpopulation A.

Figure A.5: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 7 Modes Resampled in Subpopulation B and Various Modes from Subpopulation A.
### A.2 Bootstrapping Rotor PCA Subset Scores

Figure A.6: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 2 Rotors Selected in Population B and Various Rotors from Population A.

Figure A.7: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 3 Rotors Selected in Population B and Various Rotors from Population A.
Figure A.8: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 4 Rotor Selected in Population B and Various Rotors from Population A.

Figure A.9: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 6 Rotor Selected in Population B and Various Rotors from Population A.
Figure A.10: Distribution by Bootstrapping IBR PCA Modes of the Entire Population, 8 Rotors Selected in Population B and Various Rotors from Population A.