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Large-Signal Analysis of Buck and Interleaved Buck DC-AC Converters

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LARGE-SIGNAL ANALYSIS OF BUCK AND INTERLEAVED BUCK DC-AC CONVERTER

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Engineering

By

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2014
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Abstract

Dey, Sourav. M.S.Egr, Department of Electrical Engineering, Wright State University, 2014. *Large-Signal Analysis of Buck and Interleaved Buck DC-AC Converters.*

With the improvement in the present day technology, power electronic engineers are more focused on systems which are fast and have better performance. Closed-loop systems with a fast response and better performance have been a priority these days. PWM converters are self-regulating voltage regulators with high efficiency. Depending upon the voltage level and the type of power conversion, different converters are used to meet the necessary demands. One such PWM converter is the buck DC-AC converter which plays an important role in the applications related to high-frequency and low-power. Large-signal analysis of buck and buck-derived converters for both open-loop and closed-loop system is the subject of study in this thesis. Small-signal transfer functions for buck DC-AC for both open-loop and closed-loop systems is derived for resistive and impedance load. Further, the large-signal analysis of buck DC-AC converter will be performed and the quality of the output signal is evaluated using total harmonic distortion (THD). It is also proven that, the small-signal analysis of buck DC-AC holds true for large-signal simulations. In real time applications, most of the loads connected to buck converters are impedance loads. Thus, in this thesis, buck DC-AC converter is to function as an amplitude modulator for Class-E radio-frequency (RF) power amplifier to achieve envelope tracking (ET). Finally, an overall conclusion will be provided on the small-signal analysis, its validation with respect to the large-signal simulations for resistive and impedance loads and the quality of the output signal in terms of THD.
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1 Introduction

1.1 Pulse-Width Modulated (PWM) Buck DC-AC Converter

Pulse-width modulation (PWM), is a modulation technique which controls the width of the pulse, based on the modulator signal. Pulse-width modulation is associated with DC-DC, and DC-AC converters for its wide range of applications, starting from analog and digital measuring devices, to power and voltage control and conversion. There are two kinds of voltage regulators, a) LinearRegulator, and b) SwitchingRegulator. A linear regulator is used to maintain a constant voltage at the output. The regulator functions in accordance with the load and a voltage divider network resulting in a constant output voltage. PWM converters falls under the category of switching regulators. In switching regulators, transistors and diodes act as switches. The power dissipated by the transistors acting as a switch is much less, than the power dissipated when transistors are acting as current sources. The filter capacitance at the output of a PWM converter affects the speed of the converter. The effective series resistance (ESR) of the output capacitor directly affect the output ripple current. Therefore, a capacitor with the lowest possible ESR is recommended for reducing the ripple current at the output and increase the speed of the system. Smaller capacitors are acceptable for light loads, or in applications where ripple is not a concern. PWM converters have various topologies like buck, boost, buck-boost, flyback, and Ćuk depending upon the application and the voltage levels. Buck converters are among one of the important topologies of PWM converters which acts a step-down switching converter. For a good DC-AC converter the output voltage of the converter should follow the modulating signal till the corner frequency of the system. Fig. 1.1 shows the circuit diagram of buck DC-AC converter. Buck DC-AC converter has a constant or fixed input voltage and the output of the converter is sinusoidally varying. Unlike buck DC-DC converter, buck DC-AC converter has a
pulse with variable width generated from the pulse-width modulator for a fixed input voltage. The duty cycle of the converter varies from a minimum to a maximum value, as a result, a sinusoidally varying voltage is obtained at the output of the converter. With the need of sinusoidally varying voltage source in the present day technology, buck DC-AC converters are widely used as very efficient voltage sources to portable devices like power amplifiers. They are used for various applications ranging from telecommunication in aerospace systems to electronic gadgets and audio amplifiers.

In a buck DC-AC converter the variable width is generated with the help of a comparator by comparing a sinusoidal voltage source with a DC offset level, against a triangular voltage. As shown in Fig. 1.1, the sinusoidal voltage source acts as the reference voltage. In order to have a good quality signal, the output voltage should be a replica of the AC reference voltage. In other words, the output voltage should follow the modulating signal or the reference voltage. Hence, in this thesis buck DC-AC converter is used for the analysis for both resistive load and impedance load.

1.2 Background

A buck converter is a PWM DC-DC switching converter which converts a high value input voltage to a low level output voltage. A switched-mode DC-DC buck converter is a non-linear system [1]. The non-linearity of the converter exists within the switching network which comprises of MOSFET and diode. Most of these converters can be analyzed using linear circuit theory or different techniques of modeling. Small-signal modeling helps to analyze the circuit completely based on its small-signal parameter. State-space averaging technique is one of the methods of modeling, already addressed by Middlebrook and Ćuk to derive the transfer functions for small-signal characteristics of PWM converters with the help of state-space equations [2]-[5]. Circuit averaging technique is another method of modeling which is again widely used for pulse-width modulated (PWM) converters. It averages the switching waveforms
and provides physical interpretation of the circuit [6]. In [7], the open-loop small signal characteristics of DC-DC buck converter are analyzed with the help of circuit averaging technique taking into account all parasitic resistances. Among the various transfer function of the open-loop buck DC-AC converter, the control-to-output voltage transfer function $T_p$ is of great importance for this thesis. The control-to-output voltage transfer function $T_p$ gives an idea about the gain and the bandwidth of the converter. As the primary objective of using buck DC-AC converter is to attain a good quality sinusoidal signal which is a replica of the reference voltage, it is very essential to know the limitations of the converter in terms of amplitude of the AC
reference voltage and its modulating frequency. Switching regulators being non-linear makes it difficult to design a stable feedback loop [24],[25]. Due to the small-signal approximations, it does not ensure complete large-signal stability. For large amplitude perturbations, the non-linear terms become dominant in the large-signal analysis. This helps to estimate the limitations of the converter in terms of the amplitude and the frequency of the reference signal. Detailed explanation about the closed-loop transfer functions of the voltage-mode controlled PWM DC-DC buck converter with a proportional controller in continuous conduction mode (CCM) is provided in [8]. Various control methods using different controllers for PWM converters have been discussed in [9],[10]. Non-linear robust control technique in DC-AC inverter with a step change in load, input voltage, and the reference voltage is presented in [11]. Due to the various types of applications of DC-AC power converters, ranging from powers supplies to aerospace power systems to AC machine drives, a good transient response and dynamic performance is required [11]. In order to understand about the dynamics and limitations of the of the in terms of the modulating frequency and amplitude of the modulating signal of the system, large-signal simulations for varying frequency and the amplitude is carried out.

1.3 Motivation

For every electronic engineer, the main priority lies in achieving a good quality signal at the output of the system. The output voltage of the system should follow the input voltage. In other words, the output voltage should be a replica of the input signal. In order to achieve a good signal at the output, it is important for an engineer to know the limitations of the system in terms of the amplitude of the input signal and its frequency. Analyzing the system in terms of varying frequency and varying amplitude becomes a challenge, as the non-linear terms becomes significant at high frequencies and amplitude of the input signal. The linear relation of the two signals will only hold
good up to a certain frequency and for a certain large-amplitude perturbation. Beyond
the cut-off frequency and amplitude of the input signal, the quality of the signal will
deteriorate and the phase difference between the two signals will be significant. In
simple terms, the output voltage will no longer follow the input signal. By performing
the same analysis on a PWM converter, like the buck DC-AC converter, will provide
information regarding the quality of the signal. As buck DC-AC converters are used
as efficient power supplies for many portable devices, it is essential for the system to
generate a good quality output signal which will replicate its input signal. Therefore,
in this thesis buck DC-AC converter with both linear and nonlinear loads are analyzed
for large-signal analysis by frequency response and large-amplitude perturbations.
The closed-loop buck DC-AC converter is also tested for the above analysis. The
quality of the signal is measured with the help of total harmonic distortion (THD).
The system is tested with a poly-phase buck converter to improve the quality of the
signal. Finally, a Class-E RF power amplifier is connected with the closed-loop buck
DC-AC converter and the overall efficiency of the system is measured.

1.4 Thesis Outline

This work is organized as follows: In Chapter 2, the buck DC-DC and buck DC-
AC PWM converters are introduced along with its circuit operation and design.
Transient-time analysis along with a design example of buck DC-AC converter is
provided in this chapter. The whole idea of performing large-signal analysis of buck
DC-AC converter is to have a good quality signal which can be used as a power supply
for linear and nonlinear loads like Class-E RF power amplifier, which is discussed in
Chapter 5. Chapter 3 presents the open-loop and closed-loop large-signal analysis of
the buck DC-AC converter for frequency response test and large-amplitude pertur-
bation for both linear and nonlinear load. The large-signal simulations holds true for
small-signal transfer functions. The control-to-output transfer function $T_p$ is of main
importance in this chapter and all the remaining open-loop transfer functions like, input-to-output voltage, input impedance, and output impedance transfer functions are presented in the appendix for reference. Chapter 3 also shows the limitations of large-signal analysis in terms of amplitude of the AC reference voltage and the frequency of the open-loop and closed-loop converters for resistive load and impedance load. Chapter 3 also provides the measure of total harmonic distortion (THD) for the output signal of the open-loop and closed-loop systems for different levels of modulating voltage. Chapter 4, provides large-signal analysis of interleaved buck DC-AC converter. The same analysis as that of conventional buck is performed for interleaved buck to compare the performance of both systems in terms of bandwidth and total harmonic distortion (THD). Chapter 5 discusses about buck DC-AC converter as an amplitude modulator to a Class-E RF power amplifier. Class-E RF power amplifier acts as a nonlinear load to the buck DC-AC converter, as a result the analysis done in Chapter 3 for impedance load is used here. Amplitude-modulated voltage at the output of the RF power amplifier is obtained based on the design of the Class-E RF power amplifier. Overall efficiency of the system including buck DC-AC converter has been analyzed for varying frequency and amplitude of the modulating signal. Finally, Chapter 6 provides the results and the conclusion of the present work along with suggestions for future work.
2 Buck DC-AC Converter

2.1 Introduction to Buck DC-AC Converter

2.1.1 Pulse-Width Modulator

A buck DC-AC converter is a pulse-width modulated step-down switching converter. The output of the buck DC-AC converter is a sinusoidal voltage for a fixed DC input voltage. The buck DC-AC converter comprises of a pulse-width modulator circuit, which provides variable pulse-width at the gate of the MOSFET. The operation of DC-AC buck converter is similar to the operation of buck DC-DC converter. In buck converters, the source of the MOSFET is not connected to the common ground hence, a two winding transformer is needed. The primary of the transformer is connected to the output of the PWM generator and the ground, whereas the secondary of the transformer are connected to the gate and source of the MOSFET. The pulse-width modulator consists of an ideal op-amp which acts as a comparator. The op-amp compares the sinusoidal modulating signal $v_R$ against a triangular voltage $V_T$. The triangular voltage is of a fixed amplitude and fixed DC offset and is of the same frequency as that of the switching frequency $f_s$. On the other hand the modulating signal has a very low amplitude and the modulating frequency $f_m$ is also small, as compared to the switching frequency. In real world the sinusoidal voltage $v_R$ is an audio signal or a voice with a certain magnitude of voltage and frequency within the audio frequency range. Fig. 2.1 shows the waveforms generated from the pulse-width modulator. The sinusoidal signal when compared against the triangular voltage source, generates a pulse with a variable width. The op-amp used here is an ideal op-amp. The characteristics of ideal op-amp are, it has infinite bandwidth, infinite open-loop gain, infinite input impedance and zero output impedance, and infinite common-mode rejection ratio (CMRR). Op-amps are also operated for closed-loop conditions. Most of the op-amp circuits are designed to use feedback. A feedback is
defined as providing a part of the output signal back to the input of the system. When the output feedback is out of phase with the input, the circuit has negative feedback. Most of the amplifiers undergo negative feedback. When a system is provided with a negative feedback, the input impedance increases, output impedance decreases and the bandwidth of the system increases for a constant gain. Due to closed-loop system, the gain of the op-amp decreases as compared to open-loop systems. In this thesis, for buck DC-AC converter the op-amp used in the circuit is analyzed for both ideal, non-ideal, open-loop and closed-loop conditions.

2.1.2 Operation of Buck DC-AC Converter

The pulse-width modulator provides a pulse of variable width as seen from Fig. 2.1. A buck converter mainly comprises of four components: a power MOSFET used as a controllable switch, a diode $D$ also commonly known as a freewheeling diode, an inductor $L$, and a filter capacitor $C$ as show in Fig 2.2. The inductor $L$, capacitor $C$, and the load together form a low-pass filter $L - C - R_L$. The resistance $R_L$ acts as a DC load. Apart from the power MOSFET, various other components like the bipolar
junction transistor (BJT), insulated gate bipolar transistors (IGBT), or GaN devices may also act as controllable switches depending upon the requirement and rating of the converter. For example, if the voltage rating is high, IGBTs will be the appropriate controllable switch for the converter. The switching action is due to the MOSFET and the diode which goes ON and OFF for the entire switching frequency and is controlled by the pulse-width modulator. The variable duty cycle $D$ of the converter generated from the pulse-width modulator is defined as

$$D = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T} = f_s t_{on}, \tag{2.1}$$

where $t_{on}$ is the time interval when the switch $S$ is ON and $t_{off}$ is the time interval...
when the switch $D_1$ is ON and $S$ is OFF. For a buck converter, the duty cycle $D$ is also the ratio of the output to the input voltage, which means the output voltage is directly proportional to the input voltage and depends on the factor $D$. Buck converter can operate in both continuous-conduction mode (CCM) and discontinuous-conduction mode (DCM) depending upon the nature of the inductor current. If the inductor current does not fall to zero during the switching cycle, then it is said that the converter is in CCM. In CCM, the inductor current will never fall below the critical value, thus never allowing the output current to be zero. Whereas, if the inductor current falls to zero during a part of the period, it is said to be operating in
DCM. In DCM, if the output current is reduced below a critical value, the inductor current would fall to zero for a portion of the switching cycle. In this thesis, the buck DC-AC converter is analyzed in continuous-conduction mode (CCM) where a non-ideal MOSFET and diode are used. The parasitic resistances $r_L$, and $r_c$ of the passive components namely the inductor and capacitor are considered. Fig. 2.3 shows the waveforms of gate-to-source voltage, input voltage, voltage across the diode, and output voltage of the buck DC-AC converter. In Fig. 2.3, the gate-to-source voltage of buck DC-AC converter is a pulse of variable width. The average value of the diode voltage is the DC value of the output voltage. Since the duty cycle is a varying pulse, the output voltage will also have a sinusoidal quantity on top of the DC voltage, as it can be seen in Fig. 2.3. Thus, the output voltage of the DC-AC buck converter consists of both DC and AC quantities. The output voltage equation is

$$v_O = V_I d_T. \quad (2.2)$$

where $v_O$ consists of both AC and DC quantity

$$V_O + v_o = V_I (D + d_T), \quad (2.3)$$

$$V_O + v_o = V_I D + V_I d. \quad (2.4)$$

Equating RHS and LHS

$$V_O = V_I D, \quad (2.5)$$

and

$$v_o = V_I d. \quad (2.6)$$

From the above equation, it can be seen that $d_T$ consists of both DC and AC terms. These terms when multiplies with a fixed input voltage gives DC output voltage.
along with an AC output voltage. Thus, the buck DC-AC converter at consists of a sinusoidally varying output with a DC offset voltage.

2.1.3 Design of Buck DC-AC Converter

The design of DC-ac buck converter is based on the required design parameters, $V_I = 20$ V, $V_{O_{\text{min}}} = 5$ V, $V_{O_{\text{nom}}} = 10$ V, $V_{O_{\text{max}}} = 15$ V, $f_s = 500$ kHz, $P_{O_{\text{max}}} = 10$ W, $V_r/V_O \leq 1\%$, and efficiency $\eta = 90\%$. The load resistance of the converter is

$$R_{L_{\text{min}}} = \frac{(V_{O_{\text{max}}})^2}{P_{O_{\text{max}}}} = 22.5\ \Omega$$

(2.7)

The maximum current of the converter $I_{O_{\text{max}}}$

$$I_{O_{\text{max}}} = \sqrt{\frac{P_{O_{\text{max}}}}{R_{L_{\text{min}}}}} = 0.667\ \text{A}$$

(2.8)

The nominal output power of the converter $P_{O_{\text{nom}}}$ is

$$P_{O_{\text{nom}}} = \frac{(V_{O_{\text{nom}}})^2}{R_{L_{\text{min}}}} = 4.444\ \text{W}$$

(2.9)

The nominal value of the current $I_{O_{\text{nom}}}$ is

$$I_{O_{\text{nom}}} = \sqrt{\frac{P_{O_{\text{nom}}}}{R_{L_{\text{min}}}}} = 0.444\ \text{A}$$

(2.10)

The minimum output power of the converter $P_{O_{\text{min}}}$ is

$$P_{O_{\text{min}}} = \frac{(V_{O_{\text{min}}})^2}{R_{L_{\text{min}}}} = 1.111\ \text{W}$$

(2.11)

The minimum output current $I_{O_{\text{min}}}$ is

$$I_{O_{\text{min}}} = \sqrt{\frac{P_{O_{\text{min}}}}{R_{L_{\text{min}}}}} = 0.2222\ \text{A}$$

(2.12)

The minimum, nominal, and maximum values of the voltage transfer functions are

$$M_{V_{\text{DC}_{\text{min}}}} = \frac{V_{O_{\text{min}}}}{V_I} = 0.25$$

(2.13)
\[ M_{V_{DC\text{nom}}} = \frac{V_{\text{nom}}}{V_i} = 0.50 \] (2.14)

\[ M_{V_{DC\text{max}}} = \frac{V_{\text{max}}}{V_i} = 0.75 \] (2.15)

The minimum, nominal, and maximum values of the duty cycle are

\[ D_{\text{min}} = \frac{M_{V_{DC\text{min}}}}{\eta} = 0.2632 \] (2.16)

\[ D_{\text{nom}} = \frac{M_{V_{DC\text{nom}}}}{\eta} = 0.5263 \] (2.17)

\[ D_{\text{max}} = \frac{M_{V_{DC\text{max}}}}{\eta} = 0.7895 \] (2.18)

The switching \( f_s = 500 \text{ kHz} \), the minimum inductance required for the converter to perform in CCM is

\[ L_{\text{min}} = \frac{V_i D_{\text{min}}(1 - D_{\text{min}})R_L}{2f_s V_{\text{Omin}}} = 17.45 \ \mu\text{H} \] (2.19)

Pick \( L = 22 \ \mu\text{H}/0.1 \ \Omega \)

The maximum ripple current \( i_{L_{\text{max}}} \) through the inductor

\[ \Delta i_{L_{\text{max}}} = \frac{V_i}{4f_s L_{\text{min}}} = 0.5730 \ \text{A} \] (2.20)

The ripple voltage across the filter capacitance \( V_r \)

\[ V_{r\text{nom}} = \frac{V_{\text{nom}}}{100} = 100 \text{ mV} \] (2.21)

The maximum ESR of the filter capacitance \( r_{c_{\text{max}}} \)

\[ r_{c_{\text{max}}} = \frac{V_i \Delta i_{L_{\text{max}}}}{0.1745 \ \Omega} \] (2.22)

The minimum value of the filter capacitance \( C_{\text{min}} \)

\[ C_{\text{min}} = \frac{1}{4f_s r_{c_{\text{max}}}} = 2.865 \ \mu\text{F} \] (2.23)

Pick \( C = 10 \ \mu\text{F}/25 \text{ V}/0.15 \ \Omega \)
2.2 Small-signal Transfer Function of the Open-Loop Buck DC-AC Converter

Power stage of any PWM converter is highly non-linear because of the presence of at least one MOSFET and a diode. Due to this non-linearity present in the system, the small-signal model of the system is obtained after linearization of the large-signal model. The following sections will discuss about the assumptions of small-signal modeling, modeling of the switching network, and the control-to-output voltage transfer function $T_p$. Small-signal analysis has been conducted assuming the transfer functions for buck DC-DC converter holds good for buck DC-AC converter also.

2.2.1 Assumptions for Small-Signal Modeling

- The output capacitance of the transistor and the capacitance of the diode is neglected, hence the switching losses are assumed to be zero.
- The on-resistance of the MOSFET $r_{DS}$ is linear and the OFF resistance is infinite.
- The transformer leakage inductance, magnetizing inductance, stray capacitance and the parallel resistance to the magnetic core are neglected[21].

2.3 Model of Ideal Switching Network

Pulse width modulated converters consists of both linear and non-linear part. The semiconductor devices such as MOSFET(s) and diode(s) are the non-linear discrete components and the capacitors, inductors along with their equivalent internal resistances comprises of the linear part. The non-linear components are replaced by an averaged circuit model. The averaged model can be linearized to obtain small signal AC model. The linear part of the PWM converter does not require averaging and linearization. Fig. 2.4 and Fig. 2.5 in shows the model of the switching network and the equivalent circuit of the switching network respectively [23].
2.3.1 Open-Loop Control-to-Output Voltage Transfer Function $T_p$

A PWM buck converter is shown in Fig. 2.6 and its corresponding small-signal low-frequency model along with the parasitic components is depicted in Fig. 2.7. This model is achieved by replacing the switching network of the buck converter with the low-frequency small-signal model shown in Fig. 2.7. In this chapter, the main focus will be on the control-to-output voltage transfer function $T_p$. Small-signal transfer $T_p$ function will be verified through large-signal simulations. Fig. 2.8 is used to derive the control-to-output voltage transfer function. The ac input voltage $v_i$ is reduced
Figure 2.6: Circuit of the PWM buck converter.

Figure 2.7: Small-signal model of the open-loop PWM buck converter for CCM.

Figure 2.8: Small-signal model of the PWM buck converter for derivation of control-to-output transfer function $T_p$.

to zero. The model is a single-input-single-output system. The $d$ in Fig. 2.8 is a small-signal quantity.
The output voltage equation using the voltage divider principle is given as

\[ v_o(s) = V_i d(s) \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \]  \hspace{1cm} (2.24)

where

\[ Z_1(s) = r + sL, \]  \hspace{1cm} (2.25)

\[ Z_2(s) = \frac{R_L(r_c + \frac{1}{sC})}{R_L + r_c + \frac{1}{sC}}, \]  \hspace{1cm} (2.26)

\[ Z_2(s) = \frac{R_L(sCr_c + 1)}{sR_L C + sr_c C + 1}. \]  \hspace{1cm} (2.27)

Reducing \( v_i = i_o = 0 \), the control to output transfer function is given as

\[ T_p(s) = \frac{v_o(s)}{d} = V_i \frac{Z_2(s)}{Z_1(s) + Z_2(s)}, \]  \hspace{1cm} (2.28)

\[ T_p(s) = V_i \frac{\frac{R_L(r_c + \frac{1}{sC})}{R_L + r_c + \frac{1}{sC}}}{r + sL + \frac{R_L(r_c + \frac{1}{sC})}{R_L + r_c + \frac{1}{sC}}} \]  \hspace{1cm} (2.29)

where \( r \) is the equivalent parasitic resistances and \( r_c \) is the series equivalent resistance if the capacitance.

\[ T_p(s) = \frac{V_i R_L r_c}{L(R_L + r_c)} \frac{(s + \frac{1}{sC_c})}{s^2 + s \frac{C(R_L r_c + R_L r_c + r_c + L)}{LC(R_L + r_c)} + \frac{R_L + r_c}{LC(R_L + r_c)}} \]  \hspace{1cm} (2.30)

\[ T_p(s) = \frac{V_i R_L r_c}{L(R_L + r_c)} \frac{s - z}{(s - p_1)(s - p_2)}, \]  \hspace{1cm} (2.31)

\[ T_p(s) = T_{p0} \frac{s + \omega_o}{s^2 + 2\xi\omega_o s + \omega_o^2}, \]  \hspace{1cm} (2.32)

where

\[ T_{p0} = \frac{V_i R_L r_c}{L(R_L + r_c)}. \]  \hspace{1cm} (2.33)

The angular frequency also called the undamped frequency is given by

\[ \omega_o = \sqrt{\frac{R_L + r}{LC(R_L + r_c)}}, \]  \hspace{1cm} (2.34)
The magnitude $|T_p|$ and phase shift $\phi_{T_p}$ of the control-to-output transfer function $T_p$ is shown with the help of Bode plots in Figs. 2.9 and 2.10 respectively, for $V_{I_{\text{nom}}} = 20$ V, $D_{\text{nom}} = 0.5$, $L = 40$ $\mu$H, $C_{\text{min}} = 10$ $\mu$F, $R_{L_{\text{min}}} = 22.5$ $\Omega$, $r = 0.1270$ $\Omega$, $r_{c_{\text{max}}} = 0.1745$ $\Omega$. The DC gain and low-frequency gain $|T_p| = 18$ dB and the bandwidth of $T_p$ is 12 kHz. For the remaining open-loop small-signal transfer functions like, input-
Figure 2.10: Variation in phase of control-to-output transfer function $T_p$ for the buck converter.

to-output voltage transfer function $M_v$, input impedance transfer function $Z_i$, and the output impedance transfer function $Z_o$ has been discussed in detail in appendix.

### 2.3.2 Importance of Control-to-Output Voltage Transfer Function $T_p$

The control-to-output voltage transfer function $T_p$ of buck DC-AC converter is used for this thesis. The control-to-output transfer function gives an insight about the bandwidth of the converter. As the buck converter model shown in Fig. 2.8 is a low-pass filter, the $T_p$ transfer function also behaves as a low pass filter as shown in the bode plots in Figs. 2.9 and 2.10. Later in this thesis, both open-loop and closed-loop buck DC-AC converters are analyzed for large-amplitude perturbations in the modulating signal and varying the modulating frequency over a wide range. In other words, large-signal analysis of buck DC-AC converter is performed to de-
termine the limitations the system in terms of amplitude of the modulating signal and modulation frequency. The bandwidth determined from the open-loop small-signal \( T_p \) transfer function is less compared to the closed-loop \( T_p \) transfer function. On the other hand the gain of the open-loop system is higher as compared to the closed-loop system. With these information obtained from the small-signal control-to-output voltage transfer function, large-signal analysis is carried out, predicting that the large-signal operation holds true for small-signal transfer function. With the help of the \( T_p \) transfer function the linearity of the system can be analyzed. Another aspect which is analyzed in this thesis is the quality of the output signal from the buck DC-AC converter for both open-loop and closed-loop systems. The linearity of the system along with a good quality of the signal can be achieved if the output voltage of the buck DC-AC converter follows its modulating signal. This relationship between the output voltage and the modulating signal is observed using the \( T_p \) transfer function.

2.4 Closed-Loop Buck DC-AC Converter

Closed-loop buck DC-AC converter has a feedback network from the output of the converter to the input of the buck converter through a compensator and a modulator. In this thesis, voltage-mode control of buck DC-AC converter is implemented. Fig. 2.11 shows the block diagram of the closed-loop small-signal model of the buck DC-AC converter along with the stages like the control-to-output voltage transfer function of the power stage of the buck converter \( T_p \), input-to-output voltage transfer function \( M_v \), the pulse-width-modulator transfer function \( T_m \), the controller transfer function \( T_c \), and the feedback network denoted by \( \beta \). It is a two-input and a single-output system and are driven by two independent voltage sources \( v_i \) and \( v_r \). Out of all these transfer functions, the closed-loop control-to-output voltage transfer function \( T_{pcl} \) is considered for further analysis. Fig. 2.12 shows the circuit for voltage-mode
controlled of buck DC-AC converter. It is a single-loop control circuit. The control circuit is combination of series-shunt negative feedback topology. The voltage derived from the reference voltage $v_R$ (modulating signal) and the feedback voltage $v_F$ controls the duty cycle. From Fig. 2.12 it can be seen that the reference voltage $v_R$ is an ac source with a DC offset. This reference voltage is given as input to the proportional-integral controller. The output voltage from the controller $v_C$ is fed as an input to the comparator and is compared against a triangular voltage. The output obtained from the comparator is a pulse with a variable duty cycle, details of which was explained earlier.

2.4.1 Proportional-Integral (P-I) Controller

A control circuit is required in any pulse-width modulated converters for the following reasons:

- To reduce the steady-state error.

- To reduce the sensitivity of the closed-loop gain over a wide-frequency range.

- To achieve a fast transient response for sudden changes in the load resistance $R_L$ and input voltage $V_I$. 

Figure 2.11: Block diagram of closed-loop voltage mode controlled buck converter.
To satisfy the conditions of relative stability, i.e. to ensure a sufficient gain margin $GM$ of the order of $-12$ dB and phase margin $PM$ of the order of $45^\circ - 60^\circ$ [1].

The Proportional-Integral (PI) controller consists of both a proportional term and an integral term. The combination of both proportional and integral terms help to increase the speed of the converter and also to eliminate the steady-state error.

The expression of proportional control is

$$u(t) = K_P e(t)$$

In proportional control, the time constant decreases hence the time response improves. There is always a steady-state offset the desired response and the output response. The steady-state offset can be reduced by increasing the proportional gain, but this may also lead to oscillations for higher order systems thus making the system
unstable[22]. The proportional gain is termed as $K_p$ and is inversely proportional to the gain.

The definition of integral control is

$$u(t) = K_I \int e(t)dt$$

(2.39)

In integral control the major advantage is, it reduces the steady state error to zero. On the other hand it reduces the response of the system i.e. the speed of the response is slow and the system becomes oscillatory in nature[22]. Thus, proper design is required to avoid instability of the system.

In Proportional-Integral (PI) controller, it is the combination of both proportional and integral terms. Hence, the complete equation is

$$u(t) = K_P e(t) + K_I \int e(t)dt$$

(2.40)

Combining proportional and integral control together it is evident that the P-I control provides a dual advantage of fast response due to the action of the proportional control and zero steady state error due to integral control. Apart from these there are few limitations associated with P-I controller. Being a linear controller, it is not effective for nonlinear systems for the entire operating point. The controller will respond to all changes in the linear region of the system. Moreover, as the operating point shifts to the nonlinear region, the controller response will be slow and oscillations would into picture [26].

The circuit diagram of the P-I controller is shown in Fig. 2.13. The reference voltage of the controller is a sinusoidal source with a DC offset. The negative feedback of the op-amp consists of a feedback resistor $R_f$ and a capacitor $C_f$.

By Kirchoff’s current law, the current entering the node is equal to current leaving the node. Therefore,

$$i_f = i_i$$

(2.41)
Figure 2.13: Circuit of proportional-integral controller

\[ \frac{v_r}{R_1} = \frac{v_C - v_R}{R_f + \frac{1}{sC}} \]  \hspace{1cm} (2.42)

\[ v_C R_1 - v_R R_1 = v_R R_f + \frac{1}{sC} \]  \hspace{1cm} (2.43)

\[ v_C R_1 = v_R (R_f + \frac{1}{sC} + v_R R_1) \]  \hspace{1cm} (2.44)

\[ v_C R_1 = v_R (R_f + R_1 + \frac{1}{sC}) \]  \hspace{1cm} (2.45)

\[ \frac{v_C}{v_R} = \frac{(R_f + R_1 + \frac{1}{sC})}{R_1} \]  \hspace{1cm} (2.46)

Hence, the voltage gain of the P-I controller is denoted by \( T_c \)

\[ T_c = \frac{v_C}{v_R} = \frac{R_f}{R_1} + 1 + \frac{1}{sR_1C} \]  \hspace{1cm} (2.47)

\[ T_c = \frac{v_C}{v_R} = K_P + \frac{K_I}{s} \]  \hspace{1cm} (2.48)
where,

\[ K_P = 1 + \frac{R_f}{R_1} \]  \hspace{1cm} (2.49)

\[ K_I = \frac{1}{R_1 C} \]  \hspace{1cm} (2.50)

In the following Chapters, large-signal operations based on varying amplitude of the modulating signal and varying the modulating frequency has been performed. Small-signal analysis will be considered to hold true for large-signal simulations.
3 Large-Signal Analysis of Open-Loop and Closed-Loop Buck DC-AC Converter

Large-signal analysis for any system is done based on its large amplitude and frequency variation of the modulating signal. Large-signal analysis gives an idea about the dynamics of the system. It also provides an information regarding the limitations of the system based on the quality of the signal, amplitude, and frequency of the modulating signal. A low-frequency perturbation is considered for analyzing the operation of PWM converter. Under low-frequency perturbation, every switching waveform of the PWM converter will have:

- DC quantity
- Low frequency quantity of the frequency \( f = \omega / (2\pi) \)

Control signals of the closed-loop PWM converters generally consists of DC and low-frequency quantities. As a result for studying the control aspects of PWM converters, only the DC quantities and the low-frequency quantities are of main concern [1]. Therefore, low-frequency quantities are used to characterize the dynamics of PWM system.

3.1 Large-Signal Equations and Linearization to Small-Signal Linear Model

Normally, low-frequency quantities are sinusoidal in nature. They are used for study of frequency response of both open-loop and closed-loop system. Fig. 3.1 shows the waveform of reference voltage \( v_R \) under a low-frequency sinusoidal perturbation. The ac low-frequency sinusoidal quantity of the input voltage is given as

\[
v_i = V_{im} \sin \omega t
\] (3.1)
The low-frequency sinusoidal quantity is superimposed on the DC quantity of the input voltage $V_I$, resulting in the large-signal input voltage.

$$v_I = V_I + v_i = V_I + V_{im} \sin \omega t$$  \hspace{1cm} (3.2)

According to Shannon's sampling theorem [1], the frequency $f_m$ of perturbation signal is much less than or half of the switching frequency $f_s$, which is known as the Nyquist frequency. Hence, the low-frequency models of PWM converters are valid only in the range $0 \leq f_m s/2$. Once the DC variables, such as the DC input voltage $V_I$ or the duty cycle $D$, are perturbed in a PWM converter circuit at low-frequency $f \leq f_s/2$, all the other variables will vary around their corresponding DC levels at a low frequency $f$. As a result, the averaged voltages, duty cycle, and currents will consists of the DC quantity and low-frequency ac quantity as follows:

$$v_{SD} = V_{SD} + v_{sd},$$  \hspace{1cm} (3.3)

$$i_L = I_L + i_l,$$  \hspace{1cm} (3.4)
Fig. 3.2 shows the nonlinear large-signal model. Large-signal model can be linearized by expanding the large-signal equations and neglecting the higher-order terms. A linear small-signal model can be achieved by assuming the magnitude of ac low-frequency quantities much lower than the DC quantities. Hence, the non-linear equations obtained are,

\[ I_S + i_s = (D + d)(I_L + i_l) = DI_L + Di_l + I_Ld + i_ld. \]  \hspace{1cm} (3.6)

\[ V_{LD} + v_{ld} = (D + d)(V_{SD} + v_{sd}) = DV_{SD} + Dv_{sd} + V_{SD}d + v_{sd}d. \]  \hspace{1cm} (3.7)

Also, the equivalent parasitic resistances is given by,

\[ r = Dr_{DS} + (1 - D)R_F + r_L. \]  \hspace{1cm} (3.8)
Fig. 3.3 shows the large-signal model, which is non-linear. To linearize this nonlinear model, the assumptions are as follows

\[ i_{ld} \ll Di_{l}, \]  

(3.9)
\[ i_{Ld} \ll I_L d, \quad (3.10) \]

\[ v_{sd} d \ll D v_{sd}, \quad (3.11) \]

\[ v_{sd} d \ll V_{SD} d. \quad (3.12) \]

Fig. 3.4 shows the linear circuit model which consists of both DC and small-signal ac quantities. Since the model is linear, the model can be split into a small-signal model and a DC model as shown in Fig. 3.5

### 3.2 Open-Loop Large-Signal Operation

Large-signal models are derived to describe the nonlinear devices in terms of their nonlinear equations. The model is applied in cases where a PWM converter is subjected to large-amplitude perturbations, whose modulating frequency is considered to be smaller than the switching frequency \( f_s \) [24]. Due to the large perturbations the non-linear terms becomes significant and the system tends to attain instability [25]. Large-signal simulations will be compared against the control-to-output voltage transfer function \( T_p \). There are two ways by which large-signal operations can be performed. There are as follows

- By varying the modulating frequency \( f_m \) over a large range keeping the amplitude of the modulating voltage \( v_r \) constant,

- By varying the amplitude of the modulating voltage \( v_r \) and keeping the modulating frequency \( f_m \) constant.

As a result, large-signal operation for buck DC-AC circuit is performed in SABER circuit simulator. A Buck DC-AC circuit is constructed based on the design example
in Chapter 2. A frequency response test is performed on the circuit by varying the modulating frequency $f_m$ over a large range, from 100 Hz to 50 kHz keeping the modulating voltage $v_R$ constant with an amplitude of 1.25 V and an DC offset level of 2.5 V. Fig 3.6 and Fig. 3.7 shows the magnitude and phase of the open-loop small-signal control-to-output voltage transfer function $T_p$ for resistive load with a DC gain of 18 dB and a bandwidth of 12kHz which is depicted by the solid line. This means, the output voltage $v_O$ follows the modulating signal $v_R$ till 12 kHz with nearly zero phase difference. After 12 kHz the signal starts attenuating and the phase difference between the output voltage $v_O$ and the modulating voltage $v_R$ becomes significant. As
shown in Fig. 3.9, the magnitude of the control-to-output voltage transfer function $|T_p|$ has a similar characteristics like the low-pass filter. As a result the output voltage will always have a linear relationship and will replicate the modulating voltage $v_R$ till 12 kHz which is the bandwidth of the open-loop PWM buck DC-AC converter. The large-signal simulations are performed for both resistive, and inductive load. There reason behind considering inductive load along with the resistive load is, buck DC-AC converters act as efficient voltage regulators and power supplies for real time applications, where the load to the buck converter is not always a linear resistive load but an impedance load, generally inductive in nature. Buck DC-AC provides power supply for applications in telecommunication systems, aerospace and motor drives. Thus, large-signal analysis is performed for different loads keeping in mind the real-
time applications of buck DC-AC converter. The load resistance at the output of the buck DC-AC converter is 22.5 Ω, hence for inductive load the simulations has been carried out keeping the overall impedance of the system $Z=R_L+X_L=22.5$ Ω, where, $R_L=10$ Ω and $L=4 \mu$H. Fig. 3.6 and Fig. 3.7 show the comparison between the magnitude and phase of the small-signal control-to-output voltage $T_p$ transfer function and large-signal simulations performed on buck DC-AC converter for a resistive load in SABER circuit simulator. From the figure it can be seen that, large-signal simulations follows the same trend as the small-signal transfer function. The difference in the DC gain between the theoretical and simulated results is due to the modulator transfer function which is not considered while deriving the theoretical $T_p$ transfer function. Whereas, in large-signal analysis the modulator is considered during the simulations.
Figure 3.8: Comparison of magnitude of open-loop small-signal control-to-output voltage transfer function $T_p$ and large-signal simulations for inductive load for varying modulating frequency $f_m$.

The phase plot in large-signal simulations has a similar response as that of the small-signal transfer function as shown in Fig. 3.7. Both the theoretical and the simulated plots remain at almost zero phase difference till they reach the bandwidth of the system. Fig. 3.8 and Fig. 3.9 shows the comparison of small-signal transfer function and large-signal simulations for inductive load. The inductive load behaves in a similar way as that of the resistive load. The trend of the graph and the gain and the bandwidth of the system remains the same.

Large-signal simulations has also been performed for large-amplitude perturbations of the modulating voltage $v_r$. The amplitude of the modulating signal has been varied over a large range from 0.5 V to 4.0 V at a step of 0.5 V. Fig 3.10 and Fig. 3.11 shows the plots for large-amplitude variations for resistive and inductive load respec-
Figure 3.9: Comparison of phase of open-loop small-signal control-to-output voltage transfer function $T_p$ and large-signal simulations for inductive load for varying modulating frequency $f_m$.

The corner frequency of the system as mentioned above is 12 kHz. The simulations for large-amplitude perturbation has been carried for $0.1 f_m$, $f_m=f_o$, and $10 f_m$. From the plot in Fig. 3.10 and Fig. 3.11, it can be seen that, for $f_m=0.1 f_m$, and $f_m=f_o$ a linear relationship is maintained throughout the large-amplitude variation for both resistive and inductive load. Hence, the linear relationship only holds true until the system reaches its bandwidth. The signal starts to attenuate beyond 12 kHz and the non-linearity between the gain $A_v$ and the reference voltage $v_R$ becomes significant.
3.2.1 Quality of the Output Signal for Open-Loop Buck DC-AC Converter

With the advancement in technology, present day engineers look for optimization of signal quality. In order to improve the quality it is very important to know what is the quality of the signal. For applications such as audio amplifiers, and microphone it is essential for a user to know or evaluate the quality and amount of noise level present in the signal. When a system is considered to be ideal, the output signal follows the input or the reference signal and a linear relationship hold between them. When a signal passes through a non-ideal system or device, non-linearity between the output and the reference signal becomes significant and additional noises gets added at the harmonics of the fundamental frequency. When an audio signal such as voice is passed through an amplifier, it gets sampled and is given in the form of...
a voltage. The amplifier itself has noise components to incorporate additional noise to the main signal. This results in deteriorating the quality of the output signal in form of distortions. There are various methods by which one can analyze the quality of the signal, like total harmonic distortion (THD), signal-to-noise ratio (SNR), and signal-to-noise-and-distortion (SINAD). In this thesis, quality of the signal has been analyzed based on the total harmonic distortion (THD). Fig. 3.12 shows the variation of total harmonic distortion w.r.t varying amplitude of the modulating signal for different modulating frequencies such as $f_m = 1$ kHz, $f_m = 5$ kHz, $f_m = 10$ kHz, and $f_m = 30$ kHz. With the increase in $v$ the THD of the output signal increases. For higher frequency such as $f_m = 30$ kHz, the output signal has a very low peak-to-peak voltage. Hence, the THD content in the signal is less compared to lower frequencies, but for all the frequencies THD follows the same trend w.r.t. $v_r$. 

Figure 3.11: Gain $A_v$ verses amplitude of the modulating signal $v_r$ for open-loop buck DC-AC converter for inductive load.
3.2.2 Total Harmonic Distortion (THD)

Total harmonic distortion (THD) of a signal is a measurement of the harmonic distortion present in it and is defined as the ratio of sum of the powers of all the harmonic components to the power of the fundamental frequency [28]. In terms of voltage, it is also defined as the ratio of the root mean square (RMS) amplitude of a set of higher harmonics to the RMS amplitude of the fundamental frequency. The formulas for THD in terms of power and voltage are as follows,

In terms of power,

\[
THD(\%) = 100 \sqrt{\frac{P_2 + P_3 + P_4 + \ldots + P_n}{P_1}}. \tag{3.13}
\]

where \( P_n \) is in Watts.
Figure 3.13: Quality of output voltage $v_O$ of the open-loop buck DC-AC converter to analyze the total harmonic distortion for $v_r = 0.5$ V.

In terms of voltage,

$$THD(\%) = 100\sqrt{\frac{V_2^2 + V_3^2 + V_4^2 + \ldots + V_n^2}{V_1}}.$$  \hfill (3.14)

where $V_n$ is the RMS voltage.

In this thesis, total harmonic distortion (THD) of the output signal is analyzed with the help of SABER circuit simulator. Fourier analysis is performed in SABER with a fundamental frequency equal to the switching frequency $f_s = 500$ kHz and number of harmonics $n=10$. Fig. 3.13, Fig. 3.14, and Fig. 3.15 shows the spectrum of the output voltage $v_O$ of open-loop buck DC-AC converter along with the continuous waveform of the output voltage $v_O$ for amplitude of the modulating signal $v_r = 0.5$ V, 1.25 V, and 3.0 V. With the increase in amplitude of the modulating signal the total harmonic distortion (THD) decreases and beyond 3.0 V the THD remains constant.
Figure 3.14: Quality of output voltage $v_O$ of the open-loop buck DC-AC converter to analyze the total harmonic distortion for $v_r = 1.25$ V.

THD for open-loop buck DC-AC converter for $v_r = 0.5$ V is 1.1 %, for $v_r = 1.25$ V is 1.83 %, and for $v_r = 2.5$ V is 12.88 %.

### 3.3 Closed-Loop Large-Signal Operation

With a feedback network from the buck DC-AC converter, connecting through the compensator and the pulse-width modulator, one can arrive at the closed-loop buck converter. The closed-loop buck converter has been analyzed with a proportional-integral (P-I) controller, where the reference voltage of the controller acts as a modulating signal. The reference voltage is a sinusoidal signal with DC offset. The modulating frequency is between the range of 1 kHz to 5 kHz, which is very less compared to the switching frequency. In order to perform the frequency response test and the large-amplitude perturbation, the modulating frequency $f_m$ and the the
amplitude of the modulating signal of the P-I controller is varied over a large range. The simulated results are compared against the closed-loop control-to-output voltage transfer function \( T_{pel} \). For a good quality signal, the output voltage has to follow the modulating signal i.e. they should be in phase with each other. The linearity between the output voltage and the modulating signal will exist till the corner frequency of the closed-loop control-to-output voltage transfer function \( T_{pel} \). Beyond the corner frequency, the signal will start attenuating and a nonlinear relationship between the output voltage and the modulating signal will start developing. Since, P-I controller is a linear controller, it will respond to all the disturbances in the load and input signal as long the operating point is in the linear region. As soon as the non-linearity starts developing in the circuit beyond the corner-frequency, the controller stops responding to the changes. Hence, the output voltage no longer is a replica of the modulating
signal. The closed-loop buck DC-AC converter is subjected to a step change in load $R_L$ which varies from 20% to 100% of its value and the analysis is performed based on the disturbance provided to the load of the converter.

Fig. 3.16 and Fig. 3.17 shows the comparison of magnitude and phase plot of small-signal closed-loop control-to-output voltage transfer function $T_{pcl}$ and large-signal simulations of frequency response test for resistive load. The modulating frequency of the controller has been varied from 100 Hz to 50 kHz. Its is observed that, the output voltage and the modulating signal remains in phase and maintains a linear relationship till the bandwidth of the system which is 35 kHz. Beyond 35 kHz the output voltage starts to attenuate and the phase difference between the modulating signal and the output voltage becomes significant. Using the closed-loop buck DC-
AC converter, has increased the bandwidth of the converter to 35 kHz unlike the open-loop system which has a bandwidth of 12 kHz. On the other hand the gain of the closed-loop system is less as compared to the open-loop system.

Fig 3.18 show the plots of gain $A_v$ verses amplitude $v_r$ of the modulating signal. Large-amplitude perturbations has been performed on the closed-loop buck DC-AC converter for three different modulating frequencies, $0.1f_m$, $f_m=f_o$, and $10f_m$. It is observed that for lower modulating frequencies $0.1f_m=3.5$ kHz and corner frequency $f_m=f_o=35$ kHz, the linearity between the output voltage and the modulating signal holds good. For higher frequency $10f_m = 350$ kHz, the non-linearity in the system becomes dominant as it can been see in Fig. 3.19. For $f_m=3.5$ kHz and $f_m=35$ kHz it is almost a linear graph, where as $f_m=350$ kHz, the gain $A_v$ of the system shows a
nonlinear trend with respect to the amplitude of the modulating signal \( v_r \).

3.3.1 Quality of the Output Signal for Closed-Loop Buck DC-AC Converter

For a closed-loop buck DC-AC converter Fourier analysis is performed in SABER circuit simulator to analyze the quality of the signal. Total harmonic distortion (THD) was calculated using the measurement tool in SABER. The specifications for fourier analysis in case of the closed-loop system was the same as that of the open-loop buck DC-AC converter. The total number of harmonics \( n = 10 \) is considered and the fundamental frequency is equal to the switching frequency \( f_s = 500 \text{ kHz} \). The end time of the simulation is specified as 5 ms. Fig. 3.19 shows the total harmonic distortion (THD) verses amplitude of the modulating signal \( v_r \) for different frequencies. Modulating frequency for \( f_m = 1 \text{ kHz}, f_m = 5 \text{ kHz}, f_m = 10 \text{ kHz}, \) and
$f_m = 30$ kHz are selected for the analysis. As the amplitude of modulating signal increases, the THD also increases. The nature and trend of the plot is similar to the open-loop system.

The total harmonic distortion is less in closed-loop buck DC-AC converter as compared to the open-loop buck DC-AC converter. Fig. 3.20, Fig. 3.21, and Fig. 3.22 shows the spectrum of the output voltage of the closed-loop buck DC-AC converter in accordance with the continuous output waveform for different levels of voltage. For amplitude of the modulating signal $v_r = 0.5$ V, THD is 1.3 %, for $v_r = 0.75$ V, THD is 1.7%, and $v_r = 1.0$ V, THD 2.12 %. The quality of the signal in open-loop system is considered to be better than closed-loop buck DC-AC converter.
Figure 3.20: Spectrum of output voltage $v_O$ of the closed-loop buck DC-AC converter in accordance with the continuous output voltage waveform $v_O$ to analyze the total harmonic distortion for $v_r = 0.5$ V.
Figure 3.21: Spectrum of output voltage $v_O$ of the closed-loop buck DC-AC converter in accordance with the continuous output voltage waveform $v_O$ to analyze the total harmonic distortion for $v_r = 0.75$ V.
Figure 3.22: Spectrum of output voltage $v_O$ of the closed-loop buck DC-AC converter in accordance with the continuous output voltage waveform $v_O$ to analyze the total harmonic distortion for $v_r = 1.0$ V.
4 Large-Signal Analysis of Interleaved Buck DC-AC Converter

Interleaved buck DC-AC converter, also known as multiphase buck converter DC-AC is a circuit topology where the conventional buck converters are placed in parallel between the input side and the load at the output. Each of the $n$ phases are turned on at equally time intervals with certain delay over the switching period. The primary advantage of this type of converter is, it helps in ripple cancellation of the output current as a result the use of small inductors improves the transient response and minimize the output capacitance [26]. It also responds to load changes as fast as $n^{th}$ times than the conventional buck, without the increase in switching losses. Another advantage of multiphase buck converter is that the load current is split among the $n$ phases. This load splitting allows the heat losses on each of the switches to be spread across a larger area, as a result the conduction losses are decreased in multiphase buck converters. Among all the advantages of multiphase buck converter, a major limitations that exists is, ripple cancellation of the inductor current doesn’t takes place at any duty ratio. For effective cancellation of the switching ripple at the output, the multiphase buck converter has to operate at a constant duty ratio of 0.5 [19],[26].

In this thesis, analysis on multiphase buck DC-AC converter has been carried out considering only two phases.

Fig. 4.1 shows the circuit of two-phase buck DC-AC converter with the pulse-width modulator circuit connected to the gate-to-source of the MOSFET. The circuit operation is similar to conventional buck DC-AC converter. The MOSFETS $S_1$ and $S_2$ are switched ON and OFF with two separate gate drive signal which are $180^\circ$ out of phase. The pulse-width modulator is an ideal op-amp which acts as a comparator. The pulse-width modulator compares the triangular voltage with the sinusoidal modulating signal to generate a pulse of varying width. In case of two-phase buck
converter two pulse-width modulators are used separately for the MOSFET(s). In time interval $0 < t \leq DT$, the MOSFET $S_1$ is ON and the diode $D_1$ is OFF. From the circuit it can be observed, as the diode $D_1$ is OFF, the switch $S_1$ is in series with the inductor. Hence, the input current flowing through the switch, flows through the inductor and therefore, the inductor stores the energy. In this interval, the diode is reverse biased since the voltage across the diode $v_D$ is equal to $-V_I$. For time interval $DT < t \leq T$ the MOSFET $S_1$ is OFF and the diode $D_1$ is ON. As the inductor current has to be continuous, diode $D_1$ is forward biased and provides a path for the load current. The same principle of operation holds good for the second phase consisting of $S_2$, $D_2$, and $L_2$. Fig. 4.2 provides the key switching waveforms pertaining to the operation of the two-phase buck converter in steady-state. As shown in Fig. 4.2, due to the switching pattern the gate-to-source voltages of $S_1$ and $S_2$ are out of phase. Similarly, the inductor currents $L_1$ and $L_2$ have a phase difference of $180^\circ$.

### 4.1 Operation of Interleaved Buck DC-AC Converter

The filter capacitor $C_f$ is the common capacitor for both the phases, as a result the ac ripple current flowing through the capacitor is the sum of the inductor currents. Due to the phase difference of $180^\circ$, the effective magnitude of the ac ripple current flowing through the filter capacitor $C_f$ is nearly zero. For effective ripple cancellation of the switching ripple at the output of the buck converter, the converter has to operate at the duty cycle of 0.5. Fig. 4.3, Fig. 4.4 and Fig 4.5 shows the simulated switching waveforms. The gate-to-source voltage $V_{GS}$ for MOSFET(s) $S_1$ and $S_2$ are shown in Fig. 4.3. Fig. 4.4 shows the ripple cancellation of the output currents which are out of phase, as a result the current through the filter capacitance is almost zero. The output voltage, input power, and the output power is shown in Fig. 4.5. The input power $P_I$ is 5.2 W and the output power $P_O$ is 4.9 W, hence the efficiency of the system is 94 %. 
4.2 Large-Signal Analysis of Interleaved Buck DC-AC Converter

As mentioned in the earlier section that, interleaved buck converter has to operate at a duty ratio of 0.5 in order to have an effective ripple cancellation. Hence, performing large-amplitude perturbation on the two-phase buck converter will lead to a change in the nominal value of the duty cycle. The purpose is to maintain a nominal value of \( D = 0.5 \) above which the the pulse width will vary from its minimum duty ratio to maximum duty ratio. In order to avoid the change in the duty ratio only frequency response test will be carried out for the two-phase buck DC-AC converter, where the modulating frequency of both the pulse-width modulators will be varied over a large range. The modulating frequency \( f_m \) is varied from 100 Hz to 40 kHz.

Fig. 4.6 and Fig. 4.7 shows the magnitude and phase of the large-signal simula-
Figure 4.2: Switching waveforms of the two-phase buck converter.

ations for the frequency response test for a resistive load. The gain of the interleaved buck DC-AC converter is shown in Fig. 4.8. There is a linear relationship between the gain of the converter and the modulating frequency $f_m$ till 10 kHz which is the
corner frequency of the interleaved buck DC-AC converter. As a result this linear relationship in the phase plot is also visible as shown in Fig. 4.9. After 10 kHz, the signal starts attenuating and the non-linearity in the system become significant. Hence, a large phase difference occurs between the voltage gain and the modulating frequency $f_m$.

Large-signal simulations are also performed for inductive load keeping the overall impedance $Z$ of the system equal to the resistive load $R_L$, where $Z=R_L+jX_L=22.5\ \Omega$. The value of $R$ is chosen as 10 Ω and the impedance reactance $X_L$ is calculated based on $R_L$ and $Z$. The inductor $L$ value calculated from the equation $Z=R_L+jX_L$ is $4\ \mu\text{H}$. The analysis is performed in SABER circuit simulator, where an inductor $L$ is connected in series with the load resistance $R_L$. As the overall output impedance of the circuit remains the same as that of the resistive load, it is expected that the nature
Figure 4.4: Inductor currents representing ripple cancellation effect and the current through the filter capacitor of the interleaved buck DC-AC converter.

and behavior of the plot for inductive load will be similar to that of the resistive load. Fig. 4.8 and Fig. 4.9 shows the magnitude and phase of the large-signal simulations respectively for the frequency response test for a inductive load. From the figures it can be observed that the large-signal simulations for the inductive load follow the same linear relationship as that of the resistive load. The corner frequency $f_0$ and the gain of the converter remains the same.

The total harmonic distortion (THD) in Fig. 4.10 for interleaved buck DC-AC converter is 4.5 % for $v_r = 0.5$ and $f_m = 1$ kHz. Whereas, for the same specifications of voltage and frequency of the modulating signal, THD for conventional buck DC-AC converter is 1.83 %. The signal quality is considered to be good in conventional buck DC-AC converter than the interleaved buck DC-AC converter. The efficiency of the interleaved buck DC-AC converter is high and offers a higher bandwidth when
Figure 4.5: Output power $P_O$, input power $P_I$, and output voltage $v_O$ of the interleaved buck DC-AC converter.

compared with conventional buck converter.
Figure 4.6: Plot for voltage gain $A_v$ verses modulating frequency for resistive load.
Figure 4.7: Plot for phase $\phi$ verses modulating frequency for resistive load.
Figure 4.8: Plot for voltage gain $A_v$ verses modulating frequency for inductive load.
Figure 4.9: Plot for voltage gain $A_v$ verses modulating frequency for inductive load.
Figure 4.10: Spectrum of output voltage $v_O$ of the interleaved buck DC-AC converter in accordance with the continuous output voltage waveform $v_O$ to analyze the total harmonic distortion for $v_r = 1.30$ V.
5 Amplitude Modulated Class-E RF Power amplifier

Radio-frequency power amplifier (RFPA) is a key element to build a wireless communication system successfully [12]. In order to minimize interference and attenuation of the signal, transmitters must be linear. Fig. 5.1 shows the block diagram of an RF power amplifier. The block diagram consists of a transistor, which can be a MOSFET or BJT, output network, input network and the choke inductance. In an RF power amplifier a transistor can be operated as (1) dependent current source; and (2) switch. RF power amplifiers can be categorized as linear and non-linear amplifiers. Power amplifiers like Class-A, B, AB, and C are linear power amplifiers where, the transistor acts as a dependent current source, whereas, in non-linear power amplifiers like Class-D, E, DE, and F the transistor acts a switch. In this thesis, class-E power amplifier is used along with a buck DC-AC converter for amplitude modulation of the carrier signal.

5.1 Class-E RF Power Amplifier

Class-E RF power amplifiers also known as Class-E DC-AC inverters offers very high DC-AC conversion efficiency. They are often categorized as: (1) Class-E zero-voltage switching (ZVS) power amplifiers, and (2) Class-E zero-current switching power amplifiers. The transistor in class-E power amplifier operates as a switch. Class-E ZVS power amplifiers are considered to be the most efficient power amplifiers till date [13]-[17]. In this thesis, Class-E ZVS power amplifiers are used, where switching action takes place at zero voltage and there is no overlap between the current and the voltage waveform leading to zero switching losses and hence, high efficiency.
5.1.1 Circuit Description

Fig. 5.2 and Fig 5.3 shows the basic circuit diagram of Class-E power amplifier and its equivalent circuit respectively. The basic circuit of class-E power amplifier consists of a MOSFET operating as a switch along with an anti-parallel diode, the $L-C-R$ series resonant circuit, the shunt capacitance $C_1$, and the choke inductor $L_f$. The MOSFET turns ON and OFF based of the operating frequency of the power amplifier $f = \frac{\omega}{2\pi}$. The shunt capacitance $C_1$ is the equivalent parasitic capacitance which includes output capacitance, choke parasitic capacitance and the stray capacitances generated within the circuit. The choke inductance $L_f$ is assumed to be high enough in order to neglect the ac ripple current from the DC supply current $I_f$. When the switch is ON, the $L-C-R$ resonant circuit gets short-circuited because of the shunt capacitance $C_1$. However, when the switch is OFF, the $L-C-R$ resonant circuit is connected in series. The capacitances $C_1$ and $C$ are in series, hence the equivalent capacitance is

$$C_{eq} = \frac{CC_1}{C + C_1}. \quad (5.1)$$

In case of hard switching, such as in semiconductor devices and PWM power converters, lot of switching losses are involved. There is an abrupt change in the voltage waveform from a high value to nearly zero when the switch in turned ON.
The switching power loss of the transistor is a function of the operating frequency $f_c$ of the power amplifier keeping the input voltage $V_I$ and the capacitance $C$ constant.

The switching power loss is given by

$$P_{sw} = \frac{1}{2} f C V_I^2. \quad (5.2)$$

In Class-E power amplifier transistor operating as a switch turns ON at zero voltage, resulting in zero switching loss and high efficiency. The switching waveforms of the Class-E power amplifiers is shown in Fig. 5.4. In Fig. 5.5, the block diagram consisting of PWM buck DC-AC converter where $V_{supply}$ is the input voltage of the converter, $V_{DD}$ is the output voltage of the converter which is given as an input to the RF power amplifier. In further sections, amplitude modulation of Class-E RF power
amplifier is achieved using buck DC-AC converter. The block diagram gives an idea about amplitude modulation as shown in Fig. 5.5.
5.1.2 Design of Class-E Power Amplifier

The specifications for the design of Class-E power amplifiers are: \( V_I = 10 \) V, \( P_{omax} = 10 \) W, \( f = 2 \) MHz. Assume \( D = 0.5 \)

Full load resistance
\[
R = \frac{8}{\pi^2 + 4 P_0} \quad \text{5.76 } \Omega
\]  
(5.3)

DC resistance of the amplifier
\[
R_{DC} = \frac{\pi^2 + 4}{8} R = 10 \quad \Omega
\]  
(5.4)

Amplitude of the output voltage
\[
V_{Rm} = \frac{4}{\sqrt{\pi^2 + 4}} V_I = 10.74 \quad \text{V}
\]  
(5.5)

Maximum voltage across the switch and the shunt capacitor \( C_1 \)
\[
V_{SM} = 3.562 V_I = 35.62 \quad \text{V}
\]  
(5.6)

DC input current
\[
I_I = \frac{8}{\pi^2 + 4} \frac{V_I}{R} = 1 \quad \text{A}
\]  
(5.7)

Maximum switch current
\[
I_{SM} = \left( \frac{\sqrt{\pi^2 + 4}}{2} + 1 \right) I_I = 2.862 \quad \text{A}
\]  
(5.8)

Amplitude of the current through the resonant circuit
\[
I_m = \frac{\sqrt{\pi^2 + 4}}{2} I_I = 1.862 \quad \text{A}
\]  
(5.9)

Assuming \( Q_L = 7 \)
\[
L = \frac{Q_L R}{\omega} = 3.21 \quad \mu\text{H}
\]  
(5.10)

The shunt capacitance \( C_1 \)
\[
C_1 = \frac{8}{\pi(\pi^2 + 4)\omega R} = 2.53 \quad \text{nF}
\]  
(5.11)
Figure 5.5: Block diagram representing PWM converter and RF Power Amplifier.

\[
C = \frac{1}{\omega R\left[Q_L - \frac{\pi^2 - 4}{16}\right]} = 2.35 \text{ nF}
\]  (5.12)

Choke inductance of the Class-E power amplifier

\[
L_f = 2\left(\frac{\pi^2}{4} + 1\right) \frac{R_f}{f} = 20 \mu\text{H}
\]  (5.13)

Peak voltage across the choke inductor

\[
V_{Lfm} = V_{sm} - V_I = 25.62 \text{ V}
\]  (5.14)

5.1.3 Amplitude Modulation (AM) in RF Power Amplifiers

Amplitude modulation (AM) is a modulation technique, most commonly used for transmitting information via a carrier wave also known as carrier signal. Amplitude modulation depends on the signal strength of the carrier signal in proportion to the information being sent. The instantaneous value of the carrier amplitude changes in accordance with the amplitude \(v_r\) and frequency \(f_m\) variations of the modulating signal. The carrier frequency \(f_c\) remains constant during modulation of the signal, but its amplitude varies in accordance with the modulating signal. The amplitude modulated waveform consists of three frequency components, carrier frequency, switching frequency, and the modulated frequency. An increase in the amplitude of the modulating signal increases the amplitude of the carrier signal. The output voltage waveform
Figure 5.6: Waveforms in Class-E ZVS RF power amplifier with amplitude modulation.

of any power amplifier without modulation is given by

\[ v_O = V_c \cos \omega_c t. \]  \hspace{1cm} (5.15)

where \( \omega_c \) is the angular frequency of the carrier signal and \( V_c \) is the amplitude of the carrier signal. The amplitude of the output voltage of the power amplifier is directly proportional to the input voltage of the Class-E power amplifier i.e. the sinusoidal output voltage with a DC offset voltage of the buck DC-AC converter. With the duty cycle \( D = 0.5 \), the amplitude of the carrier signal is

\[ V_c = \frac{4}{\sqrt{(\pi^2 + 4)}} V_I. \]  \hspace{1cm} (5.16)

The amplitude modulated voltage waveforms are shown in Fig. 5.6. The modulating voltage \( v_m \) is given by

\[ v_m = V_m \cos \omega_m t. \]  \hspace{1cm} (5.17)
where, $V_m$ is the amplitude of the modulated signal and $\omega_m$ is the angular frequency of the modulated signal. The supply voltage of the Class-E power amplifier with amplitude modulation is given by

$$v_I(t) = V_I + v_m(t) = V_I + V_m \cos(\omega_m t). \quad (5.18)$$

The amplitude of the output voltage of the Class-E power amplifier is

$$V_m(t) = \frac{4}{\sqrt{\pi^2 + 4}} [V_I + v_m(t)] = \frac{4}{\sqrt{\pi^2 + 4}} (V_I + V_m \cos \omega_m t).$$

### 5.2 Buck DC-AC Converter as an Amplitude Modulator

Pulse-width modulated (PWM) power converters plays a critical role in high-efficiency power management systems when operated as dynamic power supplies or amplitude modulators [18]. In amplitude modulated (AM) systems such as audio or low-power signal transmitters, stability and quality of the input voltage to these power amplifiers are of important concern [19]. In this thesis, a buck DC-AC converter is used for amplitude modulation in Class-E power amplifier. A lot of initiative has been taken towards different applications of DC-AC buck converter. It is well known that, buck converters have an added advantage over normal voltage regulators because they are more efficient and self-regulating power supplies. Generally, all DC-AC converters are employed using a pulse-width modulation (PWM) switching technique. The PWM switching technique transforms the input DC voltage into a series of pulse waveforms of the frequency same as that of the switching frequency, which is smoothened using a LC filter to obtain a sinusoidal waveform [20]. Unlike, buck DC-DC converters which has a constant width of the pulse waveform, DC-AC converters have variable width, due to which it offers a sinusoidal voltage at the load which is fed as an input to the Class-E power amplifier. In Fig. 5.5 [19], the block diagram of PWM power converter with Class-E power amplifier acting as a load to it.
is shown. The difference between the amplitude of the output voltage and the input voltage to the Class-E power amplifier determines the quality of the signal. In order to achieve good amplitude modulation (AM) from the output of the Class-E RF power amplifier the input signal to the Class-E RF power amplifier should be modulated. The signal can be consider of good quality if all the three components, modulating voltage $v_r$, output voltage of the buck converter, and the output of the voltage of the Class-E RF power amplifier are in phase with each other and the output of the Class-E RF power amplifier is a replica of the modulating signal. (5.19)

From the circuit diagram of Class-E RF power amplifier dynamically supplied from a buck DC-AC converter as shown in Fig. 5.7 [19], the output of the converter is given as an input to the Class-E power amplifier. In Fig. 5.8 it can be seen that, the input voltage of the Class-E amplifier $v_I = 10$ V peak-peak and having a DC value of 10 V. This input voltage is supplied to the choke inductor $L_f$. The MOSFET acting as a switch is driven by a pulse source with a carrier frequency $f = 2$ MHz. The modulating output signal at of the Class-E RF power amplifier $v_O$ is a sinusoidal signal oscillating at the carrier frequency $f = 2$ MHz with an amplitude of 11 V and peak magnitude of approximately 22 V. The amplitude of the output current through the $L - C - R$ resonant circuit is $i_O = 1.9$ A with a peak magnitude of 3.85 A.

Fig. 5.9 shows the gate-to-source voltage $V_{GS}$ and the drain-to-source voltage $V_{DS}$. The gate-to-source voltage is a pulse starting from 0 V and going up to 10 V. The drain-to-source voltage has a half wave sinusoidal output with a maximum voltage of 31.23 V. The output power and the input power are shown in Fig. 5.10. Considering the average values of both the power, the efficiency of the Class-E ZVS power amplifier is calculated as $\eta = 85.33 \%$. The simulation results on SABER closely matches to the designed specification of the Class-E ZVS power amplifier.
Amplitude modulation takes place at the output of the Class-E power amplifier. The input to the Class-E is a modulated signal as shown in Fig. 5.8. The modulating signal is a sinusoidal source connected to the pulse-width modulator with an amplitude $v_m$ of 1.25 V and DC offset voltage of 2.5 V is shown in Fig. 5.7. As far as the quality of the AM signal is considered, all the three voltages, output voltage of the Class-E RF power amplifier, output voltage of the buck DC-AC converter and the modulating signal are in phase with each other and the output of Class-E follows the modulating voltage. The quality of the input voltage to Class-E $v_I$ and the output voltage $v_O$ of the Class-E amplifier is analyzed based on total harmonic distortion.
Figure 5.8: Output voltage $v_o$ of the Class-E, input voltage $v_I$ of the Class-E, and the modulating signal $v_r$ of the Class-E RF power amplifier with amplitude modulation.

(THD). The input signal has a THD of 2.8 %. According to the THD standards, any signal with THD $\leq 1 \%$ is not audible to human ear and is consider to be a high fidelity signal. In case of power converters signals with, THD $\leq 10 \%$ is considered to be a good signal. The THD level for input to the Class-E is a good modulated signal with 2.8 % THD but due to the additional components of the Class-E the distortion level may increase. As Class-E RF power amplifier is nonlinear amplifier amplitude, distortion takes place when the peak-value of the signal gets attenuated due to the shift in the operating point which can occur for the entire time interval.
Figure 5.9: Drain-to-source voltage $V_{DS}$ and gate-to-source voltage $v_{GS}$ of the Class-E ZVS RF power amplifier with amplitude modulation.
Figure 5.10: Input power $P_I$, and the output power $P_O$ of the Class-E ZVS RF power amplifier with amplitude modulation.
Figure 5.11: Spectrum of Class-E input signal in accordance with the continuous output voltage waveform $v_O$ to analyze the total harmonic distortion.
6 Results and Conclusion

6.1 Summary

- Different components of the buck DC-AC converter has been explained along with its switching waveforms and a design example. The operation of op-amps has been discussed as pulse-width modulator with its properties, advantages and limitations. Small-signal model of open-loop buck DC-AC converter has been introduced assuming that the small-signal transfer functions for buck DC-DC converter holds good for buck DC-AC converter. One of the major transfer function used for the this thesis is the control-to-output voltage $T_p$ transfer function. Closed-loop buck DC-AC converter has been discussed. The controller used for the closed-loop system is the proportional-integral (P-I) controller.

- Large-signal analysis of open-loop and closed-loop buck DC-AC converter has been discussed. Large-signal analysis based on large-amplitude perturbations and variable frequency of the modulating signal is carried out. Waveforms related to large-signal analysis is introduced and validated against small-signal transfer functions for both open-loop and closed-loop system. Large-signal analysis is also carried for both resistive and inductive load in open-loop system but for closed-loop buck converter, simulations pertaining to only resistive load is considered. The quality of the signal for both the systems have been analyzed using total harmonic distortion (THD).

- Interleaved buck DC-AC converter has been introduced considering two phase network. The design equations are similar to the conventional buck converter. Switching waveforms are discussed along with its circuit operation. Ripple cancellations of the inductor current is discussed and the transient time analysis is performed using SABER circuit simulator. Large-signal analysis of interleaved
buck converter is carried for only variable modulating frequency \( f_m \). Large-amplitude perturbations is avoided only to maintain a constant duty cycle of \( D = 0.5 \). Quality of the signal for inter-leaved buck DC-AC converter is observed and compared against the the conventional buck converter.

- Amplitude modulated Class-E RF power amplifier is discussed with a design example along with its switching waveforms and circuit operation. Amplitude modulation (AM) is introduced with its AM waveforms. Open-loop Buck DC-AC converter has been used as a amplitude modulator for Class-E RF power amplifier. Quality of the signal is analyzed and necessary details regarding different components of the output waveform, causes of distortion and its limitations has been provided.

### 6.2 Large-Signal Results

Large-signal analysis is carried out for open-loop buck DC-AC converter, closed-loop buck DC-AC converter, and interleaved buck DC-AC converter. For all the systems the analysis is carried out for varying modulating frequency \( f_m \) and varying the amplitude of modulating signal \( v_r \). The frequency response test is analyzed for the magnitude and phase for the voltage gain was analyzed for different frequencies. The modulating frequency was varied from 100 Hz to 50 kHz. Small-signal open-loop control-to-output voltage \( T_p \) transfer function for the buck DC-AC converter has been plotted and the corner frequency \( f_o \) of the open-loop buck converter is around 12 kHz. The large-signal simulations for varying modulating frequency \( f_m \) follows the same trend as that of the small-signal \( T_p \) transfer function. The corner frequency \( f_o \) for large-signal simulations is around 10 kHz. The difference in \( f_o \) in small-signal and large-signal analysis is because of the reduced gain obtained from large-signal simulations. The reduction of gain is due to the modulator which is considered during simulations, where as in small-signal transfer function, the modulator transfer
function $T_m$ is neglected. From Fig. 6.1 and 6.2, comparison of the small-signal open-loop transfer function and large-signal simulation can be made based on the magnitude and phase of the system. Thus, large-signal simulations holds good for small-signal transfer function. With this result, the quality of the signal can also be analyzed. Ideally, good quality signal should always be in phase and maintain a linear relationship with the modulating frequency and amplitude of the modulating signal. The output should be a replica of its input, but practically there are some limitations to this theory. Buck converter being a nonlinear in nature injects non-linearity into the system after a certain frequency and amplitude of the modulating signal. This limitation can be observed through the large-signal analysis. It is observed that beyond the corner frequency which is nothing but the bandwidth of the system, non-linearity starts developing in the system and the output no longer remains a replica of the input signal.

This same analysis is observed in case of closed-loop buck DC-AC converter and the signal starts attenuating beyond the corner frequency. The corner frequency for the closed-loop system is around 35 kHz beyond which there is a significant phase difference between the output and the input signal. In case of closed-loop systems also, the large-signal simulations holds true for small-signal transfer function. Fig. 6.3 and 6.4 shows the comparison of the closed-loop control-to-output voltage $T_{pcl}$ transfer function with the large-signal simulations for varying modulating frequency $f_m$. From the figures it can be seen that, large-signal simulations holds good for small-signal transfer functions for closed-loop system too.

### 6.3 Overall Efficiency of Buck DC-AC Converter

Overall efficiency $\eta$ of the system is considered with buck DC-AC converter along with Class-E RF power amplifier. The same circuit used for amplitude modulation (AM) is also used for estimating the overall efficiency $\eta$. Before varying the modulating
Figure 6.1: Comparison of magnitude of open-loop small-signal control-to-output voltage transfer function $T_p$ and large-signal simulations for resistive load for varying modulating frequency $f_m$.

Frequency $f_m$ and amplitude of the modulating signal $v_r$, the overall all efficiency of the system is evaluated using the nominal values used for transient-time analysis. This will help in estimating the change in the efficiency $\eta$ for large-amplitude variations and varying frequency of the modulating signal. The nominal values used for amplitude of the modulating signal $v_r = 1.25$ V with $f_m = 1$ kHz. The output power $P_O$ measured at the output of Class-E power amplifier is 2.48 W and the input power $P_I$ is measured at the input of the buck converter is 2.91 W. As a result the overall efficiency of the system is around 85 %. Based on this, analysis the the frequency and amplitude of the modulating signal is varied. When the modulating frequency $f_m$ is varied, the amplitude $v_r$ is kept constant and vice verse. Table 1 summarizes the values for varying modulating frequency $f_m$, output power $P_O$, input power $P_I$,
and the overall efficiency keeping the amplitude $v_r$ of the modulating signal constant.

In Fig. 6.5, the overall efficiency is plotted with respect to the varying modulating frequency $f_m$.

From the figure, it is observed that, with the increase in modulating frequency $f_m$ there is a constant decrease in the efficiency $\eta$ till 7 kHz. Beyond 7 kHz, the efficiency remains constant and it remains unaffected for any change in the modulating frequency $f_m$. As mentioned in the previous section that the open-loop buck converter has a corner frequency $f_o$ of 12 kHz. This can be seen in case of estimating the overall efficiency, the linear relationship between the efficiency and the modulating frequency remains till 7 kHz. The difference of 3 kHz in corner frequency is because of the result of the Class-E amplifier. The corner frequency of 12 kHz is obtained from open-loop
Figure 6.3: Comparison of magnitude of closed-loop small-signal control-to-output voltage transfer function $T_p$ and large-signal simulations for resistive load for varying modulating frequency $f_m$.

buck DC-AC converter without the Class-E power amplifier connected to it. Whereas, the overall efficiency is calculated with both buck DC-AC converter and Class-E power amplifier connected together.

In Fig. 6.6, the overall efficiency is plotted with respect to the varying amplitude of the modulating signal $v_r$. From the figure, it is observed that the with increase in amplitude $v_r$ of the modulating signal, the efficiency decreases in a linear fashion until $v_r = 3.0$ V. Beyond $v_r = 3.0$ V, amplitude of the signal has no effect on the efficiency of the system. The remains constant at 85 % for the any change in $v_r$.

### 6.4 Contribution

PWM DC-AC converters are used in applications which require low power and high efficiency. With a modulating voltage at the output and good quality signal is of main
importance now-a-days. Hence, in this thesis open-loop and closed-loop buck DC-AC converter has been analyzed to estimate the quality of the signal. The limitations in terms of the amplitude of the modulating signal and the frequency of modulation for both open-loop and closed-loop buck DC-AC converter is estimated using large-signal analysis. A buck derived topology known as the interleaved buck DC-AC converter is subjected to the same operations and the results are compared with the conventional buck converter. The quality of the signal is measured using total harmonic distortion (THD) for varying amplitude and frequency of the modulating signal for all the three topologies. Finally, verifying the range of frequency for which small-signal model holds true for large-signal operations.
Table 1: ESTIMATING OVERALL EFFICIENCY OF THE SYSTEM WITH VARYING MODULATING FREQUENCY AND KEEPING $v_r = 1.25$ V.

<table>
<thead>
<tr>
<th>Modulating Frequency $f_m$</th>
<th>Overall Efficiency $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 Hz</td>
<td>88.6 %</td>
</tr>
<tr>
<td>700 Hz</td>
<td>86.4 %</td>
</tr>
<tr>
<td>1 kHz</td>
<td>85.2 %</td>
</tr>
<tr>
<td>2 kHz</td>
<td>83.5 %</td>
</tr>
<tr>
<td>3 kHz</td>
<td>82.8 %</td>
</tr>
<tr>
<td>5 kHz</td>
<td>82.4 %</td>
</tr>
<tr>
<td>7 kHz</td>
<td>82 %</td>
</tr>
<tr>
<td>10 kHz</td>
<td>82 %</td>
</tr>
<tr>
<td>20 kHz</td>
<td>82 %</td>
</tr>
<tr>
<td>30 kHz</td>
<td>82 %</td>
</tr>
<tr>
<td>40 kHz</td>
<td>82 %</td>
</tr>
<tr>
<td>50 kHz</td>
<td>82 %</td>
</tr>
</tbody>
</table>

6.5 Future Work

Since, the analysis is done based continuous conduction mode (CCM) operation, the applicability of this method can be used for discontinuous conduction mode (DCM). The controller used in this thesis is a proportional-integral (PI) controller, hence other controllers like proportional-integral-derivative (PID) controller can be tested for this system. In this thesis, pulse-width modulator of the converter has been analyzed based on ideal op-amps. Hence, non-ideal op-amps can been used for the same converter to check their limitations based on the gain and the bandwidth. Also closed-loop buck converter can be used for amplitude modulation of Class-E RF power amplifier.
Figure 6.5: Overall efficiency $\eta$ of buck DC-AC converter with Class-E RF power amplifier for varying modulating frequency $f_m$.

Table 2: ESTIMATING OVERALL EFFICIENCY OF THE SYSTEM WITH VARYING AMPLITUDE OF THE MODULATING SIGNAL AND KEEPING $f_m = 1$ kHz.

<table>
<thead>
<tr>
<th>Amplitude of Modulating Signal $v_r$</th>
<th>Overall Efficiency $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 V</td>
<td>88 %</td>
</tr>
<tr>
<td>0.75 V</td>
<td>87.6 %</td>
</tr>
<tr>
<td>1.0 V</td>
<td>87 %</td>
</tr>
<tr>
<td>1.25 V</td>
<td>86.2 %</td>
</tr>
<tr>
<td>2.0 V</td>
<td>85.8 %</td>
</tr>
<tr>
<td>2.5 V</td>
<td>85.1 %</td>
</tr>
<tr>
<td>3.0 V</td>
<td>85 %</td>
</tr>
<tr>
<td>3.5 V</td>
<td>85 %</td>
</tr>
<tr>
<td>4.0 V</td>
<td>85 %</td>
</tr>
</tbody>
</table>
Figure 6.6: Overall efficiency $\eta$ of buck DC-AC converter with Class-E RF power amplifier for varying amplitude of the modulating signal $v_r$. 
7 Bibliography

References


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Appendix A

A.1 Open Loop Input-to-Output Transfer Function ($M_v$)

The small signal model of the buck converter is shown in Fig. A.1 which is obtained by setting $d = 0$ and output current $i_o = 0$. The model is a single-input single-output system which is used to derive the open loop input to output transfer function.

\[ v_o(s) = Dv_i(s) \frac{Z_2(s)}{Z_1(s) + Z_2(S)}, \quad (A.1) \]

\[ M_v(s) = \frac{v_o(s)}{v_i(s)} = D \frac{Z_2(s)}{Z_1(s) + Z_2(S)} = \frac{D}{V_I} T_p(s), \quad (A.2) \]

\[ M_v(s) = \frac{DR_{Lr_c}}{L(R_L + r_c)} \frac{(s + \frac{1}{C_{r_c}})}{s^2 + \frac{C(R_{Lr_c} + R_{Lr_c} + R_{r_c} + L)}{LC(R_L + r_c)} + \frac{R_{Lr_c}}{LC(R_L + r_c)}}, \quad (A.3) \]

\[ M_v(s) = M_{vo} \frac{s + \omega_z}{s^2 + 2\xi \omega_o s + \omega_o^2}, \quad (A.4) \]

where

\[ M_{vo} = \frac{DR_{Lr_c}}{L(R_L + r_c)}. \quad (A.5) \]

The input to output transfer function is a second order transfer function which consists of two complex conjugate poles and one zero in the left half plane (LHP). $M_v$ is independent on the input voltage $V_I$ and increases with increase in D. Figs. A.2 and A.3 shows the Bode plots for magnitude $M_v$ and phase shift $\phi_{M_v}$ respectively.

A.2 Open Loop Input Impedance of Buck Converter

From the Fig. A.1, the small signal model of buck converter is obtained and the open-loop input impedance of the converter is derived. The current through the inductor is
Figure A.1: Small-signal model of the buck converter for the derivation of input-to-output transfer function $M_v$ and input impedance transfer function $Z_i$.

\[ i_i = \frac{D v_i}{Z_1(s) + Z_2(s)}, \quad \text{(A.6)} \]

input current $i_i = D i_l$

\[ i_i = D i_l = \frac{D^2 v_i}{Z}, \quad \text{(A.7)} \]

where $Z = Z_1 + Z_2$

\[ Z_i = \frac{v_i(s)}{i_i(s)} = \frac{Z}{D^2}, \quad \text{(A.8)} \]

\[ Z_i = \frac{L}{D^2} \frac{s^2 + \frac{C(R_L r_c + R_L r_e + r_e r_c) + L}{L C (R_L + r_c)}}{s + \frac{1}{C (R_L + r_c)}}, \quad \text{(A.9)} \]

\[ Z_i = \frac{L}{D^2} \frac{s^2 + 2 \xi \omega_o s + \omega_o^2}{s + \omega_{cr}}. \quad \text{(A.10)} \]

From the input impedance transfer function $Z_i$ is dependent on square of the duty cycle. As $D$ increases the input impedance decreases. Bode plots for magnitude $|Z_i|$ and phase shift $\phi_{Z_i}$ is shown in Figs. A.4 and A.5.
Figure A.2: Variation in magnitude of input-to-output transfer function $M_v$ for the buck converter.

### A.3 Open Loop Output Impedance

The output impedance transfer function of a DC-ac buck converter is the ratio of output voltage $v_o(s)$ to output current $i_o(s)$. For deriving the open loop output impedance, the small signal model of buck converter is shown in Fig. A.6. Making $v_i = d = i_o = 0$ and applying a voltage source to the output of the small signal model the output impedance transfer function is obtained.

Making $V_i = d = i_o = 0$, we get the output impedance as,

$$Z_o(s) = \frac{v_o(s)}{i_o(s)},$$  \hspace{1cm} (A.11)
The output impedance of the buck converter is achieved by taking the parallel combination of $Z_1$ and $Z_2$,

$$Z_o(s) = \frac{(r+sL)(R_L(r_c+\frac{1}{sC})}{R_L+r_c+\frac{1}{sC}} \bigg) \frac{R_L(r_c+\frac{1}{sC})}{R_L+r_c+\frac{1}{sC}} + (r+sL),$$

(A.14)

Figure A.3: Variation in phase of input-to-output transfer function $M_v$ for the buck converter.
Figure A.4: Variation in magnitude of input impedance transfer function $Z_i$ for the buck converter.

\[ Z_o(s) = \frac{R_L r_c}{(R_L + r_c)} \frac{(s + \frac{1}{C r_c})(s + \frac{1}{r_c})}{s^2 + s \frac{C(R_L r_c + R_L r + r_c) + L}{LC(R_L + r_c)} + \frac{R_L + r}{LC(R_L + r_c)}} , \quad (A.15) \]

\[ Z_o(s) = Z_{ox} \frac{(s + \omega_z)(s + \omega r L)}{s^2 + 2\xi \omega_o s + \omega_0^2} , \quad (A.16) \]

where,

\[ Z_{ox} = \frac{R_L r_c}{R_L + r_c} . \quad (A.17) \]

The output impedance is formed by a parallel resonant circuit. It is the parallel combination of the impedance formed due to inductance, capacitance and the load.
Figure A.5: Variation in phase of input impedance transfer function $Z_i$ for the buck converter.

\[
\begin{align*}
\phi_{Z_i} \text{(deg)} \\
90 & \quad 135 \\
180 & \quad 225 \\
270 &
\end{align*}
\]

\[
10 \\
10 \\
10 \\
4
\]

\[
f \text{(Hz)}
\]

Figure A.6: Small-signal model of buck converter for the derivation of output impedance transfer function $Z_o$.

\[
\begin{align*}
\nu_i &= 0 \\
d &= 0 \\
i_o &= 0
\end{align*}
\]
Figure A.7: Variation in magnitude of output impedance transfer function $Z_o$ for the buck converter.

The Bode plots for magnitude $|Z_o|$ and the phase shift $\phi_{Z_o}$ for the output impedance is shown in Figs. A.7 and Fig. A.8.
Figure A.8: Variation in phase of output impedance transfer function $Z_o$ for the buck converter.