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An Adapted Approach to Process Mapping across Alloy Systems and Additive Manufacturing Processes

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AN ADAPTED APPROACH TO PROCESS MAPPING ACROSS ALLOY SYSTEMS
AND ADDITIVE MANUFACTURING PROCESSES

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Mechanical Engineering

By

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B.S., Wright State University, 2015

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The continually growing market for metal components fabricated using additive manufacturing (AM) processes has called for a greater understanding of the effects of process variables on the melt pool geometry and microstructure in manufactured components for various alloy systems. Process Mapping is a general approach that traces the influence of process parameters to thermal behavior and feature development during AM processing. Previous work has focused mainly on Ti-6Al-4V (Ti64), but this work uses novel mathematical derivations and adapted process mapping methodologies to construct new geometric, thermal, and microstructural process maps for Ti64 and two nickel superalloy material systems. This work culminates in the production of process maps for both Inconel 718 (IN718) and Inconel 625 (IN625) that were developed via both experimental and analytical data, and the tools used in the established process mapping approach have been thoroughly explored. This has resulted in a non-dimensional template for solidification behavior in terms of material solidification parameters and AM process parameters. The optimized non-dimensional approach presented here will increase the efficiency of future process map development and will facilitate the comparison of process maps across alloy systems and AM processes, laying the ground work for integrated AM feature control and evaluation of current and future materials for AM application.
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1 Introduction
Additive Manufacturing (AM), a technology that the manufacturing community has been developing for decades, has, over the past ten years, gained traction and garnered industry’s attention as a potentially viable player in the development of lighter, stronger metal components. Two of the advantages of AM are its geometric versatility and economic friendliness. Components manufactured using traditional subtractive methods may consist of multiple subcomponents and assemblies. Traditional machining practices also typically involve large quantities of waste material. AM processes allow the designer and manufacturer to develop a similar component as one solid part with little to no waste material, thus increasing the integrity and lowering the cost of the part as a whole.

The aerospace and biomedical industries have taken a large interest in AM processes in light of the constant desire for stronger, lighter materials, as well as very complex geometries and custom parts. The high level factor of safety necessary to meet standards in these two industries has created a push for extensive understanding and high precision control of the microstructure (and thus the material properties) of AM metal components.

Previous work has found that in metal systems, microstructure is directly linked to process variables: machine inputs for each process [1]. Two variables previously explored that have proven to have a large effect on microstructure control are the power and the velocity of the concentrated heat source in each process [2]. The effects of the power and velocity process variables for both the Ti-6Al-4V (Ti64) and Inconel ® 718 (IN718) alloy systems for various AM processes have been analyzed and documented [2-4]. This thesis expands on the work performed previously in these alloy systems and adapts the methods developed in prior work to further explore the effect of beam power and velocity on melt geometry and microstructure in the IN718 and IN625 alloy systems. This work is yet another step in applying microstructure control to AM processes across multiple alloys.
1.1 Material Summary

1.1.1 Ti-6Al-4V
Ti64 is currently in widespread use in lightweight, load bearing applications. Its low weight to strength ratio and corrosion resistance make it ideal for aerospace applications, but its tendency to oxidize at high temperatures limits its use to cooler structural components [5]. This $\alpha$-$\beta$ titanium alloy contains a nominal 6 wt. % aluminum and 4 wt. % vanadium, which helps to strengthen the material.

1.1.1.1 Microstructure
Ti64 is an $\alpha - \beta$ alloy, which means that at room temperature two phases are present. The $\beta$ matrix is the high temperature phase strengthened by the vanadium alloying element. The $\alpha$ phase is similar to unalloyed titanium, but is strengthened by the addition of the aluminum alloying element [5]. Ti64 is characterized by two different lattice structures, one for each of the phases. The $\beta$ phase is body centered cubic while the $\alpha$ phase is hexagonal close-packed.

In as-built additive components, the $\alpha$ phase generally forms inside the prior $\beta$ grains. The $\alpha$ formations can grow either in colonies with several laths or in basketweave (Widmanstätten) patterns with a complex “crisscross” structure. It has been shown that process variables do play an effect on the size of both the $\alpha$ and $\beta$ grain formations [3, 6]. Other characterization of AM Ti64 has been performed extensively in the literature as well [7-10] and has examined the influences of process variables on grain morphology, growth, and texture.

1.1.1.2 Material Properties
The materials used in this investigation exhibit temperature dependent thermal properties that were included in finite element (FE) modeling of multiple additive processes. The properties of special interest are the thermal conductivity ($k$), specific heat ($c$), density ($\rho$), and latent heat of fusion. The values of these properties as used in FE models for Ti64 are listed in Table I.
### Ti64 Thermal Properties

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Density ((\text{kg/m}^3))</th>
<th>Specific Heat ((\text{J/kg} \cdot \text{K}))</th>
<th>Thermal Conductivity ((\text{W/m} \cdot \text{K}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>4467.49</td>
<td>611.13</td>
<td>6.80</td>
</tr>
<tr>
<td>498</td>
<td>4439.37</td>
<td>618.77</td>
<td>10.02</td>
</tr>
<tr>
<td>698</td>
<td>4411.25</td>
<td>632.01</td>
<td>13.24</td>
</tr>
<tr>
<td>898</td>
<td>4383.13</td>
<td>650.85</td>
<td>16.46</td>
</tr>
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<td>1098</td>
<td>4355.01</td>
<td>675.29</td>
<td>19.68</td>
</tr>
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<td>1298</td>
<td>4326.89</td>
<td>705.33</td>
<td>22.90</td>
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<td>4298.77</td>
<td>740.97</td>
<td>26.12</td>
</tr>
<tr>
<td>1698</td>
<td>4270.65</td>
<td>782.21</td>
<td>29.34</td>
</tr>
<tr>
<td>1898</td>
<td>4242.53</td>
<td>829.05</td>
<td>32.56</td>
</tr>
<tr>
<td>Latent Heat ((\text{J/kg}))</td>
<td>286000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table I—Thermal properties for Ti-6Al-4V [11]*

#### 1.1.2 Nickel Superalloys

The materials used in the experimental builds were IN718 and IN625, two nickel superalloys containing primarily Nickel and Chromium. Both materials are high temperature alloys with good corrosion, strength, and fatigue properties. IN625 is a solution strengthened alloy typically used in naval and nuclear applications [12, 13] while IN718 is a precipitation strengthened alloy that has found its niche in widespread aerospace applications, particularly within high temperature regions such as the hot section of a turbine engine. Operating ranges for both alloys are from -423° to 1300°F [14], far surpassing the usability of Titanium in similar applications. As shown in Table II, IN718 and IN625 contain similar percentages of Nickel and Chromium, but dissimilar amounts of other alloying elements. For example, IN718 has much higher iron content while IN625 has a much higher molybdenum content.

<table>
<thead>
<tr>
<th>Ni</th>
<th>Cr</th>
<th>Fe</th>
<th>Nb</th>
<th>Mo</th>
<th>Ti</th>
<th>Al</th>
<th>Co</th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>Ph</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN625</td>
<td>58</td>
<td>20</td>
<td>5</td>
<td>3.15</td>
<td>8</td>
<td>0.4</td>
<td>0.4</td>
<td>1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.015</td>
</tr>
<tr>
<td>IN718</td>
<td>50</td>
<td>17</td>
<td>Bal.</td>
<td>4.75</td>
<td>2.8</td>
<td>0.65</td>
<td>0.2</td>
<td>1</td>
<td>0.08</td>
<td>0.35</td>
<td>0.35</td>
<td>0.015</td>
</tr>
</tbody>
</table>

*Table II—Limiting Alloy Composition for IN625 [13] and IN718 [14]*
1.1.2.1 Microstructure
Wrought nickel-based superalloys exhibit three main intermetallic phases that precipitate out of the face centered cubic nickel $\gamma$ matrix: $\gamma'$ (face centered cubic), $\gamma''$ (body centered cubic) and $\delta$ (orthorhombic simple) [15-16] shown in Figure 1 (a-c).

Figure 1 – Crystal structure of the three main phases observed in wrought nickel superalloys: a. $\gamma'$ ordered face-centered cubic (FCC), b. $\gamma''$ body-centered cubic (BCC), c. $\delta$ orthorhombic structure [17]

The $\gamma'$ phase ($NiAl$) and $\gamma''$ ($Ni_3Nb$) phase are the main strengthening phases in IN718 and the $\delta$ ($Ni_3Nb$) orthorhombic phase seen in IN718 is, in many cases, found as an intergranular precipitate. IN718 has been characterized extensively in the literature [18-26], and attempts have also been made to control microstructure in AM components through the use of novel scan strategies and controlling process parameters [27, 28].

1.1.2.2 Material Properties
As with Ti64, nickel-based super alloys exhibit temperature dependent thermal properties that were included in FE modeling of AM processes for this investigation. The properties of special interest are once again thermal conductivity ($k$), specific heat ($c$), density ($\rho$), and latent heat of fusion. The values of these properties as used in FE models for both IN625 and IN718 are listed in Table III and Table IV.
1.2 Additive Manufacturing Processes

AM processes come in a variety of scales and types and are designed for multiple material systems. In beam-based metal AM, the different types of processes are distributed between two main categories: directed energy deposition and powder bed fusion. Within these two categories, the processes are distinguished by their material delivery system and beam type. Four example processes that exhibit these different characteristics are displayed below in Figure 2. NASA Langley’s Electron Beam Freeform Fabrication (EBF3) and Optomec’s Laser Engineered Net Shaping (LENS) processes are both directed
energy deposition processes. The EBF3 process is an electron beam, wire-feed process, and the LENS process is a laser beam, powder stream process. Alternatively, the ARCAM and EOS processes are both powder bed deposition processes, with the ARCAM process employing an electron beam and the EOS process employing a laser beam as the heat source.

Figure 2 – Four characteristic metal AM processes: NASA Langley’s Electron Beam Freeform Fabrication (EBF3) [29], Optomec’s Laser Engineered Net Shaping (LENS) [30], ARCAM’s Electron Beam Melting (EBM) [31], and the EOS Direct Metal Laser Sintering (DMLS) [32]

Each of the processes described above operate in different power and velocity ranges. This means that components manufactured via different AM processes will most likely experience different thermal histories and consequently will exhibit different size scales, microstructures, and material performance. Dr. Jack Beuth et al. of Carnegie Mellon University (CMU) originally described the process variable capability of each of the above machines in the 2013 SFF conference proceedings [1], and Colt
Montgomery updated the original figure in 2015 (as shown in Figure 3) to reflect a broader range of power and velocity each process could reach.

**Figure 3 – Various well-known AM processes and their respective process space capabilities [1, 33]**

Figure 3 indicates various regions of process space that characterize each of the above systems. The fact that each process has different power and velocity capabilities indicates that the built component will be subject to different thermal histories depending on the process. The thermal history of the build dictates the microstructure and therefore the mechanical properties of the finished component. Thus, linking the influence of the process variables to the resulting melt pool geometry and microstructure allows for the improvement and optimization of additive processes based on desired material features.

### 1.3 Process Mapping

Process mapping [34] is an approach developed by Beuth *et al.* to “capture the dependence of process characteristics on primary processing variables under steady-state and transient conditions” [1], with the goal of reducing the need for expensive, time-consuming post-processing procedures and increasing the feasibility and affordability of AM. Process mapping sets the stage for in-situ process monitoring, with the potential to predict and/or control process outcomes based on measureable primary input parameters.
In the literature, the primary input variables of interest have included beam power, beam speed, material feed rate, substrate preheat, and component geometry while process outcomes include melt pool geometry measurements (length, width, depth, cross-sectional area) [3, 33, 35] microstructure morphology (feature width, feature aspect ratio) [3, 6], and component performance (fracture toughness, ultimate strength, fatigue life) [36]. The beauty of process mapping is that the approach may be applied to virtually any thermally based AM process. Additionally, it will be shown in this thesis that process outcomes such as microstructure may be compared across alloy systems for any number of beam based AM processes by implementing the novel non-dimenional process space. Utilizing this approach will open opportunities for new AM processes and material design. Furthermore, as the library of process maps as a whole is developed and the physics behind AM processes are more fully understood, the need for expensive, time-consuming post-processing procedures such as heat treating and Hot Isostatic Pressing (HIP) will become more unnecessary, and the feasibility and affordability of AM will continue to grow.

A large amount of previous work conducted in the area of AM has focused on Ti64, but as the development of process maps has become more streamlined and the need for AM components in other material systems has become necessary, researchers have begun developing maps for other alloy systems including IN718 and now IN625. Ideally, process map development in the field of AM may begin by examining simple, single-bead geometries and moving to more complex geometries and scan strategies including single layer pads, thin walls, multi-layer pads, beam rastering, and beam pulsing [6, 33, 37].

### 1.3.1 Geometric Process Mapping

Previous work performed at CMU has investigated the effects of process variables on steady state melt pool geometries. Soylemez, *et al.* examined the effect of varying deposition rates on the melt pool size and shape of single bead geometries in electron beam processes [38]. Similarly, Vasinonta *et al.*, explored the effect of laser power and velocity on melt pool geometry in thin-walled and bulky 3D geometries [35]. Gockel *et al.*, worked to relate melt pool geometry to microstructure morphology in Ti64, and observed
that, for Ti64 manufactured via the EBF3 process, the cross-sectional area of the melt pool loosely correlated with grain size and that melt pools of constant cross-sectional area exhibited grains of constant thickness [3].

1.3.2 Microstructure Process Mapping
Klingbeil, et al. and Bontha et al. were some of the first to investigate the direct effects of process variables on microstructure in Ti64 by utilizing point source solutions and solidification maps in conjunction with finite elements and cellular automaton modeling [39-41] employing previous work by Kobryn et al. showing that solidification curves for castings may be used to accurately predict microstructure in AM deposits [42]. Gockel et al. went on to produce the first comprehensive microstructure process map for Ti64 deposited via the EBF3 process [3].

![Solidification Process Map for Ti64 manufactured via NASA’s EBF3 AM Process](image)

**Figure 4 – Solidification Process Map for Ti64 manufactured via NASA’s EBF3 AM Process [3]**

Figure 4 displays the solidification process map developed by Gockel. The red dashed line represents the transition from fully columnar to mixed microstructure, and the blue solid line represents the transition from mixed to fully equiaxed microstructure. Recently, the development of process maps has moved to
other alloy systems including IN718. Thompson took the work initiated in the Ti64 alloy system and applied the insight to IN718 by developing a process map for the Sciacky process to help predict grain morphology as a function of beam power and velocity [4]. A few errors have since been discovered in that work, however, that have been rectified in this thesis.

A tool that has proven useful in the construction of AM process maps is the Rosenthal point-source solution [43], which provides a linear solution to the heat equation for a moving point heat source across an infinite substrate. The temperature information gathered at discrete locations in space has traditionally allowed the user to obtain a first-order approximation of melt pool dimensions and thermal conditions throughout the melt pool geometry. This first order approximation has historically been used to guide a non-linear finite element simulation taking into account more of the “real to life” physics, which provides insight into expected thermal behavior throughout the substrate. The thermal behavior then allows the investigator to accurately predict melt pool geometry including length, width, and depth. Microstructure behavior throughout the melt pool may also be predicted and mapped by coupling thermal history data with pre-determined solidification behavior. Behavior predicted analytically has also been verified using experimental data, and the numerical data have been shown to reasonably predict actual thermal and solidification behavior [3].

1.4 Traditional Process Mapping Approach
The approach that has been employed historically for process mapping in AM is outlined below in Figure 5.
Process parameters are initially chosen using a first order estimate like the Rosenthal point source solution or past experience. A finite element model employing as much of the real physics as desired for the end application is developed, and a simulation is performed. The results from the finite element simulation are extracted and compared with other observations or experimental data. For a microstructure process map, the thermal gradients and solidification rates extracted from the simulation are compared to the Hunt criterion solidification curves. For a geometric process map, the measured geometric dimensions are compared to the desired values for each dimension. If the desired behavior, whether it is microstructure morphology or melt pool geometry, is not obtained, a new set of process parameters are specified, and the procedure is repeated. If the desired outcomes are produced, however, the process parameters giving the desired results are noted and plotted to create the process map.

This process typically involves many finite element simulations with relatively long computation times. The precision of the methodology described above is limited by the amount of computation time and power available and the complexity of the modeling being conducted. The work described in this thesis,
however, re-examines the fundamental physics governing AM processes and develops an adaptation to the traditional procedure that can greatly enhance the efficiency of the process mapping approach without compromising precision or reliability.

### 1.5 Hunt Solidification Map

In 1984, Hunt produced closed form equations for a material’s solidification map boundary curves based on multiple solidification parameters and material composition [44]. This classical approach was originally developed for the casting community, but Kobryn, \textit{et al.} showed that the maps were capable of predicting microstructure obtained in Ti64 via a laser AM process [42, 45]. Gockel \textit{et al.} expanded the scope of using Hunt’s curves for process mapping microstructure in AM processes producing solidification morphology curves for Ti64 in process variable space [3], and provided experimental evidence to support their findings.

The equations Hunt derived describe the solidification morphology boundaries for dendrite and grain growth separating three morphology regions: Fully equiaxed, mixed, and fully columnar. The curves are given as

\[
G_{eq} = 0.617N_0^{\frac{1}{3}}\left(1 - \Delta T_N^3\left(\frac{RC_0}{A}\right)^{-\frac{2}{3}}\right)\left(\frac{RC_0}{A}\right)^{\frac{1}{2}}
\]

and

\[
G_{col} = 0.617(100N_0)^{\frac{1}{3}}\left(1 - \Delta T_N^3\left(\frac{RC_0}{A}\right)^{-\frac{3}{2}}\right)\left(\frac{RC_0}{A}\right)^{\frac{1}{2}}
\]

where the \textit{eq} and \textit{col} subscripts indicate the mixed to equiaxed boundary curve and the columnar to mixed boundary curve respectively. Variable definitions and base units are given in Table V.
Table V – Hunt Curve Variable Definitions and Base Units

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>Thermal Gradient</td>
<td>K/m</td>
</tr>
<tr>
<td>N_0</td>
<td>Nucleation Sites per unit Volume</td>
<td>m⁻³</td>
</tr>
<tr>
<td>ΔT_N</td>
<td>Undercooling</td>
<td>K</td>
</tr>
<tr>
<td>V</td>
<td>Solidification Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>A</td>
<td>Experimental Constant</td>
<td>(m-Wt.%)/(s-K²)</td>
</tr>
<tr>
<td>C₀</td>
<td>Material Composition</td>
<td>Wt.%</td>
</tr>
</tbody>
</table>

The solidification maps for Ti64 [42] and IN718 [46] used in the following investigation were developed for each material using experimental and analytical data over a range of thermal gradients and cooling rates. Therefore, the process maps have only been validated for a small range of solidification space, and the behavior outside of that region may be characterized by different solidification and grain nucleation behavior. For the low velocity AM processes that have been investigated before, the thermal behavior at the trailing edge of the melt pool has fallen into the validated range, but many, if not all, of the processes modeled in this thesis induce thermal behavior that falls outside of the range for which experimental specimens were analyzed. Further, it is safe to assume that rapid solidification of the materials due to extremely high process speeds make the solidification more dependent on other physical phenomenon, such as constitutional undercooling and compositional variance, which might influence the thermodynamics of the material solidification.

While it is recognized that the physics based model predicting microstructure size and morphology for rapidly solidified melt pool regions may need to be amended to accurately represent the actual rapid solidification behavior of the metal, the work presented in this thesis uses the solidification maps as defined in the literature to represent the solidification behavior in AM processes. This approach maintains continuity with previous process mapping efforts, and allows for comparison with past work. The goal of the investigation at hand is to develop a methodology for process mapping that is applicable to multiple regions of process space. Past results have shown that the Hunt criterion curves reasonably predict solidification microstructure of low velocity processes, but little work has been performed for high velocity
processes. When other relationships are obtained for extended regions of solidification space, the methodology developed in this thesis may be adapted to produce other process mapping tools that predict microstructure size and morphology in rapidly solidified AM materials due to changes in process parameters.

1.6 Rosenthal Point Source Solution

The three dimensional Rosenthal solution is the linear, quasi-steady state solution to the heat equation with a point heat source travelling at a constant velocity along a semi-infinite substrate [43] as illustrated by Bontha et al. in Figure 6 [41].

![Figure 6 – Bulky 3D geometry considered in Rosenthal solution [2]](image)

The solution describes the temperature field surrounding the heat source, and it predicts the thermal behavior throughout a substrate with width, length, and depth infinitely large compared to the length, width and depth of the melt pool. Vasinonta et al. [47] expressed the solution in dimensionless form as shown in (1.3):

\[
T = \frac{e^{-\left(x_0^+\sqrt{x_0^2+y_0^2+z_0^2}\right)}}{2\sqrt{x_0^2+y_0^2+z_0^2}},
\]

where
\[ T = \frac{T - T_0}{\frac{\alpha Q}{\pi k} \cdot \frac{pcv}{2k}}. \]  

In (1.4) \( T \) is the temperature of interest, \( T_0 \) is the substrate preheat temperature, \( \alpha \) is the absorptivity of the heat source, \( Q \) is the incident machine power, \( v \) is the beam travel speed, and \( \rho, c, \) and \( k \) are the density, specific heat, and thermal conductivity of the material, respectively.

The non-dimensional coordinates \( \bar{x}_0, \bar{y}_0 \), and \( \bar{z}_0 \) are defined as

\[ \bar{x}_0 = x_0 \left( \frac{pcv}{2k} \right), \quad \bar{y}_0 = y_0 \left( \frac{pcv}{2k} \right), \quad \text{and} \quad \bar{z}_0 = z_0 \left( \frac{pcv}{2k} \right), \]

where \( x_0, y_0, \) and \( z_0 \) are the dimensional spatial coordinates in the substrate.

2 Contributions

The contributions of this thesis are as follows:

1. Uses the Rosenthal solution for a moving point heat source to derive closed-form solutions for melt pool length, thermal gradient, and cooling rate at the top of the trailing edge of a melt pool that may be used as tools to predict thermal behavior in AM single beads

2. Uses (1) in conjunction with the Hunt criterion equations to derive closed form microstructure process maps for any beam based AM process and alloy system

3. Use (1) and (2) to adapt previously existing process mapping methodologies across multiple alloy systems and processes (Ti64, IN718, IN625, ARCAM, EOS, and EBF3), including a comparison with available experimental data

4. Uses (1) and (2) to derive non-dimensional microstructure process maps for any material system and beam-based AM process and illustrates the utility of these process maps for Ti64 and IN718
3 Development and Application of Rosenthal Closed-form Equations

3.1 Rationale and Procedure for Fitting the Rosenthal Solution
The Rosenthal solution by definition is a linear solution that does not take into account various aspects of the physical AM process including temperature-dependent properties, latent heat of fusion, added material, melt pool fluid flow, and surface convection or radiation. As a result, the Rosenthal solution is somewhat limited in its capability to predict the actual thermal behavior in a substrate in additive manufacturing. However, calibrating the Rosenthal solution to complex, non-linear finite element results or experimental data can expand the utility of the Rosenthal solution to accurately predict actual behavior. This can be achieved by applying a correction that helps account for phenomenon not included in the Rosenthal solution, effectively increasing the approximation from a first order to a second or even third order approximation.

Multiple approaches have been proposed to fit the Rosenthal solution to experimental or non-linear finite element data, including modification of the temperature at which the thermal properties are defined [48]. One of the challenges that has been encountered with this fitting procedure is that it does not allow for the most accurate fitting for certain processes. Additionally, this approach does not represent the physical thermal properties acting at the melting temperature.

In particular, the thermal gradients and cooling rates induced by the ARCAM and EOS processes as modeled using Finite Element Analysis (FEA) (see following sections) increase very rapidly at the solidus temperature, reaching values that are not represented well using the unfitted Rosenthal solution. In order to fit the Rosenthal solution for processes such as ARCAM and EOS using the fitting procedure mentioned above, the thermal properties obtained in the fitting must be extrapolated from the data provided in Table I, Table III, and Table IV, essentially estimating the property values at temperatures much larger than the melting temperature. Examination of (1.4) and (1.5) shows that $\rho$, $c$, and $k$ are grouped together in the
Rosenthal solution as $\frac{\rho c}{k}$, indicating that the actual values of $\rho$, $c$, and $k$ are not as important as their relationship to each other, so theoretically this fitting method might work. However, using the extrapolation method for these parameters introduces a problem, because $k(T)$ dramatically decreases in value at temperatures beyond the melting temperature which means that $\frac{\rho c}{k}$ increases accordingly. Using this method the below plot of error produced by the Rosenthal solution was developed.

![Rosenthal Error for Property Extrapolation](image)

**Figure 7 – Error between FEA simulation results and the Rosenthal solution with material properties ($\rho$, $c$, and $k$) defined at different temperatures**

It may be observed that the error in the Rosenthal calculation decreases up until about 3250° C, but then the error starts to increase. Further, the lowest level of error that the Rosenthal solution is capable of reaching using this procedure is only about 10%, which is not accurate enough for the task at hand. It is
important to note that it IS possible to reach an error of 0% with this method for some processes, however as is seen in Figure 7, there are certainly many occasions when this is not the case.

The virgin Rosenthal solution follows very closely to linear FEA simulations; however, the physical phenomena that characterize “real life” additive processes necessitate the incorporation of a correction factor in the Rosenthal solution that reduces the error between the linear Rosenthal solution and non-linear simulation results. This correction factor, from here on defined as $\phi$, can be represented as a function of all the modeled physics in a given simulation as

$$\phi = f(\rho(T), c(T), k(T), q_f, m_a, \ldots). \quad 3.1$$

In (3.1), $\rho(T)$, $c(T)$, and $k(T)$ represent the temperature dependent thermal properties; $q_f$ represents the latent heat of fusion; and $m_a$ is the added material. The complexity of the $\phi$ function is dependent on the number of non-Rosenthal physical variables accounted for in a simulation. As will be discussed in section 4.2, the only non-Rosenthal variables included in the FEA models for this thesis were latent heat and temperature dependent material properties. It was assumed that, for a single isotherm, the material properties were constant and in accordance with the tables above. Therefore, for this investigation, $\phi = f(q_f)$ for a single isotherm. Further reasoning revealed that latent heat of fusion at the solidus temperature is nothing more than a power term during the phase change modifying the amount of heat delivered by the beam that is actually conducted through the material. For this reason, the $\phi$ term for this thesis was paired with the power variable ($Q$), redefining (1.4) to be

$$\bar{T} = \frac{T - T_0}{\frac{\phi \alpha Q}{\pi k} \star \frac{\rho cv}{2k}}. \quad 3.2$$

$\phi$ has not been defined functionally in regards to latent heat ($q_f$), but it was determined that the Rosenthal solution could be fit by modifying the $\phi$ term to optimally represent the FEA results. By fitting
the Rosenthal solution in this way, the error was guaranteed to reach a minimum value of zero as shown in Figure 8.

Figure 8 – Sample error between FEA simulation results and the Rosenthal solution with different values for $\phi$

This approach is much more accurate because the Rosenthal solution is proportional to $\phi$. The error is easily reduced to zero by modifying the value of the correction factor which makes fitting the Rosenthal solution much easier and much more accurate than the approach discussed previously.

The procedure in this thesis for fitting the Rosenthal solution has been adapted from methods documented in the literature [4] and in private communications with previous researchers. Initially, a single finite element simulation is conducted under process conditions specified for the AM process of interest. Identical process parameters are specified in the Rosenthal solution, and the isotherm of interest
(T) is isolated. For the proceeding work, the isotherm of interest is taken to be the solidus isotherm to maintain consistency with the literature [3]. The material properties in the Rosenthal solution are also specified at this isotherm. Once the output data of interest is extracted from the FE simulation, the \( \phi \) parameter is modified until the Rosenthal solution matches the extracted data. It is important to note that each output variable obtained from the FEA simulation must be fit independently (i.e. the length must be fitted differently from the thermal gradient, which must be fitted differently from the cooling rate, etc.).

### 3.2 Derivation of Closed-form Rosenthal Equations

Many of the output variables of interest in this research (i.e. melt pool depth, melt pool length, and thermal gradient, etc.) are functionally related to the non-dimensional temperature. While most of these relationships must be determined numerically throughout the depth of the melt pool, the relationships at the top of the trailing edge of the melt pool can be derived in closed form. Prior work has shown that the top point on the trailing edge of the melt pool is a critical point because it represents the first point of microstructure transition (i.e. columnar to equiaxed) in the melt pool. In essence, any ability to obtain an equiaxed microstructure within an AM build is dependent on the ability to reach the necessary thermal conditions at the top of the melt pool. By using the closed-form equations derived in the proceeding chapters, computation time in creating process maps may be considerably reduced, and the efficiency of the process mapping approach may be greatly enhanced. Determining closed-form relationships between process parameters and thermal behavior greatly simplifies process mapping efforts and facilitates process map comparisons for multiple alloys across various AM processes.

#### 3.2.1 Melt Pool Length

Due to the nature of the Rosenthal Solution and the assumptions made in its derivation, the temperature distribution induced by the point source is axisymmetric around the x-axis. In Figure 6, the region of interest for this analysis is every location where \( \bar{x}_0 \leq 0 \) and \( \bar{z}_0 < 0 \). The axisymmetry of the distribution
indicates that a single isotherm cross-section provides all the data necessary for every point lying on the three-dimensional isotherm surface. In order to take the solution to its simplest form, \( \bar{y}_0 \) is set equal to zero leaving the distribution only in terms of \( \bar{x}_0 \) and \( \bar{z}_0 \). Equation 1.3 then reduces to

\[
\bar{T} = e^{-\left(\bar{x}_0 + \sqrt{\bar{x}_0^2 + \bar{z}_0^2}\right)}
\]

At the top of the melt pool the \( \bar{z}_0 \) component of the equation is equal to zero, and the \( \bar{x}_0 \) is unknown. Given this information, (3.3) reduces to

\[
\bar{T} = e^{-\left(\bar{x}_0 + \sqrt{\bar{x}_0^2}\right)}
\]

It must be remembered that the value of \( \bar{x}_0 \) at the length of the melt pool is negative. To this end, the location of the length of the melt pool is defined as \( \bar{L} \) where

\[
\bar{L} = -\bar{x}_0 .
\]

Therefore,

\[
\bar{T} = e^{-(\bar{L} + \sqrt{(-\bar{L})^2})}
\]

which gives

\[
\bar{T} = \frac{e^{\bar{L}-\bar{L}}}{2\bar{L}},
\]

or

\[
\bar{T} = \frac{1}{2\bar{L}}.
\]

By solving for \( \bar{L} \), it is shown that the non-dimensional length is functionally related to the non-dimensional temperature such that
\[ \bar{L} = \frac{1}{2\bar{T}} \]  

Using (1.5), the dimensional \( L \) variable is defined as

\[ L = \bar{L} \left( \frac{2k}{\rho c v} \right) \]

Therefore, substituting \( T = T_m \), where \( T_m \) is the melting temperature, into (3.2), substituting (3.2) into (3.9), and dimensionalizing using 3.10, the melt pool length is given by

\[ L = \frac{\varphi \alpha Q}{2\pi k (T_m - T_0)} . \]

It is important to note that in this thesis, the melt pool length is equal to the distance from the power source to the tip of the tail. From (3.11), it may be observed that the length of the melt pool is directly related to the power and the thermal conductivity. A unit analysis verifies initially that this solution is reasonable. While intuition may indicate that a faster beam speed will produce a longer melt pool, according to (3.11), the beam power and thermal conductivity are actually the key players while velocity plays no role.

One way to visualize the reality of this phenomenon is to think about the roles of both beam power and thermal conductivity in the formation of the melt pool. The beam power provides a continuous supply of energy to the build while the thermal conductivity indicates how quickly the material is capable of distributing the absorbed energy throughout the substrate. Larger powers decrease the gradient of energy throughout the material by forcing the energy to spread throughout the substrate. The lower gradient means that the temperature would decrease over a larger distance allowing a single isotherm to reach farther away from the point source, which results in a longer melt pool.
Similarly, the thermal conductivity characterizes the material’s ability to dissipate heat throughout the material volume. As the conductivity decreases, the material is not able to conduct the heat as efficiently to regions farther away from the source, so the heat collects near the surface, resulting in a long, flat melt pool. If the thermal conductivity increases, the material is able to disperse the absorbed energy into the bulk geometry, which, in turn, causes a wide distribution of heat throughout the entire substrate.

3.2.2 Thermal Gradient at Top of the Melt Pool

The non-dimensional thermal gradient is obtained using a similar derivation. Following Bontha et al., the thermal gradient at the top of the melt pool is simply the x-component.

\[
\tilde{G} = \frac{\partial \tilde{T}}{\partial \tilde{x}_0} = \frac{e^{-\left(\tilde{x}_0 + \sqrt{\tilde{x}_0^2 + \tilde{y}_0^2 + \tilde{z}_0^2}\right)}}{2\sqrt{\tilde{x}_0^2 + \tilde{y}_0^2 + \tilde{z}_0^2}} \left\{ 1 + \frac{-\tilde{x}_0}{\sqrt{\tilde{x}_0^2 + \tilde{y}_0^2 + \tilde{z}_0^2}} + \frac{-\tilde{x}_0}{\tilde{x}_0^2 + \tilde{y}_0^2 + \tilde{z}_0^2} \right\}.
\]

3.12

The same assumptions made in section 3.2.1 are made here which allows for the thermal gradient at the top of the melt pool to easily be derived. Assuming that \(\tilde{y}_0 = \tilde{z}_0 = 0\) in (3.12) gives

\[
\frac{\partial \tilde{T}}{\partial \tilde{x}_0} = \frac{e^{-\left(\tilde{x}_0 + \sqrt{\tilde{x}_0^2}\right)}}{2\sqrt{\tilde{x}_0^2}} \left\{ 1 + \frac{-\tilde{x}_0}{\sqrt{\tilde{x}_0^2}} + \frac{-\tilde{x}_0}{\tilde{x}_0^2} \right\}
\]

3.13

By defining

\[
\bar{L} = -\tilde{x}_0,
\]

3.14

it follows that

\[
\frac{\partial \tilde{T}}{\partial \tilde{x}_0} = \frac{e^{-\left(-\bar{L} + \sqrt{(-\bar{L})^2}\right)}}{2\sqrt{(-\bar{L})^2}} \left\{ 1 + \frac{-\bar{L}}{\sqrt{(-\bar{L})^2}} + \frac{-\bar{L}}{(-\bar{L})^2} \right\}
\]

3.15

which gives

\[
\frac{\partial \tilde{T}}{\partial \tilde{x}_0} = \frac{1}{2\bar{L}} \left\{ -\frac{1}{\bar{L}} \right\}
\]

3.16

or
\[
\frac{\partial \bar{T}}{\partial x_0} = -\frac{1}{2L^2}.
\]

From Equation 3.8,

\[
\frac{\partial \bar{T}}{\partial x_0} = -2\bar{T}^2.
\]

Since,

\[
|\bar{G}| = \left| \frac{\partial \bar{T}}{\partial x_0} \right| = 2\bar{T}^2,
\]

using the definition of \(\bar{T}\) given in (3.2) with \(T = T_m\) gives

\[
|\bar{G}| = 2 \left( \frac{T_m - T_0}{\frac{\phi \alpha Q}{\pi k}} \right)^2 \left( \frac{\rho c_v}{2k} \right)^2 \left( \frac{\phi \alpha Q}{\pi k} \right).
\]

The thermal gradient at the top of the melt pool may be dimensionalized as shown in (3.21-3.24) as

\[
|G| = |\bar{G}| \left( \frac{\rho c_v}{2k} \right)^2 \left( \frac{\phi \alpha Q}{\pi k} \right).
\]

Substituting 3.19 gives

\[
|G| = (2\bar{T}^2) \left( \frac{\rho c_v}{2k} \right)^2 \left( \frac{\phi \alpha Q}{\pi k} \right),
\]

or

\[
|G| = 2 \left( \frac{T_m - T_0}{\frac{\phi \alpha Q}{\pi k} \cdot \rho c_v \frac{2k}{2k}} \right)^2 \left( \frac{\rho c_v}{2k} \right)^2 \left( \frac{\phi \alpha Q}{\pi k} \right).
\]

Cancelling terms gives the thermal gradient at the top of the melt pool as

\[
|G| = \frac{2\pi k (T_m - T_0)^2}{\phi \alpha Q}.
\]
In (3.24), it is seen that an increase in power results in a decrease in thermal gradient while an increase in thermal conductivity results in an increase in thermal gradient. It is intuitive then that the thermal gradient is inversely proportional to the length of the melt pool, as indirectly indicated by (3.17).

3.2.3 Cooling Rate at the Top of the Melt Pool
Due to the shape of the melt pool and the definition of the cooling rate at the top of the melt pool, it may be observed that the cooling at the top of the melt pool is purely in the x-direction. Additional comparison of the non-dimensional forms of the cooling rate and thermal gradient shows that these values are equal at the top of the melt pool, i.e.,

\[
\frac{\partial \tilde{T}}{\partial \tilde{t}} = \tilde{G} = e^{-\frac{x_0 + \sqrt{x_0^2 + y_0^2 + z_0^2}}{2\sqrt{x_0^2 + y_0^2 + z_0^2}}} \left(1 + \frac{x_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} + \frac{x_0}{x_0^2 + y_0^2 + z_0^2}\right).
\]

As a result, the method described in section 3.2.2 may be used to define the cooling rate at the top of the melt pool in terms of \( \tilde{T} \), which gives

\[
\frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{\partial \tilde{T}}{\partial x_0} = -2\tilde{T}^2,
\]

where \( \frac{\partial \tilde{T}}{\partial \tilde{t}} \) is negative because the material is cooling in the melt pool tail. Substituting for \( \tilde{T} \) and dimensionalizing the cooling rate

\[
\frac{\partial T}{\partial t} = -2\left(\frac{T_m - T_0}{\phi \alpha Q \frac{\rho c v}{2k}}\right)^2 \left(\frac{\phi \alpha Q v}{\pi k}\right)
\]

results in a closed form solution for cooling rate at the top of the melt pool:

\[
\frac{\partial T}{\partial t} = -2\left(\frac{T_m - T_0}{\phi \alpha Q}\right)^2 \pi k v
\]

The non-dimensional solidification rate (\( \tilde{R} \)) is defined as
\[ \bar{R} = \frac{1}{|G|} \left| \frac{\partial T}{\partial t} \right| . \]  \hspace{1cm} (3.29)

From (3.29), it may be seen that the non-dimensional solidification rate at the top of the melt pool is always equal to one because the values for non-dimensional thermal gradient and non-dimensional cooling rate are equal to each other. However, dimensionalizing \( \bar{G} \) and \( \frac{\partial T}{\partial t} \) and substituting (3.24) and (3.28) gives

\[ R = \frac{1}{|G|} \left| \frac{\partial T}{\partial t} \right| = v. \]  \hspace{1cm} (3.30)

Hence, the dimensional solidification speed is equal to the beam velocity. This is intuitively validated due to the steady-state assumption of the Rosenthal solution. If the melt pool is at steady state, then the melt pool geometry does not change with time. For this to be true, the tail of the melt pool must solidify at a rate equal to the beam travel speed.

### 3.3 Closed-Form Solidification Process Maps

As stated previously, the top of the melt pool has been analyzed to produce process maps for multiple additive manufacturing processes due to the fact that the top of the melt pool represents the first point of morphology transition at the melt pool. Another benefit of looking at the top of the melt pool is the fact that the thermal behavior at the top of the melt pool can be easily derived from the Rosenthal point source solution, as described in Sections 3.2.2 and 3.2.3.

#### 3.3.1 Closed-Form Microstructure Process Map

Replacing the thermal gradient in the dimensional Hunt’s solidification equations with (3.24) and the solidification rate with (3.30) results in a relationship between process variables, solidification parameters, and morphology regions. In so doing, (1.1) and (1.2) give

\[ \frac{2\pi k(T - T_0)^2}{\phi \alpha Q_{eq}} = 0.617 N_0 \left( 1 - \Delta T N \left( \frac{\nu C_0}{A} \right)^{-\frac{3}{2}} \right) \left( \frac{\nu C_0}{A} \right)^{\frac{1}{2}} \]  \hspace{1cm} (3.31)

and
\[
\frac{2\pi k(T - T_0)^2}{\phi\alpha Q_{col}} = 0.617(100N_0)^\frac{1}{3} \left\{ 1 - \Delta T_N^3 \left( \frac{vC_0}{A} \right)^{-\frac{3}{2}} \right\} \left( \frac{vC_0}{A} \right)^{\frac{1}{2}}
\]

3.32

Here \( Q_{eq} \) and \( Q_{col} \) represent the powers lying on the mixed to equiaxed and the columnar to mixed boundary curves, respectively. Solving for the \( Q \) variables in (3.31) and (3.32) gives the closed-form equations for solidification morphology process map curves for the top point of a melt pool in terms of thermal properties, material solidification parameters and AM process variables as

\[
Q_{eq} = \frac{2\pi k(T - T_0)^2}{\phi\alpha \left( 0.617N_0^\frac{1}{3} \left\{ 1 - \Delta T_N^3 \left( \frac{vC_0}{A} \right)^{-\frac{3}{2}} \right\} \left( \frac{vC_0}{A} \right)^{\frac{1}{2}} \right)}
\]

3.33

and

\[
Q_{col} = \frac{2\pi k(T - T_0)^2}{\phi\alpha \left( 0.617(100N_0)^\frac{1}{3} \left\{ 1 - \Delta T_N^3 \left( \frac{vC_0}{A} \right)^{-\frac{3}{2}} \right\} \left( \frac{vC_0}{A} \right)^{\frac{1}{2}} \right)}
\]

3.34

Non-dimensionalizing (3.31) and (3.32) allows for comparison across alloy systems and across AM processes. The non-dimensional velocity \( (\bar{v}) \) here is defined as

\[
\bar{v} = \frac{vC_0}{A\Delta T_N^2},
\]

3.35

and the non-dimensional power is defined as

\[
\bar{Q} = \frac{(0.617)\phi\alpha Q N_0^\frac{1}{3} \Delta T_N}{2\pi k(T - T_0)^2}.
\]

3.36

Substituting (3.35) and (3.36) into (3.33) and (3.34) results in a non-dimensional closed-form solution representing the Hunt’s solidification curves in terms of non-dimensional process variables.

\[
\bar{Q}_{eq} = \frac{\bar{v}}{\bar{v}^2 - 1}
\]

3.37

and

\[
\bar{Q}_{col} = \frac{\bar{v}}{(100)^\frac{1}{3} (\bar{v}^2 - 1)}
\]

3.38

27
3.3.2 Lines of Constant Cooling Rate

The equations for cooling rate in terms of both dimensional and non-dimensional process variables may also be derived from the original Rosenthal solution. From (3.30),

\[ R = \frac{1}{|G|} \left| \frac{\partial T}{\partial t} \right|. \]

Therefore,

\[ \frac{\partial T}{\partial t} = |G|R \] \hfill (3.39)

At the top of the melt pool, it follows from (3.24) and (3.30) that

\[ \frac{\partial T}{\partial t} = \frac{2\pi k(T - T_0)^2}{\phi\alpha Q} v. \] \hfill (3.40)

Solving for \( Q \) gives a linear relationship between process variables \( Q \) and \( v \) for any given cooling rate,

\[ Q = \frac{2\pi k(T - T_0)^2}{\phi\alpha \frac{\partial T}{\partial t}} v. \] \hfill (3.41)

This equation may be non-dimensionalized using the definitions of non-dimensional power and non-dimensional velocity given in (3.35) and (3.36) as

\[ \frac{2\bar{Q}\pi k(T - T_0)^2}{0.617\phi\alpha N_0^3\Delta T_N^3} = \frac{2\pi k(T - T_0)^2}{\phi\alpha \frac{\partial T}{\partial t}} \left( \frac{\bar{v}A\Delta T_N^2}{C_0} \right). \] \hfill (3.42)

Solving for \( \bar{Q} \) and simplifying gives a linear relationship between non-dimensional process variables for comparison across multiple alloy systems as

\[ \bar{Q} = \frac{0.617N_0^3A\Delta T_N^3}{C_0 \left( \frac{\partial T}{\partial t} \right)} \bar{v}. \] \hfill (3.43)

It is important to note that the non-dimensional form of the lines of constant cooling rate contains solidification parameters from the Hunt equations only because the process power and velocity were non-
dimensionalized with respect to these parameters. Re-dimensionalizing (3.43) will cancel out any Hunt criterion variables, and will result in (3.41).

3.4 Verification of Closed-Form Process Map Equations
To verify the accuracy of the derived process map equations in regards to overall shape and ability to describe actual process maps, $\phi$ was optimized in (3.33) and (3.34) to provide “best-fit” curves for Gockel’s previously published process map for Ti64 manufactured via NASA’s EBF3 Process. The AM process variables used in this investigation were the same as specified in Gockel’s 2014 paper [3]. The resulting process map overlayed over Gockel’s process map is given below, where the blue line is (3.33) and the red line is (3.34).

![Microstructural Process Map for Ti64](image)

**Figure 9 – Comparison of Gockel’s Process Map to the process map using the closed form process mapping equations**
Figure 9 indicates that the closed form process mapping equations derived from the Rosenthal equation and the Hunt’s criterion curve equations produce very accurate microstructure process maps, as compared to process maps developed solely using FEA. Statistical analysis shows that the $R^2$ values for the closed-form equiaxed and columnar boundary curves produced are approximately 0.96 and 1.0 respectively indicating a sufficient goodness of fit to Gockel’s published results. This bolsters the reasoning that the Rosenthal solution, despite its simplifying assumptions, can be a useful and accurate tool in predicting thermal behavior and grain growth at the top of the melt pool.

4 Finite Element Modeling
By itself the Rosenthal point source solution is limited in its ability to predict behavior in additive processes as it is a purely linear model that does not take temperature dependent properties, latent heat, added material or anything else directly into account. It has been shown above, however, that the Rosenthal solution may be calibrated to reflect results obtained via non-linear FE simulations and experimental methods. The “fitted” Rosenthal solution may then be used to estimate the thermal behavior for an additive process in the vicinity of the fitted process variable set, which is helpful in process mapping applications.

![Sample Finite Element melt pool region for an axisymmetric single pass model](image)

Figure 10 – Sample Finite Element melt pool region for an axisymmetric single pass model

Axisymmetric thermal models used to simulate AM processes were created specifically for this project. The model geometries and grid meshes (shown in Figure 10) used in the models were produced via a Matlab script specifying DCAX4 axisymmetric continuum heat transfer elements for the axisymmetric
geometry for use in the conventional software package ABAQUS. The DCAX4 element type is a two-
dimensional, 4 node axisymmetric element.

No added material was included in the modeling of the powder bed processes because previous work has
shown that the addition of material for powder bed processes does not substantially affect heat transfer
throughout the substrate material [33]. Models for the directed energy deposition processes also did not
include powder because it was found that an axisymmetric model provided sufficient accuracy to neglect
the behavior of added material. A single FE model was run for each material and process, and the
Rosenthal solution was fit to accurately represent the cross-sectional area, thermal gradient, and cooling
rate at that process variable combination. Microstructure process maps were developed using the closed-
form equations derived from the Rosenthal point source solution as described in section 1.6, and
geometric process maps were constructed with the Rosenthal solution using an optimization approach.
Experimental data from across process space was compared to the analytical predictions for IN625 and
IN718 in the EOS and ARCAM processes, respectively.

4.1 Linear Modeling
The FE Method is an approach to numerically solve a governing differential equation in a discrete manner.

Ideally, a closed-form solution to a differential equation will yield the same result as a FE simulation if the
assumptions made for the analytical solution are also taken into account in the FE numerical solution.
Therefore, an initial FE simulation was performed to compare to the Rosenthal solution to verify that the
geometry defined in the simulation would approximate a “semi-infinite” Rosenthal geometry.

In the model, the melt pool length was calculated from (3.11) and the melt pool depth was approximated
from the Rosenthal solution. The axisymmetric model substrate was defined as about 20 melt pool depths
by about 7 melt pool lengths. The substrate was meshed to include a biased region with a large element
size at the edge of the material leading to a smaller element size closer to a steady state region of interest.
The steady state region was meshed with a fine mesh of about 30 elements through the approximate depth of the melt pool and about 70 elements through the length of the melt pool. Constant material properties were defined for the material of interest, and a concentrated heat flux was applied to node for a time step correlating to the velocity of the beam travel. In this way, the continuous velocity of the beam was discretized.

The thermal behavior at the top of the melt pool and the geometric dimensions were extracted from the FE simulation, and the resulting measurements were compared to the derived Rosenthal solution assuming the same power, velocity, and preheat temperature. If the FE results and Rosenthal calculations matched, it was verified that the linear FE model accurately approximated the Rosenthal solution, and the Rosenthal fitting process would only take into account the temperature dependent properties and latent heat of the corresponding non-linear FE model.

4.2 Non-Linear Modeling
After the equality of the linear FE substrate dimensions and the Rosenthal solution were verified, non-Rosenthal physical variables including temperature dependent properties and latent heat, were incorporated into the model. The substrate length was increased by at least one extra melt pool length to account for the increase in melt pool length due to latent heat effects. The mesh resolution along the length of the substrate was also increased by 50-100 elements to maintain the required mesh resolution.

5 Application of Closed Form Process Mapping Method
The ability to combine the accuracy and robust nature of finite elements with the flexibility and simplicity of a closed-form analytical solution makes the process map equations a promising alternative to the traditional process mapping method depending on the level of accuracy desired. The traditional method as described in section 1.4 typically utilizes many FE simulation iterations and large amounts of computation time and power to determine the entire microstructure and thermal process maps. Using the closed form process mapping equations, however, greatly simplifies the traditional approach by
requiring only one finite element simulation with a single set of process variables, which, in turn, provides enough information to predict thermal behavior and microstructure.

Three other analyses were performed using the closed-form equations to prove the versatility of the new process mapping tools in producing and comparing process maps across alloy systems and AM processes. Two of these investigations were also compared to experimental data to show the ability to predict actual geometric and microstructure data. For this investigation, a single non-linear axisymmetric simulation was performed at the midrange of the process space being mapped, and the thermal data at the top of the substrate surface was obtained from the solidus isotherm as has been the procedure in the past [3]. When using the Rosenthal equations, it was assumed for this section that the material absorbed all the incident beam energy, thus the absorptivity factor ($\alpha$) was specified as one. Due to the assumptions made in the Rosenthal solution, the cross-sectional area of any melt pool are assumed to be semi-circular. Thus, the cross-sectional areas for the geometric process maps were calculated by taking the numerically determined depth and using (5.1).

\[ A = \frac{\pi d^2}{2} \]  \hspace{1cm} (5.1)

where $d$ is the melt pool depth.

The National Science Foundation (NSF) grant under which this work was conducted specified that the goal of this research would be to investigate the effects of process parameters for powder bed processes across alloy systems. An additional facet was funded by the National Institute of Standards and Technology to examine the effects of process parameters on IN625 manufactured via a laser powder bed process. To this end, additional analytical models were developed for Ti64 in the ARCAM process range, IN718 in the EBF3 and ARCAM process ranges, and IN625 manufactured via the EOS process range. A comparison of process maps for multiple processes and material systems was then possible using the closed form process mapping equations and the non-dimensionalization of those process maps.
5.1 Titanium – ARCAM Process
An axisymmetric model was developed for Ti64 single beads manufactured via the ARCAM powder bed electron beam AM process. The initial temperature throughout the substrate was set to 1023 K, and one simulation was run with a power of 1111 W and a beam velocity of 500 mm/s. Geometry and thermal data was extracted from the simulation, and the following process maps were created.

5.1.1 Geometry
The length (from the heat source to the tip of the melt pool) and the depth of the melt pool were extracted from the finite element model at the solidus isotherm (1893 K), and a numerical root finding optimization technique was applied to fit the Rosenthal solution to the FEA depth. The length obtained from the FEA was fitted separately via (3.11). The curves of constant area were obtained numerically, and the curves of constant length to depth ratio were obtained using (3.11).
Figure 11 – Geometric process map predicting curves of constant area and curves of constant L/D ratio for Ti64 manufactured via the ARCAM process at a 1023 K preheat.

It is important to note that the curves of constant L/D ratios contain only four points each with each point intersecting a curve of constant area. The four points included for each ratio, however, give a pretty good approximation of the shape of the curves themselves.

5.1.2 Microstructure
The thermal gradient and cooling rate at the solidus temperature were also obtained. The thermal gradient and cooling rate equations were fit using (3.24) and (3.28), and the microstructure process map was constructed as shown below.
Figure 12 – Microstructure process map for Ti64 manufactured via the ARCAM process at a preheat of 1023 K.

An initial examination of the process map shows that the lines of constant cooling rate, represented by the three dashed lines are very similar in slope to the lines of constant area. This could indicate that maintaining a constant cross-sectional area will produce constant cooling rate and therefore, constant grain size at the top of the melt pool. This agrees with Gockel et al.’s experimentally corroborated conclusion for Ti64 in the EBF3 process range that constant cross-sectional area may indicate constant grain size.

It is also noted that the slope of the line decreases as the cooling rate increases. This indicates that at higher cooling rates the power does not play as big of a role as velocity while at lower cooling rates the
power has a more prevalent role in influencing the cooling rate. This is due to the fact that $\frac{\partial T}{\partial t}$ is in the denominator of (3.43).

The red line in the process map above represents the transition from columnar to mixed morphology (eq. (3.33)) while the blue line represents the transition from mixed morphology to equiaxed morphology (eq. (3.34)). The microstructure boundary curves for this process and material both occur below 600 W, which corresponds to the lowest twenty percent of the actual power capability of the process. Realistically, the prediction that any power setting above 600 W will result in equiaxed microstructure for Ti64 does not pass a sanity check when considering experimental observations. To date, no literature has been produced indicating the production of equiaxed microstructure for Ti64 single beads manufactured via the ARCAM process using nominal process parameters and scan patterns.

There are probably multiple reasons that the process mapping equations may not accurately represent what is taking place in the manufacture of Ti64 single beads, but the main reason is that the Hunt Criterion curves may not accurately represent the rapid solidification phenomena that are taking place at high speeds. Kobryn, as mentioned above, developed the Hunt’s criterion curves for castings which induce relatively low cooling rates and solidification rates. At higher speeds, however, the solidification curves have been unverified and may not accurately represent the physics typical of the process.

5.2 Inconel 718 – ARCAM Process
The process mapping procedure conducted for Ti64 was also conducted for IN718. An axisymmetric model was developed for an IN718 single bead manufactured via the ARCAM powder bed electron beam AM process. The initial temperature throughout the substrate was set to 1023 K, and a single simulation was performed with a power of 1111 W and a beam velocity of 500 mm/s to match the Ti64 analysis. Geometry and thermal data was extracted from the simulation at the trailing edge of the solidus isotherm, and the following process maps were created.
5.2.1 Geometry
The length and the depth of the melt pool were measured in the finite element model, and a numerical root finding optimization technique was applied to fit the Rosenthal solution to the FEA depth. The length obtained from the FEA was also fitted via (3.11). The curves of constant area were obtained via the numerical root finding optimization technique, and the curves of constant length to depth ratio were obtained using (3.11).

A comparison of the geometric process maps for Ti64 and IN718 manufactured via the ARCAM process shows that the slopes of the curves of constant area for IN718 are slightly smaller than those for Ti64. This indicates that the power is slightly less sensitive to changes in velocity while being slightly more sensitive to power. Further observation shows that for IN718 L/D ratios equal to those in Ti64 occur at lower powers and velocities. The slight change in the area curves compared to the drastic change in the L/D curves between the two alloys indicates that for a certain set of process parameters in ARCAM manufactured IN718, the melt pool length is much smaller than that of Ti64 at the same set of parameters. This is corroborated by (3.11) and the fact that the thermal conductivity for IN718 is slightly higher than for Ti64.
5.2.2 Microstructure

The thermal gradient and cooling rate at the solidus temperature of 1533 K was obtained. The thermal gradient and cooling rate equations were fit, and the microstructure process map was constructed as shown below.

The process map shows that the mixed region covers a much larger area of process space than in Ti64. Once again, the red line represents the transition from columnar to mixed morphology. The blue line, however, is outside of the region of process space shown. This indicates that, according to this process map, equiaxed microstructure at the top of the melt pool is not possible under the specified conditions. A comparison of the lines of constant cooling rate between the two materials shows that the IN718 lines
rotate clockwise compared to the Ti64 lines, indicating that lower powers or higher velocities are needed for IN718 to obtain the same cooling rates as Ti64. This indicates that cooling rate in IN718 is less sensitive to changes in velocity than Ti64, but it is more sensitive to power.

![Geometric Process Map for IN718](image)

Figure 14 – Microstructure process map for IN718 manufactured via the ARCAM process at a preheat of 1023 K.

5.3 Inconel 625 – EOS Process

Finally, an axisymmetric FE model was developed for IN625 manufactured via the EOS selective laser melting process. A substrate preheat of 353 K was defined, and a 150 W point heat source traveled along the substrate at a rate of 600 mm/s. Geometric dimensions and thermal data from the top of the melt pool were extracted from the simulation at the solidus isotherm (1564 K), and the Rosenthal solution was fit to this data. Unlike Ti64 and IN718, no literature to date has produced a solidification map for IN625.
As a result, direct relationships between thermal behavior and resulting microstructure may not be constructed using the methodology described in the previous chapters. It has been shown, however, that the thermal behavior does indeed have a direct relationship to resulting microstructure [3, 6], so creating a thermal process map for IN625 will inform future process mapping efforts once solidification maps for IN625 are made available.

5.3.1 Geometry

![Geometric Process Map for IN625](image)

**Figure 15 – Analytical geometric process map for IN625 manufactured via the EOS process at a preheat of 353 K**

Geometry trends were developed using the fitted Rosenthal as shown in Figure 15. The cross-sectional areas and the L/D ratios are much smaller than the ARCAM manufactured Ti64 and IN718. This is due to
the lower powers characterizing the EOS process which decreases the length and the depth of the melt pools.

5.3.2 Thermal Conditions
Because previous work has not looked into the development of a solidification map for IN625, the solidification parameters necessary to construct a comprehensive microstructure process map are not readily available. Additionally, the nature of the EOS process by which the IN625 was manufactured is characterized by small melt pools. This may increase the influence of factors such as location specific material properties and compositional undercooling that the Hunt criterion curves do not generally take into account. For this reason, only the thermal behavior simulated by the FE model was determined, and that thermal behavior may be related to microstructure in future work.
Figure 16 – Analytical thermal process map constructed for IN625 manufactured via the EOS process at a preheat of 353 K

6 Experimental Procedure

Collaborative work between Carnegie Mellon University (CMU) and Wright State University (WSU) has sought to explore effects of beam power and velocity on both the geometric and microstructure characteristics of IN718 and IN625 in both laser and electron beam powder bed processes. For this thesis, the geometry measurements obtained from experimental single bead specimens are highlighted. All power settings plotted for the experimental data are the power settings at which the machine was set (hereafter referred to as machine power).

All optical images were obtained via a Keyence VHS30K digital optical microscope. Images obtained via scanning electron microscopy (SEM) and were produced using a Quanta ® 600F scanning electron
microscope. All microscopes and training were provided courtesy of the Air Force Research Laboratory’s (AFRL) Materials Characterization Facility at Wright Patterson Air Force Base. Measurements obtained from any experimental samples were obtained via the FIJI v.1.49t software that is provided as an open source software courtesy of the National Institutes of Health.

6.1 IN718 – ARCAM ®
CMU provided twenty-four IN718 single-beads with varying power and velocity combinations using laser parameters corresponding to the ARCAM process power and velocity range. The single bead specimens were deposited on a rolled IN718 plate obtained from McMaster-Carr, Inc., and the specimens were manufactured without powder. The substrate composition was assumed to be the standard composition as specified by Special Metals Inc. (Table II)

The experimental layout is shown below in Figure 17.

![Figure 17– Experimental setup for IN718 plate](image)

Initially the plate was cut in half cross ways using a water-cooled abrasive saw as indicated by the red dotted line. It was desired that the melt pool regions to be examined had reached a steady state condition
during processing, so that the melt pool geometry and thermal conditions would be fully developed and allow for repeatable data sets.

The specimens were sectioned with a low RPM diamond saw and the cross-sectional surface was polished. All twenty four lines of geometry were etched using Waterless Kalling’s Etchant (5gr CuCl₂+100cc HCl+100cc Ethyl Alcohol), exposing the melt pool geometries as illustrated by Table VI.

![Images of melt pools at different speeds and power levels](image)

**Table VI – Various etched melt pools for different machine power velocity combinations formed using the ARCAM process with no powder (100x magnification)**

**6.1.1 Experimental Geometry Data for IN718**

The width and depth of each melt pool was measured as shown in Figure 18 and the cross-sectional area was specified as the etched region. The data was plotted as shown in Figure 19-Figure 21 below.
It was initially observed that the relationship between melt pool width and beam power was a linear relationship for all velocities. It was also observed that each of the velocity curves had similar slopes with very little scatter. In order to obtain a clearer view of the relationship between width and velocity the same data points were plotted for different powers as a function of velocity in Figure 20.
Figure 20 – Process map for IN718 melt pool width trends for different powers

Figure 20 indicates that for powers with more than three data points, melt pool widths at the same machine powers remain relatively constant. There is a possibility that if more data points were to be included in the power ranges having only three data points, then a constant average melt pool width would be obtained.

Figure 21 – Process map for IN 718 melt pool depth with trends for different velocities
Figure 21 indicates a linear relationship between the power and the depth of melt pool geometry in this velocity range. A variance from that linear trend is noticed at about four power and velocity combinations, specifically in the 106 mm/s range and the 250 mm/s range. A comparison of these irregularities with the geometry shapes shown in Table VI indicates that these irregularities correspond to keyhole geometries (Figure 22-Figure 23). For example, in the 106 mm/s range, two data points at 556 W and 1111 W deviate from the linear trend.

![Etched keyhole melt pool geometries](image)

(a) (P,V) = (556 W, 106 mm/s),

(b) (P,V) = (1111 W, 106 mm/s)

Irregularities in the 250 mm/s velocity trend also indicate some keyhole geometries forming, as shown in Figure 23.
Figure 23 – Etched keyhole melt pool geometry, \((P,V) = (556 \text{ W}, 250 \text{ mm/s})\)

The geometry displayed in Figure 23 has some slight elongation in the depth dimension, which may account for the slight variance from linearity in Figure 21.

For modeling purposes the cross sectional area of the melt pool has been assumed to be directly related to the melt pool depth. Both the Rosenthal solution and finite element heat transfer models have resulted in semi-circular melt pool areas; therefore, the area is functionally related to the depth of the melt pool. It is expected then that the experimental area process map would follow similar trends to the process maps for melt pool depth (Figure 24).

Both process maps do show similar trends to the depth process maps. Interestingly, Figure 24 shows a linear trend with very little variation at the keyhole geometries. This contrasts with Figure 21, which does show the deviances from the linear trend for the depth measurements.
The linearity without scatter in the area process map shows that the experimental melt pool area is not as sensitive to the keyhole effect as is depth. The area remains linear while the depth experiences deviations from linearity at keyhole geometries.

6.1.2 Experimental Process Mapping IN718
Process maps were constructed to approximate lines of constant cross-sectional area based on the actual experimental data. The process maps were constructed by curve fitting lines to the experimental data and determining the power and velocity combinations that result in a single cross-sectional area. These combinations were noted, and plotted as shown in Figure 25.
Figure 25 – Experimentally produced process map for IN718 manufactured via the ARCAM process at a preheat of 1023 K.

The melt pool length data was not monitored in this experiment, so the curves of constant L/D ratio could not be constructed from the experimental data. Initial observations show a linear trend at higher velocities, which is consistent with observations made in the literature for Ti64 [3]. Very low slopes indicate a large influence of power on the cross-sectional area and a small influence of velocity. This compares well to the analytical data as will be discussed in the upcoming chapters.

6.2 IN625 – EOS ®
In addition to producing the IN718 AM specimens, CMU also provided IN625 single bead specimens manufactured by the National Institute of Standards and Technology (NIST) using the EOS process. The 42 different power and velocity combinations used in the manufacturing of the samples ranged from 50 W to 195 W and from 200 mm/s to 1200 mm/s. Each specimen was sintered using EOS laser technology, with a single 20 micron powder layer spread over a stock rolled IN625 plate. Each power and velocity track configuration was in the shape of a rectangle, with three power and velocity combinations “concentrically” oriented as shown below in Figure 26.
The plate was cut so that each concentric rectangle set was separated, and then each rectangle set was cut in half as indicated by the dotted line. The plate was sectioned in this way to allow for two separate data points for each power and velocity combination to be obtained.

6.2.1 Experimental Geometry Data for IN625

The cross-sectional areas of the single beads were polished to a 0.05 um finish, and they were imaged via scanning electron microscopy to measure the geometric features of interest. Example cross-sectional areas for IN625 are displayed in Figure 27.
The width, depth, and cross-sectional area measurements were taken and documented using the same procedure as for IN718, and similar trends were observed. Melt pool width measurements (shown in Figure 28) indicate that for IN625 manufactured via the EOS process, the width has a relatively linear, negative relationship with respect to velocity. At velocities on the high end of the spectrum, there is an upward turn for some of the power sets, but the general trend is a negative, linear relationship.
Figure 28 – Experimental width measurements for IN625 manufactured via the EOS process

This observation is different from what was seen in IN718. For IN718, the width stays pretty constant with respect to velocity. The difference in the trends could be an artifact of the process itself. The depth with respect to velocity follows a trend similar to that found in IN718. The melt pool depth follows an inversely proportional relationship to velocity for each of the different powers, as shown in Figure 29.

Figure 29 – IN625 melt pool depth measurements for the EOS process as a function of velocity for different machine powers
Plotting the depth as a function of machine power for multiple velocities resembles the linear trends produced by IN718 via the ARCAM process (Figure 30). Some of the measurements deviate from the linear trend, but there does not seem to be a correlation between any process-based phenomenon and resulting measurements that would explain this deviation. It is possible that the very small size scale of the EOS melt pools could be affected by small variations in beam power, beam velocity, material property variation, or powder layer thickness, causing the variations in the depth measurements. The cross-sectional area, however, does not diverge as much from the linear trend, as shown in Figure 31.

**Experimental Depth Measurements**

![Experimental Depth Measurements](image)

**Figure 30** – IN625 melt pool depth measurements for the EOS process as a function of machine power for different velocities
Figure 31 – IN625 melt pool cross-sectional area measurements for the EOS process as a function of power for different velocities

6.2.2 Experimental Process Mapping IN625
As described above, curves were fit to the experimentally collected data, and power and velocity combinations intersecting those lines for a constant area were noted. This produced the process map for IN625 shown in Figure 32.

Figure 32 – Experimentally constructed geometric process map for IN625 manufactured via the EOS AM process
As stated previously, the melt pool length data was not monitored in this experiment, therefore the curves of constant L/D ratio could not be constructed. Initial observations for IN625 show a linear trend at higher velocities which is consistent to observations made in the literature for Ti64 [3, 47, 33] as well as those discussed previously. The cross-sectional areas that the EOS process is capable of producing for IN625 are much smaller than the areas produced in IN718 via the ARCAM process, but a reduction in cross-sectional area also correlates with much higher cooling rates, as was indicated in Chapter 5.

7 Results and Discussion
The investigations described above were designed to facilitate the comparison of process maps constructed for multiple alloys and multiple processes. In the following sections, the validity of the fitting procedure prescribed above and the limitations of the models developed for this investigation are discussed in detail. Initial comparisons are made between the experimental and analytical process maps to gauge the accuracy of the finite element models in predicting actual geometry and thermal behavior. In addition, comparisons are performed between process maps constructed via the same process but different alloy systems, and those constructed via different processes with the same alloy system. The process maps are shown in both dimensional and non-dimensional space to give some insight into how the region of process space scales and shifts due to changes in material properties and process parameters.

7.1 Comparison of Analytical and Experimental Process Maps
It was necessary to compare results obtained experimentally and analytically to provide insight into the accuracy of the models and their ability to predict actual behavior. Comparisons were made for both IN718 and IN625, and the process variables used in the FE simulations corresponded with one of the experimental data points for the purpose of model validation.
7.1.1 IN718

7.1.1.1 Geometry
Comparing the single FEA simulation that was performed with the corresponding experimental melt pool shows that the FEA simulation predicted a melt pool with a cross-sectional area 10% smaller than that of the experimental melt pool for the same process variable. The reason for this discrepancy is probably because not all of the power impinging on the substrate was absorbed in the experiment. The model assumes that all energy is absorbed, but it has been stated previously in the literature that for electron beam processes, only 90% of the incident energy is absorbed [3]. All the data presented in Chapter 5 was presented as absorbed power, but in order to directly compare the analytical data to the experimental data a unit conversion was performed to change the absorbed power to machine power.

Figure 33 shows a comparison of lines of constant area developed using the two different methods described in Chapters 5 and 6. The solid lines represent the lines of constant area developed via the fitted Rosenthal approach, while the single points represent the areas observed experimentally. The violet line and data points represent predictions for 0.4 m$^2$, the yellow line and data points represent predictions for 0.3 mm$^2$, the red lines and data points represent predictions for 0.2 mm$^2$, and the blue line and data points represent predictions for 0.1 mm$^2$. 
A comparison of geometric process maps developed from analytical and experimental data shows that the analytical predictions are rotated slightly clockwise. This means that the analytical process map is slightly underpredicting the actual behavior. Although underprediction is not completely desirable, it does mean that the analytical model is going to give a lower bound of cross-sectional area, which could be useful in some applications.

It is also recognized that the experimental data points for the 0.30 mm$^2$ and 0.40 mm$^2$ data sets are slightly non-linear at lower velocities, which is not observed in the analytical prediction. From this plot, the analytical model as displayed above produces very good general approximations for cross-sectional area.
and presumably L/D ratio information, which may inform future efforts to predict and control melt pool size in AM processes.

7.1.1.2 Microstructure
Comparing microstructure observed experimentally to analytically calculated process maps is very difficult because the literature has not standardized the definition of “mixed” grain morphology. The characterization of the experimental IN718 microstructure was attempted for this thesis to show actual microstructure morphology compared to the analytical predictions.

!!Figure 34 – Microstructural comparison between experimental melt pools and analytically constructed process map.!!

The image on the left of Figure 34 is the melt pool produced with a machine power of 106 W and a velocity of 500 mm/s. The microstructure at the top shows long, columnar grains. The image on the right is the melt pool produced with a machine power of 2778 W and a velocity of 500 mm/s, and the microstructure is slightly shorter. This is a qualitative assessment, and future work must be performed to verify these results. If further review of IN718 microstructure reveals that the curves produced in this thesis do not accurately predict the microstructure, the first step to improving the accuracy would be to modify Hunt’s
solidification model to predict the correct microstructure. Once this model is improved, similar derivations may be used to construct more accurate process mapping equations.

7.1.2 IN625
A similar comparison was made between the experimental and analytical geometry data obtained for IN625. As was performed for the IN718, the analytical data was converted from absorbed power to machine power by changing the value of the absorptivity factor to 0.57. This value was chosen to maintain consistency with the literature [33]. The experimental data was then overlaid on the analytical process map, and the lines of constant area were compared.

![Geometric Process Map for IN625](image)

Figure 35 - Comparison of lines of constant area developed using analytical methods (solid line) and experimental data (*) for IN625 manufactured via the EOS AM process with a 353 K preheat.
The experimental data shows that at higher velocities, the relationship between power and velocity is nearly linear for lines of constant area, and these lines have a very small slope when compared with the Ti64 and IN718 process maps. This difference is due mainly to the region of process space in which the IN625 was manufactured. The relatively flat slope may be the reason that the analytical model is not as accurate for IN625 as it is for the IN718 process map. The slopes of the curves are flat and slightly non-linear which means that any inconsistencies in the nature of the Rosenthal solution are going to be amplified in the process map. The analytical prediction however, is still a good first order approximation of lines of constant area across the entire process, and may be coupled with further FEA validation to provide a better prediction.

7.1.3 Process Map Comparison Across Multiple AM Alloys and Processes

Comparison of process maps constructed for various AM processes for both Ti64 and IN718 was conducted in non-dimensional process space. Performing the comparison in this way allows for the size and location of the actual material and process specific process spaces to be viewed in relation to each other using a single set of solidification curves described by (3.37) and (3.38). Just as Figure 2 describes the process capability of each AM process in terms of process variables, Figure 36 and Figure 37 exhibit the microstructural capability of the same AM processes in terms of non-dimensional process variables which include thermal Rosenthal behavior, non-Rosenthal correction, and material solidification parameters. Axisymmetric FEA simulations were conducted to represent Ti64 and IN718 manufactured via the various AM processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>Machine Power Range (W)</th>
<th>Velocity Range (mm/s)</th>
<th>Nominal Preheat (K)</th>
<th>Absorbed Power, $\alpha$</th>
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</thead>
<tbody>
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<td>50</td>
<td>3000</td>
<td>100</td>
<td>1023</td>
</tr>
<tr>
<td>EBF3</td>
<td>50</td>
<td>6000</td>
<td>0</td>
<td>42.3</td>
</tr>
<tr>
<td>EOS</td>
<td>25</td>
<td>250</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td>LENS</td>
<td>25</td>
<td>500</td>
<td>0</td>
<td>42.3</td>
</tr>
</tbody>
</table>

Table VII – Process map summary plotted in Figure 36 and Figure 37
Using the thermal data from the simulations, the closed form Rosenthal solution was fit, and rectangles representing the respective process spaces of interest (summarized in Table VII) were overlaid on a log-log plot of the non-dimensional microstructure curves.

\[
\bar{u} = \frac{vC_0}{AC_0^2}
\]

Figure 36 – Process Map comparison for IN718 between AM processes representative of a wide range of process space
The non-dimensional process maps for IN718 and Ti64 displayed in Figure 36 and Figure 37 are qualitatively very different from each other. The Ti64 process space reaches higher non-dimensional powers while IN718 maintains lower ranges of non-dimensional power. This indicates that processes using Ti64 will be more likely to produce mixed or equiaxed microstructure than those using IN718. All of the process spaces examined for the Ti64 material cross over the red columnar to mixed boundary curve, and three out of the four crossed over the blue mixed to equiaxed boundary curve. Conversely, processes manufacturing IN718 components will most likely not be able to obtain equiaxed microstructure. In fact, only one of the four AM processes described in this thesis is predicted to have some region of process space capable of obtaining even mixed microstructure. The other three processes remain in the columnar...
region. It must be noted here that Thompson published a process map for IN718 manufactured via the Sciacky (same process space as EBF3) process in his Master thesis [4] that does not agree with the above non-dimensional representation. After a private communication with Thompson, it was determined that errors were made in his thesis in representing the Hunt criterion solidification curves and in modeling the actual AM process which were propagated to the process map curves he created. The process map represented by the pink box in Figure 36 is considered to be the accurate process map in non-dimensional process space, and it predicts that for the given process space, only columnar microstructure should be expected.

In general, the Ti64 space reaches higher non-dimensional powers than IN718 mainly due to the fact that the number of nucleation sites ($N_0$) is much higher than that of IN718. Similarly, the IN718 space reaches higher velocities than Ti64 mainly due to the fact that the undercooling in IN718 is smaller than that of Ti64. It seems that for this comparison, the process space size and shape is much more dependent on the process variable range than on the material. The location of the box, however, is dependent on the solidification and thermal behavior induced by the process.

7.1.4 A Discussion on the Role of Modeling in Process Mapping
Over the course of many years, the modeling capability available to the scientific community has progressed from simple, closed-form solutions to differential equations to discretized numerical solvers to even more complicated “all-physics-included” models that all are characterized by different levels of complexity. The type of model that is used in a given application is determined by a balance of accuracy of the final product and efficiency in the calculation process.

The methodology developed in this thesis provides a way to more efficiently predict thermal behavior in AM processes, and opportunities for greater accuracy are controlled by the complexity and reliability of the modeling approaches. In this investigation, axisymmetric substrate geometries were heated by a
concentrated heat flux moving to discrete locations along the substrate edge. It is recognized that many assumptions were made in the construction of the model, and opportunities to increase the capability of the model are available including:

1. Increasing mesh resolution
2. Distributing the heat flux to account for beam spot size
3. Modeling Beam absorption and material emissivity
4. Accounting for surface convection
5. Accounting for powder density and powder/beam interactions
6. Modeling fluid and gas flow within the melt pool
7. Including material composition and thermodynamic effects

As each of these factors are included in the models and validated with experimental data the accuracy of process maps resulting from the methodology prescribed above will continue to increase. While the Rosenthal solution and the related derivations produced in this thesis are not meant to completely replace numerical solving methods, they do have the ability to provide a sufficient approximation that can complement numerical methods to inform more in-depth process mapping investigations.

8 Contributions
Summarizing the conclusions made in the reported work, the contributions of this thesis include:

1. Using the Rosenthal solution for a moving point heat source to derive closed-form solutions for melt pool length, thermal gradient, and cooling rate at the top of the trailing edge that may be used as tools to predict thermal behavior in AM single beads
2. Using (1) in conjunction with the Hunt criterion equations to derive closed form microstructure process maps for any beam based AM process and alloy system
3. Using (1) and (2) to adapt previously existing process mapping methodologies across multiple alloy systems and processes (Ti64, IN718, IN625, ARCAM, EOS, and EBF3), including a comparison with available experimental data

4. Using (1) and (2) to derive non-dimensional microstructure process maps for any material system and beam-based AM process and illustrates the utility for Ti64 and IN718

9 Future Work
While the work in this thesis has presented a thorough investigation of a novel approach for predicting melt pool geometry and microstructure in AM single beads, little work has been performed in verifying those predictions and increasing their accuracy. This work spawns multiple opportunities for future work including:

1. Tuning FEA models to more accurately predict experimental behavior at points of interest.
2. Investigating how the Hunt solidification curves change when entering the rapid solidification domain
3. Analyzing microstructure in experimental specimens for validation of microstructure process maps
4. Utilizing the closed-form microstructure boundary curves in other applications such as AM material design, process development, and welding applications

10 Conclusion
The work presented in this thesis has produced a thorough investigation into the applicability of a closed-form solution for a moving point heat source to accurately model AM processes. The adapted methodology for process mapping and the closed-form microstructure process mapping equations developed in this thesis greatly increase the temporal and computational efficiency of the process mapping approach. This is based on a substantial reduction in the number of FEA simulations and iterative loops necessary to produce a high-resolution process map for any alloy. Using these adapted methods,
various process maps have been constructed for multiple different materials and AM processes, and the process maps have been compared to each other and to experimental results providing insight into mechanisms driving solidification in AM single beads across alloy systems. The results and conclusions of this thesis provide insight into the governing physics inherent in AM processes and will help inform future endeavors in process mapping of AM processes. Ultimately, the results of this work will help facilitate the development of feedback controls and aiding in the pursuit of in-situ microstructure and property control.
Bibliography


[38] E. Soylemez, J. Beuth and K. Taminger, "Controlling Melt Pool Dimensions Over a Wide Range of Material Deposition Rates in Electron Beam Additive Manufacturing".


Appendix A – Code to Fit Thermal Data

```matlab
function [beta,G_fit,CR_fit,G_error,CR_error]=Fit_Therm_beta(Q,alpha,v,...
    T_iso,T0,G,CR,Material_Name)

%This code was designed to fit the Rosenthal solution to analytically
%determine thermal gradients and cooling rates. To use this code, you
%must run one FEA simulation under a nominal set of process parameters.
%Extract the thermal gradient and cooling rate from the top of the melt
%pools trailing edge and define the variable G and CR as those values.
%Define the process variables used and the absorptivity.
%NOTE: All values input into this code must be in standard base SI units
%(m,kg,s,K,etc.)

%This code was developed by Luke Sheridan for his Master's thesis entitled:
%"An Adapted Approach to Process Mapping Across Alloy Systems and Additive
%Manufacturing Processes".
%NOTE:The variable beta that is used in this function file is the same as
%the variable phi used in the above thesis.

beta=linspace(0.001,5,100); %Defines a vector of beta values that will be
%used to minimize the error between the Rosenthal solution and the input
%thermal gradient

%Defines the thermal conductivity used in the calculations.
if strcmp(Material_Name,'IN718')
    k=0.0144*(T_iso-273)+10.415;
elseif strcmp(Material_Name,'Ti64')
    k=0.0161*(T_iso-273)+6.3976;
elseif strcmp(Material_Name,'IN625')
    k=0.0155*(T_iso-273)+9.2811;
else
    error('This material is not included in the default material list')
end

G_fit=2*pi*k*(T_iso-T0)^2./(beta*alpha*Q); %Calculates a value for thermal
%gradient for every value of beta
CR_fit=G_fit*v; %Calculates a value for cooling rate (dT/dt) for every
%value of beta
CR_error=abs((CR-CR_fit)/CR); %Error between the Rosenthal value and the
%input value
G_error=abs((G-G_fit)/G); %Error between the Rosenthal value and the input
%value

[xG,IG]=min(G_error); %Minimum error and location in the vector
[xCR,ICR]=min(CR_error); %Minimum error and location in the vector

if IG==1 %Defines cases for if there is a min at the beginning of vector
    IG=2;
elseif IG==length(G_error)
    IG=length(G_error)-1;
else
end
```

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if ICR==1 %Minimum error and location in the vector
    ICR=2;
elseif ICR==length(CR_error)
    ICR=length(CR_error)-1;
else
end

beta=linspace(beta(IG-1),beta(IG+1),100); %Refined beta vector

G_fit=2*pi*k*(T_iso-T0)^2./(beta*alpha*Q); %Thermal gradient for each beta
CR_fit=G_fit*v; %Cooling rate for each beta
CR_error=abs((CR-CR_fit)/CR); %Error between Rosenthal and input value
G_error=abs((G-G_fit)/G); %Error between Rosenthal and input value

[xG,IG]=min(G_error); %Location in vector of minimum value
[xCr,ICr]=min(CR_error); %Location in vector of minimum value

beta=mean([beta(IG),beta(ICr)]); %Beta is the average of the betas obtained %for cooling rate and thermal gradient

G_fit=2*pi*k*(T_iso-T0)^2./(beta*alpha*Q); %Value of the Rosenthal thermal %gradient (This should be the same as the input thermal gradient.)
CR_fit=G_fit*v; %Value of the Rosenthal cooling rate (This should be the %same as the input cooling rate.)

G_error=xG; %Error in the Rosenthal thermal gradient calculation
CR_error=xCr; %Error in the Rosenthal cooling rate calculation
Appendix B - Plot Non-Dimensional Process Spaces

```matlab
%NDMap_Ti64
%This code was designed to plot multiple process spaces on a single set of
%non-dimensional microstructure process map curves. To use this code, you
%must run one FEA simulation under a nominal set of process parameters.
%Extract the thermal gradient from the top of the melt pool's trailing edge
%and define the variable G as that value. Define the process variables used
%and the absorptivity, and then define the region of process space that you
%wish to map. NOTE: All values input into this code must be in standard
%base SI units (m,kg,s,K,etc.)

%This code was developed by Luke Sheridan for his Master's thesis entitled:
%"An Adapted Approach to Process Mapping Across Alloy Systems and Additive
%Manufacturing Processes".
%NOTE:The variable beta that is used in this function file is the same as
%the variable phi used in the above thesis.

Initialization

close all
clear all
c1c

Comparison 1

Material_Name='Ti64'; %Name of Material for definition of properties
Process_Name='ARCAM'; %Name of the Process being mapped
Power=1111; %Power value at which thermal gradient was obtained
alpha=0.9; %Absorptivity factor for the material and process
Velocity=0.5; %Velocity at which thermal gradient was obtained
T_iso=1893; %Temperature of interest
T0=1023; %Substrate preheat temperature
G=160664; %Thermal gradient extracted from top of the melt pool in FEA
P_range=[50,3000]; %Process power range
V_range=[0.100,1.000]; %Process velocity range
file_name='Process_Comparison_Ti64'; %What you would like to name the image

CR=G*Velocity; %R=1/G*dT/dt -> dT/dt=G*R
[beta_fit,G_fit,CR_fit,G_error,CR_error]=Fit_Therm_beta(Power,alpha,...
Velocity,T_iso,T0,G,CR,Material_Name); %This function fits the
%Rosenthal solution to the extracted data. (Beta is the fitting factor)

Definition of Variables

if strcmp(Material_Name,'IN718')==1
    NO=7e8; %Nucleation sites per unit volume
    TN=7e-1; %Undercooling
```
C0=4e-1; %Material Composition
A=1.875e-5; %Experimental Constant
k=5e-6*(T_iso-273)^2+0.0085*(T_iso-273)+11.26; %Thermal Conductivity

elseif strcmp(Material_Name,'Ti64')==1
N0=4e12;
TN=1;
C0=4e-1;
A=1.875e-5;
k=0.0161*(T_iso-273)+6.3976;
else
    error('The material properties for this material have not been defined.')
end

Pbar_range=0.617*alpha*beta_fit*P_range*N0^(1/3)*TN/(2*pi*k*(T_iso-T0)^2); %Converts the process power range to non-dimensional power range
Vbar_range=V_range*C0/(A*TN^2); %Converts the process power range to non-dimensional power range

Vbar=logspace(0,5,10^5); %Define the ND velocity range for the curve plots
Qbar_eq=Vbar./(Vbar.^1.5-1); %Calculate the Q values for the eq curve
Qbar_col=Vbar./(100^(1/3)*(Vbar.^1.5-1)); %Calculate the Q values for the col curve

Comparison 2

Material_Name='Ti64';
Process_Name='EBF3';
Power=1500;
alpha=0.9;
Velocity=40*0.000423333;
T_iso=1893;
T0=373;
G=349019;
P_range=[50,6000];
V_range=[0.0000001,100*0.000423333];

CR=G*Velocity;
[beta_fit,G_fit,CR_fit,G_error,CR_error]=Fit_Therm_beta(Power,alpha,...
    Velocity,T_iso,T0,G,CR,Material_Name);
Pbar_range=0.617*alpha*beta_fit*P_range*N0^(1/3)*TN/(2*pi*k*(T_iso-T0)^2);
Vbar_range=V_range*C0/(A*TN^2);
lime=[Vbar_range(1),Pbar_range(1),(Vbar_range(2)-Vbar_range(1)),...
Comparison 3

Material_Name = 'Ti64';
Process_Name = 'EOS';
Power = 150;
alpha = 0.57;
Velocity = 0.6;
T_iso = 1893;
T0 = 353;
G = 6.8419e6;
P_range = [50, 250];
V_range = [0.1, 1.000];

CR = G * Velocity;
[beta_fit, G_fit, CR_fit, G_error, CR_error] = Fit_Therm_beta(Power, alpha, ...
    Velocity, T_iso, T0, G, CR, Material_Name);
Pbar_range = 0.617 * alpha * beta_fit * P_range * N0^(1/3) * TN / (2 * pi * k * (T_iso - T0)^2);
Vbar_range = V_range * C0 / (A * TN^2);
lim = [Vbar_range(1), Pbar_range(1), (Vbar_range(2) - Vbar_range(1)), ...
    Pbar_range(2) - Pbar_range(1)];
rectangle('Position', lim, 'LineStyle', '--', 'LineWidth', 2, 'EdgeColor', 'm');

Comparison 4

Material_Name = 'Ti64';
Process_Name = 'LENS';
Power = 225;
alpha = 0.35;
Velocity = 40 * 0.000423333;
T_iso = 1893;
T0 = 298;
G = 1.37e6;
P_range = [10, 500];
V_range = [0.00000001, 100 * 0.000423333];

CR = G * Velocity;
[beta_fit, G_fit, CR_fit, G_error, CR_error] = Fit_Therm_beta(Power, alpha, ...
    Velocity, T_iso, T0, G, CR, Material_Name);
V = linspace(0.0001, 2, 10^5);
Pbar_range = 0.617 * alpha * beta_fit * P_range * N0^(1/3) * TN / (2 * pi * k * (T_iso - T0)^2);
Vbar_range = V_range * C0 / (A * TN^2);
lim = [Vbar_range(1), Pbar_range(1), (Vbar_range(2) - Vbar_range(1)), ...
    Pbar_range(2) - Pbar_range(1)];
rectangle('Position', lim, 'LineStyle', '--', 'LineWidth', 2, 'EdgeColor', 'b');

xlabel 'Non-Dimensional Velocity'
ylabel 'Non-Dimensional Machine Power'
msg = sprintf('Microstructural Process Map Comparison for %s', Material_Name);
title(msg)
axis tight;
ylim([10^-5, 10^1])
legend('Mixed to Equiaxed', 'Columnar to Mixed')
cd('Images')
cd('Microstructure')
print('-dbmp', file_name)
Appendix C - Plot Geometric Process Maps

%Fit_Plot_Area_PM_Absorbed

This code was designed to plot geometric process maps in dimensional process space. To use this code, you must run one FEA simulation under a nominal set of process parameters. Extract the depth and length and define the variables z and L as those values. Define the process variables used and the absorptivity, and then define the region of process space that you wish to map. NOTE: All values input into this code must be in standard base SI units (m, kg, s, K, etc.)

This code was developed by Luke Sheridan for his Master's thesis entitled: "An Adapted Approach to Process Mapping Across Alloy Systems and Additive Manufacturing Processes".

material_name='IN625'; %Material Name
process='EOS'; %Process Name
source='Analytical'; %Was the measured data analytical or experimental
Q=150; %Machine Power
v=0.6; %Velocity
alpha=1; %Absorptivity (1 for absorbed, other for machine)
phi=linspace(0.001,2,50); %Vector of phi values for optimization
T_iso=1564; %Isotherm of interest
T0=80+273; %Preheat
z=9.374e-5; %Measured depth
L=1.182e-3; %Measured length
A_desired=[0.0202 0.0101 0.00505 0.002525]/1000^2; %Values of desired lines of constant area
LD_desired=[4 6 8]; %Values of desired curves of L/D ratio
PM_limits=[0.1,1,1,250]; %Process limits [Vel1,Vel2,Pow1,Pow2]

dim='Area'; %What dimension are you fitting?
Q_l=Q;
v_l=v;

h=waitbar(0,'Obtaining Fitting Parameters');
for i=1:length(phi) %Fits the Rosenthal solution to the measured data
    value(i)=TBAR(Q,alpha*phi(i),v,T_iso,T0,-z,material_name,dim);
    %Evaluates the Rosenthal solution with the given process parameters
end

plot(phi,value);[x,I]=min(value);%If the minimum is
if I==length(phi)
    msg1=sprintf('The error in the Area fit is %f',x); %If the minimum is
    %not an absolute minimum, the error in the fit is recorded
    disp(msg1)
else
clc
close(h)
disp('Fit Temperature Found')
end

phi=phi(I); \%phi is defined as the value of the array phi that gives the
\%smallest error

z_desired=-(A_desired*2/pi).^0.5; \%The desired depth is calculated from the
\%desired area assuming a semi-circular profile

counter=0;

Q=linspace(PM_limits(3),4*PM_limits(4),50); \%An array of Q is defined for
\%optimization
v=linspace(PM_limits(1),PM_limits(2),4); \%An array of v is defined for
\%optimization

h=waitbar(counter,'Assembling Area Process Map');
for k=1:length(v)
    for j=1:length(z_desired)
        if k==1 && j==4
            Q=linspace(PM_limits(3),PM_limits(4),100);
        else
            counter=counter+1;
            waitbar(counter/(length(v)*length(z_desired)),h)
            for i=1:length(Q)
                [value(i),Tbar,depth]=TBAR(Q(i),alpha*phi,v(k),T_iso,T0,z_desired(j),material_name,dim);
            end
            [x,I]=min(value);
            clear value
            if I==1|I==length(Q)
                else
                Q=linspace(Q(I-1),Q(I+1),100);
            for i=1:length(Q)
                value(i)=TBAR(Q(i),alpha*phi,v(k),T_iso,T0,z_desired(j),material_name,dim);
            end
            [x,I]=min(value);
            clear value
            for i=1:length(Q)
                [err_A(j,k),Tbar(j),depth(j,k)]=TBAR(Q(i),alpha*phi,v(k),T_iso,T0,z_desired(j),material_name,dim);
            end
            Power(j,k)=Q(I);
            Q=linspace(PM_limits(3),3*PM_limits(4),50);
        end
    end
end

close(h)
figure
[R,C]=size(Power);
for i=1:R
    plot(1000*v,Power(i,:),'LineWidth',2)
    hold on
end
axis(PM_limits)
xlabel('Velocity (mm/s)')
ylabel('Absorbed Power (W)')
axis([PM_limits(1:2)*1000,PM_limits(3:4)])

beta_A_absorbed=phi

disp('Assembling L/D Process Map')
h=waitbar(0,'Fitting the Length for L/D Process Map');
clear value error
dim='Length'

phi=linspace(0.001,5,100)
for i=1:length(phi)
    waitbar(i/(length(phi)),h)
    [err_l(i), Tmbar, dim_calc]=TBAR(Q_l,alpha*phi(i),v_l,T_iso,T0,L,material_name,dim);
end

[x,I]=min(err_l)
if I==length(phi)
    msg2=sprintf('The error in the Length fit is %f. Please change the beta range above.',x);
    error(msg2)
else
    phi=phi(I);
end
close(h)
counter=0;
h=waitbar(counter,'Constructing L/D Process Map');

for j=1:length(LD_desired)
    for i=1:length(z_desired)
        counter=counter+1;
        waitbar(counter/(length(LD_desired)*length(z_desired)),h)
        Q_LD(i,j)=-LD_ratio(LD_desired(j),z_desired(i),alpha*phi,T_iso,T0,L,material_name);
        [value,I]=min(abs(Q_LD(i,j)-Power(i,:)));
        v1=v(I)
        Q1=Power(i,I);
        if Q_LD(i,j)>Power(i,I)
            if I+1>length(v)
                error('The specified L/D ratios are out of range.')
            else
                v1=v1
            end
        end
    end
end
end
v2=v(I+1);
Q2=Power(i,I+1);
else
    if I==1
        v2=v(2);
        Q2=Power(i,2);
    else
        v2=v(I-1);
        Q2=Power(i,I-1);
    end
end

v_LD(i,j)=(v2-v1)*(Q_LD(i,j)-Q1)/(Q2-Q1)+v1; %Linear interpolation
%along lines of constant Area to find the point for the L/D curve

end
end

close(h)

[R,C]=size(Q_LD);
for i=1:C
    plot(v_LD(:,i)*1000,Q_LD(:,i),'x-','Linewidth',2)
    hold on
end

a=sprintf('%0.3f mm^2',A_desired(1)*1000^2);
b=sprintf('%0.3f mm^2',A_desired(2)*1000^2);
c=sprintf('%0.3f mm^2',A_desired(3)*1000^2);
d=sprintf('L/D=%d',LD_desired(1));
e=sprintf('L/D=%d',LD_desired(2));
f=sprintf('L/D=%d',LD_desired(3));
g=sprintf('%0.3f mm^2',A_desired(4)*1000^2);
% h=sprintf('L/D=%d',LD_desired(4));
legend(a,b,c,g,d,e,f,'Location','NorthWest')
title(sprintf('Geometric Process Map for %s',material_name))

if strcmp(source,'Analytical')==1
    title=sprintf('%s %s Geometry_absorbed',process,material_name);
else
    title=sprintf('%s %s (EXP) Geometry_absorbed',process,material_name);
end

cd('/Users/LSheridan/Google Drive/Master Thesis Work/Rosenthal/Images')
print('-dbmp16m',title)
cd('../')

phi_L_absorbed=phi

Fit_Plot_Area_PM_Machine %Fit_Plot_Area_PM_Machine is the same as this
%script with an absorptivity factor not equal to zero
Appendix D - Determine the Error in the Rosenthal Solution with a Fit

```matlab
function [value, Tmbar, dim_calc]=TBAR(Q, alpha, v, T_iso, T0, dim_value, mat, dim)
    %This code was designed to determine the error predicted by the Rosenthal solution and the measured data input into the function.
    %To use this code, you must run one FEA simulation under a nominal set of %process parameters. Q is the power, alpha is the absorptivity, v is the %velocity, T_iso is the temperature of interest, T0 is the substrate %temperature, dim_value is the length or depth measurement, mat is the %material, dim is the dimension of interest (Area or Length) NOTE: All %values input into this code must be in standard base SI units
    %(m,kg,s,K,etc.)

    %This code was developed by Luke Sheridan for his Master’s thesis entitled:
    %“An Adapted Approach to Process Mapping Across Alloy Systems and Additive %Manufacturing Processes”.

    if strcmp(mat, 'Ti64')==1 %Define thermal properties based on material
        rho=-0.1406*(T_iso-273)+4471;
        c=0.00007*(T_iso-273)^2+0.0207*(T_iso-273)+610.57;
        k=0.0161*(T_iso-273)+6.3976;
        elseif strcmp(mat, 'IN718')==1
            rho=-4e-4*(T_iso-273)^2-0.049*(T_iso-273)+8133;
            c=0.1993*(T_iso-273)+422.44;
            k=0.0144*(T_iso-273)+10.415;
        elseif strcmp(mat, 'IN625')==1
            rho=-5e-5*(T_iso-273)^2-0.0892*(T_iso-273)+8440.7;
            c=0.244*(T_iso-273)+405.28;
            k=0.0155*(T_iso-273)+9.2811;
        else
            end

    Tmbar=(T_iso-T0)/((alpha*Q/(pi*k))*(rho*c*v/(2*k)));
    if strcmp(dim, 'Area')==1
        ND=-1/sqrt(Tmbar); %Approximation for non-dimensional depth
        A=100; %Resolution
        x0bar=-2/(2*Tmbar); %Approximation of length of melt pool
        error=1;
        D = linspace(0,ND,A)';

        t = 1;

        while abs(error)>0.000001 %Iterate through root finding routine until
            %error reaches necessary level
                clear z0bar y0bar x0bar(2:length(x0bar)) z0barnorm

            % Initializations
            m = 1;
            n = 1;

                while n <= A
```
x0bar(m) = D(m);
y0bar(m) = 0;

x(m) = fzero(@(x,x0bar,optimset('display','off'),Tmbar,z0bar),x0bar,m);
x0bar(m+1)=x(m);

if isnan(x(m))==0
    m=m+1;
    n=n+1;
else
    n=A+1;
end

end

if m==101
    error=(ND-z0bar(m-1))/z0bar(m-1);
    ND=z0bar(m-1);
elseif isnan(x(m))==0
    ND=ND-1;
    ND=z0bar(m);
else
    error=(ND-z0bar(m))/z0bar(m);
    ND=z0bar(m);
end
x0bar=x0bar(m-1);
D=linspace(z0bar(m-1),ND,A);
t=t+1;
end

depth=ND*2*k/(rho*c*v);
value=abs((dim_value-depth)/dim_value);
dim_calc=depth;
elseif strcmp(dim,'Length')==1
    length=alpha*Q/(2*pi*k*(T_iso-T0));
    value=abs((dim_value-length)/dim_value);
dim_calc=length;
else
    error('This dimension is not supported by this function');
end
Appendix E – Generate ABAQUS Input File

clc
clear all
close all

%INPUTS%
format short
Power=150; %Power (W)
velocity=0.6; %Velocity (m/s)
melting_temp=1533; %Melting Temp (K) (Preferably the lowest temperature in
% the melting range)
preheat=80+273; %Base Preheat Temp (K)

Material_Name='IN718'

rho=-0.0004*(melting_temp-273)^2-0.049*(melting_temp-273)+8133; %Density
%(kg/m^3)
c=0.1993*(melting_temp-273)+422.44; %Specific Heat (J/kg-K)
k=5e-6*(melting_temp-273)^2+0.0085*(melting_temp-273)+11.26; %Thermal
%Conductivity (W/m-K)
Tmbar=((melting_temp-preheat)/(((Power)/(pi*k))*((rho*c*velocity)/
...(2*k)))); %Non-Dimensional Temperature

RosLength=1/(2*Tmbar)*(2*k/(rho*c*velocity)); %Non-Dimensional Length
RosDepth=0.7/sqrt(Tmbar)*(2*k/(rho*c*velocity)); %Non-Dimensional Depth
% Estimate (Just an estimate. Not the real thing.)
TotalSteps=350; %Total Number of Steps across the Length of the Substrate
% (Must be multiple of 100)
finedepthN=25; %Total number of nodes in the fine region of the depth
biasdepthN=15; %Total number of nodes in the biased region of the depth
% Generate Geometry******************************************************%
% Calculates required geometry dimensions
Depth=RosDepth*20; %Substrate geometry depth is 20X the Rosenthal Depth
Length=RosLength*8; %Substrate geometry length is at least 4X the Rosenthal
%Depth (6X for simulations with Temp dependent properties)

rho=rho; %Redefine the thermal properties for insertion into material
% definitions
Cp=c;
K=k;

% File Name
filename=sprintf('%s_P%d_V%d_Ph%d_Rosenthal.inp',Material_Name,Power,...
velocity*1000,preheat-273); %writes the input file name
fid=fopen(filename, 'w'); %Opens the input file for writing

% Concentrated Area Coordinates**************************************
% Predefine Matrices
xc=[];
yc=[];
zc=[];

% Total number of steps split into regions
LeftBiasSteps=round(TotalSteps*.15); % 15% of the steps are in the bias
% leading up to the fine mesh region
ConcentratedCenterSteps=round(TotalSteps*.75); % 75% of the steps are in
% the fine mesh region
RightBiasSteps=round(TotalSteps*0.1); % 10% of the steps are in the bias
% leading away from fine mesh region. These steps are not included in
% the actual analysis

% Bias parameter
b=0.95; %elemental bias
% length of the left bias region
X(LeftBiasSteps)=Length/2; %The bias region leading to fine region takes up
%half of the substrate length

% Calculate the first distance in the biased region
% NOTE: Starts from the right side
sum=0;
for i=1:LeftBiasSteps-1 % Create Biased Mesh leading up to fine mesh region
    sum=sum+1/(b^i);
end
FirstZDistance=x1=(X(LeftBiasSteps))/(1+sum);
X(LeftBiasSteps-1)=X(LeftBiasSteps)-x1;
% Starts from the right side, so define k as the step number to correct this
k=LeftBiasSteps-1;
for j=2:LeftBiasSteps-1
    dx=x1/(b^(j-1));
    X(k)=X(k+1)-dx;
    j=j+1;
    k=k-1;
end
X(LeftBiasSteps)=Length/2;

% Creates the Fine Mesh Region of the length
% First/last Index
k1=LeftBiasSteps;
k2=LeftBiasSteps+ConcentratedCenterSteps;
dx=((Length)/3)/ConcentratedCenterSteps;
k2a=k2-1;
for m=k1:k2a
    X(m+1)=X(m)+dx;
end
plot(X)
hold

% Creates the Bias region leading away from the fine mesh region
clear j
% First/last index
n1=k2;
n2=n1+RightBiasSteps;
b = 0.9;
sum = 0;
for i = n1:n2-1
    sum = sum + 1/(b^i);
end
x1 = (Length/6)/(1 + sum);
for j = n1:n2-1
    dx = x1/(b^j);
    X(j+1) = X(j) + dx;
end
plot(X)

%*****************************************************************************
% The depth of the fine mesh region is approx. 1 melt pool deep
Y = 0;
dy = fineYlength/finedepthN; % There are N number of nodes through the depth
    % of the melt pool (defined above)
    % fine region
for i = 2:finedepthN+1
    Y(i) = Y(i-1) + dy;
end

b = 0.8; % Create Bias Mesh leading away from fine region in the depth
    % direction
sum = 0;
biasYlength = Depth - fineYlength;
Y(finedepthN+1+biasdepthN) = Depth;
k1 = finedepthN+1;
k2 = finedepthN+biasdepthN;

for i = k1:k2-1
    sum = sum + 1/b^i;
end
y1 = biasYlength/(1+sum);
for j = k1:k2-1
    dy = y1/(b^j);
    Y(j+1) = Y(j) + dy;
end
plot(Y)

%*****************************************************************************
% CreateNodes
[L,D] = meshgrid(X,Y); % Create a mesh grid from the length and depth vectors
    % that specifies coordinates in the length and depth directions
U = [D(:,1),L(:,1)]; % Take apart the Depth and Length Matrices
figure
scatter(U(:,1),U(:,2),'.') %Create Plot to show the resultant mesh
xlabel 'Depth'
ylabel 'Length'

%%%%%%%%%%%%%%%%Begin Creation of Input File%%%%%%%%%%%%%%%%%%%%%%%

fprintf(fid,'*HEADING
 **3D Analysis \n');

%********************************************************************
%****************************Create Nodes*****************************
%********************************************************************
fprintf(fid,'*NODE, NSET=ALLN \n');
for i=1:length(U)
    fprintf(fid, '%d, %e, %e\n', i, U(i,:));
end

% %********************************************************************
% %****************************Create Elements****************************
% %********************************************************************
fprintf(fid,'**\n');
fprintf(fid,'*ELEMENT, ELSET=ALLE, TYPE=DCAX4 \n');
e=0;
for j=0:length(Y)-3
    for i=[1:length(Y):length(U)-length(Y)]
        e=e+1;
        a=i+j;
        b=a+1;
        c=b+length(Y);
        d=c-1;
        elem=[e,a,b,c,d];
        fprintf(fid,'%d,%d,%d,%d,%d\n',elem);
    end
end

% %********************************************************************
% %****************************Material Properties****************************
% %********************************************************************
% %********************************************************************
fprintf(fid, '**Material Properties\n');
fprintf(fid, '*MATERIAL, NAME=%s \n', Material_Name);
fprintf(fid, '*DENSITY \n');
fprintf(fid, '%f \n', rho);
fprintf(fid, '*SPECIFIC HEAT \n');
fprintf(fid, '%f', Cp);
fprintf(fid, '*CONDUCTIVITY\n');
fprintf(fid, '%f\n', K);

fprintf(fid, '*SOLID SECTION, MATERIAL=%s, ELSET=ALLE\n', Material_Name);
fprintf(fid, '*INITIAL CONDITIONS, TYPE=TEMPERATURE\n');
fprintf(fid, 'ALLN, %d\n', preheat);

for i=2:LeftBiasSteps-5
    Q=Power*2;
    le=abs(X(i)-X(i-1));
    time=le/velocity;
    fprintf(fid, '*STEP, INC=10000, AMPLITUDE=STEP\n');
    fprintf(fid, '*HEAT TRANSFER, DELTMX=1000\n');
    fprintf(fid, '%e, %e, 1e-10\n', time/20, time);
    fprintf(fid, '*CFLUX, op=new\n');
    q=[1+(i-2)*length(Y),11, Q];
    fprintf(fid, '%d, %d, %f\n', q);
    fprintf(fid, '*OUTPUT, FIELD, variable=preselect, FREQUENCY=100\n');
    fprintf(fid, '*END STEP\n\n');
end

for i=i+1:TotalSteps-RightBiasSteps
    Q=Power*2;
    le=abs(X(i)-X(i-1));
    time=le/velocity;
    fprintf(fid, '*STEP, INC=10000, AMPLITUDE=STEP\n');
    fprintf(fid, '*HEAT TRANSFER, DELTMX=500\n');
    fprintf(fid, '%e, %e, 1e-10\n', time/20, time);
    fprintf(fid, '*CFLUX, op=new\n');
    q=[1+(i-2)*length(Y),11, Q];
    fprintf(fid, '%d, %d, %f\n', q);
    fprintf(fid, '*OUTPUT, FIELD, variable=preselect, FREQUENCY=100\n');
    fprintf(fid, '*END STEP\n\n');
end
fclose(fid);