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High Temperature Transient Creep Analysis of Metals

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HIGH TEMPERATURE TRANSIENT CREEP ANALYSIS OF METALS

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering

By

SARA MIRMASOUDI
B.S.M.E., Wright State University, 2011

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY SARA MIRMASOUDI ENTITLED HIGH TEMPERATURE TRANSIENT CREEP ANALYSIS OF METALS BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN ENGINEERING

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ABSTRACT

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The ability to design vehicles capable of reaching hypersonic speeds has become a necessity to satisfy industry requirements, hence requiring the need for better understanding of creep behavior of materials. Although the steady state creep of metals has been analyzed rigorously, there is little known about transient creep of many metals. Understanding transient creep behavior of metals is crucial in analysis and design of short term hypersonic flight applications. Hence, a transient creep analysis of 304SS, Al7075-T6, Al2024-T6, Inconel 625, Inconel 718, and Rene N4 is carried out focusing on the microstructural behavior of these metals undergoing high temperature operating conditions. In doing so, the material properties that were unknown in literature were determined by parameter fitting techniques using existing steady state experimental data and also previous parametric studies determining critical parameters affecting strain values. A transient creep deformation map for each metal is produced including the required design space of the application.
NOMENCLATURE

A: Dorn constant

$A_1$: Constant

$A_2$: Dimensionless constant

$\alpha_c D_{oc}$: Pre-exponential of core diffusion ($m^4/s$)

$\alpha_c$: Cross sectional area of dislocation core

b: Burger’s vector (m)

$B_m$: Mobility coefficient in Argon’s model

$\beta$: Dimensionless constant

c: Concentration of vacancy in Hirth and Lothe model

$C_s$: Transient constant

d: Grain size ($\mu$m)

D: Diffusion coefficient ($m^2/s$)

$D_{0v}$: Lattice diffusion pre exponential ($m^2/s$)

$D_b$: Grain boundary diffusion coefficient ($m^2/s$)

$D_c$: Core diffusion coefficient ($m^2/s$)

$D_{eff}$: Effective diffusion coefficient

$D_{gb}$: Grain boundary diffusion coefficient in Coble model

$D_{sd}$: Self diffusion coefficient in Nabarro-Herring model

$D_v$: Lattice diffusion coefficient ($m^2/s$)

$\frac{dc}{dx}$: Concentration Gradient (mol/mm$^4$)

$\delta D_{0b}$: Pre-exponential of boundary diffusion ($m^3/s$)

$\delta$: Effective boundary diffusion thickness (m)

$\Delta F_P$: Helmholtz free energy (J)

E: Young’s modulus (GPa)

$\dot{\varepsilon}_{ss-c}$: Coble creep strain rate (1/s)
\( \dot{\varepsilon}_{ss-NH} \): Nabarro-Herring creep strain rate (1/s)

\( \varepsilon_p \): Plastic tensile strain (mm/mm)

\( \varepsilon_t \): Tensile transient strain (mm/mm)

F: Force per unit length (J/m)

\( f_c \): Fraction of atom site associated with core diffusion

\( f_v \): Fraction of atom site associated with lattice diffusion

G: Gibb’s free energy (J)

\( \dot{\gamma} \): Strain rate (1/s)

\( \dot{\gamma}_1 \): Strain rate for elastic collapse (1/s)

\( \dot{\gamma}_2 \): Strain rate for obstacle limited plasticity (1/s)

\( \dot{\gamma}_3 \): Strain rate for plasticity limited by lattice resistance (1/s)

\( \dot{\gamma}_4 \): Strain rate for power law creep (high temperature plasticity) (1/s)

\( \dot{\gamma}_5 \): Harper-Dorn creep strain rate (1/s)

\( \dot{\gamma}_6 \): Power law breakdown strain rate (1/s)

\( \dot{\gamma}_7 \): Strain rate for diffusional flow (1/s)

\( \dot{\gamma}_{Net} \): Total steady state strain rate (1/s)

\( \dot{\gamma}_{ss} \): Steady state strain rate (1/s)

\( \gamma_0 \): Obstacle controlled glide pre exponential (s\(^{-1}\))

\( \gamma_1 \): Elastic Strain (mm/mm)

\( \gamma_2 \): Low temperature plasticity strain (mm/mm)

\( \gamma_3 \): Power law creep strain (mm/mm)

\( \gamma_4 \): Diffusional flow strain (mm/mm)

\( \gamma_t \): Transient shear strain (mm/mm)

\( \gamma_{total} \): Total strain (mm/mm)

J: Flux of atoms (mol/m\(^2\) s)

k: Boltzmann constant (J/K)
$K$: Hardening constant (Pa)

$m$: Hardening exponent

$M$: Mobility

$\mu$: Shear modulus ($MN/m^2$)

$\mu_0$: Shear modulus at 300$^\circ$K ($MN/m^2$)

$n$: Power law exponent

$N$: Density of moving dislocation ($m^{-2}$)

$\nu$: Poisson’s ratio

$\Omega$: Atomic volume ($m^3$)

$Q_b$: Activation energy for boundary diffusion (Kj/mole)

$Q_c$: Core diffusion activation energy (Kj/mole)

$Q_v$: Activation energy for lattice diffusion (Kj/mole)

$R$: Gas constant ($J/mole \cdot K$)

$\rho_m$: Mobile dislocation density ($m^{-2}$)

$\sigma$: Tensile stress (MPa)

$\sigma_n$: Normal stress (MPa)

$\sigma_s$: Shear stress (MPa)

$\sigma_{xy}$: Stress encountered by a moving dislocation in Argon’s model

$T$: Temperature (Kelvin)

$T_m$: Melting Temperature (Kelvin)

$\dot{\tau}$: Flow strength of solid at 0$^\circ$K

$t$: Time (s)

$\bar{\nu}$: Average velocity of dislocations (m/s)
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CHAPTER I
INTRODUCTION

The world of hypersonic flight, at speeds of Mach 5 or higher, is growing rapidly, and with a large variety of vehicles comes a wide range of requirements. One basic, yet challenging requirement that most designers and researchers face is the extremely high temperatures and stresses that such vehicles experience during flight, hence causing metals to creep. Failing to consider material creep behavior in hypersonic flight design can result in large deformation of components, and eventually catastrophic failures. In general, there are three main stages of creep as shown in figure (1).

![Stages of Creep](image)

Figure (1): Stages of Creep

The primary phase, also known as transient phase, starts instantaneously at a very high deformation rate. This rate decreases with time as a result of work hardening during the primary phase until it reaches a constant rate as it enters the secondary stage, also known as steady state. The deformation rate stays relatively steady at its specific value for some duration of time until it reaches the tertiary phase of creep, at which the deformation rate starts to increase with time until rupture. The secondary and tertiary phases of creep have been the main focus of many creep
studies over the years; as such information is crucial in design of vehicles with long and repetitive flights that expose the vehicle to the high temperature and high stress conditions of hypersonic flight. However, this information may not necessarily be helpful when dealing with very short flight events. In such cases, understanding the primary phase creep becomes a necessity.

Material creep has been studied extensively for decades, and a wide variety of metals have been tested to help determine their creep behavior and rupture life. There are two main approaches to creep analysis. One is a purely mathematical point of view similar to the use of the differential equations produced by Kelvin-Voigt relations, and the second is characterized by a more experimental approach related to the micro-mechanics referred to as the deformation mechanism technique. This second approach (micro-mechanical approach) is meant for creep analysis in the range of temperatures greater than 50% of the melting temperature of the metal, and it captures the phases present in a creep environment. Many scientists have analyzed creep of materials, determined microstructural behaviors that occur during creep, and helped implement the empirical equations that describe each mechanism and quantify the amount of resulting strain. In order to better convey such large amounts of information about each material, Ashby introduced the deformation mechanism maps [1]. These maps are a summarized way of communicating creep information about a material and its dominating mechanisms.

This thesis is to study the transient creep characteristics of a variety of metals that are considered for a short term, hypersonic flight application. In this case, the design safety factor is to be 1.25, and the plastic deformation is to be limited to 0.2 percent. Hence, the purpose of this research is to produce transient deformation mechanism maps along with design space curves for Inconel 718, Inconel 625, Rene 4, Aluminum alloys 7075 and 2024, and 304SS. In doing so, strain calculations were carried out using the micromechanical approach discussed by Frost and Ashby [1]. In order to determine unavailable material properties, base metal properties were used as a starting point and were then fine-tuned by parameter fitting techniques using existing steady
state experimental data and existing parametric study results [10] that determine the critical parameters affecting transient maps.

In this document, after this introduction, the creep mechanisms and their respective equations are discussed in chapter two. Chapter three includes information regarding the methodology and the approach taken towards generating the transient deformation maps. Chapter four provides resulting maps and information for each metal along with discussion of the results, and finally chapter five discuss concluding remarks for this thesis. All references used to generate this document are listed in the references section followed by the appendix which includes all computer codes used to generate the maps.
CHAPTER II

BACKGROUND

Much has been written about creep that is in the range of elastic strain with temperatures close to 30% of the material’s melting temperature. One can find work using the Kelvin-Voigt solution for time dependent differential equations. The approach has limitations once the event requires a greater thermal environment consideration producing significant plastic strain. This more thermal characteristic requires a more micro mechanic approach. This approach has become very much associated with experimental evaluation. One will see features dealing with such items as lattice sliding, twinning, internal imperfection, climbing of micro asperities, and diffusivity within the crystal arrangement. A great deal of these considerations do not fit into the arena of a tidy differential equation. This thesis will explore the features of high temperature creep using an approach followed by Ashby and Frost in order to determine transient creep represented for several metals that are being considered in a hypersonic flight design. A design space has been set which considers the maximum plastic strain of 0.2% with a design factor of safety of 1.25 and a possible potential temperature of 1300 degrees centigrade. Transient maps will be produced for each of these materials with a curve representing the overall design space.

The study of material deformation has been researchers’ point of focus for decades. Different metals have been studied undergoing low temperature and high temperature deformation tests, and the results have been analyzed rigorously. Many of such tests focused on steady state creep studies or rupture characteristics, and a few studied the transient creep characteristics of metals. The information available in the literature, however has provided enough background to produce a systematic approach for analyzing the short term and high temperature transient creep of metals.
There has been a large number of theories suggested describing the creep behavior of metals, many of which are based on empirical equations focusing on the macroscopic behavior of the metals. This empirical approach was the first method of creep analysis, and it was mainly concerned with the mathematical description of creep. Due to this high focus on the mathematical analysis, there was little attention paid to the true physical behavior of the metal undergoing creep. The goal here was to develop mathematical relations that would correlate well with the experimental data. Works of Freudenthal, Mott and Nabarro, and Spderberg are a few of such equations [2]. Despite the extensive amount of work and the large quantity of different relations and equations introduced by different scientists, none were found to be applicable to all cases and materials, and the equations were limited significantly. The most common limitation amongst these mathematical relations was the fact that they had assumed constant temperature or constant stress. To eliminate such limitations, further studies were done and more equations were produced to consider temperature dependence or stress dependence. Dorn was among those researchers who attempted develop equations for both temperature dependence and stress dependence. The most comprehensive relation of the empirical equations was Graham’s equation, where an extensive amount of work was done to reduce an equation which was most compatible with experimental data, and finally Stowell’s relation that took into account elastic and thermometric contributions [2]. However, most empirical relations proved insufficient to describe creep of materials as a whole, and were more applicable for special case studies.

The second type of approach was developed later to allow for more considerations of the physical behavior of metals. This quasi-empirical method was a continuation of the empirical method while focusing on the physical properties and characteristics as well. Although this method too overlooked some of the material characteristics and behaviors on a microstructural level, it was a step forward from the purely mathematical method of describing creep. The most
complete quasi-empirical method was proposed by Andrade, defining constants representing physical mechanisms and resulting in temperature and stress dependence [2].

A number of different empirical relations and theories attempting to describe the creep of materials is great, but none can be used as a single equation that would fit all. As Kennedy explains in his book “Processes of Creep and Fatigue in Metals” [2], usually several different processes take place in material creep; each method requiring a different time function and dependent on different microstructural characteristics, hence making it impossible to find one mathematical equation to describe creep as a whole. As a result of this realization, researchers started considering different microstructural characteristics of materials. One of the early researchers that investigated different microstructural characteristics was McLean who considered contributions by slip and grain boundary displacement [2]. Many researchers have since studied creep of materials from a microstructural view point, each providing insight into the material behavior undergoing deformation and hence, determining different mechanisms of creep at various conditions. However, most usual approaches taken towards creep analysis are related to temperatures below 30% of the melting temperature and hence do not address the high temperature creep of the metals very well. Hence, in high temperature cases, especially above 50% of melting temperature, further investigation is required. The focus of this thesis is to address the high temperature creep behavior of metals following the microstructural approach taken by Frost and Ashby [1].

In this study of short term and high temperature flight application, there are several creep mechanisms present in the process. In order to be able to study and calculate transient strain, first the steady state strain rates need to be calculated as the steady state stage is the upper bound of primary stage and the steady state strain rate values are used in the total transient strain calculations. Hence, the equations used for each mechanism is discussed in two sections. In section 2.1, each mechanism is discussed in detail along with its respective steady state rate
equation. Section 2.2 will represent the strain equations that will ultimately be used for map generation. All calculations and equations in this study are in shear for simplicity of work.

2.1. Steady State

Creep is the time dependent, permanent plastic deformation of materials under constant loads. Plastic flow of fully dense solids is caused by shearing of the stress field. This shear stress applies forces to defects such as dislocations and vacancies and forces them to move. Therefore, material defects are the carriers of the deformation, and hence their velocity and density effect the shear strain rate. In the study of creep, stress, temperature and strain rate are the macroscopic variables of plastic deformation. In this study, strain rate is considered the independent variable, and hence each mechanism of deformation is described by a rate equation that is dependent on stress and temperature. On the other hand, the microstructural characteristics that affect strain rate and vary with time, and stress and temperature changes are called the state variables. Hence, strain rate in general is a function of stress, temperature, state variables, and material properties [1]. In this section, different mechanisms are described and the steady state strain rate equations are discussed.

The first mechanism to consider is the elastic collapse, which is when deformation of a crystal becomes plastic, and as a result the structure becomes mechanically unstable. This elastic collapse happens due to application of stresses above the ideal shear strength of the material, which is stresses above which the flow becomes plastic. This ideal strength can vary with temperature and hence is a kinetic problem requiring the calculations of the frequency at which the dislocation loops form and expand [1]. As a result of this nucleation, small new dislocation loops appear in the structure [6]. However, in this case and for the sake of simplifying the problem, it is assumed that the ideal strength is the same as the shear modulus of the material [1].
\[ \dot{\gamma}_1 = \infty \text{ When } \sigma_s \geq \alpha \mu \]  

\[ \dot{\gamma}_1 = 0 \text{ When } \sigma_s < \alpha \mu \]

where \( \alpha \) is constant between the values of 0.05 and 0.1. Its value for face centered cubic (f.c.c.) metals is found to be 0.06, for body centered cubic (b.c.c.) metals to be 0.1, and 0.1 for all other metals [1]. The shear modulus, \( \mu \), is related to elastic modulus \( E \), commonly known as Young’s modulus, by the following mathematical relation.

\[ \mu = \frac{E}{2(1+\nu)} \] (2)

where \( \nu \) is the Poisson’s ratio, the ratio between lateral and longitudinal strains [5]. Young’s modulus, \( E \), is the linear ratio of stress over strain [6]. The shear modulus however is also dependent on the temperature since the amount of strain in the metal changes with temperature. The temperature dependence of the shear modulus is shown in equation (3) where \( T_m \) is the melting temperature of the metal and \( \mu_0 \) is its shear strength at 300°K [1].

\[ \mu = \mu_0 \left(1 + \frac{(T-300)T_m}{T_m \mu_0} \frac{d\mu}{dT} \right) \] (3)

The next mechanism is plasticity. Plasticity in general is the result of obstacle limited dislocation glide in response to an applied stress [1]. In this process, the interatomic bonds are ruptured and then recreated [6]. A dislocation is a linear defect around which some atoms are misaligned as a result of a movement in the direction of the burger’s vector. There are different types of dislocations: edge dislocation, screw dislocation, and mix dislocation which is a mixture of the edge and screw dislocation [6]. An image of a mobile edge dislocation is shown in figure (2) [6]. In this image, the edge dislocation moves in the crystalline solid as a result of an applied shear stress causing deformation in material.
Under normal conditions, dislocations are locked inside the metal, however, once additional stress is applied and the metal yields, the dislocations become mobile by either breaking interatomic bonds or nucleating new dislocations, hence increasing the mobile dislocation density \([48]\). The strain rate produced by a density \(\rho_m\) of mobile dislocations can be calculated as shown

\[
\dot{\gamma} = \rho_m b \ddot{v}
\]  

where \(b\) is the Burger’s vector, representing the magnitude and direction of lattice distortion associated with a dislocation \([6]\), and \(\ddot{v}\) is the average velocity of the mobile dislocations depending on mobility \(M\), and force per unit length.

\[
\ddot{v} = MF
\]

where \(F = \sigma_s b\). The mobility \(M\) of the material is mainly dependent on the Peierls stress, which is the minimum stress required to move a dislocation at temperature of 0 Kelvin, and also the orientation of the dislocation in the structure. On the other hand, since the density of mobile dislocations is dependent on stress and temperature, an extensive study was carried out by Argon \([7]\) determining strain rate’s dependence on dislocation density. This research was originally done due to observation of discrepancies between dynamics of individual dislocations and the response of the macroscopic strain rate to stress and temperature. These discrepancies were previously attributed to the stress dependence of the mobile dislocation density or the internal stress present in the strained state. On the other hand, a third condition could also be the reason for this
discrepancy and that is the dynamic internal stress arising from the interaction of mobile dislocations with themselves. This was studied by first considering a total mobile dislocation density where half of the dislocations were assumed to be stationary while the other half moving through the stationary half. The image shown in figure (3) represents Argon’s model for one moving dislocation ($T_3$) moving towards two stationary dislocations with a mobile dislocation density of $N$ and mean spacing of $\frac{1}{\sqrt{N}}$.

![Image of Argon's Dislocations Interaction](image)

Figure (3): Argon’s Dislocations Interaction [7]

The stress that the moving half would encounter was found to be

$$\sigma_{xy} = \frac{\mu b}{8\pi(1-v)} \left\{ \frac{\sin 4\theta_1}{\frac{\lambda}{2} + y} + \frac{\sin 4\theta_2}{\frac{\lambda}{2} - y} \right\}$$

(6)

where $\lambda$ is the distance between two stationary dislocations, and $y$ is the distance between the moving dislocation to the center as shown on figure (3), and $\frac{\tan \theta_1}{\lambda + 2y} = \frac{\tan \theta_2}{\lambda - 2y}$. Next the maximum resistance applied to dislocation 3 was approximated to be

$$\sigma_{xy_{max}} = D \frac{\lambda}{\left(\frac{\lambda}{2}\right) - y^2}$$

(7)

where in this study only, $D$ is

$$D = \frac{\mu b}{8\pi(1-v)}$$

(8)

Assuming that the velocity of an isolated dislocation in the crystal obeys
\[ v = B_m \sigma^m \] (9)

And that internal stresses are insignificant, meaning that mobile dislocations only encounter stresses from interactions with their own kind, and also using curves for average velocity at different strain exponents, \( m \), the simplified strain rate in the crystal is found by

\[ \dot{\gamma} = \frac{b \rho_m B_m \sigma^m}{2} f_m (\xi_1) \] (10)

where \( B_m \) is the mobility coefficient, and \( \xi_1 = \frac{4D}{\lambda} \). Now, assuming \( m = 1 \) and that \( \xi_1 \) is not too near zero, and knowing that \( \lambda = \sqrt{\frac{2}{N}} \), the strain rate becomes

\[ \dot{\gamma} = \frac{2.18 b^2 \rho_m \sigma}{\beta} \sqrt{1 - \frac{N \cdot 4D}{2 \sigma}} \] (11)

where \( \beta = \frac{b}{B_1} \), and \( B_1 \) is the mobility coefficient assuming strain exponent \( m \) of 1. A plot of mobile dislocation density versus strain rate at constant stress (using equation (11)), shows that the strain rate reaches its maximum when the density is \( \frac{2\sigma^2}{25D^2} \), and it becomes zero when density is 0.125(\( \sigma/D \))^2 [7].

![Figure (4): Plot of Mobile Dislocation Density versus Strain Rate from Argon [7]](image)
Hence if the dislocation generation is not a rate limiting process, the flux of the mobile dislocations governs the rate of deformation. Equation (10) above was experimentally confirmed by Li [64] by using the exponent m and strain rate in equation (10), and plotting % strain rate versus exponent m for single crystal NaCl [64]. Note that in studies performed by Argon, “D” was used as a coefficient in this section. However, from here on, D is considered the diffusion coefficient as described in the nomenclature. Now, using the relations for density and equation 8, the expression for dislocation density becomes [7]

\[ \rho_m = \alpha \left( \frac{\sigma_b}{\mu b} \right)^2 \]  

(12)

where \( \alpha \) holds all other constants in the simplification process, and is a constant of order unity.

Hence, the mobile dislocation density depends on the stress directly, and on the temperature knowing that the shear modulus is a temperature dependent variable. However, in order to develop rate equations for plasticity, one must determine the shape and nature of the obstacles limiting dislocations. Although there are many different types of obstacles, they can generally be thought of as two main groups.

First group is discrete obstacles which are bypassed or cut individually by a moving dislocation. The average velocity of the dislocations limited by discrete obstacles is

\[ \bar{v} = \beta b v e^{\left( \frac{-\Delta G}{kT} \right)} \]  

(13)

where \( \beta \) is a dimensionless constant, and k is the gas constant, also known as Boltzmann constant. This equation is a temperature dependent equation that takes on the form of a solution to a one dimensional time dependent heat equation. Hence, it has an important role in the high temperature creep analysis. \( \Delta G \) is a thermodynamic parameter called the Gibbs free energy of activation, which is the energy needed to initiate a microstructural action such as passing or cutting an obstacle by a dislocation. Hence, its value depends on the distribution of the obstacles.
and their shape. Gibbs free energy for general form and random distribution of obstacles has the form

\[
\Delta G(\sigma_s) = \Delta F \left[ 1 - \left( \frac{\sigma_s}{\tau^\ast} \right)^P \right]^q
\]  

(14)

where \( \Delta F \) is the total free energy, the energy needed to pass or cut the obstacle without any external stress, and \( \tau^\ast \) is the flow strength of the solid at 0°K, which is the shear strength in the absence of thermal energy, which reduces the Gibbs free energy to zero, hence forcing the dislocation to overcome the obstacle without aid from thermal energy. Gibb’s free energy is used to represent volume change due to the occurrences outside of the mechanical energy present; hence, the reason to start with the present stress. For widely spaced discrete obstacles \( \tau^\ast \) is proportional to \( \mu b \l / l \), where \( l \) is the obstacle spacing. Both \( 0 \leq p \leq 1 \) and \( 1 \leq q \leq 2 \) are constants and their value depends on magnitude of \( \Delta F \), but for analysis of discrete obstacles, they are both equal to unity [1]. The value of \( \Delta F \) however, is divided into three categories based on the obstacle strength as shown in table (1).

<table>
<thead>
<tr>
<th>Obstacle Strength</th>
<th>( \Delta F )</th>
<th>( \tau^\ast )</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>( 2\mu b^3 )</td>
<td>&gt; ( \mu b / l )</td>
<td>Dispersions; Large of strong precipitates</td>
</tr>
<tr>
<td>Medium</td>
<td>( 0.2 - 1.0\mu b^3 )</td>
<td>( \approx \mu b / l )</td>
<td>Forest dislocations, radiation damage;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>small or weak precipitates</td>
</tr>
<tr>
<td>Weak</td>
<td>&lt; ( 0.2\mu b^3 )</td>
<td>( \ll \mu b / l )</td>
<td>Lattice resistance; solution hardening</td>
</tr>
</tbody>
</table>

In order to get the rate equation for discrete obstacle limited plasticity, equations 4, 12, 13, and 14 are combined to result in

\[
\dot{\gamma}_2 = \dot{\gamma}_0 e^{-\frac{\Delta F}{kT}\left(1 - \frac{\sigma_s}{\tau^\ast} \right)}
\]  

(15)

where \( \dot{\gamma}_0 \) is

\[
\dot{\gamma}_0 = \frac{\alpha}{b} \left( \frac{\sigma_s}{\mu} \right)^2 \beta b \nu
\]  

(16)
Equation 15 represents the strain rate produced by discrete obstacle limited plasticity, and has the form of Arrhenius equation, found by the Dutch chemist, Svante Arrhenius in 1889 [17].

Arrhenius equation was developed to describe the temperature dependence of reaction rates, and both activation energy and pre-exponential parameters were originated in this equation. The activation energy is the minimum amount of energy needed to overcome the barrier and allow for movement, and it can be found experimentally from the slope of an Arrhenius plot of $\ln(D_v)$ versus $\frac{1}{T}$. The pre-exponential, also known as the frequency factor, is determined by extrapolating the y intercept of that same plot which will give the natural logarithm of the pre-exponential [17]. It is important to note that while the activation energy depends on the shape of the forcing function $Q$, the pre-exponential depends on its magnitude [17].

Therefore, equation 16 represents the pre-exponential of equation 15 and describes its stress dependence. However, in this case where $\Delta F$ is large, the stress dependence of the exponent in equation 15 becomes so large that $\dot{\gamma}_0$ can be treated as a constant and its variation can be ignored. Based on experimental data fitting, $\dot{\gamma}_0$ is set to $10^6/s$ and hence equation 16 is not used in calculations [1].

The second group is plasticity limited by a lattice resistance, which is caused by the interactions of the dislocations with the atomic structure itself [1]. The resistance between an array of atoms and the dislocations creates a barrier against the dislocation movement. The energy required to overcome this lattice resistance, also called Peierls force, fluctuates depending on the location of the dislocation within the structure. The dislocation ultimately overcomes the barrier by throwing forward kink pairs with aid from applied stress and thermal energy [1]. In this process, a short segment of the dislocation crosses into the next Peierls valley and is connected to the remaining dislocation by two kinks of opposite sign [14]. A kink displaces the dislocation line
locally by a unit distance in the slip plane and continue on with the movement [13] as shown in figure (5).

![Figure (5): A kink pair [20]](image)

The general form of the activation energy for all shapes of lattice resistance is

\[
\Delta G(\sigma_s) = \Delta F_P [1 - \left(\frac{\sigma_s}{\tilde{\tau}_P}\right)^p]^q
\]

(17)

where \(\tilde{\tau}_P\) is the approximated flow stress at 0°K, and \(\Delta F_P\) is Helmholtz free energy of an isolated pair of kinks. The Helmholtz free energy is the thermodynamic potential in order to produce the kink pairs to overcome the lattice resistance. Gibbs free energy includes the Helmholtz free energy and has an additional term dealing with the change in volume for a constant pressure. Therefore, Gibbs free energy does not include the heat energy in the system and is dependent on internal energy, minus temperature, and entropy. Using the best fit experimental data, \(p\) and \(q\) are determined as shown in equation (18) [1]. Hence, equation (17) becomes

\[
\Delta G(\sigma_s) = \Delta F_P [1 - \left(\frac{\sigma_s}{\tilde{\tau}_P}\right)^{3/4}]^{4/3}
\]

(18)

Combining equations (4), (12), (13), and (18) leads to the rate equations for plasticity limited by lattice resistance.

\[
\dot{\gamma}_3 = \dot{\gamma}_P \left(\frac{\sigma_s}{\mu}\right)^2 e^{\frac{\Delta F_P}{kT} \left(1 - \left[\frac{\sigma_s}{\tilde{\tau}_P}\right]^{3/4}\right)4/3}
\]

(19)
Similar to equation (15), equation (19) is also an Arrhenius type rate equation representing the temperature dependence of the strain rate produced by plasticity limited by lattice resistance. In this case the low temperature strength of b.c.c metals and ceramics is described by $\dot{\gamma}_P = 10^{11}/s$ using experimental data fitting, however since the value of $\dot{\gamma}_P$ isn’t critical in the final results, this value is used as a constant for all cases.

The next mechanism is high temperature plasticity also known as power law creep. At high temperatures, rate-dependent plasticity (creep) is highly apparent, and flow strength is highly dependent on the strain rate at temperatures higher than 30% of melting temperature for pure metals and at 40% of melting temperature for alloys and most ceramics. This strain rate dependence has the form of

$$\dot{\gamma} \propto \left(\frac{\sigma_s}{\mu}\right)^n$$  \hspace{1cm} (20)

where n has a value between 3 and 10, and is the reason this region is called “power law creep”. However, at high temperatures dislocations can both climb and glide due to the additional degree of freedom that is available [1]. A dislocation climb is where the dislocation changes slip planes to bypass a microstructural barrier [20]. This can happen due to addition of an atom or vacancy to an edge dislocation, or the intersection of dislocations. Hence, a minimal climb can release a gliding dislocation held up by discrete obstacles [1]. In this region the glide step is responsible for most of the strain, while the climb step is used to determine the average velocity. In climb-controlled creep, the rate controlling process is the diffusive motion of single ions or vacancies to or from the climbing dislocation. At higher than 0.6 melting temperatures, climb is lattice diffusion controlled, where lattice diffusion is the movement and transport of lattice, usually due to point defects and vacancies [15]. The velocity $v_c$ that an edge dislocation climbs at under local normal stresses is parallel to its Burgers’ vector and is equal to
\[ v_c \approx \frac{D \sigma \Omega}{b k T} \]  \hspace{1cm} (21)

where \( \Omega \) is the atomic or Ionic volume, and \( \sigma_n \) is the normal stress and is proportional to the applied stress \( \sigma_s \) \([1]\). \( D_v \) is the lattice diffusion coefficient, the measure of the mobility of diffusing lattice \([16]\). The diffusion coefficient, also known as the proportionality constant, in a general sense comes from the steady state diffusion law, also known as Fick’s law \([52]\).

\[ J = -D \left( \frac{dc}{dx} \right) \]  \hspace{1cm} (22)

where \( J \) represents the flux of atoms and \( \frac{dc}{dx} \) is the concentration gradient. The proportionality constant between concentration gradient and flux of substance is related to the root mean square distance and time by \( \bar{x}^2/t \), where the root mean square distance is dependent on the distance at which random jumps occur \( (\lambda) \) and the number of jumps \( (n) \) as \( \bar{x} = \lambda \sqrt{n} \) \([56]\). The lattice diffusion coefficient however is calculated by an Arrhenius type equation as shown in equation (23).

\[ D_v = D_{0v} e^{\frac{Q_v}{k T}} \]  \hspace{1cm} (23)

where \( Q_v \) is the activation energy for lattice diffusion, and \( D_{0v} \) is temperature independent pre exponential \([1]\). The relation in equation (21) however is the simplified form of the climb velocity of a mixed dislocation found by Hirth and Lothe \([8]\), through analysis of diffusion controlled climb rate. This velocity is needed to obtain the rate oriented relation with respect to time. In their studies, they assumed the application of a shear stress to a mixed dislocation near the center of a cylinder resulting in the local equilibrium vacancy concentration expressed as

\[ c = c^0 e^{\frac{\sigma b v_a}{b e k T}} \]  \hspace{1cm} (24)
where \( c^0 \) is the concentration of vacancies in local equilibrium with the surface when only
tangential forces are required to maintain the stress, and hence the hydrostatic component of the
external stress is zero. \( \nu_a \) is the frequency of the applied stress, \( b_e \) is the edge component of force,
and \( \sigma b_s \) is the force per unit length applied to the dislocation by the shear stress. As a result of
this vacancy, and the chemical potential gradient between the dislocation and the surface, a
diffusion flux of vacancies exists. This flux is expressed as

\[
J = -\frac{D_v c}{kT} \nabla \tilde{G} = -D_v c \nabla \ln \left( \frac{c}{c^0} \right)
\]

where \( D_v \) here is the vacancy diffusivity. However the divergence of \( J \) vanishes in the steady state
condition resulting in

\[
\nabla \cdot J = -\nabla \cdot \left( D_v c \nabla \ln \left( \frac{c}{c^0} \right) \right) = 0
\]

This equation is however very difficult to deal with due to variations of \( D_v \) and \( c^0 \), hence it is
assumed here that \( c^0 \) and \( D_v \) are both constants. This assumption is good except in regions near
the dislocation core. Applying these assumptions, equation (26) becomes Laplace’s equation
where derivatives with respect to \( \theta \) and \( z \) axis are zero by symmetry, as shown in equation (27).

\[
\nabla^2 c = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial c}{\partial r} = 0
\]

Assuming \( \sigma b_s \nu_a \ll b_e kT \), the boundary conditions are

\[
\begin{align*}
\begin{cases}
    c - c^0 = 0 & \text{at } r = R \\
    c - c^0 = c' = \frac{\sigma b_s \nu_a}{b_e kT} c^0 & \text{at } r = b
\end{cases}
\end{align*}
\]

The solution to equation (27) that would satisfy these boundary conditions is

\[
c - c^0 = c' \frac{\ln \left( \frac{r}{R} \right)}{\ln \left( \frac{r}{b} \right)}
\]
The net vacancy current per unit length away from the dislocation is

\[ I = \frac{2\pi D_0 \sigma b \nu a c^0}{b_e kT \ln \left( \frac{F_e}{F} \right)} \]  

(30)

And the velocity of climb of the dislocation is

\[ v_y = \frac{1}{b_e} \frac{\nu a}{\nu a} \]  

(31)

Now, using equations (30) and (31), and also the definition of the atomic self-diffusion \( D_s = D_0 \nu a c^0 \), the velocity becomes

\[ v_y = \frac{2\pi D_0 \sigma b \nu a}{b_e^2 kT \ln \left( \frac{F_e}{F} \right)} \]  

(32)

Finally, generalizing this equation and assuming \( R \sim 10^4 b \), the velocity equation becomes [8]

\[ v = \frac{D_0 \nu a \left( \frac{F_e}{T} \right)}{b_e^2 kT} \]  

(33)

Now, combining equations (4), (12), and (21), and also assuming that \( \sigma_n \) is proportional to \( \sigma_s \) and approximating \( \Omega \) by \( b^3 \), the following rate equation is obtained for high temperature plasticity (power law creep).

\[ \dot{\gamma} = A_1 \frac{D_0 \mu b}{kT} \left( \frac{\sigma_s}{\mu} \right)^3 \]  

(34)

where \( A_1 \) is a constant including all constants of proportionality [1]. Equation (34) however doesn’t apply to all materials and very few exceptions exhibit power law creep with a power of 3 and \( A_1 \) constant of 1 [9]. In addition, applied stress and local normal stress proportionality assumption is not necessarily correct for all materials. As a result, a modified equation was found using experimental data by Mukherjee, Bird, and Dorn. Their goal was to study high temperature creep and the significance of climb on it [32]. To begin, they focused on the effects of
independent variables of stress and temperature on the creep rate. They started with the general steady state creep rate expressed as

\[ \dot{\gamma}_s = \sigma^n f\{T\} \]  

(35)

where values of n for several metals was found experimentally ranging from 3 to 7 [32]. To find n, the logarithmic plot of stress versus strain need to be created using experimental stress and strain values obtained from a creep test [65]. The value of n is the slope of the resulting curve of the experimental data. Now, since creep is a thermally activated process, its temperature dependence was expressed using the Arrhenius equation as [32]

\[ \dot{\gamma}_s = f\{\sigma\} e^{\frac{Q_c'}{RT}} \]  

(36)

where the activation energy \( Q_c' \), was insensitive to stress and in very close agreement with activation energy of self diffusion, \( H_D \)[32]. Combining the two equations (35) and (36), results in the empirical relation as follows.

\[ \dot{\gamma}_s e^{\frac{Q_c'}{RT}} = A' \sigma^n \]  

(37)

with \( A' \) being dependent on the modulus of elasticity of the metal. Using this knowledge, equation (37) was rewritten to include the effects of the shear modulus and hence resulting in an improvement in the correlation of secondary creep rates among different metals and alloys.

\[ \dot{\gamma}_s e^{\frac{Q_c'}{RT}} = A''\left(\frac{\sigma}{\mu}\right)^n \]  

(38)

Since high temperature creep is diffusion controlled, this equation was rewritten to include the burgers vector as a significant dimension in diffusion or deformation [32]. Using these findings, Ashby and Frost used the more general form of the rate equation as expressed in equation (39) with the exponent n varying between 3 and 10 [1].
\[
\dot{\gamma} = A_2 \frac{D_v \mu b}{kT} \left( \frac{\sigma_s}{\mu} \right)^n
\]  

(39)

Where \(A_2\) is a dimensionless constant, and is related to the tensile constant “A” by \((A_2 = \sqrt[3]{\frac{n+1}{n}} A)\). Although there aren’t convincing explanations for the observed values of \(n\) and \(A_2\), the equation still provides good description for the experimental observations and serves as a good general equation. In order to explain some of the experimental facts (especially at low temperatures), it is assumed that the dislocation core diffusion contributes to the overall transport of matter [1]. To include core diffusion contributions an effective diffusion coefficient was defined [19]

\[
D_{\text{eff}} = D_v f_v + D_c f_c
\]  

(40)

where \(D_c\) is the core diffusion coefficient which is about equal to \(D_b\), the grain boundary diffusion coefficient, and \(f_v \& f_c\) are fractions of atom sites associated with each type diffusion, note \(f_v\) is unity, and \(f_c\) is determined by dislocation density [1]

\[
f_c = a_c \rho
\]  

(41)

where \(a_c\) is the cross sectional area of the dislocation core in which fast diffusion is taking place, and is taken to be \(2\delta^2\), where \(\delta\) is the effective boundary diffusion thickness. Hence using equation 12, the effective diffusion coefficient becomes

\[
D_{\text{eff}} = D_v \left[ 1 + \frac{10a_c}{b^2} \left( \frac{\sigma_s}{\mu} \right)^2 \frac{D_c}{D_v} \right]
\]  

(42)

Using \(D_{\text{eff}}\) (equation42) and inserting into the rate equation (equation 39), the power law creep rate equation is

\[
\dot{\gamma}_4 = \frac{A_2 D_{\text{eff}} \mu b}{kT} \left( \frac{\sigma_s}{\mu} \right)^n
\]  

(43)
This power law creep rate equation in reality represents two rate equations: High temperature creep region where lattice diffusion is dominant at lower stresses, and the low temperature creep region where core diffusion is dominant at higher stresses [1].

The last mechanism is diffusional flow, the phenomenon of material transport by atomic motion [6]. This means that the atoms within the metal physically move as a result of an increase in their energies, which increases the amount of vibration in the atom, until this energy is enough to make the atom jump to another site [49]. Many reasons could contribute to this behavior such as the existence of vacant lattice sites or vacancies that other atoms can occupy, or interstitial sites which are open spaces within a closed packed arrangement of atoms in which a positively charged ion can fit. [6]. Now, when stress without a deviatoric component is applied to the material, it changes the chemical potential, hence the hydrostatic pressure changes by the same amount to prevent from any potential gradient [1]. However, a stress field with deviatoric component, which is the stress tensor component responsible for shape change, causes different amounts of potential variation on some grain surfaces, hence resulting in a potential gradient. This potential gradient at high temperatures induces flux of matter through and around grains. This gradient combined with sliding displacements leads to strain. Assuming that both lattice and grain boundary diffusion are present, the strain rate equation produced by diffusional flow is

\[ \dot{\gamma} = \frac{42\sigma \Omega}{kTd^2} D_{eff} \]  

(45)

where \( d \) is the grain size, and \( D_{eff} \) is

\[ D_{eff} = D_v \left[ 1 + \frac{n\delta}{d} \frac{D_b}{D_v} \right] \]  

(46)

where \( \delta \) is the effective thickness of the boundary, and \( D_b \) is the boundary diffusion coefficient.

Although equation 45 ignores kinetics of detaching and reattaching vacancies from grain boundary sites, it is an acceptable simplified equation to be used to produce general deformation...
maps. Just like the strain rate equation for power law creep, equation (45) is also representing two rate equations: Lattice diffusion controlled rate at high temperatures, also known as Nabarro-Herring creep, and boundary diffusion controlled rate at lower temperatures, also known as Coble creep [1]. Equation (45) is in fact a combination of the two rate equations with $D_{eff}$ holding all uncommon parameters between the two rate equations, along with all constants generated during the simplification process. Nabarro-Herring creep is named after the two researchers that determined creep characteristics at high temperatures and low stresses. In their studies which were started by Nabarro and then further completed by Herring, the creep of metals were thought to be controlled by self-diffusion. Meaning that the self-diffusion within the grains of the polycrystalline solid can cause the solid to yield under an applied shearing stress [50]. The diffusive flux of atoms in the absence of pressure gradients is due to migration of lattice vacancies or interstitial atoms under an applied stress. Hence, this flux is proportional to the gradient of the concentration of these lattice defects. Now, in the presence of a pressure gradient, the lattice defects move in whichever direction that would relieve the inequality of pressure [50]. The strain rate caused by this lattice diffusion is described by Nabarro-Herring creep strain as

$$ \dot{\varepsilon}_{ss-NH} = \frac{D_{sd} a b^3}{kT d^2} $$

(46)

where $D_{sd}$ is the self-diffusion diffusion coefficient. On the other hand, Coble proposed that creep is controlled by boundary diffusion at lower temperatures, temperatures less than 70% of melting temperature [51]. Grain boundary is the boundary separating two small grains due to atomic mismatch within the region where two grains meet [6]. As a result, atoms are bonded less regularly along the grain boundary. In Coble creep, the diffusion of atoms can happen along the grain boundaries with less activation energy compared to the Nabarro-Herring mechanism, hence allowing this type of diffusion to occur at lower temperatures [23]. The strain rate suggested by Coble is [51]
\[ \dot{\varepsilon}_{ss-L} = \frac{\alpha_3 D_{gb} \sigma b^4}{kT d^3} \]  

(47)

where \( \alpha_3 \) is a constant of unity and \( D_{gb} \) is the diffusion coefficient for grain boundary diffusion [51]. A schematic figure shown in figure (6) demonstrates the difference between Nabarro-Herring and Coble creep mechanisms [23]. On the left is the Nabarro-Herring creep where diffusion occurs through the main body of the grains while on the right is the coble creep where diffusion occurs through the grain boundaries.

Figure (6): (a) Nabarro-Herring Creep – (b) Coble Creep [23]

Having discussed the above mechanisms, since the steady state region has a constant strain rate, the total strain rate is the superposition of the strain rates of the most effective mechanism in each group of similar mechanisms, as shown in equations (48) and (49) [1].

\[ \dot{\gamma}_{Net} = \dot{\gamma}_1 + Max (\dot{\gamma}_{plas}, \dot{\gamma}_4, or \ \dot{\gamma}_6) + Max(\dot{\gamma}_5, \dot{\gamma}_7) \]  

(48)

\[ \dot{\gamma}_{plas} = Least \ of \ (\dot{\gamma}_2, \dot{\gamma}_3) \]  

(49)

In equation (48), \( \dot{\gamma}_5 \) and \( \dot{\gamma}_6 \) represent Harper-Dorn creep and Power law breakdown respectively. However, their effect on the final results were found to be insignificant. Hence they are not discussed here any further.
Other deformation mechanisms such as mechanical twinning and plasticity limited by phonon or electron drag, are not discussed here because they do not necessarily apply to the operating conditions this thesis is focused on. For example, electron drag limited plasticity applies to explosive or shock loading environments, and mechanical twinning applies to low temperature operating conditions [1]. Not to mention mechanical twinning is more important in h.c.p. (hexagonal close packed) and b.c.c. metals while most metals studied in this research have face centered cubic (f.c.c.) structure.

2.2. Transient

Having the steady state rate equations available will enable the calculation of the transient strain values for each mechanism. Just like the steady state region, there are several mechanisms to be considered in the transient phase. The first mechanism is elastic deformation caused by stress. Elastic strain is not permanent, and will disappear instantaneously once the forces that cause it are removed [5]. However, a component of elastic strain exist in the plastic range from the initial stress application prior to entering the plastic region [1], [2]. Elastic shear strain produced by a shear stress $\sigma_s$, is calculated as followed.

$$\gamma_1 = \frac{\sigma_s}{\mu}$$  \hspace{1cm} (50)

The second mechanism is low temperature plasticity. The stress-strain relationship fitted to a work hardening law for tensile loading is similar to the equation for one dimension creep based on time and temperature, and has the form

$$\sigma = \sigma_0 + K \varepsilon_p^m$$  \hspace{1cm} (51)

where $\varepsilon_p$ is the plastic tensile strain, $m$ is the hardening exponent, $K$ is the hardening constant, and $\sigma_0$ is the yield strength at 300°K. To calculate m, one must calculate the slope of the
logarithmic curve of strain versus stress, and \( K \) is the value of stress when strain is equal to unity on that same plot [18].

Using equation (51) and converting the tensile stress and strain to equivalent shear stress and strain using the factor of \( \sqrt{3} \), and also solving for strain, the equation will have the form

\[
\gamma_2 = \left( \frac{\sigma_s - \sigma_{s0}}{K_s} \right)^{\frac{1}{m}} \\
K_s = \frac{K}{\frac{1}{3} \left( \frac{m+1}{2} \right)}
\]

which represents calculated strain produced by plasticity.

The next mechanism considers the effect of work hardening on metals that is due to generation of dislocations in the power law creep studies. As a result of this work hardening, the dislocation density is increased and the metal hardens. During the work hardening process, the dislocations move and rearrange in the metal until they reach steady state. However, prior to this steady state stage, while the dislocations are still rearranging, the metal tends to creep faster, which is known as the normal transient creep. The strain can then be calculated by

\[
\varepsilon = \varepsilon_t \left\{ 1 - e^{-(C \dot{\varepsilon}_{ss} t)} \right\} + \dot{\varepsilon}_{ss} t
\]

where \( \varepsilon_t \) is the total transient strain and \( \dot{\varepsilon}_{ss} \) is the steady state creep rate, and \( C \) is a constant. Equation (53) is then manipulated and converted to shear stress and strain to get the form

\[
\gamma_3 = \gamma_t \left\{ 1 - e^{-(C_s \dot{\gamma}_{ss} t)} \right\} + \dot{\gamma}_{ss} t
\]

where \( \dot{\gamma}_{ss} \) is the steady state strain rate calculated by equation (43), in the earlier section. \( \gamma_t \) and \( C_s \) are the transient strain and transient constant and can be found by the following two equations respectively

\[
\gamma_t = \sqrt{3} \varepsilon_t
\]
The last mechanism, diffusional flow, is the result of grain boundary sliding under an applied stress, due to crystalline defects and vacancies. The strain due to diffusion creep is calculated using

\[ \gamma_4 = \frac{\sigma_s}{\mu} \left\{ 1 - e^{-\left(\frac{\dot{\gamma}_{ss} t}{\sigma_s/\mu}\right)} \right\} + \dot{\gamma}_{ss} t \]  

(57)

where \( \dot{\gamma}_{ss} \) is the steady state strain rate by diffusional flow calculated by equation (45). The relaxation time in this case is the diffusion time over a distance comparable with the grain size

\[ \text{Relaxation time} = \frac{\sigma_s}{\mu \dot{\gamma}_{ss}} \]  

(58)

The total strain for the material is then found by \[1\]

\[ \gamma_{total} = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 \]  

(59)

2.3. Deformation Mechanism Maps

Having discussed the equations needed to evaluate creep of metals in high temperature and short term flight applications, it is important to determine the method of communication. In order to communicate a large body of information efficiently, Ashby [1], introduced deformation mechanism maps. An example of a deformation mechanism map is shown in figure (7).
These maps are essentially a summarized way of communicating information about material deformation at different stress and temperature levels, in addition to letting the user know of the dominating creep mechanism in each area. Such information becomes handy in different applications including design of new components, determining life of existing components in use, alloy design and selection, and design of experiments.

The transient maps represent the total strain of the material on a normalized temperature versus normalized stress map at a certain elapsed time. In addition, the regions of dominance of each mechanism are shown for different stress and temperature combinations. A mechanism is dominant when the amount of strain contributed to the total strain by that specific mechanism is greater than all other methods. It is important to note that the lines separating the dominant mechanisms, also known as boundaries, are regions where contributions by two different mechanisms are almost equal. In other words, their values are within a small tolerance. This tolerance however is not a constant value as it can vary for different metals, and hence will be determined separately for each case. Having a constant tolerance, 0.01 for example, could result in 3 different cases. The ideal case is where a perfect line would appear on the map representing the boundary, but the other not so desirable possibilities are either the appearance of a fully colored region on the map, or the opposite, where no lines appear at all. This of course depends...
on the amount of overlap there is between two dominant regions. In cases where two mechanisms almost equally dominate over a large range of stress and temperature combinations, the fully colored region appears. On the other hand, when strains produced by two neighboring dominant mechanisms are not close in value, no lines would appear and hence the tolerance need to be opened up to include enough data points between the two regions that would form a line. Hence, the tolerance will have to be tweaked in some cases to allow for appearance of perfect lines on the maps. Examples of such cases along with corrective actions are provided in chapter 3. Finally, the thin lines shown on the plot in figure (7) represent the total strain curves of different values such as $10^{-2}$ and $10^{-3}$ strain curves. Since the maps show stress, temperature, and strain, one must have two of the pieces of information in order to get the third from the map. For example, if the operating temperature and stress are available, as shown in example in figure (8), then the intersection of the perpendicular orange dotted lines from those values will represent the design space. In this case, the dominant mechanism is elastic deformation and the amount of strain is $10^{-4}$.

![Figure (8): Example of the transient map usage](image)
CHAPTER III

METHODOLOGY

In order to produce transient deformation maps, both steady state and transient values are needed as discussed in the previous chapter. The material properties required to perform all calculations for each metal is listed in table (2).

Table (2): Required Parameters

<table>
<thead>
<tr>
<th>Crystallographic Data</th>
<th>Power-law Creep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic Volume - $\Omega$ ($m^3$)</td>
<td>Exponent - $n$</td>
</tr>
<tr>
<td>Burger's Vector - $b$ (m)</td>
<td>Dorn Constant - $A$</td>
</tr>
<tr>
<td>Melting Temperature - $T_m$ (°K)</td>
<td>Obstacle Controlled Glide</td>
</tr>
<tr>
<td>Modulus</td>
<td>0K flow stress, $\dot{\varepsilon}/\mu_0$</td>
</tr>
<tr>
<td>Shear Modulus at 300 °K - $\mu_0$ (MN/m$^2$)</td>
<td>Pre-exponential - $\gamma_0^*(s^{-1})$</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
<td>Activation Energy - $\Delta F/\mu_0 b^3$</td>
</tr>
<tr>
<td>Lattice Diffusion</td>
<td>Work Hardening</td>
</tr>
<tr>
<td>Pre-exponential - $D_{0L}$ (m$^2$/s)</td>
<td>Initial Yield Stress - $\sigma_{0S}/\mu_0$</td>
</tr>
<tr>
<td>Activation Energy - $Q_L$ (Kj/mole)</td>
<td>Hardening Exponent - $m$</td>
</tr>
<tr>
<td>Boundary Diffusion</td>
<td>Hardening Constant - $K_S/\mu_0$</td>
</tr>
<tr>
<td>Pre-exponential - $\delta D_{0B}$ (m$^3$/s)</td>
<td>Transient Power-law Creep</td>
</tr>
<tr>
<td>Activation Energy - $Q_B$ (Kj/mole)</td>
<td>Transient Strain - $\gamma_t$</td>
</tr>
<tr>
<td>Core Diffusion</td>
<td>Transient Constant - $C_s$</td>
</tr>
<tr>
<td>Pre-exponential of core diffusion - $a_c D_{0C}$ (m$^4$/s)</td>
<td>Others</td>
</tr>
<tr>
<td>Core diffusion activation energy - $Q_c$ (Kj/mole)</td>
<td>Gas Constant - $R$</td>
</tr>
<tr>
<td></td>
<td>Boltzmann Constant - $K$</td>
</tr>
</tbody>
</table>

These values however are not available for most of the metals studied in this thesis. Hence the following step by step approach is taken.

1- Create computer codes to produce transient and steady state maps

2- Determine sensitivity levels of the transient maps to different parameters.

3- Use base metal parameters for parameters that are not available in the literature.

4- Use existing steady state creep data for comparison and parameter fitting.

5- Use adjusted parameters to produce transient deformation maps.
3.1. Computer Codes

In order to perform all calculations and produce transient maps, steady state and transient computer codes needed to be written. These computer codes were originally written by Royue Song [10], and were then modified and optimized further to increase efficiency of the code. In order to optimize the code, one first needs to understand strengths and weaknesses of the programming software and the way the code is compiled. MATLAB in general is not as time efficient or space efficient as other programming languages such as C or FORTRAN. On the other hand, one of MATLAB’s strengths is its ability to handle large arrays of data in a matrix. As a result of this, it is more efficient to use matrices to handle large arrays of data rather than loops. Therefore, MATLAB’s capability of handling large matrices was employed to handle calculations using logical indexing. By employing this method, the program run time was reduced from roughly an hour to about 5~7 minutes for the 3 hour transient map of 316 stainless steel. A section of the code using this method is demonstrated in figure (9).

```
mu_0 = 4.1*10^10;
muTd = -4.85;
T = 0:5:1010;
sigma_s = 0:10^5:5:10^10;
mu = (mu_0+(T-300)/T_m*muTd).*ones(1,length(s sigma_s));
%% Elastic Deformation %
eps_1 = (sigma_s.*ones(1,length(T)))./mu; % elastic shear strain produced by shear stress

%% Low-temperature Plasticity %
sigma_os = (mu_0*(7.5*10^-4-2.2*10^-7*(T-273.15))).*ones(1,length(s sigma_s));
m = (6.31+6*10^-5*(T-273.15)).*ones(1,length(s sigma_s));
K_s = (mu_0*(2.5*10^-3-5.7*10^-7*(T-273.15))).*ones(1,length(s sigma_s));
sigma_s_array = (sigma_s.*ones(1,length(T))).;'''
eps_2 = (((sigma_s_array-sigma_os)./K_s).*(1./m)); % Strain produced by plasticity
As=(sigma_s_array-sigma_os)./K_s;
Bs=(eps_1(1,:)>0);
eps_2=eps_2.*Bs;
```

Figure (9): Section of modified transient code using logical indexing
The algorithm to produce the transient map is shown in figure (10). In this algorithm, time and the material properties listed in table (2) are the inputs to the program. The code then uses the four equations shown in figure (10) and also discussed in chapter 2, to calculate the strain produced by each mechanism. The summation of the resulting strain values will then provide the total strain for the metal at that specific time. Once the total strain is calculated, the code will compare every two mechanism to determine the dominant mechanism in each region. This is determined by comparing the strains produced by each method as shown in the section of the code shown in figure (11). The highest contributing mechanism is considered dominant, and the areas in between, where two dominant mechanisms produce almost the same amount of strain is considered the boundary region. The reason that the values are almost equal in the boundary region is that strain values that are calculated at each of the finite number of data points may not necessarily be equal. Hence, the number of data points where the strain values are equal will not be enough to produce a line on the curve to represent the boundary. As a result, instead of setting strains produced by different mechanisms equal to one another, their difference is set to be within a very small range. For example, in figure (11), different values such as 0.1 and 0.001 are used. Possible adjustments to this range is further discussed in section (3.3).

The total strain and associated stress and temperature values are then plotted on a logarithmic normalized temperature versus normalized stress while featuring the boundaries between dominant regions, curves of constant strains, and the design space curve. Figure (13) represent an example of such work where the thin black curves on the plot represent the strain curves.
In addition to the transient codes, the steady state code is needed to allow for steady state parameter fitting. In order to ensure the steady state and transient codes produced correct maps,
the results were compared [10] with the existing Nickel steady state map and 316SS transient maps provided by Frost and Ashby [1]. Plots of the steady state and transient curves produced by MATLAB codes along with ones from literature [1], are shown in figures (12) and (13).

Figure (12): a) Steady state map of Nickel by Ashby [1] – b) Steady state map of Nickel by code

Figure (13): a) Transient map of 316SS by Ashby [1] – b) Transient map of 316SS by code

Once the accuracy of the codes was confirmed [10], the modified steady state map codes were used for steady state parameter fitting purposes. The algorithm for the steady state parameter fitting code is shown in figure (14) below.
3.2. Parameter Fitting

In order to determine sensitivity levels of the transient maps to different parameters, parametric studies were done [10][11] using existing transient maps of 316SS [1]. In this parametric study, different parameters were varied one at a time, and the effects of changes on the transient maps were recorded [10][11]. Same practice was then carried out using multiple parameter variations to study the effects [10]. Based on these studies [10][11], most variables could be varied by 50% before there were any significant effects on the transient maps, and a few parameters could be varied as much as 1000 times while the final results were only affected by 10% [11]. The table shown in figure (15) represents information on some of the parametric studies done by Moore [11].
Hence the parameters that the transient maps are most sensitive to are power law exponent, and boundary diffusion activation energy [10]. Furthermore, it was determined that the transient constant and transient exponent variations do not affect the transient maps significantly [1][10]. This information allows for using base metal values for these parameters without concern regarding their effects on the final transient results since these values are not readily available in literature for many of the metals studied here. The base metal is one that the alloy is mainly made of. For example Al7075 is an aluminum base metal and Inconel 625 is a nickel base metal.

Once the effective parameters were determined and the codes were ready, available material properties of the metals along with property values of the based metals for the unavailable properties were used to start the process. Existing steady state creep data available in
the literature was then used for parameter fitting. The steady state data were results of traditional creep tests were several samples of the same size and material were tested at different temperatures that were held relatively constant, and under different applied stress levels. The amount of strain for each sample at different elapsed times were then recoded. This data was then used to determine the steady state creep rate which is used to perform the parameter fitting process. In doing so, the experimental values were plotted along with the analytical temperature contours of the test temperatures to allow comparison between experimental and analytical data. An example of steady state parameter fitting curves is shown in figure (16) representing initial and final states. In this example, the “x” are the experimental data points, each color representing a different test temperature, and the solid lines represent the test analytical data for the corresponding temperatures. The goal here is to get the analytical curves to line up with the experimental data points as close and possible.

**Figure (16): Example parameter fitting curves (a) Initial (b) Final**

In the parameter fitting process, a few of the parameters from the base metal values including ones that the transient maps are sensitive to, are adjusted to get a better fit with the experimental data, hence resulting in values that are closer to the actual value of the parameter for the metal. Once the parameters are adjusted, the fitted values are then used along with available
parameters and base metal values of unavailable parameters in the transient code to produce transient deformation maps.

It was observed during the parameter fitting process that the power law exponent “n” and Dorn constant “A”, act in a coupled manner for most metals, meaning that they affect the results in similar manners, and could be more effective when varied together. It was also noticed that they affect the vertical position of the analytical curves the most amongst all other parameter variations. Hence, power law exponent and Dorn constant where the first parameters to be changed for most of the metals. Besides, these two parameters, the activation energies were the most effective in curve fitting adjustments, and this is in line with statements made by Frost and Ashby [1], regarding the fact that the activation energies for alloys are different from that of the pure metals. Hence, these parameters were the ones to be experimented with before trying out any other parameters.

In the parameter fitting process, power law exponent was first increased by about 10% to determine the effects. Next, it would be decreased by about 10% from the original value and the results would be compared to determine the correct direction of variation, and the proper value for the parameter. Next, keeping the first parameter adjustments to (n), the same process was done for Dorn constant except in this case, the values were varied by a power of 10 knowing that large variations in Dorn constant wouldn’t affect the transient results significantly. In most cases, after determining the correct direction of variation for Dorn constant, larger variations were required such as variations by 100 or even 1000. In most cases, the power law exponent and Dorn constant variations would move the analytical curves closer to the experimental values. Next, keeping the previous changes, activation energies would be changed to study the effects in a similar manner. In cases that the steady state curves were not sensitive to either of the properties discussed so far, the pre-exponential values were tested by once increasing and then decreasing the values to determine the effectiveness. For each metal, once the effective properties and the
correct direction of variation were determined, the values of the parameters would require additional fine tuning in order to get the best fit with the experimental data. An example of such process is demonstrated in the image in figure (17) below where several parameters were adjusted for Rene N4 starting with increasing the power law exponent (n). Next, the value of Dorn constant (A) was decreased, and finally the activation energies for core diffusion and lattice diffusion were changed simultaneously since it was observed that the best fitting occurs when the variation occurs at the same time.

![Figure (17): Example of parameter fitting process (Rene N4)](image)

Once the final results of the parameter fitting process were established, those values along with the rest of the material properties were input into the transient code to produce the transient maps of each metal. The run time for all metals was set to 30 seconds to account for the short operation time and also allow for a side by side comparison of the metals.
3.3. Boundary Line Adjustments

Once the transient maps were produced, the boundary lines were checked to ensure proper line formation as in some cases fully colored regions could appear or no lines would show on the map as it was discussed in section 2.3. In cases where proper boundary lines weren’t present, additional variation of the boundary thickness was performed. An example of a map missing a boundary line was Inconel 625 where no apparent boundary line could be seen between the diffusional flow and elastic regions when the code was originally ran. In figure (18), the range for the boundary between diffusional flow and elastic flow was set to be less than 0.001, resulting in the transient map as shown in figure (19) where the boundary line was missing.

```
146 # Diffusional flow dominant & boundary with elastic deformation
147 test=((epsa_4>=epsa_1),* (epsa_4>=epsa_3),* (epsa_4>=epsa_2)),* (abs(epsa_4-epsa_1) < 0.001*epsa_1);
148 Y_B = y(test==1);
150 X_B = x(test==1);
```

Figure (18): Section of code resulting no apparent boundary line

Figure (19): Transient map missing a boundary line

Based on this result it was apparent that the range listed between the boundaries wasn’t wide enough to include enough data points that would result in a boundary line, hence the value was
increased to allow for more data points to be included. The updated section of the code and the resulting transient map of Inconel 625 are then shown in figures (20) and (21) respectively.

Figure (20): Correction on section of code resulting no apparent boundary line

![Figure 20: Correction on section of code resulting no apparent boundary line](image)

Figure (21): Transient map with visible boundary line that was previously missing

Another example where similar type modification was required was 304 stainless steel where very thick lines appeared on the transient map between power law creep and plasticity regions. Figures (22) and (23) below represent the section of the code for this boundary along with the resulting map.

Figure (22): Section of code resulting in a fully colored region instead of a boundary line

![Figure 22: Section of code resulting in a fully colored region instead of a boundary line](image)
Figure (23): Transient map with a fully colored region instead of a boundary line

Obviously, the range between power law creep and plasticity was set too wide allowing for too many data points to be plotted and hence not forming a proper line between the two regions. Hence, this range was reduced from 0.1 to 0.001 to eliminate the additional data points on the 304 stainless steel map. The resulting code and figure are shown in figure (24) and (25) respectively.

Figure (24): Correction on the code resulting in a fully colored region instead of a boundary line

```plaintext
data
124 % plasticity dominant & boundary with power-law creep
data
125 - test= ((eps_2==eps_1)* (eps_2==eps_0) .* (eps_2==eps_4).* (abs(eps_2-eps_3) < 0.001.*eps_3));
data
126 - y_2 = y(test==1);
data
127 - X_2 = x(test==1);
data
```

Figure (25): Transient map with proper boundary line replacing the fully colored region
3.4. Design Space

Once the transient maps are produced, the required design space must be considered for this application of hypersonic short term flight. In order to do so, an additional curve is plotted on the transient map which represents the design space. The stress values for this curve are divided by 1.25 as it is shown in line 180 of the code in figure (26), to take into consideration the 1.25 required safety factor, and a conditional statement is added to show the maximum plastic strain of 0.002 which represents the 0.2% plastic deformation limitation. However, since an exact value of 0.002 may not be present at the stress and temperature combinations, a very narrow range of values up to 0.002 are used as shown in line 179 in the code in figure (26) to create the design space line shown in magenta in the example plot in figure (27). In the section of the code shown in figure (26), eps1 represents total plastic strain, Y_7 represents stress used to plot the design space curve, and X_7 represents the normalized temperature.

```
179 -   test = (eps1 > 0.00195 & eps1<= 0.002);
180 -   Y_7 = y(test==1)/1.25;
181 -   X_7 = x(test==1);
```

Figure (26): Section of transient code that applies the design space factors

![Transient Map](image)

Figure (27): Example transient map (Al7075 in this case) showing design space curve in cyan
In this case, the design space curve shown in cyan, informs designers of the maximum design conditions based on the specified limiting factors, in addition to providing insight as to which mechanisms are dominant in that specific design space. In this case, the boundaries between plasticity, elastic deformation, power law creep, and diffusional flow are shown in red, dark blue, magenta, green, and black curves. Moreover, several constant strain curves are presented on the plot in thinner blue lines. Similar maps with design space curves are generated for all metals. Finally, a temperature versus design stress curve of the design space is produced for each metal as it is shown in the example in figure (27). In the next chapter, the parameter fitting process for each metal is discussed, and the final transient curves along with design space curves are presented.

The last step to the process is developing a separate design space curve for ease of usage. In order to produce this curve, the same conditional range stated in line 179 of figure (26) that specifies the 0.2% plastic deformation limitation is used to gather the data points that would specify the upper limit for the design space, however this time, the actual stress and temperature values are plotted rather than the normalized stress and temperature values for simplicity of usage. In addition, the stress values are divided by 1.25 to take into account the design safety factor. In this case, the temperature was converted to Fahrenheit and the design stress is MPa as shown in figure (28) below. Using this map, one could determine the maximum design stress at a known temperature, or the maximum temperature at a known design stress by tracing perpendicular lines from the known value on either axis to the design space curve, and then tracing another perpendicular line to the other axis to determine the corresponding maximum design value. For example in figure (28) below, the maximum design stress at 305 degrees Fahrenheit is about 546 MPa.
Figure (28): Example design space curve (AL7075-T6)

All steps explained in this chapter regarding steady state parameter fitting process, transient map construction process, boundary line adjustments, and design space curve production are performed as explained in this chapter for every single metal listed in this study. The results for each metal are provided and discussed in chapter 4.
CHAPTER IV

RESULTS AND DISCUSSIONS

This chapter presents the details of the parameter fitting process, transient map production, and the acceptable design space for 304 Stainless steel, Al7075-T6, Al2024-T6, Inconel 625, Inconel 718, and Rene N4. In addition, the general information regarding each metal such as chemical composition and crystalline structure are provided.

4.1 Stainless Steel 304

In this section the process to develop the 304 stainless steel (304SS) transient map and the design space are discussed. 304 Stainless Steel is in the martensitic range which has f.c.c. structure. As discussed before, in order to produce transient curves and discuss the design space, the steady state parameters are required. In this rare case, all steady state parameters are available, hence eliminating the need to perform the parameter fitting process discussed in the previous chapter. This metal is the only material in the group of required metals with this condition. Since there is no parameter fitting for this metal, the next step is discussed here, which is the production of transient curves and discussion on design space. In order to do so, all material properties required to produce the maps need to be found. In an investigation of the 304SS parameters, it is observed that most of the 304SS steady state parameters happen to be the same as 316SS except for power law parameters, power law exponent and Dorn constant. This observation is indeed intriguing, and the fact that the difference in the value of the power law parameters between 304 and 316 stainless steel is minimal is only a confirmation that these metals have very similar characteristics. In addition, knowing that small variation in most of the parameters does not affect the transient results, allows for the use of 316SS parameters for the few parameters that are not available for 304. Hence, the material properties used to produce the 304SS maps are listed in table (3).
Table (3): 304SS Properties

<table>
<thead>
<tr>
<th>Crystallographic Data</th>
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<tbody>
<tr>
<td>Atomic Volume - Ω (m³)</td>
<td>1.21x10⁻²⁹ (a)</td>
</tr>
<tr>
<td>Burger's Vector - b (m)</td>
<td>2.58x10⁻¹⁰ (a)</td>
</tr>
<tr>
<td>Melting Temperature - Tm (°K)</td>
<td>1680 (a)</td>
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<table>
<thead>
<tr>
<th>Modulus</th>
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<tbody>
<tr>
<td>Shear Modulus at 300 °K - μ₀ (MN/m²)</td>
<td>8.1x10⁴ (a)</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
<td>-0.85 (a)</td>
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<th>Lattice Diffusion</th>
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<td>Pre-exponential - D₀v (m²/s)</td>
<td>3.7x10⁻⁵ (a)</td>
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<td>Activation Energy - Qv (Kj/mole)</td>
<td>280 (a)</td>
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<td>Pre-exponential - δD₀₀ (m³/s)</td>
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<td>Activation Energy - Q₀ (Kj/mole)</td>
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<tbody>
<tr>
<td>Pre-exponential of core diffusion - a₀D₀₀ (m⁴/s)</td>
<td>10⁻²⁴ (b)</td>
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<tr>
<td>Core diffusion activation energy - Qₐ (Kj/mole)</td>
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<table>
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<th>Power-law Creep</th>
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<tr>
<td>Exponent - n</td>
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<td>Dorn Constant - A</td>
<td>1.5x10¹² (a)</td>
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<table>
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<tr>
<th>Obstacle Controlled Glide</th>
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<tr>
<td>0K flow stress, 𝜏/μ₀</td>
<td>6.5x10⁻³ (a)</td>
</tr>
<tr>
<td>Pre-exponential - 𝜌₀ (s⁻¹)</td>
<td>10⁶ (a)</td>
</tr>
<tr>
<td>Activation Energy - ΔF/μ₀b³</td>
<td>0.5 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Work Hardening</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Yield Stress - σ₀₀/μ₀</td>
<td>3.827x10⁻³ (c)</td>
</tr>
<tr>
<td>Hardening Exponent - m</td>
<td>0.31 + 6x10⁻⁵T (b)</td>
</tr>
<tr>
<td>Hardening Constant - K₀/μ₀</td>
<td>2.5x10⁻³ - 5.7x10⁻⁷T (b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transient Power-law Creep</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Strain - 𝜎ₜ</td>
<td>0.087 (b)</td>
</tr>
<tr>
<td>Transient Constant - Cₛ</td>
<td>46 (b)</td>
</tr>
</tbody>
</table>

(a) 304SS data is taken from Frost and Ashby [1], table 8.1
(b) 316SS data is taken from Frost and Ashby [1]
(c) 304SS Yield [26]

In addition, detailed yield strength information for 304SS at various temperatures is provided in the table (4) [26].
Using these material properties, the transient map for 304SS at 30 seconds operating time is produced.

![Transients of 304SS at 30 seconds](image)

**Figure (29): Transient Deformation Map for 304SS at 30 seconds**

As it is observed in the transient map of 304SS, the design space curve (curve in light blue) is mainly in the elastic deformation dominant region with very short range in the plasticity dominant region at higher stress and lower temperatures. However, it never enters the power law creep or the diffusional flow dominant regions. It is also important to note that diffusional flow never becomes dominant for 304SS during the 30 second period and stresses decrease quickly as the temperature reaches 60% of melting temperature (around 1000 °K), which is consistent with the yield strength data for the metal. Based on this information, the design space for 304SS at 30 seconds operating time is as shown in figure (30). The maximum elastic strain in regions with less than 0.2% plastic deformation for 304SS is 0.0032 (mm/mm). The design space figure featuring the equation fitted to the curve is provided in figure (31).
4.2 Aluminum Alloys

Aluminum is mainly known for its lightweight and somewhat corrosion resistant characteristics in the industry [6]. However, its low melting temperature and strength make it undesirable for many applications. Alloying is one way of eliminating these issues, where certain amounts of other metals are added to create an aluminum alloy. Alloying changes some of the
characteristics of the metal and hence make it more desirable for a variety of applications. The increased melting temperature and strength in many aluminum alloys in addition to their lightweight characteristics have been a motivation for their high usage in the aerospace industry.

4.2.1 **AL-7075-T6**

Aluminum alloy 7075 is basically strengthened aluminum through zinc alloying [24]. This alloy was first introduced by Alcoa in 1943 as a high strength Aluminum alloy that could be used in the aerospace industry [22]. The nominal chemical composition for alloy AL-7075 is shown in table (5) [22].

| Table (5): Chemical composition of AL-7075 (WT. %) |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Si              | Fe (max)       | Cu (max)       | Mn (max)       | Mg (max)       | Cr (max)       | Zn (max)       | Ti (max)       |
| 0.40            | 0.50           | 1.2-            | 0.30           | 2.1-            | 0.18-          | 5.1-           | 0.20           |

However, the alloy we are interested in is AL7075-T6, where T6 is the tempering designation. This means that the alloy was solution heat treated and then artificially aged [52]. Aluminum Alloy 7075-T6 has f.c.c. structure, and is one of the strongest aluminum alloys with yield strength characteristics as shown in table (6) [25].

| Table (6): Al7075-T6 Yield Strength [25] |
|-----------------|----------------|
| T (°C)          | Yield Strength (Mpa) |
| 25              | 485              |
| 100             | 460              |
| 150             | 415              |
| 177             | 345              |
| 205             | 275              |
| 230             | 195              |
| 260             | 125              |
| 315             | 55               |
| 370             | 32               |
| 482             | 15               |
| 538             | 2                |
In order to produce deformation maps for Al7075-T6 alloy, one must investigate the material properties of this alloy. Since only a few of Al7075-T6 properties are available in literature, parameter fitting needs to be done at the steady state phase as it was discussed in the previous chapter. The material properties used to produce the maps are listed in table (7). Properties with “*” next to them are those to be fitted to the experimental data.

<table>
<thead>
<tr>
<th>Crystallographic Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic Volume - Ω (m³)</td>
<td>1.66x10^{-29} (a)</td>
</tr>
<tr>
<td>Burger's Vector - b (m)</td>
<td>2.86x10^{-10} (a)</td>
</tr>
<tr>
<td>Melting Temperature - T_m (°K)</td>
<td>908 (b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modulus</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Modulus at 300 °K - μ₀ (MN/m²)</td>
<td>2.69x10^4 (c)</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
<td>-0.5 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lattice Diffusion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential - D₀ᵇ (m²/s) *</td>
<td>1.7x10^-4 (a)</td>
</tr>
<tr>
<td>Activation Energy - Q₀ᵇ (Kj/mole)</td>
<td>142 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boundary Diffusion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential - δD₀ᵇ (m³/s)</td>
<td>5.0x10^-14 (a)</td>
</tr>
<tr>
<td>Activation Energy - Q₀ᵇ (Kj/mole)</td>
<td>84 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Core Diffusion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential of core diffusion - a_cD₀𝑐 (m⁴/s)</td>
<td>7x10^-25 (a)</td>
</tr>
<tr>
<td>Core diffusion activation energy - Q_c (Kj/mole) *</td>
<td>82 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power-law Creep</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponent – n *</td>
<td>4.4 (a)</td>
</tr>
<tr>
<td>Dorn Constant – A *</td>
<td>3.4x10^6 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Obstacle Controlled Glide</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0K flow stress, ῦ/μ₀</td>
<td>7.2x10^-3 (a)</td>
</tr>
<tr>
<td>Pre-exponential - ῦ₀ (s⁻¹)</td>
<td>10⁶ (a)</td>
</tr>
<tr>
<td>Activation Energy - ΔF/μ₀b³</td>
<td>0.5 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Work Hardening</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Yield Stress - σ₀₀/μ₀</td>
<td>0.0185 (d)</td>
</tr>
<tr>
<td>Hardening Exponent - m</td>
<td>0.11 (d)</td>
</tr>
<tr>
<td>Hardening Constant - Kₛ/μ₀</td>
<td>0.0195 (d)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transient Power-law Creep</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Strain - γₜ</td>
<td>0.1818 (e)</td>
</tr>
<tr>
<td>Transient Constant - Cₛ</td>
<td>47.34 (e)</td>
</tr>
</tbody>
</table>

(a) Pure Aluminum values from Frost and Ashby [1]
(b) 7075-T6 value from google [31]
(c) 7075-T6 value from [28]
(d) 7075-T6 value from [29]
(e) Pure Aluminum value from Amin [30]
Having initial material properties, steady state experimental data is needed in order to fit parameters. The experimental data for Al7075-T6 is summarized in table (8) [27].

<table>
<thead>
<tr>
<th>T (°F)</th>
<th>T (°K)</th>
<th>Stress (Ksi)</th>
<th>Stress (Mpa)</th>
<th>Min creep rate (1/hr)</th>
<th>Steady State creep rate (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>422.0389</td>
<td>32.75</td>
<td>225.8034</td>
<td>0.000061</td>
<td>1.69E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37.5</td>
<td>258.5535</td>
<td>0.000067</td>
<td>1.86E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>46</td>
<td>317.159</td>
<td>0.00039</td>
<td>1.08E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>344.738</td>
<td>0.0019</td>
<td>5.28E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>117.2109</td>
<td>0.000053</td>
<td>1.47E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>137.8952</td>
<td>0.0107</td>
<td>2.97E-06</td>
</tr>
<tr>
<td>375</td>
<td>463.7056</td>
<td>22.5</td>
<td>155.1321</td>
<td>0.00022</td>
<td>6.11E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>172.369</td>
<td>0.00038</td>
<td>1.06E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>206.8428</td>
<td>0.0005</td>
<td>1.39E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>62.05284</td>
<td>0.000078</td>
<td>2.17E-08</td>
</tr>
<tr>
<td>450</td>
<td>505.3722</td>
<td>10</td>
<td>68.9476</td>
<td>0.000066</td>
<td>1.83E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>89.63188</td>
<td>0.00031</td>
<td>8.61E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>103.4214</td>
<td>0.00075</td>
<td>2.08E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>27.57904</td>
<td>0.000055</td>
<td>1.53E-08</td>
</tr>
<tr>
<td>600</td>
<td>588.7056</td>
<td>5</td>
<td>34.4738</td>
<td>0.00016</td>
<td>4.44E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>41.36856</td>
<td>0.00045</td>
<td>1.25E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>48.26332</td>
<td>0.006</td>
<td>1.67E-06</td>
</tr>
</tbody>
</table>

In order to fit parameters, first the experimental data were inputted into the steady state parameter fitting code and all values were updated to match values shown in table (8). Next, the temperatures that the experimental data were collected at were selected for the code to plot, and finally, the experimental data points along with temperature contours were plotted on a normalized stress versus normalized strain figure to allow for comparison. The initial plot for AL7075-T6 is shown in figure (32). In this plot, the testing temperatures of each group of data points are shown beneath the data points, and each group is represented with a different color.
Figure (32): Original temperature contours of Al7075-T6 along with experimental data

As it is shown in figure (32), the analytical curves are quite far from the experimental data points. In this case parameter fitting was started by varying the Dorn constant since it proved to affect the results significantly. The optimum value for Dorn constant was found to be $2.5 \times 10^2$ by several rounds of trial and error. The resulting plot is shown in figure (33).

Figure (33): Parameter fitting of Al7075-T6 – Varying Dorn constant (A)
Next, the core diffusion activation energy ($Q_c$) was varied to get a better fit. After several trial and errors, the optimum value was found. The resulting plot is shown in figure (34).

![Figure (34): Parameter fitting of Al7075-T6 – Varying “A” and “$Q_c$”](image)

To enhance the improvements, the lattice diffusion pre-exponential was decreased to $1.7 \times 10^{-5}$. This value was selected because it would have the best effect on the parameter fitting results. The resulting plot of the additional change is shown in figure (35).

![Figure (35): Parameter fitting of Al7075-T6 – Varying “A” and “$Q_c$” and “$D_0\nu$”](image)
Although changing Dorn constant (A), core diffusion activation energy ($Q_c$), and lattice diffusion pre-exponential ($D_{0v}$) improved the results, it didn’t provide a good correlation with the experimental data at lower temperatures. It was noticed at this point that no additional parameter adjustments would result in significant improvements on the results. The last change that resulted in a very minor improvement was increasing power law exponent (n) to 4.5. The plot shown in figure (36), on the left, shows the final stage of parameter fitting for Al7075-T6 which includes the 4 parameter changes discussed.

![Plot showing parameter fitting curve for Al7075-T6](image)

**Figure (36):** (a)-Final parameter fitting curve for Al7075-T6 (b)-Initial state

In figure (36), the initial and final states of the steady state temperature contours and experimental data are shown side by side to allow for better comparison. Although there is very poor correlation between the analytical and experimental data at lower temperatures, it is not considered an issue for two main reasons. First is that the objective of this analysis is to understand high temperature transient creep, and second that in most cases, as shown by parametric studies, variations in many of the parameters don’t affect the final transient results significantly. Hence, the final adjusted values of the parameters are shown in table (9).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power law exponent (n)</td>
<td>4.4</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table (9): Al7075-T6 Initial and Final Values of Parameters Fitted to Experimental Data
Dorn Constant (A) \(3.4 \times 10^6\) \(2.5 \times 10^2\)
Activation energy for Core Diffusion \((Q_c)\) - Kj/mole 82 115
Lattice Diffusion Pre-exponential \((D_{0v})\) - m\(^2\)/s \(1.7 \times 10^{-4}\) \(1.7 \times 10^{-5}\)

Using the final values and all other parameters discussed earlier in this section, the transient curve for Al7075-T6 was generated at 30 seconds operating time as shown in figure (37). On this figure, the red line represents the border between elastic deformation and plasticity dominant regions. The dark blue line represents the bordering area between power law creep and plasticity dominant regions. The curve in magenta represents the border between power law creep and elastic deformation dominant regions. The bold black curve represents the border between elastic deformation and diffusional flow dominant regions. The green line represents the border between power law creep and diffusional flow, and finally the curve in cyan represents the design space curve with limiting factors as discussed in chapter 3. As it is shown, the design space falls in the elastic deformation dominant regime for the most part and enters the diffusional flow dominant region at roughly 75% of it’s melting temperature which is at around 680 °K or 764 °F.

Figure (37): Al7075-T6 Transient map at 30 seconds
The corresponding stress and temperature combinations representing the design space are shown on a temperature versus design stress plot as shown in figure (38). The design space plot was broken down into three sections for equation fitting, and are shown in figures (39), (40), and (41).

![Figure (38): Al7075-T6 Design space](image)

![Figure (39): Al7075-T6 Design space Featuring Fitted Equation up to 350°K](image)
Based on the analysis of Al7075-T6, the design stresses decrease significantly (from 600 MPa to almost 100 MPa) at 300 °F to 500 °F operating temperature including the 1.25 safety factor and limiting plastic deformation to 0.2%. The maximum elastic strain at regions were plastic strain is less than 0.002% is 0.0182 (mm/mm).
4.2.2 Aluminum Alloy 2024-T6

Aluminum alloy 2024 is strengthened aluminum through copper alloying [24]. The nominal chemical composition for alloy AL-2024 is [33]

| Table (10): Chemical composition of AL-2024 (WT. %) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Si              | Fe              | Cu              | Mn              | Mg              | Cr              | Zn              | Ti              | Others          | Al              |
| 0.50            | 0.50            | 3.8-            | 0.30-           | 1.2-            | 0.1             | 0.25            | 0.15            | 0.15            | Balance         |
| (max)           | (max)           | 4.9             | 0.9             | 1.8             | (max)           | (max)           | (max)           | (max)           |                  |

However, the alloy we are interested in is AL2024-T6, where T6 is the tempering designation, which is the same type of heat treatment as the AL7075-T6 discussed in the previous section. In addition, aluminum alloy 2024-T6 has f.c.c. structure similar to the Al7075 alloy.

In order to produce deformation maps for Al2024-T6 alloy, one must investigate the material properties of this alloy. Since only a few of Al2024-T6 properties are available in literature, parameter fitting needs to be done at the steady state phase as it was discussed in the previous chapter. The material properties used to produce the maps are listed in table (11). Properties with “*” next to them are those to be fitted to the experimental data.

<table>
<thead>
<tr>
<th>Table (11): AL2024-T6 Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Crystallographic Data</strong></td>
</tr>
<tr>
<td>Atomic Volume - $\Omega$ ($m^3$)</td>
</tr>
<tr>
<td>Burger's Vector - $b$ (m)</td>
</tr>
<tr>
<td>Melting Temperature - $T_m$ (°K)</td>
</tr>
<tr>
<td><strong>Modulus</strong></td>
</tr>
<tr>
<td>Shear Modulus at 300 °K - $\mu_0$ ($MN/m^2$)</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
</tr>
<tr>
<td><strong>Lattice Diffusion</strong></td>
</tr>
<tr>
<td>Pre-exponential - $D_{0v}$ ($m^2/s$) *</td>
</tr>
<tr>
<td>Activation Energy - $Q_v$ (Kj/mole)*</td>
</tr>
<tr>
<td><strong>Boundary Diffusion</strong></td>
</tr>
<tr>
<td>Pre-exponential - $\delta D_{0b}$ ($m^3/s$)</td>
</tr>
<tr>
<td>Activation Energy - $Q_b$ (Kj/mole)</td>
</tr>
<tr>
<td><strong>Core Diffusion</strong></td>
</tr>
<tr>
<td>Pre-exponential of core diffusion - $\alpha_c D_{oc}$ ($m^4/s$)</td>
</tr>
<tr>
<td>Core diffusion activation energy - $Q_c$ (Kj/mole) *</td>
</tr>
</tbody>
</table>
Power-law Creep
Exponent – n 4.4 (a)
Dorn Constant – A 3.4x10^6 (a)

Obstacle Controlled Glide
0K flow stress, \( \tilde{\tau}/\mu_0 \) 7.2x10^{-3} (a)
Pre-exponential - \( \gamma_0 (s^{-1}) \) 10^6 (a)
Activation Energy - \( \Delta F/\mu_0 b^3 \) 0.5 (a)

Work Hardening
Initial Yield Stress - \( \sigma_{0s}/\mu_0 \) 0.0143 (d)
Hardening Exponent - m 0.093 (e)
Hardening Constant - \( K_s/\mu_0 \) 0.0254 (e)

Transient Power-law Creep
Transient Strain - \( \gamma_t \) 0.1818 (f)
Transient Constant - \( C_s \) 47.32 (f)

(a) Pure Aluminum values from Frost and Ashby [1]
(b) 2024-T6 value from google [31]
(c) 2024-T62 value from [34]
(d) 2024-T6 value from [25]
(e) 2024-T6 value from [35]
(f) Pure Aluminum value from Amin [30]

The yield strength of this alloy at different temperatures is listed in table (12).

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>Yield Strength (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>395</td>
</tr>
<tr>
<td>100</td>
<td>370</td>
</tr>
<tr>
<td>150</td>
<td>360</td>
</tr>
<tr>
<td>177</td>
<td>340</td>
</tr>
<tr>
<td>205</td>
<td>295</td>
</tr>
<tr>
<td>260</td>
<td>170</td>
</tr>
<tr>
<td>315</td>
<td>95</td>
</tr>
<tr>
<td>370</td>
<td>45</td>
</tr>
</tbody>
</table>

Having initial material properties, steady state experimental data is needed in order to fit parameters. However, most experimental deformation data available for Al2024 are either for different tempering conditions other than T6, or they do not include the steady state strain rate information that is needed here. For example, there is rupture information available for the T6 heat treatment, in addition to specific %strain data points which are not sufficient to calculate steady state rate information. The only experimental data for Al2024-T6 found in literature that included steady state rate information is shown in table (13) [36].
Table (13): Al2024-T6 Experimental Data [36]

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>T (°K)</th>
<th>Stress (Mpa)</th>
<th>Steady State creep rate (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>423</td>
<td>300</td>
<td>8.1018E-09</td>
</tr>
</tbody>
</table>

However, one data point is not enough for parameter fitting. Out of all other heat treatments, the most information available is for T3, which means that alloy was solution heat treated and then cold worked [23]. Hence, the experimental data points for this heat treatment are used for parametric fitting process. The experimental data for Al2024-T3 used here included several test temperatures, however the upper two temperatures were used here since our focus is a high temperature analysis. This information is listed in table (14).

Table (14): Al2024-T3 Experimental Data [27]

<table>
<thead>
<tr>
<th>T (°F)</th>
<th>T (°K)</th>
<th>Stress (Mpa)</th>
<th>Steady State creep rate (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>505</td>
<td>137.9</td>
<td>4.167E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>172.4</td>
<td>9.444E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>189.6</td>
<td>2.028E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>206.8</td>
<td>7.222E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27.6</td>
<td>1.389E-08</td>
</tr>
<tr>
<td>600</td>
<td>588</td>
<td>41.3</td>
<td>8.611E-08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>55.2</td>
<td>3.889E-07</td>
</tr>
</tbody>
</table>

Using the information presented so far, the original curve was produced as shown in figure (42).
Power law creep exponent was the first parameter that was varied resulting in the following plot.

Next, the activation energy for core diffusion was varies resulting in the plot shown in figure (44).
Continuing the parameter fitting process, the activation energy for lattice diffusion \( Q_v \) was varied.

Figure (45): Al2024-T6 parameter fitting – Varying “n” & “Qc” & “\( Q_v \)”
Finally the pre-exponential for lattice diffusion “$D_{0v}$” was varied to provide the best fit possible. This final parameter fitting state is presented along with the original state prior to this process.

Figure (46): Al2024-T6 parameter fitting – a) Final state “n” & “Qc” & “$Q_v$” & “$D_{0v}$” – b) Initial

Although there is very poor correlation between the analytical and experimental data at the lower temperature, it is not considered an issue for two main reasons. First is that the objective of this analysis is to understand high temperature transient creep, and second that in most cases, as shown by parametric studies, variations in many of the parameters don’t affect the final transient results significantly. Hence, the final adjusted values of the parameters are shown in table (15).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power law exponent (n)</td>
<td>4.4</td>
<td>5.2</td>
</tr>
<tr>
<td>Activation energy for lattice Diffusion ($Q_v$) - Kj/mole</td>
<td>142</td>
<td>195</td>
</tr>
<tr>
<td>Activation energy for Core Diffusion ($Q_c$) - Kj/mole</td>
<td>82</td>
<td>160</td>
</tr>
<tr>
<td>Lattice Diffusion Pre-exponential ($D_{0v}$) - m²/s</td>
<td>$1.7 \times 10^{-4}$</td>
<td>$9.7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Using the final values and all other parameters discussed earlier in this section, the transient curve for Al2024-T6 was generated at 30 seconds operating time as shown in figure
In this figure, the curve in cyan represents the design space curve with limiting factors as discussed in chapter 3. As it is shown, the design space falls in the elastic deformation dominant regime at all times.

The corresponding stress and temperature combinations representing the design space are shown on a temperature versus design stress plot as shown in figure (48). As it is shown, design stressed decrease significantly at a steep rate as the temperatures get closer to 400°F. The maximum elastic strain in regions were plastic strain is less than 0.2% is 0.0151 (mm/mm). The design space plot was broken down into three sections for equation fitting, and are shown in figures (49), (50), and (51).

Figure (47): Al2024-T6 Transient map at 30 seconds
Figure (48): Al2024-T6 Design space at 30 seconds

Figure (49): Al2024-T6 Design space Featuring Fitted Equation up to 350°K
Figure (50): Al2024-T6 Design space Featuring Fitted Equation - 350°K-500°K

Figure (51): Al2024-T6 Design space Featuring Fitted Equation - 500°K & up
4.3 Inconel

Inconel is a trademark name for a Nickel-Chromium based superalloy. It is known for its high strength and resistance to corrosion at very high temperatures and is very suitable for extreme operating conditions. In this study, Inconel 625 and Inconel 718 are investigated.

Age hardened nickel based super alloys include austenitic f.c.c. matrix dispersed with coherent precipitation of an Ni3(Al,Ti) intermetallic with an f.c.c. structure. This coherent intermetallic phase is called $\gamma'$ and is responsible for high temperature strength and creep resistance of these alloys. Aluminum and Titanium are added in amounts required to precipitate the $\gamma'$ f.c.c. coherent phase [49].

On the other hand, the combination of iron, nickel and niobium in specific amounts forms the body centered tetragonal (b.c.t) Ni3Nb, which is coherent with gamma matrix. This phase is called $\gamma''$ and provides high strength at low to intermediate temperatures, but it becomes unstable at temperatures above 650$^\circ$C (1200F). This precipitate is found in the nickel-iron alloys [49].

4.3.1 Inconel 625

Inconel 625 is one of the nickel based superalloys used in aerospace, petrochemical, and marine applications [54]. It’s high strength is primarily associated with the $\gamma''$ phase [54]. At temperatures above 600$^\circ$C, the precipitates of the equilibrium $Ni_3Nb$ ($\delta$) phase replace those of the metastable $\gamma''$ phase, hence affecting metal characteristics negatively [40]. To begin the study of Inconel 625 transient creep, let’s start with chemical composition and material characteristics.

The chemical composition and material characteristics of Inconel 625 are in tables (16) and (17).
Table (17): Inconel 625 Properties

<table>
<thead>
<tr>
<th>Crystallographic Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic Volume - $\Omega$ ($m^3$)</td>
<td>1.09x10^{-29} (a)</td>
</tr>
<tr>
<td>Burger's Vector - b (m)</td>
<td>2.49x10^{-10} (a)</td>
</tr>
<tr>
<td>Melting Temperature - $T_m$ (°K)</td>
<td>1622 (b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modulus</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Modulus at 300 °K - $\mu_0$ ($MN/m^2$)</td>
<td>8.14x10^4 (b)</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
<td>-0.64 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lattice Diffusion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential - $D_{0v}$ ($m^2/s$) *</td>
<td>1.9x10^{-4} (a)</td>
</tr>
<tr>
<td>Activation Energy - $Q_v$ (Kj/mole)*</td>
<td>284 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boundary Diffusion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential - $\delta D_{0b}$ ($m^3/s$)</td>
<td>3.5x10^{-15} (a)</td>
</tr>
<tr>
<td>Activation Energy - $Q_b$ (Kj/mole)</td>
<td>115 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Core Diffusion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential of core diffusion - $a_c D_{oc}$ ($m^4/s$)</td>
<td>3.1x10^{-23} (a)</td>
</tr>
<tr>
<td>Core diffusion activation energy - $Q_c$ (Kj/mole) *</td>
<td>170 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power-law Creep</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponent – n</td>
<td>4.6 (a)</td>
</tr>
<tr>
<td>Dorn Constant – $A$</td>
<td>3.0x10^{6} (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Obstacle Controlled Glide</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0K flow stress, $\dot{\tau}/\mu_0$</td>
<td>6.3x10^{-3} (a)</td>
</tr>
<tr>
<td>Pre-exponential - $\gamma_0$($s^{-1}$)</td>
<td>10^{6} (a)</td>
</tr>
<tr>
<td>Activation Energy - $\Delta F/\mu_0 b^3$</td>
<td>0.5 (a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Work Hardening</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Yield Stress - $\sigma_{0s}/\mu_0$</td>
<td>0.00444 (b)</td>
</tr>
<tr>
<td>Hardening Exponent - $m$</td>
<td>0.22 (c)</td>
</tr>
<tr>
<td>Hardening Constant - $K_s/\mu_0$</td>
<td>0.0132 (d)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transient Power-law Creep</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Strain - $\gamma_t$</td>
<td>0.0727 (e)</td>
</tr>
<tr>
<td>Transient Constant - $C_t$</td>
<td>199.7 (e)</td>
</tr>
</tbody>
</table>

(a) Nickel values from Frost and Ashby [1]
(b) Inconel 625 value [37] – annealed value of shear modulus is used – Initial yield strength (annealed at 1093°C)
(c) Inconel 625 value taken to be the average value of the B3 and D1 heats at 60°C [40]
(d) Inconel 625 value for a service aged case, bottom value is used [47]
(e) Nickel value from Amin [30]

The experimental data used for parameter fitting in this case is for Inconel 625 annealed at 1100 °C which is appropriate for high temperature operating conditions [38]. Hence, the yield strength reported here is for annealed condition as shown in table (18).

Table (18): Inconel 625 Yield Strength [39]

<table>
<thead>
<tr>
<th>T (°K)</th>
<th>Yield Strength (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>295</td>
<td>479</td>
</tr>
</tbody>
</table>
The experimental data for Inconel 625 is shown in table (19) [38].

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Stress Mpa</th>
<th>SS creep rate (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1073</td>
<td>103.4214</td>
<td>9.11111E-07</td>
</tr>
<tr>
<td></td>
<td>137.8952</td>
<td>3.83333E-06</td>
</tr>
<tr>
<td></td>
<td>34.4738</td>
<td>3.29167E-07</td>
</tr>
<tr>
<td>1173</td>
<td>68.9476</td>
<td>7.12778E-06</td>
</tr>
<tr>
<td></td>
<td>20.68428</td>
<td>3.88889E-08</td>
</tr>
<tr>
<td></td>
<td>13.78952</td>
<td>3.13889E-08</td>
</tr>
<tr>
<td>1223</td>
<td>20.68428</td>
<td>3.86111E-07</td>
</tr>
<tr>
<td></td>
<td>10.34214</td>
<td>1.33333E-07</td>
</tr>
<tr>
<td>1273</td>
<td>13.78952</td>
<td>8.36111E-07</td>
</tr>
</tbody>
</table>

The parameter fitting process started by plotting analytical curves of temperature contours along with the experimental values using the material properties listed in table (19). The resulting plot is shown in figure (52) below.
In order to get a better correlation between experimental and analytical data, first the activation energy for core diffusion ($Q_c$) was changed. The resulting plot is shown in figure (53).

![Figure (52): Inconel 625 Parameter fitting – Original](image)

![Figure (53): Inconel 625 Parameter fitting – Varying “Qc”](image)
Next, the activation energy for lattice diffusion ($Q_v$) was increased, resulting the plot shown in figure (54).

![Inconel 625 of grain size 1μm graph](image)

Figure (54): Inconel 625 Parameter fitting – Varying “Qc” and “$Q_v$”

The last parameter that was fitted to the experimental data was the lattice diffusion pre-exponential ($D_{0v}$) which resulted in the closest overall fitting of the analytical curves to the experimental data. The final plot of the parameter fitting results is shown in figure (55) alongside the original curve to allow for comparison.

![Inconel 625 of grain size 1μm graphs](image)

Figure (55): (a) Inconel 625 Final parameter fitting– Varying “$Q_v$” & “$Q_c$” & “Dov” (b) Initial
The final values of the parameters fitted to the experimental data are listed in the table (20) alongside the original values for comparison. In this case, the typical approach of varying power law exponent and Dorn constant discussed in chapter 3, proved to be inefficient. Lattice diffusion pre-exponential and activation energies where the parameters adjusted for curve fitting.

<table>
<thead>
<tr>
<th>Table (20): Inconel 625 Initial and Final Values of Parameters Fitted to Experimental Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Lattice diffusion pre-exponential - $D_0v$</td>
</tr>
<tr>
<td>Activation energy for Core Diffusion ($Q_c$) - Kj/mole</td>
</tr>
<tr>
<td>Activation energy for Lattice Diffusion ($Q_v$) - Kj/mole</td>
</tr>
</tbody>
</table>

Using the parameter fitting results shown in table (20) and the rest of the material properties, the transient deformation map for Inconel 625 was produced featuring the design space curve in cyan as shown in figure (56).

**Figure (56): Inconel 625 Transient Deformation Map at 30 seconds**
Based on the resulting transient and design space curves, it is observed that elastic deformation is dominant over most of the high temperature operating region with a maximum value of 0.0051 (mm/mm) when plastic strain is less than 0.2%, until around 87% of melting temperature range. However, at this point, design stresses are very low. The temperature at which the design stresses start to decline at a steep rate is at about 50% of the melting temperature, ~810°C (538°C). The design space plot was broken down into three sections for equation fitting, and are shown in figures (58), (59), and (60).
Figure (58): Inconel 625 Design space Featuring Fitted Equation up to 800°F

Figure (59): Inconel 625 Design space Featuring Fitted Equation - 800°F-1160°F
Inconel 718 is an iron-nickel superalloy with superior strength at high temperatures (up to 650°C) [53]. This strength is due to the existence of the $\gamma'$ and $\gamma''$ phases in the metal [53]. At higher temperatures however, a nickel and niobium rich phase, also known as $\delta$ phase appears which introduces very brittle characteristics, and hence causing the temperature limitation on this metal [53]. The chemical composition of Inconel 718 is as followed.

Table (21): Chemical composition of Inconel 718 (WT. %) [41]

<table>
<thead>
<tr>
<th>Element</th>
<th>Ni</th>
<th>Fe</th>
<th>BAL</th>
<th>Mo</th>
<th>Mn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>50-55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph</td>
<td>17-21</td>
<td></td>
<td>Nb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co</td>
<td>0.015 (max)</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0 (max)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure (60): Inconel 625 Design space Featuring Fitted Equation - 1160°K & up
The initial material properties used to start the parameter fitting process as discussed in the previous chapter is listed in table (22). Just like Inconel 625, many of the properties that were not found in literature are pure nickel properties.

<table>
<thead>
<tr>
<th>Crystallographic Data</th>
<th>Table (22): Inconel 718 Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic Volume - $\Omega$ (m$^3$)</td>
<td>1.09x10$^{-29}$ (a)</td>
</tr>
<tr>
<td>Burger's Vector - b (m)</td>
<td>2.75x10$^{-10}$ (h)</td>
</tr>
<tr>
<td>Melting Temperature - $T_m$ (°K)</td>
<td>1609 (b)</td>
</tr>
<tr>
<td><strong>Modulus</strong></td>
<td></td>
</tr>
<tr>
<td>Shear Modulus at 300 °K - $\mu_0$ (MN/m$^2$)</td>
<td>8.1x10$^4$ (c)</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
<td>-0.036 (c)</td>
</tr>
<tr>
<td><strong>Lattice Diffusion</strong></td>
<td></td>
</tr>
<tr>
<td>Pre-exponential - $D_{0v}$ (m$^2$/s)</td>
<td>1.9x10$^{-4}$ (a)</td>
</tr>
<tr>
<td>Activation Energy - $Q_v$ (Kj/mole) *</td>
<td>284 (a)</td>
</tr>
<tr>
<td><strong>Boundary Diffusion</strong></td>
<td></td>
</tr>
<tr>
<td>Pre-exponential - $\delta D_{0b}$ (m$^3$/s)</td>
<td>3.5x10$^{-15}$ (a)</td>
</tr>
<tr>
<td>Activation Energy - $Q_b$ (Kj/mole)</td>
<td>115 (a)</td>
</tr>
<tr>
<td><strong>Core Diffusion</strong></td>
<td></td>
</tr>
<tr>
<td>Pre-exponential of core diffusion - $a_c D_{0c}$ (m$^4$/s)</td>
<td>3.1x10$^{-23}$ (a)</td>
</tr>
<tr>
<td>Core diffusion activation energy - $Q_c$ (Kj/mole) *</td>
<td>170 (a)</td>
</tr>
<tr>
<td><strong>Power-law Creep</strong></td>
<td></td>
</tr>
<tr>
<td>Exponent – n *</td>
<td>4.6 (a)</td>
</tr>
<tr>
<td>Dorn Constant – A *</td>
<td>3.0x10$^6$ (a)</td>
</tr>
<tr>
<td><strong>Obstacle Controlled Glide</strong></td>
<td></td>
</tr>
<tr>
<td>0K flow stress, $\dot{\gamma}/\mu_0$</td>
<td>6.3x10$^{-3}$ (a)</td>
</tr>
<tr>
<td>Pre-exponential - $\dot{\gamma}_0$ ($s^{-1}$)</td>
<td>10$^6$ (a)</td>
</tr>
<tr>
<td>Activation Energy - $\Delta F/\mu_0 b^3$</td>
<td>0.5 (a)</td>
</tr>
<tr>
<td><strong>Work Hardening</strong></td>
<td></td>
</tr>
<tr>
<td>Initial Yield Stress - $\sigma_{0s}/\mu_0$</td>
<td>0.0139 (g)</td>
</tr>
<tr>
<td>Hardening Exponent - $m$</td>
<td>0.44 (e)</td>
</tr>
<tr>
<td>Hardening Constant - $K_s/\mu_0$</td>
<td>0.0214 (d)</td>
</tr>
<tr>
<td><strong>Transient Power-law Creep</strong></td>
<td></td>
</tr>
<tr>
<td>Transient Strain - $\gamma_t$</td>
<td>0.0727 (f)</td>
</tr>
<tr>
<td>Transient Constant - $C_s$</td>
<td>199.7 (f)</td>
</tr>
</tbody>
</table>

(a) Nickel values from Frost and Ashby [1]
(b) Inconel 718 value [41]
(c) Inconel 718 value [43]
(d) Inconel 718 value heat treated at 973K for one hour - [40]
(e) Inconel 718 value [46]
(f) Nickel value from Amin [30]
(g) Inconel 718 value [45]
(h) Inconel 718 value from [58], and adjusted by 10% to consider temperature effects
Furthermore, the yield strength of Inconel 718 at different temperatures is listed in the table (23).

<table>
<thead>
<tr>
<th>T (°K)</th>
<th>Yield Strength (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>311</td>
<td>1124</td>
</tr>
<tr>
<td>422</td>
<td>1103</td>
</tr>
<tr>
<td>588</td>
<td>1075</td>
</tr>
<tr>
<td>755</td>
<td>1034</td>
</tr>
<tr>
<td>811</td>
<td>1013</td>
</tr>
<tr>
<td>922</td>
<td>965</td>
</tr>
<tr>
<td>977</td>
<td>924</td>
</tr>
</tbody>
</table>

The experimental data for Inconel 718 was provided for different grain sizes. In this case the 10 μm grain size data were used as show in table (24).

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Stress Mpa</th>
<th>Steady state creep rate (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>873</td>
<td>700</td>
<td>2.04E-09</td>
</tr>
<tr>
<td>898</td>
<td>690</td>
<td>7.74E-09</td>
</tr>
</tbody>
</table>

The parameter fitting process for Inconel 718 started by plotting analytical curves of temperature contours along with the experimental values using the material properties listed in table (24). The resulting plot is shown in figure (61).
As it is observed, the analytical curves are quite far from experimental data points. Hence the parameter fitting was started by first varying and optimizing the Dorn constant (A). The resulting plot is shown in figure (62).
Next, the power law creep exponent (n) was increased resulting the plot shown in figure (63).

![Inconel 718 of grain size 10μm](image)

Figure (63): Inconel 718 Parameter fitting – Varying “n” and “A”

Next, the activation energy for core diffusion ($Q_c$) was changed which resulted in a much better fit to the experimental data as shown in figure (64). In this figure, the final plot of the parameter fitting results is shown alongside the original curve to allow for comparison. Note that the plot axis were adjusted between the initial and final states to allow for better visual capability.

![Inconel 718 of grain size 10μm](image)

Figure (64): (a) Inconel 718 Final parameter fitting – Varying “n” & “$Q_c$” & “A”  (b) Initial
As one can see in the figure (64) above, the results of the parameter fitting process are by no means close to the experimental data. Despite extensive work and also proper adjustments to variables that improved the results, proper fitting was not possible. It was however observed that the logarithm of the analytical stress values was roughly off by a factor of 2. In addition, burgers vector values proved very influential in the lateral location of the curves, and this is the reason that the burgers vector value was adjusted and increased by 10% to improve results, but also to take into account possible temperature effects on the burgers vector. The final values of the parameters fitted to the experimental data are listed in table (25) alongside the original values for comparison.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power law exponent (n)</td>
<td>4.6</td>
<td>5.1</td>
</tr>
<tr>
<td>Dorn Constant (A)</td>
<td>3.0x10^6</td>
<td>3.0x10^3</td>
</tr>
<tr>
<td>Activation energy for Core Diffusion (Q_c) - Kj/mole</td>
<td>170</td>
<td>253</td>
</tr>
</tbody>
</table>

Using the parameter fitting results shown in table (25) and the rest of the material properties, the transient deformation map for Inconel 718 was produced featuring the design space curve in magenta as shown in figure (65).
Based on the transient plot of Inconel 718 as shown in figure (65), and design space curve in figure (66), it is observed that the design space curve starts in the low temperature plasticity dominant region. As the temperature increases on the plot, the stress is reduced slightly all the way to around 0.57 of normalized temperature on the map. This temperature happens to be the
57% of the melting temperature, which is 923 °K (650°C - 1200°F). At temperatures above this limit, the stress values decrease at a rapid rate entering the elastic region and finally going through the diffusional flow regime at temperature just below the 80% of melting. It is noteworthy to point out that these results are in line with the characteristics described by Columbia University research team (reference 53) as described in the beginning of this section where it was pointed out that the high strength of Inconel 718 holds up to 650°C temperature range and decreases at temperatures above this value. The maximum elastic strain in regions where plastic strain is less than 0.2% is 0.01 (mm/mm). The design space plot was broken down into three sections for equation fitting, and are shown in figures (67), (68), and (69).

![Inconel 718 Design space featuring fitted equation up to 600°C](image)

Figure (67): Inconel 718 Design space Featuring Fitted Equation up to 600°C
Figure (68): Inconel 718 Design space Featuring Fitted Equation - 600°K-900°K

Figure (69): Inconel 718 Design space Featuring Fitted Equation - 900°K & up
### 4.4 Rene N4

Rene N4 is a Nickel based, single crystal superalloy developed by GE known for its very high strength and great properties at high temperatures. Hence it has been considered for many aerospace applications where high temperature endurance is of importance. All modern single crystal materials are two phase alloys with a large volume fraction of $\gamma'$ phase [55]. The strength of the alloy is a function of the $\gamma'$ size and percentage [55]. It has been experimentally determined that the peak creep strength is achieved at roughly 60% volume fraction of $\gamma'$ [55]. In such cases, deformation must occur by shearing of the precipitates [55]. The chemical composition of Rene N4 is listed in table (26).

| Table (26): Chemical composition of Rene N4 (WT. %) [59] |
|---|---|---|---|---|---|---|---|---|---|---|
| Ni | Cr | Co | Mo | W | Ta | Nb | Al | Ti | C | B | Hf |
| 62 | 9.8 | 7.5 | 1.5 | 6 | 4.8 | 0.5 | 4.2 | 3.5 | 0.06 | 0.004 | 0.15 |

The initial material properties used to start the parameter fitting process as discussed in the previous chapter is listed in table (27). Just like the Inconel alloys discussed previously, many of the properties that were not found in literature are pure nickel properties.

<table>
<thead>
<tr>
<th>Table (27): Rene N4 Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Crystallographic Data</strong></td>
</tr>
<tr>
<td>Atomic Volume - $\Omega$ ($m^3$)</td>
</tr>
<tr>
<td>Burger's Vector - $b$ (m)</td>
</tr>
<tr>
<td>Melting Temperature - $T_m$ (°K)</td>
</tr>
<tr>
<td><strong>Modulus</strong></td>
</tr>
<tr>
<td>Shear Modulus at 300 °K - $\mu_0$ ($MN/m^2$)</td>
</tr>
<tr>
<td>Temperature dependence of Modulus</td>
</tr>
<tr>
<td><strong>Lattice Diffusion</strong></td>
</tr>
<tr>
<td>Pre-exponential - $D_{0\nu}$ ($m^2/s$)</td>
</tr>
<tr>
<td>Activation Energy - $Q_\nu$ (Kj/mole)</td>
</tr>
<tr>
<td><strong>Boundary Diffusion</strong></td>
</tr>
<tr>
<td>Pre-exponential - $\delta D_{0b}$ ($m^3/s$)</td>
</tr>
<tr>
<td>Activation Energy - $Q_b$ (Kj/mole)</td>
</tr>
<tr>
<td><strong>Core Diffusion</strong></td>
</tr>
<tr>
<td>Pre-exponential of core diffusion - $a_cD_{oc}$ ($m^4/s$)*</td>
</tr>
</tbody>
</table>
Core diffusion activation energy - $Q_c$ (Kj/mole) * 170 (a)

**Power-law Creep**
- Exponent – n 4.6 (a)
- Dorn Constant – A * $3.0 \times 10^6$ (a)

**Obstacle Controlled Glide**
- 0K flow stress, $\dot{\gamma}/\mu_0$ $6.3 \times 10^{-3}$ (a)
- Pre-exponential - $\gamma_0 (s^{-1})$ $10^6$ (a)
- Activation Energy - $\Delta F/\mu_0 b^3$ 0.5 (a)

**Work Hardening**
- Initial Yield Stress - $\sigma_{05}/\mu_0$ 0.0078 (d)
- Hardening Exponent - m 0.2 (e)
- Hardening Constant - $K_s/\mu_0$ 0.0336 (f)

**Transient Power-law Creep**
- Transient Strain - $\gamma_t$ 0.0727 (g)
- Transient Constant - $C_s$ 199.7 (g)

(a) Nickel values from Frost and Ashby [1]
(b) Rene N4 value [60]
(c) Rene N4 value [62]
(d) Rene N4 value [62]
(e) Rene N4 value [63]
(f) Rene N4 value calculated using experimental data
(g) Nickel value from Amin [30]

Furthermore, the yield strength of Rene N4 at different temperatures is listed in table (28). Note the interesting behavior of Rene N4 yield strength at temperatures in the 920 to 1030 Kelvin range. Unlike most other metals where the yield strength decreases with temperature, the yield strength for Rene N4 increases for a short temperature range at relatively high temperatures starting roughly at 57% of melting temperature before it starts to decrease again.

| Table (28): Rene N4 Yield Strength [60] |
|----------|------------------|
| T (°K)   | Yield Strength (Mpa) |
| 294      | 1000             |
| 811      | 828              |
| 922      | 952              |
| 1033     | 1028             |
| 1144     | 690              |
| 1255     | 428              |

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The experimental data for Rene N4 was provided for [0 0 1] load direction tensile loading at 1144°K. The loading direction is shown in figure (70), and the data is tabulated in table (29).

![Loading direction](image)

Figure (70): Loading direction

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Stress Mpa</th>
<th>SS creep rate (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>379</td>
<td>6.94E-10</td>
<td></td>
</tr>
<tr>
<td>276</td>
<td>1.11E-10</td>
<td></td>
</tr>
<tr>
<td>345</td>
<td>2.77E-10</td>
<td></td>
</tr>
<tr>
<td>414</td>
<td>1.38E-09</td>
<td></td>
</tr>
</tbody>
</table>

The parameter fitting process for Rene N4 started by plotting analytical curves of temperature contours along with the experimental values using the material properties listed in table (27). The resulting plot is shown in figure (71).
In the first step, power law exponent “n” was changed and its value was optimized to result in the following plot in figure (72).

Next, Dorn constant “A” was changed resulting in the plot shown in figure (73).
The last change is with regards to the core diffusion activation energy and lattice diffusion activation energy. It was observed that although individual changes of these variables would only result in minor improvements of the curve fittings, the simultaneous adjustment of these parameters would result in a significant change in the overall results on the parameter fitting process. Hence, figure (74) includes changes to both the activation energies resulting in the final state of the parameter fitting process.

Figure (73): Rene N4 Parameter fitting – Varying “A” & “n”

Figure (74): (a) Rene N4 Final parameter fitting – Varying “n” & “Q_v” & “Q_c” & “A” (b) Initial
In figure (74), the initial and final states of the parameter fitting process for Rene N4 are shown side by side. Note that the vertical axes on the two plots and not the same. The axis has been adjusted to in the final state to provide better resolution. One of the reasons for obtaining such great fitting results for Rene N4 compared to previous metals could be the very high testing temperature. The adjusted values of the parameters for Rene N4 are shown in table (30).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power law exponent (n)</td>
<td>4.6</td>
<td>6.3</td>
</tr>
<tr>
<td>Dorn Constant (A)</td>
<td>3.0x10^6</td>
<td>3.0x10^4</td>
</tr>
<tr>
<td>Activation energy for Core Diffusion (Q_c) - Kj/mole</td>
<td>170</td>
<td>278</td>
</tr>
<tr>
<td>Activation energy for Lattice Diffusion (Q_v) - Kj/mole</td>
<td>284</td>
<td>365</td>
</tr>
</tbody>
</table>

Using the adjusted values and the remaining of the material parameters, the transient deformation maps of Rene N4 was constructed featuring the design space curve in magenta as shown in figure (75).

![Figure (75): Rene N4 Transient Map at 30 seconds](image)
The resulting transient map, figure (75), for Rene N4 shows a quite different type of characteristic if compared to all other metals. This is due to the intriguing yield strength properties of the metal. The design space curve in this case stays in the plasticity dominant region for the most part all the way to about 65% of the melting temperature of the metal (~1000°K). It
then enters the elastic region for a short range while oscillating significantly and closely to the power law creep border, as the stresses finally begin to decrease. This behavior is unique to Rene N4 and was not observed in any of the other metals studied here. At this time, 30 seconds, diffusional flow never becomes dominant. It was however noticed that at 1 hour study time, the diffusional flow becomes dominant at very high temperature (above 90% of melting).

On the other hand, the design space curve in figure (77), exhibits very odd behaviors at temperatures near (1400 – 1500 °F) range. Although this behavior is somewhat in line with the yield strength behavior of the metal, the extremely high peak is rather odd. However, its existence could be associated with the existence of large gaps between the temperatures at which yield strength values are available. Hence, it is recommended that values near this peak to be discarded, and instead the slightly positive sloped line (starting at 1000°F) to be continued until intercept with the negative slope curve near 1500°F temperature, similar to the red dotted line, to allow for a more realistic design space. The data point shown on the design space curve at 1565°F is a more realistic intersection point between the two curves. In fact, this point with 1200 MPa should be treated as the maximum design stress for this metal, and all higher stress values between 1385°F and 1565°F (the peak) should be discarded. The red dotted line connecting the two curves is a more realistic design space curve between these two temperatures and its fitted equation is the same as the one shown in figure (79). The maximum elastic strain in the region where plastic strain is less than 0.2% is 0.0112 (mm/mm). The design space plot was broken down into three sections for equation fitting, and are shown in figures (78), (79), and (80).
Figure (78): Rene N4 Design space Featuring Fitted Equation up to 810°K

Figure (79): Rene N4 Design space Featuring Fitted Equation - 810°K-1000°K
Figure (80): Rene N4 Design space Featuring Fitted Equation - 1040°K & up
CHAPTER V

CONCLUSION

In this thesis, the high temperature transient creep behavior of 304SS, Al7075-T6, Al2024-T6, Inconel 625, Inconel 718, and Rene N4 were studied, and the transient deformation maps for short operating times were generated along with the required design space, limiting plastic deformation to 0.2% and using a safety factor of 1.25. These maps represent a close resemblance to the actual behavior of the metals, however, it is important to note that values are not exact and some variation is expected due to assumptions that were made.

In order to produce transient maps, the wide range of material properties required for the process were broken down into two main groups of properties that were available in literature, and those properties that were not available. Since determining all missing properties for the metal is an extremely difficult and time consuming task, the results of a parametric study were employed to determine the most effective parameters in the results of the transient maps. Hence, the missing properties were divided into two groups of more effective and less effective parameters. For each alloy, the material properties of the base metals were used for the less effective properties. For example, pure aluminum properties of the less effective parameters were used for aluminum alloys. In order to determine values of the more effective material properties, a parameter fitting process was done using available steady state creep data. During this steady state parameter fitting process, it was determined that power law exponent, Dorn constant, activation energies and diffusion pre-exponential values were the most effective parameters to fit analytical steady state data to experimental values. Hence, a combination of some of these parameters were used for each metal using trial and error. This process was done by observing the effects of increasing and decreasing each parameter values and based on the results, determining the most optimized value for each metal. During the steady state parameter fitting process
however, it was observed that there was poor correlation between analytical and experimental data at low temperatures. This could be due to the fact that the steady state codes were written with a focus towards high temperature creep analysis and hence, a few deformation mechanisms that are very effective at lower temperatures were not included. This poor correlation at low temperatures that was observed for several of the metals is not a concern for this study, and a good high temperature correlation is the desired outcome of the parameter fitting process. In addition, the variations in the adjusted values and the material properties do not affect the transient maps significantly according to the parametric study results. Therefore, the maps are a good representation of the high temperature transient creep behavior of the metals, and the design space curves will provide valuable information to hypersonic vehicle designers.
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APPENDIX

Design Space Curves for all metals

304 Stainless Steel Design Space at 30 seconds (page 49)

Al7075-T6 Design space at 30 seconds (page 57)
Al2024-T6 Design space at 30 seconds (page 64)

Inconel 625 Design Space at 30 seconds (page 70)
Inconel 718 Design Space at 30 seconds (page 76)

Rene N4 Design Space at 30 seconds (page 83)
**Transient algorithm (page 33) and code**

```matlab
clc
clear
close all

%% TIME %%
t = 30;

%% Crystallographic and thermal data %
omega = 1.09*10^-29; % Atomic Volume (m^3)
b = 2.49*10^-10; % Burgers Vector (m)
T_m = 1622; % Melting Temperature (Kelvin)-635C
k = 1.381*10^-23; % Boltzman's constant (J/Kelvin)
R = 8.314; % Gas constant
d = 1*10^-6; % grain size

%% Modulus %
mu_0 = 8.14*10^4*10^6; % shear Modulus at 300 K (MN/m^2)-value from NSRDS-NBS-61-V
mu_td = -0.64; % Rate of change of shear modulus.

Temperature dependence of Modulus - value from Dr. Palazotto's word doc

T = 0:5:1622;
sigma_s = 0:2*10^5:2*10^9;

mu = (mu_0*(1+(T-300)/T_m*mu_td)).'*ones(1,length(sigma_s)); % Table 17.1 of Ashby -- Tm*d(mu)/mu0*dT
```
%% Elastic Deformation %
\[ \text{eps}_1 = \left( (\text{sigma}_s.' \times \text{ones}(1, \text{length}(T))).' \right) \! \div \! \mu; \quad \text{Eqn 17.5 of Ashby} \]

\[ \Gamma = \text{sigma}_s \div \mu \]

%% Low-temperature Plasticity %
\[ \text{test} = (T < 700); \]
\[ \text{sigma}_\text{yield}_1 = (560.76 - 0.2762 \times T(\text{test} == 1)) \times 10^6; \]

\[ \text{test} = (T \geq 700 \& T < 922); \]
\[ \text{sigma}_\text{yield}_2 = (460.86 - 0.1334 \times T(\text{test} == 1)) \times 10^6; \]

\[ \text{test} = (T \geq 922); \]
\[ \text{sigma}_\text{yield}_3 = (910 - 0.6205 \times T(\text{test} == 1)) \times 10^6; \]

\[ \text{sigma}_\text{yield} = [\text{sigma}_\text{yield}_1 \ \text{sigma}_\text{yield}_2 \ \text{sigma}_\text{yield}_3]; \]

\[ \text{sigma}_\text{os} = (\text{sigma}_\text{yield} / \sqrt{3}).' \times \text{ones}(1, \text{length}(	ext{sigma}_s)); \quad \text{%Initial Yield Stress} \]

\[ m = (0.22).' \times \text{ones} \left( \text{length} (T), \text{length} (\text{sigma}_s) \right); \quad \text{% Hardening Exponent - value from lec21_notes} \]

\[ K_s = (1070 \times 10^6 / (3^((0.5+1)/2))).' \times \text{ones} \left( \text{length} (T), \text{length} (\text{sigma}_s) \right); \quad \text{% Hardening Constant - value from lec21_notes} \]

\[ \text{sigma}_s_\text{array} = (\text{sigma}_s.' \times \text{ones}(1, \text{length}(T))).'; \]

\[ \text{eps}_2 = \left( \left( (\text{sigma}_s_\text{array} - \text{sigma}_\text{os}) / K_s \right) \right)^{1/m}; \quad \text{% Eqn 17.6 of Ashby. Transient} \]

\[ \text{Aa} = (\text{sigma}_s_\text{array} - \text{sigma}_\text{os}) / K_s; \quad \text{%This is to calculated the value within the paranthesis for eps 2. next step gets rid of all negative values.} \]
\[ \text{Bb} = \text{Aa}(::,:) >= 0; \quad \text{% Logical (shows 1's where the value of Aa is positive or equal to zero and shows 0's when value is negative)} \]
\[ \text{eps}_2 = \text{eps}_2 \times \text{Bb}; \]

\[ \text{clearvars} \quad \text{sigma}_\text{os} \ \text{m} \ \text{K_s} \ \text{Aa} \ \text{Bb} \]

%% Power-law Creep %

\[ A = 3 \times 10^6; \quad \text{% Dorn Constant, final value} \]
\[ D_{ov} = 0.4 \times 10^{-4}; \quad \text{% Lattice diffusion pre-exponential (m^2/s), final value} \]
\[ \text{acD}_{oc} = 3.1 \times 10^{-23}; \quad \text{% Pre-exponential of core diffusion, final value} \]

\[ n = 4.6; \quad \text{% Power law creep Exponent} \]
\[ Q_v = 336 \times 10^3; \quad \text{% Lattice diffusion Activation Energy (Kj/mole)} \]
\[ A_{2} = \text{sqrt}(3)^{(n+1)} \times A; \quad \text{% Below equation 2.21 of Ashby's book} \]
(According to Dr. Palazotto it is the slope of the log-log curve of $\gamma_{dot}/D_v\mu b/R^4 T = A_2 (\sigma_s/\mu)^n$)
\[ D_v = (D_{ov} \cdot \exp(-Q_v/(R \cdot T))) \cdot \text{ones(1, length(sigma_s))}; \] % Lattice diffusion coefficient (Equation is below table 8.1 of Ashby's)

\[ Q_c = 258 \cdot 10^3; \] % Core diffusion activation energy
\[ aCD_c = (aCD_{oc} \cdot \exp(-Q_c/(R \cdot T))) \cdot \text{ones(1, length(sigma_s))}; \] % cross section area of dislocation core and the core diffusion coefficient
\[ D_{eff} = D_v \cdot (1 + 10/\beta^2 \cdot (sigma_s_array./mu)^2 \cdot aCD_c./D_v); \] % Equation 2.20 (Is ac taken to be unity here? in the book it shows 1+10*ac/b^2 ....)

\[ gamma_3 = (A_{D} \cdot D_{eff} \cdot \mu \cdot b/(k \cdot T \cdot \text{ones(1, length(sigma_s))}).*(sigma_s_array./mu).^n); \] % Eqn 2.19 Ashby's book

\[ \text{eps}_t = 0.0727; \] % Total transient strain
\[ C_s = 199.7; \] % Transient constant
\[ \text{eps}_3 = (\text{eps}_t \cdot (1 - \exp(-C_s \cdot gamma_3 \cdot t)) + gamma_3 \cdot t); \] % Eqn 17.10 of Ashby's book

\[ \text{gamma}_4 = 42 \cdot \text{sigma_s_array} \cdot \omega/(k \cdot T \cdot \text{ones(1, length(sigma_s))}.d^2) \cdot \text{D_{eff1}}; \] % Eqn 2.29 of Ashby's book
\[ \text{eps}_4 = \text{sigma_s_array}/\mu \cdot (1 - \exp(-(gamma_4 \cdot t)7/(\text{sigma_s_array}/\mu))) + gamma_4 \cdot t; \] % Eqn 7.11 of Ashby's book

\[ \text{clearvars} \ D_{eff} \] % Diffusional Flow %
\[ Q_b = Q_c; \]
\[ \text{deltaD}_ob = 3.5 \cdot 10^{-15}; \] % Boundary diffusion pre-exponential (m^3/s)
\[ \text{deltaD}_b = (\text{deltaD}_ob \cdot \exp(-Q_b/(R \cdot T))) \cdot \text{ones(1, length(sigma_s))}; \] % below table 8.1 of Ashby's book
\[ D_{eff1} = D_v \cdot (1 + \pi/d \cdot \text{deltaD}_b./D_v); \] % Eqn 2.30 of Ashby's book

\[ \text{gamma}_4 = 42 \cdot \text{sigma_s_array} \cdot \omega/(k \cdot T \cdot \text{ones(1, length(sigma_s))}.d^2) \cdot \text{D_{eff1}}; \] % Eqn 2.29 of Ashby's book
\[ \text{eps}_4 = \text{sigma_s_array}/\mu \cdot (1 - \exp(-(gamma_4 \cdot t)7/(\text{sigma_s_array}/\mu))) + gamma_4 \cdot t; \] % Eqn 7.11 of Ashby's book

\[ \text{clearvars} \ D_{eff1} \] % Total strain %
\[ \text{eps} = (\text{eps}_1 + \text{eps}_2 + \text{eps}_3 + \text{eps}_4)/\sqrt{3}; \]
\[ \text{eps1} = (\text{eps}_2 + \text{eps}_3 + \text{eps}_4)/\sqrt{3}; \]
\[ \text{sigma_array} = \sqrt{3} \cdot \text{sigma_s_array}; \]
\[ y = \log10(\text{sigma_array}/\mu); \]
\[ y_d = \log10(\text{sigma_array}/1.25/\mu); \]
\[ y_1 = \text{sigma_array}; \]
\[ x = (\text{T}/\text{T_m}) \cdot \text{ones(1, length(sigma_s))}; \]
\[ x1 = \text{T} \cdot \text{ones(1, length(sigma_s))}; \]

% elasticity deformation dominant & boundary with plasticity
test=((eps_1>=eps_2).* (eps_1>=eps_3) .* (eps_1>=eps_4)).*( abs(eps_1-eps_2) < 0.1.*eps_2);

    Y_1 = y(test==1);
    X_1 = x(test==1);

% plasticity dominant & boundary with power-law creep
    test= ((eps_2>=eps_1).* (eps_2>=eps_3) .* (eps_2>=eps_4)).* (abs(eps_2-eps_3) < 0.001.*eps_3);
    Y_2 = y(test==1);
    X_2 = x(test==1);

% power-law creep dominant & boundary with elastic deformation (use this
% when time is small)
    test= ((eps_3>=eps_1).* (eps_3>=eps_2) .* (eps_3>=eps_4)).* (abs(eps_3-eps_1) < 0.01.*eps_1);
    Y_3 = y(test==1);
    X_3 = x(test==1);

% power-law creep dominant & boundary with diffusional flow
    test=((eps_3>=eps_1).* (eps_3>=eps_4) .* (eps_3>=eps_2)).* ( abs(eps_3-eps_4) < 0.1.*eps_4);
    Y_4 = y(test==1);
    X_4 = x(test==1);

% Diffusional flow dominant & boundary with elastic deformation
    test=((eps_4>=eps_1).* (eps_4>=eps_3) .* (eps_4>=eps_2)).* (abs(eps_4-eps_1) < 0.1*eps_1);
    Y_5 = y(test==1);
    X_5 = x(test==1);

% between Nabarro-Herring creep and Coble creep
    test= (D_v > pi/d*deltaD_b) .* (abs(D_v-pi/d*deltaD_b) < 0.05*pi/d*deltaD_b) .*((eps_4>eps_1).* (eps_4>eps_3) .* (eps_4>eps_2));
    Y_6 = y(test==1);
    X_6 = x(test==1);

% test = (eps1 > 0.00195 & eps1<= 0.002);
    Y_7 = y(test==1)/1.25;
    X_7 = x(test==1);
    X_71 = x1(test==1);
    Y_71 = y_d(test==1);
    Y_72 = y_l(test==1)/1.25;
    [c,pos] = sort(X_7);
    Y_7 = Y_7(pos);
    Y_71 = Y_71(pos);
    Y_72 = Y_72(pos);
    X_7 = c;
    mul = mu_0*(1+(X_71-300)/T_m*mu_td);
clearvars Aa Bb test D_v deltaD_b d

```matlab
figure
v = [10^(-5) 10^(-4) 10^(-2) 10];
```

% Next few lines of code change the contour line colors
```matlab
myColors = [0,0,1;0,1,0;1,0,0;0,0,0]; % an n-by-3 array of different colors' RGB values
colormap(myColors)
```

% Contour data
```matlab
cs = contour(x,y,eps,v);
clabel(cs);
hold on
plot(X_1,Y_1,'r','LineWidth',2)
hold on
plot(X_2,Y_2,'b','LineWidth',2)
hold on
plot(X_3,Y_3,'magenta','LineWidth',2)
hold on
plot(X_4,Y_4,'g','LineWidth',2)
hold on
plot(X_5,Y_5,'k','LineWidth',2)
hold on
plot(X_6,Y_6,'c--','LineWidth',2)
hold on
plot(X_7,log10(Y_72./mu1),'c','LineWidth',2)
hold on
```

% % axis([0 1 -6 -1])

% for 30s
text(0.025,-3.7,'ELASTIC DEFORMATION')
text(0.1,-1.4,'PLASTICITY')
text(0.6,-1.5,'POWER-LAW CREEP')
text(0.945,-4.5,'D-F')

title('Inconel 625 of grain size 1 \um, for a time of 30 seconds')

xlabel('Homologous Temperature \(T/T_m\)')
ylabel('Normalized Uniaxial Stress \(\log(\sigma/\mu)\)')
grid on

```matlab
figure
plot(((X_7*T_m-273.15)*9/5+32),Y_72/10^6,'b','LineWidth',2)
hold on
title('Inconel 625 of grain size 1 \um, for a time of 30 Seconds')
xlabel('Temperature \(T\) (F)')
ylabel('Design Stress \(\sigma_D\) (MPa)')
grid on
```
clc
clear
close all

%% Crystallographic and thermal data %
omega = 1.09*10^-29;
b = 2.49*10^-10;
T_m = 1622;
k = 1.381*10^-23;
R = 8.314;
d = 1*10^-6;

%% Modulus %
mu_0 = 8.14*10^4*10^6;
mu_td = -.64;

T = 0:5:1622;
sigma_s = 0:2*10^5:2*10^9;
mu = (mu_0*(1+(T-300)/T_m*mu_td)).'*ones(1,length(sigma_s));

%% Plasticity limited by discrete obstacles %
gamma_0 = 10^6;
delta_F = 0.5*mu_0*b^3;
tau = 6.3*10^-3*mu_0;  

sigma_s_array = (sigma_s.'*ones(1,length(T))).';
\[ \gamma_2 = \gamma_0 \cdot \exp\left(-\frac{\Delta_F}{(k \cdot T) \cdot \text{ones}(1, \text{length}(\sigma_s))) \cdot (1 - \frac{\sigma_s_array}{\tau})}\right) \]  

% Equation 2.9 Ashby's handbook

%% Power law creep % % section 2.4 Ashby's book

\[ n = 4.6; \]
\[ Q_v = 336 \times 10^3; \text{ change to } 336 \]
\[ Q_c = 258 \times 10^3; \text{ changed to } 258 \]
\[ A = 3000000; \]
\[ D_{ov} = 0.4 \times 10^{-4}; \text{ changed to } 0.4 \times 10^{-4} \]
\[ acD_{oc} = 3.1 \times 10^{-23}; \]
\[ A_2 = \sqrt{3}^{(n+1)} A; \]
\[ D_v = (D_{ov} \cdot \exp\left(-\frac{Q_v}{(R \cdot T)}\right)) \cdot \text{ones}(1, \text{length}(\sigma_s)); \]
\[ acD_c = (acD_{oc} \cdot \exp\left(-\frac{Q_c}{(R \cdot T)}\right)) \cdot \text{ones}(1, \text{length}(\sigma_s)); \]
\[ D_{eff} = D_v \cdot (1+10/\mu^2 \cdot \sigma_s_array/\mu)^2 \cdot acD_c/D_v; \]

% Equation 2.20 Ashby's Handbook

\[ \gamma_4 = A_2 \cdot D_{eff} \cdot \mu \cdot b/(k \cdot T) \cdot \text{ones}(1, \text{length}(\sigma_s))) \cdot (\sigma_s_array/\mu)^n; \]

%% Diffusional flow % % Section 2.5 Ashby's Handbook

\[ \DeltaD_{ob} = 3.5 \times 10^{-15}; \]
\[ Q_b = Q_c; \]
\[ \DeltaD_{b} = (\DeltaD_{ob} \cdot \exp(-Q_b/(R \cdot T))) \cdot \text{ones}(1, \text{length}(\sigma_s)); \]
\[ D_{eff1} = D_v \cdot (1+\pi/d \cdot \DeltaD_{b}/D_v); \]

% Equation 2.30 Ashby's Handbook

\[ \gamma_7 = 42 \cdot \sigma_s_array \cdot \omega/(k \cdot T) \cdot \text{ones}(1, \text{length}(\sigma_s))) \cdot \DeltaD_{eff1}; \]

% Equation 2.29 Ashby's Handbook

clearvars D_{eff1}

clearvars D_eff1

% Calculate for net strain rate %
\[ \gamma_{net} = (\max(\gamma_2, \gamma_4) + \gamma_7)/\sqrt{3}; \]
\[ \sigma_array = \sqrt{3} \cdot \sigma_s_array; \]
\[ y = \log10(\sigma_array/\mu); \]
\[ x = T \cdot \text{ones}(1, \text{length}(\sigma_s)); \]

%% Deformation mechanism map % %

\[ v = [1073 1173 1223 1273]; \]

% Next few line of code change the contour line colors
myCM = [1,0,0;0,1,0;0,0,1;0,1,1;0,0,0];
% create an n-by-3 array of RGB values in the colormap(myCM)
% Contour data
\[ \text{cs} = \text{contour}(y, \log10(\gamma_{net}, x, v)); \]
\[ \text{hold on}; \]
\[ \text{axis}([-4 -2.4 -10 0]); \]
\[ \text{clabel}(\text{cs}); \]
\[ \text{grid on}; \]
```matlab
% %
a = \log_{10}(\frac{68.9, 103.4, 137.9 \times 10^6}{\mu_0(1+(1073-300)/T_m*\mu_td)})
b = \log_{10}([2.069 \times 10^{-8} 9.11 \times 10^{-7} 3.83 \times 10^{-6}])

plot(a, b, 'xb', 'LineWidth', 2)
hold on
% %
c = \log_{10}(\frac{34.47, 68.9, 20.68 \times 10^6}{\mu_0(1+(1173-300)/T_m*\mu_td)})
d = \log_{10}([3.29 \times 10^{-7} 7.127 \times 10^{-6} 3.888 \times 10^{-8}])

plot(c, d, 'xr', 'LineWidth', 2)
hold on
% %
e = \log_{10}(\frac{13.789, 20.68 \times 10^6}{\mu_0(1+(1223-300)/T_m*\mu_td)})
f = \log_{10}([3.138 \times 10^{-8} 3.86 \times 10^{-8}])

plot(e, f, 'xg', 'LineWidth', 2)
hold on
% %
g = \log_{10}(\frac{10.34, 13.789 \times 10^6}{\mu_0(1+(1273-300)/T_m*\mu_td)})
h = \log_{10}([1.333 \times 10^{-7} 8.36 \times 10^{-7}])

plot(g, h, 'xm', 'LineWidth', 2)
hold on

title('Inconel 625 of grain size 1 \mu m')
xlabel('Normalised Stress log(\sigma/\mu)')
ylabel('Strain Rate log(\varepsilon/\dot{\varepsilon})')```
**List of Matlab Codes Attached**

304 Stainless Transient: transient_304

Al7075-T6 steady state parameter fitting: Al7075steady_state_parameter_fitting

Al7075-T6 Transient: transient_Al_7075

Al2024-T6 steady state parameter fitting: Al2024steady_state_parameter_fitting

Al2024-T6 Transient: transient_Al_2024

Inconel 625 steady state parameter fitting: Inconel625steady_state_parameter_fitting

Inconel 625 Transient: transient_625

Inconel 718 steady state parameter fitting: steady_state_parameter_fitting718

Inconel 718 Transient: transient_718

Rene N4 steady state parameter fitting: steady_state_parameter_fittingReneN4

Rene N4 Transient: transient_ReneN4