2017

On the Thermal and Contact Fatigue Behavior of Gear Contacts under Tribo-dynamic Condition

Anusha Anisetti
Wright State University

Follow this and additional works at: https://corescholar.libraries.wright.edu/etd_all

Part of the Engineering Commons

Repository Citation
https://corescholar.libraries.wright.edu/etd_all/1711

This Dissertation is brought to you for free and open access by the Theses and Dissertations at CORE Scholar. It has been accepted for inclusion in Browse all Theses and Dissertations by an authorized administrator of CORE Scholar. For more information, please contact corescholar@www.libraries.wright.edu.
ON THE THERMAL AND CONTACT FATIGUE
BEHAVIOR OF GEAR CONTACTS UNDER
TRIBO-DYNAMIC CONDITIONS

A dissertation submitted in partial fulfillment of
the requirements for the degree of Doctor of
Philosophy

by

ANUSHA ANISETTI
M.S.E.G., Wright State University, 2008
B.Tech, Jawaharlal Nehru Technological University, 2004

2017
Wright State University
WRIGHT STATE UNIVERSITY

GRADUATE SCHOOL

March 24, 2017


________________________________________
Sheng Li, Ph.D.
Dissertation Director

________________________________________
Frank W. Ciarallo, Ph.D.
Director, Ph.D. in Engineering Program

________________________________________
Robert E. W. Fyffe, Ph.D.
Vice President for Research and Dean of the Graduate School

Committee on Final Examination

________________________________________
Sheng Li, Ph.D.

________________________________________
Joseph C. Slater, Ph.D., PE

________________________________________
Ahmet Kahraman, Ph.D.

________________________________________
Ha-Rok Bae, Ph.D.

________________________________________
Nikolai V. Priezjev, Ph.D.

Gears are vital power transmitting mechanical components, in both automotive and aerospace applications, and commonly operate within relatively high rotational speed ranges. Therefore, the dynamic behavior of gears is inevitable and can be quite significant under certain circumstances. The gear dynamics introduces not only noises and vibrations, but also large tooth force amplitudes, and consequently large amplitudes of bending stresses and contact stresses, and high surface temperatures, promoting the failures of tooth bending fatigue, contact fatigue, and scuffing. This study focuses on the mechanism by which the gear dynamic responses affect the flash temperature rise and contact fatigue life using a gear tribo-dynamic formulation. The significance of this work is that it connects the gear dynamics and gear tribology disciplines and shows the importance of dynamic response on the two critical failure modes; scuffing and pitting.

A six degree-of-freedom transverse-torsional discrete gear dynamics equation set is coupled with a thermal mixed elastohydrodynamic lubrication formulation to include the interactions between the gear dynamics and the gear tribological behavior. The flash temperature rises are quantified within a wide speed range under the different operating and surface conditions. The results indicate evident deviations of flash
temperature rise between quasi-static condition and tribo-dynamic condition especially in the vicinities of the resonances.

The interactive model of gear dynamics and gear tribological behavior is bridged through an iterative numerical scheme to determine the surface normal pressure and tangential shear under the tribo-dynamic condition. The resultant multi-axial stress fields (from these surface tractions) on and below the surface are then used to assess the fatigue damage. A comparison between the tribo-dynamic and quasi-static life predictions is performed to demonstrate the important role of the gear tribo-dynamics in the fatigue damage. The impacts of the input torque, surface roughness and lubricant temperature on the gear contact fatigue under the tribo-dynamic condition are also investigated. The results show that the fatigue life under tribo-dynamic conditions show large deviations at the vicinities of the resonances when compared to the quasi-static conditions.
## Contents

1  Introduction ........................................................................................................1

1.1  Overview ........................................................................................................1

1.2  Background and Motivation ...........................................................................4

1.3  Literature Review ..........................................................................................4

1.3.1  Elastohydrodynamic Lubrication .............................................................4

1.3.2  Scuffing Failure .........................................................................................6

1.3.3  Contact Fatigue Crack Nucleation .........................................................8

1.3.4  Gear Tribo-dynamics ..............................................................................12

1.4  Scope and Objectives .....................................................................................16

1.5  Dissertation Outline .....................................................................................18

2  Gear Thermal Tribo-Dynamics Model .............................................................19

2.1  Gear Load Distribution Model ...................................................................22

2.2  Gear Dynamics Model ...............................................................................25

2.3  Gear Contact Tribological Model ...............................................................33

2.4  Gear Tribo-Dynamics Model ......................................................................41

2.4.1  Discretization ........................................................................................41

2.5  Gear Thermal Model ..................................................................................45

3  Gear Stress Prediction and Fatigue Model .....................................................51
3.1 Gear Contact Stress Prediction Model ..........................54
3.2 Gear Contact Multi-Axial Fatigue Life Model ..........62
4 Results ..............................................................................66
  4.1 Thermal Behavior .....................................................66
  4.2 Contact Fatigue Behavior ......................................86
5 Conclusions and Future Work .....................................133
  5.1 Conclusion ..............................................................133
  5.2 Future Work ............................................................137
Bibliography .................................................................138
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e(t)$</td>
<td>static transmission error (unloaded)</td>
</tr>
<tr>
<td>$\hat{e}(t)$</td>
<td>static transmission error (loaded)</td>
</tr>
<tr>
<td>$T$</td>
<td>torque</td>
</tr>
<tr>
<td>$r$</td>
<td>base radius</td>
</tr>
<tr>
<td>$k_m(t)$</td>
<td>gear mesh stiffness</td>
</tr>
<tr>
<td>$r_1, r_2$</td>
<td>base radius of gears 1 and 2</td>
</tr>
<tr>
<td>$\omega_1, \omega_2$</td>
<td>angular velocity of gears 1 and 2</td>
</tr>
<tr>
<td>$m_1, m_2$</td>
<td>mass of disks 1 and 2</td>
</tr>
<tr>
<td>$J_1, J_2$</td>
<td>polar moment of inertia of disks 1 and 2</td>
</tr>
<tr>
<td>$l$</td>
<td>half backlash</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>static transmission error</td>
</tr>
<tr>
<td>$\varepsilon_y(t)$</td>
<td>dynamic transmission error</td>
</tr>
<tr>
<td>$\delta(t)$</td>
<td>non-linear displacement function</td>
</tr>
<tr>
<td>$k_x, k_y$</td>
<td>spring stiffness in OLOA and LOA directions</td>
</tr>
<tr>
<td>$c_x, c_y$</td>
<td>damping coefficient in OLOA and LOA directions</td>
</tr>
<tr>
<td>$c_t$</td>
<td>torsional damping of the bearing</td>
</tr>
<tr>
<td>$\theta_1, \theta_2$</td>
<td>rotational displacements in OLOA and LOA directions</td>
</tr>
<tr>
<td>$x, y$</td>
<td>translational displacements in OLOA and LOA directions</td>
</tr>
<tr>
<td>$R_1, R_2$</td>
<td>radii of curvature of disks 1 and 2</td>
</tr>
</tbody>
</table>
$F_1, F_2$ frictional forces on disks 1 and 2

$F_s, F_r$ sliding and rolling friction forces

$u_s, u_r$ sliding and rolling velocities

$u_1, u_2$ surface velocities

$E$ elastic modulus

$c_m(t)$ gear mesh viscous damping

$W_T$ dynamic gear mesh tooth force

$W_T^s$ quasi-static tooth force

$Z_1, Z_2$ number of teeth on gears 1 and 2

$f(x,t)$ flow coefficient

$p(x,t)$ pressure

$p_s$ solidification pressure

$p_e$ evaporation pressure

$h(x,t)$ fluid film thickness

$\rho$ density of lubrication film

$t$ time

$\eta(x,t)$ lubricant viscosity

$\tau_m$ viscous shear stress

$\tau_0$ reference shear stress

$g_0(x,t)$ curvature gap

$h_0$ rigid body approach
\[ V(x,t) \] elastic deflection

\[ S_1, S_2 \] surface roughness height distributions along surface 1 and 2

\[ R_{eq} \] equivalent radius of curvature

\[ K(x) \] influence function

\[ E' \] reduced elastic modulus

\[ \nu \] poisson’s ratio

\[ \tau''(x,z,t) \] viscous shear

\[ L \] face width

\[ \mu_b \] boundary lubrication coefficient

\[ a_{\text{max}} \] maximum half-Hertzian contact width

\[ A \] area of tooth surface grid element

\[ k_f \] lubricant thermal conductivity

\[ c_f \] lubricant specific heat

\[ \phi_f \] lubricant temperature

\[ \gamma \] lubricant shear strain rate

\[ \kappa_s \] gear material thermal diffusivity

\[ k_s \] gear material conductivity

\[ \Gamma \] computational domain

\[ Q \] frictional heat flux

\[ \vartheta \] heat partition coefficient

\[ \mu \] friction coefficient
\( \alpha_1, \alpha_2 \) pressure-viscosity coefficients

\( p_t \) transition pressure

\( \beta \) thermal expansion coefficient

\( \sigma_x, \sigma_y, \sigma_z \) normal stress fields

\( \sigma_{xz} \) shear stress field

\( X_0 \) position of x-z frame at t=0

\( \alpha \) angle between characteristic and macro fatigue fracture planes

\( s \) fatigue strength ratio

\( S_{N}^{b} \) fully reversed bending fatigue strength

\( S_{N}^{t} \) fully reversed torsion fatigue strength

\( N \) fatigue life cycles

\( \kappa \) material property

\( S_r \) reference stress
List of Figures

FIGURE 1-1 (A) POINT CONTACT BETWEEN BALL AND DISK (B) LINE CONTACT BETWEEN ROLLER AND PLATE ................................................................. 1
FIGURE 1-2 MIXED EHL CONDITIONS ............................................................... 2
FIGURE 1-3 MICROSCOPE IMAGES OF THE SURFACE (A) BEFORE AND (B) AFTER SCUFFING FAILURE AT THE × 100 MAGNIFICATION ............................ 1
FIGURE 1-4 PITTING FAILURE AT THE ROOT OF GEAR TOOTH ........................................ 3
FIGURE 2-1 THE COMPUTATIONAL METHODOLOGY FOR THE MODELING OF THE THERMAL UNDER THE TRIBO-DYNAMIC CONDITION ............................................ 21
FIGURE 2-2 A SPUR GEAR PAIR MESH SHOWING AN INSTANTANEOUS CONTACT POINT C ALONG ITS LOA ........................................................................ 28
FIGURE 2-3 (A) THE SIX-DOF DISCRETE DYNAMICS MODEL FOR SPUR GEAR, AND (B) THE TOOTH MESHING INTERFACE FRICTION ........................................... 29
FIGURE 3-1 THE COMPUTATIONAL METHODOLOGY FOR THE MODELING OF THE CONTACT FATIGUE BEHAVIOR UNDER THE TRIBO-DYNAMIC CONDITION ........................................ 53
FIGURE 3-2 TWO-DIMENSIONAL COMPUTATIONAL DOMAIN AND THE MESHING SCHEME .... 56
FIGURE 3-3 ELASTIC HALF PLANE (A) UNDER POINT LOAD (B) UNDER DISTRIBUTED PRESSURE ... 57
FIGURE 4-1 THE VARIATIONS OF (A) THE STATIC TRANSMISSION ERROR AND (B) THE MESH STIFFNESS WITH THE GEAR MESH CYCLE ......................................................... 68
FIGURE 4-2 THE MEASURED ROUGHNESS PROFILES ALONG THE TOOTH PROFILE DIRECTION FOR (A) GROUND, AND (B) POLISHED SURFACE FINISHES ................................................................... 70
FIGURE 4-3 (A) THE COMPARISON OF $\Delta \theta_1^{\text{max}}$ (BLACK) BETWEEN THE TRIBO-DYNAMIC (SOLID CURVE) AND QUASI-STATIC (DASHED CURVE) CONDITIONS PLOTTED TOGETHER WITH $W_M^{\text{rms}}$ (RED SOLID CURVE) AND $W_{Bx}^{\text{rms}}$ (RED DASHED CURVE), AND (B) THE COMPARISONS
OF $\Phi_A$ (BLACK), $\Phi_W$ (BLUE) AND $\Phi$ (RED) BETWEEN THE TRIBO-DYNAMIC (SOLID CURVES) AND QUASI-STATIC (DASHED CURVES) CONDITIONS, UNDER THE BASELINE CONDITION………………………………………………………………………………………………………………………73

FIGURE 4-4 THE COMPARISONS OF THE MAXIMUM HERTZIAN PRESSURE DISTRIBUTIONS ALONG THE GEAR 1 ROLL ANGLE BETWEEN THE TRIBO-DYNAMIC AND QUASI-STATIC CONDITIONS FOR THE BASELINE AT (A) $\Omega_{IV}$ AND (B) $\Omega_{V}$ AS SPECIFIED IN FIGURE 4-3. ..................... 74

FIGURE 4-5 THE INSTANTANEOUS DISTRIBUTIONS OF (A) PRESSURE (BLACK) AND FILM THICKNESS (RED), (B) SURFACE 1 (BLACK) AND SURFACE 2 (RED) ROUGHNESS HEIGHTS, AND (C) SURFACE 1 (BLACK) AND SURFACE 2 (RED) FLASH TEMPERATURE RISES ACROSS THE EHL CONJUNCTION AT THE $16^\circ$ GEAR 1 ROLL ANGLE AND $\Omega_I$ ROTATIONAL SPEED (SPECIFIED IN FIGURE 4-3) FOR THE BASELINE………………………………………………………………………………………………………………………………75

FIGURE 4-6 THE DISTRIBUTIONS OF (A) SPECIFIC ASPERITY CONTACT PRESSURE, AND (B) ASPERITY CONTACT AREA RATIO (BLACK) AND LOAD RATIO (RED) ALONG THE GEAR 1 ROLL ANGLE UNDER THE TRIBO-DYNAMIC CONDITION FOR THE BASELINE AT $\Omega_{II}$ AS SPECIFIED IN FIGURE 4-3………………………………………………………………………………………………………………………………79

FIGURE 4-7 (A) THE COMPARISON OF $\Delta \phi_{1}^{\max}$ (BLACK) BETWEEN THE TRIBO-DYNAMIC (SOLID CURVE) AND QUASI-STATIC (DASHED CURVE) CONDITIONS PLOTTED TOGETHER WITH $W_{M}^{rms}$ (RED SOLID CURVE) AND $W_{Bx}^{rms}$ (RED DASHED CURVE), AND (B) THE COMPARISONS OF $\Phi_A$ (BLACK), $\Phi_W$ (BLUE) AND $\Phi$ (RED) BETWEEN THE TRIBO-DYNAMIC (SOLID CURVES) AND QUASI-STATIC (DASHED CURVES) CONDITIONS, UNDER THE HIGHER TORQUE CONDITION………………………………………………………………………………………………………………………………80

FIGURE 4-8 (A) THE COMPARISON OF $\Delta \phi_{1}^{\max}$ (BLACK) BETWEEN THE TRIBO-DYNAMIC (SOLID CURVE) AND QUASI-STATIC (DASHED CURVE) CONDITIONS PLOTTED TOGETHER WITH $W_{M}^{rms}$ (RED SOLID CURVE) AND $W_{Bx}^{rms}$ (RED DASHED CURVE), AND (B) THE COMPARISONS
FIGURE 4-14 THE CRACK NUCLEATION FATIGUE LIFE DISTRIBUTIONS UNDER THE BASELINE CONDITION FOR (A) SPEED CASE A AND (B) SPEED CASE B, WHICH ARE DEFINED IN FIGURE 4-12. .............................................................. 94

FIGURE 4-15 THE CRACK NUCLEATION FATIGUE LIFE DISTRIBUTIONS UNDER THE BASELINE CONDITION FOR (A) SPEED CASE C AND (B) SPEED CASE D, WHICH ARE DEFINED IN FIGURE 4-12. .............................................................. 95

FIGURE 4-16 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT LOW INPUT TORQUE (1000 N-M), HRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE. .................................................................................................................. 97

FIGURE 4-17 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT LOW INPUT TORQUE (1000 N-M), HRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE. 98

FIGURE 4-18 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT LOW INPUT TORQUE (1000 N-M), MRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE. .................................................................................................................. 102

FIGURE 4-19 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT LOW INPUT TORQUE (1000 N-M), MRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE. .................................................................................................................. 103

FIGURE 4-20 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT LOW INPUT TORQUE (1000 N-M), LRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE. .................................................................................................................. 104
FIGURE 4-21 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT LOW INPUT TORQUE (1000 N-M), LRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

................................................................. 105

FIGURE 4-22 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT MEDIUM INPUT TORQUE (1700 N-M), HRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

................................................................. 107

FIGURE 4-23 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT MEDIUM INPUT TORQUE (1700 N-M), HRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

................................................................. 108

FIGURE 4-24 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT MEDIUM INPUT TORQUE (1700 N-M), MRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

................................................................. 109

FIGURE 4-25 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT MEDIUM INPUT TORQUE (1700 N-M), MRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

................................................................. 110

FIGURE 4-26 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT MEDIUM INPUT TORQUE (1700 N-M), LRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

................................................................. 111

FIGURE 4-27 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT MEDIUM INPUT TORQUE (1700 N-M), LRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE
CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

FIGURE 4-28 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT HIGH INPUT TORQUE (2400 N-M), HRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

FIGURE 4-29 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT HIGH INPUT TORQUE (2400 N-M), HRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

FIGURE 4-30 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT HIGH INPUT TORQUE (2400 N-M), MRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

FIGURE 4-31 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT HIGH INPUT TORQUE (2400 N-M), LRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 50°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

FIGURE 4-32 THE EFFECT OF DYNAMIC FORCE (RMS) ON THE FATIGUE LIFE AT HIGH INPUT TORQUE (2400 N-M), LRA SURFACE ROUGHNESS PROFILE AND INLET TEMPERATURE CONTROLLED AT 90°C. THE RMS DYNAMIC FORCES ARE ALSO INCLUDED IN THE FIGURE.

FIGURE 4-33 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT TORQUE LEVELS FOR (A) HIGH ROUGHNESS AMPLITUDE, (B) MEDIUM
ROUGHNESS AMPLITUDE AND (C) LOW ROUGHNESS AMPLITUDE SURFACES. INLET LUBRICANT TEMPERATURE IS 90°C .......................... 123

FIGURE 4-34 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT TORQUE LEVELS FOR (A) HIGH ROUGHNESS AMPLITUDE, (B) MEDIUM ROUGHNESS AMPLITUDE AND (C) LOW ROUGHNESS AMPLITUDE SURFACES. INLET LUBRICANT TEMPERATURE IS 90°C .......................... 124

FIGURE 4-35 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT ROUGHNESS AMPLITUDE LEVELS FOR (A) 2400 N-M, (B) 1700 N-M AND (C) 1000 N-M INPUT TORQUES. INLET LUBRICANT TEMPERATURE IS 90°C .......... 126

FIGURE 4-36 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT ROUGHNESS AMPLITUDE LEVELS FOR (A) 2400 N-M, (B) 1700 N-M AND (C) 1000 N-M INPUT TORQUES. INLET LUBRICANT TEMPERATURE IS 50°C .......... 127

FIGURE 4-37 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT ROUGHNESS TEMPERATURE LEVELS FOR (A) HRA, (B) MRA AND (C) LRA SURFACES UNDER 2400 N-M INPUT TORQUE .......................................................... 130

FIGURE 4-38 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT ROUGHNESS TEMPERATURE LEVELS FOR (A) HRA, (B) MRA AND (C) LRA SURFACES UNDER 1700 N-M INPUT TORQUE .......................................................... 131

FIGURE 4-39 THE COMPARISONS OF THE TRIBO-DYNAMIC CRACK NUCLEATION FATIGUE LIVES BETWEEN DIFFERENT ROUGHNESS TEMPERATURE LEVELS FOR (A) HRA, (B) MRA AND (C) LRA SURFACES UNDER 1000 N-M INPUT TORQUE .......................................................... 132
List of Tables

TABLE 1 ............................................................................................................................................... 67
TABLE 2 ............................................................................................................................................... 67
Acknowledgment

My deepest gratitude goes first to my advisor, Dr. Sheng Li for expertly guiding me with patience through this laborious dissertation journey and for the financial support. It has been a great pleasure to work under his supervision over the past three years. I would also like to thank Dr. Ahmet Kahraman, Dr. Joseph C. Slater, Dr. Ha-Rok Bae and Dr. Nikolai Priezjev, who agreed to sacrifice their time and serve on my dissertation committee. I appreciate their intellectual comments and suggestions.

My appreciation extends to Dr. Joseph C. Slater to motivate and help me get started on the path of Ph.D. and his inspirational words from time to time.

Aboveground, I am indebted to my family. My parents for their love and inspiration throughout my life, my sister for an excellent example she set, my brother-in-law for his unfailing continuous support. This accomplishment would not have been possible if not for them. I would also like to thank my best friends and my fellow doctoral students for their moral support and encouragement.

Finally, special thanks to my husband for his unselfish support, love, understanding and sacrifices.
Dedicated to

My Family
Chapter 1

1 Introduction

1.1 Overview

When two non-conformal elastic bodies come in contact with each other, the contact is made at a point or along a line as shown in Figure 1-1 and as the load applied increases, an area of contact is formed. In point contacts, the contact zone forms a finite elliptical region and in line contacts, the contact zone is an infinitely long strip. The contact pressure (or applied load per area) in gears reduces continuously to zero towards the end of contact zone. This characteristic makes the contact zone incomplete.

Figure 1-1 (a) Point contact between ball and disk (b) Line contact between roller and plate
When the contact pressure between the contact surfaces is very high, the contact surfaces deform elastically and provides a gap for lubricant to flow through separating the surfaces. The elastic deformation is in the same order of magnitude as lubricant fluid film thickness. This type of hydrodynamic lubrication is called elasto-hydrodynamic lubrication (EHL). Some examples of EHL contacts are gears, rolling element bearings and cam-followers.

Initially in EHL theory, the surfaces of the contact bodies are assumed to be smooth [1]. However, in realistic operating conditions, the surfaces are not ideally smooth and the surface roughness of the contact surfaces effects the contact pressure distribution, the lubricant film thickness and the contact temperatures creating severe lubrication conditions. Under heavy load and low speed conditions, the asperity contacts penetrate through the lubricant film and asperities come into direct contacts. This transition between asperity contacts and lubricant film is called mixed EHL conditions as shown in Figure 1-2.

![Figure 1-2 Mixed EHL conditions](image-url)
An important parameter associated with the lubricant film thickness is lubricant viscosity. In Newtonian fluids, the shear stress and shear rate are linearly proportional where the constant of proportionality is called the coefficient of viscosity. To improve the lubricant properties and to enhance performance at high temperatures, additives are added to the lubricant. The addition of these additives, makes the lubricant to behave as non-Newtonian fluid even at normal conditions i.e., the coefficient of viscosity is non-linear.

Often, gears operate under conditions where load, speed and radius of curvature of contact points vary with time. Under these operating conditions, all the variables involved in EHL contact are time dependent and thus formulate a transient EHL problem.

Under mixed EHL conditions as described above, when the asperities come in contact, an instantaneous rise in temperature at the surfaces occur known as flash temperature. The total temperature at the contact surface is thus the sum of the bulk temperature and flash temperature. The bulk temperature is easily measured using thermocouples while flash temperature is usually calculated as it is not easily measurable.
1.2 Background and Motivation

Scuffing and pitting are two important failure modes commonly observed in gearing applications. The former appears as a sudden and catastrophic failure shortly after the gears start operating, while the latter is a high-cycle contact fatigue phenomenon and takes millions of contact cycles to take place. Although these two failure modes differ in their appearances, they both are tightly related to the tribological behavior occurring at the gear tooth contact interfaces.

The onset of scuffing failure is dictated by the surface temperature of the contacting element, which consists of the bulk temperature component and the flash temperature component. When the surface temperature exceeds a critical value, the solid surfaces weld at the high temperature spot and then are torn apart by the relative motion of the surfaces, resulting in the severe damage as shown in Figure 1-3. The bulk temperature component is influenced mainly by the macro-scale frictional heat produced within the contact zone, the heat convection between the gear surface and the surrounding air-lubricant mixture, and any heat conduction through the bearings and shafts. The determination of this temperature component can be performed through contact or non-contact infrared thermocouples [2]. As for the assessment of the micro-scale flash temperature rise within the contact zone, there is currently no reliable experimental instruments for
such application and can only be evaluated numerically using sophisticated thermal mixed elastohydrodynamic lubrication (EHL) modeling technique [3].
Figure 1-3 Microscope images of the surface (a) before and (b) after scuffing failure at the × 100 magnification.
The pitting failure is a progressive failure dictated by the severe cyclic contact stresses as the tooth surfaces roll and slide against each other. Owing to the gear finishing processes, including shaving and grinding, significant tool marks, i.e. surface roughness profiles, are left on the tooth surfaces. These surface irregularities interrupt the lubrication film within the contact zone and introduce asperity contacts, where the contact pressures can be significantly elevated. Even under the full film lubrication condition, for instance when the rolling (entraining) velocity is high and/or the lubricant viscosity is large, the abrupt surface topography introduces large surface profile gradients and consequently high hydrodynamic pressures [4]. This explains the experimental observation by Hoffman et al. [5] that the fatigue crack nucleated at the surface, even when the contacting surfaces are separated from each other by a full lubrication film. Therefore, the surface roughness has been well identified to be one of the most important contact fatigue parameters in addition to load, velocity, sliding, and lubricant properties.

Gears commonly operate under the high rotational speed condition, where the gear dynamics is inevitable. The direct consequence of the gear dynamics in a transmission system is the vibration and noise. This aspect, usually referred as noise, vibration and harshness (NVH), has been extensively studied in vehicles. The other important consequence of the gear dynamics that has been rarely considered in the literature is the large dynamic force amplitude, which significantly changes the tribological
behavior of the contact. Additionally, the viscous power dissipation mechanism and frictions occurring within the contact provides the gear mesh damping and the friction excitations to impact the gear dynamic response, forming the mutual interactions between the gear dynamics and the gear tribology. This newly emerging research field is referred as the gear tribo-dynamics.

Figure 1-4 Pitting failure at the root of gear tooth.
1.3 Literature Review

1.3.1 Elastohydrodynamic Lubrication

The tribology literature contains a wide spectrum of elastohydrodynamic lubrication (EHL) models. Building on the smooth surface EHL formulations [6], contacts of rough surfaces were analyzed by either using micro EHL models [7-11] where a continuous fluid film is maintained between the contacting surfaces or using mixed EHL models [12-16] that are capable of handling the actual asperity contacts as part of the lubrication analysis. These models vary in several aspects in terms of their ability in handling line or point contacts, Newtonian or non-Newtonian fluids, and isothermal or thermal contact conditions. Since the solution of the highly nonlinear EHL governing equations is computationally demanding and often subject to numerical difficulties, these models tend to differ in their discretization schemes (symmetric [7-8, 10-14] or asymmetric [16] control volumes) as well as their solution methodologies (Fast Fourier Transforms [12, 15-16], Multi-level Multi-integration [7-9, 11, 13, 14], etc.). Additional differences can be noted amongst the mixed EHL models in the way they handle asperity contacts, including separate treatment of wet and dry areas [12, 15] and the unified schemes [13-14, 16] where the asperity contact regions are handled simultaneously with the lubricated regions by employing a reduced form of the Reynolds equation. In spite of such differences, these models are all designed to analyze the contact of two rough surfaces having
constant speeds, a constant normal load, and a time-invariant geometry. This is sufficient for many fundamental EHL problems such as the contacts between two cylinders, two balls, or a ball and a disk, however, is inadequate for gear contacts where the contact curvature, surface velocities and tooth force all vary as the gears rotate.

A small number of published studies investigated such time-varying effects of spur gear contacts. In one of these studies, Wang and Cheng [17-18] predicted the minimum film thickness and thermal characteristics of spur gears with ideally smooth surfaces. In their model, the elastic deformation was approximated by that of a simple Hertzian contact. Larsson [19] and Wang et al. [20] proposed involute spur gear models for isothermal non-Newtonian and thermal Newtonian fluids, respectively, by employing an assumed time-varying normal tooth force as the contact moves along the line-of-action. They showed certain transient variations of minimum film thickness that are attributable to the change in the normal load. These three studies, while establishing the need for a specialized EHL model for spur gears, lacked the ability to handle rough surface conditions. In the analysis of gear contacts, boundary and mixed EHL conditions are rather common, especially in high-load and low-speed automotive applications [21]. Inclusion of rough tooth surfaces in the EHL analysis is a must for gear contact fatigue (such as pitting and scuffing failures) and efficiency (mechanical power loss) simulations. In addition, these studies limited their treatment of the gear mesh deformations to Hertzian effects. Yet, other
effects due to tooth bending, base rotation and shear deformations were shown to be equally important in defining gear mesh compliance [22]. These models were also not able to include any deviation from the involute tooth profile due to intentional tooth modifications or unavoidable manufacturing errors, which are common in real-life gear systems. Li and Kahraman [23], thus, proposed a transient gear EHL model that includes all these unique characteristics of gear contacts. By comparing to earlier models, they showed that the tribological behaviors are indeed impacted by these influential parameters. Additionally, the friction and power loss of a gear pair operating under various loading and speed conditions with different surface roughness conditions were predicted using the model and compared to the experimental measurements to show good agreement, demonstrating the model capability and accuracy.

1.3.2 Scuffing Failure

The failure of scuffing, whose onset is tightly related to the extreme surface local temperatures of the contacting components [2] and [24], has been frequently observed in the aerospace gearing applications due to the very high operating speeds. In automotive transmission systems, continuously increasing power density also imposes the high risk of this thermal failure mode. Surface local temperature is the sum of the surface bulk temperature and the instantaneous temperature rise (flash temperature) caused by the local frictional heat flux. In an early study, Blok [25] proposed a closed-form flash temperature formula by assuming smooth contact surfaces and
uniform heat flux. Ling [26] showed that even a limited number of asperity contacts can largely influence the surface temperature. To investigate the roughness effect on the instantaneous temperature rise, deterministic thermal mixed EHL models have been proposed in the recent years. Uncoupling the thermal analysis from the mixed lubrication analysis, Qiu and Cheng [24] and Lai and Cheng [27] evaluated the temperature rise induced by simulated surface roughness. Cioc et al. [28] solved the energy equations together with the EHL governing equations iteratively to predict the flash temperature for line contacts having very limited asperity interactions. Zhu and Hu [29] and Wang et al. [30] introduced a reduced Reynolds equation into the thermal mixed EHL formulations, successfully eliminating the numerical instabilities under the severe asperity contact condition. Deolalikar et al. [31] treated the fluid regions and the asperity contact regions separately considering computer generated surface roughness profiles. In these studies the frictional heat generation was determined through assumed friction coefficients instead of using the surface traction predicted by the EHL model itself. Additionally, the bulk temperatures of the contact surfaces were assumed to be known. Using a heat transfer formulation, Li et al. [2] predicted the surface bulk temperature rise, onto which the flash temperature was added to determine the maximum surface temperature. In the process the frictional heat flux was directly evaluated from the predicted viscous shear or boundary friction without any subjective friction coefficient selection. The other factors that may contribute to scuffing failure include wear or fatigue debris in the
lubricant, wear out of the protective tribo-film, lubricant degradation, etc. [32].

In regards to the experimental studies on scuffing failure, four-ball [33], ball-on-disk or twin-disk type of set-up [34-38] has been widely used due to the relatively easy and accurate control of the contact parameters. These studies focused on investigating the influence of lubricants [34-35], surface finish characteristics [36-37], and coating [38] on the scuffing performance of lubricated contacts. The commonly used scuffing test procedure is to increase the load stepwise while maintaining the surface velocities (rolling and sliding) constant until the scuffing failure occurs. The measurements during the test are usually limited to the friction force and the bulk temperature of the contacting surfaces as the localized maximum surface temperatures of non-transparent contact pairs are not feasible to measure. As such, the critical scuffing temperature was estimated theoretically in the works such as Lai and Cheng [27].

1.3.3 Contact Fatigue Crack Nucleation

The contact fatigue in the form of macro-pitting has been one of the most common surface damage processes that occur in gearing systems due to recurring contacts. Rolling contact fatigue includes pitting, spalling, micro-pitting etc. Contact fatigue life of a crack in the contacting component has two parts, initiation and propagation. Some of the earlier attempts in predicting rolling contact fatigue (RCF) crack nucleation life considered the
contact between two ideally smooth surfaces with no lubrication [39-42]. The finite element (FE) method was used in these studies to evaluate the strain fields for low cycle fatigue (LCF) or the stress fields for high cycle fatigue (HCF) within the contacting bodies, which were then utilized to predict the life of the component according to various fatigue criteria. The phenomenon of surface crack formation was claimed to be mainly due to the friction shear along the contacting surfaces [39-41] captured by the product of the Hertzian pressure and the friction coefficient. These studies ignored any effect associated with the surface and lubrication conditions and their reliance on a user-defined friction coefficient limited the accuracy significantly. In order to avoid the time-consuming FE analyses and the associated convergence issue under heavy loading condition, Jiang and Sehitoglu [43] used a semi-analytical approach to determine the elastic-plastic stress and strain fields, and proposed a multi-axial critical plane criterion for ratcheting type of rolling fatigue failure considering dry contact of smooth surfaces under pure rolling condition. Cheng et al. [44] investigated the contact crack formation mechanism on the grain scale using the persistent slip band dislocation pile-up theory and proposed a semi-analytical approach. Glodež et al. [45-46] included the crack propagation into the RCF modeling while considering only subsurface cracks under smooth contact condition with a user-defined friction coefficient. Flašker et al. [47] studied the surface crack propagation including the EHL effects in the form of the empirical smooth surface lubrication formulae.
The model of Qiao et al. [48] provided improvements over those earlier studies by including the lubrication of the contacting rough surfaces forming a line contact. In the process, full-film (micro) EHL condition with no asperity contacts was considered. They compared a variety of multi-axial critical plane fatigue criteria to claim that all yielded similar predictions. This study also showed that the crack nucleation location was moved up towards the surface with the introduction of the surface roughness even when no asperity contacts were present. Another group of studies considered the RCF under the more challenging mixed EHL condition [49-50]. A single equivalent stress quantity (orthogonal shear stress [49] or von Mises stress [50]) was computed from the stress fields induced by the surface normal pressure and shear distributions. The contacting component life was then evaluated by adopting the fatigue model developed by Zaretsky [51] and later extended by Epstein et al. [49]. The predicted fatigue lives of the contact of a spur gear pair at the lowest point of single tooth contact with different surface finishes were shown to agree with the spur gear pitting test results [50]. In these simulations, the residual stress effect was included indirectly by adjusting the stress exponent (a material parameter) used in the fatigue criterion such that the predicted gear pitting lives match the test results. The mixed EHL model of these studies used a discretization scheme that was sensitive to discretization errors unless the computational grid is sufficiently fine. The Weibull model based fatigue criterion used in Refs. [49-50] provides a fatigue life value at the given probability of failure while the instantaneous surface roughness effect was not captured fully.
Also missing in these predictions was the location of the crack nucleation site. Using the novel linearization and discretization scheme developed by Li and Kahraman [16] to exclude the numerical instability and including the evaluation of both the crack nucleation life and position, Li and Kahraman [52] presented a contact fatigue model for point contacts, proposing a Lagrangian-Eulerian approach to include the surface roughness effect on pitting fatigue in a statistical way. The predictions showed evident competition between the surface and the subsurface crack nucleation and correlated well with the experimental measurements. This approach was later extended to gear contacts [53] and was shown to be able to correlate well with the experimental measurements.

With respect to contact fatigue considering the dynamic condition, the majority of the studies have been focused on the detection of the faults through the analysis of the dynamic signals in terms of either the vibration [54-61] or the acoustic emission [61-63] induced by the surface pits or wear. These signals are post processed for frequency analysis, joint time-frequency analysis and time-statistical analysis using signal processing techniques as continuous wavelet transforms [54-56, 58, 61], Fourier transforms [56, 59], Wigner-Ville distribution [61] etc., to detect faults and fault locations. Relying on the features of vibration analysis post processing, it can be deduced that Wavelet analysis is better used to identify localized defects [64], Fourier analysis to identify distributed defects and Wigner-Ville distribution to identify defect propagation [61]. From the acoustic analysis,
it is concluded that acoustic data can be effectively used for detection of micro damages such as fatigue, fretting wear etc. and early detection of cracks when compared to the vibration signals. However, the acoustic signals are sensitive to the gearbox environment, background noise, loading conditions and speed etc. Gear systems often do not work in constant loading environment. Stander et al. [65] followed a statistical approach using pseudo-Wigner-Ville-Distribution on test data obtained from vibration measurements to include the fluctuating loading conditions in the process of the local gear tooth fault detection. Besides these experimental works, Choy et al. [64], Fakhfakh et al. [66] and Chen et al. [67] utilized the numerical modeling approach to simulate the gear dynamic behavior owing to the variation in gear mesh stiffness induced by localized (pitting) and distributed (wear or spalling) defects. Regarding the impact of the dynamic behavior (before the occurrence of any surface fault) on the contact fatigue, however, the related works seem to be very limited. For instance, Ramanathan et al. [68] experimentally investigated the influence of vibration on the rolling contact fatigue using eccentric specimens with different hardness levels. The occurrence of surface pitting failure is observed to be promoted due to the extent of stresses induced due to vibrations.

1.3.4 Gear Tribo-dynamics

As mentioned earlier, gear pairs often operate within a high speed range, where the gear dynamic behavior is evident and alters the tooth contact force significantly. It has to be noted that gear dynamics does not play
alone. It interacts with the gear tribological behavior, which is a critical factor dominating the gear contact fatigue [4, 49-53, 69]. The gear dynamics has been a research field that attracts extensive amount of modeling efforts. The early studies, including those reviewed in Refs. [70-71], mostly focused on the noise and vibration aspect of gears, using deformable finite element approaches [72-73] or discrete lumped-parameter description [74-77] or for the description of the dynamic behavior of single or multi degree-of-freedom (DOF) gear systems. For simplification purpose, these works excluded the friction along the contact surfaces, which was found to be important in the dynamic response along the off-line-of-action (OLOA) direction (such as dynamic bearing forces) although its impact on the vibratory motion along the line-of-action (LOA) direction is negligible [72-74, 78]. The influence of the gear dynamics goes beyond noises and vibrations. The tooth surface frictions that point along the off-line-of action (OLOA) direction, in turn, produce main excitation for the OLOA gear vibration, which is coupled with the motion in the line-of action (LOA) direction by the corresponding frictional moments. Li and Kahraman [75] showed significant variations in the contact pressure, frictional shear and lubrication film thickness under the dynamic tooth force in comparison to those under the quasi-static condition. Paouris et al. [79] investigated the sub-surface contact stresses under the dynamic condition. Li [76] and Mohammadpour et al. [80] examined the impacts of the gear dynamic behavior on the friction and power losses. It was shown the quasi-static assumption became invalid in
the vicinity of the resonances for the accurate prediction of the mechanical efficiency [23].

An important element in the gear dynamics modeling is the gear mesh damping. To determine this damping value, experimental measurements or empirical estimation has been widely implemented. Recognizing the power dissipation mechanism at the gear mesh is due to the viscous shearing occurring in the lubrication film due to sliding motion of the tooth surfaces, Li and Kahraman [77] proposed the gear mesh damping formulation under the mixed elastohydrodynamic lubrication (EHL) condition. It was demonstrated the gear mesh damping is dependent on both the viscosity and the film thickness of the lubricant. Instead of being a constant that had been widely assumed in the literature, the gear mesh damping was shown to be varying periodically. For lightly loaded gears operating under the hydrodynamic lubrication condition, Liu et al. [81] proposed another approach for the gear mesh damping estimation. The dependences of the gear mesh damping and the OLOA dynamics on the gear contact tribological behavior [77], and the dependence of the EHL film thickness and contact pressure on the gear dynamic load, form the interaction mechanism between the gear dynamics and gear tribology fields. To model this tribo-dynamic interaction, Li and Kahraman [74] combined the gear dynamics governing equations with an isothermal mixed EHL formulation [23] for spur gears. Utilizing an iterative numerical procedure, the converged solutions showed significant influences of the
surface roughness amplitude, lubricant viscosity and operating conditions on the gear dynamic responses.
1.4 Scope and Objectives

In view of the literature, most of the EHL models neglected the dynamic behavior of the contacting components. Although the studies on the gear dynamics that exclude the interaction between the tribology and the dynamics has been extensive, the tribo-dynamics ones that include such interaction are still sparse in the literature. The research activities on the flash temperature rise and the contact fatigue crack nucleation of gear contacts under the tribo-dynamic condition are missing. This study, thus, propose to develop a tribo-dynamic contact model for the determination of the tooth surface flash temperature rise and the contact fatigue crack nucleation behaviors, considering the interactions between the gear tribology and the gear dynamics. The main objectives and scopes of this study are summarized accordingly as:

1) Develop a thermal tribo-dynamics model for gear contacts with rough surfaces, including the impacts of the gear tribological behavior on the gear dynamic forces and velocities, and the reverse effects of the gear dynamic response on the lubrication film thickness, contact pressure, tangential shear, and flash temperature rise.

2) Incorporate a multi-axial stress formulation to determine the transient multi-axial stress fields as the gears rotate in mesh.
3) Incorporate a multi-axial fatigue criterion to assess the fatigue crack nucleation life and site of gear teeth using the predicted stress histories.

4) Perform a parametric study to investigate the gear contact performances in terms of flash temperature and contact fatigue life under various loading, rotational speed, lubricant viscosity, and surface roughness conditions to examine the roles of these contact parameters in the scuffing and contact fatigue failures under the tribo-dynamic condition.

5) The results will be also be compared to those assuming the quasi-static condition to demonstrate the importance of the gear tribo-dynamic behavior in both the thermal scuffing and contact fatigue failure modes.

6) The scope of this work is limited to the mesh of spur gears. However, the methodology of this study is general, allowing the replacement of the dynamic model or the EHL model with other ones of varying sophistication.
1.5 Dissertation Outline

1) Chapter 2: Tribo-dynamic formulation with thermal mixed EHL model will be discussed in detail. The iterative scheme used, gear design parameters, gear mesh stiffness, damping and input conditions considered etc. will be discussed.

2) Chapter 3: Multi-axial contact stresses prediction from the surface normal pressure and tangential shear and multi-axial fatigue criterion to determine fatigue life will be discussed in detail.

3) Chapter 4: The effect of gear dynamics on flash temperature rise and contact fatigue life are provided in this chapter.

4) Chapter 5: The research activity will be summarized. Conclusions and recommendations for future work are offered.
Chapter 2

2 Gear Thermal Tribo-Dynamics Model

The methodology for the modeling of the thermal behavior under the tribo-dynamic condition involves three models:

1) A gear load distribution model for the determination of the gear mesh stiffness and the static transmission error.

2) A gear dynamics model for the computation of the gear dynamic mesh force and surface velocities.

3) A gear thermal mixed EHL model for the evaluation of the normal and tangential surface tractions, the gear mesh viscous damping, and the flash temperature.

The above models are coupled to quantify the surface flash temperature rises under the effects of interaction between the tribology and the dynamics. The assembly and flow of these models are illustrated in Figure 2-1. According to the gear design parameters and the input torque, the gear load distribution model [22] that is available from The Ohio State University yields the mesh stiffness as well as the static transmission error, both of which serve as the excitations for the gear dynamic behavior. An iterative scheme is then implemented to couple the gear dynamics model and the
gear thermal mixed EHL model. The initial conditions for this tribo-dynamic iteration, including the friction excitations and the gear mesh damping are set to be none, with which the equations of motion are solved by using a Fortran ODE solver to find the dynamic responses. The dynamic surface velocities and the dynamic mesh force are then fed into the gear EHL simulation to update the frictions and viscous damping. This iterative process is continued till the convergence of dynamic mesh force. The flash temperature rise under the tribo-dynamic condition is then arrived.
Figure 2-1 The computational methodology for the modeling of the thermal under the tribo-dynamic condition.
2.1 Gear Load Distribution Model

When the gears are in mesh, the distribution of load acting on the gear teeth in the contact zone is uneven. Such uneven load distribution is effected by various factors such as manufacturing errors, assembly errors, elastic deformations in the contact zone, intentional profile modifications (transmission error), backlash non-linearity etc. For example, due to axis misalignment, a concentrated contact load at one end of gear causes bending. To avoid such concentrated loads, the gears are crowned (intentional tooth modifications) to ensure the concentrated load acts at the center of the face width. Former studies [82], used simple models of linear theory of elasticity, simplified Navier’s equation and Hertzian contact assuming uniform load distribution along LOA. Pedrero [83] developed a non-uniform load distribution model using minimum elastic potential energy theory based on the assumption that the potential energy is minimum on the line of contact. It was concluded that the load distribution is highly impacted by the factors that affect the length of contact: transverse contact ratio. Conry and Seirig [22] developed a deformation analysis considering all the important factors that affect the load distribution. Load distribution factors are added into the empirical formulations to compensate for the errors and modification; and thus load distribution.
The elastic deformation when the gears are in mesh is the sum of deformations originated due to bending and torsion of the shafts and bearings, the gears are built on, gear tooth bending and local contact loads in the contact region. A simply supported beam deflections are used to calculate the deflections due to bending and torsion of the shaft. Gear tooth is approximated as a cantilever plate with concentrated load acting at the tip of the tooth. A simplified general equation for approximate deflection of cantilever plate with bending load is simulated to obtain the deflection due to tooth bending. The local contact zone is considered to be Hertzian contact by assuming two infinitely long cylinders in contact. The effects of curvature of contacting cylinders is taken into consideration from the model generated by Loo [84].

In this work, we utilize Gear Load Distribution Program (LDP) [85] that is established based on the Conry and Seirig’s [22] model of elastic bodies in contact incorporating the effects of radius of curvature [85]. The load distribution program is entered with dimensions of profile geometry and any profile modifications as per design requirements. LDP designs and analyses the gear pair to compute load distribution, transmission errors, contact length, gear mesh stiffness, backlash, fluid film thickness and tooth contact force under quasi-static conditions. However, when the amplitude of the forces acting exceeds the quasi-static forces, non-linear behavior of the spur gear pair occurs. These dynamic forces result in bending and contact stresses reducing the fatigue lives of the gear sets.
The time variation in gear mesh stiffness is due to the variation of the number of tooth pairs in contact as the gears roll in mesh. A dynamic model similar to the model of Tamminana et al. [86] is discussed in Section 2.2 to enclose the gear mesh stiffness fluctuation, displacement excitation due to manufacturing errors and intentional tooth corrections, and gear backlash non-linearity in to the model and obtain dynamic tooth force. In other words, Gear load distribution program is basis to dynamic model to obtain time-varying gear mesh parameters. The backlash is computed from the effective center distance and tooth thickness values. The quasi-static transmission error excitation under loaded and unloaded conditions respectively are predicted using Gear LDP over a one period of mesh cycle (several discrete positions). The difference between static transmission errors under unloaded \( e(t) \) and loaded \( \dot{e}(t) \) conditions, torque \( T \) and base radius \( r \) is used to estimate the mesh stiffness at discrete mesh positions in a mesh cycle as [77, 86]

\[
k_m(t) = \frac{T}{r} \left[ \frac{1}{\dot{e}(t) - e(t)} \right]
\] (1)
2.2 Gear Dynamics Model

The study of dynamic behavior of gear pairs and gear trains has been studied for two main reasons. The noise and vibration generated by a gear system is consequence of its dynamic behavior. A vibratory model predicts the impacts on gear mesh and bearing forces due to tooth profile modifications etc. Such Dynamic gear tooth forces are typically larger than the loads predicted under quasi-static conditions, possibly effecting the thermal conditions of the lubricant, tooth bending and contact fatigue lives of the gear system as mentioned in Section 2.1.

A large number of dynamic models have been developed over the past 5 decades [70-71]. These models typically include two rigid disks to represent the gears. The gear mesh interface model along the line of action that connects the two rigid disks consists of four main modules.

i. A periodically time-varying gear mesh stiffness that represents the overall gear mesh flexibility.

ii. A parametrically time-varying gear backlash allowing tooth separations to take place.

iii. A periodic displacement excitation, due to the disturbances caused by intentional tooth profile modifications such as tip and root relief or profile crown and any unintended manufacturing eccentricities from the actual tooth profile, known as transmission error (TE) excitation.
iv. And, a gear mesh damping module due to frictional forces at the gear mesh interfaces.

Most of the previous studies predicted varying gear mesh stiffness and TE excitation using load distribution model [21] under quasi-static condition or an empirical formulation of gear mesh damping. Some of the works used FE based approach [86-87] where the gear mesh stiffness and TE excitation were included implicitly through a deformable multi-body formulation. These models assume that the gear mesh stiffness matrix is proportional to the stiffness and mass matrices with two empirically determined proportionality constants. Li and Kahraman [77] published a formulation to obtain a dynamic model with an approximate equivalent damper in the LOA direction as a function of operating conditions (speed and torque) and surface conditions (lubricant parameters). In the later studies [74], it is observed that OLOA gear mesh damping is not insignificant.

With the review of the works above, and aiming towards an interactive Gear tribo-dynamics model, a six degree-of-freedom (DOF) transverse-torsional dynamic model is used to demonstrate the dynamic behavior of the gear pair under various operating conditions of speed and torque. Details of the dynamic model is discussed below in this section.

Figure 2-3 shows a transverse plane view of spur gears in contact. The base circle radius of gear 1 (driving gear) is \( r_1 \) and of gear 2 (driven gear) is \( r_2 \). \( x \) is the off-line-of-action direction and \( y \) is the line-of-action direction of
the gear pair that is defined by the line segment $\overline{B_1B_2}$, a tangent to the base circles. The resultant tooth point of contact $C$ shown is at particular time instant is represented by two equivalent cylinders, one with radius $R_1(t) = B_1C$ with center at $B_1$ and the other with radius $R_2(t) = B_2C$ with center at $B_2$. An external contact torque in counter-clockwise direction, $T_1$ is applied on the pinion (gear 1), to rotate the gear pair, is balanced by $T_2$, torque on the gear (gear 2) in counter-clockwise direction.
Figure 2-2 A spur gear pair mesh showing an instantaneous contact point C along its LOA.
Figure 2-3 (a) The six-DOF discrete dynamics model for spur gears, and (b) the tooth meshing interface friction.
As illustrated in Figure 2-3 (a), the spur gear pair that consists of gear 1 (driving gear) and gear 2 (driven gear) is modeled as a rigid disk pair that is composed of disk 1 and disk 2. The mass and the polar moment inertia of disk 1 and 2 are denoted as $m_1$, $J_1$ and $m_2$, $J_2$ respectively. The radius of disks, are denoted as $r_1$ and $r_2$; and are equal to the base radii of gears 1 and 2. The discrete dynamic model that describes both the torsional motion and the translational motions in the $x$ (OLOA) and $y$ (LOA) directions is illustrated in Figure 2-3 (a) for a general spur gear pair. The gear mesh is represented by a periodically time-varying mesh spring element ($k_m(t)$) applied in the LOA direction. This periodicity of the gear mesh stiffness is mainly due to the fluctuation of the tooth pairs in contact in addition to other secondary effects [85]. The gear mesh stiffness is subject to a clearance element of magnitude $2\ell$, and a static transmission error element ($\varepsilon_s$) connected in parallel i.e. along the LOA direction, representing the tooth mesh stiffness, the backlash ($\ell$ equals the half backlash), and the geometric deviation from the involute profile caused by manufacturing errors and/or intentional modifications, respectively. The gear mesh damping is introduced through the friction forces exerted along the tooth surfaces in the OLOA direction as shown in Figure 2-3 (b).

To model the bearing and shaft supports for gears, a set of spring-damping elements is implemented in the LOA direction ($k_y$ and $c_y$) and another set
is applied in the OLOA direction ($k_x$ and $c_x$). In addition, the torsional damping of the bearing caused by the viscous power loss is included as $c_t$ for each gear, while not shown in the Figure 2-3.

With the positive directions of the alternating rotational displacements, $\theta_1(t)$ and $\theta_2(t)$ the translational motions in the $x$ and $y$ directions, and constant external torques, $T_1$ and $T_2$, the equations of motion [74] are then arrived for spur gear pair as [1]

\begin{align*}
J_1\ddot{\theta}_1(t) + c_{t1}\dot{\theta}_1(t) + \eta_1k_{m1}(t)\delta(t) &= T_1 + \sum_{n=1}^{N} [F_{1n}(t)R_{1n}(t)]_n \\
m_1\ddot{y}_1(t) + c_{y1}\dot{y}_1(t) + k_{y1}y_1(t) + k_{m1}(t)\delta(t) &= 0 \\
m_1\ddot{x}_1 + c_{x1}\dot{x}_1(t) + k_{x1}x_1(t) &= \sum_{n=1}^{N} [F_{1n}(t)]_n \\
J_2\ddot{\theta}_2 + c_{t2}\dot{\theta}_2 - r_2k_{m2}(t)\delta(t) &= -T_2 - \sum_{n=1}^{N} [F_{2n}(t)R_{2n}(t)]_n \\
m_2\ddot{y}_2 + c_{y2}\dot{y}_2(t) + k_{y2}y_2(t) - k_{m2}(t)\delta(t) &= 0 \\
m_2\ddot{x}_2 + c_{x2}\dot{x}_2(t) + k_{x2}x_2(t) &= -\sum_{n=1}^{N} [F_{2n}(t)]_n
\end{align*}

Where $R_{1n}(t)$ and $R_{2n}(t)$ are radii of curvature of the nominal involute profile [23, 74] at the contact pair of $n$th meshing tooth pair corresponding to the friction forces $F_{1n}(t)$ and $F_{2n}(t)$, respectively. Here $N$ represents the total
number of tooth pairs in contact at certain mesh position. For most spur gears, \( N \) fluctuates between 1 and 2 periodically. The frictional force was derived by Li and Kahraman [74] as

\[
F_1 = c_m \ddot{X} + F_s + F_r 
\]

(3a)

\[
F_2 = c_m \ddot{X} + F_s - F_r 
\]

(3b)

where \( \ddot{X} = (R_2 \dot{\theta}_2 - \dot{x}_2) - (R_1 \dot{\theta}_1 - \dot{x}_1) \), \( F_s \) and \( F_r \) are the sliding and rolling friction forces. As illustrated in Figure 2-3 (b), the tooth surface velocities of \( u_1 \) and \( u_2 \) point in the OLOA direction of \( x \). The sliding velocity, \( u_s = u_1 - u_2 \) introduces gear mesh viscous damping, \( c_m(t) \) through power dissipation mechanism of shear heating within the lubrication film.

The non-linear displacement function \( \delta(t) \) that is used to model the circumstances of tooth separations has the piece-wise linear expression of [74]

\[
\delta(t) = \begin{cases} 
\varepsilon_d(t) - \varepsilon_s(t) - \ell; & \varepsilon_d(t) - \varepsilon_s(t) > \ell; \\
0; & |\varepsilon_d(t) - \varepsilon_s(t)| \leq \ell; \\
\varepsilon_d(t) - \varepsilon_s(t) + \ell; & \varepsilon_d(t) - \varepsilon_s(t) < -\ell; 
\end{cases}
\]

(4)

\[
\varepsilon_d(t) = \left( \theta_1(t) + y_1(t) \right) - \left( \theta_2(t) + y_2(t) \right)
\]

(5)

where \( \varepsilon_d(t) \) is the relative dynamic gear mesh displacement (or dynamic transmission error). In Equation (4), the first condition represents the linear motion with no tooth separations, while the second and third conditions
represent the tooth separation (single-sided impacts) and the back side contacts (double-sided impacts) conditions, respectively. Although tooth separations (single-sided contact) were demonstrated to occur commonly in spur gears [85], there has been no experimental evidence of back contacts under loaded steady-state conditions. As such, the third condition in Equation (4) is maintained for completeness purposes only.

It is noted $W_T$ is the dynamic gear mesh tooth force is approximated from the quasi-static tooth force, $W_{T}^s$ as [74-75]

$$W_T(t) = W_{T}^s(t) \left[ k_m(t)\delta(t) \frac{P_1}{T_1} \right]$$

where $W_{T}^s$ is determined by using the tooth contact analysis in gear load distribution program [22-23] and contact radii. The dynamic tooth force is used in the EHL analysis to determine the expressions for $c_m$, $F_s$, and $F_r$, in the process capturing the most critical influence of dynamic behavior on the lubricant characteristics and vice versa.

2.3 Gear Contact Tribological Model

Under heavy loads with rough gear contacts operating at low or moderate speeds, a full elasto-hydrodynamic (EHL) lubrication film does not exist. The asperities share the load with the fluid film and gears operate under mixed EHL conditions. Initial studies paid attention to only full EHL or smooth surfaces due to computational challenges [5]. Moreover, as the
gears roll in mesh, the contact radii of curvatures, surface velocities and normal tooth force are all time dependent. These transient effects alters the lubricant film thickness and the load sharing between the gear contacts. Hence, studies were extended to introduce transient effects on artificial surface roughness by Venner and Lubrichet [7-8], Ai et al. [11] and other authors. More complicated lubrication conditions were later simulated by Jiang [12] using a transient mixed EHL model for Newtonian fluids. Jiang’s model [12] predicted significant contact load fluctuations induced by measure surface roughness profiles.

Based on the assumption of a smooth transition between the asperity contact regions from lubricated EHL regions, Hu et al. [13-14], proposed a contact equation for the asperity contact regions employing a reduced form of Reynolds equation. The transient Reynolds equation governs the fluid flow in the wet contact areas with no asperity interactions and the reduced Reynolds form governs the dry contact areas i.e. asperity contact region.

In the lumped-parametric gear model illustrated in Figure 2-3 based on the gear involute geometry and kinematics, the contact radii of curvature varies with time and is given as

\[ R_1(t) = \eta \theta_1(t) \]  \hspace{1cm} (7a)

\[ R_2(t) = r_2 \theta_2(t) \]  \hspace{1cm} (7b)
Under the quasi-static condition, the kinematic tangential surface velocity of gear 1 and 2, whose angular velocity is $\omega_1$ and $\omega_2 = Z_1\omega_1/Z_2$, varies with time, $t$ [23] and is given as

\begin{align}
    u_1^m(t) &= R_1(t)\omega_1 \quad \text{(8a)} \\
    u_2^m(t) &= R_2(t)\omega_2 \quad \text{(8b)}
\end{align}

with $Z_1$ and $Z_2$ being the number of teeth on gears 1 and 2 respectively. When the gear dynamic response is evident, an additional alternating component is introduced due to both the torsional and the $x$ direction vibratory motions as

\begin{align}
    u_1^a(t) &= R_1(t)\dot{\omega}_1(t) - \dot{x}_1(t) \quad \text{(9a)} \\
    u_2^a(t) &= R_2(t)\dot{\omega}_2(t) - \dot{x}_2(t) \quad \text{(9b)}
\end{align}

These transient mean and amplitude components constitute the total tangential surface velocity as

\begin{align}
    u_1(t) &= R_1(t)\omega_1 + R_1(t)\dot{\omega}_1(t) - \dot{x}_1(t) \quad \text{(10a)} \\
    u_2(t) &= R_2(t)\omega_2 + R_2(t)\dot{\omega}_2(t) - \dot{x}_2(t) \quad \text{(10b)}
\end{align}

The movement of gear tooth surfaces entrains the lubricant into the contact. Under different operating conditions the lubricant entrained with surface roughness of the contact surfaces form full EHL or mixed EHL conditions.
The fluid flow within the contact zone is governed by one dimensional transient Reynolds equation

\[
\frac{\partial}{\partial x} \left[ f(x,t) \frac{\partial p(x,t)}{\partial x} \right] = u_r(t) \frac{\partial}{\partial x} \left[ \rho(x,t) h(x,t) \right] + \frac{\partial}{\partial t} \left[ \rho(x,t) h(x,t) \right]
\]  

(11a)

In Equation (11), \( p, h, \) and \( \rho \) respectively represent the pressure, thickness and density of the lubrication film at position \( x \) and time \( t \), \( u_r \) is the rolling velocity that is defined as \( u_r = \frac{1}{2} (u_1 + u_2) \), and \( f \) is the flow coefficient.

To improve the lubricant properties and to enhance performance at high temperatures, additives are added to the lubricant. The addition of these additives, makes the lubricant to behave as non-Newtonian fluid even at normal conditions. A Ree-Eyring fluid is assumed to model the EHL problem and the expression of \( f \) is referred for Eyring fluid [23, 25] is

\[
f = \frac{\rho(x,t) h(x,t)^{3}}{12 \eta(x,t)} \cosh \left( \frac{\tau_m(x,t)}{\tau_0} \right). \eta \text{ is the lubricant viscosity,}
\]

\[
\tau_m = \tau_0 \sinh^{-1} \left( \frac{\eta(x,t) u_s(t)}{\tau_0 h(x,t)} \right) \text{ is the viscous shear stress.} \tau_0, \text{ lubricant reference shear stress, is linearly dependent on pressure [88-89]. If} \ p_s \text{ is solidification pressure and} \ p_e \text{ is the evaporation pressure limit, and if} \ p > p_e \text{ then} \ \tau_0 \text{ depends non-linearly on pressure.}
\]
Owing to the significant surface irregularities caused by the gear finishing processes, a continuous lubrication film across the entire EHL conjunction is often not achievable. The film thickness breaks down wherever metal-to-metal asperity contact takes place. Within these local boundary lubrication areas, the separation between the surfaces is constant i.e., a small number that is close to zero. Thus, the contact can be described as [21, 23]

\[
\frac{\partial h(x,t)}{\partial x} = 0
\]  

(11b)

At the boundaries between the hydrodynamic areas and the asperity contact areas, the local film shape is considered to preserve and travel at the rolling velocity such that

\[
\frac{\partial h(x,t)}{\partial x} = -\frac{\partial h(x,t)}{u_r(t) \partial t}
\]  

(11c)

Limiting the analysis to elastic deformations only, the transient local film thickness in Equation (12) consists of the rigid body approach, \( h_0 \), the curvature gap, \( g_0 \), the elastic deflection, \( V \), assuming sufficiently high surface hardness to prevent any plasticity, and the roughness height distributions along surface 1, \( S_1(x,t) \), and surface 2, \( S_2(x,t) \), as

\[
h(x,t) = h_0(t) + g_0(x,t) + V(x,t) - S_1(x,t) - S_2(x,t)
\]  

(12)
The time-varying unloaded geometric gap between two tooth surfaces, is defined as

\[ g_0(x,t) = \frac{x^2}{2R_{eq}(t)} \]  

(13)

where \( R_{eq}(t) = R_1(t)R_2(t)/(R_1(t) + R_2(t)) \). Additionally, the elastic deformation due to normal load \( W \) applied is given as

\[ V(x,t) = \int_{x_s}^{x_c} K(x-x')p(x',t)dx' \]  

(14)

where \( x_s \) and \( x_c \) are the limits of the computational domain of the contact zone and \( K(x) = -4\ln|x|/(\pi E') \) is the influence function with \( E' = 2\left(\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}\right)^{-1} \) where \( \nu \) and \( E \) are Poisson’s ratio and Young’s Modulus respectively.

To enforce the equilibrium condition between the normal tooth force, \( W_1 \) and the contact force (due to the contact pressure distribution within the contact zone), the rigid body approach in Equation (12) is iteratively adjusted till the difference between these two action and reaction forces is within a small tolerance [23]. This load balance between the contact force and the pressure distribution is given as using the tooth force intensity along the contact line, \( W'(t) \), normal force per unit width. This tooth force contact intensity varies in a specific way according to tooth-to-tooth load carrying characteristics and is given in Equation (15)
\[ W'(t) = \int_{x_2}^{x_1} p(x,t) dx \quad (15) \]

To obtain the viscous gear mesh damping \( c_m \), the sliding friction force \( F_s \) and the rolling friction force \( F_r \), both Poiseuille and Couette flows are included to obtain the total viscous shear stress. This viscous shear is written as [77]

\[ \tau^v(x,z,t) = \frac{\eta^*(x,t)}{h(x,t)} [u_2(t) - u_1(t)] + \left( z - \frac{1}{2} h(x,t) \right) \frac{\partial p(x,t)}{\partial x} \quad (16) \]

The viscous shear stress acting on the tooth surfaces in, within lubricant film thickness, of pinion \( z = 0 \) and gear \( z = h \) is thus written as

\[ \tau_1^v(x,t) = \frac{\eta^*(x,t)}{h(x,t)} [u_2(t) - u_1(t)] - \frac{1}{2} h(x,t) \frac{\partial p(x,t)}{\partial x} \tag{17a} \]

\[ \tau_2^v(x,t) = \frac{\eta^*(x,t)}{h(x,t)} [u_2(t) - u_1(t)] + \frac{1}{2} h(x,t) \frac{\partial p(x,t)}{\partial x} \tag{17b} \]

Within the asperity contact region, the shear stress is product of \( \mu_b \) is the boundary lubrication friction coefficient, and the local contact pressure \( \tau_b = \mu_b p(x,t) \). Frictional forces is the product of contact area and the total shear stress. Substituting equations (8) and (9) in (17) and integrating over the contact area where \( L \) is the face width of the gear tooth, \( \eta^* = \eta / \cosh(\tau_m/\tau_0) \) is the effective viscosity of non-Newtonian fluid, \( \Gamma_b \) and \( \Gamma_f \) represent the portion of computational domain where the hydrodynamic fluid film exists.
\[ F_1(t) = L \int_{\Gamma_f} \frac{\eta^*(x,t)}{h(x,t)} \left[ u_2^g(t) - u_1^g(t) \right] dx + L \int_{\Gamma_b} \mu_b p(x,t) dx + L \int_{\Gamma_f} \frac{h(x,t)}{2} \frac{\partial p(x,t)}{\partial x} dx \]  
\tag{18a}

\[ F_2(t) = L \int_{\Gamma_f} \frac{\eta^*(x,t)}{h(x,t)} \left[ u_2^m(t) - u_1^m(t) \right] dx + L \int_{\Gamma_b} \mu_b p(x,t) dx + L \int_{\Gamma_f} \frac{h(x,t)}{2} \frac{\partial p(x,t)}{\partial x} dx \]  
\tag{18b}

From the above tribological formulation Equation (18), the sliding friction force \( F_s \) and the rolling friction force \( F_r \) and viscous gear mesh damping force \( c_m \) can be calculated as below [74]

\[ F_s(t) = L \int_{\Gamma_b} \mu_b p(x,t) dx + L \int_{\Gamma_f} \frac{\eta^*(x,t)}{h(x,t)} \left[ u_2^m(t) - u_1^m(t) \right] dx \]  
\tag{19}

\[ F_r(t) = L \int_{\Gamma_f} \frac{h(x,t)}{2} \frac{\partial p(x,t)}{\partial x} dx \]  
\tag{20}

\[ c_m = L \int_{\Gamma_f} \frac{\eta^*(x,t)}{h(x,t)} dx \]  
\tag{21}

These calculated frictional forces and gear mesh damping are looped back into gear dynamics model to form an interactive gear tribo-dynamics model.
2.4 Gear Tribo-Dynamics Model

2.4.1 Discretization

In order to ensure the consistency of the EHL analysis along the line of action, the size of the computational domain is fixed through the entire simulation starting at SAP and ending at the tip of the tooth of gear 1. A discretized computational scheme [23] in a computational domain $-1.5a_{\text{max}} \leq x \leq 1.5a_{\text{max}}$ in the direction of rolling ($x$) where $a_{\text{max}}$ is the maximum half-Hertzian contact width along the entire LOA is employed. A refined mesh with $N$ grid elements is applied. It is ensured that the mesh captures the surface roughness geometry sufficiently. At a given time, $t_n$, it is assumed that the lubricant viscosity, density, film thickness and pressure are uniform with in each grid element, $i$ ($i \in [1, N]$) and is represented at the center point of the grid cell. These Poiseuille term of Reynold’s equation is discretized as followed

$$
\frac{\partial}{\partial x} \left( f \frac{\partial p(x,t)}{\partial x} \right) = \frac{f(i-1/2),t_n}{\Delta x} \frac{p(i-1),t_n}{\Delta x^2} - \frac{f(i-1/2),t_n + f(i+1/2),t_n}{\Delta x} \frac{p(i),t_n}{\Delta x^2} + \frac{f(i+1/2),t_n}{\Delta x} \frac{p(i+1),t_n}{\Delta x^2}
$$

(22a)
With the flow coefficients approximated by

\[
f_{(i-1/2),t_n} = \frac{1}{2} \left[ f_{(i-1),t_n} + f_{i,t_n} \right] \tag{22b}
\]

\[
f_{(i+1/2),t_n} = \frac{1}{2} \left[ f_{i,t_n} + f_{(i+1),t_n} \right] \tag{22c}
\]

Because the central difference discretization might introduce oscillations to the solution [23], a second order backward scheme is employed for the Couette term as

\[
\frac{\partial (\rho h)}{\partial x} = \frac{3}{2} \rho_{i,t_n} h_{i,t_n} - 2 \rho_{(i-1),t_n} h_{(i-1),t_n} + \frac{1}{2} \rho_{(i-2),t_n} h_{(i-2),t_n} \Delta x
\]

(23)

The squeeze term is linearized using the second order backward scheme as

\[
\frac{\partial (\rho h)}{\partial t} = \frac{3}{2} \rho_{i,t_n} h_{i,t_n} - 2 \rho_{i,t_{n-1}} h_{i,t_{n-1}} + \frac{1}{2} \rho_{i,t_{n-2}} h_{i,t_{n-2}} \Delta t
\]

(24)

To solve the governing equations of gear tribo-dynamic model, the total traction forces exerted on the surfaces of the \(i\)th tooth pair, where \(A\) denotes the area of the tooth surface grid element are written as

\[
F_{1n} = A \left[ \sum_{j=1}^{N} \tau_{1i} \right] \quad \text{and} \quad F_{2n} = A \left[ \sum_{j=1}^{N} \tau_{2i} \right]
\]

(25)

Where \(\tau_{1i} = \tau_1(x_i)\) and \(\tau_{2i} = \tau_2(x_i)\). Substituting Equations (10) and (17) in (25), the following equations of frictional forces are arrived
\[ F_{1n} = -[R_1 \dot{\theta}_1 - R_2 \dot{\theta}_2] A \left[ \sum_{i=1}^{N} \left( \frac{\eta^*}{h} \right)_{i,n} \right] + A \left[ \sum_{i=1}^{N} \left( \eta^* \frac{(\bar{u}_2 - \bar{u}_1) - h \frac{\partial p}{\partial x}}{2 \frac{\partial x}{\partial x}} \right)_{i,n} \right] \]

(26a)

\[ F_{2n} = -[R_1 \dot{\theta}_1 - R_2 \dot{\theta}_2] A \left[ \sum_{i=1}^{N} \left( \frac{\eta}{h} \right)_{i,n} \right] + A \left[ \sum_{i=1}^{N} \left( \eta \frac{(\bar{u}_2 - \bar{u}_1) + h \frac{\partial p}{\partial x}}{2 \frac{\partial x}{\partial x}} \right)_{i,n} \right] \]

(26b)

From these expressions the viscous damping associated with the dynamic components of the velocities, \( R_1 \dot{\theta}_1 \) and \( R_2 \dot{\theta}_2 \), for the \( n \) th contacting tooth pair is extracted as

\[ c_m = A \left[ \sum_{i=1}^{N} \left( \frac{\eta^*}{h} \right)_{i,n} \right] \]

(27)

Substituting frictional forces and damping into the equations of motion (2) become

\[ J_1 \ddot{\theta}_1(t) + \left\{ c_1 + \sum_{n=1}^{N} \left[ R_1^2(t)c_m(t) \right]_{n} \right\} \ddot{\theta}_1(t) - \sum_{n=1}^{N} \left[ R_1(t)R_2(t)c_m(t) \right]_{n} \dot{\theta}_2(t) \]

\[- \sum_{n=1}^{N} \left[ R_1(t)c_m(t) \right]_{n} [\dot{x}_1(t) - \dot{x}_2(t)] + \eta k_m \delta(t) = T_1 + \sum_{n=1}^{N} \left\{ [F_s(t) - F_f(t)]_{n} \right\} \]

(28a)

\[ m_1 \ddot{y}_1(t) + c_{y1} \dot{y}_1(t) + k_{y1} y_1(t) + k_m(t) \delta(t) = 0 \]

(28b)

\[ m_1 \ddot{x}_1 - \sum_{n=1}^{N} \left[ R_1(t)c_m(t) \right]_{n} \dot{\theta}_1(t) + \sum_{n=1}^{N} \left[ R_2(t)c_m(t) \right]_{n} \dot{\theta}_2(t) \]

\[ + \left\{ c_{x1} + \sum_{n=1}^{N} \left[ c_m(t) \right]_{n} \right\} \dot{x}_1(t) - \sum_{n=1}^{N} \left[ c_m(t) \right]_{n} \dot{x}_2(t) + k_{x1} x_1(t) = \sum_{n=1}^{N} \left[ F_s(t) - F_f(t) \right]_{n} \]

(28c)
\begin{align*}
J_2 \ddot{\theta}_2 - \sum_{n=1}^{N} \left[ R_1(t)R_2(t)c_m(t) \right]_n \dot{\theta}_1(t) + \left\{ c_{r2} + \sum_{n=1}^{N} \left[ R_2^2(t)c_m(t) \right]_n \right\} \dot{\theta}_2(t) \\
+ \sum_{n=1}^{N} \left[ R_2(t)c_m(t) \right]_n \left[ \dot{x}_1(t) - \dot{x}_2(t) \right] - r_2k_m(t)\delta(t) = -T_2 - \sum_{n=1}^{N} \left\{ [F_s(t) + F_r(t)]R_2(t) \right\}_n
\end{align*}

(28d)

\begin{align*}
m_2 \ddot{y}_2 + c_{y2} \dot{y}_2(t) + k_{y2}y_2(t) - k_m(t)\delta(t) = 0
\end{align*}

(28e)

\begin{align*}
m_2 \ddot{x}_2 + \sum_{n=1}^{N} \left[ R_1(t)c_m(t) \right]_n \dot{\theta}_1(t) - \sum_{n=1}^{N} \left[ R_2(t)c_m(t) \right]_n \dot{\theta}_2(t) - \sum_{n=1}^{N} \left[ c_m(t) \right]_n \dot{x}_1(t) \\
+ \left\{ c_{x2} + \sum_{n=1}^{N} \left[ c_m(t) \right]_n \right\} \dot{x}_2(t) + k_{x2}x_2(t) = - \sum_{n=1}^{N} \left[ F_s(t) + F_r(t) \right]_n
\end{align*}

(28f)

The discretized computational scheme is solved in FORTRAN where the models are interlinked through viscous damping term and frictional terms in Equation (28).
2.5 Gear Thermal Model

Under heavy loads and speed operating conditions of gears, one of the important parameters for scuffing failure is temperature rise due to an unexpected failure of fluid film when asperities of rough surfaces come in contact. Other parameters that cause scuffing are lubricant properties, gear surface material properties and the surrounding atmosphere. As mentioned in Section 1.3.2, scuffing is a thermal failure mode, dictated by the local surface temperatures. Local surface temperature is the sum of bulk temperature and the instantaneous flash temperature rise. Bulk surface temperature can be easily measured experimentally by measuring the contact temperatures using thermocouples. On the other hand, flash temperature rise measurement is more difficult experimentally as it exists for a short duration of time.

In primitive studies, Blok [25] proposed a flash temperature equation that depends on material properties of surfaces, sliding speed, coefficient of friction and the contact geometry. The Blok formulation is further extended and new models were formulated that include set of asperity contacts in the contact [26], computerized surface roughness profiles [90], different contact conditions [91] etc. The majority of scuffing failures, occur at higher operating conditions, mixed EHL film formations, and highly transient conditions. Wang et al. [30] introduced a reduced Reynolds equation into the thermal mixed EHL formulations under asperity contact conditions using
a deterministic numerical model. One of the major drawbacks of the above studies is the bulk temperatures of the contact surfaces were assumed to be known and frictional coefficient is predefined. In this study, a heat transfer formulation by Li et al. [2] that predicts the surface bulk temperature rise, onto which the flash temperature was added to determine the maximum surface temperature, is used and discussed below.

As the gears rotate in mesh, the frictional heat rises the lubricant temperature, $\phi_f$ and consequently impact the lubricant viscosity as well as the viscous damping [74, 77]. Under heavy loading conditions and high sliding, the shear heating within the contact is considered to be dominant and the compressive heating/cooling is neglected. Also, the heat convection across the fluid film and the heat conduction along the rolling direction are ignored. The thermal behavior is implemented with the fluid energy equation [2, 92]

$$k_f \frac{\partial^2 \phi_f}{\partial z^2} + \tau \dot{\gamma} = \rho c_f \left( u \frac{\partial \phi_f}{\partial x} + \frac{\partial \phi_f}{\partial t} \right)$$

(29)

to determine the $\phi_f$ variation within the EHL conjunction. In this equation, $k_f$ and $c_f$ are the thermal conductivity and specific heat of the fluid, $\tau$ and $\dot{\gamma}$ are the shear and shear strain rate, and $u$ is the fluid velocity. The $z$ axis points from surface 1 ($z=0$) to surface 2 ($z=h$), representing the position along the film thickness. For high speed gearing applications, the
shear flow dominates and the fluid velocity becomes a linear function along the film thickness direction $z$ as

$$u(z,t) = u_1(t) \left(1 - \frac{z}{h}\right) + u_2(t) \left(\frac{z}{h}\right)$$

(30)

For an Eyring fluid the shear strain rate is given by $\dot{\gamma} = (\tau_0/\eta)\sinh(\tau/\tau_0)$. It is also assumed that the temperature distribution across the fluid film can be approximated as the parabolic shape of [93]

$$\phi_f = (3\phi_1 + 3\phi_2 - 6\phi_m)\left(\frac{z}{h}\right)^2 - (4\phi_1 + 2\phi_2 - 6\phi_m)\left(\frac{z}{h}\right) + \phi_1$$

(31)

where $\phi_m$ is the mean fluid temperature across the film, and $\phi_1, \phi_2$ are the temperatures of the bounding surfaces.

To determine the flash temperature rise of tooth surface $j$ that is denoted as $\Delta\phi_j$ is described by the energy equation for the bounding solids [94]

$$\Delta\phi_j(x, t) = \int_{t}^{t'} d\tau' \exp\left\{-\frac{[(x-x')-u_j(t-t')]^2}{4\kappa_s(t-t')}\right\} \frac{Q_j(x', t') dx'}{2\pi k_s(t-t')}$$

(32)

is applied. In the integral, $\kappa_s$ and $k_s$ are the gear material thermal diffusivity and conductivity, $\Gamma$ represents the computational domain. $Q_j$ is the frictional heat flux going into surface $j$ ($j = 1, 2$), both which constitute the total local frictional heat $Q$ in the way of
Here, $\vartheta$ is the heat partition coefficient [25] is described as

$$\phi_1 - \phi_2 = \frac{h}{2k_f} (1 - 2\vartheta)Q$$

Equation (34) reduces to $\phi_1 = \phi_2$, indicating a continuous temperature transition at the interface. The total local frictional heat flux is given by sliding viscous shear, $q = \eta^* u_s / h$ that depends on the sliding velocity $u_s$, effective viscosity, $\eta^*$ and fluid film thickness, $h$. Neglecting the rolling component of heat generation, we have heat generation for any fluid region [95]

$$Q(x,t) = \eta^* \frac{u_s^2}{h(x,t)}$$

For the boundary lubrication regions, the surface shear is $q = \mu_b p$ and $Q = \mu_b p |u_s|$, where $\mu_b$ is the boundary lubrication friction coefficient, and is assumed to be 0.1 [93, 95], due to lack of measurements for the specific lubricant additive and steel combination used in this research. For the entire contact, the friction coefficient is determined as
\[
\mu = \int_{\Gamma} q(x,t)dx
\]  
(36)

Under mixed EHL conditions, the contact pressure changes are abrupt and significant. This change effects lubricant viscosity significantly. Thus, a precise viscosity-pressure relationship is essential to result an accurate EHL model. A two-slope viscosity-pressure relationship is modified to incorporate the effect of flash temperature rise, \( \Delta \phi \) [2]

\[
\eta = \begin{cases} 
\eta_0 \exp\left( \alpha_1 p - \phi_f \Delta \phi_f \right), & p < p_a \\
\eta_0 \exp\left( c_0 + c_1 p + c_2 p^2 + c_3 p^3 - \phi_f \Delta \phi_f \right), & p_a \leq p \leq p_b \\
\eta_0 \exp\left( \alpha_1 p + \alpha_2 (p - p_t) - \phi_f \Delta \phi_f \right), & p > p_b 
\end{cases}
\]  
(37)

where \( \alpha_1 \) and \( \alpha_2 \) are the pressure-viscosity coefficients for the low \( p < p_a \) and high \( p > p_b \) pressure ranges, respectively, and \( p_t \) is the transition pressure between these two ranges. The constants \( c_0, c_1, c_2, c_3 \) are determined such that both \( \eta \) and \( \partial \eta / \partial p \) are continuous at \( p = p_a \) and \( p = p_b \). The temperature-viscosity coefficient \( \phi \) describes the slope of \( \ln(\eta) \) versus the temperature rise.

To consider the compressibility of the lubricant in thermal conditions, a density-pressure relationship used by Dowson and Higginson [1] is used,

\[
\rho = \rho_0 \frac{1 + \lambda_1 p}{1 + \lambda_2 p} \left( 1 - \beta \Delta \phi_f \right)
\]  
(38)
where $\lambda_1 = 2.266 \times 10^{-9} \text{ Pa}^{-1}$, $\lambda_2 = 1.683 \times 10^{-9} \text{ Pa}^{-1}$, and $\beta$ is the thermal expansion coefficient.
Chapter 3

3 Gear Stress Prediction and Fatigue Model

A theoretical and numerical model to study the influence of gear tribo-dynamics on the gear contact fatigue is lacking. An interdisciplinary model that bridges the gear dynamics, the gear tribology and the contact fatigue fields is required for the appropriate physical description of the gear contact fatigue failure. Gear dynamics model and tribo-dynamics model are discussed in previous section.

The methodology to compute crack nucleation life in order to understand the gear contact fatigue failure (pitting) involves two models:

1) A gear stress prediction model for the determination of the gear surface stresses and subsurface stresses.

2) A multi-axial fatigue model to predict multi-axial fatigue crack nucleation life.

Gear Thermal Tribo-Dynamics Model in the previous section is carried out till the convergence of the dynamic mesh force is reached [74]. Under this converged tribo-dynamic condition, the yielded normal pressure and tangential shear serve as the inputs of a stress prediction model to evaluate
the multi-axial stress fields, onto which, any residual stress can be superimposed which are then used in a multi-axial fatigue criterion to determine the fatigue life according to the material tension and torsion fatigue strength. The modeling methodology for the gear contact fatigue is shown in Figure 3-2.
Figure 3-1 The computational methodology for the modeling of the contact fatigue behavior under the tribo-dynamic condition.
3.1 Gear Contact Stress Prediction Model

The shear traction between the contact surfaces consists of (a) the viscous shear within the lubricated areas of the contact and (b) the contact friction due to any direct asperity interactions. Figure 3-2 illustrates the two-dimensional (2D) computational domain of a single gear tooth with $p(x,t)$ and $q(x,t)$ applied, where $q$ represents the surface shear, taking the form [95]

$$
q(x,t) = -\eta_s \frac{u_s(t)}{h(x,t)} \pm \frac{1}{2} h(x,t) \frac{\partial p(x,t)}{\partial x}
$$  

(39)

in the hydrodynamic areas and the form $q(x,t) = \mu_b p(x,t)$ in the asperity contact areas. Equation (39) consists of both the Poiseuille and Couette flow terms assuming no slip between the lubricant and tooth surfaces. And, $p(x,t)$ represents the pressure acting on tooth surface; it is obtained by solving Reynold’s equation (11) in Section 2.3. The $x-z$ reference frame attaches to the contact zone and moves with it as the gears roll in mesh. The $x$ axis denotes the rolling direction and the $z$ axis points down into the surface. The computational contact domain is load dependent that is defined by the maximum Hertzian contact half width $a_{max}$ as of $-2.5a_{max} \leq x \leq 1.5a_{max}$ and $0 \leq z \leq a_{max}$. It is discretized into grid elements. The $x$ direction element size is set to be constant and on the order of microns to accurately capture the surface roughness effects in the mixed
EHL simulation. A non-uniform increment in the z direction is applied, specifically increases as z increases, such that the finer mesh towards the surface allows a better resolution for the surface irregularity induced near surface stress concentrations.

As the contact zone moves along the tooth profile, with the plane strain assumption, the material points on and below the surface experience transient stress fields of the normal components, and the shear component (that is a reasonable assumption since the gear face width is usually thick). The direction is perpendicular to the surface and axes in a Cartesian coordinate system orientation. Since the contact zone is usually small in comparison to the gear tooth, the perfectly smooth elastic half space assumption is adopted [49] to determine the stress components induced by the distributions of and on the grid nodes along the tooth surface as

\begin{equation}
\sigma_x(x, 0, t) = -p(x, t) - \frac{2}{\pi x_i} \int_{-x_i}^{x_i} \frac{q(s, t)}{(x - s)} ds
\end{equation}  \tag{40a}

\begin{equation}
\sigma_z(x, 0, t) = -p(x, t)
\end{equation}  \tag{40b}

\begin{equation}
\sigma_{xz}(x, 0, t) = -q(x, t)
\end{equation}  \tag{40c}
Figure 3-2 Two-dimensional computational domain and the meshing scheme.
Figure 3-3 Elastic half plane (a) under point load (b) under distributed pressure
The stress components at the grid nodes into the material (below the surface) are formulated from the elastic half-space assumption and expanding the line load on a surface to a distributed pressure.

From Figure 3-3 (a), the normal and tangential forces, of magnitude $P$ and $Q$ per unit length is used to solve for the radial stress fields induced into the material consecutively [88] are given in polar coordinates as

$$\sigma_r = -\frac{2P}{\pi r} \cos \theta \quad \text{and} \quad \sigma_r = -\frac{2Q}{\pi r} \cos \theta$$

(41a)

$$\sigma_\theta = \tau_{r\theta} = 0$$

(41b)

Transforming into Cartesian coordinates using the following formulation we obtain equivalent stress components.

$$\sigma_x = \sigma_r \sin^2 \theta = -\frac{2P}{\pi} \frac{x^2 z}{(x^2 + z^2)^2}$$

(42a)

$$\sigma_y = \sigma_r \cos^2 \theta = -\frac{2P}{\pi} \frac{z^3}{(x^2 + z^2)^2}$$

(42b)

$$\sigma_{xz} = \sigma_r \sin \theta \cos \theta = -\frac{2P}{\pi} \frac{xz^2}{(x^2 + z^2)^2}$$

(42c)

using $\theta = \arctan \left( \frac{z}{x} \right)$, $r^2 = x^2 + z^2$, $\cos \theta = \frac{x}{\sqrt{x^2 + z^2}}$ and $\sin \theta = \frac{z}{\sqrt{x^2 + z^2}}$. 

58
These equations form basis to finding the elastic-half space stress fields induced into the contact body by any arbitrary pressure distribution \( p(s) \) and shear distribution \( q(s) \) loaded over a strip from \( x_s \) (start point) to \( x_e \) end point. Consider a small element \( ds \) located at distance \( s \) from the origin such that \( P = -p(s)ds \) and \( Q = q(s)ds \) as shown in Figure 3-3 (b) and expanding the stress fields (42) from the point loading equations, by integrating over the loaded regions and replacing \( x \) by \( (x-s) \) is given as

\[
\sigma_x(x,z,t) = -\frac{2z}{\pi} \int_{x_s}^{x_e} \frac{p(s,t)(x-s)^2}{(x-s)^2 + z^2} ds - \frac{2z}{\pi} \int_{x_s}^{x_e} \frac{q(s,t)(x-s)^3}{(x-s)^2 + z^2} ds \quad (43a)
\]

\[
\sigma_z(x,z,t) = -\frac{2z^3}{\pi} \int_{x_s}^{x_e} \frac{p(s,t)}{(x-s)^2 + z^2} ds - \frac{2z^2}{\pi} \int_{x_s}^{x_e} \frac{q(s,t)(x-s)}{(x-s)^2 + z^2} ds \quad (43b)
\]

\[
\sigma_{xz}(x,z,t) = -\frac{2z^2}{\pi} \int_{x_s}^{x_e} \frac{p(s,t)(x-s)}{(x-s)^2 + z^2} ds - \frac{2z}{\pi} \int_{x_s}^{x_e} \frac{q(s,t)(x-s)^2}{(x-s)^2 + z^2} ds \quad (43c)
\]

Equations (42) and (43) apply under the plane strain condition. Here, the influence of local asperity geometry on the near surface stress concentration is not considered. Only a portion of the surface roughness effect on the stress fields is included through the normal pressure and tangential shear yielded from the mixed EHL analysis. To include the surface topography variation in the stress evaluation, the more sophisticated while also more computationally involved boundary element approach developed by Li [96] can be used in the place on the cost of much.
elevated computational efforts. In this work, Equations (42) and (43) are used to avoid the overwhelming computational demand.

In addition, the stress component $\sigma_y$ is not evaluated since the multi-axial fatigue criteria do not require it under the line contact condition. It is noted here that Equations (42) and (43) have the form of convolution and can be numerically evaluated using the DFT convolution technique.

For most of the fatigue criteria, the fatigue damage assessment requires the mean and alternating components of the shear and normal stresses on a certain plane. Therefore, the multi-axial stress time histories for each material point of interest must be determined while the contact passes by.

Considering gear $j$ ($j = 1, 2$), a $X - Z$ reference frame that is fixed on the gear tooth is defined with its origin positioned at the start-of-active-profile (SAP) of the straightened tooth profile with the $X$ axis being tangent to the surface pointing towards the tooth tip and the $Z$ axis representing the depth into the material. The fixed $X - Z$ coordinate frame and the moving $x - z$ coordinate frame are related according to

$$ X = X_0 + \int u_j(t) \, dt \quad - x, \quad Z = z $$

(44a, b)

where $X_0$ is the position of the $x - z$ frame at $t = 0$. With this, the histories of the stress components $\sigma_i$ ($i = x, z$ and $xz$) of any arbitrary grid point fixed in the $X - Z$ frame can be defined as

60
\[ \Sigma_f(X, Z, t) = \sigma_i \left( \left[ X_0 + \int u_j(t) dt - X \right], Z, t \right) \]  

(45)

where \( I = X, Z \) or \( XZ \). Any residual stresses caused by the surface machining and heat treatment processes (measured along the \( z \) axis) can be superimposed onto the predicted elastic stress fields, which alters the mean values while leaving the alternating stress amplitudes unchanged.
3.2 Gear Contact Multi-Axial Fatigue Life Model

Contact fatigue life of a crack in gear components has two parts: crack initiation and propagation. The crack propagation life is shown to be small in comparison to the crack nucleation (initiation) life for high speed rolling contact fatigue [5, 63], hence this work is limited to the crack formation life prediction. The critical plane approach has been frequently used in the multi-axial fatigue life prediction, showing reasonable correlations to fatigue experiments [97-101]. The critical plane approach evolved from experimental observations of nucleation and growth of cracks during loading. Various forms of critical plane fatigue criteria have been proposed according to the fatigue failure mechanisms observed and the damage parameters selected. For rough surface gear contacts subjected to mixed EHL condition, the fracture mode is rather complicated for surface nucleated local failures, making it difficult to select the most appropriate form of the critical plane fatigue criteria. One of the drawbacks of critical plane approach is it depends on the experimental observations of the crack. Although many fatigue criteria use these same stress parameters to assess the fatigue damage, they took different forms [97, 99, 101-104] depending on the different materials, loading conditions, fracture modes, etc. Among those, the criterion proposed by Liu and Mahadevan [102] targeting the wheel-rail contact fatigue was shown to perform better in terms of the crack nucleation life as well as the crack formation position [52-53, 105] in comparison to the critical plane based fatigue criteria [52]. This is an
alternative fatigue approach that does not require the pre-knowledge of the fracture mode is the so-called characteristic plane approach [102] that evaluates the fatigue damage on a material plane, on which the contribution of the hydrostatic stress on fatigue is minimum. This plane may or may not represent the fracture plane. The characteristic plane based multi-axial fatigue method was used earlier to generic point contacts under mixed EHL condition [52]. The fatigue predictions were shown to correlate well with the pitting experiments using a twin-disk setup, not only in terms of the life cycles but also in terms of the critical failure locations. Therefore, the characteristic plane fatigue criterion [102] will be implemented for the rough surface gear contact fatigue analysis. It is noted here any other suitable multi-axial fatigue criterion can also be used with the proposed methodology.

The gear material properties, namely the fully reversed pure bending fatigue strength and fully reversed pure torsion fatigue strength, define the angle $\alpha$ at which the characteristic plane is positioned from the macro fatigue fracture plane (the plane experiencing the maximum normal stress amplitude). This angle has the expression of [102]

$$\alpha = \frac{1}{2} \cos^{-1} \left[ \frac{-s^2 + \sqrt{s^4 - (1 - 3s^2)(-1 + 5s^2 - 4s^4)}}{-1 + 5s^2 - 4s^4} \right]$$

(46)

Here, $s = S_N^b / S_N^t$ is the fatigue strength ratio where $S_N^b$ and $S_N^t$ are the fully reversed bending and fully reversed torsion fatigue strength of the material.
at finite fatigue life cycles of $N (s < 1$ for non-extremely brittle materials [101]). Assuming that the mean shear stress effect is negligible, the fatigue criterion is defined on the characteristic plane as [102]

$$\beta \left(1 - \frac{\sigma_{m,\max}}{S_r}\right) \left[\sigma_a^2 + \left(\frac{S_N^b}{S_N^l}\right)^2 \tau_a^2 + \kappa \sigma_a^{2,H}\right]^{\frac{1}{2}} = S_N^b$$

(47)

where $\beta = \sqrt{s^2 \cos^2(2\alpha) + \sin^2(2\alpha)}$, and $\sigma_a$, $\tau_a$ and $\sigma_{a,H}$ are the normal stress amplitude, shear stress amplitude and the hydrostatic stress amplitude acting on the characteristic plane, respectively. $\kappa$ is the material property obtained from uniaxial and torsional fatigue limits. As the fatigue damage increases gradually, $\kappa$ is positive. The mean normal stress effect is included through the correction term $(1 - \frac{\sigma_{m,\max}}{S_r})$ where, $\sigma_{m,\max}$ is the mean normal stress on the macro fracture plane and $S_r$ is the reference stress that defines the extent of the mean stress effect and is determined through the uniaxial fatigue data or approximated using the ultimate tensile strength of the material. For different types of materials (ductile or brittle), the hydrostatic stress amplitude varies and hence the characteristic plane cannot be fixed as opposed to some of the critical plane approaches. Therefore, the characteristic planes passing through the material points of interest are searched in the 2D plane with a $2^\circ$ increment on which the hydrostatic stress is minimum. The fatigue lives in terms of contact cycles are then determined from numerical simulation of Equation (47). $S_N^b$ and $S_N^l$
take the initial values of the equation. Iterative convergence of Equation (47) predicts the fatigue life $N_f$. 
4 Results

4.1 Thermal Behavior

The design parameters of the example spur gear pair considered in this study are listed in Table 1.

For this gear pair, gear 1 and gear 2 are identical, and both have the tip relief of 10 µm starting at the roll angle of 20.9°. This micro geometry modification leads to the static transmission error as shown in Figure 4-1(a), obtained by using the gear load distribution program (LDP) [85]. The bearing supporting stiffness and damping as specified Figure 2-3 take the same values as those in Ref. [74] of \( k_{yj} = 1.15 \times 10^9 \) N/m and \( c_{yj} = 5360 \) Ns/m in the LOA direction, and \( k_{xj} = 8.0 \times 10^8 \) N/m and \( c_{xj} = 2980 \) Ns/m in the OLOA direction (\( j = 1, 2 \)). The torsional damping of the bearing takes the value of \( c_{ij} = 10 \) Nms/ rad [8]. The turbine fluid, Mil-L23699, is used as the lubricant, whose density and viscosity properties are listed in Table 2 [105].
Table 1
Example spur gear design parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Teeth</td>
<td>50</td>
</tr>
<tr>
<td>Module [mm]</td>
<td>3.0</td>
</tr>
<tr>
<td>Pressure Angle [deg]</td>
<td>20.0</td>
</tr>
<tr>
<td>Outside Diameter [mm]</td>
<td>156.0</td>
</tr>
<tr>
<td>Pitch Diameter [mm]</td>
<td>150.0</td>
</tr>
<tr>
<td>Root Diameter [mm]</td>
<td>140.0</td>
</tr>
<tr>
<td>Center Distance [mm]</td>
<td>150.0</td>
</tr>
<tr>
<td>Face Width [mm]</td>
<td>20.0</td>
</tr>
<tr>
<td>Backlash [mm]</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 2
Basic parameters of the lubricant Mil-L23699.

<table>
<thead>
<tr>
<th>Temperature T [°C]</th>
<th>Dynamic viscosity $\eta_0$ [Pa.s]</th>
<th>Pressure-viscosity Coefficient $\alpha_1$ [GPa⁻¹]</th>
<th>Pressure-viscosity Coefficient $\alpha_2$ [GPa⁻¹]</th>
<th>Density $\rho_0$ [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.01502</td>
<td>15.8</td>
<td>8.83</td>
<td>977.80</td>
</tr>
<tr>
<td>75</td>
<td>0.00703</td>
<td>13.7</td>
<td>7.17</td>
<td>962.80</td>
</tr>
<tr>
<td>100</td>
<td>0.00398</td>
<td>12.2</td>
<td>6.01</td>
<td>947.80</td>
</tr>
<tr>
<td>125</td>
<td>0.00256</td>
<td>11.1</td>
<td>5.17</td>
<td>932.80</td>
</tr>
</tbody>
</table>
Figure 4-1 The variations of (a) the static transmission error and (b) the mesh stiffness with the gear mesh cycle.
Two engineering surface roughness profiles with the root-mean-square roughness amplitudes of 0.6 μm [Figure 4-2 (a)] and 0.1 μm [Figure 4-2 (b)] are adopted to investigate the surface roughness effect on the flash temperature rise. To examine the influences of the load and the lubricant viscosity on the flash temperature, two input torques of 700 N-m and 1500 N-m, and two lubricant temperatures of 90°C and 60°C are implemented. The corresponding mesh stiffness under these two torques are determined using LDP [85] and plotted in Figure 4-1 (b), where the variation of $k_m$ with the mesh cycle is mainly due to the fluctuation of the number of the loaded tooth pairs as the gears rotate. In this study, the baseline operating condition is defined to have the 700 N-m input torque, the 90°C lubricant temperature, and the surface roughness profile displayed in. For each of the simulations, the gear rotational speed, $\Omega$, increases from 500 RPM to 4200 RPM with the increment of 25 RPM.
Figure 4-2  The measured roughness profiles along the tooth profile direction for (a) ground, and (b) polished surface finishes.
Under the baseline condition, the maximum flash temperature rise across the entire EHL conjunction of surface 1 (surface 2 has the similar thermal behavior), denoted as $\Delta \phi_{i}^{\text{max}}$, is compared between the tribo-dynamic condition (black solid curve) and the quasi-static condition (black dashed curve) in Figure 4-3 (a). To provide metrics for the LOA and OLOA direction gear dynamics, the RMS dynamic gear mesh force and the RMS dynamic bearing force in the OLOA direction, which are respectively defined as

$$W_{M}^{\text{rms}} = \sqrt{\sum_{i=1}^{10} (W_{M}^{i})^2} \quad \text{and} \quad W_{Bx}^{\text{rms}} = \sqrt{\sum_{i=1}^{10} (W_{Bx}^{i})^2}$$

where $W_{M}^{i}$ and $W_{Bx}^{i}$ are the $i^{\text{th}}$ harmonic amplitudes of $W_{M}$ and $W_{Bx}$, are plotted in the same figure. It is observed that $\Delta \phi_{i}^{\text{max}}$ is not only affected by the LOA dynamic response of $W_{M}^{\text{rms}}$ (red solid curve), such as at the LOA resonances of $\Omega_{II} = 1725$ RPM and $\Omega_{IV} = 3525$ RPM, but also influenced by the OLOA dynamic response of $W_{Bx}^{\text{rms}}$ (red dashed curve), for instance at the OLOA resonance of $\Omega_{I} = 1025$ RPM that is almost half of $\Omega_{III} = 2025$. In between $\Omega_{II} = 1725$ RPM and $\Omega_{III} = 2025$ RPM, $\Delta \phi_{i}^{\text{max}}$ is impacted by the combination of the downtrend of the LOA resonance at $\Omega_{II}$ and the uptrend of the OLOA resonance at $\Omega_{III}$. Therefore, the translational vibratory motion in the OLOA direction plays an important role in the surface flash temperature rise. The simple one-DOF torsional gear dynamics model widely used in the literature [70-71] is not sufficient for the accurate prediction of $\Delta \phi_{i}^{\text{max}}$. In
Figure 4-3 (a), the first LOA resonance at $\Omega_{H} = 1725$ RPM is associated with the mode that the gear LOA transverse motions and the gear torsional motions are out of phase such that they offset each other; while the second LOA resonance at $\Omega_{V} = 3525$ RPM is associated with the mode that the translational and the torsional motions are in phase to add upon each other, leading to the much larger dynamic mesh force peak at $\Omega_{V} = 3525$ RPM.

Another observation in Figure 4-3 (a) is that the gear dynamic responses not necessarily increase the flash temperature. For instance at $\Omega_{IV} = 3000$ RPM, $\Delta\phi_{1}^{\max}$ has a larger value under the quasi-static condition.

The underlying mechanism is shown in Figure 4-4 (a), where the maximum Hertzian pressure, $p_{h}$, is compared between the tribo-dynamic and quasi-static conditions. It is seen the tribo-dynamic $p_{h}$ becomes smaller than its quasi-static counterpart within a large portion of the double-tooth-contact (DTC) region where the sliding is high and the scuffing failure usually occurs, resulting in the reduced flash temperature rise.
Figure 4-3 (a) The comparison of $\Delta\phi_A^{\text{max}}$ (black) between the tribo-dynamic (solid curve) and quasi-static (dashed curve) conditions plotted together with $W_A^{\text{rms}}$ (red solid curve) and $W_B^{\text{rms}}$ (red dashed curve), and (b) The comparisons of $\Phi_A$ (black), $\Phi_W$ (blue) and $\Psi$ (red) between the tribo-dynamic (solid curves) and quasi-static (dashed curves) conditions, under the baseline condition.
Figure 4-4 The comparisons of the maximum Hertzian pressure distributions along the gear 1 roll angle between the tribo-dynamic and quasi-static conditions for the baseline at (a) $\Omega_1$ and (b) $\Omega_2$ as specified in Figure 4-3.
Figure 4-5 The instantaneous distributions of (a) pressure (black) and film thickness (red), (b) surface 1 (black) and surface 2 (red) roughness heights, and (c) surface 1 (black) and surface 2 (red) flash temperature rises across the EHL conjunction at the 16° gear 1 roll angle and $\Omega_I$ rotational speed (specified in Figure 4-3) for the baseline.
The flash temperature rise is dictated by two main factors: the sliding velocity, and the shear along the contact interface. Under the mixed EHL condition, the asperity contact activities play a critical role in the surface shear. Figure 4-5 shows the instantaneous tribological behavior, including the contact pressure and film thickness distributions, the contacting surface roughness height variations, and the flash temperature rises, at the 16° gear 1 roll angle and the \( \Omega_I \) rotational speed under the baseline condition. It is seen the film thickness breaks down when the surface asperities come into contact, where the very high contact pressure leads to the large flash temperature rises. To relate the extent of asperity contact to the flash temperature, the specific asperity contact pressure that is defined as 
\[ \Psi = \frac{p_a^{avg}}{p_h^{avg}}, \] 
where \( p_a^{avg} \) is the average asperity contact pressure, i.e. the ratio of the normal load supported by the asperity contacts to the total area of asperity contacts, and \( p_h^{avg} \) is the average Hertzian pressure, is introduced.

Figure 4-6 (a) shows an example distribution of \( \Psi \) along the gear 1 roll angle at \( \Omega_{II} \) for the baseline. Additionally, the asperity contact area ratio, \( \Phi_A \), and load ratio, \( \Phi_W \), which are defined as the ratio of the total asperity contact area to the nominal Hertzian area, and the ratio of the total asperity contact force to the tooth force, respectively, are included in Figure 4-6 (b). The fluctuations of \( \Psi \), \( \Phi_A \), and \( \Phi_W \) are due to the transient surface roughness profiles, whose amplitude is quite significant as shown in Figure
4-3 (b) and Figure 4-5 (b), moving across the EHL conjunction [16]. The average of the specific asperity contact pressure, and the averages of the asperity contact area and load ratios along the gear 1 roll angle, denoted as \( \bar{\Psi} \), \( \bar{\Phi}_A \), and \( \bar{\Phi}_W \), respectively, are plotted in Figure 4-3 (b) for the tribo-dynamic condition (solid curves) and the quasi-static condition (dashed curves). It is observed, as the rotational speed increases, \( \Phi_A \) and \( \Phi_W \) decrease owing to the increase in the lubrication film thickness. Neither \( \Phi_A \) nor \( \Phi_W \) reflects the behavior of \( \Delta \phi_1^{\text{max}} \). The behavior of \( \bar{\Psi} \), however, is found to be generally in line with that of \( \Delta \phi_1^{\text{max}} \), except at \( \Omega_{V} \). This only deviation is due to the tooth separation as shown in Figure 4-4 (b), where the load becomes zero between 18° and 20° gear 1 roll angle, introduced by the large vibratory motion at the resonance peak.

Raising the torque from 700 N-m to 1500 N-m while keeping the other operating conditions the same as those of the baseline, the tribo-dynamic simulation results of this higher torque case are shown in Figure 4-7 in the format of Figure 4-3. It is observed, the higher torque leads to the higher resonance amplitudes for both the LOA \( W_{M}^{\text{rms}} \) and the OLOA \( W_{B_{x}}^{\text{rms}} \). For instances, the amplitude of \( W_{M}^{\text{rms}} \) is increased from 6 kN to 14 kN at \( \Omega_{II} \), and from 18 kN to 61 kN at \( \Omega_{V} \); and the amplitude of \( W_{B_{x}}^{\text{rms}} \) is increased from 1.7 kN to 2.9 kN at \( \Omega_{I} \), and from 2.2 kN to 4.2 kN at \( \Omega_{III} \). In the vicinities of these resonances, evident fluctuations are found in the
maximum flash temperature rise. Comparing $\Delta \phi_1^{\text{max}}$ between the higher torque case and the baseline, the former corresponds to the $\Delta \phi_1^{\text{max}}$ range of 43 to 264°C, whose upper limit is significantly higher than that of the baseline, where $\Delta \phi_1^{\text{max}}$ ranges from 30 to 154°C. In view of behavior of $\bar{\Psi}$, $\Phi_A$, and $\Phi_W$, $\bar{\Psi}$ is again identified to be a better roughness contact activity parameter that reflects the variation of $\Delta \phi_1^{\text{max}}$. 
Figure 4-6 The distributions of (a) specific asperity contact pressure, and (b) asperity contact area ratio (black) and load ratio (red) along the gear 1 roll angle under the tribo-dynamic condition for the baseline at $\Omega_{H}$ as specified in Figure 4-3.
Figure 4-7 (a) The comparison of $\Delta \phi_1^{\text{max}}$ (black) between the tribo-dynamic (solid curve) and quasi-static (dashed curve) conditions plotted together with $W_{M}^{\text{rms}}$ (red solid curve) and $W_{Bx}^{\text{rms}}$ (red dashed curve), and (b) The comparisons of $\bar{\Phi}_A$ (black), $\bar{\Phi}_W$ (blue) and $\bar{\Psi}$ (red) between the tribo-dynamic (solid curves) and quasi-static (dashed curves) conditions, under the higher torque condition.
Figure 4-8 and Figure 4-9 show the simulation results under the higher lubricant viscosity condition and the lower roughness amplitude condition, respectively. The viscosity increase is realized by reducing the lubricant temperature from $90^\circ C$ to $60^\circ C$, resulting in the ambient dynamic viscosity increase from the baseline of 0.005 Pa-s to 0.011 Pa-s. For the lower roughness amplitude case, the roughness profile of Figure 4-2 (b) is used, replacing the baseline one of Figure 4-2 (a). In the case of Figure 4-8, the lubrication film thickness is improved through the viscosity increase. Both the asperity contact area ratio and the asperity contact load ratio are reduced in comparison to Figure 4-3. In the case of Figure 4-9, the decreased roughness amplitude decreases the asperity contact activities even further. In the high speed range, both $\Phi_A$ and $\Phi_W$ are approaching zero. With the hydrodynamic film more prevalent in the EHL conjunction for these two cases, the resemblance between $\Delta \phi_A^{\max}$ and $\Psi$ becomes less evident, while $\Psi$ is still a better asperity contact parameter relating to the behavior of $\Delta \phi_A^{\max}$ in comparison to $\Phi_A$ and $\Phi_W$.  

81
Figure 4-8 (a) The comparison of $\Delta \phi_1^{\text{max}}$ (black) between the tribo-dynamic (solid curve) and quasi-static (dashed curve) conditions plotted together with $W_{M}^{\text{rms}}$ (red solid curve) and $W_{Bx}^{\text{rms}}$ (red dashed curve), and (b) The comparisons of $\Phi_A$ (black), $\Phi_W$ (blue) and $\bar{\Psi}$ (red) between the tribo-dynamic (solid curves) and quasi-static (dashed curves) conditions, under the higher viscosity condition.
Figure 4-9 (a) The comparison of $\Delta \phi^\text{max}_1$ (black) between the tribo-dynamic (solid curve) and quasi-static (dashed curve) conditions plotted together with $W_{Bx}^{\text{rms}}$ (red solid curve) and $W_{M}^{\text{rms}}$ (red dashed curve), and (b) The comparisons of $\Phi_A$ (black), $\Phi_W$ (blue) and $\bar{\Psi}$ (red) between the tribo-dynamic (solid curves) and quasi-static (dashed curves) conditions, under the lower roughness amplitude condition.
Lastly, the maximum flash temperature rise of surface 1 is compared between the four operating conditions in Figure 4-10. It is observed that the torque increase leads to the large increase in $\Delta \phi_1^{\text{max}}$ within the entire rotational speed range by imposing larger friction. When the viscosity is increased, the lubrication film thickness increases to reduce the asperity contact activities and consequently decrease the friction and the flash temperature rise. At $\Omega_V = 3525 \text{ RPM}$, for instance, the asperity contact area ratio for the baseline and the higher viscosity cases are 0.074 Pa-s and 0.037 Pa-s, respectively. The half asperity contact activities under the higher viscosity condition results in the reduced amplitude at this resonance. When the roughness RMS amplitude is decreased to 0.1 μm from the baseline whose RMS roughness amplitude is 0.6 μm, the lubrication film thickness becomes sufficiently thick to eliminate most of the asperity contacts. For instance, the asperity contact area ratio is only 0.00053 at $\Omega_V = 3525 \text{ RPM}$. Thus, the reduced roughness amplitude leads to the minimum $\Delta \phi_1^{\text{max}}$ among the four cases.
Figure 4-10 The comparison of the flash temperature rise under the tribo-dynamic condition between the baseline, higher torque, higher lubricant viscosity, and lower roughness amplitude conditions.
4.2 Contact Fatigue Behavior

The design parameters of the example spur gear pair listed in Table 1, presented in Section 4.1 are employed in this study as well.

The micro-geometry modification applied on each gear includes a 10 μm tip relief starting at the 20.9° roll angle to eliminate any tip corner contact induced excessive stress. The stiffness and damping that are equivalent to the shaft and bearing flexibility (Figure 2-3) [79] are $k_{yj} = 1.15 \times 10^9$ N/m and $c_{yj} = 5360$ Ns/m in the LOA direction, and $k_{xj} = 8.0 \times 10^8$ N/m and $c_{xj} = 2980$ Ns/m in the OLOA direction for gear $j$. The bearing torsional viscous damping is estimated to be $c_{tj} = 10$ N ms/rad $(j = 1,2)$ [74]. For the lubrication of the gear tooth contact, the turbine fluid, Mil-L23699, whose density and viscosity properties are presented in Table 2 [105], is used. In order to assess the surface roughness impact on the contact fatigue crack nucleation, three surface roughness profiles, namely high roughness amplitude (HRA) [Figure 4-11 (a)], medium roughness amplitude (MRA) [Figure 4-11 (b)] and low roughness amplitude (LRA) [Figure 4-11 (c)] profiles, are applied. Both the HRA and MRA surfaces are produced through grinding. The LRA surface is achieved through polishing after grinding. The root-mean-square (RMS) roughness amplitudes for these three surface finishes are 0.66 μm, 0.30 μm and 0.11 μm, respectively.
Figure 4-11 The measured roughness profiles along the tooth profile direction for (a) high roughness amplitude, (b) medium roughness amplitude, and (c) low roughness amplitude surfaces.
Defining the baseline condition, the input torque is set at 2400 N-m, the inlet lubricant temperature is controlled at 90°C, and the MRA surface roughness profile is implemented. For one continuous tribo-dynamic simulation, the gear rotational speed \( \Omega \) increases in a stepwise way from 500 RPM to 4200 RPM with the 25 RPM increment. It is noted, however, the multi-axial stress and fatigue analyses are performed every four rotational speed increments i.e., at \( \Omega = 500, 600, \ldots, 4100, 4200 \) RPM, for the purpose of avoiding the overwhelming computational efforts. In order to provide metrics for the dynamic tooth mesh force, the dynamic bearing force in the LOA direction and the dynamic bearing force in the OLOA direction over the entire rotational speed span, their respective RMS parameters are defined as

\[
W_M^{\text{RMS}} = \sqrt{\sum_{i=1}^{10} (W_M^i)^2}, \quad W_{By}^{\text{RMS}} = \sqrt{\sum_{i=1}^{10} (W_{By}^i)^2} \quad \text{and} \quad W_{Bx}^{\text{RMS}} = \sqrt{\sum_{i=1}^{10} (W_{Bx}^i)^2}
\]

with \( W_M^i \), \( W_{By}^i \) and \( W_{Bx}^i \) representing the \( i \)-th harmonic amplitude of the respective dynamic forces.

The contact fatigue responses under the baseline condition are constructed in Figure 4-12, where the variation of the crack nucleation fatigue life \( N_f \) with the rotational speed \( \Omega \) is compared between the tribo-dynamic (solid circles) and the quasi-static (hollow circles and dashed line, obtained using the model proposed in Ref. [102]) predictions. The RMS dynamic forces are also included in the figure to determine which dynamic force matters the most in the gear contact fatigue. It is observed the deviations between the tribo-dynamic and quasi-static fatigue lives are evident, especially in the
vicinities of the resonance peaks at $\Omega_I = 1750$ RPM and $\Omega_{II} = 3500$ RPM. At the first resonance speed, the torsional vibrational motion and the LOA transverse vibrational motion of the gears are out-of-phase and neutralize each other, while the opposite is true, i.e. these two motions are in-phase and promote each other, at the second resonance speed, resulting in the much larger LOA dynamic force peak amplitudes (mesh force and bearing force) at $\Omega_{II}$ in comparison to those at $\Omega_I$.

Owing to the large dynamic mesh force around the first resonance peak, the fatigue life decreases from $10^{6.7}$ contact cycles under the quasi-static condition to as low as $10^{6.3}$ contact cycles under the tribo-dynamic condition, i.e. a 60% fatigue life reduction is produced by the gear dynamic behavior. It is noticed in Figure 4-12 that the fatigue life at the speed case A is smaller than that at the speed case B, although the former case corresponds to a smaller value of the RMS dynamic mesh force. To investigate the mechanism behind this observation, the distribution of the maximum Hertzian pressure, denoted as $p_h$, along the gear 2 roll angle are compared between these two speed cases in Figure 4-13 (black curve versus red curve). It is seen the speed case A has its maximum $p_h$ (2.8 GPa) that is larger than that of the speed case B (2.5 GPa), both occurring at the $20^\circ$ roll angle. This greater loading condition of the speed case A, thus, leads to the lower crack nucleation fatigue life in comparing to that at the speed case B.
Figure 4-12 The comparison of the fatigue life between the tribo-dynamic and quasi-static predictions under the baseline condition. The RMS dynamic forces are also included in the figure.
The distributions of the maximum Hertzian pressure under the tribo-dynamic condition along the gear 2 roll angle for speed cases A (black), B (red), C (green) and D (blue), which are defined in Figure 4-12. The dashed line represents the maximum Hertzian pressure under the quasi-static condition.
Figure 4-14 shows the fatigue life distributions of the cases A and B along the x-z plane, where the x axis points along the tooth profile direction and the z axis points into the tooth surface, i.e. representing the depth. For both cases, the critical position is located at the 20° roll angle where the maximum Hertzian pressure peaks and at the surface due to the surface irregularity induced near surface stress concentrations [52-53, 69]. Additionally, the significant fatigue damage is also observed around the 26° roll angle for the speed case B as shown in Figure 4-14 (b). This corresponds to a local $p_h$ peak near the tooth tip as displayed in Figure 4-13 (red curve).

At the second resonance peak in Figure 4-12, the RMS dynamic mesh force is seen to drop abruptly while the RMS LOA bearing force peaks. This drop of $W^\text{RMS}_M$ is due to the tooth separation caused by the significant vibratory motions of the gears described by Equation (4). As shown by the blue curve in Figure 4-13, the gear teeth loses contact between the 19.9° and 21.7° roll angles where the contact pressure becomes zero. This nonlinear dynamic behavior at the resonance is largely influenced by the periodically time-varying gear mesh stiffness and the gear mesh viscous damping [3, 74, and 76]. As stated in Equation (21), the viscous damping is a function of the lubricant film thickness and viscosity, who are dictated by the normal load, the surface velocities and the surface roughness profiles, pointing to
a very complicated tribo-dynamic phenomenon. Under this reduced loading condition at the speed case \( D \), the fatigue life is significantly lengthened from \( 10^{6.8} \) contact cycles under the quasi-static condition to \( 10^{7.9} \) contact cycles under the tribo-dynamic condition.

Figure 4-15 (b) illustrates the corresponding tribo-dynamic fatigue damage distribution, which correlates well with the \( p_h \) distribution in Figure 4-13 (blue curve). It must be noted, however, although the gear contact fatigue life becomes longer at \( \Omega_{II} \), the bearing fatigue failure can be promoted by the jump in the LOA dynamic bearing force, and consequently leads to the entire system fatigue failure. Except at the speed case \( D \), the fatigue lives at the other rotational speeds around the second resonance peak, where no tooth separation is observed, are substantially shortened under the tribo-dynamic condition in comparison to the quasi-static predictions as shown in Figure 4-12. For the speed case \( C \) that is in the middle of the two resonances, the tribo-dynamic and the quasi-static fatigue lives overlap in Figure 4-12. Examining the corresponding tribo-dynamic maximum Hertzian pressure distribution in Figure 4-13 (green curve), it fluctuates very limitedly around its quasi-static counterpart (dashed black curve), thus, resulting in the negligible difference between the tribo-dynamic and quasi-static fatigue lives. The fatigue life distribution at the speed case \( C \) under the tribo-dynamic condition is shown in Figure 4-15 (a).
Figure 4-14 The crack nucleation fatigue life distributions under the baseline condition for (a) speed case A and (b) speed case B, which are defined in Figure 4-12.
Figure 4-15  The crack nucleation fatigue life distributions under the baseline condition for (a) speed case C and (b) speed case D, which are defined in Figure 4-12.
The importance of dynamic force effects on contact fatigue life at the two resonant speeds is studied further at different levels of torques, surface roughness amplitudes and temperatures. Firstly, the contact fatigue response under low torque (1000 N-m), HRA surface roughness profile and with inlet temperature set at $50^\circ C$ is shown in Figure 4-16, where the variation of the crack nucleation fatigue life ($N_f$) with the rotational speed ($\Omega$) under tribo-dynamic conditions (solid circles) is plotted. The RMS dynamic forces are included in the figure to determine which dynamic force matters the most in gear contact fatigue. Similar to the baseline condition, at the first resonance speed, the torsional vibrational motion and the LOA transverse vibrational motion of the gears are out-of-phase and neutralize each other, while the opposite is true, i.e. these two motions are in-phase and promote each other, at the second resonance speed, resulting in the larger LOA dynamic force peak amplitudes (mesh force and bearing force) at $\Omega_{II} = 3500$ RPM in comparison to those at $\Omega_I = 1750$ RPM.

It is observed that the fatigue life at the speed case A is equal to that at the speed case B owing to the small difference in dynamic mesh forces at these speed cases. At the second resonance peak, it is noticed that the dynamic mesh force rises while the RMS LOA bearing force drops as opposed to the baseline condition, showing no nonlinear dynamic behavior.
Figure 4-16 The effect of dynamic force (RMS) on the fatigue life at low input torque (1000 N-m), HRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-17 The effect of dynamic force (RMS) on the fatigue life at low input torque (1000 N-m), HRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Figure 4-17 performs the same type of comparison as that of Figure 4-16 under 90°C inlet lubricant temperature. However, at the first resonant speed, the fatigue life at speed case A is greater than that at speed case B due to the corresponding dynamic mesh forces. At the second resonant speed the dynamic RMS mesh force rises as opposed to the baseline condition implying that the gear teeth do no separate.

Under low torque, the contact fatigue response at MRA surface roughness profile with inlet temperature set at 50°C is shown in Figure 4-18. Similar observation as in HRA surface roughness profile case is made at the resonant speeds $\Omega_I = 1750$ and $\Omega_{II} = 3500$ RPM. A visible difference is observed in the increase of contact fatigue life at both resonant peaks as compared to the HRA surface roughness profile owing to low occurrence of asperity contacts. The fatigue life at speed case A is smaller than that at speed case B, although the former case corresponds to a smaller value of the RMS dynamic mesh force. The mechanism behind this is explained earlier in Figure 4-13. At the second resonant speed, no nonlinear behavior is observed.

Figure 4-19 shows the crack nucleation fatigue life ($N_f$) under low torque, medium roughness amplitude and at inlet temperature of 90°C. The difference in fatigue life at speed case A and at speed case B is not as significant as in the case of low torque, medium roughness amplitude at low
inlet temperature (50°C). At the second resonant speed, the reduction in contact fatigue life is due to the large LOA bearing force as opposed to the gear dynamic meshing force as in previous cases at low torque conditions. This drop is due to the tooth separation caused by the significant vibratory motions of the gears described by Equation (4). At low inlet temperature, the contact fatigue life is marginally higher as compared to that of at high inlet temperature; due to decrease in viscosity at high temperatures.

Under low torque, the contact fatigue responses at LRA surface roughness profile with inlet temperature set at 50°C is shown in Figure 4-20. When compared to MRA and HRA roughness profiles, significant improvement in the contact fatigue life is detected for the LRA surface roughness profile owing to the low asperity contacts between contact surfaces. Figure 4-21 shows the crack nucleation fatigue life ($N_f$) at LRA surface roughness profile with inlet temperature set at 90°C.

At the first resonance, the difference in fatigue life at speed cases A and B is significant as compared to the other low torque cases. The fatigue life at speed case A is smaller than that at speed case B, due to the greater loading condition caused by the larger maximum Hertzian pressure at speed case A as observed in Figure 4-13. At the second resonant speed, the reduction in contact fatigue life is due to the large LOA bearing force as opposed to the gear dynamic meshing force in previous cases. This drop of is due to the tooth separation caused by the significant vibratory motions
of the gears described by Equation (4). This nonlinear dynamic behavior at the second resonance is largely influenced by periodically time-varying gear mesh stiffness and the gear mesh viscous damping. It is to be duly observed that the contact fatigue life at the second resonance peak is due to the RMS dynamic mesh force.
Figure 4-18 The effect of dynamic force (RMS) on the fatigue life at low input torque (1000 N-m), MRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-19 The effect of dynamic force (RMS) on the fatigue life at low input torque (1000 N-m), MRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Figure 4-20 The effect of dynamic force (RMS) on the fatigue life at low input torque (1000 N-m), LRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4.21 The effect of dynamic force (RMS) on the fatigue life at low input torque (1000 N·m), LRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Secondly under medium torque (1700 N-m), the contact fatigue response at HRA surface roughness profile with inlet temperature set at 50°C is shown in Figure 4-22, where the variation of the crack nucleation fatigue life ($N_f$) with the rotational speed ($\Omega$) under tribo-dynamic conditions (solid circles) is plotted. Figure 4-23 shows the crack nucleation fatigue life ($N_f$) higher at inlet temperature of 90°C. At the first resonant speed, as opposed to the baseline condition, the fatigue life at speed case A is greater than that of at the speed case B due to the respective dynamic RMS mesh forces at both cases.

Figure 4-24 and Figure 4-25 show the fatigue life responses under medium torque, MRA surface roughness profile with inlet temperature set at 50°C and 90°C respectively. Figure 4-26 and Figure 4-27 show the fatigue life responses under medium torque, LRA surface roughness profile with inlet temperature set at 50°C and 90°C respectively. Similar observations as in base line condition are made at the resonant speed $\Omega = 1750$ RPM. At second resonant speed interestingly no nonlinear behavior or tooth separation occurs under medium torque conditions in all cases. The distortion at peaks are induced by the dynamic behavior.
Figure 4-22 The effect of dynamic force (RMS) on the fatigue life at medium input torque (1700 N-m), HRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-23 The effect of dynamic force (RMS) on the fatigue life at medium input torque (1700 N-m), HRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Figure 4-24 The effect of dynamic force (RMS) on the fatigue life at medium input torque (1700 N-m), MRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-25 The effect of dynamic force (RMS) on the fatigue life at medium input torque (1700 N-m), MRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Figure 4-26 The effect of dynamic force (RMS) on the fatigue life at medium input torque (1700 N-m), LRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-27 The effect of dynamic force (RMS) on the fatigue life at medium input torque (1700 N-m), LRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Figure 4-28 and Figure 4-29 illustrate the dynamic force effects on the contact fatigue life under high torque (2400 N-m), HRA surface roughness profile and at low and high temperatures respectively. At the first resonance peak, the fatigue life at speed case A is greater than that of at speed case B due to lower RMS dynamic mesh force at speed case A than that of at speed case B. At the second resonance peak, the rise in fatigue life is noticed due to swift decline in the RMS dynamic mesh force due to the tooth separation caused by the tribo-dynamic non-linear behavior. Although the gear contact fatigue life lasts long at the second resonance speed, $\Omega_{II}$, the bearing fatigue failure (LOA) is expected to drop abruptly and consequently leads to the entire system fatigue failure. It is also observed that OLOA bearing force is not significant yet substantial on fatigue life under high torque conditions as compared to that of at low torque conditions.
Figure 4-28 The effect of dynamic force (RMS) on the fatigue life at high input torque (2400 N-m), HRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-29 The effect of dynamic force (RMS) on the fatigue life at high input torque (2400 N-m), HRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
Figure 4-30 illustrates the dynamic force effects on the fatigue life under high torque (2400) N-m, MRA surface roughness profile with inlet temperature maintained at 50°C. Under high torque and medium surface roughness profile with inlet temperature maintained at 90°C is the baseline condition described in Figure 4-12. At the first resonance peak, similar observations as in baseline condition are made where the fatigue life at speed case A, is greater although the corresponding dynamic mesh force is lower compared to that of at speed case B. At the second resonance peak, no nonlinear behavior is observed. The peak distortions of the bearing force in LOA direction is due to the dynamic behavior. The nonlinearity under the high torque at the second resonance appears when the inlet lubricant temperature is increased to 90°C due to occurrence of tooth separation that is aided by the reduction in lubricant viscosity at high temperatures.

Under high torque, LRA surface roughness profile conditions with inlet temperature set at 50°C and 90°C are shown in Figure 4-31 and Figure 4-32 respectively. At \( \Omega_l \), the contact fatigue life at speed case A and speed case B behave similarly as seen in Figure 4-12 (baseline condition). RMS dynamic mesh force and OLOA dynamic bearing force also behave similarly at both low and high inlet lubricant temperatures as compared with MRA surface roughness profile respectively showing tooth separation.
The effect of dynamic force (RMS) on the fatigue life at high input torque (2400 N-m), MRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
Figure 4-31 The effect of dynamic force (RMS) on the fatigue life at high input torque (2400 N-m), LRA surface roughness profile and inlet temperature controlled at 50°C. The RMS dynamic forces are also included in the figure.
The effect of dynamic force (RMS) on the fatigue life at high input torque (2400 N-m), LRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.

Figure 4-32 The effect of dynamic force (RMS) on the fatigue life at high input torque (2400 N-m), LRA surface roughness profile and inlet temperature controlled at 90°C. The RMS dynamic forces are also included in the figure.
From the above discussion, the following can be concluded:

- Fatigue life generally decreases with the increase of the dynamic mesh force;
- The more detailed dynamic load distribution along the tooth profile (such as in Figure 4-13) offers a better measure of the dynamic mesh force impact on the fatigue damage;
- Gear contact fatigue life may be lengthened when tooth separations occur under severe vibration conditions;
- Although the LOA dynamic bearing force does not affect the gear fatigue damage directly, it can impact indirectly by promoting the bearing fatigue failure, therefore, has to be considered as well;
- No tight correlation between the OLOA dynamic bearing force and the gear tooth contact fatigue is observed;
- In addition, the speed effect on the crack nucleation fatigue life under the quasi-static condition is quite small with the adopted speed range and input torque, considering the roughness profile of Figure 4-11. The specific film thickness that is defined as the ratio of the smooth surface minimum film thickness to the RMS roughness amplitude is found to increase from 0.11 to 0.49 when the rotational speed increases from 500 to 4200 RPM. Therefore, the severe asperity contact condition has not been very much relieved as the rotational speed increases. As a
result, the fatigue life is only slightly improved from contact cycles to contact cycles when \( \omega \) is increased from 500 RPM to 4200 RPM under the quasi-static condition, which is in line with the twin-disk rolling contact fatigue experiments [54] where the dynamic behavior is trivial.

Figure 4-33 investigates the effect of the input torque on the tribo-dynamic contact fatigue life under different surface roughness amplitude conditions with the inlet lubricant temperature set at. Considering the high surface roughness amplitude in Figure 4-33 (a), as the input torque increases from 1000 N-m (low) to 1700 N-m (medium) and 2400 N-m (high), the fatigue life is decreased \( 10^{7.2} \) to \( 10^{6.5} \) and \( 10^{6.1} \) contact cycles, respectively, at the speed case A (first resonance), representing 80% and 92% life reductions. In between the two resonances at the speed case C, the fatigue life is observed to decrease by 75% and 94% from \( 10^{7.3} \) contact cycles as the torque increases from low to medium and to high, respectively. In Figure 4-33 (b) when the MRA surface is considered, 84% and 95% life decreases from \( 10^{7.6} \) at the speed case A, and 94% and 97% life decreases from \( 10^{8.4} \) at the speed case C are found.

In Figure 4-33 (c) where the LRA surface is used, 87% and 96% life reductions from \( 10^{8.1} \) at the speed case A, and 96% and 99% life reduction from \( 10^{8.9} \) at the speed case C are recorded. It is observed the torque effect becomes more significant as the contact surface becomes smoother. At
the second resonance, the fatigue life is also shown to decrease when the torque increases from low to medium in Figure 4-33 (a-c). However, at the high torque, the gear contact fatigue life is seen to shoot up due to the tooth separation caused by the nonlinear dynamic behavior. Under such a condition, the potential bearing failure has to be taken into account in view of the very large bearing force as discussed earlier.

Figure 4-34 performs the same type of comparison as that in Figure 4-33 under the $50^\circ C$ inlet lubricant temperature. Very similar observations can be obtained, except that the nonlinearity under the high torque at the second resonance disappears for the medium and low surface roughness amplitudes in Figure 4-34 (b) and (c). The reduced lubricant temperature in Figure 4-34 triples the lubricant low-shear viscosity from 0.005 Pa-s at $90^\circ C$ to 0.015 Pa-s at $50^\circ C$, resulting in the increase of the numerator of the integral kernel in Equation (21). On the other hand, the asperity contact regions within the contact zone under the medium and low roughness amplitude surface conditions are larger than that under the HRA condition, such that the integral in Equation (21) involves more fluid areas and leads to the larger viscous damping, suppressing the nonlinear dynamic behavior in Figure 4-34 (b) and (c).
Figure 4-33  The comparisons of the tribo-dynamic crack nucleation fatigue lives between different torque levels for (a) high roughness amplitude, (b) medium roughness amplitude and (c) low roughness amplitude surfaces. Inlet lubricant temperature is 90°C
Figure 4-34 The comparisons of the tribo-dynamic crack nucleation fatigue lives between different torque levels for (a) high roughness amplitude, (b) medium roughness amplitude and (c) low roughness amplitude surfaces. Inlet lubricant temperature is 50°C
The impact of the surface roughness amplitude on the gear contact crack nucleation fatigue life is examined in Figure 4-35 and Figure 4-36 for the high and low lubricant temperatures, respectively. The method of surface roughness amplitude reduction is shown to be very effective in improving the contact fatigue life for both temperature conditions within the entire speed range, either in the vicinities of or far away from the resonances. For instance, at the speed case A, the fatigue life is increased by 58% and 298% under the high torque, 100% and 400% under the medium torque, and 151% and 694% under the low torque when the HRA surface is replaced by the MRA and LRA surfaces with the 90°C lubricant temperature. At the speed case C, the corresponding life increases are recorded as 400% and 531%, 216% and 531%, and 1160% and 3881%. When the inlet lubricant temperature is reduced to in Figure 4-36, similar conclusions can be drawn, except at the second resonance where the fatigue life with the HRA surface is observed to be the longest under the high torque condition in Figure 4-36 (a). This is due to the tooth separation induced by the nonlinear gear dynamics.
Figure 4-35  The comparisons of the tribo-dynamic crack nucleation fatigue lives between different roughness amplitude levels for (a) 2400 N-m, (b) 1700 N-m and (c) 1000 N-m input torques. Inlet lubricant temperature is 90°C.
Figure 4-36 The comparisons of the tribo-dynamic crack nucleation fatigue lives between different roughness amplitude levels for (a) 2400 N-m, (b) 1700 N-m and (c) 1000 N-m input torques. Inlet lubricant temperature is 50°C
Figure 4-37, Figure 4-38 and Figure 4-39, carry out the comparison of the fatigue lives between the low and high lubricant temperatures under the high, medium and low input torques, respectively. In general, the fatigue life is lengthened when the inlet lubricant temperature is reduced within the entire rotational speed span. This life improvement is due to the increased lubricant viscosity at the decreased temperature, which elevates the lubricant film thickness and reduces the asperity contact activity [51]. Although the viscosity is tripled from 0.005 Pa-s to 0.015 Pa-s when the temperature is reduced from 90°C to 50°C, the observed life improvement is quite limited under the 2400 N-m and 1700 N-m input torques in Figure 4-37 and Figure 4-38. For instance, at the speed case C, 0.07%, 25% and 50% life increases are found for the HRA, MRA and LRA surfaces under the high torque. When the medium torque is considered, the life improvement at the same speed are recorded as 11%, 45% and 4.4% for the HRA, MRA and LRA surfaces, respectively. Only when the torque is relatively low, say 1000 N-m, the fatigue life elongation becomes relatively significant as in Figure 4-39, where the corresponding life increases are 208%, 155% and 105%.

It is very interestingly noted the lubricant temperature reduction not necessarily improves the fatigue performance under the tribo-dynamic condition. At the second resonance under the high torque in Figure 4-37, the nonlinear dynamic behavior results in the tooth separation at the high lubricant temperature as shown in Figure 4-37 (a-c). This nonlinearity is suppressed under the low temperature by the increased viscous damping
as shown in Figure 4-37 (b) and (c). As a result, the gear contact fatigue life is actually decreased when the lubricant temperature is reduced. Certainly, the potential bearing fatigue failure owing to the large LOA bearing force should be considered at the high temperature when the tooth separation occurs.
Figure 4-37 The comparisons of the tribo-dynamic crack nucleation fatigue lives between different roughness temperature levels for (a) HRA, (b) MRA and (c) LRA surfaces under 2400 N-m input torque.
Figure 4-38 The comparisons of the tribo-dynamic crack nucleation fatigue lives between different roughness temperature levels for (a) HRA, (b) MRA and (c) LRA surfaces under 1700 N-m input torque.
Figure 4-39 The comparisons of the tribo-dynamic crack nucleation fatigue lives between different roughness temperature levels for (a) HRA, (b) MRA and (c) LRA surfaces under 1000 N-m input torque.
5 Conclusions and Future Work

5.1 Conclusion

A novel and first of its kind tribo-dynamic thermal mixed EHL model for spur gear contacts is presented in this work. The model included the tribo-dynamic effects, using the governing motion equations and mixed EHL equations which are coupled together utilizing an iterative method. This interactive model is used to investigate the flash temperature rise and contact fatigue life. In order to validate the gear tribo-dynamics model of this study, it is suggested to measure the dynamic transmission error and the LOA and OLOA gear vibrations using the approach of Kang and Kahraman [106] and compare to the model predictions.

The flash temperature rises quantified within a wide speed range and compared between the tribo-dynamic condition and the quasi-static condition, showed evident deviations, especially in the vicinities of the resonances. It was very interesting to observe that not only the LOA direction gear dynamics, but also the OLOA direction transverse vibratory motion influences the flash temperature rise. Therefore, the single DOF torsional gear dynamics model in literature is not sufficient for the accurate prediction of the gear surface flash temperature. Additionally, a parametric study was carried out by varying the torque, the lubricant viscosity and the roughness amplitude from the baseline condition to examine the influences
of these contact parameters on the flash temperature rise. It was shown the increase of the load largely increased the flash temperature by imposing more friction. The flash temperature was reduced by increasing the lubricant viscosity or decreasing the surface roughness amplitude. The latter was shown to be more effective in the reduction of the roughness contact activities, and thus reached the minimum flash temperature rise among the four operating conditions considered. For the comparisons on the flash temperature aspect, the in-situ flash temperature measurement of meshing steel gears is overwhelmingly challenging. There are neither measurement techniques nor temperature data available in the literature that could be used for the validation purpose. It probably could be only partially validated by comparing the friction and power loss between the model predications and the experimental measurements [95], since the flash temperature is dictated by the frictional heat produced within the contact zone.

The converged normal pressure and tangential shear from the tribo-dynamic model are then used to determine the multi-axial stress fields on and below the surface, provided which, the contact fatigue crack nucleation life is then determined according to a multi-axial fatigue criterion [107]. Employing an example unity ratio spur gear pair, the fatigue lives under the tribo-dynamic and the quasi-static conditions are compared to show large deviations, especially in the vicinities of the resonances where the RMS dynamic mesh force either peaks or drops (because of the tooth separation due to the nonlinear gear dynamics) abruptly. The elevated dynamic mesh
force shortens the life and the reduced dynamic mesh force lengthens the life, i.e. a resonance not necessarily leads to the premature gear fatigue failure. However, it is noted, the large magnitudes of the LOA dynamic bearing force at the resonances may cause the bearing fatigue failure and then results in the gear failures indirectly even when the nonlinear tooth separation occurs. It is shown the quasi-static assumption can be valid only when the rotational speed is far away from the resonances, pointing to the necessity of the inclusion of the tribo-dynamic description in the high speed gear contact fatigue modeling.

In addition, the influences of the input torque, the surface roughness amplitude and the lubricant temperature on the contact fatigue under the tribo-dynamic condition are examined through a parametric simulation. It is observed, the increase of the input torque largely decreases the fatigue life. The surface roughness amplitude reduction is an effective method to improve the fatigue performance. The lubricant temperature is shown to be able to lengthen the life evidently only when the input torque is relatively low. In short,

- The main focus of the above presented work is to better understand the thermal mixed EHL behavior under tribo-dynamic spur gear contacts, which dictates the thermal and contact fatigue failure modes commonly observed in gearing applications.
• As a result of this study, many realistic applications, such as the study of the gear friction, power loss and efficiency in addition to the flash temperature under the tribo-dynamic condition, became possible.

• Multi-axial stress formulation provided a better understanding on pit formation by determining the location of the failure initiation, the corresponding life, and the potential crack propagation direction.

• The inclusion of the tribo-dynamic behavior offers a powerful design tool for the determination of the scuffing and contact fatigue performances considering the more realistic dynamic operating condition.
5.2 Future Work

- The study can be extended to other commonly used gears such as helical and bevel gears.

- The contact fatigue methodology, which includes the mixed EHL analysis, stress tensor prediction, and the multi-axial fatigue damage evaluation, will provide guidelines for lubrication, material selection, surface finish process, hardening depth and so on.

- To include other effects such as thermal deformations of the gear surfaces due to high operating temperatures, or gears made of composite materials etc., this model can be incorporated with a corresponding mathematical model incorporating thermal stresses.
Bibliography


[38] Snidle, R. W., Dhulipalla, A. K., Evans, H. P. and Cooper, C.V. (2008), “Scuffing performance of a hard coating under EHL conditions at sliding speeds up to 16 m/s and contact pressures up to 2.0 GPa,” Journal of Tribology, 130 (2), 021301 (10 pages).


[85] LDP, Gear Load Distribution Program, Gear and Power Transmission Research Laboratory, Columbus, Ohio, USA: The Ohio State University; 2012


